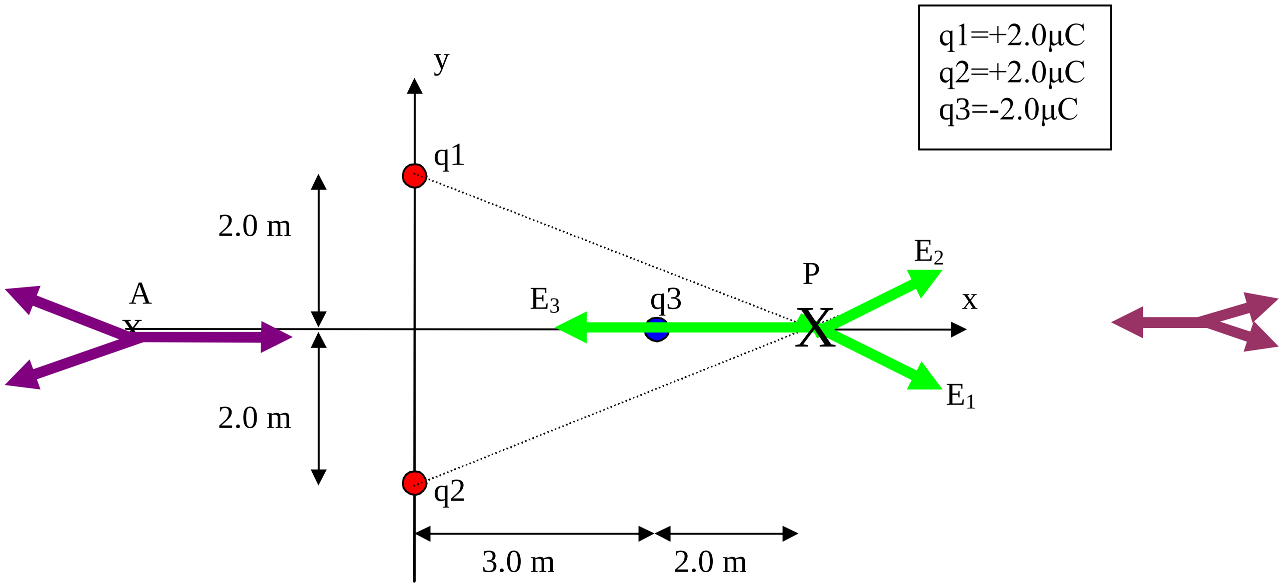
$\mu=10^{-6}$ $n=10^{-9}$ $\varepsilon_0=8.85\text{x}10^{-12}\ NC^{-2}m^2$

Solution: Quiz for January 19 2005 - Physics 151-001 - Prof. Thomson

(1) Three point charges are arranged in the *xy* plane as shown below. Charge q3, located on the x-axis 3.0m to the right of the origin, is negative. The other two charges, located on the y-axis 2.0m above and below the origin, are positive. All three stationary charges have an equal magnitude of 2.0 μC.



(a) Indicate the direction of the electric field from each stationary charge at point P. *See green lines in figure above!*

(b) Find the two components of the electric field in the *xy* plane at point P, which is located 5.0m to the right of the origin and 2.0m to the right of q3 on the x-axis.

i. Ex = ?

ii. Ey =?

(c) Calculate the magnitude of the force on a test charge of 1.0μC placed at point P.

(d) Considering only the three stationary charges, at how many points in the *xy* plane is the total electric field zero? Do not include points at infinity. (Hint: Do not try to solve this analytically! Try drawing the direction of the electric field from each stationary charge at several points, for example at point A and at point B.)

(b) Electric field at P is the VECTOR sum of the electric fields from q1, q2 and q3. Calculate each component separately.

(i) The x-component

$$E_x = E_{x1} + E_{x2} + E_{x3}$$

$$E_x = \frac{kq_1 \cos\alpha}{r_1^2} + \frac{kq_2 \cos\alpha}{r_2^2} + \frac{kq_3}{r_3^2}$$

$$= kq\left(\frac{2\cos\alpha}{r^2} - \frac{1}{r_3^2}\right) \qquad (1)$$

$$where\ r = r_1 = r_2 = \sqrt{(3+2)^2 + 2^2} = \sqrt{29}m$$

$$r_3 = 2.0m$$

$$q = q_1 = q_2 = -q_3 = 2\mu C$$

$$\cos\alpha = x / r = 5 / \sqrt{29}$$

$$k = 9x10^9 NC^{-2}m^2$$

$$Evaluation\ of\ E_x = -3.35x10^3 NC^{-1}$$

In version 1, r=√29 m, x=5 m q=2μC and Ex=-3.35x10^3 N/C.
In version 2, r=√58 m, x=7 m q=5μC and Ex=-9.82x10^3 N/C.
In version 3, r=√80 m, x=8 m q=7μC and Ex=-14.3x10^3 N/C.

(ii)The y-component. By symmetry, the electric field component in the y-direction is zero, since the electric fields from q1 and q2 have equal magnitude at point P and their y-components point in opposite directions and therefore cancel. The electric field from q3 has zero y-component at point P.

$$E_y = E_{y1} + E_{y2} + E_{y3}$$

$$= \frac{kq \sin\alpha}{r^2} - \frac{kq\ \sin\alpha}{r^2}$$

$$= 0$$

(c) Force on charge q is F=qE. We just worked out the electric field in part (b).

$$F_x = qE_x = 1x10^{-6}x(-3.35x10^3) = -3.35x10^{-3}N$$

$$F_y = qE_y = 0$$

Therefore, magnitude of |F| is 3.35x10^{-3}N and direction is along the –ve x-axis.

(d) In (b) we noticed that E_y=0 anywhere along the x-axis – this is the axis of symmetry of the problem. Due to the fast drop-off of the electric field strength with distance, at point P the electric field from the single –ve charge is stronger than the electric field from the more distant two +ve charges: the electric field points in the –ve x-direction. The net charge of the system is (+q+q-q)=+q, meaning that at large distances the field simplifies to that from a positive point charge. Mathematically, you can notice that for r>>a, the term $1/r^2$ is approximately the same as $1/r_3^2$ and $\cos\alpha = x/r$ tends to 1, so the expression for the electric field E_x in (1) above reduces to that for a single +ve point charge. Therefore, at some point along the positive x-axis, the direction of the resultant electric field must switch from pointing in the –ve direction to the +ve direction. At this point the electric fields from all three charges cancel and the total electric field is zero. You can make a similar argument for a point on the –ve x-axis. So there are two points where the electric field is zero. The purple arrows (not to scale!) indicate how the electric field from the three different charges cancels at two points on the x-axis.