A Quantitative Theory of the Credit Score

Satyajit Chatterjee (Philly Fed)
Dean Corbae (Wisconsin)
Kyle Dempsey (The Ohio State)
José-Víctor Ríos-Rull (Penn)

Prepared for Econ 8200, 2023,

Intro

What we do

- Credit scores are a central object in the allocation of credit.

- Credit scores are a central object in the allocation of credit.
- Yet they do not play a central role in macro models.

What we do

- Credit scores are a central object in the allocation of credit.
- Yet they do not play a central role in macro models.
- Here we provide a joint theory of unsecured credit and credit scores that is motivated by what we see in the world.

What we do

- Credit scores are a central object in the allocation of credit.
- Yet they do not play a central role in macro models.
- Here we provide a joint theory of unsecured credit and credit scores that is motivated by what we see in the world.

- Credit scores are a central object in the allocation of credit.
- Yet they do not play a central role in macro models.
- Here we provide a joint theory of unsecured credit and credit scores that is motivated by what we see in the world.
- In a world where agents have persistent unobservable characteristics, credit scores can help allocate credit among risky borrowers.

What we do

- Credit scores are a central object in the allocation of credit.
- Yet they do not play a central role in macro models.
- Here we provide a joint theory of unsecured credit and credit scores that is motivated by what we see in the world.
- In a world where agents have persistent unobservable characteristics, credit scores can help allocate credit among risky borrowers.
- We don't assume exogenous exclusion or stigma following default, but instead people's credit score falls and subsequently face worse borrowing terms.

What we do

- Credit scores are a central object in the allocation of credit.
- Yet they do not play a central role in macro models.
- Here we provide a joint theory of unsecured credit and credit scores that is motivated by what we see in the world.
- In a world where agents have persistent unobservable characteristics, credit scores can help allocate credit among risky borrowers.
- We don't assume exogenous exclusion or stigma following default, but instead people's credit score falls and subsequently face worse borrowing terms.

What we do

- Credit scores are a central object in the allocation of credit.
- Yet they do not play a central role in macro models.
- Here we provide a joint theory of unsecured credit and credit scores that is motivated by what we see in the world.
- In a world where agents have persistent unobservable characteristics, credit scores can help allocate credit among risky borrowers.
- We don't assume exogenous exclusion or stigma following default, but instead people's credit score falls and subsequently face worse borrowing terms.
- Our theory matches the data from unsecured consumer credit markets and the age profile of credit scores.

Our contribution

- We show there is a one-to-one mapping between type scores in our theory and a person's ranking in the credit score distribution in the data (equivalence result).

- We show there is a one-to-one mapping between type scores in our theory and a person's ranking in the credit score distribution in the data (equivalence result).

- We show there is a one-to-one mapping between type scores in our theory and a person's ranking in the credit score distribution in the data (equivalence result).

- After estimating parameters from data on the age profile of scores, we infer a considerable amount of pooling of unobservable type when young and partial separation over one's lifetime.

Our contribution

- We show there is a one-to-one mapping between type scores in our theory and a person's ranking in the credit score distribution in the data (equivalence result).

- After estimating parameters from data on the age profile of scores, we infer a considerable amount of pooling of unobservable type when young and partial separation over one's lifetime.

- We show there is a one-to-one mapping between type scores in our theory and a person's ranking in the credit score distribution in the data (equivalence result).

- After estimating parameters from data on the age profile of scores, we infer a considerable amount of pooling of unobservable type when young and partial separation over one's lifetime.

- We use our model to run counterfactuals to understand the role of hidden information on credit access and find:

- We show there is a one-to-one mapping between type scores in our theory and a person's ranking in the credit score distribution in the data (equivalence result).
- After estimating parameters from data on the age profile of scores, we infer a considerable amount of pooling of unobservable type when young and partial separation over one's lifetime.
- We use our model to run counterfactuals to understand the role of hidden information on credit access and find:
  - The median newborn prefers to live in our hidden info (baseline) economy than one with full info.

Our contribution

- We show there is a one-to-one mapping between type scores in our theory and a person's ranking in the credit score distribution in the data (equivalence result).
- After estimating parameters from data on the age profile of scores, we infer a considerable amount of pooling of unobservable type when young and partial separation over one's lifetime.
- We use our model to run counterfactuals to understand the role of hidden information on credit access and find:
    - The median newborn prefers to live in our hidden info (baseline) economy than one with full info.
    - Our baseline economy is also preferred by the median newborn to a hidden info economy where credit market behavior is not tracked.

- Signaling vs. Screening: Our framework combines both.

- Signaling vs. Screening: Our framework combines both.
  - As in screening models, lenders offer loans conditional on loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation.

- Signaling vs. Screening: Our framework combines both.
    - As in screening models, lenders offer loans conditional on loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation.
    - As in signaling models, there are actions a person can take that have no effect on the payoff to any lender (e.g. saving) but which convey valuable info to them about one's type.

- Signaling vs. Screening: Our framework combines both.
    - As in screening models, lenders offer loans conditional on loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation.
    - As in signaling models, there are actions a person can take that have no effect on the payoff to any lender (e.g. saving) but which convey valuable info to them about one's type.

Relation to some literature

- Signaling vs. Screening: Our framework combines both.
  - As in screening models, lenders offer loans conditional on loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation.
  - As in signaling models, there are actions a person can take that have no effect on the payoff to any lender (e.g. saving) but which convey valuable info to them about one's type.
- Unobservable Extreme Value Shocks:

Relation to some literature

- Signaling vs. Screening: Our framework combines both.
  - As in screening models, lenders offer loans conditional on loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation.
  - As in signaling models, there are actions a person can take that have no effect on the payoff to any lender (e.g. saving) but which convey valuable info to them about one's type.
- Unobservable Extreme Value Shocks:
  - Ensure lender beliefs following any feasible action are determined in eqm. (no need for off-eqm. beliefs)

- Signaling vs. Screening: Our framework combines both.
    - As in screening models, lenders offer loans conditional on loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation.
    - As in signaling models, there are actions a person can take that have no effect on the payoff to any lender (e.g. saving) but which convey valuable info to them about one's type.
- Unobservable Extreme Value Shocks:
    - Ensure lender beliefs following any feasible action are determined in eqm. (no need for off-eqm. beliefs)
    - Different types may choose the same action analogous to a "semi-separating" eqm. clouding inference on types.

- Signaling vs. Screening: Our framework combines both.
  - As in screening models, lenders offer loans conditional on loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation.
  - As in signaling models, there are actions a person can take that have no effect on the payoff to any lender (e.g. saving) but which convey valuable info to them about one's type.
- Unobservable Extreme Value Shocks:
  - Ensure lender beliefs following any feasible action are determined in eqm. (no need for off-eqm. beliefs)
  - Different types may choose the same action analogous to a "semi-separating" eqm. clouding inference on types.
  - Deliver tractability.

1. Model
2. Equilibrium
3. Taking the Model to Data
4. Model Properties
5. The Role of Persistent Hidden Information for Credit Access

Model

- Two types of HH with persistent hidden characteristics (patience).

- Two types of HH with persistent hidden characteristics (patience).
- Demographics enter into the evolution of type and earnings.

- Two types of HH with persistent hidden characteristics (patience).
- Demographics enter into the evolution of type and earnings.
- Hidden type and earnings endogenously affect the likelihood of default.

- Two types of HH with persistent hidden characteristics (patience).
- Demographics enter into the evolution of type and earnings.
- Hidden type and earnings endogenously affect the likelihood of default.
- Lenders try to infer a HH's type from their credit market actions, updating their assessments using Bayes' law.

Model basics

- Two types of HH with persistent hidden characteristics (patience).
- Demographics enter into the evolution of type and earnings.
- Hidden type and earnings endogenously affect the likelihood of default.
- Lenders try to infer a HH's type from their credit market actions, updating their assessments using Bayes' law.
- Lenders use those assessments of creditworthiness to price their loans.

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$
- transitory: additive, action-specific shocks $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$, GEV
  ▸ GEV

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$
- transitory: additive, action-specific shocks $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$, GEV

▸ GEV

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$
- transitory: additive, action-specific shocks $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$, GEV
  ▸ GEV

Earnings are comprised of 2 idiosyncratic components:

- persistent (observed): e, follows $Q^e(e'|e)$

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$
- transitory: additive, action-specific shocks $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$, GEV

▸ GEV

Earnings are comprised of 2 idiosyncratic components:

- persistent (observed): e, follows $Q^{e}(e'|e)$
- transitory (unobserved): z, drawn from $H(z)$

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$
- transitory: additive, action-specific shocks $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$, GEV

▸ GEV

Earnings are comprised of 2 idiosyncratic components:

- persistent (observed): e, follows $Q^{e}(e'|e)$
- transitory (unobserved): z, drawn from $H(z)$

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$
- transitory: additive, action-specific shocks $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$, GEV

▸ GEV

Earnings are comprised of 2 idiosyncratic components:

- persistent (observed): e, follows $Q^{e}(e'|e)$
- transitory (unobserved): z, drawn from $H(z)$

Each period, HH take **action** (d, a')

- a' : asset (or debt) position for next period

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$
- transitory: additive, action-specific shocks $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$, GEV

GEV

Earnings are comprised of 2 idiosyncratic components:

- persistent (observed): e, follows $Q^{e}(e'|e)$
- transitory (unobserved): z, drawn from $H(z)$

Each period, HH take **action** (d, a')

- a' : asset (or debt) position for next period
- $d \in \{0, 1\}$: default decision. If HH defaults ($d = 1$), then

Model environment: households

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$
- transitory: additive, action-specific shocks $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$, GEV

▸ GEV

Earnings are comprised of 2 idiosyncratic components:

- persistent (observed): e, follows $Q^{e}(e'|e)$
- transitory (unobserved): z, drawn from $H(z)$

Each period, HH take **action** (d, a')

- a' : asset (or debt) position for next period
- $d \in \{0, 1\}$: default decision. If HH defaults ($d = 1$), then
  - HH cannot save that period ($a' = 0$)

HH die with prob. $(1-\rho)$ with **preferences** $u(c)$, s.t. to 2 unobservable shocks

- persistent: discount rate $\beta \in \{\beta_L, \beta_H\} \sim Q^{\beta}(\beta'|\beta)$
- transitory: additive, action-specific shocks $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$, GEV

▸ GEV

Earnings are comprised of 2 idiosyncratic components:

- persistent (observed): e, follows $Q^{e}(e'|e)$
- transitory (unobserved): z, drawn from $H(z)$

Each period, HH take **action** (d, a')

- a' : asset (or debt) position for next period
- $d \in \{0, 1\}$: default decision. If HH defaults ($d = 1$), then
  - HH cannot save that period ($a' = 0$)
  - and pays filing cost κ ($c = e + z - \kappa$) → "static" punishment

- risk neutral, perfectly competitive (free entry)

Inference problem: can't observe β, z or $\epsilon^{(d,a')}$ to price loans:

Solution: assign a "type score", subjective prior $s = \Pr(\beta = \beta_H)$

Pricing: offer discount loans at prices $q^{(0,a')}(\omega)$, where

$$q^{(0,a')}(\omega) = \begin{cases} \frac{\rho p^{(0,a')}(\omega)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0, \end{cases}$$

where $p^{(0,a')}(\omega)$ is the prob of repayment on a loan of size $a' < 0$.

- risk neutral, perfectly competitive (free entry)
- borrow at exogenous interest rate r

Inference problem: can't observe β, z or $\epsilon^{(d,a')}$ to price loans:

Solution: assign a "type score", subjective prior $s = \Pr(\beta = \beta_H)$

Pricing: offer discount loans at prices $q^{(0,a')}(\omega)$, where

$$q^{(0,a')}(\omega) = \begin{cases} \frac{\rho p^{(0,a')}(\omega)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0, \end{cases}$$

where $p^{(0,a')}(\omega)$ is the prob of repayment on a loan of size $a' < 0$.

- risk neutral, perfectly competitive (free entry)
- borrow at exogenous interest rate r
- observe earnings/assets (e, a) and choices (d, a')

Inference problem: can't observe β, z or $\epsilon^{(d,a')}$ to price loans:

Solution: assign a "type score", subjective prior $s = \Pr(\beta = \beta_H)$

Pricing: offer discount loans at prices $q^{(0,a')}(\omega)$, where

$$q^{(0,a')}(\omega) = \begin{cases} \frac{\rho p^{(0,a')}(\omega)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0, \end{cases}$$

where $p^{(0,a')}(\omega)$ is the prob of repayment on a loan of size $a' < 0$.

- risk neutral, perfectly competitive (free entry)
- borrow at exogenous interest rate r
- observe earnings/assets (e, a) and choices (d, a')

Inference problem: can't observe β, z or $\epsilon^{(d,a')}$ to price loans:

- β persistent $\implies$ actions can signal type

Solution: assign a "type score", subjective prior $s = \Pr(\beta = \beta_H)$

Pricing: offer discount loans at prices $q^{(0,a')}(\omega)$, where

$$q^{(0,a')}(\omega) = \begin{cases} \frac{\rho p^{(0,a')}(\omega)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0, \end{cases}$$

where $p^{(0,a')}(\omega)$ is the prob of repayment on a loan of size $a' < 0$.

- risk neutral, perfectly competitive (free entry)
- borrow at exogenous interest rate r
- observe earnings/assets (e, a) and choices (d, a')

Inference problem: can't observe β, z or $\epsilon^{(d,a')}$ to price loans:

- β persistent $\implies$ actions can signal type
- z, ϵ transitory $\implies$ adds confusion

Solution: assign a "type score", subjective prior $s = \Pr(\beta = \beta_H)$

Pricing: offer discount loans at prices $q^{(0,a')}(\omega)$, where

$$q^{(0,a')}(\omega) = \begin{cases} \frac{\rho p^{(0,a')}(\omega)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0, \end{cases}$$

where $p^{(0,a')}(\omega)$ is the prob of repayment on a loan of size $a' < 0$.

- risk neutral, perfectly competitive (free entry)
- borrow at exogenous interest rate r
- observe earnings/assets (e, a) and choices (d, a')

Inference problem: can't observe β, z or $\epsilon^{(d,a')}$ to price loans:

- β persistent $\implies$ actions can signal type
- z, ϵ transitory $\implies$ adds confusion
  - $\epsilon \sim$ GEV $\implies$ all budget feasible actions chosen with prob > 0

Solution: assign a "type score", subjective prior $s = \Pr(\beta = \beta_H)$

Pricing: offer discount loans at prices $q^{(0,a')}(\omega)$, where

$$q^{(0,a')}(\omega) = \begin{cases} \frac{\rho p^{(0,a')}(\omega)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0, \end{cases}$$

where $p^{(0,a')}(\omega)$ is the prob of repayment on a loan of size $a' < 0$.

- risk neutral, perfectly competitive (free entry)
- borrow at exogenous interest rate r
- observe earnings/assets (e, a) and choices (d, a')

Inference problem: can't observe β, z or $\epsilon^{(d,a')}$ to price loans:

- β persistent $\implies$ actions can signal type
- z, ϵ transitory $\implies$ adds confusion
  - $\epsilon \sim$ GEV $\implies$ all budget feasible actions chosen with prob > 0

Solution: assign a "type score", subjective prior $s = \Pr(\beta = \beta_H)$

- update via Bayes rule using observables (d, a') and $\omega = (e, a, s)$ to revise posterior score $\psi^{(d,a')}(\omega)$

Pricing: offer discount loans at prices $q^{(0,a')}(\omega)$, where

$$q^{(0,a')}(\omega) = \begin{cases} \frac{\rho p^{(0,a')}(\omega)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0, \end{cases}$$

where $p^{(0,a')}(\omega)$ is the prob of repayment on a loan of size $a' < 0$.

1. HH begin period with state (β, e, a, s)
2. HH receive transitory earnings and preference shocks
   - $z \sim H(z)$
   - $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$
3. Given price schedule $q = \{q^{(0,a')}(\omega)\}$, agents choose (d, a')
4. Intermediaries revise type scores from $s \to \psi^{(d,a')}(\omega)$ via Bayes rule.
5. Next period states are drawn:
   - Individuals who survive with prob ρ draw $\beta' \sim Q^{\beta}(\beta'|\beta)$, $e' \sim Q^{e}(e'|e)$, and $s' \sim Q^{s}(s'|\psi)$.
   - Newborns draw $\beta' \sim G_{\beta}$, $e' = \underline{e}$, $a' = 0$, and s' consistent with G_{β}.

Estimation, Demographics, Both

Timing

1. HH begin period with state (β, e, a, s)
2. HH receive transitory earnings and preference shocks
   - $z \sim H(z)$
   - $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$
3. Given price schedule $q = \{q^{(0,a')}(\omega)\}$, agents choose (d, a')
4. Intermediaries revise type scores from $s \rightarrow \psi^{(d,a')}(\omega)$ via Bayes rule.
5. Next period states are drawn:
   - Individuals who survive with prob ρ draw $\beta' \sim Q^{\beta}(\beta'|\beta)$, $e' \sim Q^{e}(e'|e)$, and $s' \sim Q^{s}(s'|\psi)$.
   - Newborns draw $\beta' \sim G_{\beta}$, $e' = \underline{e}$, $a' = 0$, and s' consistent with G_{β}.

Estimation, Demographics, Both

Timing

1. HH begin period with state (β, e, a, s)
2. HH receive transitory earnings and preference shocks
    - $z \sim H(z)$
    - $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$
3. Given price schedule $q = \{q^{(0,a')}(\omega)\}$, agents choose (d, a')
4. Intermediaries revise type scores from $s \rightarrow \psi^{(d,a')}(\omega)$ via Bayes rule.
5. Next period states are drawn:
    - Individuals who survive with prob ρ draw $\beta' \sim Q^{\beta}(\beta'|\beta)$, $e' \sim Q^{e}(e'|e)$, and $s' \sim Q^{s}(s'|\psi)$.
    - Newborns draw $\beta' \sim G_{\beta}$, $e' = \underline{e}$, $a' = 0$, and s' consistent with G_{β}.

Estimation, Demographics, Both

Timing

1. HH begin period with state (β, e, a, s)
2. HH receive transitory earnings and preference shocks
   - $z \sim H(z)$
   - $\epsilon = \{\epsilon^{(d,a')}\} \sim F(\epsilon; \alpha, \lambda)$
3. Given price schedule $q = \{q^{(0,a')}(\omega)\}$, agents choose (d, a')
4. Intermediaries revise type scores from $s \to \psi^{(d,a')}(\omega)$ via Bayes rule.
5. Next period states are drawn:
   - Individuals who survive with prob ρ draw $\beta' \sim Q^{\beta}(\beta'|\beta)$, $e' \sim Q^{e}(e'|e)$, and $s' \sim Q^{s}(s'|\psi)$.
   - Newborns draw $\beta' \sim G_{\beta}$, $e' = \underline{e}$, $a' = 0$, and s' consistent with G_{β}.

Estimation, Demographics, Both

Equilibrium

Definition
A stationary **recursive competitive equilibrium (RCE)** comprises:

- pricing function $q^*(\omega)$ (vector-valued)
- type scoring function, $\psi^*(\omega)$ (vector-valued)
- choice probability function, $\sigma^*(\beta, z, \omega)$ (vector-valued)
- steady state distribution, $\mu^*(\beta, z, \omega)$

such that

- $\sigma^*(\beta, z, \omega)$ consistent with HH dynamic optimization ▸ HH
- $\psi^*(\omega)$ satisfies Bayes' Rule ▸ score
- $q^*(\omega)$ implies lenders break even, with repayment probabilities $p^{(0,a')}(\omega)$ implied by σ^* ▸ price
- $\mu^*(\beta, z, \omega)$ is a fixed point of $\mu' = T^*\mu$ ▸ Distn

Theorem
Existence: *There exists a RCE.*

Theorem
Existence: *There exists a RCE.*

- We show how a RCE with *type scores* s is equivalent to the stationary eqm (RCECS) of an economy where prices depend on *credit scores*.
- Credit scores just the prob. of repayment $m \equiv p^{(0,\bar{a})}(e, a, s)$ on loan $\bar{a}$.

Theorem

Equivalence: *If there exists a RCE and there is a one-to-one mapping between type scores* s *and credit scores* m*, then there exists a RCECS.*

▸ 1to1

Mapping the Model to Data

- A credit score is an *ordinal* measure of creditworthiness (i.e. What does 760 really mean?).

- To take the theory to data, we associate with $p^{(0,\bar{a})}(\omega) \in [0,1]$ a number that gives $p^{(0,\bar{a})}(\omega)$'s position (ranking) in the overall distn. (i.e. the fraction of people with lower credit scores)

- We check that rankings preserve order (i.e. do not depend on the specification of $\bar{a}$, which we take to be 3.5% of median earnings).

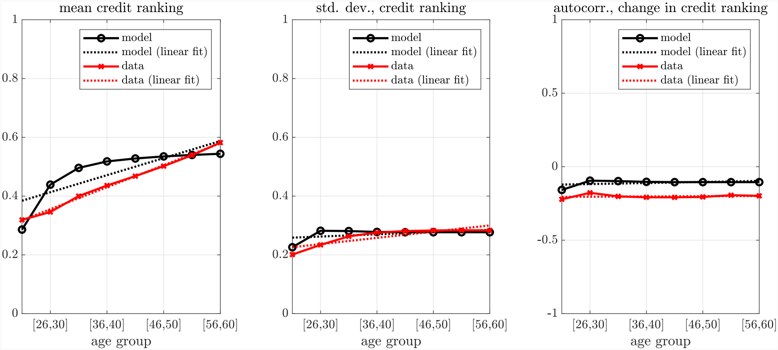
Parameter		Value	Notes (annual, 2 types)
Demographics and preferences			
survival probability	ρ	0.975	avg. 20-60 age profile
risk aversion	γ	3	CRRA preferences
persistent earnings at birth	$\underline{e}$	0.57	lowest e level
Technology			
risk-free rate (%)	r	1.0%	real annual T-Bill
filing cost	κ	0.02	2% median earnings
debt level for computing credit score	$\bar{a}$	-0.035	3.5% median earnings
Earnings processes			
persistence of log(e)	ρ_e	0.914	Floden and Linde (2001)
std. dev. of innovation to log(e)	ν_e	0.043	Floden and Linde (2001)
std. dev. of log(z)	ν_z	0.042	Floden and Linde (2001)

Estimated Parameters

Parameter		Value
Evolution of types		
high β	β_H	0.915
low β	β_L	0.886
high→low prob.	$Q^{\beta}_{HL'}$	0.011
low→high prob.	$Q^{\beta}_{LH'}$	0.013
frac β_H newborns	G_{β_H}	0.280
Extreme value parameters		
EV scale (1e-3)	α	3.387
EV correlation	λ	0.991

Moment (%)	Data	Model
Aggregate credit market statistics		
default rate	0.99	0.98
avg. interest rate	12.87	13.95
IR dispersion	6.56	7.24
fraction in debt	10.43	10.50
debt to income	0.35	0.25
Credit score life cycle moments		
intercept, mean	0.281	0.355
slope, mean	0.037	0.029
intercept, std. dev.	0.216	0.255
slope, std. dev.	0.011	0.004
avg autocorr of change	-0.202	-0.109

▸ Identification



- Mean and standard deviation of rankings rise over age
- Autocorrelation of changes fall (i.e. mean reversion of rankings).

About hidden type:

- Data consistent with small type differences $\beta_H = 0.915 > 0.886 = \beta_L$.
- Types are persistent $> 98\%$ and more likely to switch to high.
- Fraction of newborn high types is 0.28, lower than stationary fraction 0.44 and long run 0.56 (i.e. scores grow over time).

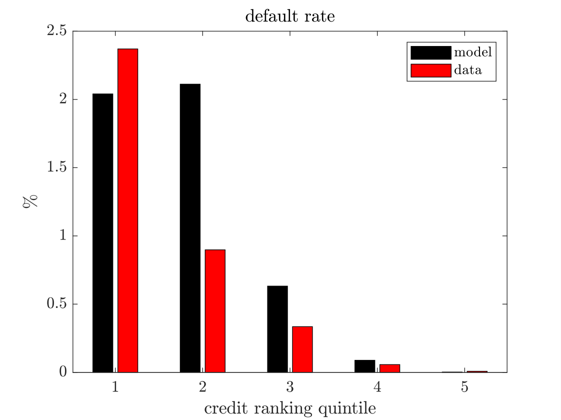
About EV shocks:

- Low variance of shocks ($\alpha = 0.003$) and high correlation ($\gamma = 0.99$) implies decisions are concentrated at the modal choice (e.g. share of modal defaulters is 87%).

Insight: Relation between $\beta_H - \beta_L$ and α important for inference problem.

- If $\beta_H - \beta_L$ large, then easy to separate types and would need large α to cloud inference.

Default rate by credit ranking quintiles (untargeted)



- Higher credit rankings have lower default rates.
- Many low earners in model Q2 leading to higher default rates than the data.

Event Study

Model Properties

Model Properties

1. As in the data, earnings, wealth, scores, cross-sectional consumption variance rise with age. ▸ Age

2. Type L default and borrow more, save less (i.e. riskier). ▸ DBS

3. Default falls with higher e, higher type β, lower debt. ▸ Default

4. Dynamic Scoring: scores fall with default and more debt, rise if pay off debt, and are mean reverting. ▸ DynScore

5. Adverse Selection: the price of not using dynamic scoring yields a riskier pool of borrowers. ▸ AS

6. Higher scores lower interest rates for given debt and more debt makes rates more sensitive to one's score. ▸ Rates

7. Conditional on default, interest rates rise. ▸ drates

8. Signalling: Type L mimicking type H is costly & grows. ▸ Mimic

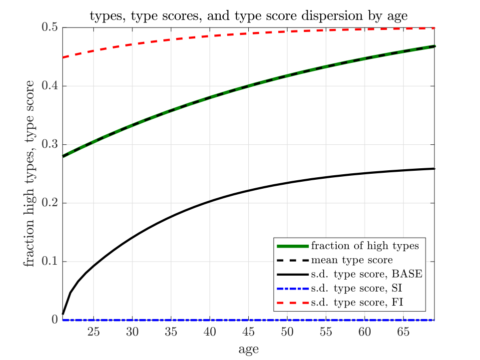
Role of Information for Credit Access

Compare with two alternative info structures:

1. **Full information (FI)**
   - Type is observable $\implies$ no inference problem $\implies$ no type scores.
   - History does not matter for pricing as in CCNR (2007).
   - Actual type directly affects prices, $q_{FI}^{(0,a')}(\beta, e)$.

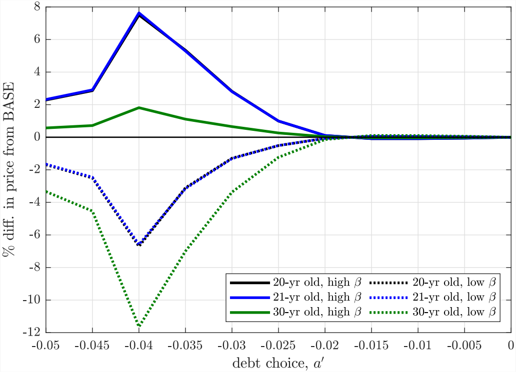
2. **No tracking (NT)**
   - Lenders cannot use the history of actions to track type so type score only reflects one's age (length of credit history).
   - Incentives are weakened for good behavior but pooling provides partial insurance (tradeoff $\longrightarrow$ quantitative question).
   - Price function $q_{NT}^{(0,a')}(age, e)$.

Joint distribution of types and scores across info structure



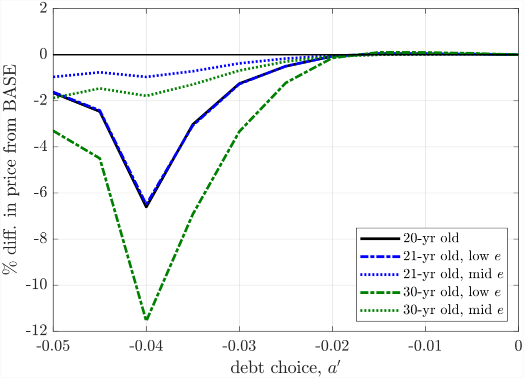
- All info structures have the same mean score given by fraction of H types: $\bar{s}' = \bar{s} \cdot Q^{\beta}(H|H) + (1 - \bar{s}) \cdot Q^{\beta}(H|L)$ over one's life with estimated initial fraction $G_{\beta_H} = 0.28$.
- Dispersion in scores weakly rise with age.
- No dispersion in NT (where type histories are pooled so nothing is learned) and highest dispersion in FI (because they are perfectly separated).

Information Effects on Prices: FI vs. BASE



- Type H face higher prices (lower rates) in FI than BASE especially when young.
- Riskier type L face lower prices (higher rates) in FI than BASE especially when old.

Incentive vs Insurance Effects on Prices: NT vs. BASE



- Lack of incentives to build a reputation in NT lead to higher interest rates than BASE.
- As time passes, earnings and adverse selection play more of a role; if an individual wants to borrow a lot, they are more likely to be type L who have not accumulated enough wealth (so face higher rates).

Credit market outcomes across info structures

	% difference relative to BASE					
model	No Tracking (NT)			Full Info (FI)		
subgroup	high β	low β	all	high β	low β	all
volume of debt	-4.20	-3.16	-3.50	+6.28	-3.14	-0.08
debt to income	-4.02	-2.97	-3.32	+5.77	-2.95	-0.09
default rate	-9.03	-6.85	-7.52	+12.2	-6.84	-1.00

Important differences:

- Type H have more debt in FI (since they face lower rates) and less debt in NT than in BASE (since they are pooled with riskier type L).
- Type L have less debt (which induces less default) in both FI and NT.
- Aggregate volume of debt and default lower in NT (negative consequences consumption smoothing).

Consumption Equivalent gain relative to BASE for newborns (in %)

model	No Tracking (NT) mean	Full Info (FI) mean
high β	-0.41	+1.04
low β	-0.55	-0.47
all	-0.51	-0.05

Important differences between young (who want to borrow against future income) and old (who have accumulated assets):

- FI: type L hurt, type H helped by more separation. Avg newborn hurt.
- NT: incentive effects dominate pooling, newborns especially hurt (1/2 of 1 percent is big in Lucas terms).

Conclusion

- We show how full info models of unsecured debt with risk of default can easily be extended to handle hidden info via EV shocks to preferences and recursive updating of type scores.

- We show how full info models of unsecured debt with risk of default can easily be extended to handle hidden info via EV shocks to preferences and recursive updating of type scores.
- We infer from data on the age profile of scores a considerable amount of pooling of persistent hidden type when young and separation over one's lifetime which can have quantitatively important effects on welfare early in the age profile.

- We show how full info models of unsecured debt with risk of default can easily be extended to handle hidden info via EV shocks to preferences and recursive updating of type scores.
- We infer from data on the age profile of scores a considerable amount of pooling of persistent hidden type when young and separation over one's lifetime which can have quantitatively important effects on welfare early in the age profile.
- Since the young tend to borrow against their future income and the old tend to accumulate precautionary balances, less info (i.e. "small data" NT economy) worsens incentives to repay (raising interest rates) which dominates insurance afforded by partial pooling, yielding lower welfare than a "big data" (BASE) economy.

- We show how full info models of unsecured debt with risk of default can easily be extended to handle hidden info via EV shocks to preferences and recursive updating of type scores.
- We infer from data on the age profile of scores a considerable amount of pooling of persistent hidden type when young and separation over one's lifetime which can have quantitatively important effects on welfare early in the age profile.
- Since the young tend to borrow against their future income and the old tend to accumulate precautionary balances, less info (i.e. "small data" NT economy) worsens incentives to repay (raising interest rates) which dominates insurance afforded by partial pooling, yielding lower welfare than a "big data" (BASE) economy.
- The intersection of information and demographics matters!

Appendix

HH problem

$$V(\beta, z, \omega, \epsilon) = \max_{(d,a')\in\mathcal{F}(z,\omega)} \underbrace{v^{(d,a')}(\beta,\omega)}_{\text{fundamental value}} + \underbrace{\epsilon^{(d,a')}}_{\sim\text{GEV}}$$

where the conditional value function is given by

$$\begin{aligned} v^{(d,a')}(\beta, z, \omega) \quad &= \quad (1-\beta\rho)u\left(c^{(d,a')}(z,\omega)\right) \\ +\beta\rho \sum_{\beta',z',\omega'} \quad & \quad Q^{\beta}(\beta'|\beta)Q^{e}(e'|e)H(z')Q^{s}(s'|\psi^{(d,a')})W\left(\beta',z',\omega'\right), \end{aligned}$$

budget feasibility $(d,a') \in \mathcal{F}(z,\omega) \iff c^{(d,a')}(z,\omega) \geq 0$ given by

$$c^{(d,a')}(z,\omega) = \begin{cases} e+z+a-q^{(0,a')}(\omega)\cdot a' & \text{if } (d,a')=(0,a') \\ e+z-\kappa & \text{if } a<0 \text{ and } (d,a')=(1,0) \end{cases},$$

and the expected future value function is given by

$$W(\beta,z,\omega) \equiv \int V(\beta,z,\omega,\epsilon)dF(\epsilon).$$

Extreme value shocks generate simple choice probabilities:

Probability of Default: If $a < 0$,

$$\sigma^{(\mathbf{1},\mathbf{0})}(\beta, z, \omega) \quad = \quad \frac{\exp\left\{v^{(\mathbf{1},\mathbf{0})}(\beta, z, \omega)/\alpha\right\}}{\exp\left\{v^{(\mathbf{1},\mathbf{0})}(\beta, z, \omega)/\alpha\right\} + \exp\left\{\lambda W_{ND}(\beta, z, \omega)/\alpha\right\}}$$

and 0 otherwise, where

$$W_{ND}(\beta, z, \omega) = \alpha \log \left(\sum_{(\mathbf{0},\tilde{a}) \in \mathcal{F}(z,\omega)} \exp\left\{v^{(\mathbf{0},\tilde{a})}(\beta, z, \omega)/\lambda\alpha\right\} \right)$$

Conditional on not defaulting, a HH chooses $(0, a') \in \mathcal{F}(z, \omega)$ via via

$$\tilde{\sigma}^{(\mathbf{0},a')}(\beta, z, \omega) = \frac{\exp\left\{v^{(\mathbf{0},a')}(\beta, z, \omega)/\lambda\alpha\right\}}{\sum_{(\mathbf{0},\tilde{a}) \in \mathcal{F}(z,\omega)} \exp\left\{v^{(\mathbf{0},\tilde{a})}(\beta, z, \omega)/\lambda\alpha\right\}},$$

with $\tilde{\sigma}^{(\mathbf{0},a')}(\beta, z, \omega) = 0$ if $(0, a') \notin \mathcal{F}(z, \omega)$. Back

- Type Scores from an action $(d, a') \in \mathcal{F}(z, \omega)$ are updated via Bayes' Rule via:

$$
\begin{aligned}
\psi_{\beta'}^{(d,a')}(\omega) &\equiv \Pr\left(\beta' | (d, a'), \omega\right) \\
&= \sum_{\beta} Q^{\beta}(\beta'|\beta) \frac{\sum_z \sigma^{(d,a')}(\beta, z, \omega) H(z) s(\beta)}{\sum_{\tilde{\beta}, z} \sigma^{(d,a')}(\tilde{\beta}, z, \omega) H(z) s(\tilde{\beta})}
\end{aligned}
$$

▸ Back

Loan Pricing and Repayment Probabilities

- The prob. of repayment on a loan a' given to an agent in observable state ω is:

$$
\begin{aligned}
p^{(\mathbf{0},a')}(\omega) &\equiv \Pr\left(d' \neq 1 | a' < 0, \omega\right) \\
&= \sum_{\beta', z', e', s'} H(z') \cdot Q^e(e'|e) \cdot Q^s(s'(\beta')|\psi_{\beta'}^{(\mathbf{0},a')}(\omega)) \cdot s'(\beta') \\
&\quad \cdot \left(1 - \sigma^{(\mathbf{1},\mathbf{0})}(\beta', z', e', a', s')\right).
\end{aligned}
$$

- The price function is:

$$
q^{(0,a')}(\omega) = \begin{cases} \frac{\rho p^{(\mathbf{0},a')}(\omega)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0, \end{cases}
$$

▸ Back

Cross-sectional Distribution

$$\mu'(\beta', z', \omega') = \sum_{\beta, z, \omega} T(\beta', z', \omega' | \beta, z, \omega) \cdot \mu(\beta, z, \omega), \tag{1}$$

where

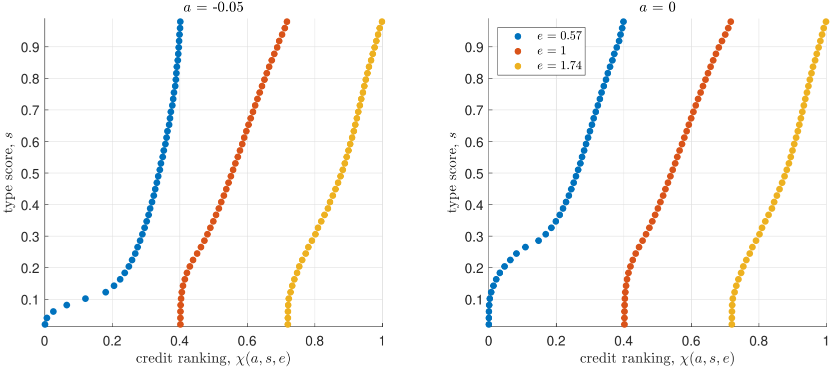
$$T(\beta', z', \omega'; \beta, z, \omega) = \tag{2}$$

$$\rho \cdot Q^{\beta}(\beta'|\beta) \cdot H(z') \cdot Q^{e}(e'|e) \cdot \sigma^{(d,a')}(\beta, z, \omega) \cdot Q^{s}(s'(\beta')|\psi_{\beta'}^{(d,a')}(\omega))$$

$$+ (1-\rho) \cdot G_{\beta}(\beta') \cdot H(z') \cdot G_{e}(e') \cdot \mathbf{1}_{\{a'=\mathbf{0}\}} \cdot \mathbf{1}_{\{s'=G_{\beta}\}}.$$

▸ Back

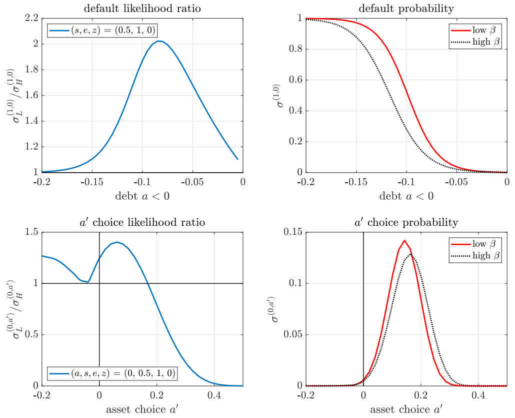
Mapping between type scores and credit scores



- One-to-one (invertibility satisfied).
- Low earners have low rankings and current assets only important for "subprime" low earners.

▸ Back

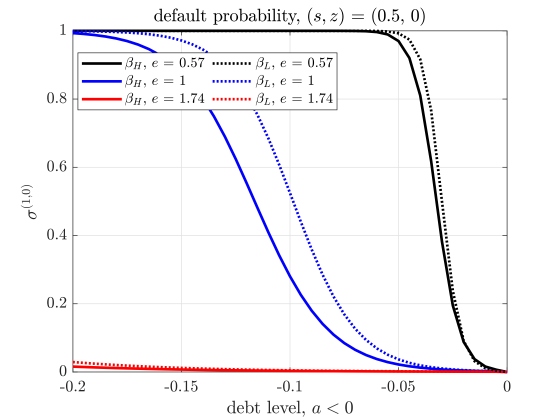
Default and borrowing / saving decisions



- β_L more likely to default and borrow, less likely to save.
- Non-monotonicity in debt choice occurs at actions w.p.1^{-8}.

Back

Factors in default

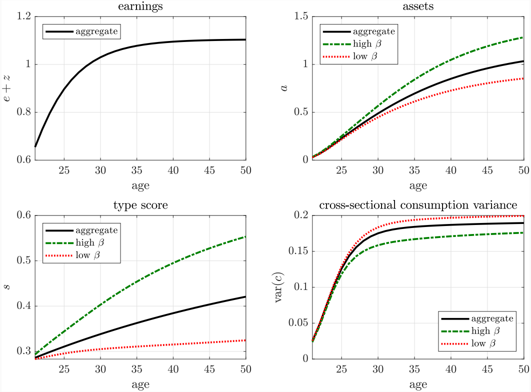


- Default falls with higher e (critical), higher type β, lower debt.

Back

Moments by Age

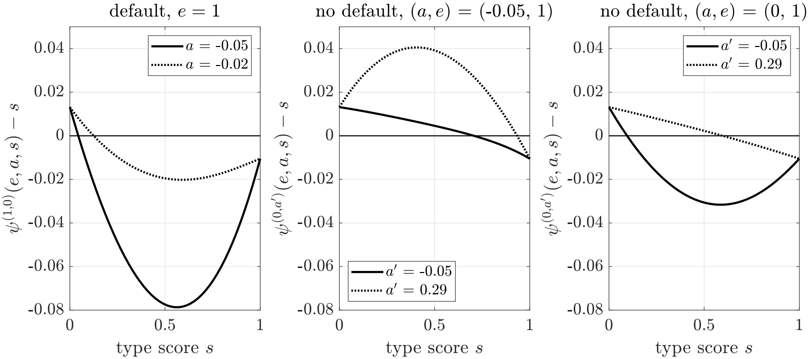
Figure 1: Moments by Age in Baseline Model



- Earnings induces wealth, scores induce cross-sect. cons. var. (rising c.c.v. age profile as in data - HPV(2010))

Rankings by Age Back

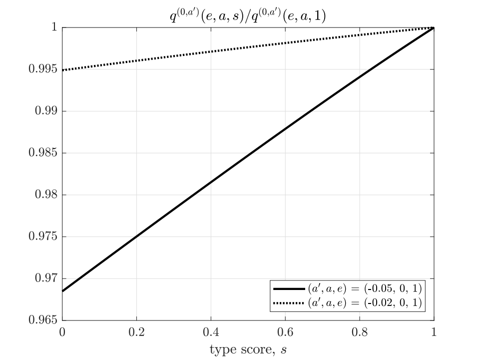
Dynamic scoring responses



- Real world credit scores: (i) are mean reverting (all) (ii) fall in bankruptcy (lhs); (iii) rise if one pays off one's debt (mid), and (iv) fall as one takes on more debt (rhs).

▸ Back

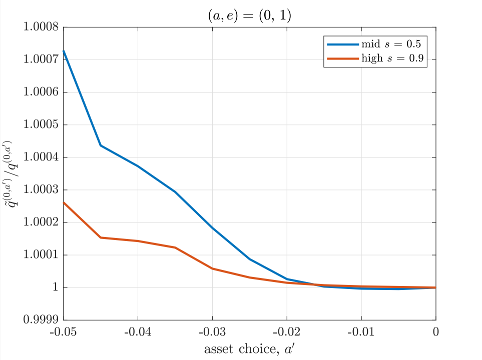
Reputation and prices



- Relative price of unsecured credit across type score: higher score → higher relative price (lower interest rate) and more debt → more sensitivity to score.

▸ Back

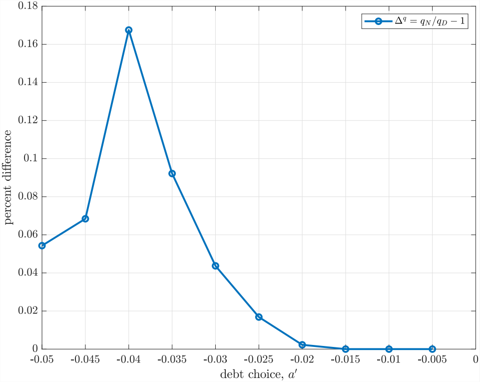
Adverse selection effects



- AS: the bigger the loan, the higher the prob. lenders assign to the borrower being less creditworthy.

- The relative price of not using dynamic scoring yields a riskier (type L) pool of borrowers.
  Back

Average price impact of default



- Given extreme value shocks, there are always two identical agents (i.e. in the same state), one of whom defaults and another who does not.
- Can quantify the impact of default on interest rates by comparing price schedules in the following period conditional on the default choice.
- $q_N - q_D > 0$: rates fall relative to a defaulter.

Back

Static Signalling Costs versus Benefits

	% Average Gain in:		
	Consumption ($\widehat{C}$)	**Wealth** ($\widehat{A}$)	**Reputation** ($\widehat{\psi}$)
All	-3.78	6.09	5.10
Newborns	-0.84	15.87	3.73

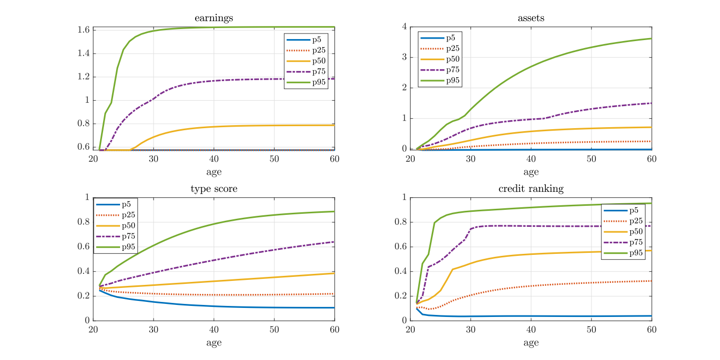
- AS: "bad" types (L) want to mimic "good" types (H) to get better borrowing terms while type H want to separate themselves from L.

- What's the $\%\Delta$ to consumption ($\downarrow$) and reputation ($\uparrow$) if type L follows $\sigma^{(d,a')}(\beta_H, z, \omega)$ instead of $\sigma^{(d,a')}(\beta_L, z, \omega)$?

- Mimicking loss costly for young since there is more pooling.

▸ Back

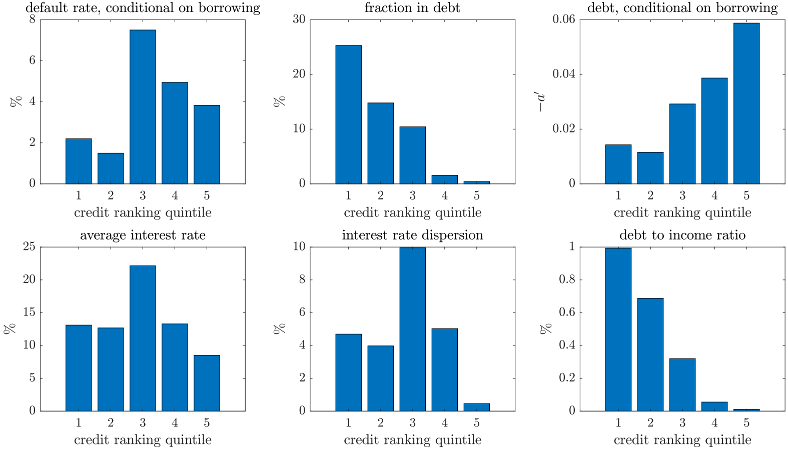
How do credit rankings vary by age?



- (tl) earnings grow with age inducing assets growing with age (tr),
- (bl) type score grows with age on average (consistent with $Q^{\beta}_{LH'} > Q^{\beta}_{HL'}$) but for some it falls with age (those whose actions reveal they are type L) inducing a similar pattern for rankings (br). Rank by Outcome
- Takeaway: More pooling when young, more separation when old.

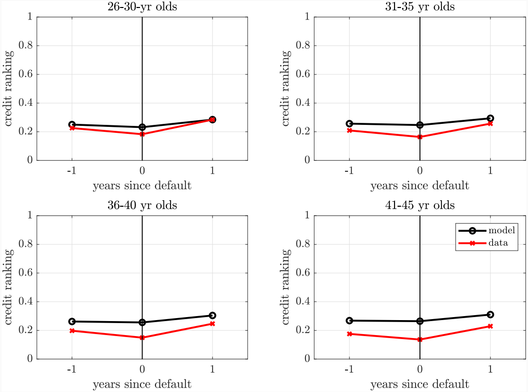
Back

How do credit outcomes vary by credit ranking?



- ▸ Back

Default event study (untargeted)



- Bankrupts are in bottom quartile of the credit score distn.
- Decline and subsequent recovery consistent with mean reversion as in Musto (2004).

Back

Identification

While all moments jointly identify the parameters, we find (see sensitivity analysis in Appendix):

- Affine age profile of credit rankings helps identify the type transition process:
  - Constant (and slope) of avg. ranking $\longrightarrow G_{\beta_H}$ (and $Q^{\beta}(H'|L) - Q^{\beta}(L'|H)$)
  - Constant (and slope) of st. dev. ranking $\longrightarrow$ extreme value parameter λ (and Q^{β})
  - Neg. autocorrelation of changes (mean reversion) $\longleftarrow$ earnings process.
- Aggregate moments help identify:
  - Default rate $\longrightarrow$ extreme value parameter α
  - Average interest rate $\longrightarrow \beta_H$
  - Interest rate dispersion $\longrightarrow \beta_H - \beta_L$

Back

Generalized Extreme Value Distribution

GEV cumulative distn. given by:

$$F(\epsilon_t;\alpha,\lambda) = \exp\left\{ \begin{array}{c} -\left[\sum_{a_{t+1}\in\mathcal{A}} \exp\left(-\frac{\epsilon(0,a_{t+1})-\mu}{\lambda\alpha}\right)\right]^{\lambda} \\ -\exp\left(-\frac{\epsilon(1,0)-\mu}{\alpha}\right) \end{array} \right\}$$

- $\alpha > 0$ governs the variance of the shocks
  - $\uparrow \alpha \implies \uparrow$ variance: decisions more "random"
- $\lambda \in [0,1]$ determines correlation between shocks associated with non-default actions
  - $\uparrow \lambda \implies \downarrow$ correlation: decisions more "random" *conditional on not defaulting*
  - $\lambda \cdot \alpha$ matters
- $\mu = -\alpha\gamma_E$ set to make the shocks mean zero in expectation

▸ return