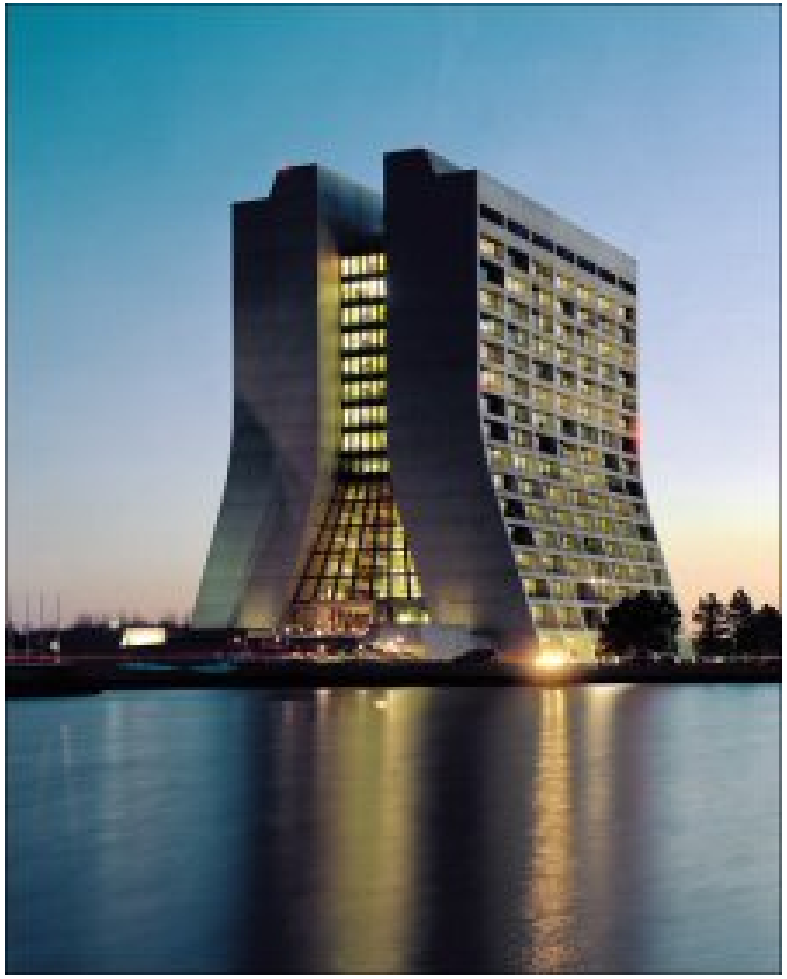
Tests of the Electroweak Theory

- History/introduction
- Weak charged current
- QED
- Weak neutral current
- Precision tests
- Rare processes
- CP violation and B decays
- Neutrino mass



References (mainly weak charged current)

- **E.D. Commins and P.H. Bucksbaum,** *Weak Interactions of Leptons and Quarks*, **(Cambridge Univ. Press, Cambridge, 1983)**

- **P. Renton,** *Electroweak Interactions*, **(Cambridge Univ. Press, Cambridge, 1990)**

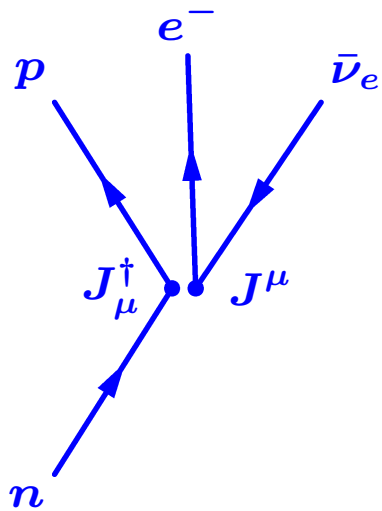
- *Precision Tests of the Standard Electroweak Model*, **ed. P. Langacker (World Scientific, Singapore, 1995)** **(especially articles by Fetscher and Gerber; Herczog; Deutsch and Quin)**

- **S. Eidelman** *et al.* **[Particle Data Group], Phys. Lett. B 592, 1 (2004)** **(Electroweak and μ Decay reviews).**

The Weak Interactions

- Radioactivity (Becquerel, 1896)
- β decay appeared to violate energy (Meitner, Hahn; 1911)
- Neutrino hypothesis (Pauli, 1930)
  - ν_e (Reines, Cowan; 1953)
  - ν_μ (Lederman, Schwartz, Steinberger; 1962)
  - ν_τ (DONUT, 2000) (τ, 1975)

- **Fermi theory (1933)**
  - **Loosely like QED, but zero range (non-renormalizable) and non-diagonal (charged current)**



$$H \sim G_F J^\dagger_\mu J^\mu$$

$$J^\dagger_\mu \sim \bar{p}\gamma_\mu n + \bar{\nu}_e \gamma_\mu e^- \qquad [n \to p,\ e^- \to \nu_e]$$

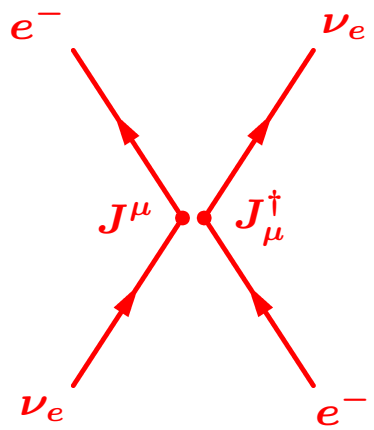
$$J_\mu \sim \bar{n}\gamma_\mu p + \bar{e}\gamma_\mu \nu_e \qquad [p \to n,\ \nu_e \to e^-\ (\ \times \to e^- \bar{\nu}_e)]$$

$$G_F \simeq 1.17 \times 10^{-5}\ \mathrm{GeV}^{-2} \qquad \text{[Fermi constant]}$$

- **Fermi theory modified to include**
  - μ, τ decay
  - strangeness (Cabibbo)
  - quark model
  - heavy quarks (CKM)
  - ν mass and mixing
  - parity violation $(V-A)$ (Lee, Yang; Wu; Feynman-Gell-Mann)

- **Fermi theory correctly describes (at tree level)**
  - Nuclear/neutron β decay $(n \to p e^- \bar{\nu}_e)$
  - μ, τ decays $(\mu^- \to e^- \bar{\nu}_e \nu_\mu; \quad \tau^- \to \mu^- \bar{\nu}_\mu \nu_\tau,\ \nu_\tau \pi^-,\ \cdots)$
  - π, K decays $(\pi^+ \to \mu^+ \nu_\mu,\ \pi^0 e^+ \nu_e; \quad K^+ \to \mu^+ \nu_\mu,\ \pi^0 e^+ \nu_e,\ \pi^+ \pi^0)$
  - hyperon decays $(\Lambda \to p \pi^-; \quad \Sigma^- \to n \pi^-; \quad \Sigma^+ \to \Lambda e^+ \nu_e)$
  - heavy quark decays $(c \to s e^+ \nu_e; \quad b \to c \mu^- \bar{\nu}_\mu,\ c \pi^-)$
  - ν scattering $(\nu_\mu e^- \to \mu^- \nu_e; \quad \underbrace{\nu_\mu n \to \mu^- p}_{\text{“elastic”}}; \quad \underbrace{\nu_\mu N \to \mu^- X}_{\text{deep-inelastic}})$

- **Fermi theory violates unitarity at high energy** (non-renormalizable)



– $\sigma(\nu_e e^- \to e^- \nu_e) \to \frac{G_F^2 s}{\pi}, \quad s \equiv E_{CM}^2$

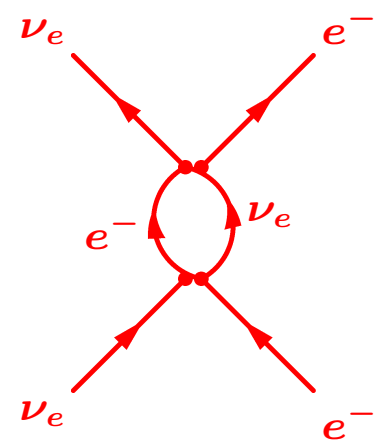
– pure S-wave unitarity: $\sigma < \frac{16\pi}{s}$

– fails for $\frac{E_{CM}}{2} \geq \sqrt{\frac{\pi}{G_F}} \sim 500$ GeV

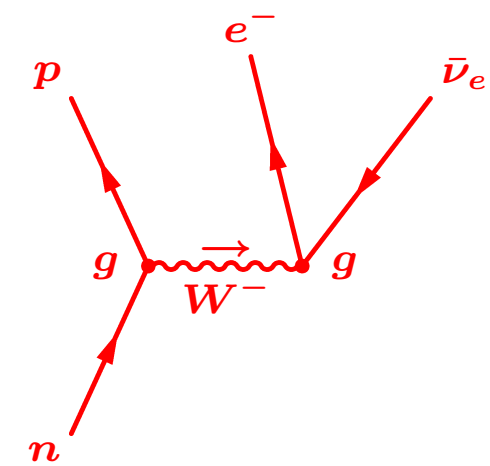
– Born not unitary; often restored by H.O.T.

– Fermi theory: divergent integrals

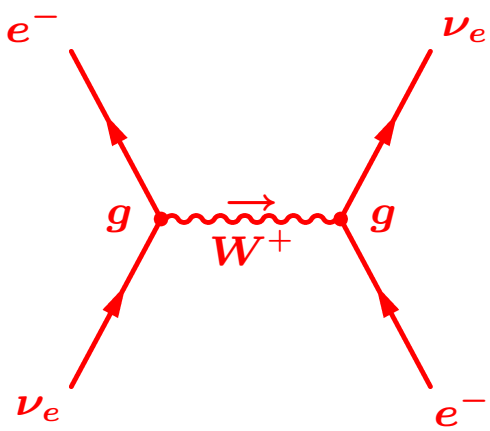
$$\int d^4k \frac{\not{k} + m_e}{k^2 - m_e^2} \frac{\not{k}}{k^2}$$



- **Intermediate vector boson theory** (Yukawa, 1935; Schwinger, 1957)

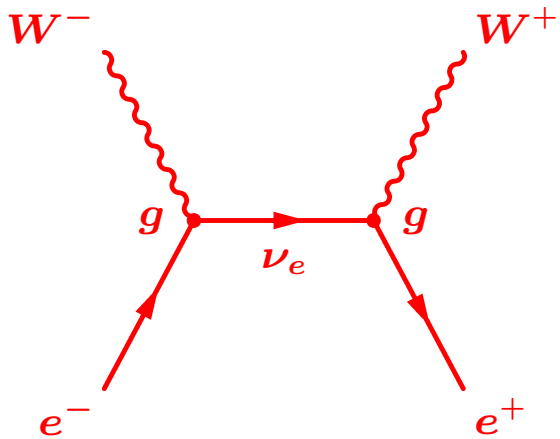


$$\frac{G_F}{\sqrt{2}} \sim \frac{g^2}{8M_W^2} \text{ for } M_W \gg Q$$



– no longer pure S-wave $\Rightarrow$

– $\nu_e e^- \rightarrow \nu_e e^-$ better behaved



– *but,* $e^+e^- \to W^+W^-$ violates unitarity for $\sqrt{s} \gtrsim 500$ GeV

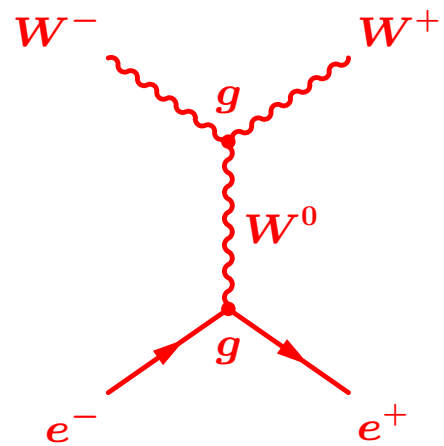
– $\epsilon_\mu \sim k_\mu / M_W$ for longitudinal polarization (non-renormalizable)

– introduce W^0 to cancel

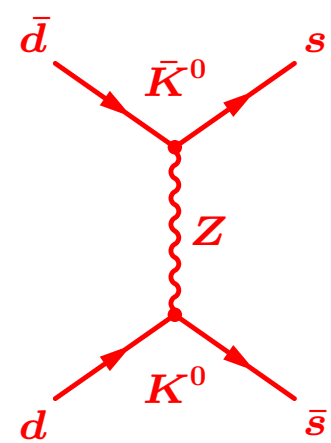
– fixes $W^0 W^+ W^-$ and $e^+e^- W^0$ vertices

– requires $\left[J, J^\dagger\right] \sim J^0$ (like $SU(2) \times U(1)$)

– not realistic

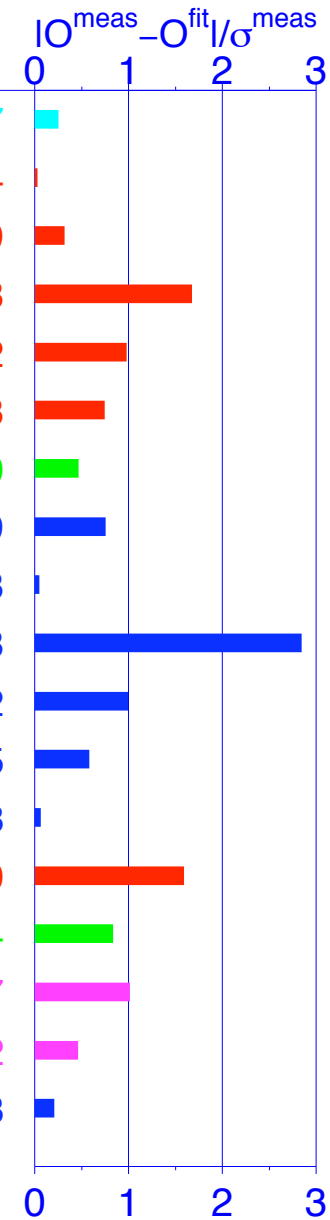


- **Glashow model (1961)** ($W^\pm, Z, \gamma$, but no mechanism for $M_{W,Z}$)
- **Weinberg-Salam (1967): Higgs mechanism** $\longrightarrow M_{W,Z}$
- **Renormalizable (1971)** ('t Hooft, $\cdots$)
- **Flavor changing neutral currents (FCNC)**
  - very large $K^0 \leftrightarrow \bar{K}^0$ mixing
  - GIM mechanism (c quark) (1970)
  - discovered (1974)



- **Weak neutral current (1973)**
- **W, Z (1983)**
- **Precision tests (1989-2000)**
- **CKM unitarity ($\sim$ 1995-)**
- **t quark (1995)**
- **ν mass (1998-2002)**

	Measurement	Fit
$\Delta\alpha^{(5)}_{had}(m_Z)$	0.02758 ± 0.00035	0.02767
m_Z [GeV]	91.1875 ± 0.0021	91.1874
Γ_Z [GeV]	2.4952 ± 0.0023	2.4959
σ^0_{had} [nb]	41.540 ± 0.037	41.478
R_l	20.767 ± 0.025	20.742
$A^{0,l}_{fb}$	0.01714 ± 0.00095	0.01643
$A_l(P_\tau)$	0.1465 ± 0.0032	0.1480
R_b	0.21629 ± 0.00066	0.21579
R_c	0.1721 ± 0.0030	0.1723
$A^{0,b}_{fb}$	0.0992 ± 0.0016	0.1038
$A^{0,c}_{fb}$	0.0707 ± 0.0035	0.0742
A_b	0.923 ± 0.020	0.935
A_c	0.670 ± 0.027	0.668
A_l(SLD)	0.1513 ± 0.0021	0.1480
$\sin^2\theta^{lept}_{eff}(Q_{fb})$	0.2324 ± 0.0012	0.2314
m_W [GeV]	80.410 ± 0.032	80.377
Γ_W [GeV]	2.123 ± 0.067	2.092
m_t [GeV]	172.7 ± 2.9	173.3



The Weak Interactions of Fermions

Group: $\underbrace{SU(3)}_{\text{QCD}} \times \underbrace{SU(2)\times U(1)}_{\text{electroweak}}$

Gauge bosons:

$SU(3)$ **(QCD):** $G^i_\mu,\ i = 1, \cdots, 8$
$SU(2)$**:** $W^i_\mu,\ i = 1, 2, 3$
$U(1)$**:** B_μ

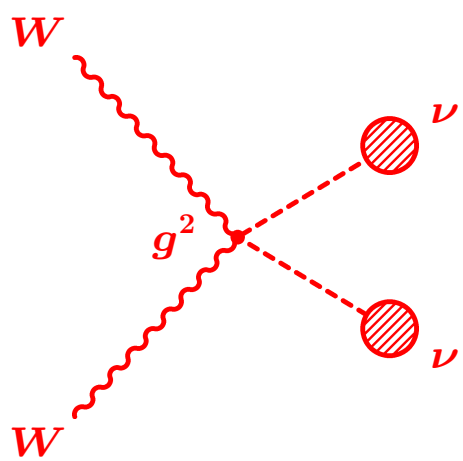
Gauge couplings: g_s **(QCD);** $g,\ g'$ **(electroweak)**

Weak angle: $\tan\theta_W \equiv g'/g$

Spontaneous Symmetry Breaking (Higgs mechanism)

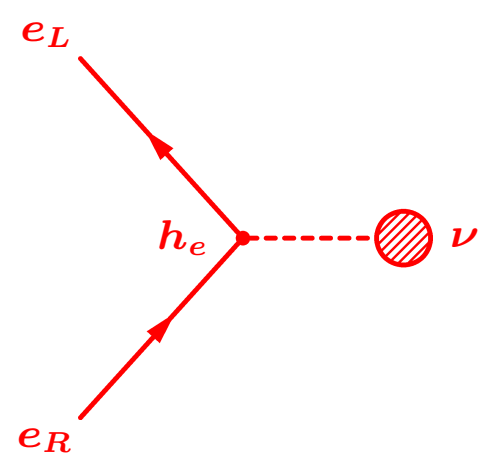
Higgs covariant kinetic energy terms:

$$
\begin{aligned}
(D_\mu\varphi)^\dagger D^\mu\varphi &= \frac{1}{2}(0\ \nu)\left[\frac{g}{2}\tau^i W^i_\mu + \frac{g'}{2}B_\mu\right]^2 \begin{pmatrix} 0 \\ \nu \end{pmatrix} + H \text{ terms} \\
&\rightarrow M_W^2 W^{+\mu} W^-_\mu + \frac{M_Z^2}{2} Z^\mu Z_\mu \\
&+ H \text{ kinetic energy and gauge interaction terms}
\end{aligned}
$$



$$M_W = \frac{g\nu}{2}$$

$$m_e = \frac{h_e\nu}{\sqrt{2}}$$



Mass eigenstate bosons: W, Z, and A (photon)

$$\begin{aligned} W^{\pm} &= \frac{1}{\sqrt{2}}(W^1 \mp iW^2) \\ Z &= -\sin\theta_W B + \cos\theta_W W^3 \\ A &= \cos\theta_W B + \sin\theta_W W^3 \end{aligned}$$

Masses:

$$M_W = \frac{g\nu}{2}, \qquad M_Z = \sqrt{g^2 + g'^2}\,\frac{\nu}{2} = \frac{M_W}{\cos\theta_W}, \qquad M_A = 0$$

(Goldstone scalars "eaten" $\longrightarrow$ longitudinal components of $W^{\pm}$, Z)

Will show: **Fermi constant** $G_F/\sqrt{2} \sim g^2/8M_W^2$
($G_F = 1.16637(1)\times 10^{-5}\ GeV^{-2}$ **from muon lifetime)**

Electroweak scale:

$$\nu = 2M_W/g \simeq (\sqrt{2}G_F)^{-1/2} \simeq 246\ GeV$$

Will show: $g = e/\sin\theta_W$ $(\alpha = e^2/4\pi \sim 1/137.036) \Rightarrow$

$$M_W = M_Z \cos\theta_W = \frac{g\nu}{2} \sim \frac{(\pi\alpha/\sqrt{2}G_F)^{1/2}}{\sin\theta_W}$$

Weak neutral current: $\sin^2\theta_W \sim 0.23 \Rightarrow M_W \sim 78\ GeV$, **and**
$M_Z \sim 89\ GeV$ **(increased by** $\sim 2\ GeV$ **by loop corrections)**

Discovered at CERN: **UA1 and UA2, 1983**

The Weak Charged Current

Fermi Theory incorporated in SM and made renormalizable

W-fermion interaction

$$\mathcal{L} = -\frac{g}{2\sqrt{2}}\left(J_W^\mu W_\mu^- + J_W^{\mu\dagger} W_\mu^+\right)$$

Charge-raising current

$$\begin{aligned} J_W^{\mu\dagger} &= \sum_{m=1}^{F}\left[\bar{\nu}_m^0\gamma^\mu(1-\gamma^5)e_m^0 + \bar{u}_m^0\gamma^\mu(1-\gamma^5)d_m^0\right] \\ &= (\bar{\nu}_e\bar{\nu}_\mu\bar{\nu}_\tau)\gamma^\mu(1-\gamma^5)\begin{pmatrix} e^- \\ \mu^- \\ \tau^- \end{pmatrix} + (\bar{u}\ \bar{c}\ \bar{t})\gamma^\mu(1-\gamma^5)V\begin{pmatrix} d \\ s \\ b \end{pmatrix}. \end{aligned}$$

Ignore ν masses for now

Pure $V - A \Rightarrow$ maximal P and C violation; CP conserved except for phases in V

$V = A_L^{u\dagger} A_L^d$ is $F \times F$ unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix from mismatch between weak and Yukawa interactions

Cabibbo matrix for $F = 2$

$$V = \begin{pmatrix} \cos\theta_c & \sin\theta_c \\ -\sin\theta_c & \cos\theta_c \end{pmatrix}$$

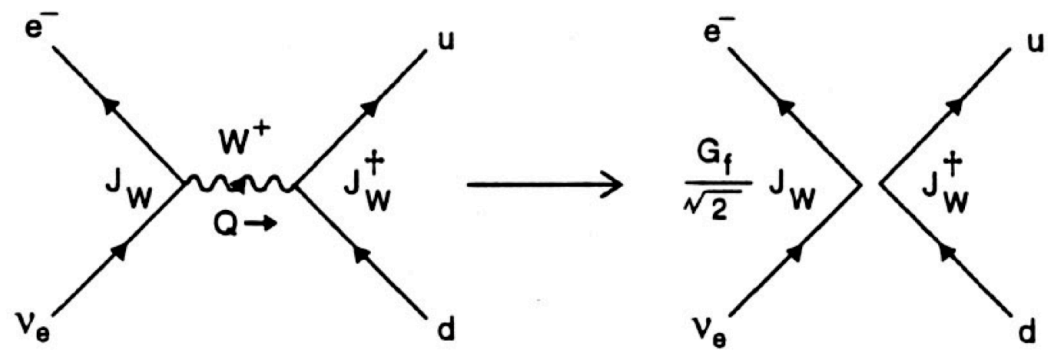
$\sin\theta_c \simeq 0.22 \equiv$ Cabibbo angle

Good zeroth-order description since third family almost decouples

Will generalize to 3 families

Effective zero- range 4-fermi interaction (Fermi theory)

For $|Q| \ll M_W$, neglect Q^2 in W propagator



$$-L^{cc}_{\text{eff}} = \left(\frac{g}{2\sqrt{2}}\right)^2 J^\mu_W \left(\frac{-g_{\mu\nu}}{Q^2 - M_W^2}\right) J^{\dagger\nu}_W \sim \frac{g^2}{8M_W^2} J^\mu_W J^\dagger_{W\mu}$$

Fermi constant: $\frac{G_F}{\sqrt{2}} \simeq \frac{g^2}{8M_W^2} = \frac{1}{2\nu^2}$

Muon lifetime: $\tau^{-1} = \frac{G_F^2 m_\mu^5}{192\pi^3} \Rightarrow G_F = 1.17\times10^{-5}$ GeV^{-2}

(exercize)

Weak scale: $\nu = \sqrt{2}\langle 0|\varphi^0|0\rangle \simeq 246$ GeV

Muon Decay

- One of most precisely studied systems
  - Test of Fermi theory, $V - A$, standard model
  - Test of renormalizable field theory
  - Constraint on new interactions, W_R, exotic fermion mixing
  - Precise G_F input to precision tests and CKM unitarity

- **Full expression for lifetime**

$$\tau_\mu^{-1} = \underbrace{\frac{G_F^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)}_{\text{Fermi, with } m_e \neq 0} \underbrace{\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)}_{\text{SM}}$$

$$\times \underbrace{\left[1 + \left(\frac{25}{8} - \frac{\pi^2}{2}\right)\frac{\alpha(m_\mu)}{\pi} + C_2 \frac{\alpha^2(m_\mu)}{\pi^2}\right]}_{\text{radiative corrections}}$$

where

$$F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2 \ln x$$

$$C_2 = \frac{156815}{5184} - \frac{518}{81}\pi^2 - \frac{895}{36}\underbrace{\zeta(3)}_{1.202} + \frac{67}{720}\pi^4 + \frac{53}{6}\pi^2 \ln(2)$$

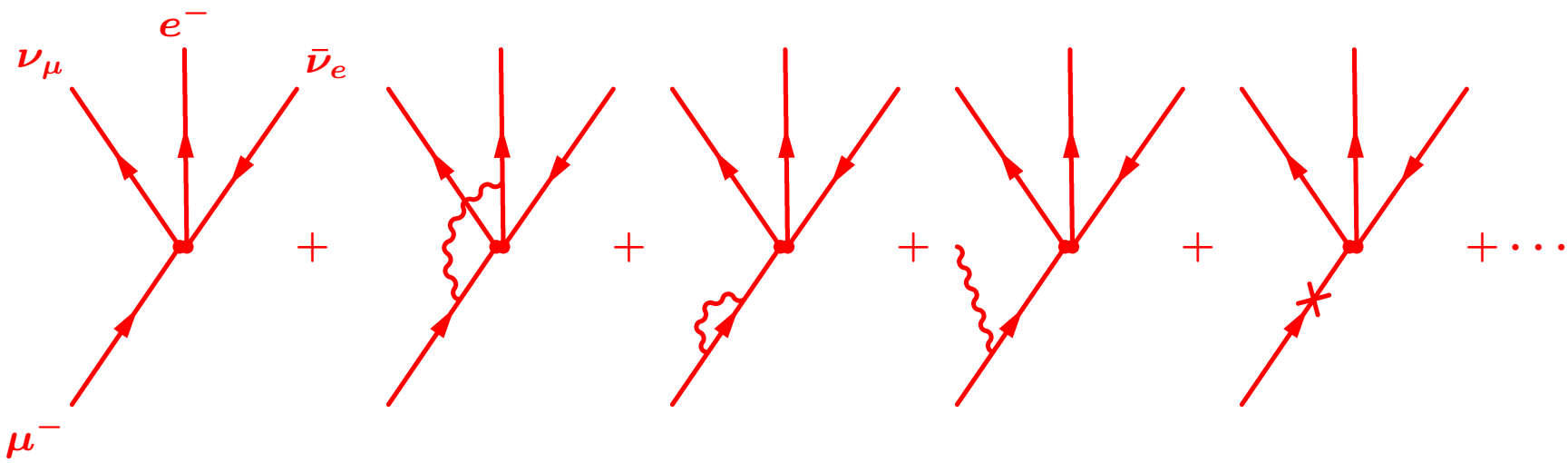
and

$$\alpha(m_\mu)^{-1} = \alpha^{-1} - \frac{2}{3\pi}\ln\left(\frac{m_\mu}{m_e}\right) + \frac{1}{6\pi} \approx 136$$

- **Order α^2 only computed recently**

- $\tau_\mu = 2.19703(4)\times 10^{-6}$ s $\rightarrow G_F \sim 1.16637(1) \times 10^{-5}$ GeV^{-2}
  (critical input to precision and universality)

- **QED radiative corrections to Fermi theory are finite to all orders in α and leading order in G_F (no $\ln M_W$)** (β decay logarithmically divergent–even for no strong interactions! Finite in SM)



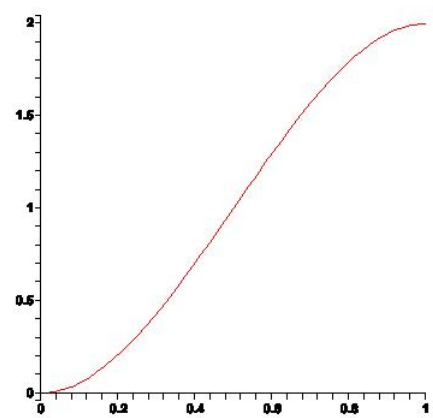
- **Spectrum** (neglecting m_e/m_μ, m_μ/M_W, radiative corrections)

$$d\Gamma = \frac{G_F^2 m_\mu^5}{192\pi^3} \underbrace{[2\epsilon^2(3-2\epsilon)]}_{\text{spectrum}} \underbrace{\left[1 + \left(\frac{1-2\epsilon}{3-2\epsilon}\right)\cos\theta\right]}_{\text{asymmetry}}$$

$$\times \underbrace{\left[\frac{1-\hat{n}_e\cdot\hat{s}_e}{2}\right]}_{e^- \text{ helicity}} d\epsilon \frac{d\cos\theta d\phi}{4\pi}$$

$$\epsilon \equiv E_e/E_{max} \sim 2E_e/m_\mu \qquad \theta \text{ w.r.t. } \mu \text{ polarization}$$

- Characteristic $V-A$ spectrum
- Angular asymmetry $\rightarrow$ polarimeter (τ polarization; $g_\mu - 2$) (reversed for μ^+)
- Electron helicity $h_e = -1$ (for $m_e = 0$) $\rightarrow V-A$



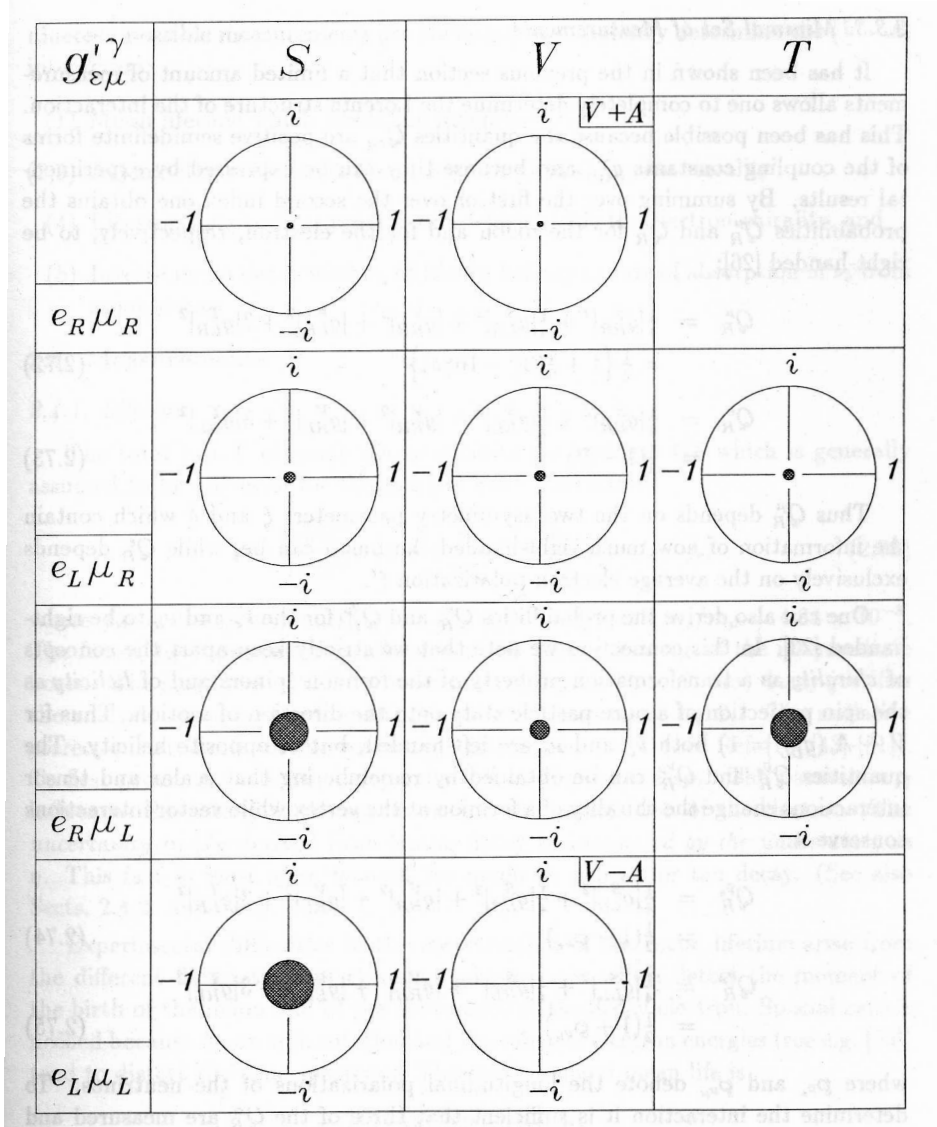
- **General four-Fermi interaction** (with L-family conservation)

$$
\begin{aligned}
H \;=\; & \frac{4G_F}{\sqrt{2}}\Big(g^V_{LL}\,\overline{e}_L\,\gamma_\lambda\,\nu_{eL}\,\overline{\nu}_{\mu L}\,\gamma^\lambda\,\mu_L \;+\; g^V_{RR}\,\overline{e}_R\,\gamma_\lambda\,\nu_{eR}\,\overline{\nu}_{\mu R}\,\gamma^\lambda\,\mu_R \\
& + \; g^V_{LR}\,\overline{e}_L\,\gamma_\lambda\,\nu_{eL}\,\overline{\nu}_{\mu R}\,\gamma^\lambda\,\mu_R \;+\; g^V_{RL}\,\overline{e}_R\,\gamma_\lambda\,\nu_{eR}\,\overline{\nu}_{\mu L}\,\gamma^\lambda\,\mu_L \\
& + \; g^S_{LL}\,\overline{e}_L\,\nu_{eR}\,\overline{\nu}_{\mu R}\,\mu_L \;+\; g^S_{RR}\,\overline{e}_R\,\nu_{eL}\,\overline{\nu}_{\mu L}\,\mu_R \\
& + \; g^S_{LR}\,\overline{e}_L\,\nu_{eR}\,\overline{\nu}_{\mu L}\,\mu_R \;+\; g^S_{RL}\,\overline{e}_R\,\nu_{eL}\,\overline{\nu}_{\mu R}\,\mu_L \\
& + \; g^T_{LR}\,\overline{e}_L\,t_{\alpha\beta}\,\nu_{eR}\,\overline{\nu}_{\mu L}\,t^{\alpha\beta}\,\mu_R \;+\; g^T_{RL}\,\overline{e}_R\,t_{\alpha\beta}\,\nu_{eL}\,\overline{\nu}_{\mu R}\,t^{\alpha\beta}\,\mu_L\Big) + \mathrm{HC}
\end{aligned}
$$

$$
\mu_L \equiv \tfrac{1}{2}(1-\gamma^5)\mu \qquad \mu_R \equiv \tfrac{1}{2}(1+\gamma^5)\mu \qquad t_{\alpha\beta} \equiv \frac{1}{\sqrt{2}}\sigma_{\alpha\beta}
$$

- Fermi theory (SM): $g^V_{LL} = 1$, others 0
- g^V from admixtures of W_L, W_R, mixing with exotic fermions
- g^S (g^T) from spin-0 (2) exchange (e.g., SUSY $\not{R}_P$)
- Coefficients relatively real if CP (T) holds

- **Determine all parameters roughly to exclude alternatives to $V-A$**



$$|g^V_{LL}| > 0.960 \qquad |g^S_{LL}| < 0.550$$
$$|g^V_{RR}| < 0.034 \qquad |g^S_{RR}| < 0.067$$
$$|g^V_{LR}| < 0.036 \qquad |g^S_{LR}| < 0.088$$
$$|g^V_{RL}| < 0.104 \qquad |g^S_{RL}| < 0.417$$
$$|g^T_{LR}| < 0.025 \qquad |g^T_{RL}| < 0.104$$

$$g'^V = g^V \quad g'^S = \frac{g^S}{2} \quad g'^T = \sqrt{3}g^T$$

(Fetscher and Gerber, PDG05; Gagliardi et al., hep-ph/0509069)

- **Precise measurements for small deviations**

$$d\Gamma_{\mu^\mp} = \frac{G_F^2 m_\mu^5}{192\pi^3}\,\frac{D}{16}\,\epsilon^2\left(12(1-\epsilon)+\frac{4}{3}\rho(8\epsilon-6)\right.$$
$$\left.\mp P_\mu\xi\,\cos\theta\left[4(1-\epsilon)+\frac{4}{3}\delta(8\epsilon-6)\right]\right)d\epsilon\frac{d\cos\theta d\phi}{4\pi}$$

  - Also, m_e, radiative corrections, $e^\mp$ helicity terms
  - Michel spectral parameters (ρ,ξ,δ,η (in m_e)) and D are functions of $g_{ab}^{V,S,T}$
  - P_μ is μ polarization from $\pi^\mp \to \mu^\mp \overset{(-)}{\nu}$
  - $V-A:\ \rho=\delta=\frac{3}{4},\ \xi=1,\ \eta=0,\ D=16,\ P_\mu=1$
  - Precise measurements at SIN (PSI), TRIUMF, $\cdots$, and recently TWIST (TRIUMF)

- **PDG (2005)**

ρ	0.7518 ± 0.0026	$\frac{3}{4}$
η	-0.007 ± 0.013	0
δ	$0.7486 \pm 0.0026 \pm 0.0028$	$\frac{3}{4}$
$P_\mu\ \xi$	$1.0027 \pm 0.0079 \pm 0.0030$	1
$P_\mu\ \xi\delta/\rho$	> 0.99682 (90%)	1
P_{e^+}	1.00 ± 0.04	1

- **New TWIST**
  - $\rho = 0.75080 \pm 0.00032 \pm 0.00097 \pm 0.00023(\eta)$ ($|\zeta_{LR}| < 0.030$)
  - $\delta = 0.74964 \pm 0.00066 \pm 0.00112$ ($M_{W_R} g_L/g_R > 380$ GeV)
- **Other tests from leptonic $\tau \to \ell\nu\bar{\nu}$ decays, νe scattering**