Advanced Macroeconomics I
ECON 525a - Fall 2009
Yale University

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Week 5 - Bubbles
Introduction

- Why a rational representative investor model of asset prices does not generate bubbles?
- **Martingale property**: LIE (Law of iterated expectations).
Introduction

- Why a rational representative investor model of asset prices does not generate bubbles?

- **Martingale property**: LIE (Law of iterated expectations).

- This is not the case with heterogeneity, since in general, average expectations fail to satisfy LIE.

- When private information is heterogeneous, agents rely excessively in public signals. Hence
  - Mean price path deviates from consensus liquidation values
  - Prices exhibit inertia.
Fail of LIE with heterogeneous information

- LIE with private information

\[ E_{it} (E_{i,t+1}(\theta)) = E_{it} (\theta) \]

- LIE with public information

\[ E_t^* (E_{t+1}^*(\theta)) = E_t^* (\theta) \]

- LIE fail in averages with asymmetric information

\[ \overline{E}_t (\overline{E}_{t+1}(\theta)) \neq \overline{E}_t (\theta) \]
Basics

- Information at all dates:
  - $\theta \sim \mathcal{N}(y, \frac{1}{\alpha})$
  - Signals: $x_i = \theta + \epsilon_i$, where $\epsilon_i \sim \mathcal{N}(0, \frac{1}{\beta})$

- Average expectation of average expectations.

$$E_{t}^{T-t}(\theta) \equiv E_{t}(E_{t+1}(...E_{T-1}(\theta))) = \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)^{T-t}\right)y + \left(\frac{\beta}{\alpha + \beta}\right)^{T-t}\theta$$

- See that

$$E_{t}^{T-t}(\theta) \neq E_{t}(\theta) = \left(1 - \left(\frac{\beta}{\alpha + \beta}\right)\right)y + \left(\frac{\beta}{\alpha + \beta}\right)\theta$$
No learning through prices

- If

\[ p_t = \bar{E}_t(p_{t+1}) \]

then

\[ p_t = \left( 1 - \left( \frac{\beta}{\alpha + \beta} \right)^{T-t} \right) y + \left( \frac{\beta}{\alpha + \beta} \right)^{T-t} \theta \]

- How to obtain the equation for \( p_t \)?

- How to deal with learning from past prices?
Model

- Single risky asset, liquidated at $T + 1$ but traded from 1 to $T$.
- Liquidation value $\theta$ is determined before date 1. $\theta \sim \mathcal{N}(y, \frac{1}{\alpha})$
- Overlapping generation of no wealth constrained traders, each living for two periods and consuming in the second period. $u(c) = -e^{-\frac{c}{\tau}}$
- Information set: $\{y, p_1, p_2, \ldots, p_t, x_{it}\}$ where $x_{it} = \theta + \epsilon_{it}$ and $\epsilon_{it} \sim \mathcal{N}(0, \frac{1}{\beta})$
- Each period exogenous net supply of assets $s_t \sim \mathcal{N}(0, \frac{1}{\gamma})$
Path of fundamental value

When each new trader is born, they do not know the true value of $C_t^0$. However, for a trader $i$ born at date $t$, there are two sources of information. First, the full history of past and current prices are available, including the ex ante mean $y$ of $C_t^0$. Second, this trader observes the realization of a private signal $x_t = C_t^0 + \varepsilon_t$.

6 In contrast, other authors such as He and Wang (1995) have examined the case where traders only consume at the terminal date and trade up to that date. We discuss this case further below.

Figure 1
Path of fundamental value

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Private Information

where \( \text{it} \) is a normally distributed noiseterm with mean 0 and variance \( \frac{1}{C_{12}} \). We assume that the noise terms \( f \text{it} \) are i.i.d. across individuals \( i \) and across time \( t \). There is no other source of information for the trader. In particular, the private signals of the previous generation of traders are not observable. Hence, the information set of trader \( i \) at date \( t \) is \( y, p_1, p_2, \ldots, p_t, x_{it} \) where \( p_t \) is the price of the asset at date \( t \) (Figure 2). As a convention, we take \( p_{T+1} = \frac{1}{C_{18}} \). All traders have the exponential utility function \( u(c) = \frac{1}{C_{28}} e^{-c} \) defined on consumption \( c \) when they are old. The parameter \( \frac{1}{C_{28}} \) is the reciprocal of the absolute risk aversion, and we shall refer to it as the traders' risk tolerance. Finally, in each trading period, we assume that there is an exogenous noisy net supply of the asset, \( s_t \), distributed normally with mean 0 and \( \frac{1}{C_{13}} \). The supply noise is independent over time, and independent of the fundamentals and the noise in traders' information. The modeling device of noisy supply is commonly adopted in rational expectations models so as to prevent the price from being a fully revealing signal of the fundamentals. Noisy supply is sometimes justified as the result of noise traders or in terms of the subjective uncertainty facing traders on the ''free float'' of the asset that is genuinely available for sale [see Easley and O'Hara (2001), footnote 9, page 52].
Price at date $T$

- Trader $i$’s demand at date $T$

$$D_{iT} = \frac{\tau}{V_{iT}(\theta)} (E_{iT}(\theta) - p_T)$$

- Market clearing is given by

$$D_T = \frac{\tau}{V_T(\theta)} (\bar{E}_T(\theta) - p_T) = s_T$$

- Then, the price at date $T$ is

$$p_T = \bar{E}_T(\theta) - \frac{V_T(\theta)}{\tau} s_T$$
Price at date $t$

- The asset price at date $T - 1$ is

$$p_{T-1} = \bar{E}_{T-1}(p_T) - \frac{V_{T-1}(p_T)}{\tau} s_{T-1} = \bar{E}_{T-1} \bar{E}_T(\theta) - \frac{V_{T-1}(p_T)}{\tau} s_{T-1}$$

- The asset price at a general date $t$ is

$$p_t = \bar{E}_t \bar{E}_{t+1} \ldots \bar{E}_T(\theta) - \frac{V_t(p_{t+1})}{\tau} s_t$$
Main results

We will answer these questions by means of three propositions. Figure 3 illustrates our results. The line labelled as $p_t$ is the mean of the price path with respect to the noisy realizations of supply. The line labelled as $E_t(\theta)$ is the mean path of the average expectation of $\theta$ with respect to the noisy realizations of supply. The mean price path deviates systematically from the mean path of average expectations, and the price path lies further away from the true value than the average expectations. So, prices not only fail to reflect the consensus on true fundamental value, but are further away from the true value. Also, the initial adjustment in price at date 1 is too sluggish, failing to reflect the true extent of the shift in fundamentals from $y$ to $\theta$. Thereafter, the price does adjust, but slowly. It only catches up with the average expectations of fundamentals at the last trading date. The line labelled by $q_t$ is the path followed by the naive iterated expectation given by Equation (3) in Section 1 that ignores the information given by prices. The mean path for price lies between this path and the mean path for average expectations. One of our results below identifies a limiting case for when the mean price path coincides with the naive iterated expectations path.

In what follows, we denote by $E_s(\theta)$ the expectation with respect to the noisy supply terms $f_{st}$. The propositions below are proved in Appendix A.

Proposition 1. For all $t < T$, 

$$E_s(p_t) > E_s(E_t(\theta))$$

It is only at the final trading date, $T$, that we have $E_s(p_T) = E_s(E_t(\theta))$. 

Figure 3
Mean of time paths of $p_t$ and $E_t(\theta)$

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Main results

- Prices deviate systematically from the average expectation of the fundamental value of the asset.
- Inertia of prices.
- Intuition: Excessive weight assigned to the public signal $y$ and previous prices.
Main results

- For risk neutral traders or infinitely precise signals, prices are fully revealing of the fundamental value. This is $p_t \to E_t(\theta) \neq \theta$.

- As investors become very risk averse ($\tau \to 0$), they are less aggressive and prices are less informative. This is $p_t \to q_t$. 
Main ideas

- Rational arbitrageurs may know the price of an asset exceeds the fundamental and still decide not to sell.
- The key is they do not know when the bubble will burst, where it is required a critical mass of speculators to do it.
- Main elements for this to work:
  - Dispersion of opinions among arbitrageurs.
  - Need for coordination.