The Standard Model

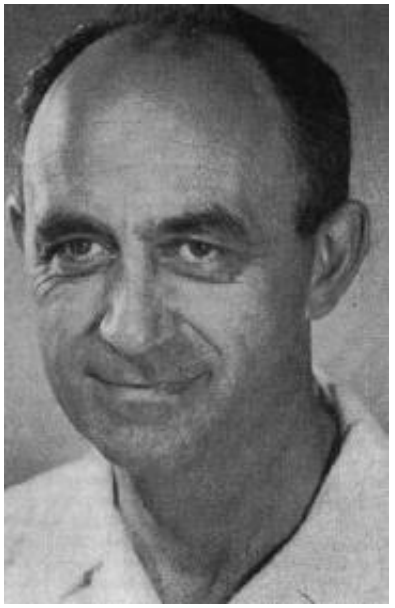
- Origins of the Electroweak Theory
- Gauge Theories
- The Standard Model Lagrangian
- Spontaneous Symmetry Breaking
- The Gauge Interactions
- Problems With the Standard Model

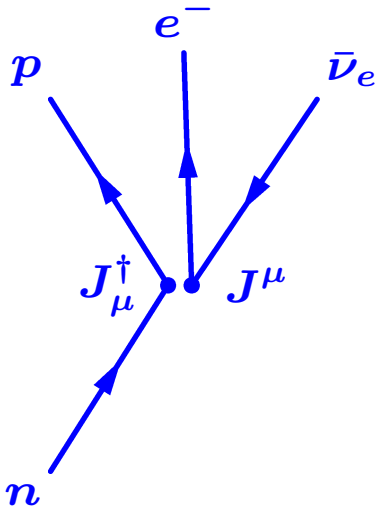
References: *2008 TASI lectures*: arXiv:0901.0241 [hep-ph] and *The Standard Model and Beyond*, CRC Press

The Weak Interactions

- **Radioactivity** (Becquerel, 1896)
- **β decay appeared to violate energy** (Chadwick, 1914)
- **Neutrino hypothesis** (Pauli, 1930)
  - ν_e (Reines, Cowan; 1956)
  - ν_μ (Lederman, Schwartz, Steinberger; 1962)
  - ν_τ (DONUT, 2000) (τ, 1975)

- **Fermi theory of β decay** $(n \to p e^- \bar{\nu})$ **(1934)**
  - **Loosely like QED, but zero range** **(non-renormalizable) and non-diagonal** **(charged current)**





$$H \sim G_F J^\dagger_\mu J^\mu$$

$$J^\dagger_\mu \sim \bar{p}\gamma_\mu n + \bar{\nu}_e \gamma_\mu e^- \qquad [n \to p,\ e^- \to \nu_e]$$

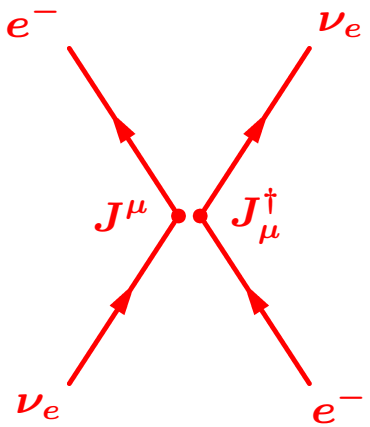
$$J_\mu \sim \bar{n}\gamma_\mu p + \bar{e}\gamma_\mu \nu_e \qquad [p \to n,\ \nu_e \to e^-\ (\times \to e^- \bar{\nu}_e)]$$

$$G_F \simeq 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2} \qquad [\text{Fermi constant}]$$

- **Fermi theory modified to include**
  - parity violation $(V-A)$ (Lee, Yang; Wu; Feynman-Gell-Mann)
  - $\mu,\ \tau$ decay
  - strangeness (Cabibbo)
  - quark model
  - heavy quarks and CP violation (CKM)
  - ν mass and mixing

- **Fermi theory correctly describes (at tree level)**
  - Nuclear/neutron β decay/inverse $(n \to p e^- \bar{\nu}_e; \quad e^- p \to \nu_e n)$
  - $\mu,\ \tau$ decays $(\mu^- \to e^- \bar{\nu}_e \nu_\mu; \quad \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau,\ \nu_\tau \pi^-,\ \cdots)$
  - $\pi,\ K$ decays $(\pi^+ \to \mu^+ \nu_\mu,\ \pi^0 e^+ \nu_e; \quad K^+ \to \mu^+ \nu_\mu,\ \pi^0 e^+ \nu_e,\ \pi^+ \pi^0)$
  - hyperon decays $(\Lambda \to p \pi^-; \quad \Sigma^- \to n \pi^-; \quad \Sigma^+ \to \Lambda e^+ \nu_e)$
  - heavy quark decays $(c \to s e^+ \nu_e; \quad b \to c \mu^- \bar{\nu}_\mu,\ c \pi^-)$
  - ν scattering $(\nu_\mu e^- \to \mu^- \nu_e; \quad \underbrace{\nu_\mu n \to \mu^- p}_{\text{"elastic"}}; \quad \underbrace{\nu_\mu N \to \mu^- X}_{\text{deep-inelastic}})$

- **Fermi theory violates unitarity at high energy** (non-renormalizable)



– $\sigma(\nu_e e^- \to e^- \nu_e) \to \frac{G_F^2 s}{\pi}$
$(s \equiv E_{CM}^2)$

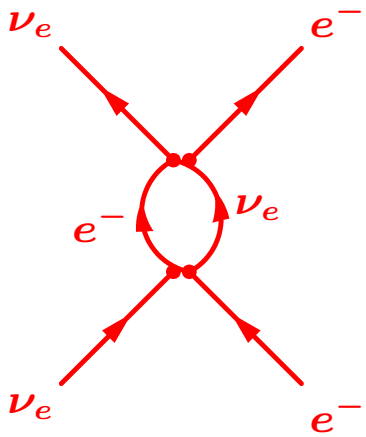
– pure S-wave unitarity: $\sigma < \frac{16\pi}{s}$

– fails for $\frac{E_{CM}}{2} \geq \sqrt{\frac{\pi}{G_F}} \sim 500$ GeV

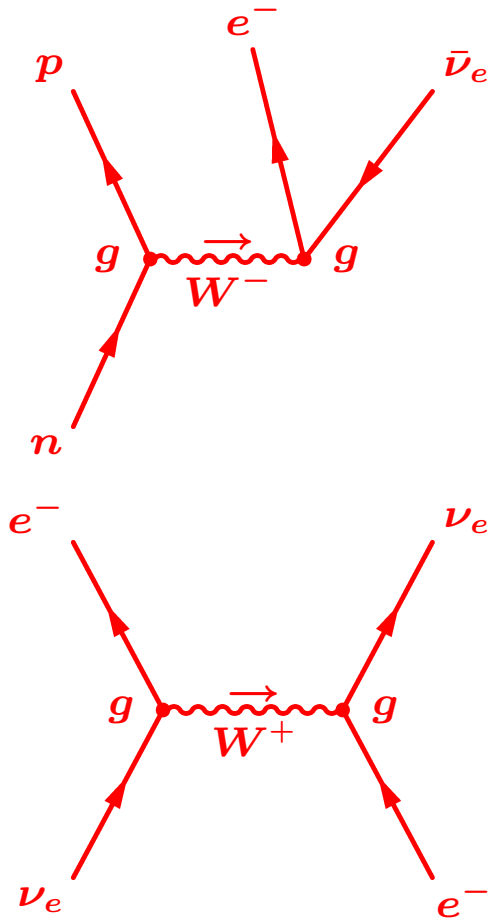
– Born not unitary; often restored by H.O.T.

– Fermi theory: divergent integrals

$$\int d^4k \left(\frac{\not{k}}{k^2}\right)\left(\frac{\not{k}}{k^2}\right)$$



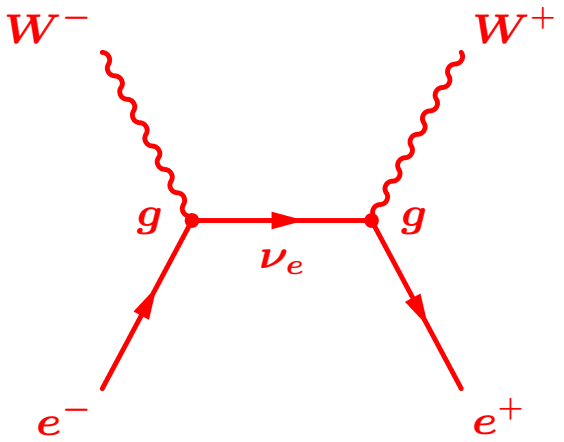
- **Intermediate vector boson theory** (Yukawa, 1935; Schwinger, 1957)



$$\frac{G_F}{\sqrt{2}} \sim \frac{g^2}{8M_W^2} \text{ for } M_W \gg Q$$

– no longer pure S-wave $\Rightarrow$

– $\nu_e e^- \to \nu_e e^-$ better behaved



– *but,* $e^+e^- \to W^+W^-$ violates unitarity for $\sqrt{s} \gtrsim 500$ GeV

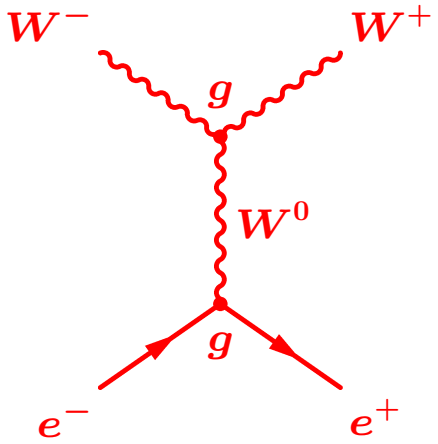
– $\epsilon_\mu \sim k_\mu/M_W$ for longitudinal polarization (non-renormalizable)

– introduce W^0 to cancel

– fixes $W^0W^+W^-$ and $e^+e^-W^0$ vertices

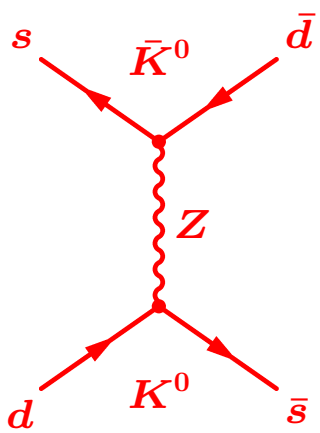
– requires $\left[J, J^\dagger\right] \sim J^0$ (like $SU(2)$)

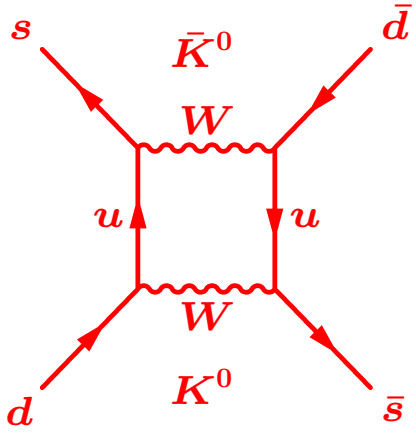
– not realistic



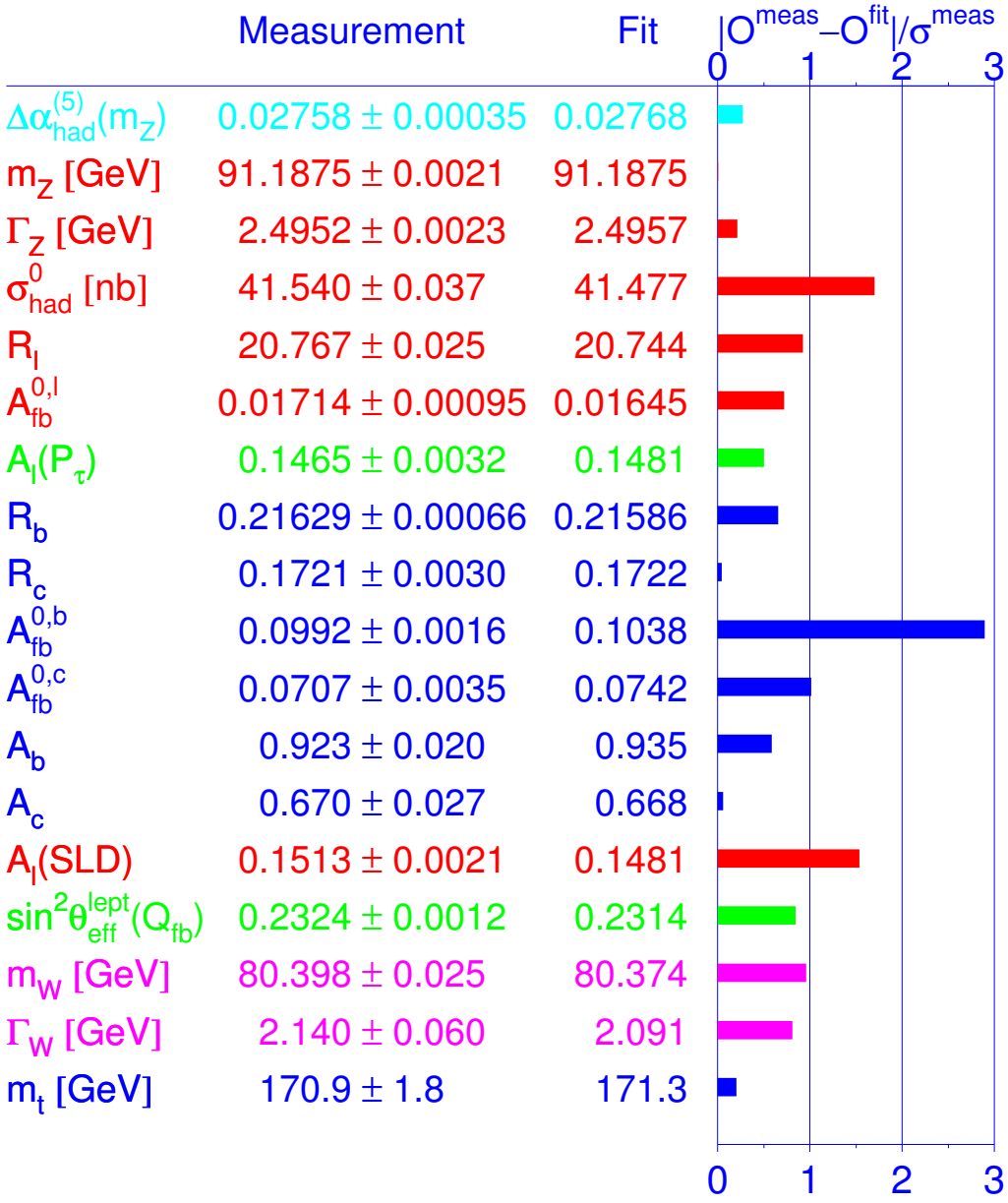
- **Glashow model (1961)** ($W^{\pm}, Z, \gamma$, but no mechanism for $M_{W,Z}$)
- **Weinberg-Salam (1967): Higgs mechanism** $\rightarrow M_{W,Z}$
- **Renormalizable (1971)** ('t Hooft, $\cdots$)
- **Flavor changing neutral currents (FCNC)**
  - very large $K^0 \leftrightarrow \bar{K}^0$ mixing
  - GIM mechanism (c quark) (1970) ($m_c \sim 1.5$ GeV (1974))
  - c discovered (1974)

$$J/\psi = c\bar{c} \text{ (BNL, SLAC)}$$
$$\nu_\mu d(s) \rightarrow \mu^- c,$$
$$c \rightarrow s\mu^+\nu_\mu \text{ (dimuons)}$$





- **Weak neutral current (1973)**
- **QCD (1970's)**
- **W, Z (1983)**
- **Precision tests (1989-)**
- **Precision K, B, D physics ($\sim$ 2000-)**
- **CKM unitarity ($\sim$ 1995-)**
- **t quark (1995)**
- **ν mass (1998-)**



Gauge Theories

Standard Model is remarkably successful gauge theory of the microscopic interactions

- **Gauge symmetry $\Rightarrow$ (apparently) massless spin-1 (vector, gauge) bosons**

- **Interactions $\Leftrightarrow$ group, representations, gauge coupling**
- **Like QED ($U(1)$), but gauge self interactions for non-abelian**
- **Application to strong (short range) $\Rightarrow$ confinement**
- **Application to weak (short range) $\Rightarrow$ spontaneous symmetry breaking (Higgs or dynamical)**
- **Unique renormalizable field theory for spin-1**

QED

- **Free electron equation,**

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m\right)\psi = 0,$$

is invariant under $U(1)$ (phase) transformations,

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m\right)\psi' = 0, \text{ where } \psi' \equiv e^{-i\beta}\psi$$

- ***Not*** **invariant under local (gauge) transf.,**

$$\psi \to \psi' \equiv e^{-i\beta(x)}\psi, \quad x \equiv (\vec{x}, t)$$

- **Introduce vector field $A^\mu \equiv (\vec{A}, \phi)$:**

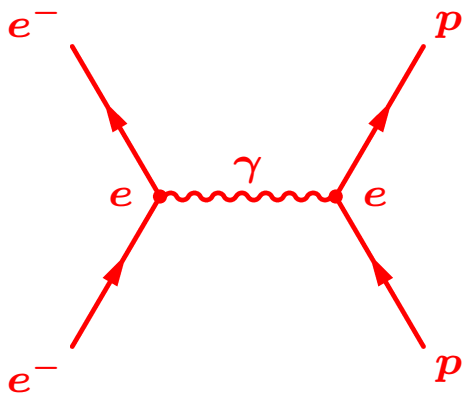
$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} + e\gamma^\mu A_\mu - m \right) \psi = 0,$$

($e > 0$ is gauge coupling) is invariant under

$$\psi \to e^{-i\beta(x)}\psi, \quad A_\mu \to A_\mu - \frac{1}{e}\frac{\partial \beta}{\partial x^\mu}$$

- **Quantization of $A_\mu \Rightarrow$ massless gauge boson**

- **Gauge invariance $\Rightarrow \gamma$, long range force, prescribed (up to e) amplitude for emission/absorption**



Non-Abelian

- n non-interacting fermions of same mass m:

$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu} - m\right)\psi_a = 0, \quad a = 1 \cdots n,$$

invariant under (global) $SU(n)$ group,

$$\begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} \rightarrow \exp(i\sum_{i=1}^{N} \beta^i L^i) \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix}.$$

L^i are $n \times n$ generator matrices ($N = n^2 - 1$); β^i are real parameters

$$[L^i, L^j] = i c_{ijk} L^k$$

(c_{ijk} are structure constants)

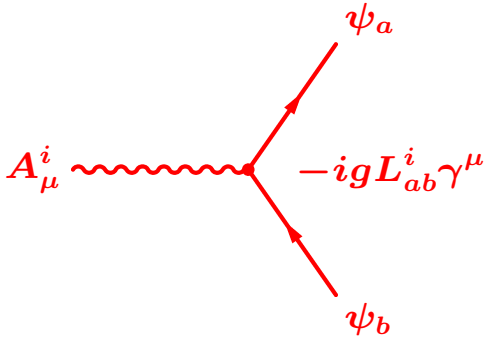
- **Gauge (local) transformation:** $\beta^i \to \beta^i(x) \Rightarrow$

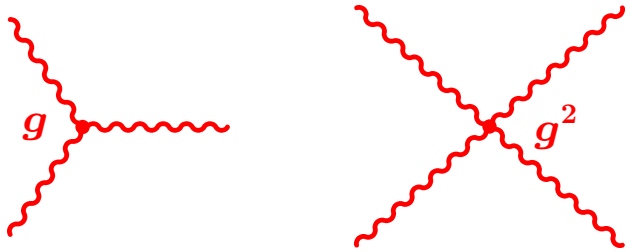
$$\left(i\gamma^\mu \frac{\partial}{\partial x^\mu}\delta_{ab} - g\sum_{i=1}^{N} \gamma^\mu A^i_\mu L^i_{ab} - m\delta_{ab} \right) \psi_b = 0$$

- **Invariant under**

$$\begin{aligned} \Phi &\equiv \begin{pmatrix} \psi_1 \\ \vdots \\ \psi_n \end{pmatrix} \to \Phi' \equiv U\Phi \\ \vec{A}_\mu \cdot \vec{L} &\to \vec{A}'_\mu \cdot \vec{L} \equiv U\vec{A}_\mu \cdot \vec{L}U^{-1} + \frac{i}{g}(\partial_\mu U)U^{-1} \\ U &\equiv e^{i\vec{\beta}\cdot\vec{L}} \end{aligned} \tag{1}$$

- **Gauge invariance implies:**
  - N (apparently) **massless gauge bosons** A^i_μ
  - **Specified interactions** (up to gauge coupling g, group, representations), **including self interactions**





- **Generalize to other groups, representations, chiral** $(L \neq R)$
  - **Chiral Projections:** $\psi_{L(R)} \equiv \frac{1}{2}(1 \mp \gamma_5)\psi$
    (Chirality = helicity up to $O(m/E)$)

The Standard Model

- **Gauge group $SU(3) \times SU(2) \times U(1)$; gauge couplings g_s, g, g'**

$$\begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$$

$$u_R \quad u_R \quad u_R \quad \nu_{eR}(?)$$
$$d_R \quad d_R \quad d_R \quad e^-_R$$

(L = left-handed, R = right-handed)

- $SU(3)$: $u \leftrightarrow u \leftrightarrow u$, $d \leftrightarrow d \leftrightarrow d$ **(8 gluons)**
- $SU(2)$: $u_L \leftrightarrow d_L$, $\nu_{eL} \leftrightarrow e^-_L$ ($W^\pm$); **phases** (W^0)
- $U(1)$: **phases** (B)
- **Heavy families** (c, s, ν_μ, μ^-), (t, b, ν_τ, τ^-)

Quantum Chromodynamics (QCD)

$$\mathcal{L}_{SU(3)} = -\frac{1}{4} F^i_{\mu\nu} F^{i\mu\nu} + \sum_r \bar{q}^\alpha_r i \not{D}^\beta_\alpha q_{r\beta}$$

F^2 term leads to three and four-point gluon self-interactions.

$$F^i_{\mu\nu} = \partial_\mu G^i_\nu - \partial_\nu G^i_\mu - g_s f_{ijk}\, G^j_\mu\, G^k_\nu$$

is field strength tensor for the gluon fields $G^i_\mu,\ i = 1, \cdots, 8$.
g_s = QCD gauge coupling constant. No gluon masses.

Structure constants f_{ijk} $(i, j, k = 1, \cdots, 8)$, defined by

$$[\lambda^i, \lambda^j] = 2 i f_{ijk} \lambda^k$$

where λ^i are the Gell-Mann matrices.

$$\lambda^i = \begin{pmatrix} \tau^i & 0 \\ 0 & 0 \end{pmatrix}, \quad i = 1,2,3$$

$$\lambda^4 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \qquad \lambda^5 = \begin{pmatrix} 0 & 0 & -i \\ 0 & 0 & 0 \\ i & 0 & 0 \end{pmatrix}$$

$$\lambda^6 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \qquad \lambda^7 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\lambda^8 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$$

The $SU(3)$ (Gell-Mann) matrices.

Quark interactions given by $\bar{q}_r^{\alpha} i \not{D}_{\alpha}^{\beta} q_{r\beta}$

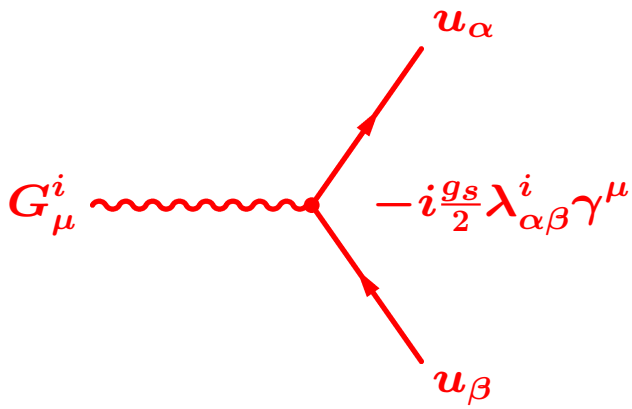
$q_r = r^{\rm th}$ quark flavor; $\alpha, \beta = 1, 2, 3$ are color indices

Gauge covariant derivative

$$D_{\alpha}^{\mu\beta} = (D^{\mu})_{\alpha\beta} = \partial^{\mu}\delta_{\alpha\beta} + i g_s \, G^{i\mu} \, L^i_{\alpha\beta},$$

for triplet representation matrices $L^i = \lambda^i/2$.

Quark color interactions:



Diagonal in flavor

Off diagonal in color

Purely vector (parity conserving)

Bare quark mass allowed by QCD, but forbidden by chiral symmetry of $\mathcal{L}_{SU(2)\times U(1)}$ (generated by spontaneous symmetry breaking)

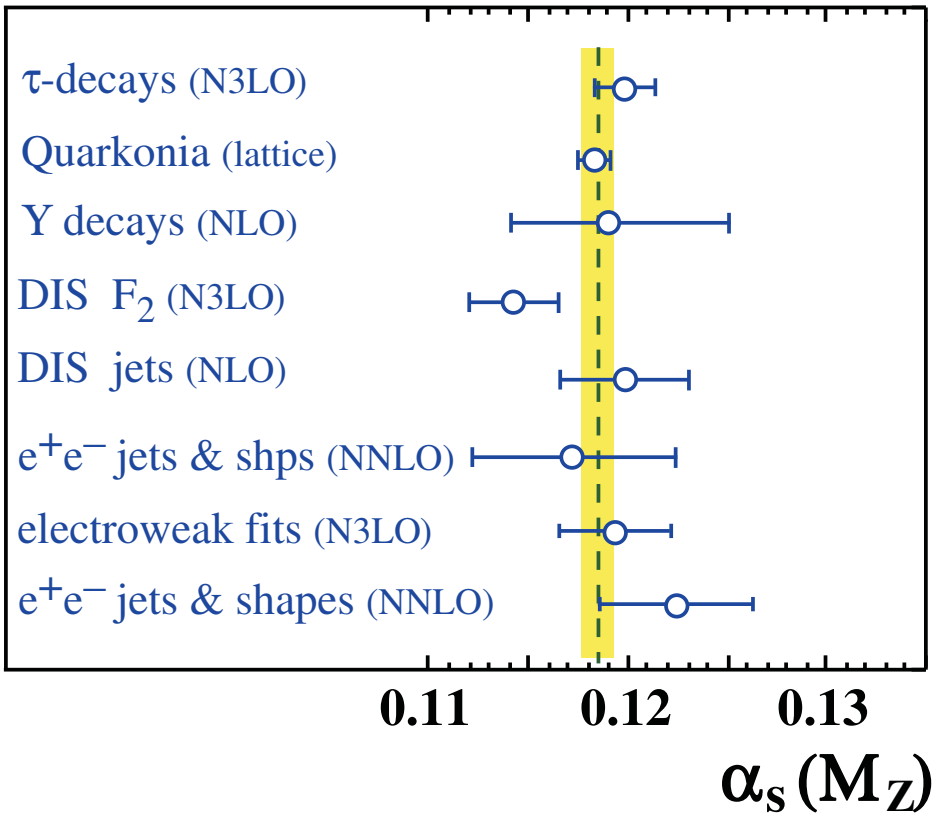
Additional ghost and gauge-fixing terms

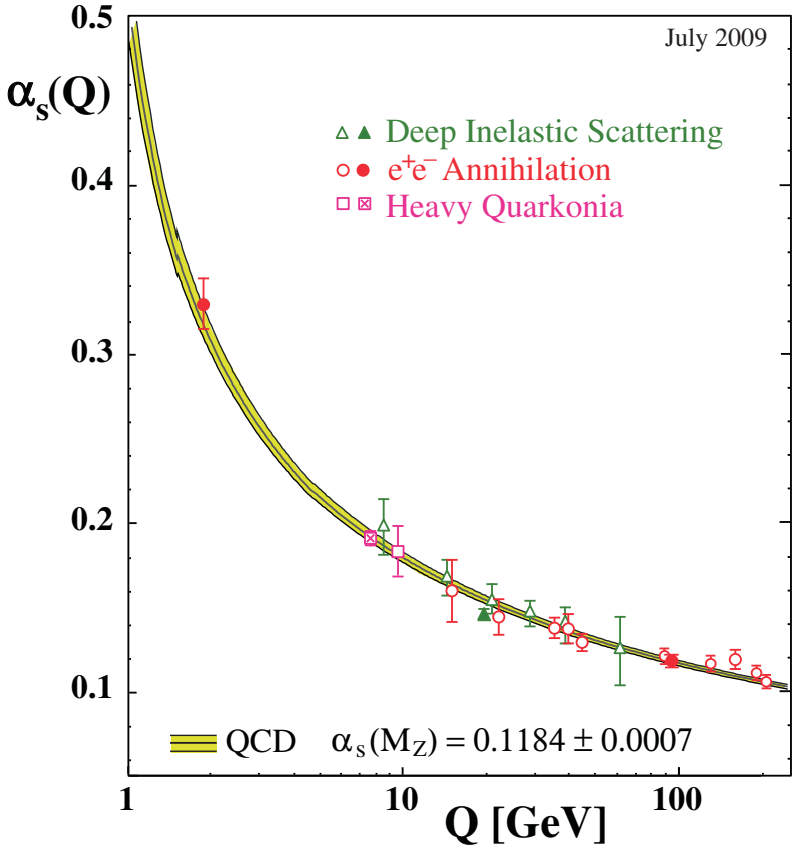
Can add (unwanted) CP-violating term

$$\mathcal{L}_\theta = \frac{\theta g_s^2}{32\pi^2} F^i_{\mu\nu}\tilde{F}^{i\mu\nu}, \quad \tilde{F}^{i\mu\nu} \equiv \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F^i_{\alpha\beta}$$

QCD now very well established

- **Short distance behavior (asymptotic freedom)**
- **Confinement, light hadron spectrum (lattice)**
  - $g_s = O(1)$ **(**$\alpha_s(M_Z) = g_s^2/4\pi \sim 0.12$**)**
  - **Strength + gluon self-interactions⇒ confinement**
  - **Yukawa model ⇒ dipole-dipole**
- **Approximate global** $SU(3)_L \times SU(3)_R$ **symmetry and breaking (**π, K, η **are pseudo-goldstone bosons)**
- **Unique field theory of strong interactions**


τ-decays (N3LO)
Quarkonia (lattice)
Υ decays (NLO)
DIS F2 (N3LO)
DIS jets (NLO)
e+e− jets & shps (NNLO)
electroweak fits (N3LO)
e+e− jets & shapes (NNLO)
0.11
0.12
0.13
αs (MZ)


July 2009
αs(Q)
Deep Inelastic Scattering
e+e− Annihilation
Heavy Quarkonia
QCD αs(MZ) = 0.1184 ± 0.0007
0.5
0.4
0.3
0.2
0.1
1
10
100
Q [GeV]

The Electroweak Sector

$$\mathcal{L}_{SU(2)\times U(1)} = \mathcal{L}_{\rm gauge} + \mathcal{L}_\phi + \mathcal{L}_f + \mathcal{L}_{\rm Yukawa}$$

Gauge part

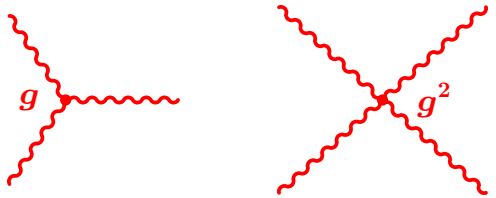
$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}F^i_{\mu\nu}F^{\mu\nu i} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

Field strength tensors

$$\begin{aligned} B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \\ F^i_{\mu\nu} &= \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g\epsilon_{ijk}W^j_\mu W^k_\nu, \quad i = 1\cdots 3 \end{aligned}$$

$g(g')$ **is** $SU(2)$ $(U(1))$ **gauge coupling;** ϵ_{ijk} **is totally antisymmetric symbol**

Three and four-point self-interactions for the W_i



B **and** W_3 **will mix to form** $\gamma,\ Z$

$U(1)$: $\Phi_j \to \exp(ig' y_j \beta)\Phi_j$, $y_j = q_j - t_j^3$ = weak hypercharge

Scalar part

$$\mathcal{L}_\phi = (D^\mu \phi)^\dagger D_\mu \phi - V(\phi)$$

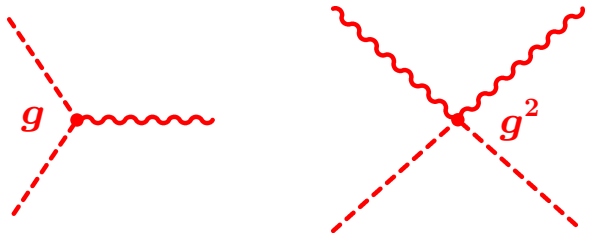
where $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ is the (complex) Higgs doublet with $y_\phi = 1/2$.

Gauge covariant derivative:

$$D_\mu \phi = \left(\partial_\mu + ig \frac{\tau^i}{2} W_\mu^i + \frac{ig'}{2} B_\mu \right) \phi$$

where τ^i are the Pauli matrices

Three and four-point interactions between gauge and scalar fields



Higgs potential

$$V(\phi) = +\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2$$

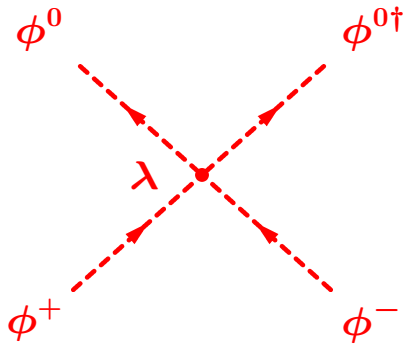
$$\phi^\dagger \phi = \underbrace{\phi^-}_{\phi^{+\dagger}} \phi^+ + \phi^{0\dagger} \phi^0$$

Allowed by renormalizability and gauge invariance

Spontaneous symmetry breaking for $\mu^2 < 0$

Vacuum stability: $\lambda > 0$.

Quartic self-interactions



Fermion part

$$\mathcal{L}_F = \sum_{m=1}^{F} (\bar{q}^0_{mL} i \not{D} q^0_{mL} + \bar{l}^0_{mL} i \not{D} l^0_{mL}$$
$$+ \bar{u}^0_{mR} i \not{D} u^0_{mR} + \bar{d}^0_{mR} i \not{D} d^0_{mR} + \bar{e}^0_{mR} i \not{D} e^0_{mR} + \bar{\nu}^0_{mR} i \not{D} \nu^0_{mR})$$

L-doublets

$$q^0_{mL} = \begin{pmatrix} u^0_m \\ d^0_m \end{pmatrix}_L \qquad l^0_{mL} = \begin{pmatrix} \nu^0_m \\ e^{-0}_m \end{pmatrix}_L$$

R-singlets

$$u^0_{mR},\ d^0_{mR},\ e^{-0}_{mR},\ \nu^0_{mR}(?)$$

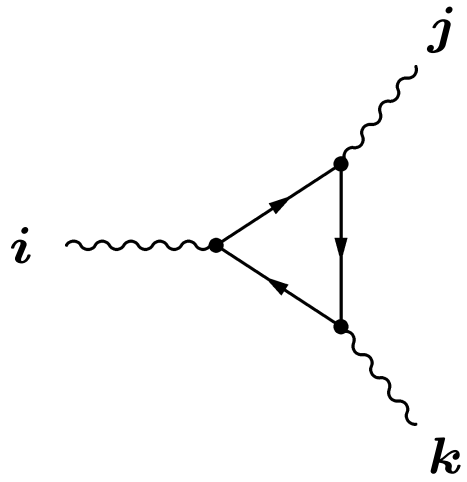
($F \geq 3$ families; $m = 1 \cdots F$ = family index;
^{0} = weak eigenstates (definite $SU(2)$ rep.), mixtures of mass eigenstates (flavors);
quark color indices $\alpha = r,\ g,\ b$ suppressed (e.g., $u^0_{m\alpha L}$).)

Can add gauge singlet ν^0_{mR} for Dirac neutrino mass term

Different (chiral) L and R representations lead to parity and charge conjugation violation (maximal for $SU(2)$)

Fermion mass terms forbidden by chiral symmetry

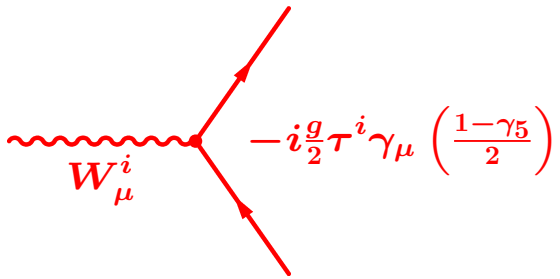
Triangle anomalies absent for chosen hypercharges and 3 colors
(includes quark-lepton cancellations)

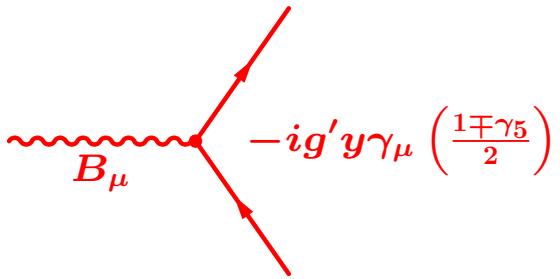


Gauge covariant derivatives

$$
\begin{aligned}
D_\mu q^0_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W^i_\mu + i\frac{g'}{6}B_\mu\right) q^0_{mL} \\
D_\mu l^0_{mL} &= \left(\partial_\mu + \frac{ig}{2}\tau^i W^i_\mu - i\frac{g'}{2}B_\mu\right) l^0_{mL} \\
D_\mu u^0_{mR} &= \left(\partial_\mu + i\frac{2}{3}g' B_\mu\right) u^0_{mR} \\
D_\mu d^0_{mR} &= \left(\partial_\mu - i\frac{g'}{3}B_\mu\right) d^0_{mR} \\
D_\mu e^0_{mR} &= \left(\partial_\mu - ig' B_\mu\right) e^0_{mR}
\end{aligned}
$$

Read off W and B couplings to fermions





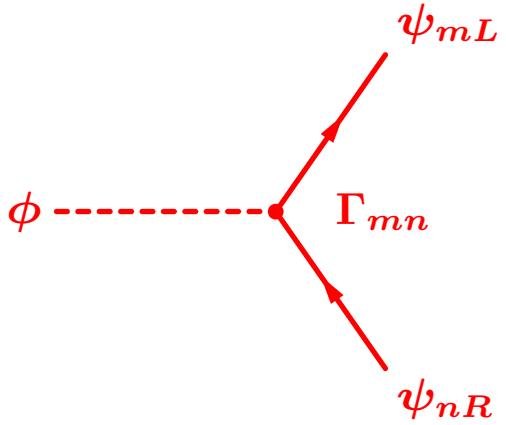
Yukawa couplings (couple L to R)

$$-\mathcal{L}_{\rm Yukawa} = \sum_{m,n=1}^{F} \Big[\Gamma^u_{mn}\bar{q}^0_{mL}\tilde{\phi}u^0_{mR} + \Gamma^d_{mn}\bar{q}^0_{mL}\phi d^0_{nR} + \Gamma^e_{mn}\bar{l}^0_{mn}\phi e^0_{nR}\ (+\Gamma^\nu_{mn}\bar{l}^0_{mL}\tilde{\phi}\nu^0_{mR})\Big] + h.c.$$

Γ_{mn} are completely arbitrary Yukawa matrices, which determine fermion masses and mixings

d, e terms require doublet $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ with $Y_\phi = 1/2$

u (and ν) terms require doublet $\Phi = \begin{pmatrix} \Phi^0 \\ \Phi^- \end{pmatrix}$ with $Y_\Phi = -1/2$



In $SU(2)$ the $\mathbf{2}$ and $\mathbf{2}^*$ are similar $\Rightarrow \tilde{\phi} \equiv i\tau^2\phi^\dagger = \begin{pmatrix} \phi^{0\dagger} \\ -\phi^- \end{pmatrix}$

transforms as a 2 with $Y_{\tilde{\phi}} = -\frac{1}{2} \Rightarrow$ only one doublet needed.

Does not generalize to $SU(3)$, most extra $U(1)'$, supersymmetry, $SO(10)$ etc $\Rightarrow$ need two doublets.

(Does generalize to $SU(2)_L \times SU(2)_R \times U(1)$ and $SU(5)$)

Spontaneous Symmetry Breaking (Higgs mechanism)

Gauge invariance implies massless gauge bosons and fermions

Weak interactions short ranged $\Rightarrow$ spontaneous symmetry breaking for mass; also for fermions

Color confinement for QCD $\Rightarrow$ gluons remain massless

Allow classical (ground state) expectation value for Higgs field

$$v = \langle 0|\phi|0\rangle = \text{constant}, \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

$\partial_\mu v \neq 0$ increases energy, but important for monopoles, strings, domain walls, phase transitions (e.g., EWPT, baryogenesis)

Minimize $V(v)$ to find v and quantize $\phi' = \phi - v$

$SU(2) \times U(1)$: introduce Hermitian basis

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\phi_1 + i\phi_2) \\ \frac{1}{\sqrt{2}}(\phi_3 + i\phi_4) \end{pmatrix}$$

where $\phi_i = \phi_i^\dagger$.

$$V(\phi) = \frac{1}{2}\mu^2 \left(\sum_{i=1}^{4} \phi_i^2\right) + \frac{1}{4}\lambda \left(\sum_{i=1}^{4} \phi_i^2\right)^2$$

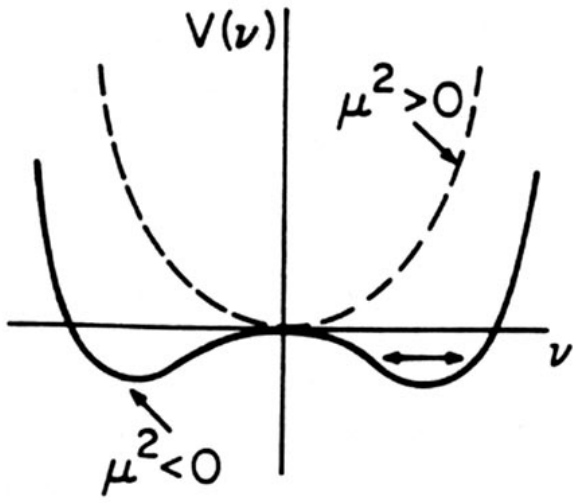
is $O(4)$ invariant.

w.l.o.g. choose $\langle 0|\phi_i|0\rangle = 0, \quad i = 1, 2, 4$ and $\langle 0|\phi_3|0\rangle = \nu$

$$V(\phi) \to V(v) = \frac{1}{2}\mu^2\nu^2 + \frac{1}{4}\lambda\nu^4$$

For $\mu^2 < 0$, minimum at

$$V'(\nu) = \nu(\mu^2 + \lambda\nu^2) = 0$$
$$\Rightarrow \quad \nu = \left(-\mu^2/\lambda\right)^{1/2}$$



SSB for $\mu^2 = 0$ also; must consider loop corrections

$\phi \rightarrow \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu \end{pmatrix} \equiv v \Rightarrow$ the generators L^1, L^2, and $L^3 - Y$ spontaneously broken, $L^1 v \neq 0$, etc $(L^i = \frac{\tau^i}{2},\ Y = \frac{1}{2}I)$

$$Qv = (L^3 + Y)v = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} v = 0 \Rightarrow U(1)_Q \text{ unbroken} \Rightarrow$$

$SU(2) \times U(1)_Y \rightarrow U(1)_Q$

Quantize around classical vacuum

- Kibble transformation: introduce new variables ξ^i for rolling modes

$$\phi = \frac{1}{\sqrt{2}} e^{i \sum \xi^i L^i} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

- $H = H^\dagger$ is the Higgs scalar
- No potential for $\xi^i \Rightarrow$ massless Goldstone bosons for global symmetry
- Disappear from spectrum for gauge theory ("eaten")
- Display particle content in unitary gauge

$$\phi \rightarrow \phi' = e^{-i \sum \xi^i L^i} \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

+ corresponding transformation on gauge fields

Rewrite Lagrangian in New Vacuum

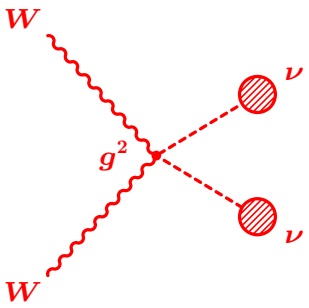
Physical Higgs scalar (oscillations around minimum): $M_H = \sqrt{2\lambda}\nu$

Higgs covariant kinetic energy terms:

$$(D_\mu\phi)^\dagger D^\mu\phi = \frac{1}{2}(0\ \nu)\left[\frac{g}{2}\tau^i W^i_\mu + \frac{g'}{2}B_\mu\right]^2\begin{pmatrix}0\\ \nu\end{pmatrix} + H \text{ terms}$$

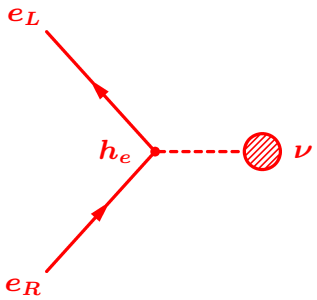
$$\rightarrow M_W^2 W^{+\mu}W^-_\mu + \frac{M_Z^2}{2}Z^\mu Z_\mu$$

$$+ \quad H \text{ kinetic energy and gauge interaction terms}$$



$$M_W = \frac{g\nu}{2}$$

$$m_e = \frac{h_e\nu}{\sqrt{2}}$$



Mass eigenstate bosons: W, Z, and A (photon)

$$\begin{aligned} W^{\pm} &= \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \\ Z &= -\sin\theta_W B + \cos\theta_W W^3 \\ A &= \cos\theta_W B + \sin\theta_W W^3 \end{aligned}$$

Weak angle: $\tan\theta_W \equiv g'/g$

Masses:

$$M_W = \frac{g\nu}{2}, \qquad M_Z = \sqrt{g^2+g'^2}\,\frac{\nu}{2} = \frac{M_W}{\cos\theta_W}, \qquad M_A = 0$$

(Goldstone scalars "eaten" $\rightarrow$ longitudinal components of $W^{\pm}$, Z)

Will show: Fermi constant $G_F/\sqrt{2} \sim g^2/8M_W^2$

($G_F = 1.166364(5) \times 10^{-5}$ GeV^{-2} from muon lifetime)

Electroweak scale:

$$\nu = 2M_W/g \simeq (\sqrt{2}G_F)^{-1/2} \simeq 246 \text{ GeV}$$

Will show: $g = e/\sin\theta_W$ ($\alpha = e^2/4\pi \sim 1/137.036$) $\Rightarrow$

$$M_W = M_Z \cos\theta_W = \frac{g\nu}{2} \sim \frac{(\pi\alpha/\sqrt{2}G_F)^{1/2}}{\sin\theta_W}$$

Weak neutral current: $\sin^2\theta_W \sim 0.23 \Rightarrow M_W \sim 78$ GeV, and $M_Z \sim 89$ GeV (increased by ~ 2 GeV by loop corrections)

Discovered at CERN: UA1 and UA2, 1983

The Higgs Scalar H

Gauge interactions: $ZZH, ZZH^2, W^+W^-H, W^+W^-H^2$

$$\phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$$

$$\begin{aligned} \mathcal{L}_\phi &= (D^\mu \phi)^\dagger D_\mu \phi - V(\phi) \\ &= \frac{1}{2}(\partial_\mu H)^2 + M_W^2 W^{\mu +} W_\mu^- \left(1 + \frac{H}{\nu}\right)^2 \\ &+ \frac{1}{2} M_Z^2 Z^\mu Z_\mu \left(1 + \frac{H}{\nu}\right)^2 - V(\phi) \end{aligned}$$

(quartic and induced cubic interactions)

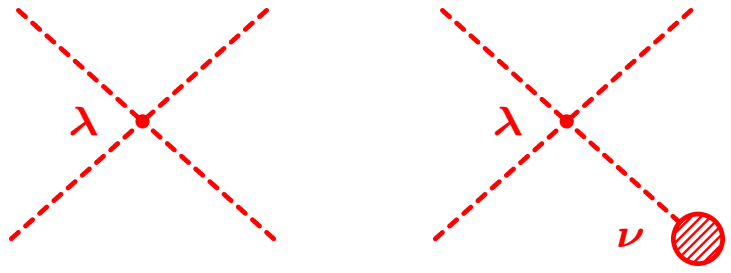
Higgs potential:

$$V(\phi) = +\mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2$$
$$\rightarrow -\frac{\mu^4}{4\lambda} - \mu^2 H^2 + \lambda\nu H^3 + \frac{\lambda}{4}H^4$$

Fourth term: Quartic self-interaction

Third: Induced cubic self-interaction

Second: (Tree level) H mass-squared, $M_H = \sqrt{-2\mu^2} = \sqrt{2\lambda}\nu$

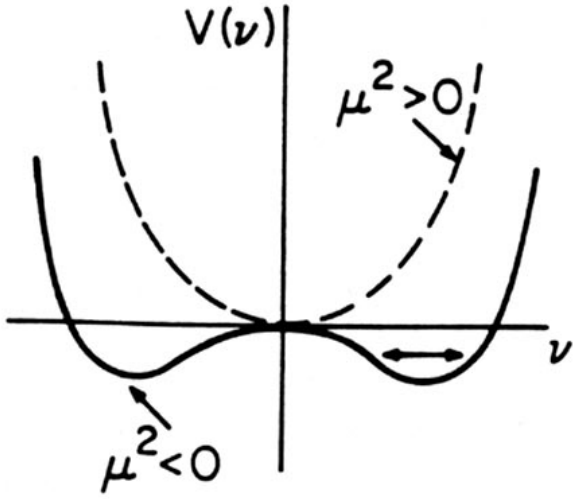


First term in V: vacuum energy

$$\langle 0|V|0\rangle = -\mu^4/4\lambda$$

No effect on microscopic interactions, but gives *negative* contribution to cosmological constant

$$|\Lambda_{\rm SSB}| = 8\pi G_N |\langle 0|V|0\rangle| \sim 10^{50} |\Lambda_{\rm obs}|$$



Require fine-tuned cancellation

$$\Lambda_{\rm cosm} = \Lambda_{\rm bare} + \Lambda_{\rm SSB}$$

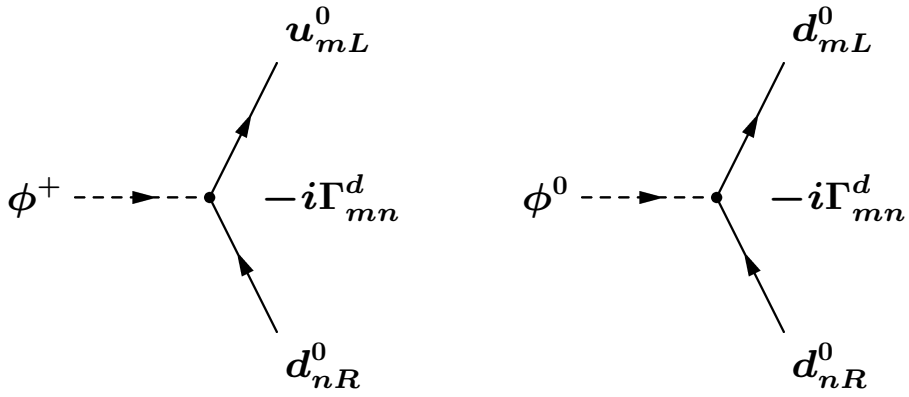
Also, QCD contribution from SSB of global chiral symmetry

Fermion Masses and Mixings

- **Yukawa (Higgs-fermion) interaction (+ analogous u, e, ν terms)**

$$-\mathcal{L}_{Yuk} = \sum_{m,n=1}^{F} \Gamma^d_{mn} \bar{q}^{\,0}_{mL} \phi d^0_{nR} + h.c.$$

$$= \sum_{m,n=1}^{F} \Gamma^d_{mn} \left[\bar{u}^0_{mL} \phi^+ d^0_{nR} + \bar{d}^{\,0}_{mL} \phi^0 d^0_{nR} \right] + h.c.$$



- For $\phi \to \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \nu + H \end{pmatrix}$ (unitary gauge)

$$-\mathcal{L}_{Yuk} \Rightarrow \sum_{m,n=1}^{F} M^d_{mn} \bar{d}^0_{mL} d^0_{nR} \left(1 + \frac{H}{\nu}\right) + h.c.$$

$$= \sum_i m_i \bar{d}_{iL} d_{iR} \left(1 + \frac{H}{\nu}\right) + h.c. = \sum_i m_i \bar{d}_i d_i \left(1 + \frac{H}{\nu}\right)$$

with $M^d \equiv \Gamma^d \nu/\sqrt{2}$

- $d_i = d_{iL} + d_{iR}$ are mass eigenstates of mass m_i

- For $F = 3$

$$A^{d\dagger}_L M^d A^d_R = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}, \qquad \begin{pmatrix} d \\ s \\ b \end{pmatrix}_{L,R} = \underbrace{A^{d\dagger}_{L,R}}_{\text{unitary}} \begin{pmatrix} d^0_1 \\ d^0_2 \\ d^0_3 \end{pmatrix}_{L,R}$$

- Higgs (H) couplings to fermions are diagonal in flavor and $\propto$ mass

Typical estimates: $m_u = 1.5 - 3$ MeV, $m_d = 3 - 7$ MeV, $m_s = 70 - 120$ MeV, $m_c \sim 1.3$ GeV, $m_b \sim 4.2$ GeV, $m_t = 172.9 \pm 1.1$ GeV

Implications for global $SU(3)_L \times SU(3)_R$ of QCD

These are current quark masses. $M_i = m_i + M_{dyn}$, $M_{dyn} \sim \Lambda_{\overline{MS}} \sim 300$ MeV from chiral condensate $\langle 0|\bar{q}q|0\rangle \neq 0$

m_t is pole mass; others, running masses at m or at 2 GeV^2

Yukawa couplings of Higgs to fermions

$$L_{\rm Yukawa} = \sum_i \bar{\psi}_i \left(-m_i - \frac{g m_i}{2 M_W} H \right) \psi_i$$

Coupling $g m_i / 2 M_W$ is flavor diagonal and small except t quark

$H \to \bar{b} b$ dominates for $M_H \lesssim 2 M_W$ ($H \to W^+ W^-$, ZZ dominate when allowed because of larger gauge coupling)

Flavor diagonal because only one doublet couples to fermions $\Rightarrow$ fermion mass and Yukawa matrices proportional

Often flavor changing Higgs couplings in extended models with two doublets coupling to same kind of fermion (not MSSM)

Stringent limits, e.g., tree-level Higgs contribution to $K_L - K_S$ mixing (loop in standard model) $\Rightarrow h_{\bar{d}s} / M_H < 10^{-6}$ GeV^{-1}

The Weak Charged Current

Fermi Theory incorporated in SM and made renormalizable

W-fermion interaction

$$\mathcal{L} = -\frac{g}{2\sqrt{2}}\left(J^{\mu}_{W} W^{-}_{\mu} + J^{\mu\dagger}_{W} W^{+}_{\mu}\right)$$

Charge-raising current (ignoring ν masses)

$$\begin{aligned} J^{\mu\dagger}_{W} &= \sum_{m=1}^{F} \left[\bar{\nu}^{0}_{m}\gamma^{\mu}(1-\gamma^{5})e^{0}_{m} + \bar{u}^{0}_{m}\gamma^{\mu}(1-\gamma^{5})d^{0}_{m}\right] \\ &= (\bar{\nu}_{e}\bar{\nu}_{\mu}\bar{\nu}_{\tau})\gamma^{\mu}(1-\gamma^{5})\begin{pmatrix} e^{-} \\ \mu^{-} \\ \tau^{-} \end{pmatrix} + (\bar{u}\ \bar{c}\ \bar{t})\gamma^{\mu}(1-\gamma^{5})V\begin{pmatrix} d \\ s \\ b \end{pmatrix}. \end{aligned}$$

Ignore ν masses for now

Pure $V-A \Rightarrow$ maximal P and C violation; CP conserved except for phases in V

$V = A_L^{u\dagger} A_L^d$ is $F \times F$ unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix from mismatch between weak and Yukawa interactions

Cabibbo matrix for $F = 2$

$$V = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

$\sin\theta_c \simeq 0.22 \equiv$ Cabibbo angle

Good zeroth-order description since third family almost decouples

General unitary 2×2: 1 angle and 3 (unobservable) q_L phases

CKM matrix for $F = 3$ involves 3 angles and 1 CP-violating phase (after removing unobservable q_L phases) (new interations involving q_R could make observable)

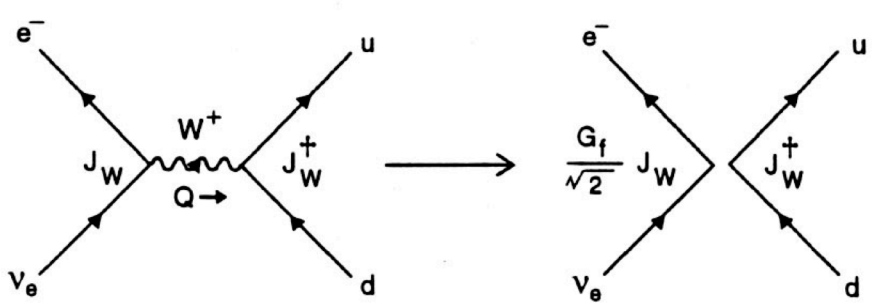
$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{td} & V_{td} \end{pmatrix}$$

Extensive studies, especially in B decays, to test unitarity of V as probe of new physics and test origin of CP violation

Need additional source of CP breaking for baryogenesis

Effective zero- range 4-fermi interaction (Fermi theory)

For $|Q| \ll M_W$, neglect Q^2 in W propagator



$$-L_{\rm eff}^{cc} = \left(\frac{g}{2\sqrt{2}}\right)^2 J_W^\mu \left(\frac{-g_{\mu\nu}}{Q^2 - M_W^2}\right) J_W^{\dagger\nu} \sim \frac{g^2}{8M_W^2} J_W^\mu J_{W\mu}^\dagger$$

Fermi constant: $\frac{G_F}{\sqrt{2}} \simeq \frac{g^2}{8M_W^2} = \frac{1}{2\nu^2}$

Muon lifetime: $\tau^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} \Rightarrow G_F = 1.17 \times 10^{-5}$ GeV^{-2}

Weak scale: $\nu = \sqrt{2}\langle 0|\phi^0|0\rangle \simeq 246$ GeV

Excellent description of β, K, hyperon, heavy quark, μ, and τ decays, $\nu_\mu e \to \mu^- \nu_e$, $\nu_\mu n \to \mu^- p$, $\nu_\mu N \to \mu^- X$

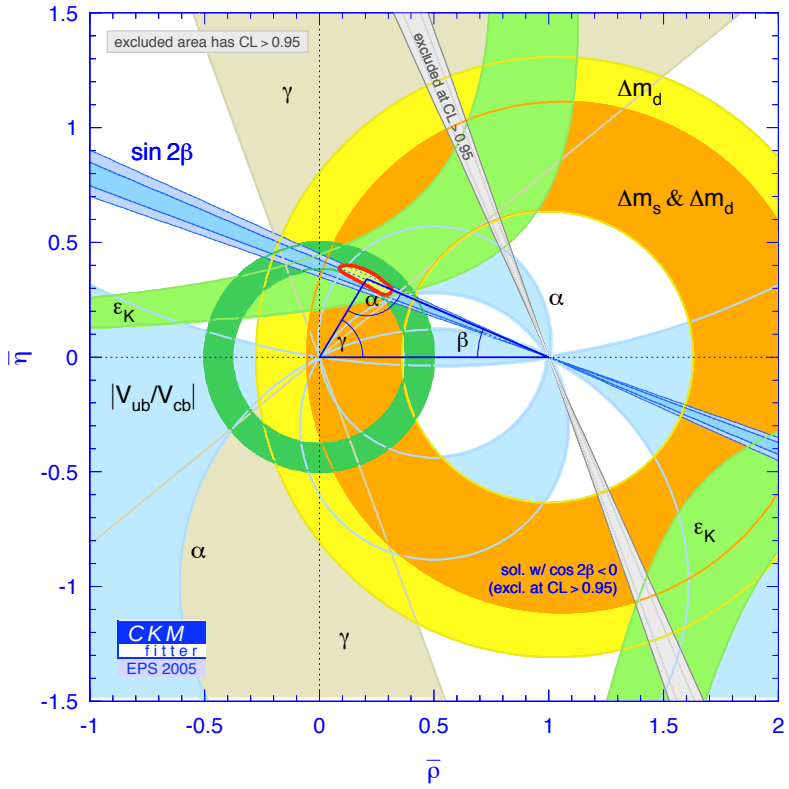
Full theory probed:

$e^{\pm}p \to \overset{(-)}{\nu}_e X$ at high energy (HERA)

Electroweak radiative corrections (loop level)

(Very important. Only calculable in full theory.)

$M_{K_S} - M_{K_L}$, kaon CP violation, $B \leftrightarrow \bar{B}$ mixing (loop level)



(CKMFITTER group:

`http://ckmfitter.in2p3.fr/)`

Quantum Electrodynamics (QED)

Incorporated into standard model

Lagrangian:

$$\mathcal{L} = -\frac{gg'}{\sqrt{g^2+g'^2}} J^\mu_Q (\cos\theta_W B_\mu + \sin\theta_W W^3_\mu)$$

Photon field:

$$A_\mu = \cos\theta_W B_\mu + \sin\theta_W W^3_\mu$$

Positron electric charge: $e = g\sin\theta_W$, **where** $\tan\theta_W \equiv g'/g$

Electromagnetic current:

$$J_Q^\mu = \sum_{m=1}^{F} \left[\frac{2}{3} \bar{u}_m^0 \gamma^\mu u_m^0 - \frac{1}{3} \bar{d}_m^0 \gamma^\mu d_m^0 - \bar{e}_m^0 \gamma^\mu e_m^0 \right]$$

$$= \sum_{m=1}^{F} \left[\frac{2}{3} \bar{u}_m \gamma^\mu u_m - \frac{1}{3} \bar{d}_m \gamma^\mu d_m - \bar{e}_m \gamma^\mu e_m \right]$$

Electric charge: $Q = T^3 + Y$, where $Y =$ weak hypercharge

(coefficient of $ig' B_\mu$ in covariant derivatives)

Flavor diagonal: Same form in weak and mass bases because fields which mix have same charge

Purely vector (parity conserving): L and R fields have same charge

($q_i = t_i^3 + y_i$ is the same for L and R fields, even though t_i^3 and y_i are not)

Experiment	Value of α^{-1}	Precision	Δ_e
$a_e = (g_e - 2)/2$	137.035 999 683 (94)	$[6.9 \times 10^{-10}]$	–
h/m (Rb, Cs)	137.035 999 35 (69)	$[5.0 \times 10^{-9}]$	0.33 ± 0.69
Quantum Hall	137.036 003 0 (25)	$[1.8 \times 10^{-8}]$	-3.3 ± 2.5
h/m (neutron)	137.036 007 7 (28)	$[2.1 \times 10^{-8}]$	-8.0 ± 2.8
$\gamma_{p,^3He}$ (J. J.)	137.035 987 5 (43)	$[3.1 \times 10^{-8}]$	12.2 ± 4.3
μ^+e^- hyperfine	137.036 001 7 (80)	$[5.8 \times 10^{-8}]$	-2.0 ± 8.0

Spectacularly successful:

Most precise: e anomalous magnetic moment $\rightarrow \alpha$

Many low energy tests to few $\times 10^{-8}$

$m_\gamma < 6 \times 10^{-17}$ eV, $\quad q_\gamma < 5 \times 10^{-30}|e|$

Running $\alpha(Q^2)$ observed

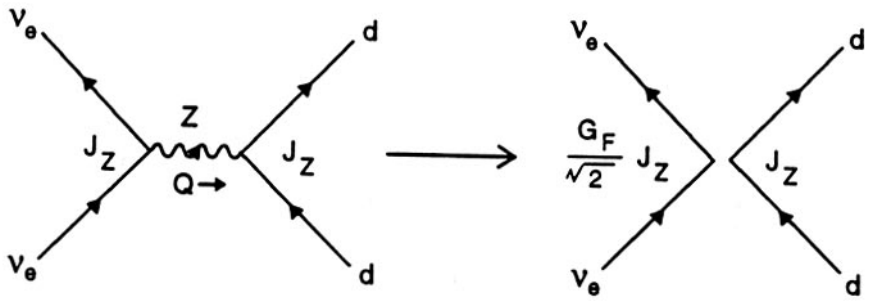
Muon $g - 2$ sensitive to new physics. Anomaly?

Muonic Lamb shift. Anomaly? Proton radius?

The Weak Neutral Current

Prediction of $SU(2) \times U(1)$

$$\begin{aligned} \mathcal{L} &= -\frac{\sqrt{g^2+g'^2}}{2} J_Z^\mu \left(-\sin\theta_W B_\mu + \cos\theta_W W_\mu^3\right) \\ &= -\frac{g}{2\cos\theta_W} J_Z^\mu Z_\mu \end{aligned}$$



Neutral current process and effective 4-fermi interaction for $|Q| \ll M_Z$

Neutral current:

$$
\begin{aligned}
J_Z^\mu &= \sum_m \left[\bar{u}^0_{mL}\gamma^\mu u^0_{mL} - \bar{d}^0_{mL}\gamma^\mu d^0_{mL} + \bar{\nu}^0_{mL}\gamma^\mu \nu^0_{mL} - \bar{e}^0_{mL}\gamma^\mu e^0_{mL}\right] \\
&\quad -2\sin^2\theta_W J_Q^\mu \\
&= \sum_m \left[\bar{u}_{mL}\gamma^\mu u_{mL} - \bar{d}_{mL}\gamma^\mu d_{mL} + \bar{\nu}_{mL}\gamma^\mu \nu_{mL} - \bar{e}_{mL}\gamma^\mu e_{mL}\right] \\
&\quad -2\sin^2\theta_W J_Q^\mu
\end{aligned}
$$

Flavor diagonal: Same form in weak and mass bases because fields which mix have same charge

GIM mechanism: c quark predicted so that s_L could be in doublet to avoid unwanted flavor changing neutral currents (FCNC) at tree and loop level

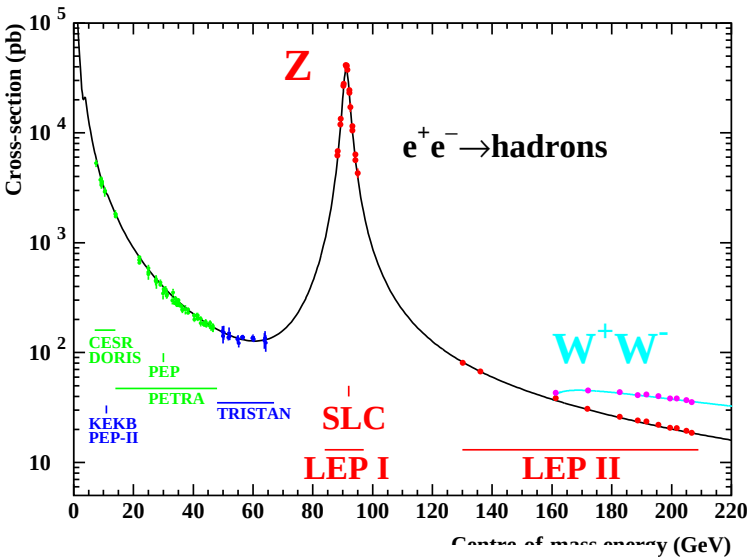
Parity and charge conjugation violated but not maximally: first term is pure $V-A$, second is V

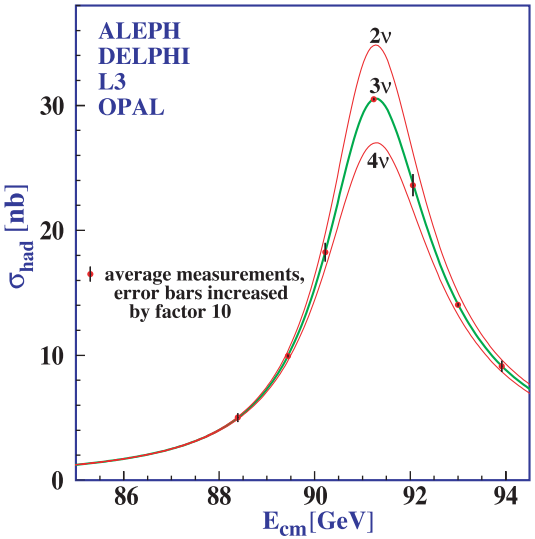
Effective 4-fermi interaction for $|Q^2| \ll M_Z^2$:

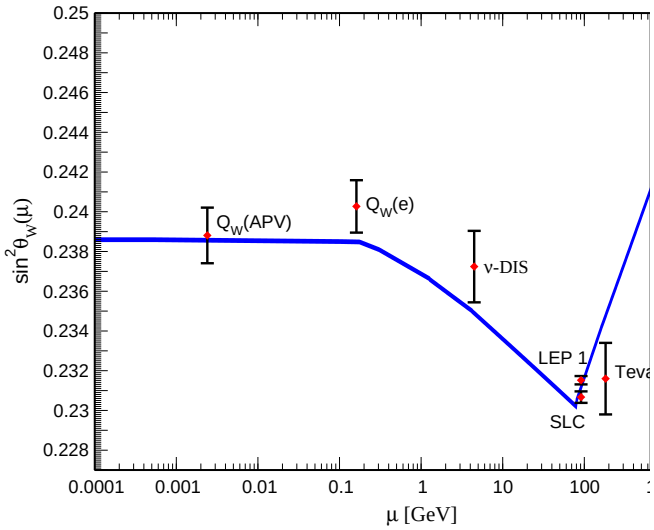
$$-\mathcal{L}_{\rm eff}^{NC} = \frac{G_F}{\sqrt{2}} J_Z^\mu J_{Z\mu}$$

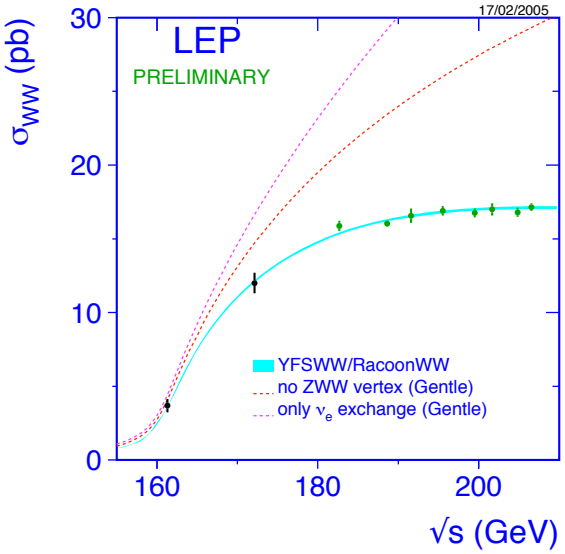
Coefficient same as WCC because

$$\frac{G_F}{\sqrt{2}} = \frac{g^2}{8M_W^2} = \frac{g^2 + g'^2}{8M_Z^2}$$


Cross-section (pb)
Z
e+e−→hadrons
W+W-
CESR
DORIS
PEP
PETRA
TRISTAN
KEKB
PEP-II
SLC
LEP I
LEP II
Centre-of-mass energy (GeV)


ALEPH
DELPHI
L3
OPAL
2ν
3ν
4ν
average measurements,
error bars increased
by factor 10
σhad [nb]
Ecm[GeV]


sin2θW(μ)
QW(APV)
QW(e)
ν-DIS
LEP 1
SLC
μ [GeV]

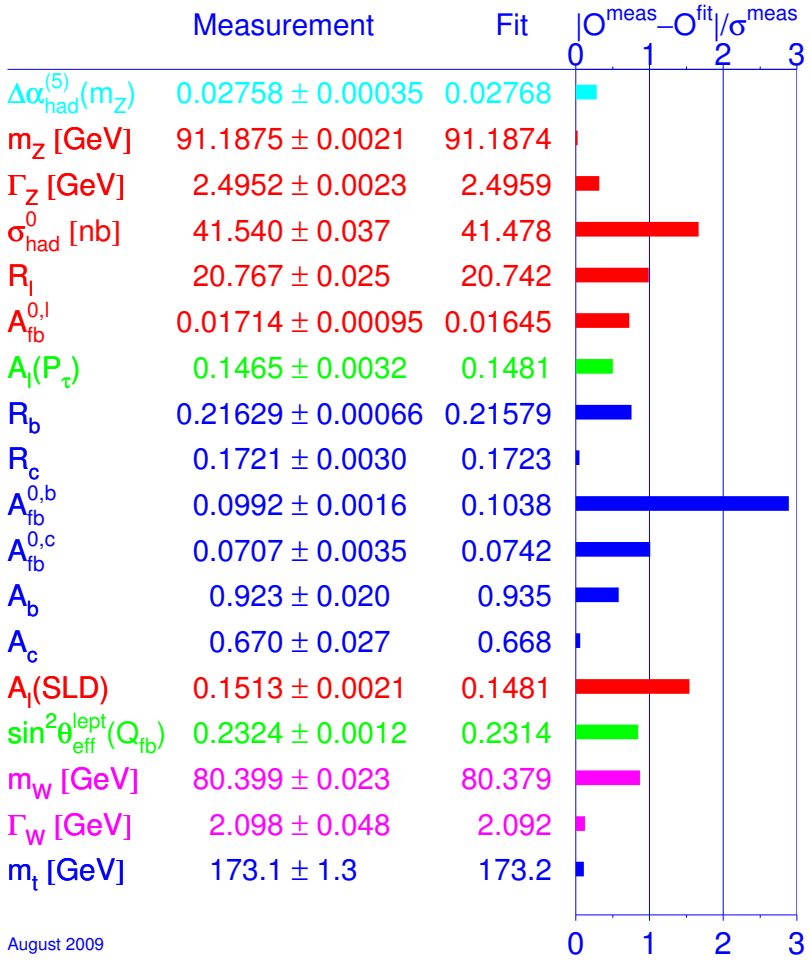

17/02/2005
LEP
PRELIMINARY
σWW (pb)
YFSWW/RacoonWW
no ZWW vertex (Gentle)
only νe exchange (Gentle)
√s (GeV)

The Z, the W, and the Weak Neutral Current

- Primary prediction and test of electroweak unification
- WNC discovered 1973 (Gargamelle at CERN, HPW at FNAL)
- 70's, 80's: weak neutral current experiments (few %)
  - Pure weak: νN, νe scattering
  - Weak-elm interference in eD, e^+e^-, atomic parity violation
  - Model independent analyses (νe, νq, eq)
  - $SU(2) \times U(1)$ group/representations; t and ν_τ exist; m_t limit; hint for SUSY unification; limits on TeV scale physics
- W, Z discovered directly 1983 (UA1, UA2)

- **90's: Z pole (LEP, SLD), 0.1%; lineshape, modes, asymmetries**
- **LEP 2: M_W, Higgs search , gauge self-interactions**
- **Tevatron: m_t, M_W, Higgs search**
- **4th generation weak neutral current experiments** (atomic parity (Boulder); νe; νN (NuTeV); polarized Møller asymmetry (SLAC))

- SM correct and unique to zeroth approx. (gauge principle, group, representations)
- SM correct at loop level (renorm gauge theory; m_t, α_s, M_H)
- TeV physics severely constrained (unification vs compositeness)
- Consistent with light elementary Higgs
- Precise gauge couplings (SUSY gauge unification)



Problems with the Standard Model

Lagrangian after symmetry breaking:

$$\mathcal{L} = \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm Higgs} + \sum_i \bar{\psi}_i \left(i \not{\partial} - m_i - \frac{m_i H}{\nu} \right) \psi_i$$
$$- \frac{g}{2\sqrt{2}} \left(J^\mu_W W^-_\mu + J^{\mu\dagger}_W W^+_\mu \right) - e J^\mu_Q A_\mu - \frac{g}{2\cos\theta_W} J^\mu_Z Z_\mu$$

Standard model: $SU(2) \times U(1)$ (extended to include ν masses) + QCD + general relativity

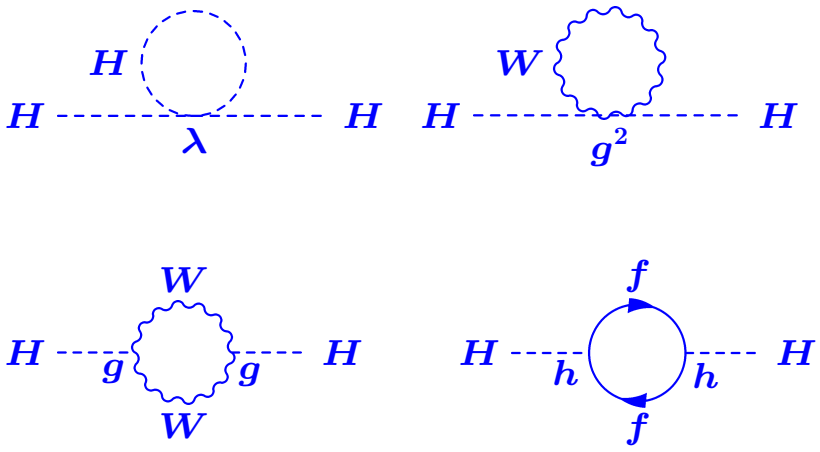
Mathematically consistent, renormalizable theory

Correct to 10^{-16} cm

However, too much arbitrariness and fine-tuning: $\mathcal{O}(27)$ parameters (+ 2 for Majorana ν) *and* electric charges

- Gauge Problem
  - complicated gauge group with 3 couplings
  - charge quantization ($|q_e| = |q_p|$) unexplained
  - Possible solutions: strings; grand unification; magnetic monopoles (partial); anomaly constraints (partial)

- Fermion problem
  - Fermion masses, mixings, families unexplained
  - Neutrino masses, nature? Probe of Planck/GUT scale?
  - CP violation inadequate to explain baryon asymmetry
  - Possible solutions: strings; brane worlds; family symmetries; compositeness; radiative hierarchies. New sources of CP violation.



- **Higgs/hierarchy problem**
  - **Expect** $M_H^2 = O(M_W^2)$
  - **higher order corrections:** $\delta M_H^2/M_W^2 \sim 10^{34}$

Possible solutions: supersymmetry; dynamical symmetry breaking; large extra dimensions; Little Higgs; anthropically motivated fine-tuning (split supersymmetry) **(landscape)**

- **Strong** CP **problem**
  - **Can add** $\frac{\theta}{32\pi^2} g_s^2 F\tilde{F}$ **to QCD (breaks,** P, T, CP**)**
  - $d_N \Rightarrow \theta < 10^{-9}$**, but** $\delta\theta|_{\rm weak} \sim 10^{-3}$
  - **Possible solutions: spontaneously broken global** $U(1)$ **(Peccei-Quinn)** $\Rightarrow$ **axion; unbroken global** $U(1)$ **(massless** u **quark); spontaneously broken** CP **+ other symmetries**

- **Graviton problem**
  - **gravity not unified**
  - **quantum gravity not renormalizable**
  - **cosmological constant:** $\Lambda_{\rm SSB} = 8\pi G_N \langle V \rangle > 10^{50} \Lambda_{\rm obs}$ **(**10^{124} **for GUTs, strings)**

  Possible solutions:

  - **supergravity and Kaluza Klein unify**
  - **strings yield finite gravity**
  - Λ**? Anthropically motivated fine-tuning (landscape)?**

- Necessary new ingredients
  - Mechanism for small neutrino masses
    . Planck/GUT scale? Small Dirac (intermediate scale)?
  - Mechanism for baryon asymmetry?
    . Electroweak transition (Z' or extended Higgs?)
    . Heavy Majorana neutrino decay (seesaw)?
    . Decay of coherent field? CPT violation?
  - What is the dark energy?
    . Cosmological Constant? Quintessence?
    . Related to inflation? Time variation of couplings?
  - What is the dark matter?
    . Lightest supersymmetric particle? Axion?
  - Suppression of flavor changing neutral currents? Proton decay? Electric dipole moments?
    . *Automatic* in standard model, but not in extensions

New Physics

- A new layer at the TeV scale
  - Compositeness, Little Higgs, twin Higgs, Higgless, dynamical symmetry breaking, strong dynamics
  - Precision electroweak constraints, FCNC, UV completions?
- Large and/or warped extra dimensions; possible low fundamental scale
- Unification at the Planck scale, $M_P = G_N^{-1/2} \sim 10^{19}$ GeV
  - Supersymmetry (between fermions and bosons), grand unification, strings?
  - Top-down remnants: Z', W', extended Higgs, exotic fermions, ...

Conclusions

- The standard model is spectacularly successful, but is incomplete
- Promising theoretical ideas at Planck and TeV scale
- Eagerly anticipate guidance from LHC