Macroeconomics of Financial Markets

ECON 712, Session 1 - Fall 2013

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Week 5 - Panics
Main ideas

- Many models are based on indeterminacy and sunspots
  - Economic fundamentals are common knowledge.
  - Agents are certain about each other’s behavior in equilibrium.
- Payoffs depend on actions, and on others’ beliefs.
- Global Games: Uncertainty about others’ beliefs lead to uniqueness.
- One’s beliefs are pinned down by the knowledge of fundamentals and that other agents are rational.
Application - Diamond and Dybvig

- Standard Diamond-Dybvig model with the following simplifying assumptions.
  - Discount rate is 1. Only the bank can invest in the illiquid project.
  - Illiquid project generates $R > 1$ at period 2.
  - If a proportion $\ell$ are withdrawn in period 1, the rate of return is reduced to $Re^{-\ell}$
  - If $0 < r = \log(R) < 1$, the rate of return can be written as $e^{r-\ell}$
- From the social optimum, $\log(c_1^*) = 0$ and $\log(c_2^*) = r$ (just plug $\rho = 1$ in our previous discussion of Diamond and Dybvig).
Payoffs and Multiplicity

- Multiple equilibria. Take the decision of a late consumer (the only decision that matters for runs)

<table>
<thead>
<tr>
<th></th>
<th>Withdraw</th>
<th>DO NOT withdraw</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\ell = 0)</td>
<td>0</td>
<td>(r &gt; 0)</td>
</tr>
<tr>
<td>(\ell = 1)</td>
<td>0</td>
<td>(r - 1 &lt; 0)</td>
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Uncertainty and Uniqueness

- Suppose $r \sim \mathcal{N} (\bar{r}, \frac{1}{\alpha})$, where $0 < \bar{r} < 1$

- Imprecise signals about $r$: $x_i = r + \epsilon_i$ where $\epsilon_i \sim \mathcal{N}(0, \frac{1}{\beta})$

- From Bayesian rule, the updated belief upon observing $x_i$ is

  $$
  \rho_i = \mathbb{E}(r | x_i) = \frac{\alpha \bar{r} + \beta x_i}{\alpha + \beta}
  $$

- Furthermore, the ex-post distribution of $r$ is,

  $$
  r | \rho_i \sim \mathcal{N} \left( \rho_i, \frac{1}{\alpha + \beta} \right)
  $$
Uncertainty and Uniqueness

- However, more than updating the fundamental it is also important to infer beliefs (and hence actions) of others.

- Others’ signals \((x_j = r + \epsilon_j)\), conditional on updated beliefs about fundamentals are

\[
x_j | \rho_i \sim \mathcal{N} \left( \rho_i, \frac{1}{\alpha + \beta} + \frac{1}{\beta} \right)
\]
Main question

When a depositor $i$ has posterior belief $\rho_i$, what is the probability that $i$ attaches to some other depositor $j$ have a posterior belief lower than himself?

$$
Pr(\rho_j < \rho_i | \rho_i) = Pr \left( x_j < \rho_i + \frac{\alpha}{\beta} (\rho_i - \bar{r}) | \rho_i \right) \\
= \Phi(\sqrt{\gamma}(\rho_i - \bar{r}))
$$

where

$$
\gamma = \frac{\alpha^2(\alpha + \beta)}{\beta(\alpha + 2\beta)}
$$
Uniqueness

- $\rho^*$ is the switching point at which the agent is indifferent between withdraw or not. This is, $E_r [r - \ell | \rho^*] = 0$

\[
E_r(r|\rho^*) = Pr(\rho_j < \rho^* | \rho^*)
\]

\[
\rho^* = \Phi(\sqrt{\gamma}(\rho^* - \bar{r}))
\]
Uniqueness

- The equilibrium will be unique as long as the slope of 
  \[ \Phi(\sqrt{\gamma}(\rho^* - \bar{r})) \] 
  is less than 1.

- This slope is just the density and achieves a maximum of 
  \[ \sqrt{\frac{\gamma}{2\pi}} \] 
  at \( \bar{r} \).

- The sufficient condition for uniqueness is then \( \gamma \leq 2\pi \), which 
  happens when \( \beta \) is big enough with respect to \( \alpha \).
Provided that $\gamma \leq 2\pi$, there is a unique equilibrium where every patient agent withdraws if and only if $\rho < \rho^*$, where

$$\rho^* = \Phi(\sqrt{\gamma}(\rho^* - \bar{r}))$$

In the limit, as $\gamma \to 0$, $\rho^* \to \frac{1}{2}$
Observable implications

- A depositor withdraws whenever $\rho_i < \rho^*$. This means.

$$x^*(\rho^*, \bar{r}) = \frac{\alpha + \beta}{\beta} \rho^* - \frac{\alpha}{\beta} \bar{r}$$

- Hence, we can obtain the equilibrium fraction of withdrawals for each realization $r$

$$\ell(r) = \Pr(x_i < x^*(\rho^*, \bar{r})|r) = \Phi \left( \sqrt{\beta}(x^*(\rho^*, \bar{r}) - r) \right)$$
Observable implications

- Gorton (1988), for example, shows fundamentals play a key role in explaining bank runs.
- Withdrawal is high when the return is low.
- Payoff relevant fundamentals generate self-fulfilling equilibrium.
Applications

- Bank runs
- Currency crises
- Riots
- Risk Taking in Credit Markets
- Debt Pricing
Limitations

- Aggregation of information through prices (Atkeson, 00).
- Formalized by Angeletos and Werning (2006).