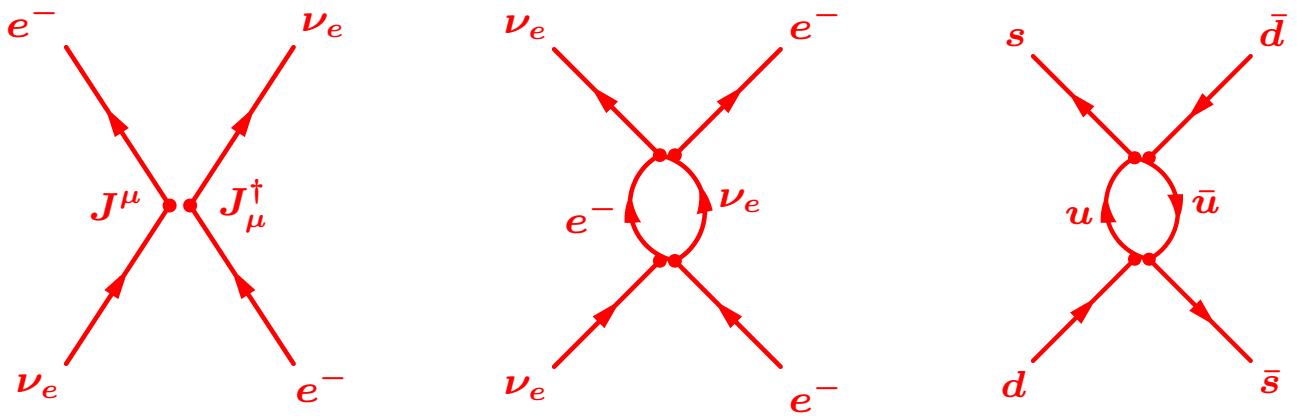
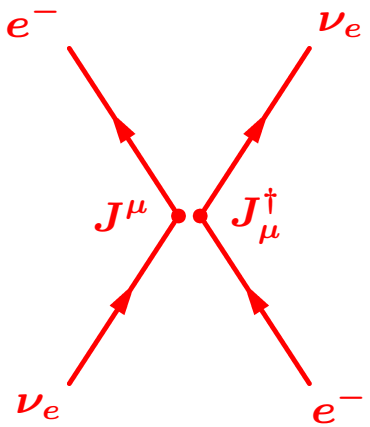
Beyond the Fermi Theory

- Fermi theory successfully describes large number of WCC processes
- However, not renormalizable
  - Radiative corrections divergent (but needed)
  - Unitarity violated at high energy ($E_{CM} \sim 1$ TeV)
  - Loop processes such as $K^0 - \bar{K}^0$ mixing meaningless/incorrect
- More complete theory needed



- **Fermi theory violates unitarity at high energy** (non-renormalizable)



– $\sigma(\nu_e e^- \to e^- \nu_e) \to \frac{G_F^2 s}{\pi}$
$(s \equiv E_{CM}^2)$

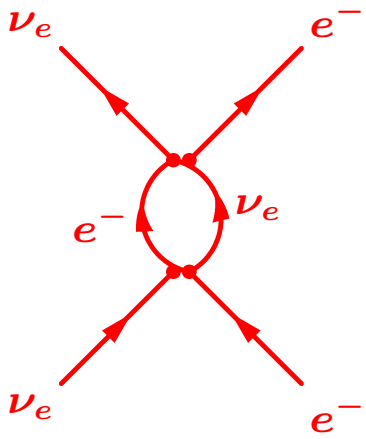
– pure S-wave unitarity: $\sigma < \frac{16\pi}{s}$

– fails for $\frac{E_{CM}}{2} \geq \sqrt{\frac{\pi}{G_F}} \sim 500$ GeV

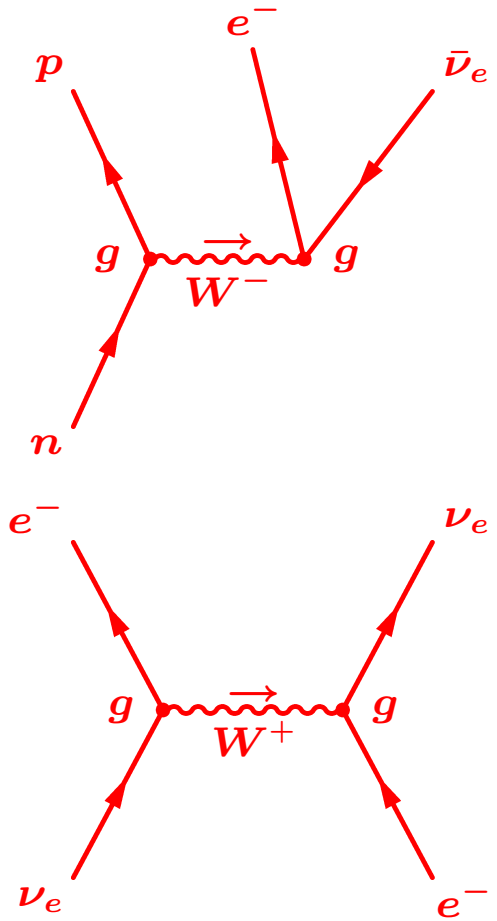
– Born not unitary; often restored by H.O.T.

– Fermi theory: divergent integrals

$$\int d^4k \left(\frac{\not{k}}{k^2}\right)\left(\frac{\not{k}}{k^2}\right)$$



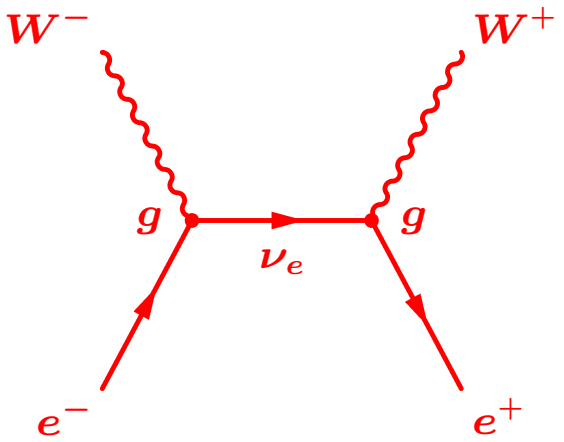
- **Intermediate vector boson theory** (Yukawa, 1935; Schwinger, 1957)



$$\frac{G_F}{\sqrt{2}} \sim \frac{g^2}{8M_W^2} \text{ for } M_W \gg Q$$

– no longer pure S-wave $\Rightarrow$

– $\nu_e e^- \to \nu_e e^-$ better behaved



– ***but,*** $e^+e^- \to W^+W^-$ **violates unitarity for** $\sqrt{s} \gtrsim 500$ **GeV**

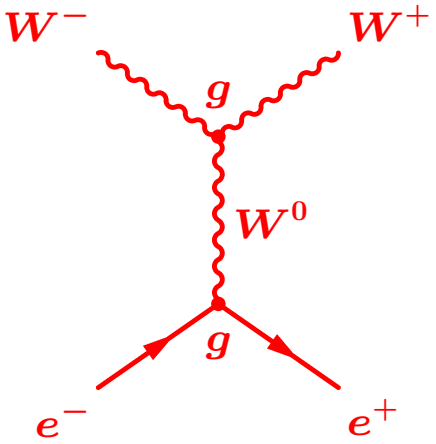
– $\epsilon_\mu \sim k_\mu/M_W$ **for longitudinal polarization** (non-renormalizable)

– **introduce** W^0 **to cancel**

– **fixes** $W^0W^+W^-$ **and** $e^+e^-W^0$ **vertices**

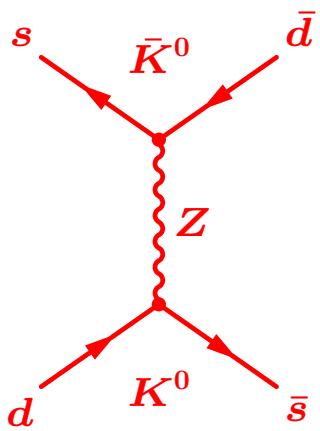
– **requires** $\left[J, J^\dagger\right] \sim J^0$ (like $SU(2)$)

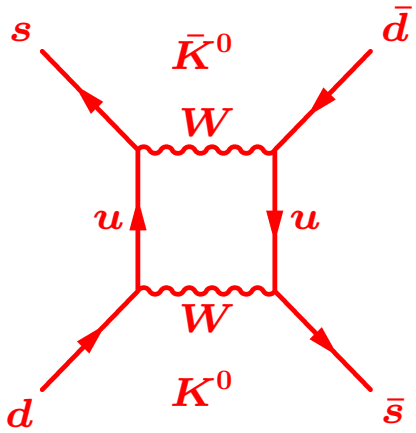
– **not realistic**



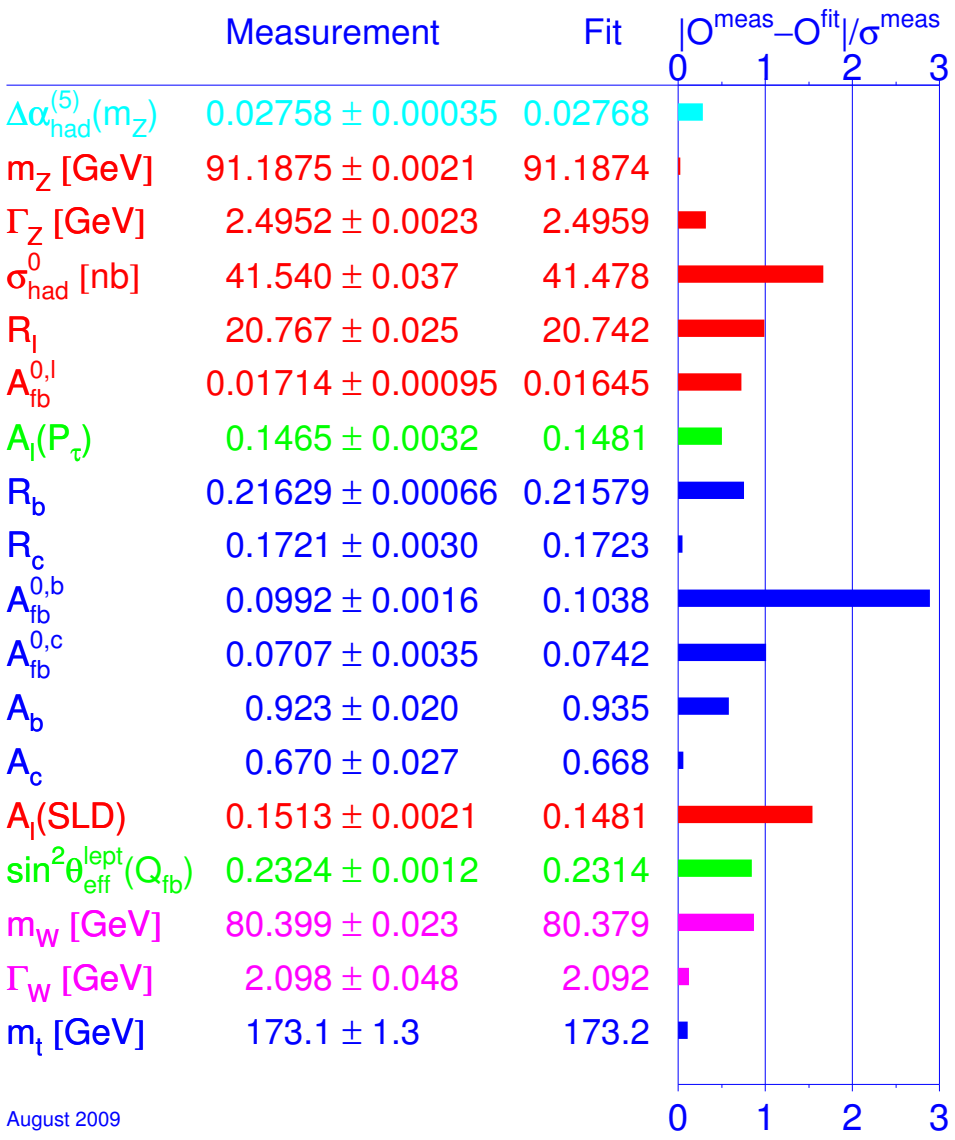
- **Glashow model (1961)** ($W^{\pm}, Z, \gamma$, but no mechanism for $M_{W,Z}$)
- **Weinberg-Salam (1967): Higgs mechanism** $\rightarrow M_{W,Z}$
- **Renormalizable (1971)** ('t Hooft, $\cdots$)
- **Flavor changing neutral currents (FCNC)**
  - very large $K^0 \leftrightarrow \bar{K}^0$ mixing
  - GIM mechanism (c quark) (1970) ($m_c \sim 1.5$ GeV (1974))
  - c discovered (1974)

$$J/\psi = c\bar{c} \text{ (BNL, SLAC)}$$
$$\nu_\mu d(s) \rightarrow \mu^- c,$$
$$c \rightarrow s\mu^+\nu_\mu \text{ (dimuons)}$$





- **Weak neutral current (1973)**
- **QCD (1970's)**
- **W, Z (1983)**
- **Precision tests (1989-)**
- **Precision K, B, D physics ($\sim$ 2000-)**
- **CKM unitarity ($\sim$ 1995-)**
- **t quark (1995)**
- **ν mass (1998-)**



The Standard Model

- **Gauge group $SU(3) \times SU(2) \times U(1)$; gauge couplings g_s, g, g'**

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

$$\begin{matrix} u_R & u_R & u_R & \nu_{eR}(?) \\ d_R & d_R & d_R & e^-_R \end{matrix}$$

(L = left-handed, R = right-handed)

- $SU(3)$: $u \leftrightarrow u \leftrightarrow u$, $d \leftrightarrow d \leftrightarrow d$ (8 gluons)
- $SU(2)$: $u_L \leftrightarrow d_L$, $\nu_{eL} \leftrightarrow e^-_L$ ($W^{\pm}$); phases (W^0)
- $U(1)$: phases (B)
- Heavy families (c, s, ν_μ, μ^-), (t, b, ν_τ, τ^-)

The Standard Electroweak Model

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{gauge} + \mathcal{L}_{\phi} + \mathcal{L}_{f} + \mathcal{L}_{Yuk}$$

Gauge part

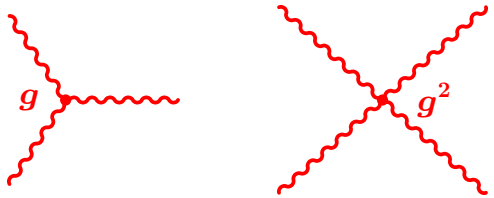
$$\mathcal{L}_{gauge} = -\frac{1}{4}F^{i}_{\mu\nu}F^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

Field strength tensors

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ F^{i}_{\mu\nu} &= \partial_\mu W^{i}_{\nu} - \partial_\nu W^{i}_{\mu} - g\epsilon_{ijk}W^{j}_{\mu}W^{k}_{\nu}, \quad i = 1\cdots 3 \end{aligned}$$

$g(g')$ **is** $SU(2)$ $(U(1))$ **gauge coupling;** ϵ_{ijk} **is totally antisymmetric symbol**

Three and four-point self-interactions for the W_i



B **and** W_3 **mix to form** γ, Z

$U(1)$: $\Phi_j \rightarrow \exp(ig'y_j\beta)\Phi_j$, $y_j = q_j - t_j^3 =$ weak hypercharge

Scalar part

$$\mathcal{L}_\phi = (D^\mu\phi)^\dagger D_\mu\phi - V(\phi)$$

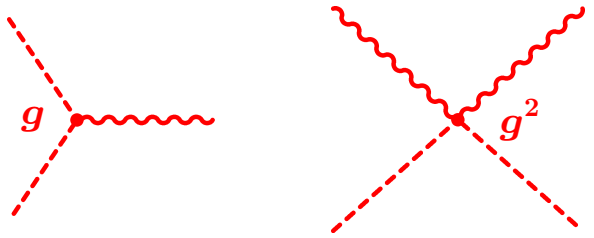
where $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ is the (complex) Higgs doublet with $y_\phi = 1/2$

Gauge covariant derivative:

$$D_\mu\phi = \left(\partial_\mu + ig\frac{\tau^i}{2}W_\mu^i + \frac{ig'}{2}B_\mu\right)\phi$$

where τ^i are the Pauli matrices

Three and four-point interactions between gauge and scalar fields



Higgs potential

$$V(\phi) = +\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

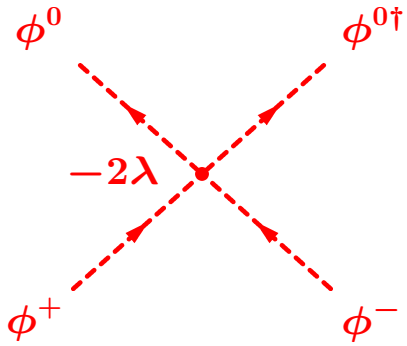
$$\phi^\dagger \phi = \underbrace{\phi^-}_{\phi^{+\dagger}} \phi^+ + \phi^{0\dagger} \phi^0$$

Allowed by renormalizability and gauge invariance

Spontaneous symmetry breaking for $\mu^2 < 0$

Vacuum stability: $\lambda > 0$

Quartic self-interactions



Fermion part

$$\mathcal{L}_f = \sum_{m=1}^{F} (\bar{q}^0_{mL} i \not{D} q^0_{mL} + \bar{l}^0_{mL} i \not{D} l^0_{mL}$$
$$+ \bar{u}^0_{mR} i \not{D} u^0_{mR} + \bar{d}^0_{mR} i \not{D} d^0_{mR} + \bar{e}^0_{mR} i \not{D} e^0_{mR} + \bar{\nu}^0_{mR} i \not{D} \nu^0_{mR})$$

L-doublets

$$q^0_{mL} = \begin{pmatrix} u^0_m \\ d^0_m \end{pmatrix}_L \qquad l^0_{mL} = \begin{pmatrix} \nu^0_m \\ e^{-0}_m \end{pmatrix}_L$$

R-singlets

$$u^0_{mR},\ d^0_{mR},\ e^{-0}_{mR},\ \nu^0_{mR}(?)$$

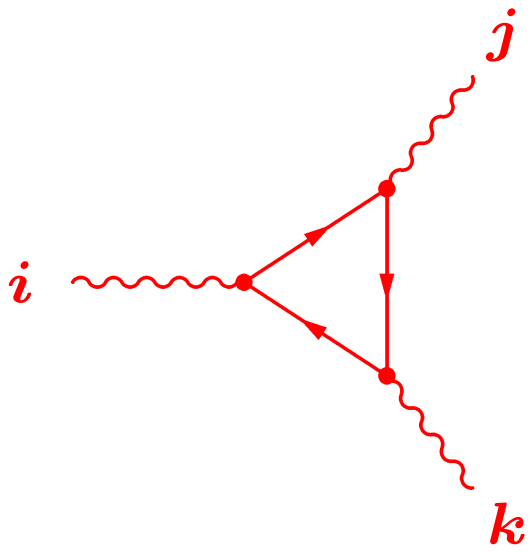
($F \geq 3$ families; $m = 1 \cdots F =$ family index;
$^0 =$ weak eigenstates (definite $SU(2)$ rep.), mixtures of mass eigenstates (flavors);
quark color indices $\alpha = r,\ g,\ b$ suppressed (e.g., $u^0_{m\alpha L}$).)

Can add gauge singlet ν^0_{mR} for Dirac neutrino mass term

Different (chiral) L and R representations lead to parity and charge conjugation violation (maximal for $SU(2)$)

Fermion mass terms forbidden by chiral symmetry

Anomalies absent for chosen hypercharges and 3 colors
(includes quark-lepton cancellations)



$$A_{ijk} = 2\mathrm{Tr}\, L_L^i\{L_L^j, L_L^k\} - 2\mathrm{Tr}\, L_R^i\{L_R^j, L_R^k\} = 0$$

$$T_i = \mathrm{Tr}\, L_L^i - \mathrm{Tr}\, L_R^i = 0$$

$SU(3) \times SU(2) \times U(1)$ assignments: $\{n_3, n_2, y\}_\Phi$ $\quad (y = q - t_L^3)$

$$\{3, 2, \frac{1}{6}\}_{q^0_{mL}} \Rightarrow q_{u^0_{mL}} = +1/2 + 1/6 = 2/3$$

$$q_{d^0_{mL}} = -1/2 + 1/6 = -1/3$$

$$\{1, 2, -\frac{1}{2}\}_{\ell^0_{mL}} \Rightarrow q_{\nu^0_{mL}} = +1/2 - 1/2 = 0$$

$$q_{e^{-0}_{mL}} = -1/2 - 1/2 = -1$$

$$\{3, 1, \frac{2}{3}\}_{u^0_{mR}} \Rightarrow q_{u^0_{mR}} = 2/3$$

$$\{3, 1, -\frac{1}{3}\}_{d^0_{mR}} \Rightarrow q_{d^0_{mR}} = -1/3$$

$$\{1, 1, -1\}_{e^{-0}_{mR}} \Rightarrow q_{e^{-0}_{mR}} = -1$$

$$\{1, 1, 0\}_{\nu^0_{mR}} \Rightarrow q_{\nu^0_{mR}} = 0,$$

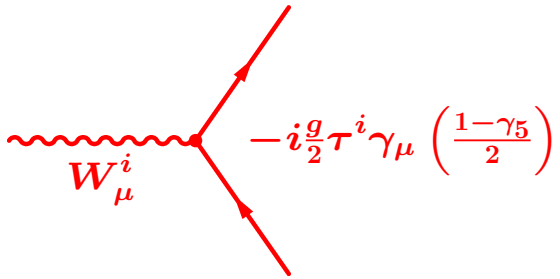
$$\{1, 2, \frac{1}{2}\}_{\phi} \Rightarrow q_{\phi^+} = +1/2 + 1/2 = 1$$

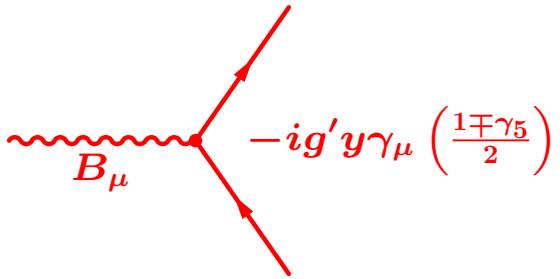
$$q_{\phi^0} = -1/2 - 1/2 = 0$$

Gauge covariant derivatives

$$
\begin{aligned}
D_\mu q^0_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W^i_\mu + i\frac{g'}{6}B_\mu\right) q^0_{mL} \\
D_\mu l^0_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W^i_\mu - i\frac{g'}{2}B_\mu\right) l^0_{mL} \\
D_\mu u^0_{mR} &= \left(\partial_\mu + i\frac{2}{3}g' B_\mu\right) u^0_{mR} \\
D_\mu d^0_{mR} &= \left(\partial_\mu - i\frac{g'}{3}B_\mu\right) d^0_{mR} \\
D_\mu e^0_{mR} &= (\partial_\mu - ig' B_\mu)\, e^0_{mR} \\
D_\mu \nu^0_{mR} &= (\partial_\mu)\, \nu^0_{mR}
\end{aligned}
$$

Read off W and B couplings to fermions



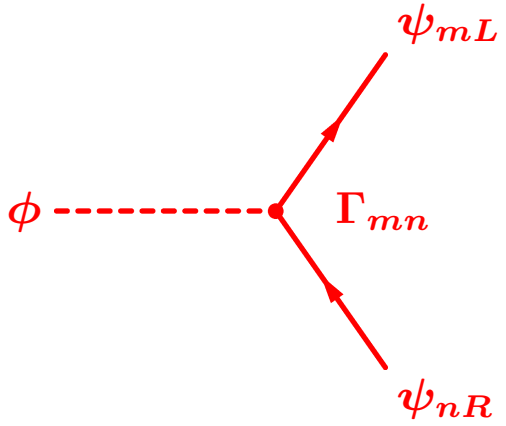


Yukawa couplings (couple L to R)

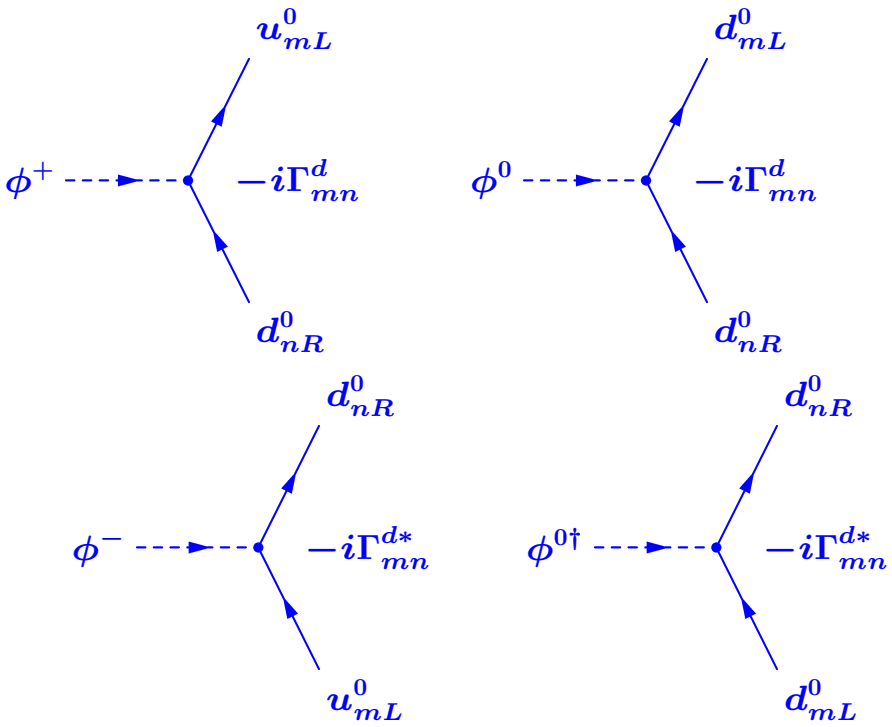
$$-\mathcal{L}_{Yuk} = \sum_{m,n=1}^{F} \left[\Gamma^{u}_{mn}\bar{q}^{0}_{mL}\tilde{\varphi}u^{0}_{mR} + \Gamma^{d}_{mn}\bar{q}^{0}_{mL}\varphi d^{0}_{nR} + \Gamma^{e}_{mn}\bar{l}^{0}_{mL}\varphi e^{0}_{nR} + \Gamma^{\nu}_{mn}\bar{l}^{0}_{mL}\tilde{\varphi}\nu^{0}_{mR}\right] + h.c.$$

Γ_{mn} are completely arbitrary Yukawa matrices, which determine fermion masses and mixings

d, e terms require doublet $\varphi = \begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix}$ with $Y_{\varphi} = 1/2$



$$\Gamma^d_{mn}\bar{q}^{\,0}_{mL}\phi d^0_{nR} = \Gamma^d_{mn}\left[\bar{u}^0_{mL}\phi^+ d^0_{nR} + \bar{d}^{\,0}_{mL}\phi^0 d^0_{nR}\right]$$



u (and ν) terms require doublet $\Phi = \begin{pmatrix} \Phi^0 \\ \Phi^- \end{pmatrix}$ with $Y_\Phi = -1/2$

In $SU(2)$ the 2 and 2^* are similar $\Rightarrow \tilde{\varphi} \equiv i\tau^2\varphi^\dagger = \begin{pmatrix} \varphi^{0\dagger} \\ -\varphi^- \end{pmatrix}$

transforms as a 2 with $Y_{\tilde{\varphi}} = -\frac{1}{2} \Rightarrow$ only one doublet needed.

Does not generalize to $SU(3)$, most extra $U(1)'$, supersymmetry, $SO(10)$ etc $\Rightarrow$ need two doublets.

(Does generalize to $SU(2)_L \times SU(2)_R \times U(1)$ and $SU(5)$)

Spontaneous Symmetry Breaking (Higgs mechanism)

Gauge invariance implies massless gauge bosons and fermions

Weak interactions short ranged $\Rightarrow$ spontaneous symmetry breaking for mass; also for fermions

Allow classical (ground state) expectation value for Higgs field

$$v = \langle 0|\phi|0\rangle = \text{constant}, \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$\partial_\mu v \neq 0$ increases energy, but important for monopoles, strings, domain walls, phase transitions (e.g., EWPT, baryogenesis)

Minimize $V(v)$ to find v and quantize $\phi' = \phi - v$

$SU(2) \times U(1)$: introduce Hermitian basis

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{pmatrix}$$

where $\phi_i = \phi_i^\dagger$.

$$V(\phi) = \mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2 \to \frac{1}{2}\mu^2 \left(\sum_{i=1}^{4} \phi_i^2 \right) + \frac{1}{4}\lambda \left(\sum_{i=1}^{4} \phi_i^2 \right)^2$$

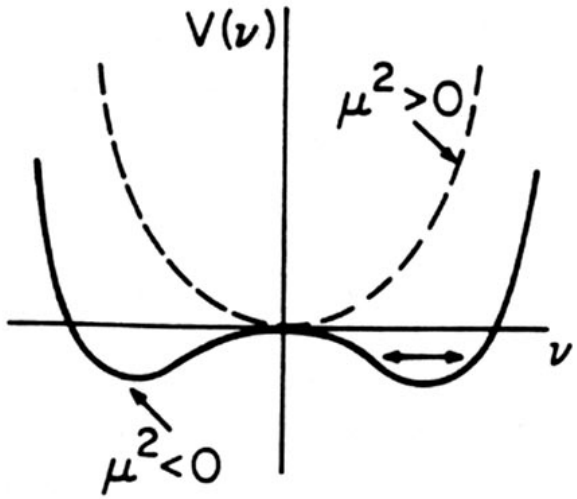
is $O(4)$ invariant.

w.l.o.g. choose $\langle 0|\phi_i|0\rangle = 0, \quad i = 1, 2, 4$ and $\langle 0|\phi_3|0\rangle = \nu$

$$V(\phi) \to V(v) = \frac{1}{2}\mu^2\nu^2 + \frac{1}{4}\lambda\nu^4$$

For $\mu^2 < 0$, minimum at

$$V'(\nu) = \nu(\mu^2 + \lambda\nu^2) = 0$$
$$\Rightarrow \quad \nu = \left(-\mu^2/\lambda\right)^{1/2}$$



SSB for $\mu^2 = 0$ also; must consider loop corrections

$\phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \equiv v \Rightarrow$ the generators L^1, L^2, and $L^3 - Y$ spontaneously broken, $L^1 v \neq 0$, etc $(L^i = \frac{\tau^i}{2},\ Y = \frac{1}{2}I)$

$$Qv = (L^3 + Y)v = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} v = 0 \Rightarrow U(1)_Q \text{ unbroken} \Rightarrow$$

$SU(2) \times U(1)_Y \rightarrow U(1)_Q$

Quantize around classical vacuum

- Kibble transformation: introduce new variables ξ^i for rolling modes

$$\phi = \frac{1}{\sqrt{2}} e^{i \sum \xi^i L^i} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix} \sim \begin{pmatrix} w^+ \\ \frac{\nu + H + iz}{\sqrt{2}} \end{pmatrix}$$

- $H = H^\dagger$ is the Higgs scalar
- $\phi^\dagger \phi = (\nu + H)^2/2 \Rightarrow$ no potential for $\xi^i \Rightarrow$ massless Goldstone bosons for global symmetry
- Disappear from spectrum for gauge theory ("eaten")
- Display particle content in unitary gauge

$$\phi \to \phi' = e^{-i \sum \xi^i L^i} \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

+ corresponding transformation on gauge fields

Rewrite Lagrangian in New Vacuum

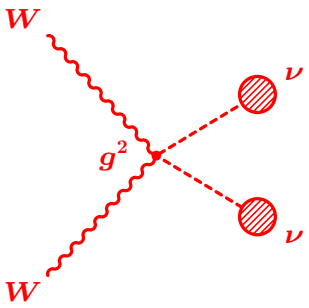
Physical Higgs scalar (oscillations around minimum): $M_H = \sqrt{2\lambda}\nu$

Higgs covariant kinetic energy terms:

$$(D_\mu\phi)^\dagger D^\mu\phi = \frac{1}{2}(0\ \nu)\left[\frac{g}{2}\tau^i W^i_\mu + \frac{g'}{2}B_\mu\right]^2\begin{pmatrix}0\\ \nu\end{pmatrix} + H \text{ terms}$$

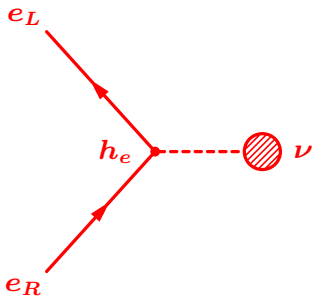
$$\rightarrow M_W^2 W^{+\mu}W^-_\mu + \frac{M_Z^2}{2}Z^\mu Z_\mu$$

$$+ \quad H \text{ kinetic energy and gauge interaction terms}$$



$$M_W = \frac{g\nu}{2}$$

$$m_e = \frac{h_e\nu}{\sqrt{2}}$$



Mass eigenstate bosons: W, Z, and A (photon)

$$\begin{aligned} W^{\pm} &= \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \\ Z &= -\sin\theta_W B + \cos\theta_W W^3 \\ A &= \cos\theta_W B + \sin\theta_W W^3 \end{aligned}$$

Weak angle: $\tan\theta_W \equiv g'/g$

Masses:

$$M_W = \frac{g\nu}{2}, \qquad M_Z = \sqrt{g^2+g'^2}\,\frac{\nu}{2} = \frac{M_W}{\cos\theta_W}, \qquad M_A = 0$$

(Goldstone scalars "eaten" $\rightarrow$ longitudinal components of $W^{\pm}$, Z)

Will show: Fermi constant $G_F/\sqrt{2} \sim g^2/8M_W^2$
($G_F = 1.166 \times 10^{-5}$ GeV^{-2} from muon lifetime)

Electroweak scale:

$$\nu = 2M_W/g \simeq (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{ GeV}$$

Will show: $g = e/\sin\theta_W$ ($\alpha = e^2/4\pi \sim 1/137.036$) $\Rightarrow$

$$M_W = M_Z \cos\theta_W = \frac{g\nu}{2} \sim \frac{(\pi\alpha/\sqrt{2}G_F)^{1/2}}{\sin\theta_W}$$

Weak neutral current: $\sin^2\theta_W \sim 0.23 \Rightarrow M_W \sim 78$ GeV, and $M_Z \sim 89$ GeV (increased by ~ 2 GeV by loop corrections)

Discovered at CERN: UA1 and UA2, 1983

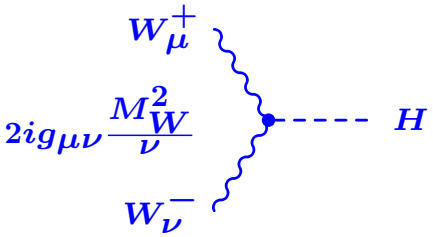
The Higgs Scalar H

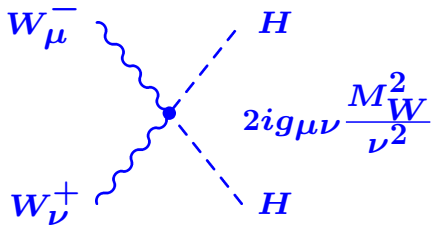
Gauge interactions: $ZZH, ZZH^2, W^+W^-H, W^+W^-H^2$

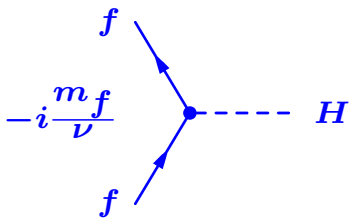
$$\varphi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix} \qquad \text{(unitary gauge)}$$

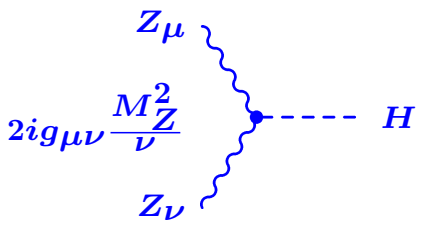
$$\begin{aligned} \mathcal{L}_\varphi &= (D^\mu \varphi)^\dagger D_\mu \varphi - V(\varphi) \\ &= \frac{1}{2}(\partial_\mu H)^2 + M_W^2 W^{\mu +} W_\mu^- \left(1 + \frac{H}{\nu}\right)^2 \\ &+ \frac{1}{2} M_Z^2 Z^\mu Z_\mu \left(1 + \frac{H}{\nu}\right)^2 - V(\varphi) \end{aligned}$$

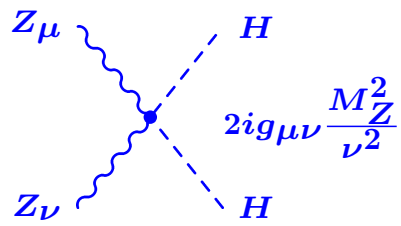
(quartic and induced cubic interactions, $\propto (\nu) M^2/\nu^2$**)**

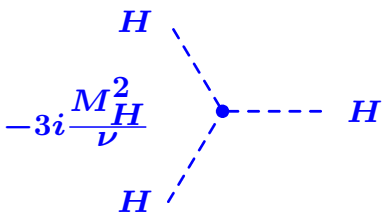

W_μ^+
$2ig_{\mu\nu}\frac{M_W^2}{\nu}$
W_ν^-
H

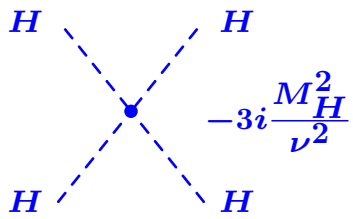

W_μ^-
H
$2ig_{\mu\nu}\frac{M_W^2}{\nu^2}$
W_ν^+
H


f
$-i\frac{m_f}{\nu}$
f
H


Z_μ
$2ig_{\mu\nu}\frac{M_Z^2}{\nu}$
Z_ν
H


Z_μ
H
$2ig_{\mu\nu}\frac{M_Z^2}{\nu^2}$
Z_ν
H


H
$-3i\frac{M_H^2}{\nu}$
H
H


H
H
$-3i\frac{M_H^2}{\nu^2}$
H
H

Higgs potential:

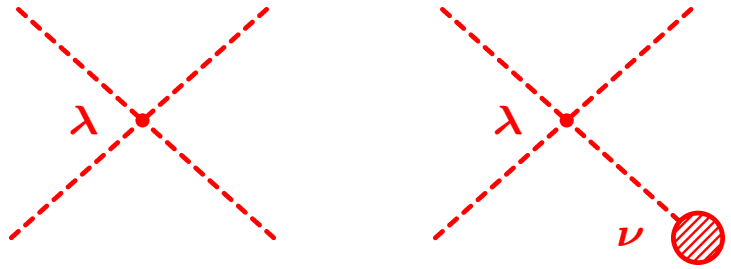
$$\begin{aligned} V(\varphi) &= +\mu^2 \varphi^\dagger \varphi + \lambda (\varphi^\dagger \varphi)^2 \\ &\rightarrow -\frac{\mu^4}{4\lambda} - \mu^2 H^2 + \lambda \nu H^3 + \frac{\lambda}{4} H^4 \end{aligned}$$

Fourth term: Quartic self-interaction

Third: Induced cubic self-interaction

Second: (tree level) H mass-squared, $M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\nu$

First: Constant (irrelevant until gravity added $\Rightarrow$ cosmological constant)

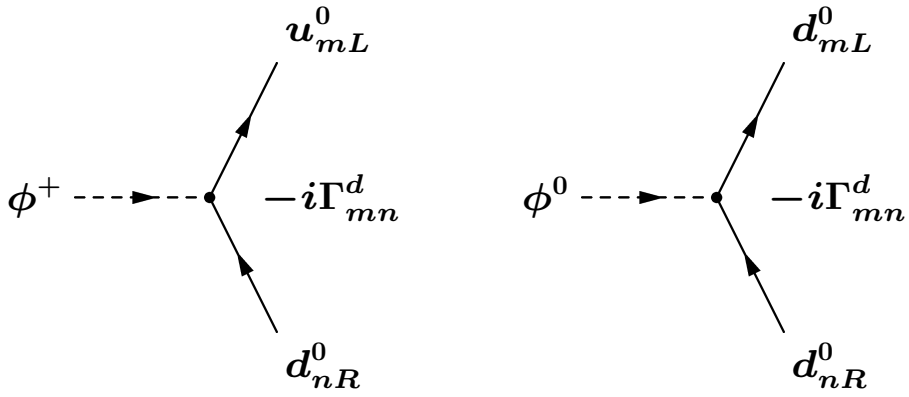


Fermion Masses and Mixings

- **Yukawa (Higgs-fermion) interaction (+ analogous u, e, ν terms)**

$$-\mathcal{L}_{Yuk} = \sum_{m,n=1}^{F} \Gamma^{d}_{mn} \bar{q}^{\,0}_{mL} \phi d^{0}_{nR} + h.c.$$

$$= \sum_{m,n=1}^{F} \Gamma^{d}_{mn} \left[\bar{u}^{0}_{mL} \phi^{+} d^{0}_{nR} + \bar{d}^{\,0}_{mL} \phi^{0} d^{0}_{nR} \right] + h.c.$$



- For $\phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$ (unitary gauge)

$$-\mathcal{L}_{Yuk} \Rightarrow \sum_{m,n=1}^{F} M^d_{mn} \bar{d}^0_{mL} d^0_{nR} \left(1 + \frac{H}{\nu}\right) + h.c.$$

$$= \sum_i m_i \bar{d}_{iL} d_{iR} \left(1 + \frac{H}{\nu}\right) + h.c. = \sum_i m_i \bar{d}_i d_i \left(1 + \frac{H}{\nu}\right)$$

with $M^d \equiv \Gamma^d \nu / \sqrt{2}$

- $d_i = d_{iL} + d_{iR}$ are mass eigenstates of mass m_i ("bare" QCD masses)

- For $F = 3 =$

$$A^{d\dagger}_L M^d A^d_R = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \qquad \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R} = \underbrace{A^{d\dagger}_{L,R}}_{\text{unitary}} \begin{pmatrix} d^0_1 \\ d^0_2 \\ d^0_3 \end{pmatrix}_{L,R}$$

- Higgs (H) couplings to fermions are diagonal in flavor and $\propto$ mass

The Weak Charged Current

Fermi Theory incorporated in SM and made renormalizable

W-fermion interaction

$$\mathcal{L} = -\frac{g}{2\sqrt{2}}\left(J^{\mu}_{W} W^{-}_{\mu} + J^{\mu\dagger}_{W} W^{+}_{\mu}\right)$$

Charge-raising current (ignoring ν masses)

$$\begin{aligned} J^{\mu\dagger}_{W} &= \sum_{m=1}^{F} \left[\bar{\nu}^{0}_{m}\gamma^{\mu}(1-\gamma^{5})e^{0}_{m} + \bar{u}^{0}_{m}\gamma^{\mu}(1-\gamma^{5})d^{0}_{m}\right] \\ &= (\bar{\nu}_{e}\bar{\nu}_{\mu}\bar{\nu}_{\tau})\gamma^{\mu}(1-\gamma^{5})\begin{pmatrix} e^{-} \\ \mu^{-} \\ \tau^{-} \end{pmatrix} + (\bar{u}\ \bar{c}\ \bar{t})\gamma^{\mu}(1-\gamma^{5})V\begin{pmatrix} d \\ s \\ b \end{pmatrix} \end{aligned}$$

Ignore ν masses for now

Pure $V-A \Rightarrow$ maximal P and C violation; CP conserved except for phases in V

$V = A_L^{u\dagger} A_L^d$ is $F \times F$ unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix from mismatch between weak and Yukawa interactions

Cabibbo matrix for $F = 2$

$$V = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

$\sin\theta_c \simeq 0.23 \equiv$ Cabibbo angle

Good zeroth-order description since third family almost decouples

General unitary 2×2: 1 angle and 3 (unobservable) q_L phases

CKM matrix for $F = 3$ involves 3 angles and 1 CP-violating phase (after removing unobservable q_L phases) (new interations involving q_R could make observable)

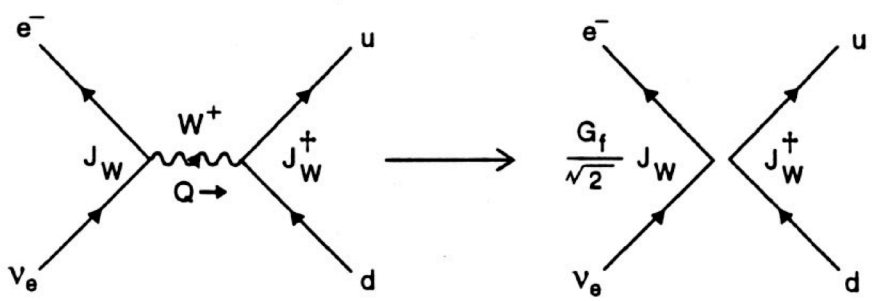
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{td} & V_{td} \end{pmatrix}$$

Extensive studies, especially in B decays, to test unitarity of V as probe of new physics and test origin of CP violation

Need additional source of CP breaking for baryogenesis

Effective zero- range 4-fermi interaction (Fermi theory)

For $|Q| \ll M_W$, neglect Q^2 in W propagator



$$-\mathcal{L}^{cc}_{eff} = \left(\frac{g}{2\sqrt{2}}\right)^2 J^{\mu}_{W} \left(\frac{-g_{\mu\nu}}{Q^2 - M^2_W}\right) J^{\dagger\nu}_{W} \sim \frac{g^2}{8M^2_W} J^{\mu}_{W} J^{\dagger}_{W\mu}$$

Fermi constant: $\frac{G_F}{\sqrt{2}} \simeq \frac{g^2}{8M^2_W} = \frac{1}{2\nu^2}$

Muon lifetime: $\tau^{-1} = \frac{G^2_F m^5_\mu}{192\pi^3} \Rightarrow G_F = 1.17 \times 10^{-5}$ GeV^{-2}

Weak scale: $\nu = \sqrt{2}\langle 0|\phi^0|0\rangle \simeq 246$ GeV

Excellent description of β, K, hyperon, heavy quark, μ, and τ decays, $\nu_\mu e \to \mu^- \nu_e$, $\nu_\mu n \to \mu^- p$, $\nu_\mu N \to \mu^- X$

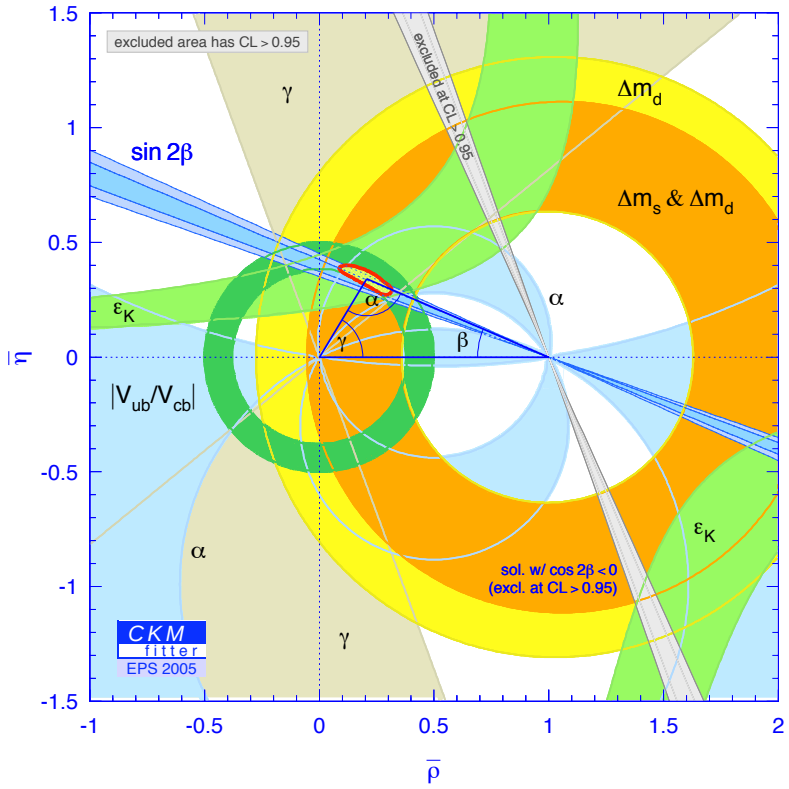
Full theory probed:

$e^{\pm}p \to \overset{(-)}{\nu}_e X$ at high energy (HERA)

Electroweak radiative corrections (loop level)

(Very important. Only calculable in full theory.)

$M_{K_S} - M_{K_L}$, kaon CP violation, $B \leftrightarrow \bar{B}$ mixing (loop level)



(CKMFITTER group:
http://ckmfitter.in2p3.fr/)

Quantum Electrodynamics (QED)

Incorporated into standard model

Lagrangian:

$$\mathcal{L} = -\frac{gg'}{\sqrt{g^2 + g'^2}} J_Q^\mu (\cos\theta_W B_\mu + \sin\theta_W W_\mu^3)$$

Photon field:

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W_\mu^3$$

Positron electric charge: $e = g\sin\theta_W$, **where** $\tan\theta_W \equiv g'/g$

Electromagnetic current:

$$J^\mu_Q = \sum_{m=1}^{F} \left[\frac{2}{3} \bar{u}^0_m \gamma^\mu u^0_m - \frac{1}{3} \bar{d}^0_m \gamma^\mu d^0_m - \bar{e}^0_m \gamma^\mu e^0_m \right]$$

$$= \sum_{m=1}^{F} \left[\frac{2}{3} \bar{u}_m \gamma^\mu u_m - \frac{1}{3} \bar{d}_m \gamma^\mu d_m - \bar{e}_m \gamma^\mu e_m \right]$$

Electric charge: $Q = T^3 + Y$, where $Y =$ weak hypercharge

(coefficient of $ig' B_\mu$ in covariant derivatives)

Flavor diagonal: Same form in weak and mass bases because fields which mix have same charge

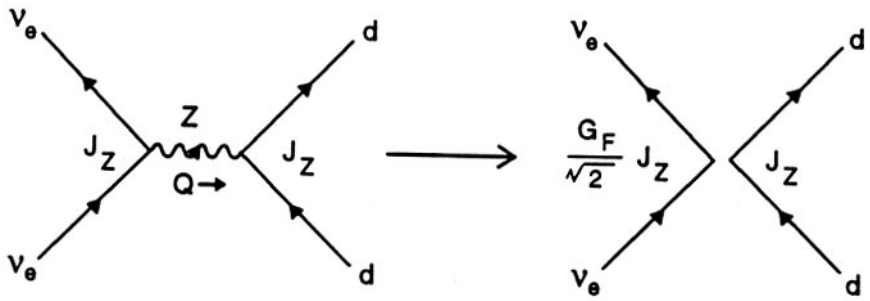
Purely vector (parity conserving): L and R fields have same charge

($q_i = t^3_i + y_i$ is the same for L and R fields, even though t^3_i and y_i are not)

The Weak Neutral Current

Prediction of $SU(2) \times U(1)$

$$\begin{aligned}\mathcal{L} &= -\frac{\sqrt{g^2+g'^2}}{2} J_Z^\mu \left(-\sin\theta_W B_\mu + \cos\theta_W W_\mu^3\right) \\ &= -\frac{g}{2\cos\theta_W} J_Z^\mu Z_\mu\end{aligned}$$



Neutral current process and effective 4-fermi interaction for $|Q| \ll M_Z$

Neutral current:

$$
\begin{aligned}
J_Z^\mu &= \sum_m \left[\bar{u}^0_{mL}\gamma^\mu u^0_{mL} - \bar{d}^0_{mL}\gamma^\mu d^0_{mL} + \bar{\nu}^0_{mL}\gamma^\mu \nu^0_{mL} - \bar{e}^0_{mL}\gamma^\mu e^0_{mL}\right] \\
&\quad -2\sin^2\theta_W J_Q^\mu \\
&= \sum_m \left[\bar{u}_{mL}\gamma^\mu u_{mL} - \bar{d}_{mL}\gamma^\mu d_{mL} + \bar{\nu}_{mL}\gamma^\mu \nu_{mL} - \bar{e}_{mL}\gamma^\mu e_{mL}\right] \\
&\quad -2\sin^2\theta_W J_Q^\mu
\end{aligned}
$$

Flavor diagonal: Same form in weak and mass bases because fields which mix have same charge

GIM mechanism: c quark predicted so that s_L could be in doublet to avoid unwanted flavor changing neutral currents (FCNC) at tree and loop level

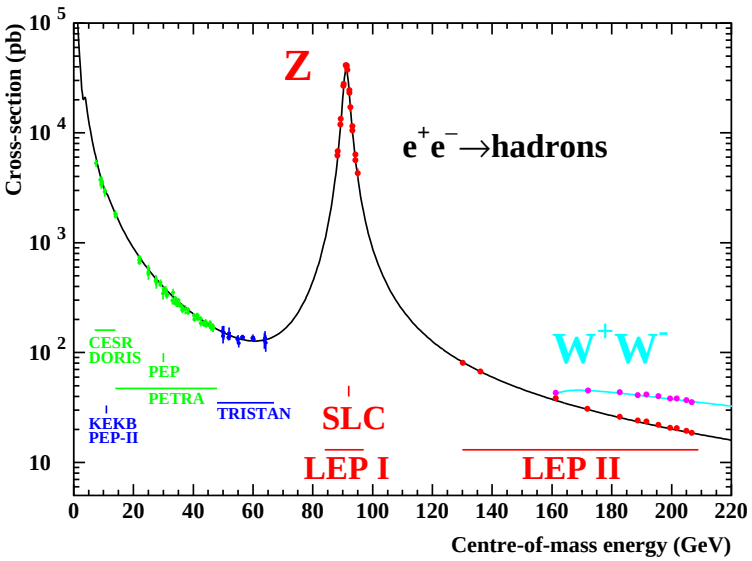
Parity and charge conjugation violated but not maximally: first term is pure $V-A$, second is V

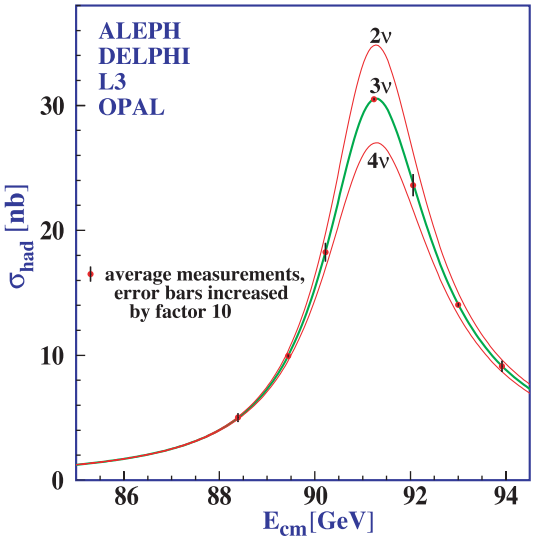
Effective 4-fermi interaction for $|Q^2| \ll M_Z^2$:

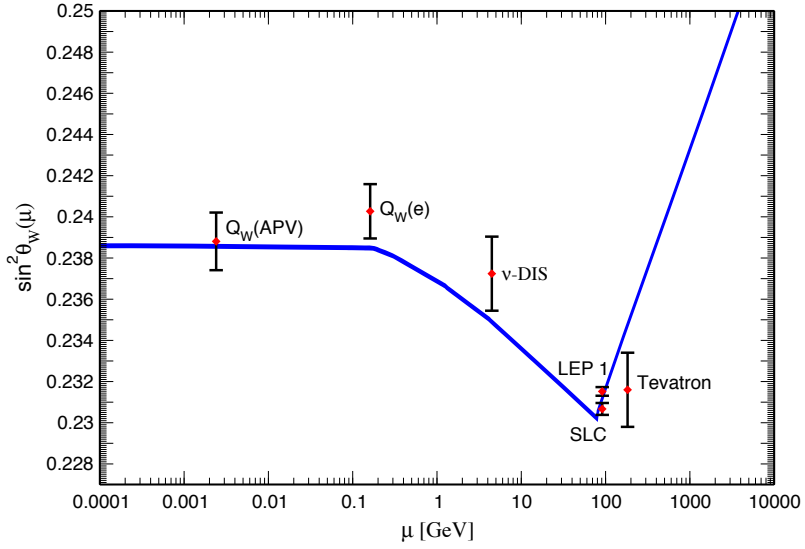
$$-\mathcal{L}_{eff}^{NC} = \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}$$

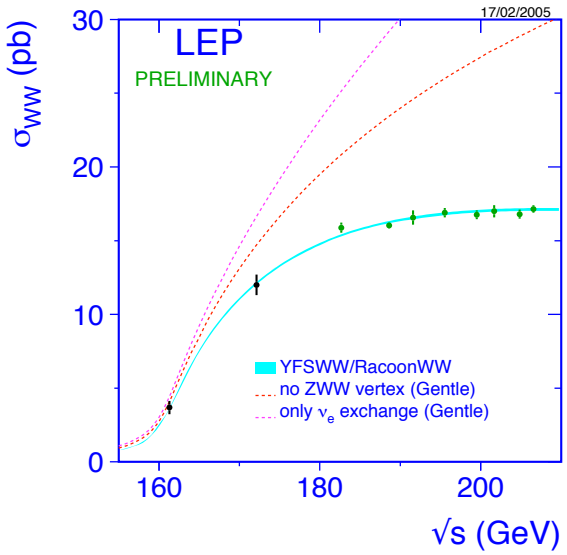
Coefficient same as WCC because

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2 + g'^2}{8M_Z^2}$$


Cross-section (pb)
Z
e+e−→hadrons
W+W-
CESR
DORIS
PEP
PETRA
TRISTAN
KEKB
PEP-II
SLC
LEP I
LEP II
Centre-of-mass energy (GeV)

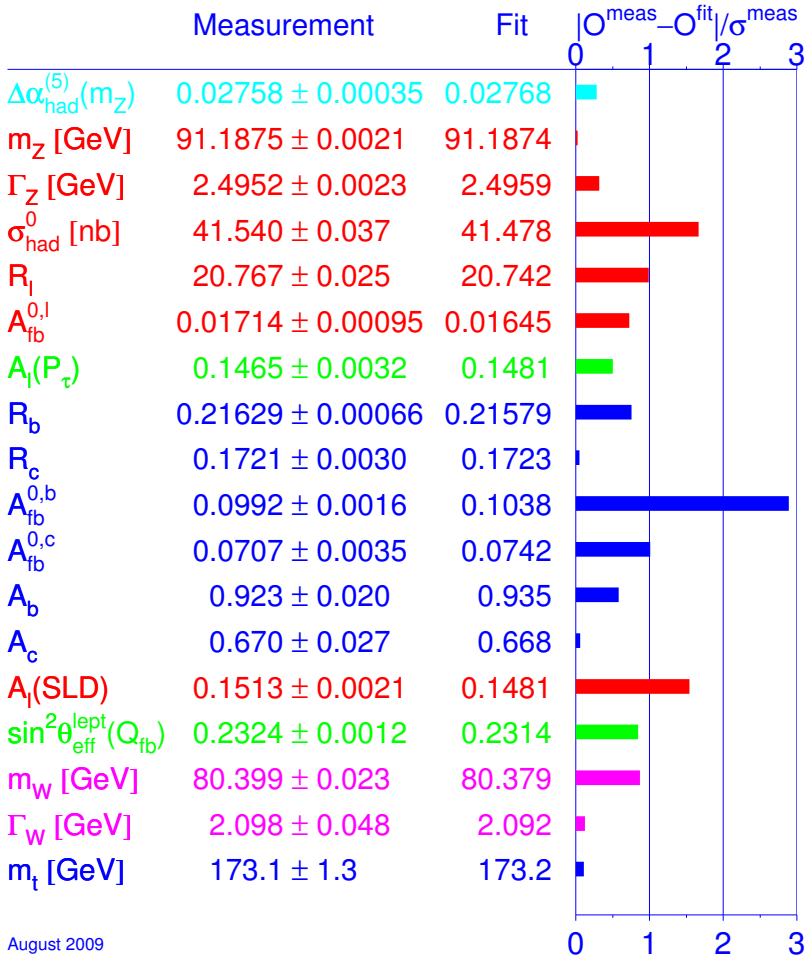

ALEPH
DELPHI
L3
OPAL
2ν
3ν
4ν
average measurements,
error bars increased
by factor 10
σhad [nb]
Ecm[GeV]


sin2θW(μ)
QW(APV)
QW(e)
ν-DIS
LEP 1
Tevatron
SLC
μ [GeV]


17/02/2005
LEP
PRELIMINARY
σWW (pb)
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only νe exchange (Gentle)
√s (GeV)

The Z, the W, and the Weak Neutral Current

- Primary prediction and test of electroweak unification
- WNC discovered 1973 (Gargamelle at CERN, HPW at FNAL)
- 70's, 80's: weak neutral current experiments (few %)
  - Pure weak: νN, νe scattering
  - Weak-elm interference in eD, e^+e^-, atomic parity violation
  - Model independent analyses (νe, νq, eq)
  - $SU(2) \times U(1)$ group/representations; t and ν_τ exist; m_t limit; hint for SUSY unification; limits on TeV scale physics
- W, Z discovered directly 1983 (UA1, UA2)

- **90's: Z pole (LEP, SLD), 0.1%; lineshape, modes, asymmetries**
- **LEP 2: M_W, Higgs search , gauge self-interactions**
- **Tevatron: m_t, M_W, Higgs search**
- **4th generation weak neutral current experiments** (atomic parity (Boulder); νe; νN (NuTeV); polarized Møller asymmetry (SLAC))

- SM correct and unique to zeroth approx. (gauge principle, group, representations)
- SM correct at loop level (renorm gauge theory; m_t, α_s, M_H)
- TeV physics severely constrained (unification vs compositeness)
- Consistent with light elementary Higgs
- Precise gauge couplings (SUSY gauge unification)