The Z, the W, and the Weak Neutral Current

$$\mathcal{L}_Z = -\frac{g}{2\cos\theta_W} J_Z^\mu Z_\mu$$

$$J_Z^\mu = \sum_r t_{rL}^3 \bar{\psi}_r^0 \gamma^\mu (1-\gamma^5)\psi_r^0 - 2\sin^2\theta_W\, J_Q^\mu$$

$$t_{uL}^3 = -t_{dL}^3 = t_{\nu L}^3 = -t_{eL}^3 = \tfrac{1}{2}, \qquad \tan\theta_W = g'/g$$

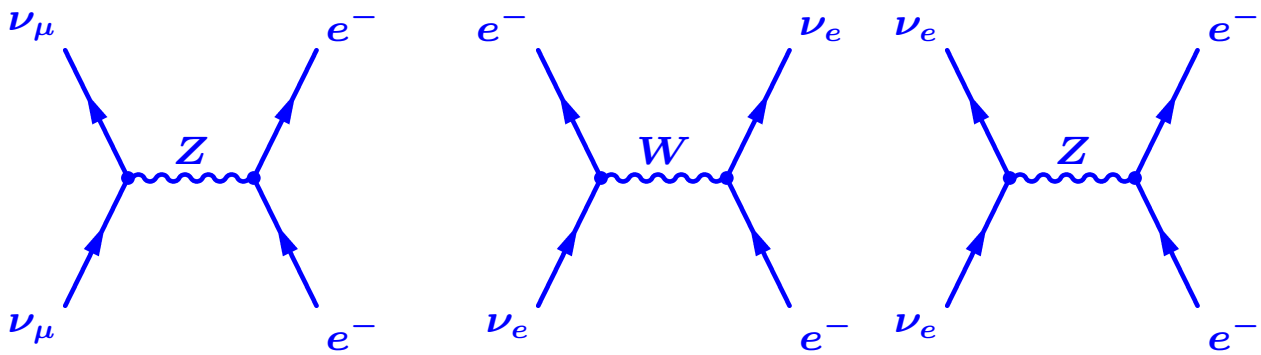
$$-\mathcal{L}_{\text{eff}}^{NC} = \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}$$

- Primary prediction and test of electroweak unification
- WNC discovered 1973 (Gargamelle at CERN, HPW at FNAL)
- W, Z discovered directly 1983 (UA1, UA2)
- 90's: Z pole (LEP, SLD), 0.1%

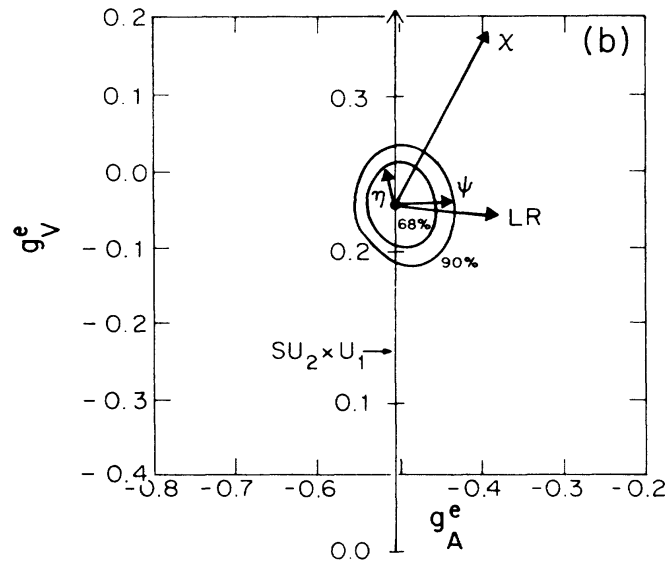
$\nu e \to \nu e$

$$-\mathcal{L}^{\nu e} = \frac{G_F}{\sqrt{2}}\,\bar{\nu}_\mu\,\gamma^\mu(1-\gamma^5)\nu_\mu\,\bar{e}\,\gamma_\mu(g_V^{\nu e} - g_A^{\nu e}\gamma^5)e$$

$$\text{SM}: \qquad g_V^{\nu e} \sim -\frac{1}{2} + 2\,\sin^2\theta_W\,, \qquad g_A^{\nu e} \sim -\frac{1}{2}$$



- **Any gauge model** (with left-handed ν) $\rightarrow$ **some** $g_{V,A}^{\nu e}$
- **Need SM rad. corr.**
- $\nu_e : g_{V,A}^{\nu e} \xrightarrow[\text{WCC}]{} g_{V,A}^{\nu e} + 1$
- **Alternative models w. disjoint parameters** *and* **perturbations on SM**



- **Amplitude-squared** $(\nu(p_1)e^-(p_2) \rightarrow e^-(p_3)\nu(p_4))$

$$\frac{1}{2}\sum_{s_1 s_2 s_3 s_4} |M|^2 = 16G_F^2\big[(g_V+g_A)^2 p_1\cdot p_2\; p_3\cdot p_4$$
$$+ (g_V-g_A)^2 p_1\cdot p_3\; p_2\cdot p_4 - (g_V^2-g_A^2)m_e^2\, p_1\cdot p_4\big]$$

- **Cross section**

$$\frac{d\sigma_{\nu_\mu,\bar{\nu}_\mu}}{dy} = \frac{G_F^2 m_e E_\nu}{2\pi} \times \Big[(g_V^{\nu e} \pm g_A^{\nu e})^2 + (g_V^{\nu e} \mp g_A^{\nu e})^2 (1-y)^2 - (g_V^{\nu e 2} - g_A^{\nu e 2}) \frac{y\, m_e}{E_\nu}\Big]$$

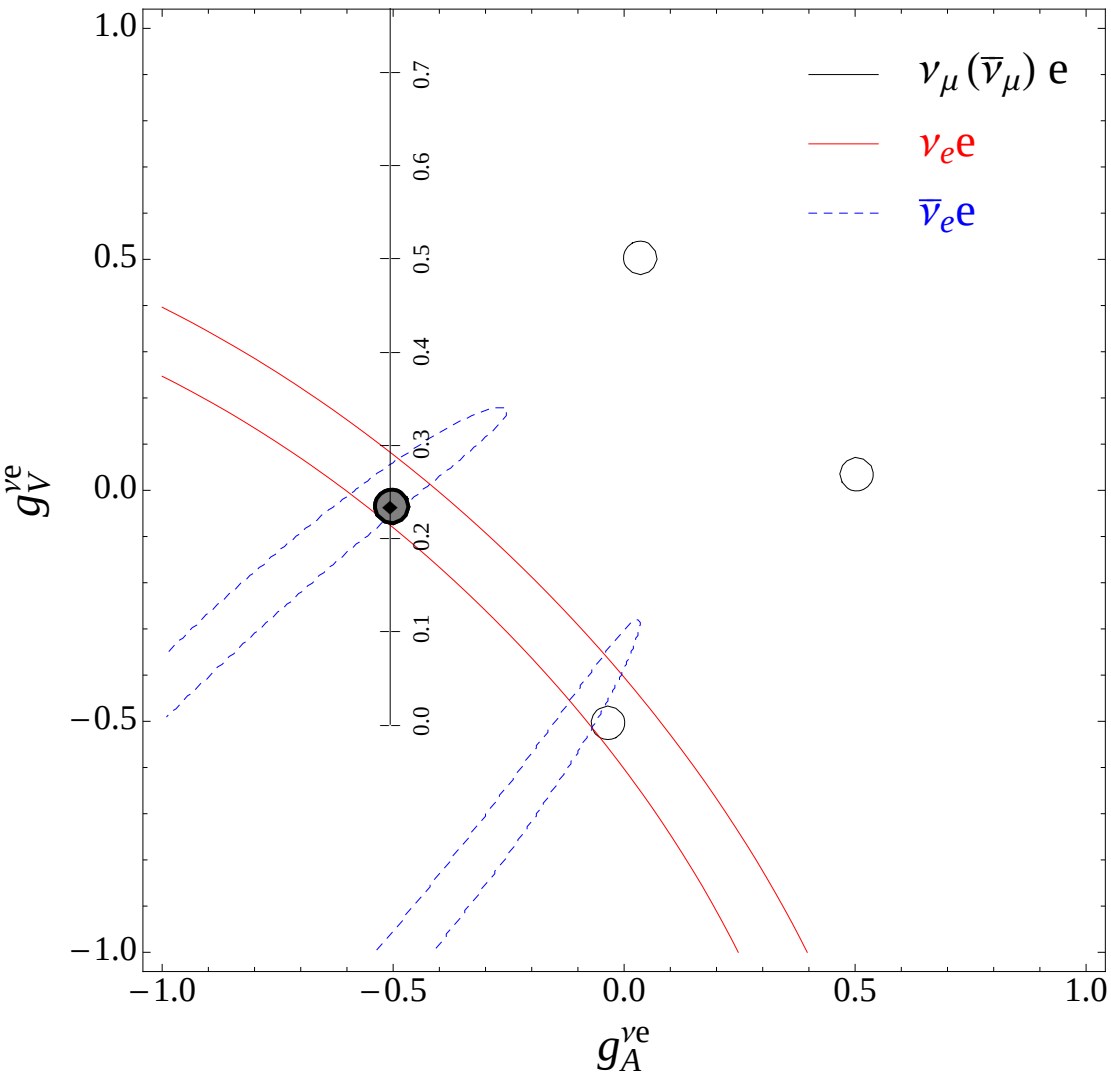
where $0 \leq y \equiv T_e/E_\nu \leq (1 + m_e/2E_\nu)^{-1}$

- **For $E_\nu \gg m_e$**

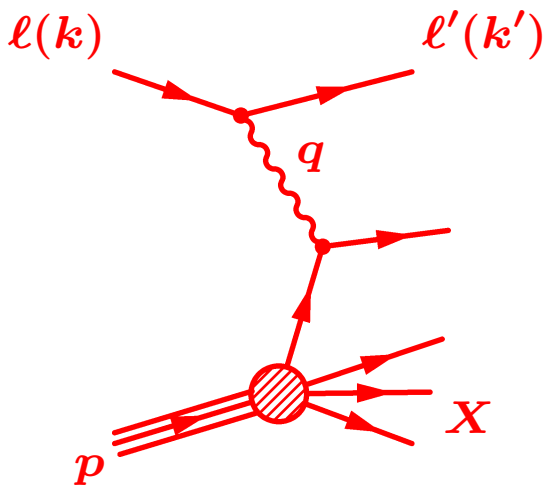
$$\sigma = \frac{G_F^2\, m_e\, E_\nu}{2\pi} \left[(g_V^{\nu e} \pm g_A^{\nu e})^2 + \frac{1}{3}(g_V^{\nu e} \mp g_A^{\nu e})^2\right]$$

- **Flux uncertainties $\sim$ cancel in $R \equiv \sigma_{\nu_\mu e}/\sigma_{\bar{\nu}_\mu e}$**

- **Most precise: CHARM II (CERN)**


$\nu_\mu\,(\bar{\nu}_\mu)$ e
ν_ee
$\bar{\nu}_e$e
$g_V^{\nu e}$
$g_A^{\nu e}$
1.0
0.5
0.0
−0.5
−1.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7

Deep Inelastic ν Scattering



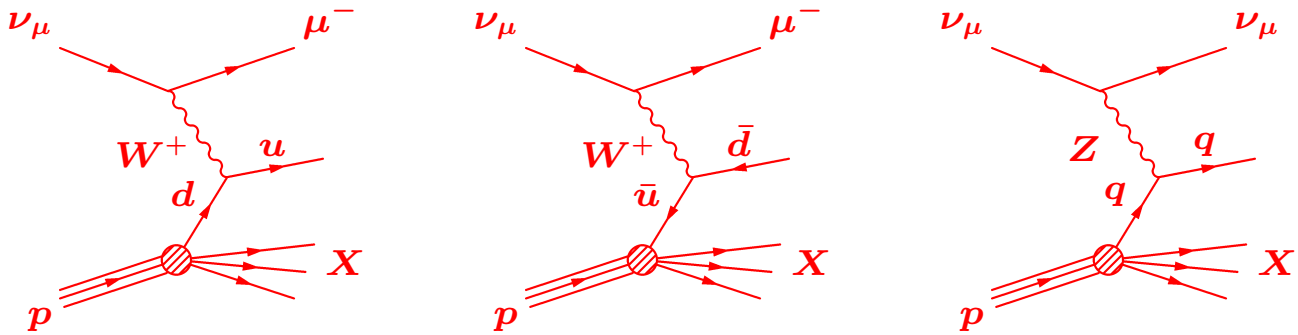
- $e^{\pm}p \to e^{\pm}X, \quad \mu^{\pm}p \to \mu^{\pm}X,$
  $\overset{(-)}{\nu}_{\mu}\, p \to \mu^{\mp}X, \ \overset{(-)}{\nu}_{\mu}\, p \to \overset{(-)}{\nu}_{\mu}\, X$
  at $Q^2 \gg M_p^2$ (or $p \to n, N$)

- WCC: test of QCD, quark model

$$Q^2 = -q^2 > 0 \underbrace{\longrightarrow}_{\text{lab}} 2kk'(1-\cos\theta)$$

$$\nu = \frac{p \cdot q}{M_p} \sim k - k'$$

$$x = \frac{Q^2}{2M_p\nu} \qquad y = \frac{\nu}{E_k} \sim \frac{k-k'}{k}$$



- **Charged current,** $\overset{(-)}{\nu}_\mu p \to \mu^\mp X$ $(Q^2 \ll M_W^2)$

$$\frac{d\bar{\sigma}_{\nu,\bar{\nu}}}{dk' d\Omega} = \frac{G_F^2}{32\pi^2}\frac{k'}{k} L^{\mu\nu}_{\nu,\bar{\nu}} W^{\nu,\bar{\nu}}_{\mu\nu}$$

$$L^{\mu\nu}_{\nu} = \text{Tr}\left[\gamma^\mu(1-\gamma^5)\ \not{k}\,\gamma^\nu(1-\gamma^5)\ \not{k}'\right]$$

$$= 8\left[k^\mu k'^\nu + k'^\mu k^\nu + g^{\mu\nu}\frac{q^2}{2} + \underbrace{i\epsilon^{\mu\rho\nu\sigma}k'_\rho k_\sigma}_{V-A\ \text{interference}}\right]$$

- **Hadronic tensor** (different for $e^-, \nu, \bar{\nu}$)

$$W_{\mu\nu}^{\nu,\bar{\nu}} = \left[-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right] W_1^{\nu,\bar{\nu}}\left(Q^2,\nu\right)$$
$$+ \frac{1}{M^2}\left[p_\mu - \frac{p\cdot q}{q^2}q_\mu\right]\left[p_\nu - \frac{p\cdot q}{q^2}q_\nu\right] W_2^{\nu,\bar{\nu}}\left(Q^2,\nu\right)$$
$$+ i\epsilon_{\mu\rho\nu\sigma}\frac{p^\rho q^\sigma}{2M^2} W_3^{\nu,\bar{\nu}}\left(Q^2,\nu\right)$$

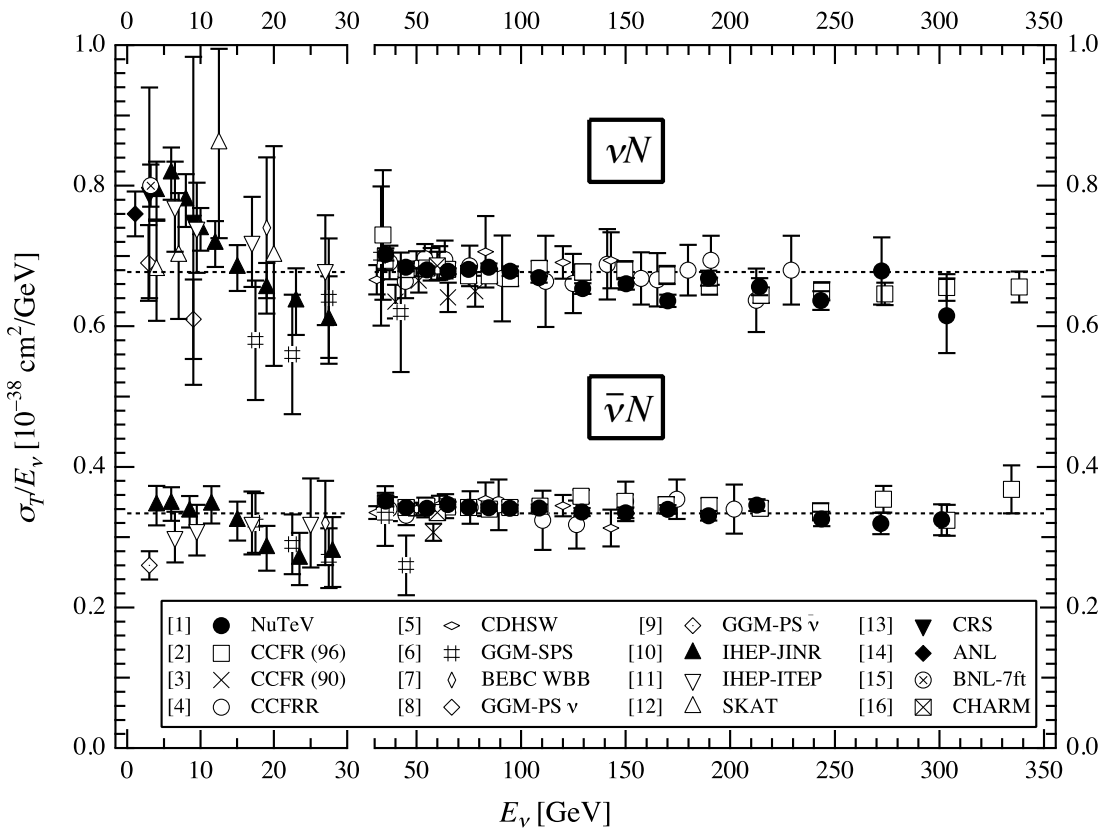
$$F_1^{\nu,\bar{\nu}}\left(x,Q^2\right) = MW_1^{\nu,\bar{\nu}}\left(Q^2,\nu\right), \qquad F_{2,3}^{\nu,\bar{\nu}}\left(x,Q^2\right) = \nu W_{2,3}^{\nu,\bar{\nu}}\left(Q^2,\nu\right)$$

- **Approximate scaling** (up to QCD corrections): $F_i^{\nu,\bar{\nu}}\left(x,Q^2\right) \sim F_i^{\nu,\bar{\nu}}\left(x\right)$

- **Cross section predicted to scale as E_ν:**

$$\frac{d^2\bar{\sigma}_{cc}^{\nu,\bar{\nu}}}{dxdy} = \frac{G_F^2 M E_\nu}{\pi}\left[xy^2 F_1^{\nu,\bar{\nu}}(x) + (1-y)F_2^{\nu,\bar{\nu}}(x) \pm xy(1-\frac{y}{2})F_3^{\nu,\bar{\nu}}(x)\right]$$

$$\bar{\sigma}_{cc}^{\nu,\bar{\nu}} = \int_0^1 dx \int_0^1 dy \frac{d^2\bar{\sigma}_{cc}^{\nu,\bar{\nu}}}{dxdy}$$



- **Simple parton model** (without mixing, heavy quarks)

$$F_2^{\nu}(x) = 2xF_1^{\nu}(x) = 2x\left[d(x) + \bar{u}(x)\right], \qquad F_3^{\nu}(x) = 2x\left[d(x) - \bar{u}(x)\right]$$
$$F_2^{\bar{\nu}}(x) = 2xF_1^{\bar{\nu}}(x) = 2x\left[u(x) + \bar{d}(x)\right], \qquad F_3^{\bar{\nu}}(x) = 2x\left[u(x) - \bar{d}(x)\right]$$

- **Isospin:**

$$W_{in}^{\nu} \sim W_{ip}^{\bar{\nu}}, \qquad W_{ip}^{\nu} \sim W_{in}^{\bar{\nu}}, \qquad W_{iN}^{\nu} = \frac{1}{2}\left[W_{ip}^{\nu} + W_{in}^{\nu}\right] \sim W_{iN}^{\bar{\nu}}$$

- **$5/18^{th}$ rule:**

$$\frac{F_{2N}}{F_{2N}^{\nu}} = \frac{\frac{1}{2}\left(F_{2p} + F_{2n}\right)}{\frac{1}{2}\left(F_{2p}^{\nu} + F_{2n}^{\nu}\right)} = \frac{\left(\frac{4}{9} + \frac{1}{9}\right)\frac{1}{2}x\left(u^p + d^p + \bar{u}^p + \bar{d}^p\right)}{x\left(d^p + \bar{u}^p + u^p + \bar{d}^p\right)} = \frac{5}{18}$$

- **Include mixing, m_c suppression factors ξ_c:**

$$\frac{d^2\bar{\sigma}^{\nu}_{cc}}{dxdy} = \frac{2G_F^2 M E_\nu}{\pi}\Big\{xd\underbrace{\left[|V_{ud}|^2 + |V_{cd}|^2\xi_c\right]}_{\lambda_d} + xs\underbrace{\left[|V_{us}|^2 + |V_{cs}|^2\xi_c\right]}_{\lambda_s}$$

$$+ x\left(\bar{u}+\bar{c}\right)\left(1-y\right)^2\Big\}$$

$$\sim \frac{2G_F^2 M E_\nu}{\pi}\left[x\left(d+s\right) + x\left(\bar{u}+\bar{c}\right)\left(1-y\right)^2\right]$$

$$\frac{d^2\bar{\sigma}^{\bar{\nu}}_{cc}}{dxdy} = \frac{2G_F^2 M E_{\bar{\nu}}}{\pi}\left[x\left(u+c\right)\left(1-y\right)^2 + x\bar{d}\lambda_d + x\bar{s}\lambda_s\right]$$

$$\sim \frac{2G_F^2 M E_{\bar{\nu}}}{\pi}\left[x\left(u+c\right)\left(1-y\right)^2 + x\bar{d} + x\bar{s}\right]$$

- **Dimuons** (extract $|V_{cd}|^2, S|V_{cs}|^2, \xi_c$, where $S = \int xs(x)dx$):

$$\nu(\bar{\nu})N \to \mu^{\mp}X + c(\bar{c}) \text{ from } d, s$$

$$c \to (s, d)\mu^+\nu_\mu, \qquad \bar{c} \to (\bar{s}, \bar{d})\mu^-\bar{\nu}_\mu$$

$\nu q \to \nu q$ (Mainly Deep Inelastic)

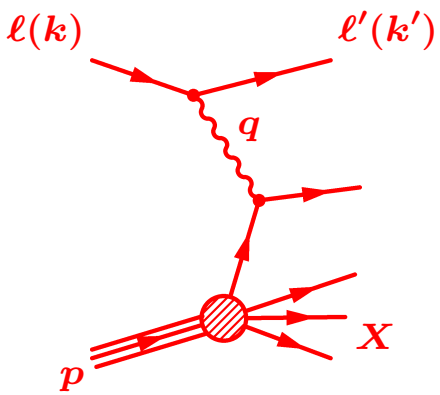
- **WNC: test of electroweak standard model**

$$
\begin{aligned}
-\mathcal{L}^{\nu\text{Hadron}} &= \frac{G_F}{\sqrt{2}}\,\bar{\nu}\,\gamma^\mu\,(1-\gamma^5)\nu \\
&\times \sum_i \left[\epsilon_L(i)\;\bar{q}_i\;\gamma_\mu(1-\gamma^5)q_i + \epsilon_R(i)\;\bar{q}_i\;\gamma_\mu(1+\gamma^5)q_i\right]
\end{aligned}
$$

- **Standard model**

$$
\begin{aligned}
\epsilon_L(u) &\sim \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \qquad & \epsilon_R(u) &\sim -\frac{2}{3}\sin^2\theta_W \\
\epsilon_L(d) &\sim -\frac{1}{2} + \frac{1}{3}\sin^2\theta_W \qquad & \epsilon_R(d) &\sim \frac{1}{3}\sin^2\theta_W
\end{aligned}
$$

- **Deep inelastic** $\overset{(-)}{\nu}_\mu N \to \overset{(-)}{\nu}_\mu X$



$$
\begin{aligned}
\frac{d^2\sigma^{NC}_{\nu N}}{dx\,dy} &= \frac{2G_F^2 M_p E_\nu}{\pi} \times \{ \\
& \left[|\epsilon_L(u)|^2 + |\epsilon_R(u)|^2(1-y)^2\right](xu + xc\,\xi_c) \\
+ & \left[|\epsilon_L(d)|^2 + |\epsilon_R(d)|^2(1-y)^2\right](xd + xs) \\
+ & \left[|\epsilon_R(u)|^2 + |\epsilon_L(u)|^2(1-y)^2\right](x\bar{u} + x\bar{c}\,\xi_c) \\
+ & \left[|\epsilon_R(d)|^2 + |\epsilon_L(d)|^2(1-y)^2\right](x\bar{d} + x\bar{s})\}
\end{aligned}
$$

($\epsilon_L(i) \leftrightarrow \epsilon_R(i)$ for $\bar{\nu}$)

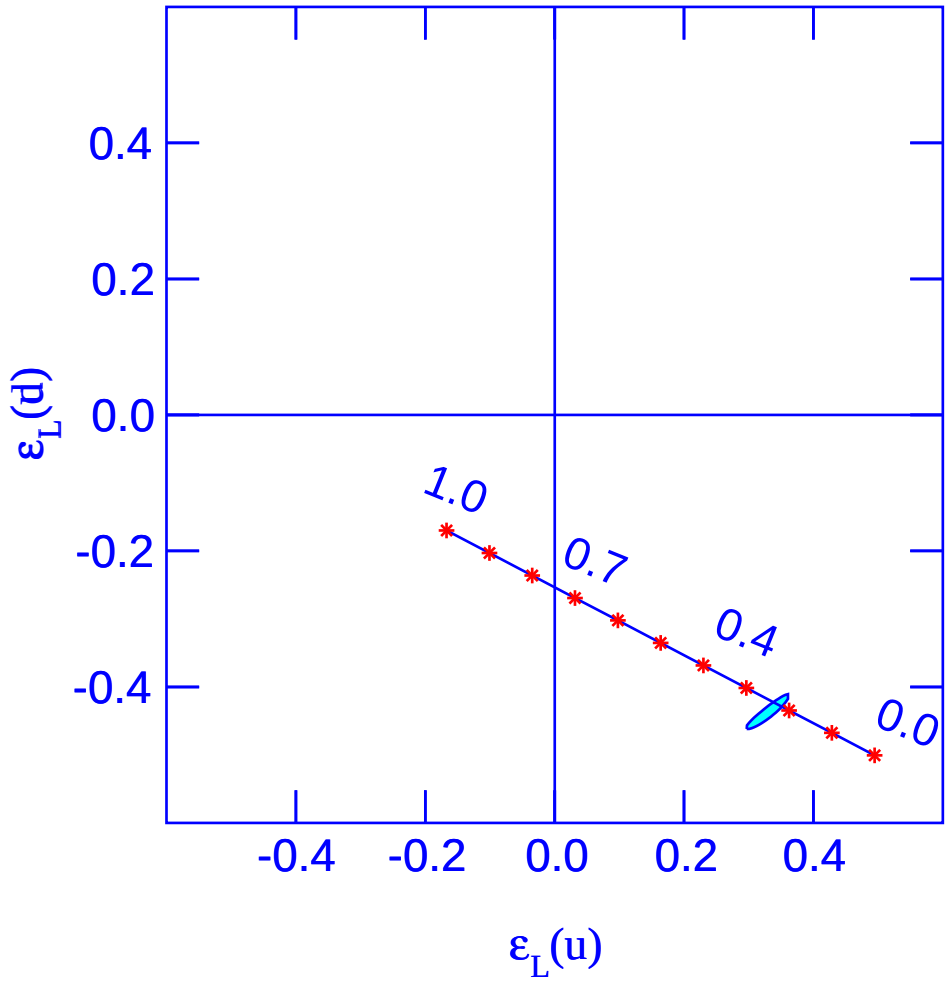
- **WNN/WCC ratios measured to 1% or better by CDHS and CHARM (CERN) and CCFR (FNAL)** (many strong interaction, ν flux, and systematic effects cancel)

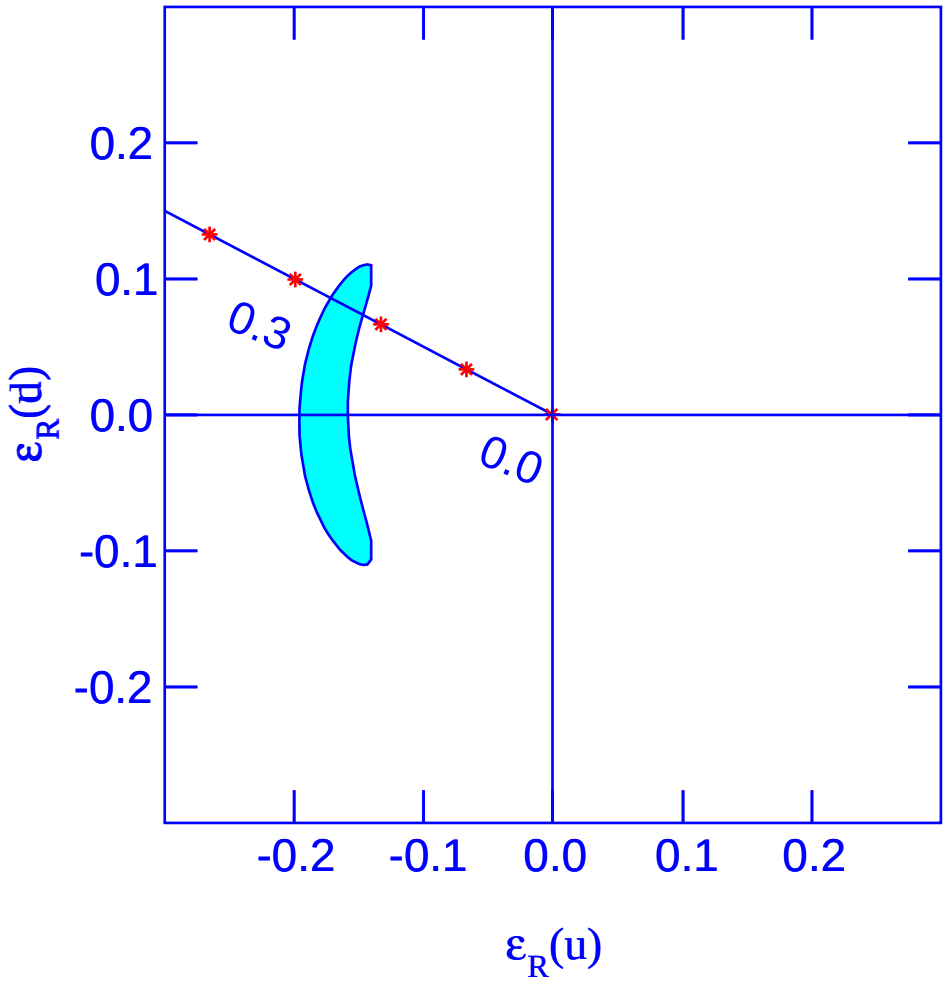
- **For isoscalar targer** ($N_p = N_n$)**; ignoring** s**,** c **and third family sea; ignoring** c **threshold correction** ($\xi_c = 1$)

$$R_\nu \equiv \frac{\sigma^{NC}_{\nu N}}{\sigma^{CC}_{\nu N}} \sim g_L^2 + g_R^2 r$$

$$R_{\overline{\nu}} \equiv \frac{\sigma^{NC}_{\overline{\nu} N}}{\sigma^{CC}_{\overline{\nu} N}} \sim g_L^2 + \frac{g_R^2}{r}$$

  - $g_L^2 \equiv \epsilon_L(u)^2 + \epsilon_L(d)^2 \approx \frac{1}{2} - \sin^2\theta_W + \frac{5}{9}\sin^4\theta_W$
  - $g_R^2 \equiv \epsilon_R(u)^2 + \epsilon_R(d)^2 \approx \frac{5}{9}\sin^4\theta_W$
  - $r \equiv \sigma^{CC}_{\overline{\nu} N}/\sigma^{CC}_{\nu N}$ **measured** ($r \to 1/3$ for $\bar{q}/q \to 0$)


0.4
0.2
0.0
-0.2
-0.4
$\varepsilon_L(d)$
1.0
0.7
0.4
0.0
-0.4
-0.2
0.0
0.2
0.4
$\varepsilon_L(u)$


0.2
0.1
0.0
-0.1
-0.2
$\varepsilon_R(d)$
0.3
0.0
-0.2
-0.1
0.0
0.1
0.2
$\varepsilon_R(u)$

- Most precise $\sin^2\theta_W$ before LEP/SLD: $s_W^2 \sim 0.233 \pm 0.003\ (\mathrm{exp}) \pm 0.005\ (m_c)$

- Must correct for $N_n \neq N_p$; $s(x), c(x)$, ξ_c, QCD, third family mixing, W/Z propagators, radiative corrections, experimental cuts

- Can separate $\epsilon_i(u)/\epsilon_i(d)$ by p and n targets, e.g., bubble chamber (less precise)

- Error dominated by charm threshold (m_c in ξ_c)

- Can reduce sensitivity using Paschos-Wolfenstein ratio

$$R^- = \frac{\sigma_{\nu N}^{NC} - \sigma_{\bar{\nu} N}^{NC}}{\sigma_{\nu N}^{CC} - \sigma_{\bar{\nu} N}^{CC}} \sim g_L^2 - g_R^2 \sim \frac{1}{2} - \sin^2\theta_W$$

Weak-Electromagnetic Interference

- **Low energy: Z exchange much smaller than Coulomb, but observe $V - A$ (parity-violating) and $A - A$ (parity conserving) effects**

- **High energy: γ and Z may be comparable (propagator effects)**

- **Observables**
  - **Polarization (charge) asymmetries in $eD \to eX$ (SLAC), $\mu C \to \mu X$ (CERN); e^-e^- Møller (SLAC); low energy elastic or quasi-elastic (Mainz, Bates, CEBAF)**
  - **Atomic parity violation in Cs (Boulder, Paris) and other atoms**
  - **Cross sections and FB asymmetries in $e^+e^- \to \ell\bar{\ell}, q\bar{q}, b\bar{b}$ (SPEAR, PEP, DORIS, TRISTAN, LEP II)**
  - **FB asymmetries in $\bar{p}p \to e^+e^-$ (CDF, D0)**

- **Parity-violating e-hadron**

$$L^{eq} = \frac{G_F}{\sqrt{2}} \sum_i \left[C_{1i}\ \bar{e}\ \gamma_\mu\ \gamma^5\ e \quad \bar{q}_i\ \gamma^\mu\ q_i + C_{2i}\ \bar{e}\ \gamma_\mu\ e \quad \bar{q}_i\ \gamma^\mu\ \gamma^5\ q_i \right]$$

- **Standard model**

$$C_{1u} \sim -\frac{1}{2} + \frac{4}{3} \sin^2 \theta_W \qquad C_{2u} \sim -\frac{1}{2} + 2\ \sin^2 \theta_W$$

$$C_{1d} \sim \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \qquad C_{2d} \sim \frac{1}{2} - 2\ \sin^2 \theta_W$$

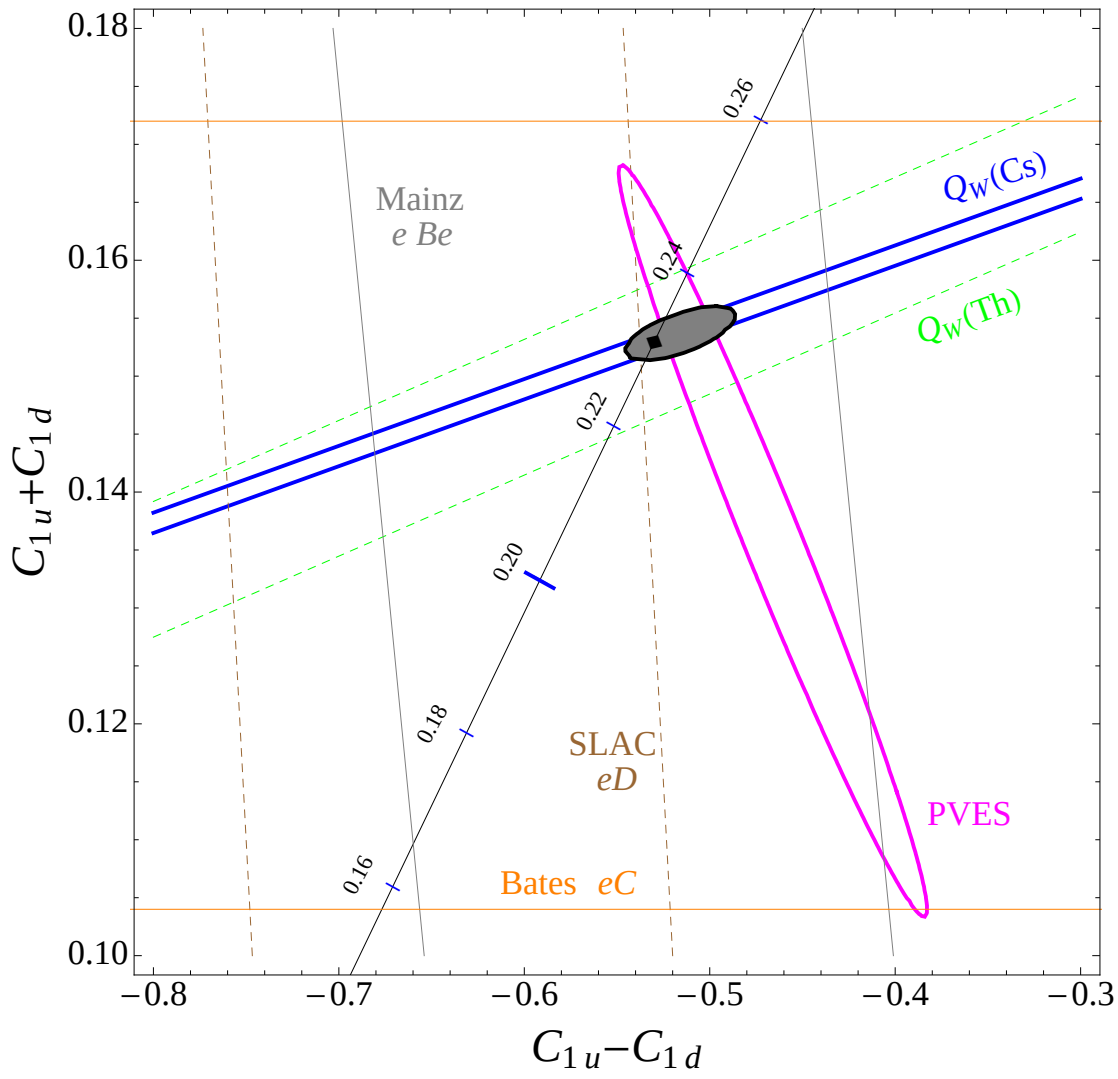
- **Atomic parity violation**
  - **Axial e^-, vector nucleon currents lead to potential**

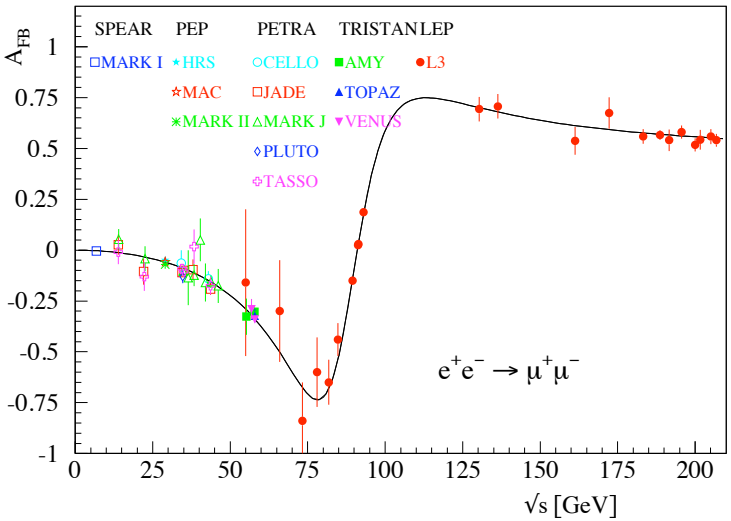
$$V(\vec{r}_e) \sim \frac{G_F}{4\sqrt{2}} Q_W \delta^3(\vec{r}_e) \frac{\vec{\sigma}_e \cdot \vec{v}_e}{c} + \text{ HC}$$

  - **Weak charge**

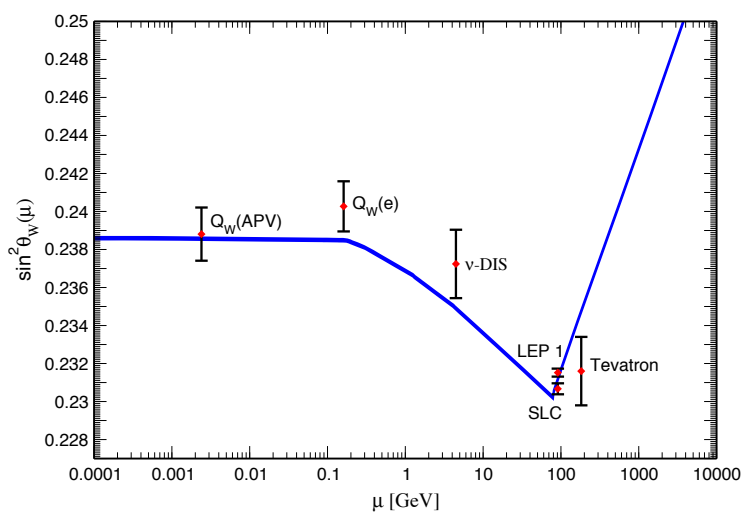
$$\begin{aligned} Q_W &= -2\left[C_{1u}\,(2Z+N) + C_{1d}(Z+2N)\right] \\ &\approx Z(1-4\sin^2\theta_W) - N \end{aligned}$$

  - **Measure in $6S-7S$ transition ($S-P$ wave mixing)**
  - **Cs is very simple atom; radiative corrections now under control**


0.18
0.16
0.14
0.12
0.10
−0.8
−0.7
−0.6
−0.5
−0.4
−0.3
$C_{1u}+C_{1d}$
$C_{1u}-C_{1d}$
Mainz
e Be
SLAC
eD
Bates eC
PVES
Q_W(Cs)
Q_W(Th)
0.16
0.18
0.20
0.22
0.24
0.26



(J. Mnich, Phys. Rep. 271, 181)



(Running $\hat{s}_Z^2$ in $\overline{MS}$ scheme)

- $e^+e^- \to \ell^+\ell^-$, hadrons below/above Z-pole
- SLAC E158 Polarized Møller Asymmetry
  - e^-e^- asymmetry, $P \sim 90\%$
  - $\sin^2\theta_W^{eff}(Q) = 0.2397(13)$ ($Q^2 = 0.026$ GeV^2)
- Future: Q_{WEAK} (JLAB): polarized ep, $\Delta s^2 \sim 0.0006$

Quantity	Value	SM	Correlation		
$\epsilon_L(u)$	0.338 ± 0.016	$0.3461(1)$			
$\epsilon_L(d)$	-0.434 ± 0.012	$-0.4292(1)$		non-	
$\epsilon_R(u)$	$-0.174\ ^{+0.013}_{-0.004}$	$-0.1549(1)$		Gaussian	
$\epsilon_R(d)$	$-0.023\ ^{+0.071}_{-0.047}$	0.0775			
g_L^2	0.3025 ± 0.0014	$0.3040(2)$	-0.18	-0.21	-0.02
g_R^2	0.0309 ± 0.0010	0.0300		-0.03	-0.07
θ_L	2.48 ± 0.036	$2.4630(1)$			0.24
θ_R	$4.58\ ^{+0.41}_{-0.28}$	5.1765			
$g_V^{\nu e}$	-0.040 ± 0.015	$-0.0398(3)$			-0.05
$g_A^{\nu e}$	-0.507 ± 0.014	$-0.5064(1)$			
$C_{1u} + C_{1d}$	0.1537 ± 0.0011	$0.1528(1)$	0.64	-0.18	-0.01
$C_{1u} - C_{1d}$	-0.516 ± 0.014	$-0.5300(3)$		-0.27	-0.02
$C_{2u} + C_{2d}$	-0.21 ± 0.57	-0.0089			-0.30
$C_{2u} - C_{2d}$	-0.077 ± 0.044	$-0.0625(5)$			
$Q_W(e) = -2\,C_{2e}$	-0.0403 ± 0.0053	$-0.0473(5)$			

Input Parameters for Weak Neutral Current and Z-Pole

- **Basic inputs**
  - $SU(2)$ and $U(1)$ gauge couplings g and g'
  - $\nu = \sqrt{2}\langle 0|\phi^0|0\rangle$ (vacuum of theory)
  - Higgs mass M_H (value unknown) (enters radiative corrections)
  - Heavy fermion masses, $m_t,\ m_b, \cdots$ (phase space; radiative corrections)
  - strong coupling α_s (enters radiative corrections)

- **Trade $g,\ g', \nu$ for precisely known quantities**
  - $G_F = \frac{1}{\sqrt{2}\nu^2}$ from τ_μ ($G_F \sim 1.166364(5) \times 10^{-5}$ GeV^{-2})
  - $\alpha = 1/137.035999084(51)$ (but must extrapolate to M_Z)
  - M_Z (or $\sin^2\theta_W$)

Definitions of $\sin^2\theta_W$

- **Several equivalent expressions for $\sin^2\theta_W$ at tree-level**

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \quad \Rightarrow \quad \text{on-shell}$$

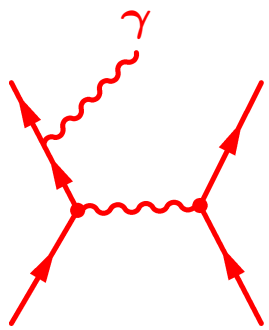
$$\sin^2\theta_W \cos^2\theta_W = \frac{\pi\alpha}{\sqrt{2}G_F M_Z^2} \quad \Rightarrow \quad Z-\text{mass}$$

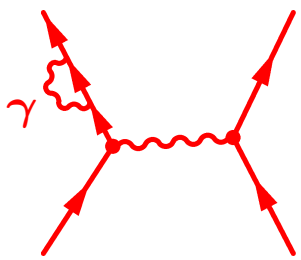
$$\sin^2\theta_W = \frac{g'^2}{g^2+g'^2} \quad \Rightarrow \quad \overline{MS}$$

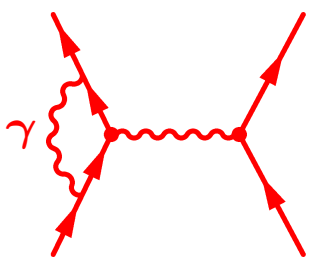
$$g_V^{Ze^+e^-} = -\frac{1}{2} + 2\sin^2\theta_W \quad \Rightarrow \quad \text{effective}$$

- **Each can be basis of *definition* of renormalized $\sin^2\theta_W$** (others related by calculable, $m_t - M_H$ dependent, corrections of $O(\alpha)$)

Radiative Corrections

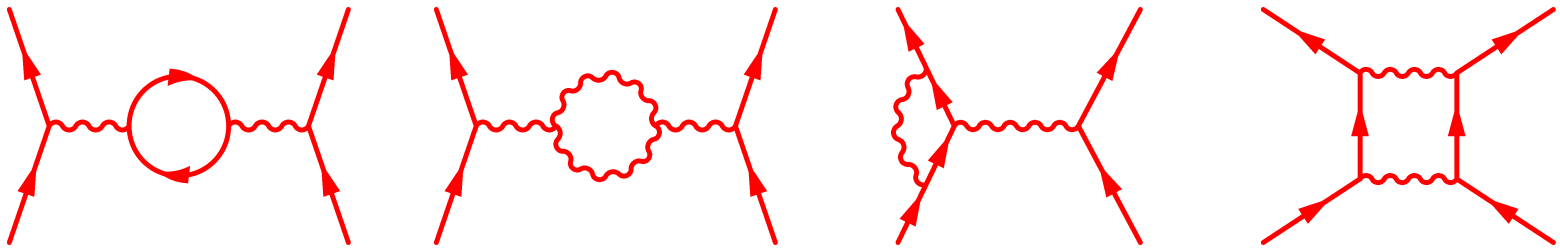






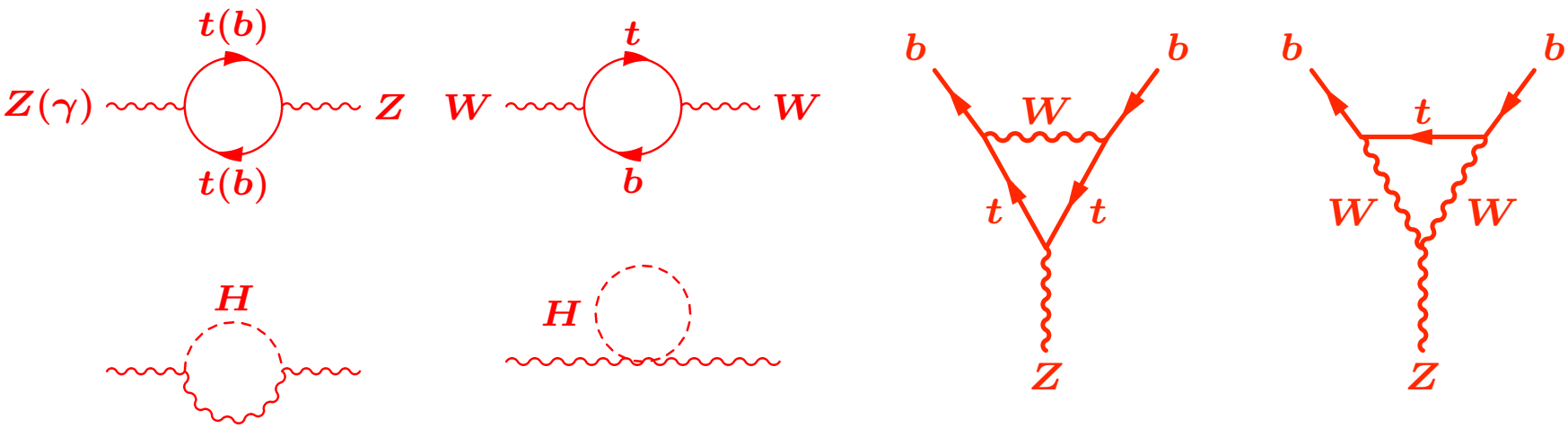
- **QED corrections to W or Z exchange**
  - **No vacuum polarization or box diagrams**
  - **Finite and gauge invariant**
  - **Depend on kinematic variables and cuts $\rightarrow$ calculate for each experiment**

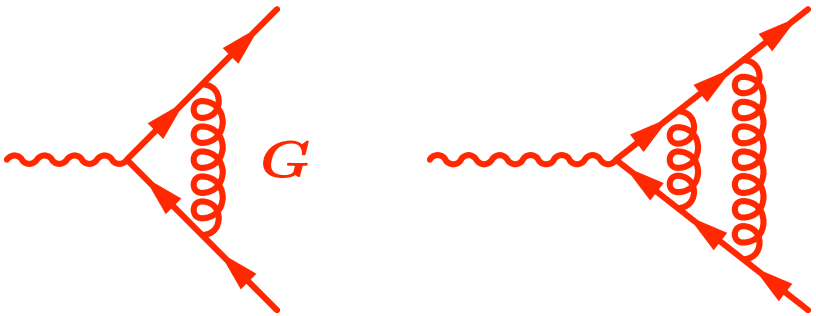
- **Electroweak at multiloop level** (include W, Z, γ)



self-energy vertex box

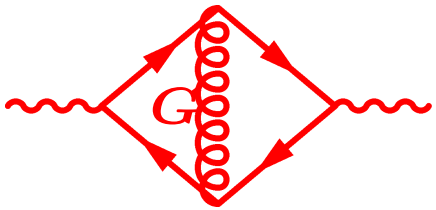
- **(quadratic) m_t and (logarithmic) M_H dependence from WW, ZZ, $Z\gamma$ self-energies** ($SU(2)$-breaking); **m_t from $Zb\bar{b}$ vertex**





- α_s from QCD vertices and mixed QCD-EW

- Mixed QCD-EW (e.g., self-energies and vertices, fermion masses)
  - Awkward in on-shell



mixed

The W and Z Masses and Decays

- **On-shell scheme,** $s_W^2 \equiv 1 - M_W^2/M_Z^2$

$$M_W = \frac{A_0}{s_W(1-\Delta r)^{1/2}} \qquad\qquad M_Z = \frac{M_W}{c_W}$$

$c_W^2 = 1 - s_W^2$, $A_0 = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.28061(8)$ **GeV**
$\Delta r \to$ **rad. corrections relating** α, $\alpha(M_Z)$, G_F, M_W, **and** M_Z

$$\Delta r \sim \underbrace{1 - \frac{\alpha}{\hat{\alpha}(M_Z)}}_{0.06655(11)} - \underbrace{\frac{\rho_t}{\tan^2\theta_W}}_{\text{artificially large}} + \text{small}$$

$$\rho_t \equiv \frac{3}{8}\frac{G_F m_t^2}{\sqrt{2}\pi^2} = 0.00939\left(\frac{m_t}{173.1\ \text{GeV}}\right)^2$$

- **Modified minimal subtraction ($\overline{MS}$) scheme**

$$M_W = \frac{A_0}{\hat{s}_Z(1-\Delta\hat{r}_W)^{1/2}} \qquad M_Z = \frac{M_W}{\hat{\rho}^{1/2}\hat{c}_Z}$$

$$\Delta\hat{r}_W \sim \underbrace{1 - \frac{\alpha}{\hat{\alpha}(M_Z)}}_{0.06655(11)} + \text{small}$$

$$\hat{\rho} \sim 1 + \underbrace{\frac{3}{8}\frac{G_F m_t^2}{\sqrt{2}\pi^2}}_{\rho_t \sim 0.00939} + \text{small}$$

- **The W decay width**

$$\Gamma(W^+ \to e^+\nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 226.31 \pm 0.07 \text{ MeV}$$

$$\Gamma(W^+ \to u_i \bar{d}_j) = \frac{C G_F M_W^3}{6\sqrt{2}\pi} |V_{ij}|^2 \approx (706.18 \pm 0.22)\, |V_{ij}|^2 \text{ MeV}$$

$$C = \begin{cases} 1, & \text{leptons} \\ \underbrace{3}_{\text{color}} \left(1 + \frac{\alpha_s(M_W)}{\pi} + 1.409\frac{\alpha_s^2}{\pi^2} - 12.77\frac{\alpha_s^3}{\pi^3} - 80.0\frac{\alpha_s^4}{\pi^4}\right), & \text{quarks} \end{cases}$$

  - Also, QED, mass; $g^2 M_W/4\sqrt{2} \to G_F M_W^3$ absorbs running α
  - $\Gamma_W \sim 2.0910 \pm 0.0007$ GeV (SM)
  - Experiment (LEP,CDF, D0): $\Gamma_W = 2.085 \pm 0.042$ GeV