Advanced Macroeconomics I

ECON 525a - Fall 2009

Yale University

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Week 6 - Global Games
Main ideas

- Many models based on indeterminacy and sunspots, assuming
  - Economic fundamentals are common knowledge.
  - Agents are certain about each other’s behavior in equilibrium.
- Payoffs depend on actions, motivated by beliefs.
- Global Games: Uncertainty about others’ beliefs lead to uniqueness.
- One’s beliefs are pinned down by the knowledge of fundamentals and that other agents are rational.
Application - Diamond and Dybvig

- Standard Diamond-Dybvig model with the following simplifying assumptions.
  - Discount rate is 1. Only the bank can invest in the illiquid project.
  - Illiquid project generates $R > 1$ at period 2.
  - If a proportion $x$ are withdrawn in period 1, the rate of return is reduced to $Re^{-\ell}$
  - If $0 < r = \log(R) < 1$, the rate of return can be written as $e^{r-\ell}$
- From the social optimum, $c_1^* = 1$ and $c_2^* = r$
Payoffs and Multiplicity

- Multiple equilibria

<table>
<thead>
<tr>
<th></th>
<th>Withdraw</th>
<th>NOT withdraw</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ell = 0$</td>
<td>0</td>
<td>$r &gt; 0$</td>
</tr>
<tr>
<td>$\ell = 1$</td>
<td>0</td>
<td>$r - 1 &lt; 0$</td>
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Uncertainty and Uniqueness

- Suppose \( r \sim \mathcal{N}(\bar{r}, \frac{1}{\alpha}) \), where \( 0 \leq \bar{r} \leq 1 \)
- Imprecise signals about \( r \): \( x_i = r + \epsilon_i \) where \( \epsilon_i \sim \mathcal{N}(0, \frac{1}{\beta}) \)
- From Bayesian rule, the updated belief upon observing \( x_i \) is

\[
\rho_i = E(r|x_i) = \frac{\alpha \bar{r} + \beta x_i}{\alpha + \beta}
\]

- Furthermore, the ex-post distribution of \( r \) is,

\[
r|\rho_i \sim \mathcal{N}\left(\rho_i, \frac{1}{\alpha + \beta}\right)
\]
Uncertainty and Uniqueness

However, more than updating the fundamental it is also important to infer beliefs (and hence actions) of others.

Others’ signals ($x_j = r + \epsilon_j$), conditional on updated beliefs about fundamentals are

$$x_j|\rho_i \sim \mathcal{N} \left( \rho_i, \frac{1}{\alpha + \beta} + \frac{1}{\beta} \right)$$
Main question

- When a depositor $i$ has posterior belief $\rho_i$, what is the probability that $i$ attaches to some other depositor $j$ have a posterior belief lower than himself?

$$Pr(\rho_j < \rho_i | \rho_i) = Pr \left( x_j < \rho_i + \frac{\alpha}{\beta}(\rho_i - \bar{r}) | \rho_i \right)$$

$$= \Phi(\sqrt{\gamma}(\rho_i - \bar{r}))$$

where

$$\gamma = \frac{\alpha^2(\alpha + \beta)}{\beta(\alpha + 2\beta)}$$
Uniqueness

Proposition

Provided that $\gamma \leq 2\pi$, there is a unique equilibrium where every patient agent withdraws if and only if $\rho < \rho^*$, where

$$\rho^* = \Phi(\sqrt{\gamma}(\rho^* - \bar{r}))$$

In the limit, as $\gamma \to 0$, $\rho^* \to \frac{1}{2}$
Uniqueness

\( \rho^* \) is the switching point at which the agent is indifferent between withdraw or not. This is, \( E_r [r - \ell | \rho^*] = 0 \)

\[ E_r(r|\rho^*) = Pr(\rho_j < \rho^* | \rho^*) \]

\[ \rho^* = \Phi(\sqrt{\gamma}(\rho^* - \bar{r})) \]
Uniqueness

- The equilibrium will be unique as long as the slope of \( \Phi(\sqrt{\gamma}(\rho^* - \bar{r})) \) is less than 1.
- This slope is just the density and achieves a maximum of \( \sqrt{\frac{\gamma}{2\pi}} \) at \( \bar{r} \).
- The sufficient condition for uniqueness is then \( \gamma \leq 2\pi \), which happens when \( \beta \) is big enough with respect to \( \alpha \).
Observable implications

- A depositor withdraws whenever $\rho_i < \rho^*$. This means.

$$x^*(\rho^*, r) = \frac{\alpha + \beta}{\beta} \rho^* - \frac{\alpha}{\beta} r$$

- Hence, we can obtain the equilibrium fraction of withdraws for each realization $r$

$$\ell(r) = Pr(x_i < x^*(\rho^*, r)|r) = \Phi \left( \sqrt{\beta}(x^*(\rho^*, r) - r) \right)$$
Observable implications

- Gorton (1988), for example, shows fundamentals play a key role in explaining bank runs.
- Withdrawal is high when the return is low.
- Payoff relevant fundamentals generate self-fulfilling equilibrium.
Applications

- Bank runs
- Currency crises
- Riots
- Risk Taking in Credit Markets
- Debt Pricing
Limitations

- Aggregation of information through prices (Atkeson, 00)