The Environment for Wealth, Wages, and Employment: A Representative Agent Version

Preliminary

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Baseline Model: Theory

Order of Events

1. A Rep Agent enters period with assets s and of members $x = \{\{x(w)\}_{w \in \mathcal{W}}, x(n)\}$ (employed at wage w or not-employed). $\int_{\mathcal{W}} dx$ consume what told c and $x(n)$ get b. Note that $\int_{\mathcal{W}} dx + x(n) = 1$.
2. Production & Consumption: Hhold heads collect $\int_{\mathcal{W}} w \; \mathrm{d}x(w)$ and choose c, a'.
3. Firm Destruction and Quits: Some Firms are destroyed at rate δ^f. Some workers quit to unemployed for exogenous reasons δ^h and also quit or switch jobs. Exogenous job destruction $\delta = 1 - (1 - \delta^h)(1 - \delta^f)$.
4. Whether to Search or not or Quit (Emp) or search or not (Nonemp):
   - **Workers** get EVS shocks $\{\eta^e, \eta^s, \eta^n\}, \; \eta \sim G\left(\mu^\eta, \alpha^\eta\right)$. Some employed choose to do nothing and remain working tomorrow in the same job. Others choose to to search for a job and yet others quit and become non-employed.
   - **Non-employed** get EVS shocks $\{\chi^o, \chi^u\}, \; \chi \sim G\left(\mu^\chi, \alpha^\chi\right)$. Some non-employed choose to do nothing and remain non-employed tomorrow while others choose to search for a job. These searchers are the unemployed.
5. Where to Search: Those that search get shocks $\{\epsilon^{w'}\}$, $\epsilon \sim G\left(\mu^\epsilon, \alpha^\epsilon\right)$ that shapes their choice of where to search $\{w', \theta(w')\}$.
6. Job Matching: $M(V, S)$: Some vacancies meet some job searchers. A match becomes operational the following period. Job finding and job filling rates $\psi^h(\theta) = \frac{M(V,S)}{S}$, $\psi^f(\theta) = \frac{M(V,S)}{V}$.

Stationary Analysis

Who chooses what and Why

- Household head chooses savings and consumption of the employed
- After production and consumption household members can be either working at various wages $w \in \mathcal{W}$ or non-employed, n.

  1. Workers draw shocks $\{\eta^e, \eta^s, \eta^n\}$, $\eta \sim G(\mu^\eta, \alpha^\eta)$, and they choose whether to not search, or to search and not quit (and if successfull switch jobs) or to quit
  2. Non-employed workers draw EVS shocks $\{\chi^o, \chi^u\}$, $\chi \sim G(\mu^\chi, \alpha^\chi)$. Some non-employed choose to do nothing and stay out of the labor force tomorrow while others choose to search for a job. These searchers are the unemployed.

- This is equivalent (proved somewhere else) to the household head choosing everything so we take advantage to the fact that it is simpler separating the decision making while ignoring the manipulation terms that would show up if the problems were not equivalent.

Household Head's Problem

State $\{a, x\}$, $x = \{x(\omega)\}_{\omega \in \{\mathcal{W} \cup n\}}$. Take as given decision job densities $h(\omega', a, x, \omega)$.

$$V(a,x) = \max_{a'} \; [1 - x(n)] \; U\left[\frac{(1+r)a + \int_{w \in \mathcal{W}} w \; dx - a'}{1 - x(n)}\right] + x(n) \; U(b) + \beta \; V(a', x')$$

$$x'(w) = (1-\delta^f) \; x(w) \; \left\{ \underbrace{\left[1 - \int_{\widehat{w} \in \mathcal{W}} \psi^h[\widehat{\theta}(w)] \; h(d\widehat{w}, a, x, w)\right]}_{\text{Not Switching to other Jobs}} \underbrace{- h(n, a, x, w)}_{\text{Not Quitting}} \right\}$$

$$+ \underbrace{(1-\delta^f) \int_{\widehat{w} \in \mathcal{W}} h(w, a, x, \widehat{w}) \; \psi^h[\theta(w)] \; dx(\widehat{w})}_{\text{Switchers from other Jobs}} + \underbrace{\psi^h[\theta(w)] \; h(w, a, x, n) \; x^n}_{\text{Hired from Unemp}}$$

$$x'(n) = \underbrace{\delta^f x(\mathcal{W})}_{\text{Layoffed}} + \underbrace{(1-\delta^f) \int h(n, a, x, w) \; x(dw)}_{\text{Quitters}} + \underbrace{x(n) \int_{\widehat{w} \in \mathcal{W}} \left[1 - \psi^h[\theta(\widehat{w})]\right] \; h(\widehat{w}, a, x, n) \; d\widehat{w}}_{\text{Unsuccesfull Job Searchers}}$$

$$+ \quad \underbrace{x(n) \; h(n, a, x, n)}_{\text{Remain Outside the Labor Force}}$$

- HH head collects all the labor income from the employed and choose the wealth next period a'.
- Unemployed all consume b and employed all consume c.
- FOC in Steady state of a closed economy or of an open economy with zero net foreign asset position implies:

$$[1 - x(\mathcal{W})] \; c = r \; a + \int_{w \in \mathcal{W}} w \; dx(w).$$

Members' Problems: The Unemployed

- Non-employed workers draw EVS shocks $\{\chi^o, \chi^u\}$, $\chi \sim G(\mu^\chi, \alpha^\chi)$. Some non-employed choose to stay out of the labor force while others chose to search for a job. These searchers are the unemployed.
- If searching there are EVS that affect which job to go to $\epsilon^w \sim G(\mu^\epsilon, \alpha^\epsilon)$ associated to applying to each wage level.
- Unemployed's flow utility is $U(b)$.

Value of Not working:

$$
\begin{aligned}
v(a,x,n) =& U(b) + \beta\ \widehat{v}(a',x',n) \\
\widehat{v}(a',x',n) =& \mathbb{E}\left[\widehat{\widehat{v}}(a',x',n,\chi)\right] = \mu + \alpha^\chi \ln\left(e^{\frac{\widehat{v}^u(a',x',n)}{\alpha^\chi}} + e^{\frac{v(a',x',n)}{\alpha^\chi}}\right) + \alpha^\chi\gamma \\
\widehat{v}^u(a',x',n) =& v(a',x',n) + \int\left[\max_{w'\in\mathcal{W}}\left\{\psi^h[\theta(w')]\left[v(a',x',w') - v(a',x',n)\right] + \epsilon^{w'}\right\}\right]\ f(d\epsilon') \\
=& v(a',x',n) + \left[\mu + \alpha^\epsilon\gamma + \alpha^\epsilon \ln \sum_{w'\in\mathcal{W}}\left(e^{\psi^h(w')\left(v(a',x',w')-v(a',x',n)\right)/\alpha^\epsilon}\right)\right]
\end{aligned}
$$

with solutions for probabilities of actions h

$$
\begin{aligned}
h(n,a,x,n) =& \frac{\exp\left\{v(a',x',n)/\alpha^\chi\right\}}{\exp\left\{v(a',x',n)/\alpha^\epsilon\right\} + \exp\left\{\widehat{v}^u(a',x',n)/\alpha^\epsilon\right\}} \\
h(w',a,x,n) =& \frac{\exp\left\{\widehat{v}^u(a',x',n)/\alpha^\chi\right\}}{\exp\left\{v(a',x',n)/\alpha^\epsilon\right\} + \exp\left\{\widehat{v}^u(a',x',n)/\alpha^\epsilon\right\}} \\
& \frac{\exp\left\{\psi^h\left[\theta(w')\right]\left[v(a',x',w') - v(a',x',n)\right]/\alpha^\epsilon\right\}}{\sum_{w\in\mathcal{W}}\exp\left\{\psi^h\left[\theta(w)\right]\left[v(a',x',w) - v(a',x',n)\right]/\alpha^\epsilon\right\}}
\end{aligned}
$$

Members' Problems: The Employed

- Employed get $U(c)$ plus the remaining wage $(w-c)$ evaluated at marginal utility $U_c(c)$ and if they do not lose their jobs they get preference shocks $\{\chi^e, \chi^s, \chi^n\}$, $\chi \sim G(\mu^\chi, \alpha^\chi)$ after work & consumption and choose whether to quit, search or do nothing

Value of working:

$$v(a,x,w) = U(c) + (w-c)U_c + \beta\left[(1-\delta^f)\ \widehat{v}(a',x',w) + \delta^f\ v(a',x',n)\right]$$

$$\widehat{v}(a',x',w) = \mathbb{E}\left[\widehat{\widehat{v}}(a',x',w,\eta)\right] = \mu + \alpha^\chi \ln\left(e^{\frac{v(a',x',w)}{\alpha^\eta}} + e^{\frac{\widehat{v}^s(a',x',w)}{\alpha^\eta}} + e^{\frac{v(a',x',n)}{\alpha^\eta}}\right) + \alpha^\eta\gamma$$

$$\widehat{v}^s(a',x',w) = v(a',x',w) + \int\left[\max_{w'\in\mathcal{W}}\left\{\psi^h[\theta(w')]\left[v(a',x',w') - v(a',x',w)\right] + \epsilon^{w'}\right\}\right]\ f(d\epsilon')$$

$$= v(a',x',w) + \left[\mu + \alpha^\epsilon\gamma + \alpha^\epsilon \ln\sum_{w'\in\mathcal{W}}\left(e^{\psi^h(w')\left(v(a',x',w') - v(a',x',w)\right)/\alpha^\epsilon}\right)\right]$$

The solution densities for quitting $h(n,a,x,w)$ and whether/where to search $h(w',a,x,w)$ are

$$h(n,a,x,w) = \frac{\exp\left\{v(a',x',n)/\alpha^\eta\right\}}{\exp\left\{v(a',x',n)/\alpha^\eta\right\} + \exp\left\{v(a',x',w)/\alpha^\eta\right\} + \exp\left\{\widehat{v}^s(a',x',w)/\alpha^\eta\right\}}$$

$$h(w',a,x,w) = \frac{\exp\left\{\widehat{v}^s(a',x',w)/\alpha^\eta\right\}}{\exp\left\{v(a',x',n)/\alpha^\eta\right\} + \exp\left\{v(a',x',w)/\alpha^\eta\right\} + \exp\left\{\widehat{v}^s(a',x',w)/\alpha^\eta\right\}}$$

$$\times \frac{\exp\left\{\psi^h\left[\theta(w')\right]\left[v(a',x',w') - v(a',x',w)\right]/\alpha^\epsilon\right\}}{\sum_{\hat{w}\in\mathcal{W}}\exp\left\{\psi^h\left[\theta(\hat{w})\right]\left[v(a',x',\hat{w}) - v(a',x',w)\right]/\alpha^\epsilon\right\}}$$

Production: Firms Post vacancies

- Firms are one worker max and are created installing k and posting a vacancy at cost $\bar{c}$ to look for workers in a $\{w, \theta(w)\}$ market. When matched with a worker they produce z (in St St $z = 1$) and pay whatever was promised (that cannot be changed). Firms get destroyed at rate δ^f, which also destroys the capital installed which when the firm is not destroyed depreciates at rate δ^k. If idle the beginning of period value of a firm is

$$\Omega = -\delta^k k + (1-\delta^f)\left\{-\bar{c} + \frac{\psi^f(w)\ \Omega(w) + [1-\psi^f(w)]\Omega}{1+r}\right\}$$

- Value of a wage-w firm when non-quitting probability is $\ell(w)$.

$$\Omega(w) = z - \delta^k k - w + \frac{1-\delta^f}{1+r}\left\{\ell(w)\ \Omega(w) + [1-\ell(w)]\ \Omega\right\}$$

- Zero profit Condition for any active market $\{w, \theta(w)\}$ is

$$k + \bar{c} = \frac{\psi^f(w)\ \Omega(w) + [1-\psi^f(w)]\ \Omega}{1+r}$$

- At any point in time the measure of firms with a worker is exactly equal to x^e. There is a measure y of unmatched firms. Let $y(w)$ the measure that post a vacancy at wage w, then $y = \sum_w\ y(w)$.

Steady State Equilibrium:

- A st-st open economy equilibrium given an exogenous world interest rate r^* satisfies $r^* = 1/\beta - 1$, consists of market tightness function, a measure of idle firms $\{a, \theta(w), y\}$ values and decisions $\{V(a,x), a, v(a,x,\omega), h(\omega, a, x, \omega), \Omega(w), \Omega\}$, a measure of idle firms y, a measure of firms going to active markets $y(w)$, and a measure x over $\mathcal{W} \cup n$ s.t.
  1. $\{V, a\}$ solve the household problem, $\{v, h\}$ solve members problems, $\{\Omega, \Omega(w)\}$ satisfy the firm's problem.
  2. Market tightness is the result of the actions of firms and workers:

$$\theta(w) = \frac{y(w)}{(1-\delta^f)\ \int_{\widehat{w}\in\mathcal{W}} h(w,a,x,\widehat{w})\ dx(\widehat{w}) + h(w,a,x,n)\ x(n)}$$

  3. Free entry condition holds for all w that are active

$$k + \bar{c} = \frac{\psi^f(w)\ \Omega(w) + [1-\psi^f(w)]\ \Omega}{1+r}$$

  4. The measures $\{x, y\}$ are stationary

- A Closed economy St St Equil no longer takes r as exogenous (although it still has to satisfy $r^* = 1/\beta - 1$,) and it requires in addition that
  4. The stock market clears

$$a = \int_w \Omega(w)\ \mathrm{d}x + y\ \Omega$$

Definition

A RCE is a function $S' = G(z', S)$, together with value functions $V(S, a, x)$ decision rules for households wealth $a' = g(S, a, x)$, the density of the application function of membbers, $h(\omega', S, a, x, \omega)$, and associated values $v(S, a, x, \omega)$, new entrants $y(S)$, iddle firms vacancy posting $y(w', S)$, a rate of return $r(S)$, and a link between wages and market tightness $\theta(w, S)$ s.t.

1. $\{V, g\}$ solve the hhold problem, given G, r and ψ functions

$$V(S, a, x) = \max_{a'} (1 - x(n))\, U \left[\frac{[1 + r(S)]a + \int_w w \ dx - a'}{1 - x(n)} \right] + x(n)\ U(b) + \beta V(S', a', x')$$

$$x'(w, S) = (1 - \delta^f)\ x(w)\ \left\{ \underbrace{\left[1 - \int_{\widehat{w}} \psi^h[\theta(\widehat{w}, S)]\ h(d\widehat{w}, S, a, x, w) \right]}_{\text{Not switching to other Jobs}} \underbrace{-h(n, S, a, x, w)}_{\text{Not Quitting}} \right\}$$

$$+ \underbrace{(1 - \delta^f) \int_{\widehat{w}} h(w, S, a, x, \widehat{w})\ \psi^h[\theta(w, S)]\ dx(\widehat{w})}_{\text{Switchers from other Jobs}} + \underbrace{\psi^h[\theta(\widehat{w}, S)]\ h(w, S, a, x, n)\ x(n)}_{\text{Hired from Unemp}}$$

$$x'(n, S) = \underbrace{\delta^f \int_w x(dw)}_{\text{Layoffed}} + \underbrace{(1 - \delta^f) \int h(n, S, a, x, w)\ x(dw)}_{\text{Quitters}} + \underbrace{x(n)\ h(n, S, a, x, n)}_{\text{Remain Outside the Labor Force}}$$

$$+ \underbrace{x(n) \int_{\widehat{w}} \left[1 - \psi^h[\theta(\widehat{w}, S)] \right]\ h(d\widehat{w}, S, a, x, n)}_{\text{Unsuccesfull Job Searchers}}$$

$$S' = G(z', S)$$

2. $\{v, h\}$ solve the members problem which is like that in the St-St except having the aggregate state as arguments

3. $\{\Omega(S, w)\}$ is the value of the firm

4. Free entry condition holds for all w that are active

$$k + \bar{c} = \frac{\psi^f[\theta(S, w)]\ \Omega(S, w) + [1 - \psi^f[\theta(S, w)]]\ \Omega(S)}{1 + r(S)}$$

5. Vacancy postings $y^w(S)$ and job applications h are consistent with $\theta(S, w)$

$$\theta(S, w) = \frac{y(S, w)}{(1 - \delta^f)\ \int_{\widehat{w}} h(w, S, a, x, \widehat{w})\ dx(\widehat{w}) + h(w, S, a, x, u)\ x(n)}$$

6. Asset market clears (wealth held by households equal the value of the stock market):

$$g(S, A, X) = G^A(S) = \int_{w \in \mathcal{W}} \Omega^*(S, w)\ dX + y(S)\ \Omega(S)$$

7. The Evolution of $\{X, Y\}$ is consistent with agents actions and representative agent conditions ($X' = x'$)