



# 8200, 2024: Labor, Topics in Quantitative Macroeconomics

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# 1 Life Cycle Stuff



$$E \left\{ \sum_{i=1}^I \beta_i u(c_i) \right\} \text{ only consumption}$$

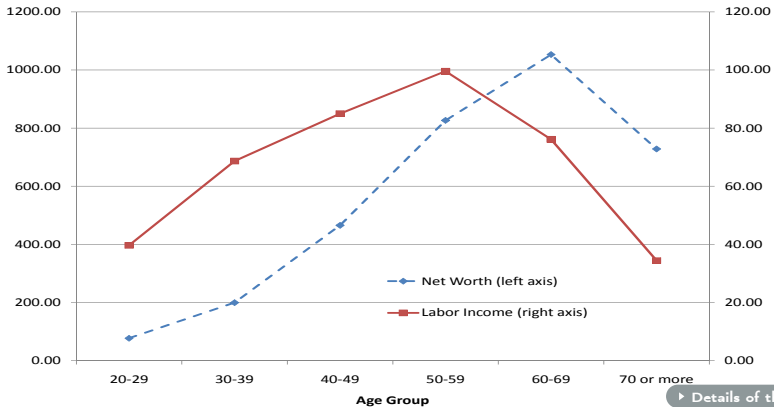
$$E \left\{ \sum_{i=1}^I \beta_i u(c_i, \ell_i) \right\} \text{ cons \& leisure}$$

$$E \left\{ \sum_{i=1}^I \beta_i u(c_i, \ell_i, h_i) \mathbf{1}_{\text{alive}} \right\} \text{ and health}$$

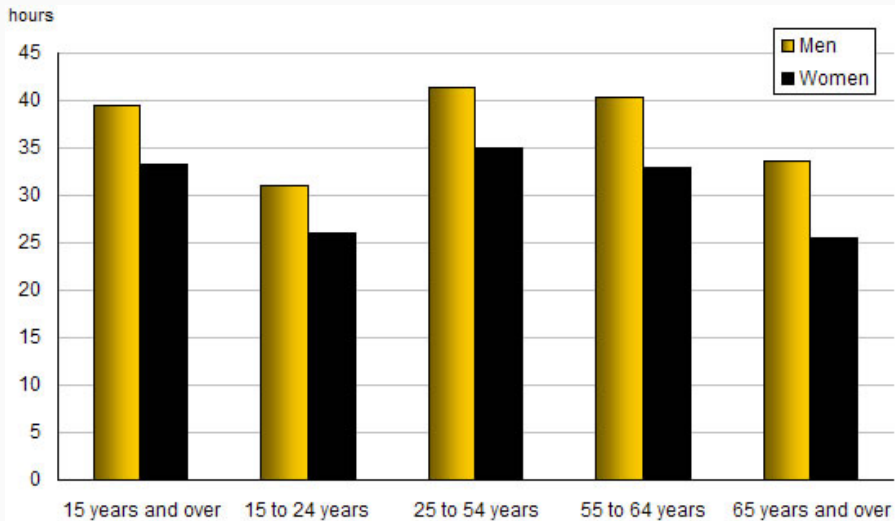
$$E \left\{ \sum_{i=1}^I \beta_i u(c_i, \ell_i, z_i, \epsilon) \right\} \text{ type and quality of household}$$



Figure: Labor Income and Net Worth by Age, SCF 2007 (\$1,000)



## HOURS WORKED BY AGE AND SEX





- Comparative advantage (leisure is more expensive) and low interest rates. It needs only  $u(c) + v(n)$
  
- Non-separability as in Cobb-Douglas
  
- Family composition details  $u(c, n, z)$  or  $u_i(c, n)$



- Need a theory to make sense of differences:
  - Different Preferences
  - Different Endowments (ability? strength? less time?)
  - Different Comparative advantage (Child rearing, soothing, caring)
  - Victims of discrimination (just coordination?)
  - Better built (longer life spans)



1. Biology, i.e. exogenous.

$$w_i = w e_i, \quad y_i = w_i n_i$$

2. Learning by doing

$$w_i = w f(e_{i-1}, n_{i-1}) = w f(e_{i-1}, 1 - l_{i-1}),$$

3. learning by watching (not) doing

$$w_i = w f(e_{i-1}, 1 - n_i - l_{i-1}).$$





$$\max_{c_i, l_i, n_i} \sum_{i=1}^I \beta_i u(c_i, l_i) \quad \text{s.t.}$$

$$\sum_{i=1}^I \frac{c_i}{(1+r)^{i-1}} = \sum_{i=1}^I \frac{w_i n_i}{(1+r)^{i-1}}$$

$$w_i = w f(e_{i-1}, 1 - l_{i-1})$$

$$w_i = w f(e_{i-1}, 1 - n_i - l_{i-1})$$

HWK: Put it in the computer and solve them for  $I = 45$ .



$$v^i(\epsilon, a, e) = \max_{c, \ell, n, a} u(c_i, \ell_i) + \beta_i \mathbb{E}\{v^{i+1}(\epsilon', a', e') \mid \epsilon\} \quad \text{s.t.}$$

$$a' = c + a(1 + r) + w e n$$

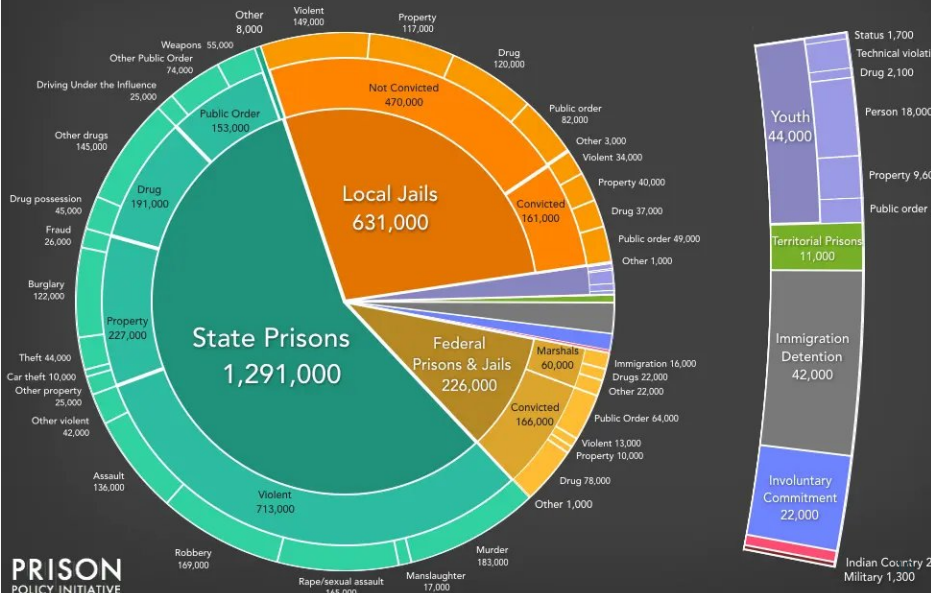
$$e' = w f(\epsilon, e, 1 - \ell)$$

$$e' = w f(\epsilon', e, 1 - n - \ell)$$

HWK: Solve it for  $I = 45$ ,  $\epsilon$  a Markov chain and a random walk.

# How many people are locked up in the United States?

The U.S. locks up more people per capita than any other nation, at the staggering rate of 698 per 100,000 residents. But to end mass incarceration, we must first consider *where* and *why* 2.3 million people are confined nationwide.





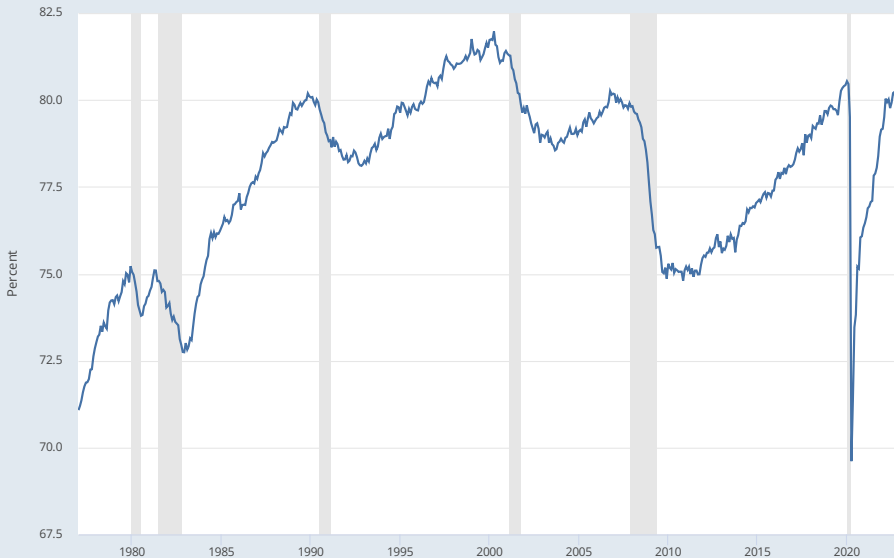
emp2pop.pdf

# EMPLOYMENT RATE 24-54. WHAT IS GOING ON?



**FRED**

— Employment Rate: Aged 25-54: All Persons for the United States



# MOTIVATING FACTS: PORTFOLIO SHARES BY AGE FROM 2007 SCF (IN %)



Age Head	(1) Stk	(2) Res. RE	(3) Non bus.	(4) Non RE	(5) Risky NW	(6) Bond +CD	(7) Car	(8) Oth.	(9) Debt	(10) Safe NW
All	30.3	47.0	12.9	3.8	94.0	17.0	3.5	4.2	-18.6	6.0
20-29	13.2	77.7	43.3	1.3	135.5	13.7	15.3	4.5	-68.9	-35.5
30-39	26.3	96.5	12.7	5.0	140.4	13.8	9.7	4.2	-68.2	-40.4
40-49	30.4	57.6	12.6	3.8	104.4	15.2	4.4	4.5	-28.5	-4.4
50-59	32.7	42.4	13.5	3.7	92.4	17.0	2.8	4.0	-16.1	7.7
60-69	32.2	35.6	13.4	4.1	85.3	17.5	2.4	4.7	-9.9	14.7
70+	27.1	39.8	9.0	3.3	79.2	19.3	1.8	3.7	-3.9	20.8

Risky Net Worth (5) is equal to sum of columns (1)+(2)+(3)+(4). Safe Net Worth (10) is sum of columns (6)+(7)+(8)+(9). Total Net Worth is sum of (5)+(10)

## 2 What do People like in Each Other (RR Seitz & Tanaka)



- U.S. **marriage** features
  1. Women Marry Younger. (22.0 v.s. 24.7)
  2. Women live Longer. (75.5 v.s. 69.4)
  3. The Ratio of Men to Women over 15 is now less than 1. (0.94)
- Why?
- Is there a Systematic Difference in what Men and Women get from marriage?





- Estimate What and When Men and Women get from Marriage
  1. Utility Modifiers from marriage by age of Spouse (diff for men and women)
  2. Dispersion. How special is a particular partner
  3. Costs of Marriage and Divorce
- We pose a fully specified equilibrium model.
- Identification strategy: large demographic changes since 1870
  1. Changes in life expectancies. (58.9 to 75.5 for females)
  2. Changes in the sex ratio. (1.04 to 0.94)
  3. yield large changes in marriage patterns.  
(age gap  $-\Delta 32\%$ , married  $\Delta 20\%$ , never-married  $-\Delta 33\%$ , divorce rate  $\Delta 642\%$  )



- Siow (1998) argues that because of the role of women as child bearers they are especially attractive at relative younger ages than men. This logic is biological.
  
- We want to use revealed preference to infer from people's behavior how large are the gains that they perceive they have from marriage and at what ages these gains accrue.



- Women's prime age starts earlier than men's one.
- Women's prime age ends earlier than men's one.
- Both male and female strongly prefer a prime-aged partner.
- Other insights we found are:
  1. Match quality process has permanent nature.
  2. Marriage is costly.
  3. Divorce is costly.

# TRENDS IN DEMOGRAPHICS AND MARITAL STATISTICS 1870-1950



	Birth Cohort			% Change in the Data
	1870	1930	1950	
<b>Demographics</b>				
Sex ratio	1.040	0.997	0.939	-9.7
Life expectancy from 15 (female)	43.4	55.0	60.4	37.8
Life expectancy from 15 (male)	43.9	51.4	54.4	25.3
<b>Age at first marriage</b>				
Females	21.9	20.3	22.0	0.5
Men	25.9	22.8	24.7	-4.6
<b>Percentage of never-married by age 50</b>				
Females	10.4	5.7	7.6	-32.6
Men	12.9	6.6	9.3	-28.1
<b>Married as a % of those aged 16-49</b>				
Females	59.7	75.4	60.8	1.8
Men	50.8	69.4	56.1	10.4
<b>Divorce rate, per 1,000 people</b>				
	0.7	2.2	5.2	642.9



1. OLG with stochastic aging. Ages:  $i \in \{a, y, o\}$ , Adolescent ( $a$ ), Young ( $y$ ), and Old ( $o$ ). Sexes:  $g \in \{m, f\}$ .
  - Aging transitions  $\Gamma_{i,j}^f$  and  $\Gamma_{i,j}^m$ .
2. New entrants: birth (in equal amounts) and men's migration
  - $n^g$  newborns are born every period.
  - Immigration rate  $i^m$ .
3. Differential mortality rates by age and sex.  $\{\pi^m, \pi^f\}$ .



- $$i' = \begin{cases} i & \text{with prob } \Gamma_{ii}^g \\ i + 1 & \text{with prob } \Gamma_{i,i+1}^g \\ \text{dead} & \text{with prob } \pi^g \end{cases}$$

- The measure of new adolescent single females is

$$n^f = \frac{[1 - \Gamma_{a,a}^f(1 - \pi^f)] [1 - \Gamma_{y,y}^f(1 - \pi^f)] \pi^f}{[1 - \Gamma_{y,y}^f(1 - \pi^f) + \Gamma_{a,y}^f(1 - \pi^f)] \pi^f + \Gamma_{a,y}^f(1 - \pi^f) \Gamma_{y,o}^f(1 - \pi^f)}$$



**Marital status:** Single, dating or married  $q \in \{0, 1, 2\}$ .

**Random dating:** Prob  $\psi^f = \min\{1, \frac{x^m}{x^f}\}$ .  $x^g$  measure of singles. Everybody that can, meets.



**Preferences:** If single  $u_i^g(0) = 0$ . If married,  $u_i^g(i^*) = \alpha_{i^*}^g + z$ .

**Match quality  $z$ :**  $z = \mu + \epsilon$ ,  $\mu \in \{\theta, 0, -\theta\}$  Markov w transition matrix  $\Lambda$ . Initial draws from  $\Lambda_0$ .  $\epsilon$ , extreme value shock.

$$\Lambda = \begin{pmatrix} 1 - \lambda_1 & \lambda_1 & 0 \\ \lambda_2 & 1 - \lambda_2 - \lambda_3 & \lambda_3 \\ 0 & \lambda_4 & 1 - \lambda_4 \end{pmatrix}$$

**Marriage:** If both agree they get married,  $q = 2$ . Else  $q = 0$ . Agent pay a cost  $c^m$  when they become married.

**Divorce:** Agents pay a cost  $c^d$  upon divorce.





- An agent is characterized by its sex and maturity  $i, g$  as well as by whether it is matched and how  $z$ , and if so, what is the maturity of the partner  $i$  and the quality of the match as assessed by itself  $q$ , and its partner  $q$ . For the purpose of record keeping in the measures of agents,  $x$ , and the value functions,  $V$ , we write the state excluding the realization of the extreme value shocks .
- State includes purely exogenous variables (age and maturity, super-indices).

$s = \{z, i, \mu, \mu^*\}$  for an agent implies  $s(s) = \{z, i, \mu, \mu^*\}$

- We have then that

$$x^{f,i}(s) = x^{f,i}(z, i, \mu, \mu^*) = x^{m,i}(z, i, \mu^*, \mu) \quad \forall z \in \{1, 2\}, i, i^*, \mu, \mu^* .$$



$$\Omega^{g,i}(0, 0) = \beta (1 - \pi^g) \sum_{i'} \Gamma_{i,i'}^g \left\{ (1 - \psi^g) V^{g,i'}(0) + \psi^g \sum_{i^*, \mu, \mu^*} \frac{x^{g^*,i^*}(1, \cdot)}{x^{g^*}(1, \cdot)} \Lambda_0(\mu) \Lambda_0(\mu^*) V^{g,i'}(1, i^*, \mu, \mu^*) \right\}.$$



$$\Omega^{g,i}(s, 1) = u^{g,i}(s, 2) + \beta (1 - \pi^g) \left[ \right.$$

$$(1 - \pi^{g^*}) \sum_{i', i^{*'}, \mu', \mu^{*'}} \Gamma_{i, i'}^g \Gamma_{i^*, i^{*'}}^{g^*} \Lambda_{\mu, \mu'}^{i'} \Lambda_{\mu^*, \mu^{*'}}^{i^{*'}} V^{g, i'}(2, i^{*'}, \mu', \mu^{*'})$$

$$+ \pi^{g^*} \sum_{i'} \Gamma_{i, i'}^g \left( (1 - \psi^g) V^{g, i'}(0) + \right.$$

$$\left. \psi^g \sum_{i^{*'}, \mu, \mu^*} p^g(i^{*'}) \Lambda_0(\mu) \Lambda_0(\mu^*) V^{g, i'}(1, i^{*'}, \mu, \mu^*) \right) \left. \right].$$



- If the matched agent gets divorced, need to add

$$\Omega^{g,i}(s, 0) = -c^d + \Omega^{g,i}(0, 0).$$

The agent will like to choose

$$\max \left\{ \Omega^{g,i}(s, 1) + \epsilon^1, -c^d + \epsilon^0 V^{g,i}(0) \right\}.$$

- But both have to agree. With extreme value shocks (Gumbel) the probability that an  $\{i, g, s\}$  agent prefers to be married is

$$p^{g,i}(s) = \frac{\exp [\Omega^{g,i}(s, 1)]}{\exp [\Omega^{g,i}(s, 0) + \Omega^{g,i}(s, 1)]}$$



- We now obtain the beginning of period value for matched or married (given the decision rule of the partner if any)  $s = \{z, i^*, \mu, \mu^*\}$  using again the convenient extreme value formulae

$$V^{g,i}(s) = \ln \left[ \exp \Omega^{g,i}(s, 1) + \exp \Omega^{g,i}(s, 0) \right] p^{g^*, i^*}(s^*(s)) + \Omega^{g,i}(s, 0) \left[ 1 - p^{g^*, i^*}(s^*(s)) \right]$$

where obviously  $p^{i,g}(0) = 0$ .



- A steady state is just a set of measures  $x^{g,i}(s)$ , values  $V^{g,i}(s)$ , and choices  $p^{g,i}(s)$  such that agents choose optimally, and their choices both generate the value functions and yield the measures as steady state distributions of agents. It is standard, double check.



- For a given cohort there are 20 parameters (plus  $\beta = .96$ )
- \* A (3) Demographic parameters, (mortality immigration). Observable.
- \* B (1) Divorce (cost)
- \* C (6) Preferences ( $\alpha_j^g$ )
- \* D (4) Aging Process  $\Gamma$
- \* E (5) Match Quality  $\lambda$ .  $\lambda$  not abrupt.  $\lambda_0$  middle, symmetric, mean zero.
- \* F (1) Cost of Marriage



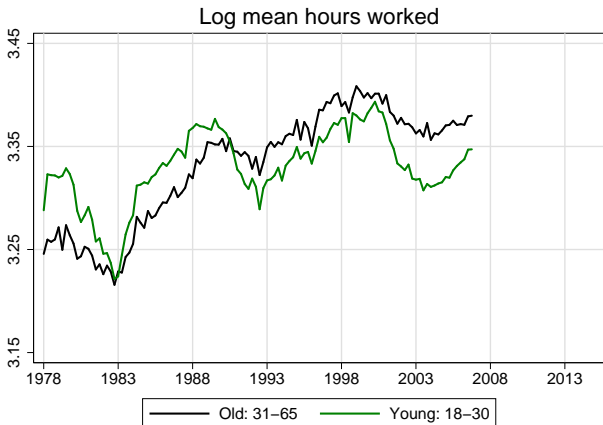
- Cohort specific: demographics (maybe others)
- Common (not always all)
- \* (12) Marriage rates for each of six groups
- \* (12) Divorce rates for each of six groups
- \* (2) Marriage Incidence
- \* (2) Average Age at First Marriage

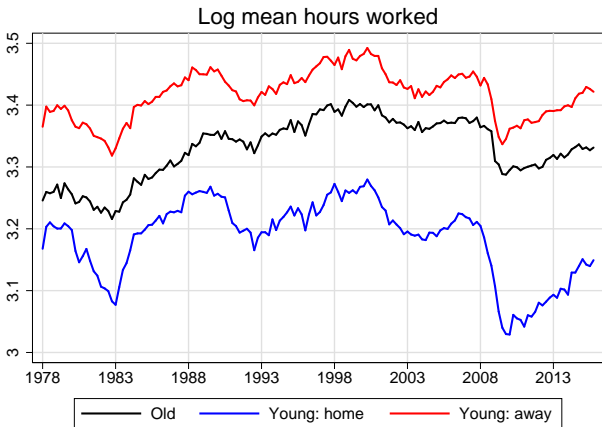


Version of the Model	(1)	(2)			(3)			(4)		
Parameters	None	(A)col:A			(A+B)col:A+B			(A+B+C)col:A+B+C		
Cohort	All	1870	1930	1950	1870	1930	1950	1870	1930	1950
<b>(A) Demographics</b>										
Sex ratio	0.992	1.040	0.997	0.939	1.040	0.997	0.939	1.040	0.997	0.939
Life expectancy (female)	52.9	43.4	54.9	60.4	43.4	54.9	60.4	43.4	54.9	60.4
Life expectancy (male)	42.9	43.9	51.4	54.4	43.9	51.4	54.4	43.9	51.4	54.4
<b>(B) Divorce cost</b>	3.33		3.02		5.55	5.37	3.25	5.63	5.41	3.11
<b>(C) Preference</b>										
$\alpha_a^f$ Fem for <i>a</i> males	-1.72		-4.12			-1.94		-2.12	-2.88	-1.90
$\alpha_y^f$ Fem for <i>y</i> males	-0.09		0.02			-0.23		-0.52	0.13	-0.50
$\alpha_o^f$ Fem for <i>o</i> males	-0.59		-0.14			-0.55		-0.65	-0.41	-0.71
$\alpha_a^m$ Males <i>a</i> females	-5.14		-5.43			-4.27		-4.46	-4.99	-4.15
$\alpha_y^m$ Males for <i>y</i> females	-0.09		-0.28			-0.06		-0.16	0.11	-0.00
$\alpha_o^m$ Males for <i>o</i> females	-0.78		-0.51			-0.64		-0.67	-0.79	-0.61
<b>(D) Aging process</b>										
<b>Female</b>										
Age of becoming prime	17.3		17.0			17.3			17.1	
Age of becoming old	29.3		29.1			27.8			29.7	
<b>Male</b>										
Age of becoming prime	18.8		17.7			17.9			17.7	
Age of becoming old	31.0		30.0			29.2			30.9	
<b>(E) Love shock process</b>										
Gain in <i>H</i> -state ( $\theta$ )	1.28		0.88			1.25			1.31	
Prob. <i>H</i> -state to <i>H</i> -state	1.00		1.00			1.00			1.00	
Prob. <i>M</i> -state to <i>H</i> -state	0.47		0.47			0.42			0.42	
Prob. <i>M</i> -state to <i>L</i> -state	0.15		0.21			0.20			0.20	
Prob. <i>L</i> -state to <i>L</i> -state	0.72		0.71			0.70			0.67	
<b>(F) Marriage cost</b>	3.11		2.74			3.11			3.10	
(WSSE): $E_j$	0.152		0.117			0.082			0.029	
Norm WSSE	0.181		0.139			0.098			0.035	
Measure of Fit: $1 - (E_j/E_1)$	0.000		0.231			0.459			0.804	

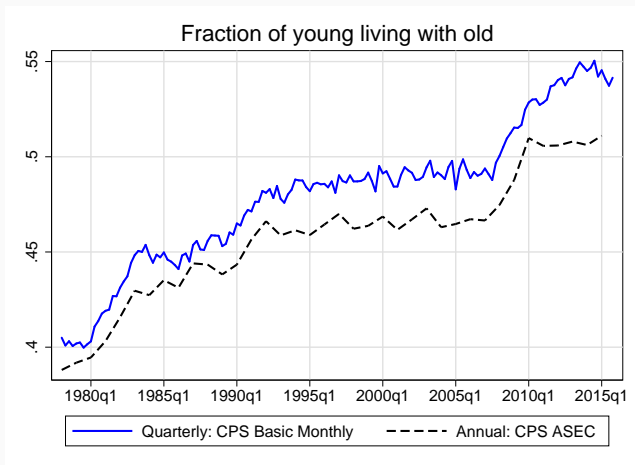
Version of the Model Parameters Cohort	(1) None All	(4)			(5)		
		(A+B+C)col:A+B+C			(A+B+D)col:A+B+D		
		1870	1930	1950	1870	1930	1950
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Life expectancy (female)	52.9	43.4	54.9	60.4	43.4	54.9	60.4
Life expectancy (male)	42.9	43.9	51.4	54.4	43.9	51.4	54.4
<b>(B) Divorce cost</b>	3.33	5.63	5.41	3.11	5.56	5.39	3.21
<b>(C) Preference</b>							
$\alpha_{af}^f$ Fem for <i>a</i> males	-1.72	-2.12	-2.88	-1.90		-1.93	
$\alpha_{yf}^f$ Fem for <i>y</i> males	-0.09	-0.52	0.13	-0.50		-0.21	
$\alpha_{of}^f$ Fem for <i>o</i> males	-0.59	-0.65	-0.41	-0.71		-0.56	
$\alpha_{am}^m$ Males <i>a</i> females	-5.14	-4.46	-4.99	-4.15		-4.32	
$\alpha_{ym}^m$ Males for <i>y</i> females	-0.09	-0.16	0.11	-0.00		0.01	
$\alpha_{om}^m$ Males for <i>o</i> females	-0.78	-0.67	-0.79	-0.61		-0.64	
<b>(D) Aging process</b>							
<b>Female</b>							
Age of becoming prime	17.3		17.1		17.3	17.3	17.3
Age of becoming old	29.3		29.7		27.4	29.0	27.7
<b>Male</b>							
Age of becoming prime	18.8		17.7		18.0	17.8	17.9
Age of becoming old	31.0		30.9		29.3	29.6	30.1
<b>(E) Love shock process</b>							
Gain in <i>H</i> -state ( $\theta$ )	1.28		1.31			1.24	
Prob. <i>H</i> -state to <i>H</i> -state	1.00		1.00			1.00	
Prob. <i>M</i> -state to <i>H</i> -state	0.47		0.42			0.42	
Prob. <i>M</i> -state to <i>L</i> -state	0.15		0.20			0.20	
Prob. <i>L</i> -state to <i>L</i> -state	0.72		0.67			0.70	
<b>(F) Marriage cost</b>	3.11		3.10			3.10	
(WSSE): $E_i$	0.152		0.029			0.068	
Norm WSSE	0.181		0.035			0.081	
Measure of Fit: $1 - (E_i/E_1)$	0.000		0.804			0.549	

### **3 Living Arrangements and Labor Market Volatility of Young Workers (Dyrda, Kaplan RR)**

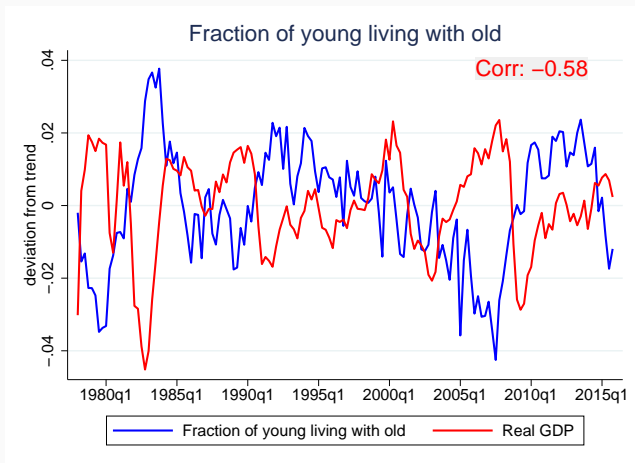




- Roughly half of 18-30 live with a 31-65 (home), half don't (away)
- Young people away: higher average hours, lower volatility
- Additional volatility concentrated among **young at home**



- Secular upward trend since 1980
- Increased by >5pp during Great Recession, barely fallen



- Counter-cyclical pre and post Great Recession



1. **Quantitative theory** of fluctuations in living arrangements and hours worked for young relative to old
  - Co-residence trade-off: **implicit transfers** vs disutility
  - Labor supply more responsive to wages: wedge between **Marshallian elasticity** of young living away vs together
2. **Estimate** model with aggregate data
  - Relative hours, wages by age and coresidence
  - Dynamics of living arrangements
  - De-trended from 1978 to 2006
  - Key identifying assumptions:
    - a. Selection: **functional forms** for dist of unobservables
    - b. Labor supply vs demand: conditional on skills, **living arrangements do not affect productivity**
3. Use estimated model as **measurement device**
  - a. Size of implicit transfers? **16.7% of consumption of the young**
  - b. Difference in Marshallian elasticity by living arrangements? **18% higher for young living with old**





- CPS Basic Monthly Surveys for hours (monthly)
- CPS ASEC for wages (annual)
- Individuals: 18-65 year olds, not in school, not in group quarters
- Households: households with at least one such person
- Household size: number of 18-65 year olds not in school
- Quarterly series: de-seasonalize using X12-ARIMA from BLS
- Detrending:
  - 1978-2006: Hodrick-Prescott and various other filters,
  - 2007-2010: Great Recession
  - 2011-2015: Great Recession recovery



## Definitions:

- Population: 18-65 yr olds not in school
- Young: 18-30
- Old: 31-65
- Young away: no old people in household
- Young together:  $\geq 1$  old person in household

## Quarterly moments *relative to old*, 1978-06:

	Young	Young Away	Young Together
Mean hours	1.00		
St dev log hours	1.58		

	Young	Young Away	Young Together
Mean hours	1.00	1.10	0.88
St dev log hours	1.58	1.32	1.89

- St dev log fraction young with old  $\approx 0.8$
- Cyclical correlation with hours worked  $\approx -0.6$



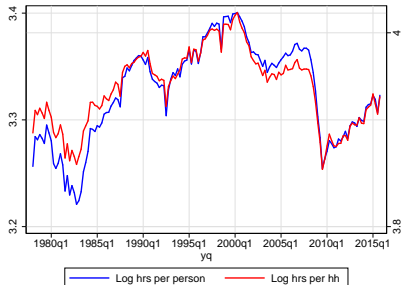
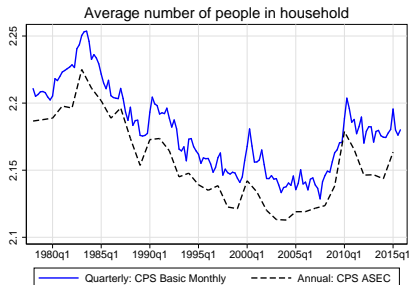
Annual moments relative to old, 1978-06:

	Young	Young Away	Young Together
Mean wages	0.65		
St dev log wages	1.07		

Annual moments relative to old, 1978-06:

	Young	Young Away	Young Together
Mean wages	0.65	0.75	0.52
St dev log wages	1.07	1.18	1.11

- Labor demand mechanism - **Jaimovich, Pruitt, Siu (2013)**:
  - Technology with imperfect substitutability between old and young
  - Quantitative argument requires Frisch for young = 7, old =  $\infty$
- Labor supply mechanism - **this paper**:
  - Selection into living arrangements
  - Imperfect substitutability by living arrangements **implausible**
  - Labor supply elasticities for old disciplined by micro estimates



- Household size moves a lot: trend and cyclical
- Hours per person more volatile than hours per household



- $H$  = total hours
- $N$  = number of individuals
- $F$  = number of households

$$\underbrace{\frac{H}{N}}_{\text{hours per person}} = \underbrace{\frac{H}{F}}_{\text{hours per household}} \div \underbrace{\frac{N}{F}}_{\text{persons per household}}$$

- Cyclical fluctuations

$$V\left(\log \frac{H}{N}\right) = \underbrace{V\left(\log \frac{H}{F}\right)}_{\text{hrs per hh}} + \underbrace{V\left(\log \frac{F}{N}\right)}_{\text{hh size}} - \underbrace{2\text{COV}\left(\log \frac{H}{F}, \log \frac{F}{N}\right)}_{\text{covariance term}}$$



$$V\left(\log \frac{H}{N}\right) = \underbrace{V\left(\log \frac{H}{F}\right)}_{\text{hrs per hh}} + \underbrace{V\left(\log \frac{F}{N}\right)}_{\text{hh size}} - \underbrace{2\text{COV}\left(\log \frac{H}{F}, \log \frac{F}{N}\right)}_{\text{covariance term}}$$

	Cyclical Variance, 78-06		Great Recession Change, 07-10	
	Quarterly	Annual	Quarterly	Annual
hrs per hh	85%	92%	84%	85%
hh size	5%	3%	16%	15%
covariance	10%	5%		

- Changes in household size offset around **8%-15%** of changes in hours per person, at the household level



- Importance of **endogeneity of coresidence**: counterfactual series for hours assuming constant  $x$  = fraction of young living with old

- All variation in hours is due to variation in hours of two groups:

$$M = \frac{V(\log h^y) - V(\log [\bar{x}h^{yT} + (1 - \bar{x})h^{yA}])}{V(\log h^y)}$$
$$\approx 5\%$$



- Old agents
- Identical
  - Live in unitary households
  - Can be invaded by a young agent
- Young agents
- Two independent idiosyncratic shocks
  - Individual productivity  $\varepsilon$
  - Distaste for living with old agents  $\eta$
  - Can invade an old households
- So three types of agents
- Old:  $\mu$
  - Young alone:  $(1 - \mu)(1 - x)$
  - Young together (with old):  $(1 - \mu)x$





$$\begin{aligned}
 V^o(a; w^o, r) &= \max_{c^o, h^o, a'} u^o(c^o, h^o) + \beta \mathbb{E} \left[ V^o(a'; w^{o'}, r') \right] \\
 \text{s.t.} \quad &c^o + a' = w^o h^o + (1+r)a
 \end{aligned}$$

Standard preferences

$$u^o(c, h) = \log c^o - \psi^o \frac{(h^o)^{1 + \frac{1}{\nu^o}}}{1 + \frac{1}{\nu^o}}$$

Aggregate uncertainty:  $w^o, r$

Non-standard preferences of old



$$V^y(\varepsilon, \eta; w^y, c^o) = \max_{A, T} \{V^A(\varepsilon; w^y), V^T(\varepsilon, \eta; w^y, c^o)\}$$

Young alone

$$V^A(\varepsilon; w^y) = \max_{c, h} \frac{c^{1-\gamma}}{1-\gamma} - \psi^y \frac{h^{1+\frac{1}{\nu^y}}}{1+\frac{1}{\nu^y}}$$

s.t.  $c = w^y \varepsilon h$

Young together

$$V^T(\varepsilon, \eta; w^y, c^o) = \max_{c, h} \frac{[c + \zeta(c^o)]^{1-\gamma}}{1-\gamma} - \psi^y \frac{h^{1+\frac{1}{\nu^y}}}{1+\frac{1}{\nu^y}} - \eta$$

s.t.  $c = w^y \varepsilon h$

Require  $\gamma < 1$  for positive co-movement of wages and hours

Implicit transfers from old (economies of scale):  $\zeta(c^o)$

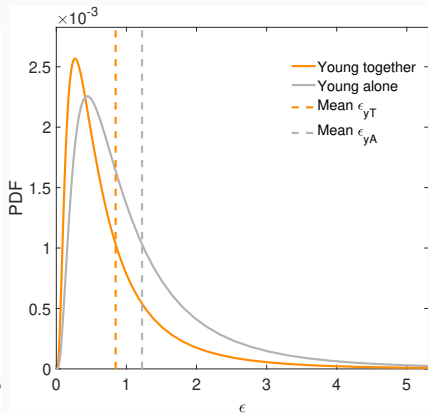
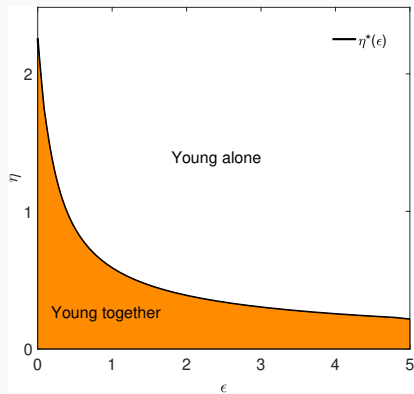


Nested CES with capital-experience complementarity  
(Jaimovich-Pruitt-Siu, AER 2013)

$$F(K, N^y, N^o; Z) = \left[ \alpha (Z_y N^y)^\sigma + (1 - \alpha) (\lambda K^\rho + (1 - \lambda) (Z_o N^o)^\rho) \frac{\sigma}{\rho} \right]^{\frac{1}{\sigma}}$$

where  $N^y$  and  $N^o$  are labor inputs of young and old

- Technology generates **higher hours and wage volatility for young**  
Technology depends on age, but **not living arrangements**  
Structure on top of standard RBC model: **shocks to  $Z_o$  and  $Z_y$**





- An equilibrium is a set functions
  - consumption  $\{c^{yA}(\varepsilon, s), c^{yT}(\varepsilon, \eta, s), c^o(s)\}$
  - hours worked  $\{h^{yA}(\varepsilon, s), h^{yT}(\varepsilon, \eta, s), h^o(s)\}$
  - threshold for staying at home  $\eta^*(s, \varepsilon)$
  - fraction of young that move in with the old  $x(s)$

such that:

- old maximize given prices
- young maximize given prices and choice of old
- factor markets clear
- **fraction of young living with old** satisfies

$$x(s) = \int_0^\infty \int_{-\infty}^{\eta^*(s, \varepsilon)} dF_\eta dF_\varepsilon$$

where  $\eta^*(s, \varepsilon)$  satisfies the indifference condition for all  $\varepsilon$ .



- Productivity heterogeneity:  $\varepsilon \sim \log N$
- Disutility heterogeneity:  $\eta \sim N$
- Implicit transfer function:  $\zeta(c^o) = \zeta_0 + \zeta_1 c^o$
- Agg. shocks:  $\log Z'_i = \rho_i \log Z_i + \xi_i$ , where  $\xi_i \sim N(0, \sigma_\xi^i)$ ,  $i = o, y$
- Demographics:  $\mu, \gamma, \zeta^o, \zeta^y$
- Preferences old  $\psi^o, \beta, \nu^o$
- Productivity dist  $\mu_\varepsilon, \sigma_\varepsilon$
- Young preferences  $\gamma^y, \nu^y, \psi^y$
- Implicit transfers  $\zeta_0, \zeta_1$
- Disutility dist:  $\mu_\eta, \sigma_\eta$
- Production parameters:  $\alpha, \lambda, \delta, \sigma, \rho$
- Agg. shocks:  $\{\rho_i, \sigma_\xi^i\}_{i=y,o}, \text{Corr}(\xi_o, \xi_y)$



1. Exogenously imposed parameters (Data + Micro Estimates):  $\nu^o, \mu, \gamma, \zeta^o, \zeta^y$
2. Directly identified from steady-state conditions:  $\beta$
3. Rest is targeted using **Simulated Method of Moments (SMM)**:
  - There are 20 parameters to determine and we use 24 targets (next slide)
  - Both first and second moments are targeted. Cannot separate their identification in the model.



## First Moments:

- Macro:  $I/Y, K/Y, h^o, w^y h^y / Y$
- Living arrangements:  $h^{yA}, h^{yT}, x, w^{yA} / w^o, w^{yT} / w^o$

## Second Moments:

- Properties of Solow Residual:  $AutoCorr(TFP), \sigma(TFP)$
- Relative hours:  $\sigma(h^{yA}) / \sigma(h^o), \sigma(h^{yT}) / \sigma(h^o), \sigma(h^y) / \sigma(h^o)$
- Relative wages:  $\sigma(w^{yA}) / \sigma(w^o), \sigma(w^{yT}) / \sigma(w^o), \sigma(w^y) / \sigma(w^o)$
- Living arrangements moments:  $\sigma(x) / \sigma(h^o), M, Contr_{HF}$
- Correlations:  $Corr(x, h), Corr(w^y, w^o), Corr(c, x), Corr(h^y, h^o)$





Moment	Weight	Data	Model
First Moments			
Capital/Output	1.0	7.50	7.67
Investment/Output	1.0	0.26	0.27
[b] Mean Hours Old	1.0	0.52	0.52
Mean Hours Young Together	1.0	0.25	0.23
Mean Hours Young Alone	1.0	0.31	0.31
Fraction of Young living with Old	20.0	0.47	0.45
Wage of young alone/Wage Old	1.0	0.75	0.70
Wage of young together/Wage Old	1.0	0.52	0.46
Share of Old Labor Income in GDP	1.0	0.53	0.49



Moment	Weight	Data	Model
Second Moments			
$\sigma(h^y)/\sigma(h^o)$	10.0	2.48	2.24
$\sigma(h^{yT})/\sigma(h^o)$	10.0	3.57	2.73
$\sigma(h^{yA})/\sigma(h^o)$	10.0	1.75	1.70
$\sigma(x)/\sigma(h^o)$	10.0	0.56	0.45
$\sigma(w^y)/\sigma(w^o)$	1.0	1.14	0.82
1.2 [b] $\sigma(w^{yA})/\sigma(w^o)$	1.0	1.40	0.77
$\sigma(w^{yT})/\sigma(w^o)$	1.0	1.23	0.77
$\text{Corr}(x, h)$	1.0	-0.59	-0.64
$\text{Corr}(w^y, w^o)$	1.0	0.62	0.80
$\text{Corr}(h^y, h^o)$	10.0	0.89	0.82
$\text{Corr}(c, x)$	1.0	-0.56	-0.23
Contribution H/F	10.0	0.15	0.17
Moment M	1.0	0.05	0.10
Persistence of the AR(1) SR	1.0	0.95	0.95
Std of the AR(1) SR	1.0	0.007	0.007



$$\zeta(c^o) = \zeta_0 + \zeta_1 c^o$$

1. Average fraction of consumption of old

$$E \left[ \frac{\zeta(c^o)}{c^o} \right] = 4.6\%$$

2. Average fraction of consumption of young together

$$E \left[ \frac{\zeta(c^o)}{\zeta(c^o) + c^{yT}} \right] = 16.7\%$$

3. Average additional hours need to work by young together

$$E \left[ \frac{\hat{h}^{yT} - h^{yT}}{h^{yT}} \right] = 18.5\%$$



- Frisch elasticity for old = 0.72
- Marshallian elasticity for young alone

$$e^{yA} = \frac{(1-\gamma)\nu^y}{1+\gamma\nu^y} e^{yA} = 0.512$$

- Marshallian elasticity for young together

$$e^{yT}(\varepsilon) = e^{yA} \times \frac{1 + \frac{1}{1-\gamma} \frac{\zeta(c^o)}{c^{yT}(\varepsilon)}}{1 + \frac{1}{1+\gamma\nu^y} \frac{\zeta(c^o)}{c^{yT}(\varepsilon)}} E[e^{yT}] = 0.603$$

- If  $\gamma < 1$ ,  $\zeta > 0$  then  $e^{yT}(\varepsilon) > e^{yA}$
- If  $\zeta = 0$  then  $e^{yT}(\varepsilon) = e^{yA}$ . Also  $e^{yT}$  increasing in  $\zeta$



## Experiment 1:

- Possibility of coresidence, no endogeneity of coresidence
- $x = \bar{x}$ : fix thresholds  $\eta^*(\varepsilon, s) = \eta^*(\varepsilon, \bar{s})$
- St dev of log total hours: 5.5% lower
- St dev of log of young hours: 6.4% lower

## Experiment 2:

- No possibility of coresidence
- $x = 0$ : all young live alone
- St dev of log total hours: 31.4% lower
- St dev of log of young hours: 37.2% lower

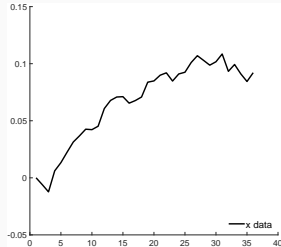
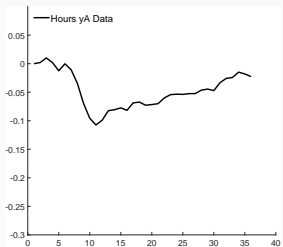
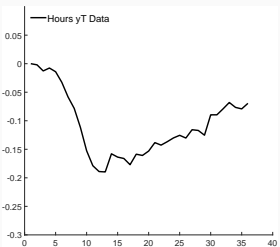
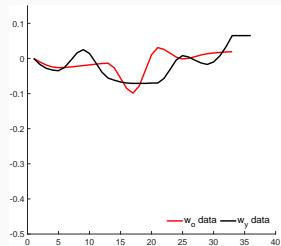
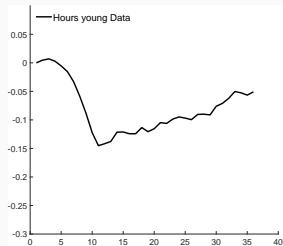
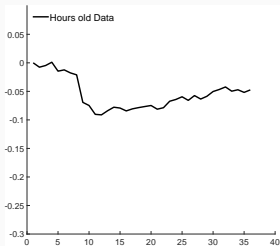


- RA models: Frisch elasticity key for volatility of aggregate hours  
→ **useful metric** for measuring strength of other channels
- What Frisch elasticity would RA model require to generate same volatility of hours as model with young people and coresidence?

Frisch elasticity for old ( $\nu^o$ )	Implied Frisch in RA RBC model	Proportional Increase
0.72	1.33	85%
0.5	0.87	75%
1.0	2.15	115%
2.0	9.62	381%

## 4 Great Recession

# THE GREAT RECESSION IN THE DATA





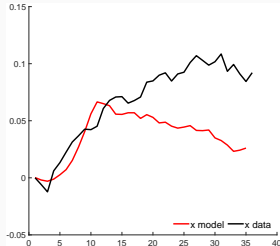
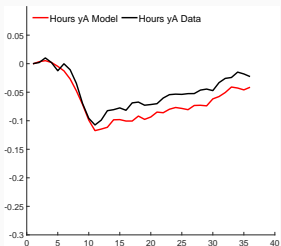
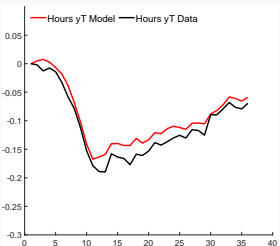
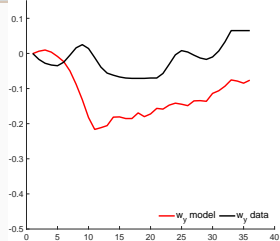
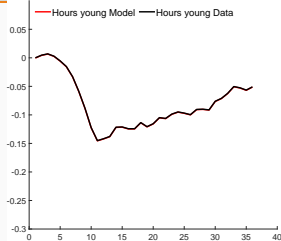
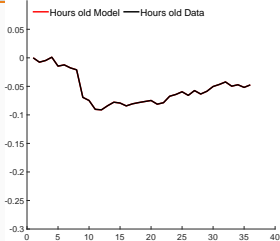


- Look through the lens of the model at hours of young (alone and together) and living arrangements during the Great Recession.
- Back out values of the shocks, so that the model replicates hours of the young and old between q1:2007 and q4:2015.
- Simulate the model forward with the implied shock values. Agents still have **rational expectations** about the shock realizations.



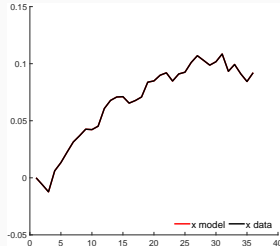
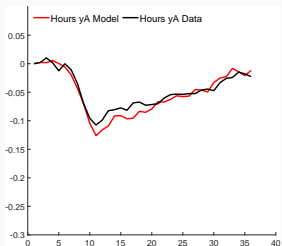
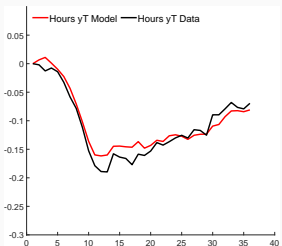
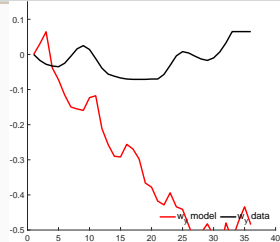
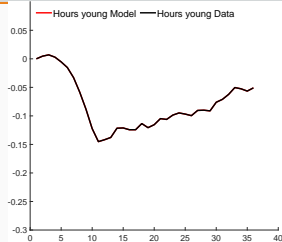
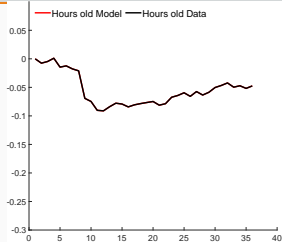
- The baseline model gets coresidence right only up to 10th quarter into the recession; misses the recovery.
- What does it take for the model to account for the data?
- **Improved leisure technology: Aguiar, Bils, Charles, Hurst (2018).** It becomes less painful to live with parents being equipped with better video games.

# ASYMMETRIC TFP SHOCKS TO MATCH HOURS RECOVERY



Model inputs

# ASYMMETRIC TFP SHOCKS + IMPROVED LEISURE ( $\psi_y$ )



Model inputs

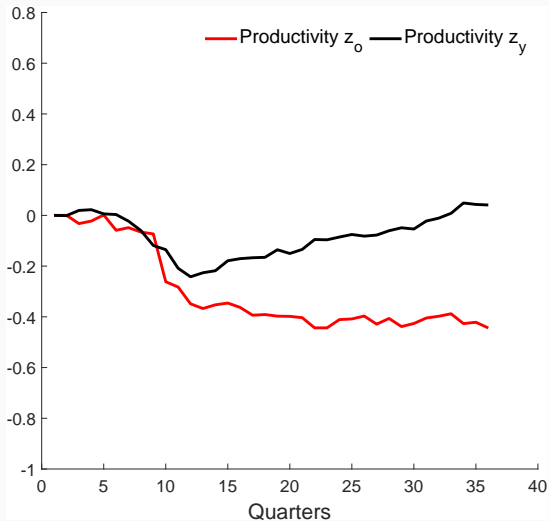
Improved leisure through  $\gamma$

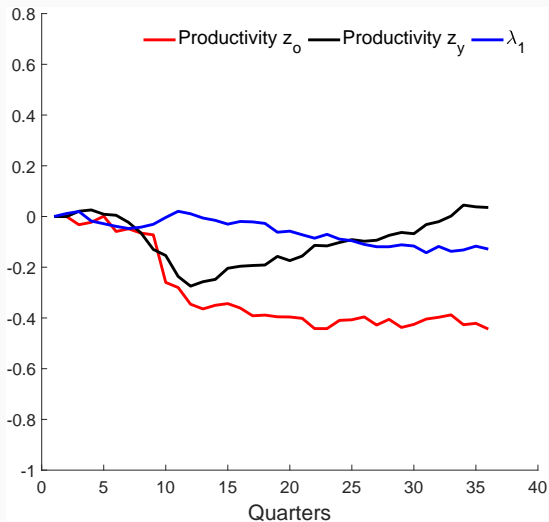


- Young and old have different labor market outcomes. Living arrangements play central role in shaping the behavior of the young.
- We have provided a theory of how it works and mapped it to the data. This theory accounts for the average and cyclical behavior of the young and the old.
- A rational for differences between the micro and the macro (which is 85% larger) Frisch elasticities.
- Our theory + Aguiar et. al. (2018) mechanism accounts for steep rise of coresidence and different outcomes of young and old during the Great Recession.

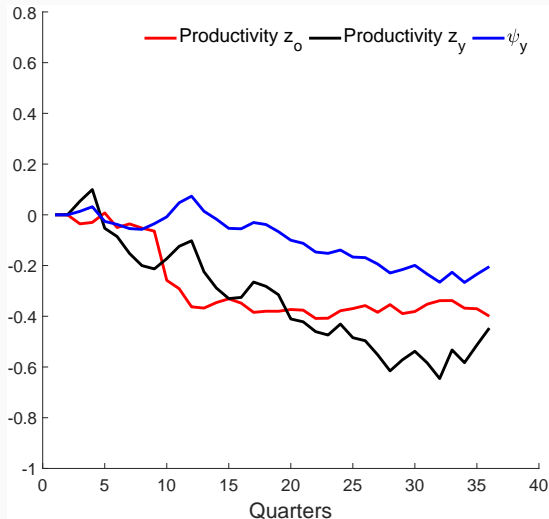


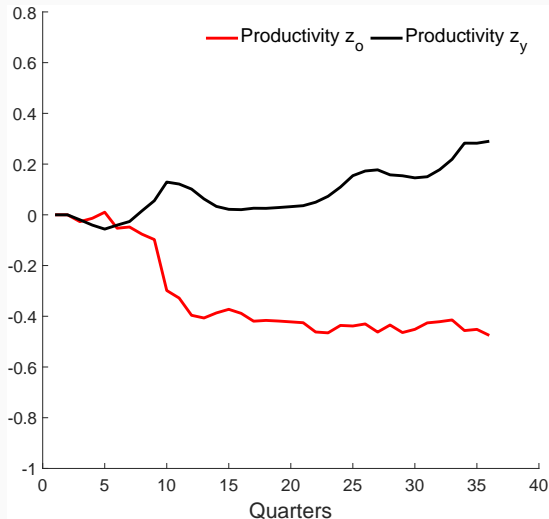
Parameter description	Symbol	Value	Discipline
<b>Params set without solving the model</b>			
Fraction of the old in the Population	$\mu$	0.709	CPS data
Frisch elasticity for the Old	$\nu^o$	0.720	Heathcote et al. (2010)
Equivalence scale within the Old	$\zeta^o$	1.700	OECD data
Equivalence scale for Old with Young	$\zeta^y$	0.500	OECD data
Size of the Old household	$\gamma$	1.831	CPS data
Discount Rate	$\beta$	0.990	$r = 0.04$
<b>Params requiring solving the model</b>			
Depreciation Rate	$\delta$	0.035	Targeted Moments - Table 57
Production technology elasticity	$\rho^f$	0.199	Targeted Moments - Table 57
Production technology elasticity	$\sigma^f$	0.007	Targeted Moments - Table 57
Disutility of labor for the Old	$\psi^o$	4.178	Targeted Moments - Table 57
Disutility of labor for the Young	$\psi^y$	4.168	Targeted Moments - Table 57
Curvature in consumption of the Young	$\phi^y$	0.409	Targeted Moments - Table 57
Labor elasticity of the Young	$\nu^y$	1.343	Targeted Moments - Table 57
Mean of the prod. distribution of the Young	$\mu_\varepsilon$	4.977	Targeted Moments - Table 57
Std of the prod. distribution of the Young	$\sigma_\varepsilon$	0.870	Targeted Moments - Table 57
Mean of the distaste for living with Old	$\mu_\eta$	0.126	Targeted Moments - Table 57
Std of the distaste for living with Old	$\sigma_\eta$	0.146	Targeted Moments - Table 57
Share of Young in production	$\mu_F$	0.015	Targeted Moments - Table 57
Share of Old in capital-labor CES	$\lambda_F$	0.261	Targeted Moments - Table 57
Constant in the transfer function	$\zeta_0$	0.027	Targeted Moments - Table 57
Slope of the transfer function	$\zeta_1$	0.023	Targeted Moments - Table 57
Persistence of the TFP shock for Old	$\rho^o$	0.970	Targeted Moments - Table 57
Std of the TFP shock for Old	$\sigma^o$	0.007	Targeted Moments - Table 57
Persistence of the TFP shock for Young	$\rho^y$	0.911	Targeted Moments - Table 57
Std of the TFP shock for Young	$\sigma^y$	0.006	Targeted Moments - Table 57

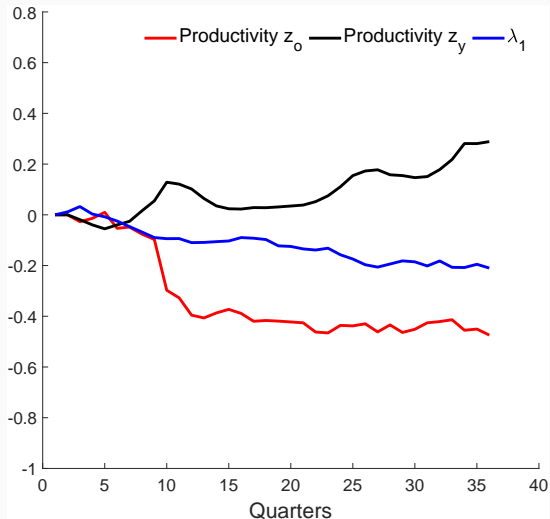




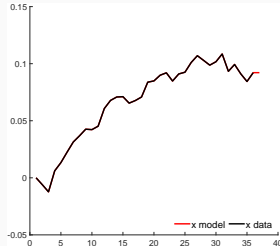
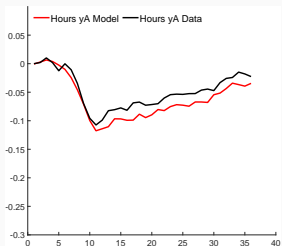
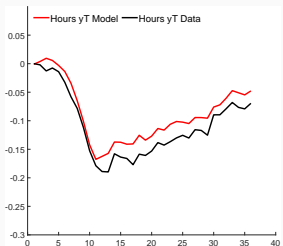
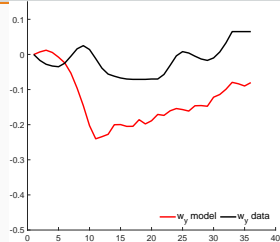
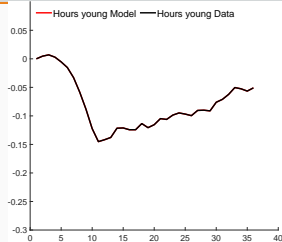
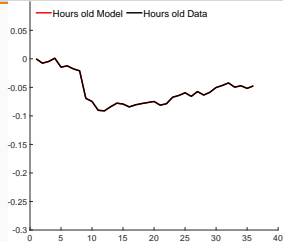








# ASYMMETRIC TFP SHOCKS + IMPROVED LEISURE ( $\eta$ )



Model inputs

Back



1. We Document **hhhold sizes and types vary over the business cycle.**
  - 1.1 In expansions hhs get smaller and the head gets younger. (mostly young people emancipate)
2. We Document **Living Arrangements of young adults shape hours worked: People that live with parents**
  - 2.1 Have higher average hours worked and lower average wages relative to their peers living alone.
  - 2.2 Have more volatile hours worked.
3. These two things together imply
  - 3.1 Volatility of hours person can be decomposed into the volatility of bodies per household and hours per household.
  - 3.2 **15%** of volatility of hours per person due to variations in household size in the data.



1. We provide a **joint quantitative theory** of the living arrangements of young adults and their labor market outcomes, which accounts for
  - 1.1 Relative differences in hours and wages between young living with old and their peers.
  - 1.2 Business cycle movements in living arrangements of young adults (vars and covars relative to macro aggregates).
  - 1.3 (First and second moments, then composition) Contribution of movements in hhs size to the variance of hours worked per person. Total hours move more than the hours of individual, in addition to the standard volatility moving in expansions makes them work more.

And it offers:

- 4.4 New **propagation and amplification (??) channel** of the aggregate shocks.



## 1. Implications of our theory:

- 1.1 We provide a rationale for **XX%** higher Frisch elasticity of labor supply in RA model due to amplification mechanism (difference between the micro and the macro elasticity).
- 1.2 We disentangle and quantify the contributions of labor demand ([Jaimovich, Pruitt, Siu \(2012\)](#)) and the **labor supply channel** to differences in hours and wages between young and old.
- 1.3 We provide **a measure of implicit transfers, the average level and dispersion of "joy" of living with parents** from old households to young living with them. We quantify the wedge in labor elasticities between young living with old and their peers resulting from different living arrangements.



- **Hhold sizes and types vary over the business cycle.**
  - In expansions hhs get smaller and the head gets younger. (mostly young people emancipate)
- **Living Arrangements of young adults shape hours worked:  
People that live with parents**
  - Work less and have lower wages than those alone
  - Have more volatile hours worked.
- These two things together imply
  - Volatility of hours person can be decomposed into the volatility of bodies per household and hours per household.
  - **15%** of volatility of hours per person due to variations in household size in the data.





- We provide a **joint quantitative theory** of the living arrangements of young adults and their labor market outcomes, which accounts for
  - Relative differences in hours and wages between young living with old and their peers.
  - Business cycle movements in living arrangements of young adults (vars and covars relative to macro aggregates).
  - (First and second moments, then composition) Contribution of movements in hhs size to the variance of hours worked per person. Total hours move more than the hours of individual, in addition to the standard volatility moving in expansions makes them work more.

And it offers:

- New **propagation and amplification channel** of aggregate shocks.



- Implications of our theory:
  - We provide a rational for **XX%** higher Frisch elasticity of labor supply in RA model due to amplification mechanism (difference between the micro and the macro elasticity).  
We disentangle and quantify the contributions of labor demand (**Jaimovich, Pruitt, Siu (2012)** who argue for imperfect substitution of hours by age) and the **labor supply channel** to differences in hours and wages between young and old.  
We provide **a measure of implicit transfers, the average level and dispersion of "joy" of living with parents** from old households to young living with them. We quantify the wedge in labor elasticities between young living with old and their peers resulting from different living arrangements.



- Productivity heterogeneity:  $\varepsilon \sim \log N$
- Disutility heterogeneity:  $\eta \sim N$
- Implicit transfer function:  $\zeta(c^o) = \zeta_0 + \zeta_1 c^o$
- Agg. shocks:  $\log Z'_i = \rho_i \log Z_i + \xi_i$ , where  $\xi_i \sim N(0, \sigma_\xi^i)$ ,  $i = o, y$
- Demographics:  $\mu, \zeta$
- Preferences old  $\psi^o, \beta, \nu^o$
- Productivity dist  $\mu_\varepsilon, \sigma_\varepsilon$
- Young preferences  $\gamma, \nu^y, \psi^y$
- Implicit transfers  $\zeta_0, \zeta_1$
- Disutility dist:  $\mu_\eta, \sigma_\eta$
- Production parameters:  $\alpha, \lambda, \delta, \sigma, \rho$
- Agg. shocks:  $\{\rho_i, \sigma_\xi^i\}_{i=y,o}$ ,  $\text{Corr}(\xi_o, \xi_y)$



- Exogenously imposed parameters (Data + Micro Estimates):  $\nu^o, \mu, \zeta, \sigma, \rho$
- Directly identified from steady-state conditions:  $\beta$
- Rest is targeted using Simulated Method of Moments (SMM):
  - There are 18 parameters and we use 24 targets (next slide)
  - Both first and second moments are targeted. Can not separate their identification in the model.



## First Moments:

- Macro:  $I/Y$ ,  $K/Y$ ,  $h^o$
- Living arrangements:  $h^{yA}$ ,  $h^{yT}$ ,  $x$ ,  $w^{yA}/w^o$ ,  $w^{yT}/w^o$

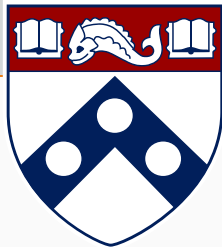
## Second Moments:

- Properties of Solow Residual:  $AutoCorr(TFP)$ ,  $\sigma(TFP)$
- Relative hours:  $\sigma(h^{yA})/\sigma(h^o)$ ,  $\sigma(h^{yT})/\sigma(h^o)$ ,  $\sigma(h^y)/\sigma(h^o)$
- Relative wages:  $\sigma(w^{yA})/\sigma(w^o)$ ,  $\sigma(w^{yT})/\sigma(w^o)$ ,  $\sigma(w^y)/\sigma(w^o)$
- Living arrangements moments:  $\sigma(x)/\sigma(h^o)$ ,  $M$ ,  $Contr_{HF}$
- Correlations:  $Corr(x, h)$ ,  $Corr(wy, wo)$ ,  $Corr(c, x)$ ,  $Corr(c, x_{-3})$ ,  $Corr(c, x_{+3})$



	Data	Model
<b>Relative hours</b>		
$E[h^Y]/E[h^O]$	1.00	0.98
$E[h^{YA}]/E[h^{YT}]$	1.24	1.35
$\sigma[h^Y]/\sigma[h^O]$	1.58	1.57
$\sigma[h^{YA}]/\sigma[h^{YT}]$	0.69	0.71
<b>Relative wages</b>		
$E[w^Y]/E[w^O]$	0.65	0.64
$E[w^{YA}]/E[w^{YT}]$	1.44	1.32
$\sigma[w^Y]/\sigma[w^O]$	1.07	1.12
$\sigma[w^{YA}]/\sigma[w^{YT}]$	1.06	1.04
<b>Living arrangements</b>		
$\sigma[x]/\sigma[h^O]$	0.75	0.75
$\text{corr}(x, h)$	-0.56	-0.56
$M$ (%)	5.0	4.5
<b>Contr <math>F/N</math> (%)</b>	15.3	16.1

\*Non-targeted moments.



$$\zeta(c^o) = \zeta_0 + \zeta_1 c^o$$

1. AVERAGE FRACTION OF CONSUMPTION OF OLD

$$E \left[ \frac{\zeta(c^o)}{c^o} \right] = 13\%$$

2. AVERAGE FRACTION OF CONSUMPTION OF YOUNG TOGETHER

$$E \left[ \frac{\zeta(c^o)}{\zeta(c^o) + c^{yT}} \right] = 49\%$$

3. AVERAGE ADDITIONAL HOURS NEED TO WORK BY YOUNG TOGETHER

$$E \left[ \frac{\hat{h}^{yT} - h^{yT}}{h^{yT}} \right] = 37\%$$



- Frisch elasticity for old = 0.72
- Marshallian elasticity for young alone

$$e^{yA} = \frac{(1-\gamma)\nu^y}{1+\gamma\nu^y} e^{yA} = 0.45$$

- Marshallian elasticity for young together

$$e^{yT}(\varepsilon) = e^{yA} \times \frac{1 + \frac{1}{1-\gamma} \frac{\zeta(c^0)}{c^{yT}(\varepsilon)}}{1 + \frac{1}{1+\gamma\nu^y} \frac{\zeta(c^0)}{c^{yT}(\varepsilon)}} E[e^{yT}] = 0.73$$

- If  $\gamma < 1$ ,  $\zeta > 0$  then  $e^{yT}(\varepsilon) > e^{yA}$
- If  $\zeta = 0$  then  $e^{yT}(\varepsilon) = e^{yA}$ . Also  $e^{yT}$  increasing in  $\zeta$





## Experiment 1:

- Possibility of coresidence, no endogeneity of coresidence
- $x = \bar{x}$ : fix thresholds  $\eta^*(\varepsilon, s) = \eta^*(\varepsilon, \bar{s})$
- St dev of log total hours: 5.5% lower
- St dev of log of young hours: 6.4% lower

## Experiment 2:

- No possibility of coresidence
- $x = 0$ : all young live alone
- St dev of log total hours: 31.4% lower
- St dev of log of young hours: 37.2% lower

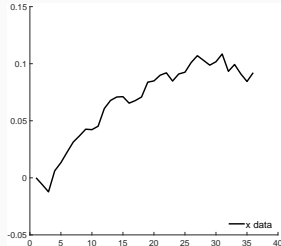
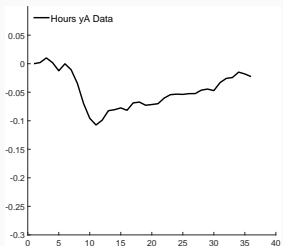
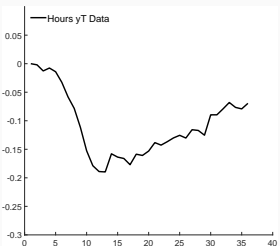
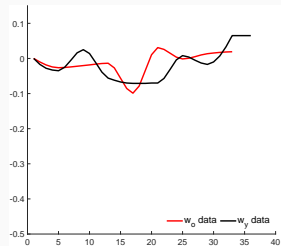
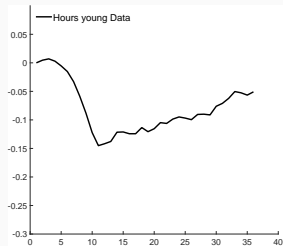
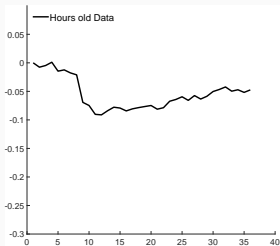


- RA models: Frisch elasticity key for volatility of aggregate hours  
→ **useful metric** for measuring strength of other channels
- What Frisch elasticity would RA model require to generate same volatility of hours as model with young people and coresidence?

Frisch elasticity for old ( $\nu^o$ )	Implied Frisch in RA RBC model	Proportional Increase
0.72	1.33	85%
0.5	0.87	75%
1.0	2.15	115%
2.0	9.62	381%

## 5 Great Recession

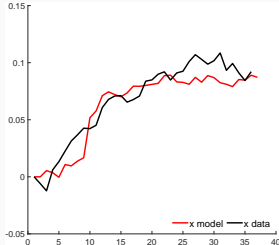
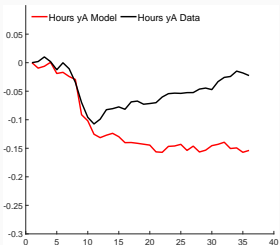
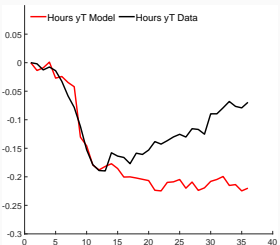
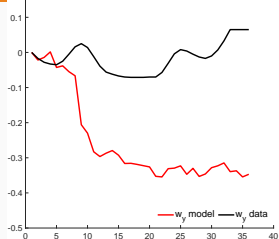
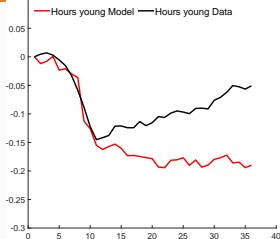
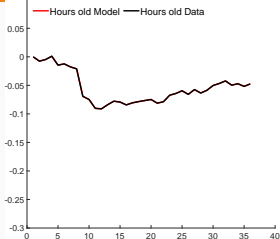
# THE GREAT RECESSION IN THE DATA





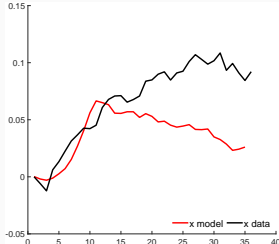
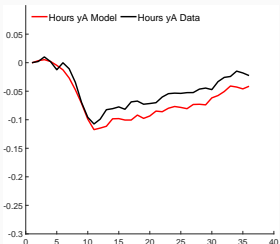
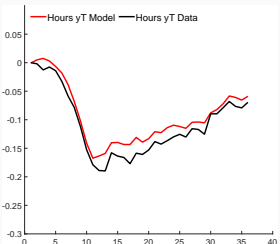
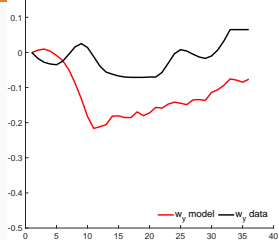
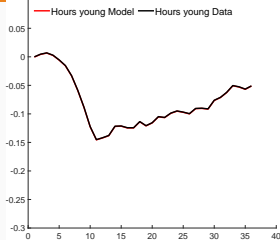
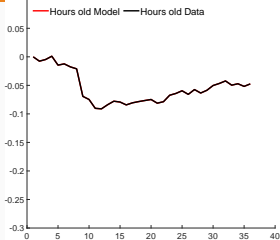
- The baseline model gets coresidence right and hours of the young only up to 10th quarter into the recession; misses the recovery.
- Reason: hours of the young recover faster than of the old
- What does it take for the model to account for these patterns?
  - **Asymmetric TFP processes** for young and old; fixes the hours but messes up the composition among young
  - **Improved leisure technology**: **Aguiar, Bils, Charles, Hurst (2018)**. It becomes less painful to live with parents being equipped with better video games.

# ONLY AIMING HOURS OF THE OLD



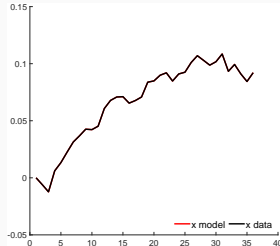
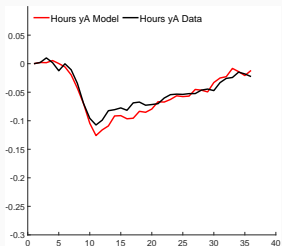
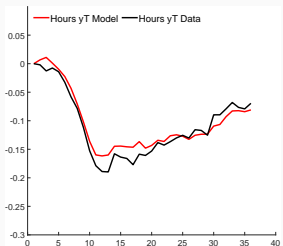
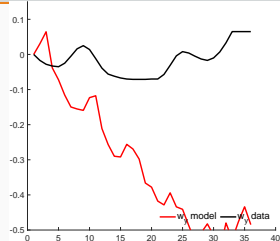
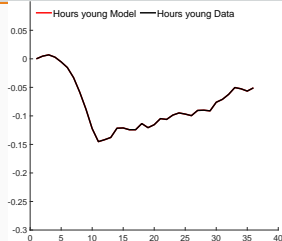
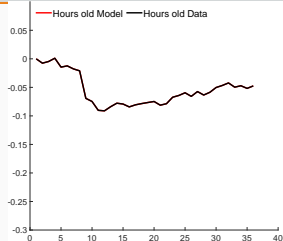
Model inputs

# ASYMMETRIC TFP SHOCKS TO MATCH HOURS RECOVERY



Model inputs

# ASYMMETRIC TFP SHOCKS + IMPROVED LEISURE ( $\psi_y$ )



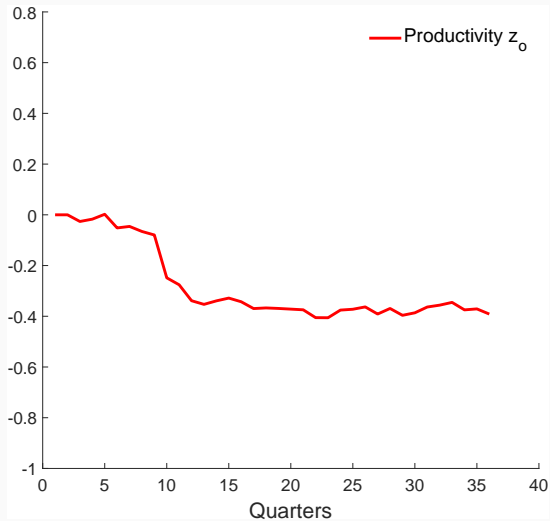
Model inputs

Improved leisure through  $\gamma$

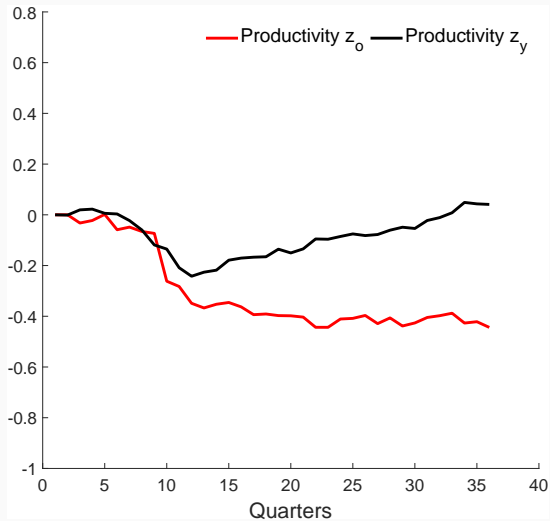


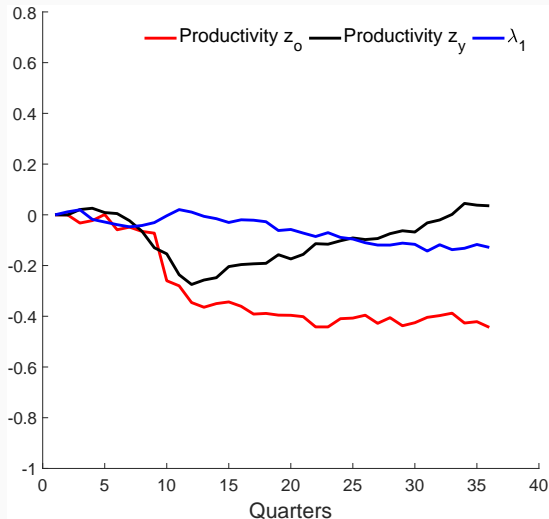


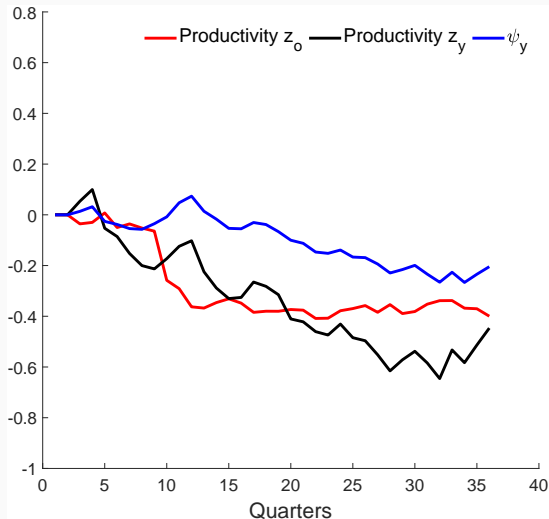
- Young and old have different labor market outcomes. Living arrangements play central role in shaping the behavior of the young.
- We have provided a theory of how it works and mapped it to the data. This theory accounts for the average and cyclical behavior of the young and the old.
- A rational for differences between the micro and the macro (which is 85% larger) Frisch elasticities.
- Our theory + Aguiar et. al. (2018) mechanism accounts for steep rise of coresidence and different outcomes of young and old during the Great Recession.



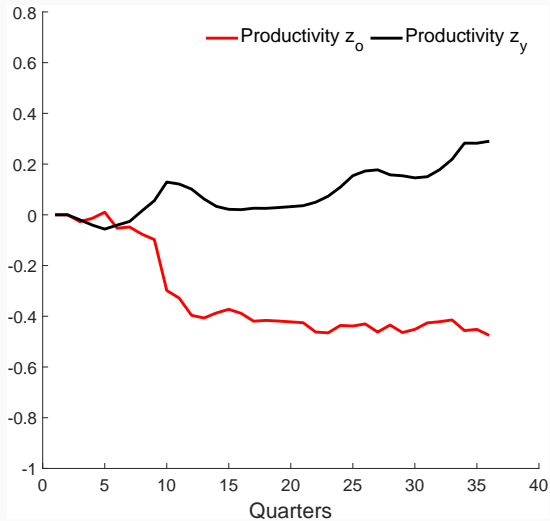
## YOUNG HIT HARDER IN THE GR, BUT RECOVER FASTER



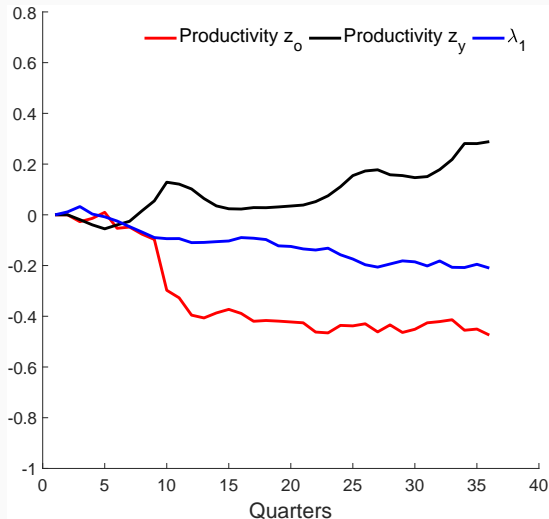




# YOUNG HIT HARDER IN THE GR, BUT RECOVER FASTER



Back





- CPS Basic Monthly Surveys for hours (monthly)
- CPS ASEC for wages (annual)
- Individuals: 18-65 year olds, not in school, not in group quarters
- Households: households with at least one such person
- Household size: number of 18-65 year olds not in school
- Quarterly series: de-seasonalize using X12-ARIMA from BLS
- Detrending:
  - 1978-2006: Hodrick-Prescott and various other filters,
  - 2007-2010: Great Recession
  - 2011-2015: Great Recession recovery





- Importance of **endogeneity of coresidence**: counterfactual series for hours assuming constant  $x$  = fraction of young living with old
  
- All variation in hours is due to variation in hours of two groups:

$$M = \frac{V(\log h^y) - V(\log [\bar{x}h^{y^T} + (1 - \bar{x}h^{y^A})])}{V(\log h^y)}$$
$$\approx 5\%$$



	Data	RBC + Imp. Subst.	RBC + Liv. Arr.	Baseline Model
<b>Relative hours</b>				
$E[h^y]/E[h^o]$	1.00	1.01	0.99	0.98
$E[h^{yA}]/E[h^{yT}]$	1.24	-	1.37	1.35
$\sigma[h^y]/\sigma[h^o]$	1.58	1.58	1.60	1.57
$\sigma[h^{yA}]/\sigma[h^{yT}]$	0.69	-	0.72	0.71
<b>Relative wages</b>				
$E[w^y]/E[w^o]$	0.65	0.87	0.63	0.64
$E[w^{yA}]/E[w^{yT}]$	1.44	-	1.33	1.32
$\sigma[w^y]/\sigma[w^o]$	1.07	1.32	1.00	1.12
$\sigma[w^{yA}]/\sigma[w^{yT}]$	1.06	-	1.15	1.04
<b>Living arrangements</b>				
$\sigma[x]/\sigma[h^o]$	0.75	-	0.77	0.75
$\text{corr}(x, h)$	-0.56	-	-0.57	-0.56
$M$ (%)	5.0	-	4.6	4.5

\*Frisch for the old

across experiments is 0.72.

[Back](#)



Two sets of parameters from outside model:

1. Production function elasticities: Jaimovich-Pruitt-Siu (2013)
2. Frisch elasticity of old: baseline = 0.72 Heathcote-Storesletten-Violante (2014)

Estimate remaining parameters using cyclical fluctuations, 1978-06

1. Standard aggregates ( $r$ ,  $I/Y$ , Capital Share, Solow residual)
2. Mean hours of old, young alone, young together
3. Mean wages of young alone, young together
4. St dev hrs of young along, young together **relative to st dev hrs old**
5. Mean fraction of young living with old
6. St dev fraction of young living with old **relative to st dev hrs old**
7. Correlation between fraction of young living with old and hours



- Standard RA intertemporal problem

$$V^o(a; w^o, r) = \max_{c^o, h^o, a'} u^o(c^o, h^o) + \beta \mathbb{E} \left[ V^o(a'; w^{o'}, r') \right]$$

$$\text{s.t.} \quad c^o + a' = w^o h^o + (1+r)a$$

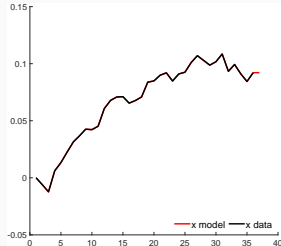
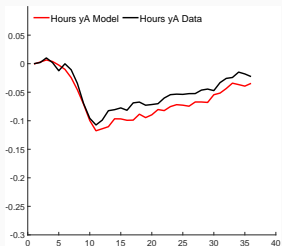
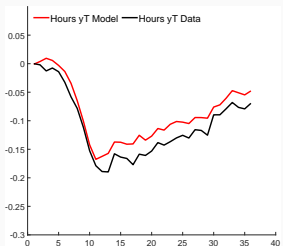
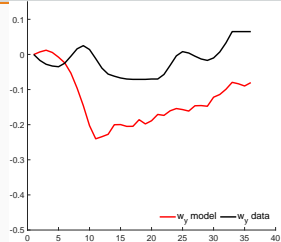
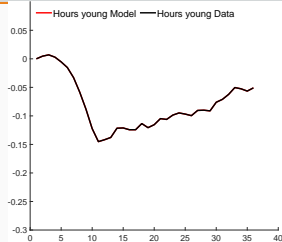
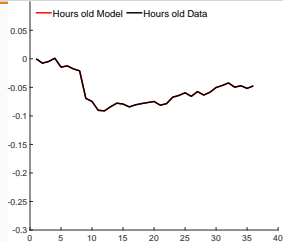
- Preferences taking into account young invasion

$$u(c^o, h^o, x) = \left[ 1 - \frac{x(1-\mu)\gamma}{\mu} \right] \left[ \frac{1}{1-\sigma^o} \left( \frac{c^o}{\zeta^o} \right)^{1-\sigma^o} - \psi^o \frac{(h^o)^{1+\frac{1}{\nu^o}}}{1+\frac{1}{\nu^o}} \right]$$

$$+ \frac{x(1-\mu)\gamma}{\mu} \left[ \frac{1}{1-\sigma^o} \left( \frac{c^o}{\zeta^o + \zeta^y} \right)^{1-\sigma^o} - \psi^o \frac{(h^o)^{1+\frac{1}{\nu^o}}}{1+\frac{1}{\nu^o}} \right]$$

- Aggregate uncertainty:  $w^o, r$

# ASYMMETRIC TFP SHOCKS + IMPROVED LEISURE ( $\eta$ )



Model inputs

Back

# Wealth, Wages, and Employment

Per Krusell, Jinfeng Luo José-Víctor Ríos-Rull

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# INTRODUCTION

- We want a theory of the joint distribution of employment, wages, and wealth, where
  - Workers are risk averse, so only use self-insurance.
  - Employment and wage risk are endogenous. (More concerned about whether people work than about how long they work.)
  - The economy aggregates into a modern economy (total wealth, labor shares, consumption/investment ratios)
  - Business cycles can be studied. In particular, we want to study employment flows jointly with the other standard objects.
- The most sophisticated version compares well with fluctuations data.

- The steady state of this economy has as its core [Aiyagari \(1994\)](#) meets [Merz \(1995\)](#), [Andolfatto \(1996\)](#) meets [Moen \(1997\)](#).
- Related [Lise \(2013\)](#), [Hornstein, Krusell, and Violante \(2011\)](#), [Krusell, Mukoyama, and Şahin \(2010\)](#), [Ravn and Sterk \(2016, 2017\)](#), [Den Haan, Rendahl, and Riegler \(2015\)](#).
- Specially [Eeckhout and Sepahsalari \(2015\)](#), [Chaumont and Shi \(2017\)](#), [Griffy \(2017\)](#).
- Developing empirically sound versions of these ideas compels us to
  - Add extreme value shocks as a form of accommodating quits and on the job search as choices.
  - Use new potent tools to address the study of fluctuations in complicated economies [Boppart, Krusell, and Mitman \(2018\)](#)



## WHAT ARE THE USES?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force
- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.
- Get some insights into the extent of wage rigidity
- Life-Cycle versions of these ideas (under construction) will allow us to assess how age dependent policies fare.

# TODAY: BUILD THE THEORY SEQUENTIALLY AND DISCUSS & FLUCTUATIONS FROM TWO TYPES OF SHOCKS

- ① **No Quits:** Exogenous Destruction, no Quits. Built on top of Growth Model. (GE version of **Eeckhout and Sepahsalari (2015)**): Not a lot of wage dispersion. Not a lot of job creation in expansions.
- ② Add **Endogenous Quits:** Higher wage dispersion may arise to keep workers longer (quits via extreme value shocks).
- ③ **On the Job Search** workers may get outside offers and take them. (Similar but not the same as in **Chaumont and Shi (2017)**).
- ④ **Outside of the Labor Force**
- ⑤ **All of the Above**
  - Employers commit both to either a wage or a wage schedule  $w(z)$  that depends on the aggregate shock.

## KEY FINDINGS

- If wages are fully fixed and committed (Drastic Wage rigidity)
  - Both endogenous quits and on-the-job yield counter factual procyclical unemployment and massive on the job search.
  - Allowing the wage of an already formed job match to respond some to aggregate shocks corrects this.
  - Getting the right relative volatility of old and new wages and the amount of job-to-job moves and quits provides a way to measure wage rigidity.
- With partial wage rigidity the model fares reasonably well with the data. A few things still to improve. (Excessive Job-to-JOB transitions)
- Similar behavior to that in the Shimer/Hagedorn-Manowski debate. Here we can try to move towards an accommodation of both points of view.

# A Brief Look At Data

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## RELEVANT PROPERTIES IN U.S. DATA

---

	Mean	St Dev	Relt	Correl	
	Perc	to Output	w Output	Source	
Average Wage	-	0.44-0.84	0.24-0.37	Haefke et al. (2013)	
New Wage	-	0.68-1.09	0.79-0.83	Haefke et al. (2013)	
Unemployment	4-6	4.84	-0.85	Campolmi&Gnocchi (2016)	
Annual Quits (All)	10-40	4.20	0.85	Brown et al. (2017)	
Annual Switches	25-35	4.62	0.70	Fujita&Nakajima (2016)	
Consumption	75	0.78	0.86	NIPA	
Investment	25	4.88	0.90	NIPA	

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# Model 1: No (Endogenous) Quits Model

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## No (ENDO) QUILTS: PRECAUTIONARY SAVINGS, COMPETITIVE SEARCH

- Jobs are created by firms (plants). A plant with capital plus a worker produce one ( $z$ ) unit of the good ( $z$  is the aggregate state of the economy).
  - Firms pay flow cost  $\bar{c}$  to post a vacancy in market  $\{w, \theta\}$ .
  - Firms cannot change wage (or wage-schedule) afterwards.
  - Think of a firm as a machine programmed to pay  $w$  or  $w(z)$
  - Plants (and their capital) are destroyed at rate  $\delta^f$ .
  - Workers quit exogenously at rate  $\delta^h$ .
- Households differ in wealth and wages (if working) but not in productivity. There are no state contingent claims, nor borrowing.
  - If employed, workers get  $w$  and save.
  - If unemployed, workers produce  $b$  and search in some  $\{w, \theta\}$ .
- General equilibrium: Workers own firms.

## ORDER OF EVENTS OF **No QUILTS** MODEL

- 1 Households enter the period with or without a job:  $\{e, u\}$ .
- 2 **Production & Consumption**: Employed produce  $z$  on the job. Unemployed produce  $b$  at home. They choose savings.
- 3 **Firm Destruction and Exogenous Quits** :  
Some Firms are destroyed (rate  $\delta^f$ ) They cannot search this period.  
Some workers quit their jobs for exogenous reasons  $\delta^h$ . Total job destruction is  $\delta$ .
- 4 **Search**: Firms and the unemployed choose wage  $w$  and tightness  $\theta$ .
- 5 **Job Matching** :  $M(V, U)$  : Some vacancies meet some unemployed job searchers. A match becomes operational the following period.  
Job finding and job filling rates  $\psi^h(\theta) = \frac{M(V, U)}{U}$ ,  $\psi^f(\theta) = \frac{M(V, U)}{V}$ .



## No QUILTS MODEL: HOUSEHOLD PROBLEM

- Individual state: wealth and wage
  - If employed:  $(a, w)$
  - If unemployed:  $(a)$
- Problem of the employed: (Standard)

$$V^e(a, w) = \max_{c, a'} u(c) + \beta [(1 - \delta)V^e(a', w) + \delta V^u(a')]$$
$$\text{s.t. } c + a' = a(1 + r) + w, \quad a \geq 0$$

- Problem of the unemployed: Choose which wage to look for

$$V^u(a) = \max_{c, a', w} u(c) + \beta \{ \psi^h[\theta(w)] V^e(a', w) + [1 - \psi^h[\theta(w)]] V^u(a') \}$$
$$\text{s.t. } c + a' = a(1 + r) + b, \quad a \geq 0$$

$\theta(w)$  is an equilibrium object

# FIRMS POST VACANCIES: CHOOSE WAGES & FILLING PROBABILITIES

- Value of wage- $w$  job: uses constant  $\bar{k}$  capital that depreciates at rate  $\delta^k$  ( $\Omega = \bar{k}$ )

$$\Omega(w) = z - \bar{k}\delta^k - w + \frac{1 - \delta^f}{1 + r} [(1 - \delta^h)\Omega(w) + \delta^h\Omega]$$

- Affine in  $w$ : 
$$\Omega(w) = \left[ z + \bar{k} \left( \frac{1 - \delta^f}{1 + r} \delta^h - \delta^k \right) - w \right] \frac{1 + r}{r + \delta^f + \delta^h - \delta^f \delta^h}$$

**Block Recursivity Applies** (firms can be ignorant of Eq)

- Value of creating a firm:  $\psi^f[\theta(w)] \Omega(w) + [1 - \psi^f[\theta(w)]] \Omega$
- Free entry condition requires that for all offered wages

$$\bar{c} + \bar{k} = \psi^f[\theta(w)] \frac{\Omega(w)}{1 + r} + [1 - \psi^f[\theta(w)]] \frac{\Omega}{1 + r},$$

## No (ENDO) QUIT MODEL: STATIONARY EQUILIBRIUM

- A stationary equilibrium is functions  $\{V^e, V^u, \Omega, g'^e, g'^u, w^u, \theta\}$ , an interest rate  $r$ , and a stationary distribution  $x$  over  $(a, w)$ , s.t.

①  $\{V^e, V^u, g'^e, g'^u, w^u\}$  solve households' problems,  $\{\Omega\}$  solves the firm's problem.

② Zero profit condition holds for active markets

$$\bar{c} + \bar{k} = \psi^f[\theta(w)] \frac{\Omega(w)}{1+r} + [1 - \psi^f[\theta(w)]] \frac{\bar{k}(1 - \delta - \delta_k)}{1+r}, \quad \forall w \text{ offered}$$

③ An interest rate  $r$  clears the asset market

$$\int a \, dx = \int \Omega(w) \, dx.$$

## CHARACTERIZATION OF A WORKER'S DECISIONS

- Standard Euler equation for savings

$$u_c = \beta (1 + r) E \{u'_c\}$$

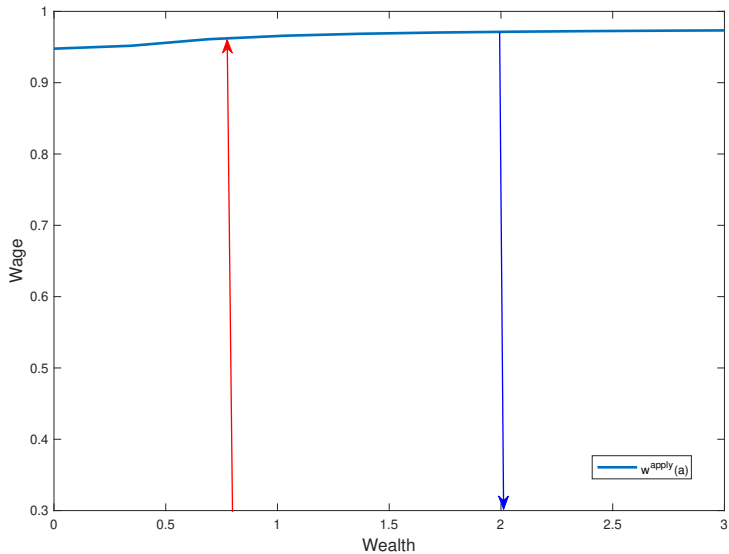
- A F.O.C for wage applicants

$$\psi^h[\theta(w)] V_w^e(a', w) = \psi_\theta^h[\theta(w)] \theta_w(w) [V^u(a') - V^e(a', w)]$$

- Households with more wealth are able to insure better against unemployment risk.
- As a result they apply for higher wage jobs and we have dispersion

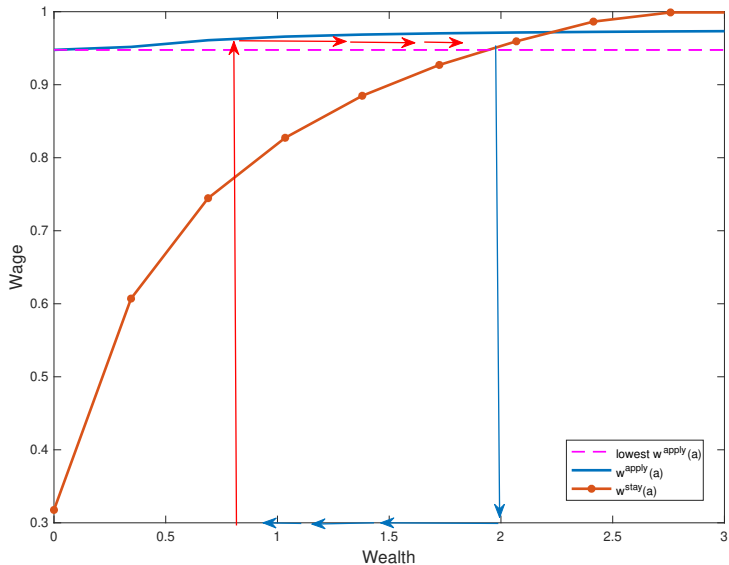
# HOW DOES THE MODEL WORK

## WORKER'S WAGE APPLICATION DECISION



# HOW DOES THE MODEL WORK

## WORKER'S SAVING DECISION



## SHORTCOMINGS OF THIS MODEL

---

- Silent on Quits and Job-To-Job Movements.
- Low Wage Dispersion
- Small differences in volatility between average and new wages
- Low unemployment volatility

## SUMMARY: No (ENDO) QUILTS MODEL

- ① Easy to Compute Steady-State with key Properties
  - ❶ Risk-averse, only partially insured workers, endogenous unemployment
  - ❷ Can be solved with aggregate shocks too
  - ❸ Policy such as UI would both have insurance and incentive effects
  - ❹ Wage dispersion small—wealth doesn't matter too much
  - ❺ ...so almost like two-agent model (employed, unemployed) of Pissarides despite curved utility and savings
  
- ② In the following we examine the implications of a quitting choice



# Endogenous Quits

---

## ENDOGENOUS QUILTS: BEAUTY OF EXTREME VALUE SHOCKS

- Temporary Shocks to the utility of working or not working: Some workers quit. (in addition to any intrinsic taste for leisure)
- Adds a (smoothed) quitting motive so that higher wage workers quit less often: Firms may want to pay high wages to retain workers.
- Conditional on wealth, high wage workers quit less often.
- But Selection (correlation 1 between wage and wealth when hired) makes wealth trump wages and those with higher wages have higher wealth which makes them quite more often: Wage inequality collapses.
- We end up with a model with little wage dispersion but with endogenous quits that respond to the cycle.

# QUITTING MODEL: TIME-LINE

- 1 Workers enter period with or without a job:  $\{e, u\}$ .
- 2 Production occurs and consumption/saving choice ensues:
- 3 Exogenous job/firm destruction happens.
- 4 **Quitting:**
  - $e$  draw shocks  $\{\epsilon^e, \epsilon^u\}$  and make quitting decision. Job losers cannot search this period.
  - $u$  draw shocks  $\{\epsilon_1^u, \epsilon_2^u\}$ . No decision but same expected means.
- 5 **Search:** New or **Idle** firms post vacancies. Choose  $\{w, \theta\}$ .  
Wealth is not observable. (Unlike **Chaumont and Shi (2017)**).  
Yet it is still **Block Recursive**
- 6 Matches occur

## QUITTING MODEL: WORKERS

- Workers receive i.i.d shocks  $\{\epsilon^e, \epsilon^u\}$  to the utility of working or not
- Value of the employed right before receiving those shocks:

$$\widehat{V}^e(a', w) = \int \max\{V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u\} dF^\epsilon$$

$V^e$  and  $V^u$  are values after quitting decision as described before.

- If shocks are Type-I Extreme Value dbtn (Gumbel), then  $\widehat{V}$  has a closed form and the ex-ante quitting probability  $q(a, w)$  is

$$q(a, w) = \frac{1}{1 + e^{\alpha[V^e(a, w) - V^u(a)]}}$$

higher parameter  $\alpha \rightarrow$  lower chance of quitting.

- Hence higher wages imply longer job durations. Firms could pay more to keep workers longer.

## QUITTING MODEL: WORKERS PROBLEM

- Problem of the employed: just change  $\widehat{V}^e$  for  $V^e$

$$V^e(a, w) = \max_{c, a'} u(c) + \beta \left[ (1 - \delta) \widehat{V}^e(a', w) + \delta V^u(a) \right]$$

s.t.  $c + a' = a(1 + r) + w, \quad a \geq 0$

- Problem of the unemployed is like before except that there is an added term  $E\{\max[\epsilon_1^u, \epsilon_2^u]\}$

So that there is no additional option value to a job.

## QUITTING MODEL: VALUE OF THE FIRM

- $\Omega^j(w)$ : Value with with  $j$ -tenured worker.

Free entry condition requires that for all offered wages

$$\bar{c} + \bar{k} = \frac{1}{1+r} \{ \psi^f[\theta(w)] \Omega^0(w) + [1 - \psi^f[\theta(w)]] \Omega \},$$

- Probability of retaining a worker with tenure  $j$  at wage  $w$  is  $\ell^j(w)$ . (One to one mapping between wealth and tenure)

$$\ell^j(w) = 1 - q^e[g^{e,j}(a, w), w]$$

$g^{e,j}(a, w)$  savings rule of a  $j$  – tenured worker that was hired with wealth  $a$

- Firm's value

$$\Omega^j(w) = z - \bar{k}\delta^k - w + \frac{1 - \delta^f}{1+r} \{ \ell^j(w)\Omega^{j+1}(w) + [1 - \ell^j(w)] \Omega \}$$

## QUITTING MODEL: SOLVING FORWARD FOR THE VALUE OF THE FIRM

$$\Omega^0(w) = (z - w - \delta^k k) Q^1(w) + (1 - \delta^f - \delta_k) k Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[ \left( \frac{1 - \delta^f}{1 + r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[ \left( \frac{1 - \delta^f}{1 + r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left( \prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- New equilibrium objects  $\{Q^0(w), Q^1(w)\}$ . Rest is unchanged.
- It is Block Recursive because wealth can be inferred from  $w$  and  $j$ . (No need to index contracts by wealth (as in [Chaumont and Shi \(2017\)](#))).

## DO WE GET MORE WAGE DISPERSION?

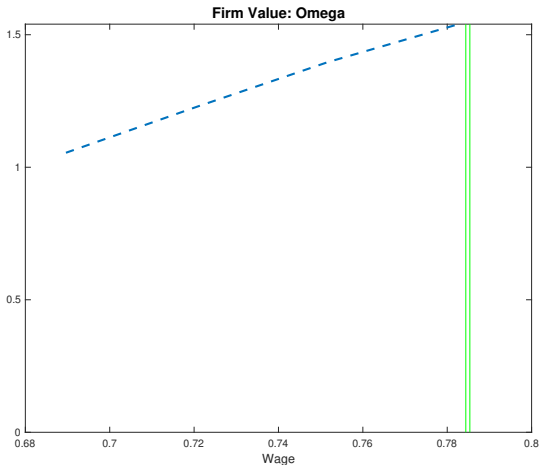
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- This Model has the potential to get more wage dispersion
- Conditional on wealth higher wages lead to less quitting.
- So firms are willing to pay more to keep workers longer
- **BUT** we will see a problem



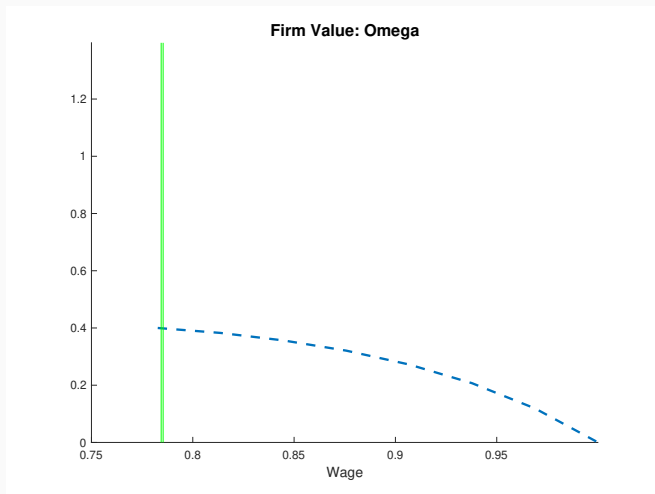
# VALUE OF THE FIRM AS WAGE VARIES: THE POOR

- For the poorest, employment duration increases when wage goes up.
- Firms value is increasing in the wage



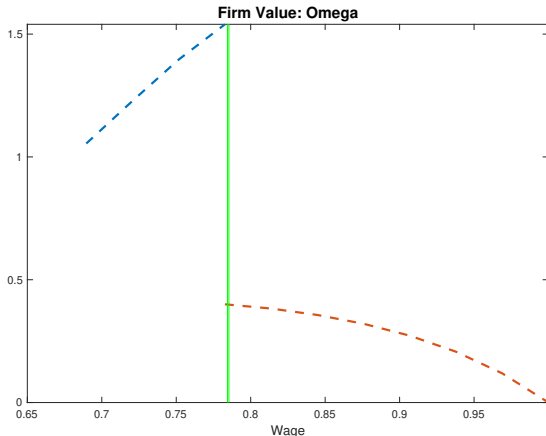
## VALUE OF THE FIRM AS WAGE VARIES: THE RICH

- For the richest, employment duration increases but not fast enough.
- Firm value is slowly decreasing in wages (less than static profits).



# VALUE OF THE FIRM: ACCOUNTING FOR WORKER SELECTION

- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion **COLLAPSES** due to selection.



- Related to the Diamond dispersion paradox but for very different reasons.

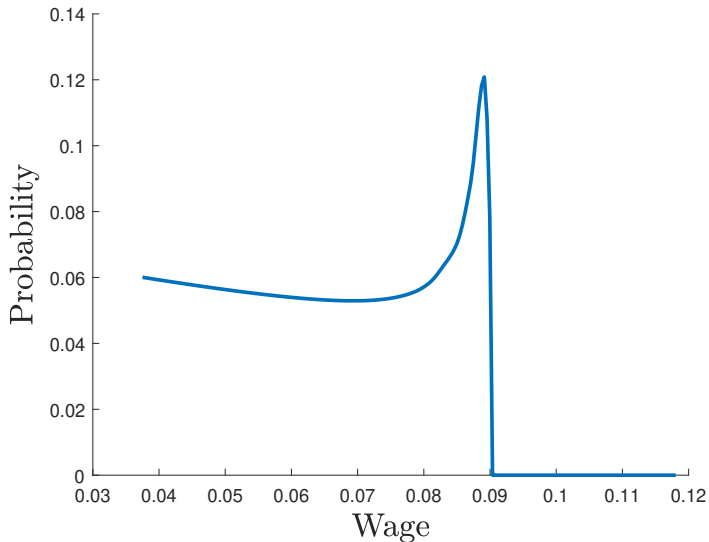
## EFFECT OF QUITTING: THE MECHANISM

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- Two forces shape the dispersion of wages
  - Agents quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay (for a given range of vacancy filling probability).
  - However, by paying higher wages, firms attract workers with more wealth.
- Wealthy people quit more often, shrink employment duration.
- In equilibrium, the wage gap is narrow (disappears?) and the effect of wealth dominates.

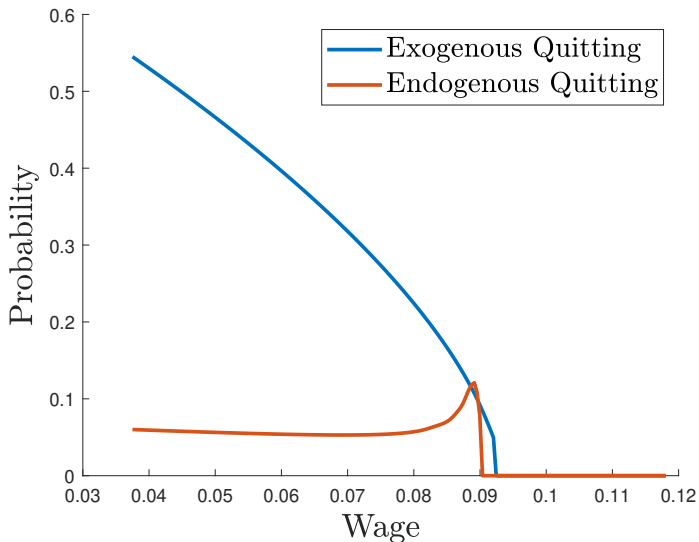
## VALUE OF THE FIRM: ZERO PROFIT JOB FINDING PROBABILITY

- Increasing in Wage (up to Grid calculation): Unique wage.



# QUITTING MAKES A BIG DIFFERENCE

- Job finding prob with Endo



- Wage Dispersion Collapses
- Silent on Job-To-Job Movements.
- Unemployment Moves little (but more than the previous one) over the cycle
- No difference in volatility between average and new wages
- Correlation 1 between Wealth when starting to work and wage

# A DETOUR ON HOW TO IMPROVE THE CORRELATION BETWEEN WEALTH AND WAGES

- Pose *aiming* (extreme value) shocks).
- This reduces the correlation between wages and wealth when first hired.
- It will have many uses, we think.



# On the Job Search

---

# ON THE JOB SEARCH MODEL: TIME-LINE

- 1 Workers enter period with or without a job:  $V^e, V^u$ .
- 2 Production & Consumption:
- 3 Exogenous Separation
- 4 **Quitting? Searching? Neither?:** Employed draw shocks  $(\epsilon^e, \epsilon^u, \epsilon^s)$  and make decision to quit, search, or neither. Those who quit become  $u'$ , those who search join the  $u$ , in case of finding a job become  $\{e', w'\}$  but in case of no job finding remain  $e'$  with the same wage  $w$  and those who neither become  $e'$  with  $w$ .  $\hat{V}^E(a', w)$ , is determined with respect to this stage.
- 5 **Search :** Potential firms decide whether to enter and if so, the market ( $w$ ) at which to post a vacancy;  $u$  and  $s$  assess the value of all wage applying options, receive match specific shocks  $\{\epsilon^{w'}\}$  and choose the wage level  $w'$  to apply. Those who successfully find jobs become  $e'$ , otherwise become  $u'$ .
- 6  $\hat{V}^u(a'), \{\Omega^j(w)\}$  are determined with respect to this stage.
- 7 Match

- After saving, the unemployed problem is

$$\widehat{V}^u(a') = \int \max_{w'} \left[ \psi^h(w') V^e(a', w') + (1 - \psi^h(w')) V^u(a') + \epsilon^{w'} \right] dF^\epsilon$$

- After saving, the employed choose whether to quit, search or neither

$$\widehat{V}^e(a', w) = \int \max \{ V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u, V^s(a', w) + \epsilon^s \} dF^\epsilon$$

- The value of searching is

$$V^s(a', w) = \int \max_{w'} \left[ \psi^h(w') V^e(a', w') + [1 - \psi^h(w')] V^e(a', w) + \epsilon^{w'} \right] dF^\epsilon$$

- The probabilities of quitting and of searching

$$q(a', w) = \frac{1}{1 + \exp(\alpha[V^e(a', w) - V^u(a')]) + \exp(\alpha[V^s(a', w) - V^u(a') + \mu^s])},$$

$$s(a', w) = \frac{1}{1 + \exp(\alpha[V^u(a') - V^s(a', w)]) + \exp(\alpha[V^e(a', w) - V^s(a', w) - \mu^s])}.$$

$\mu^s < 0$  is the mode of the shock  $\epsilon^s$  which reflects the search cost.

- Households solve

$$V^e(a, w) = \max_{a' \geq 0} u[a(1+r) + w - a'] + \beta \left[ \delta V^u(a') + (1-\delta) \widehat{V}^e(a', w) \right]$$

$$V^u(a) = \max_{c, a' \geq 0} u[a(1+r) + b - a'] + \beta \widehat{V}^u(a')$$

- The value of the firm is again given like in the **Quitting** Model

$$\Omega^0(w) = (z - w - \delta^k k) Q^1(w) + (1 - \delta - \delta_k) k Q^0(w),$$

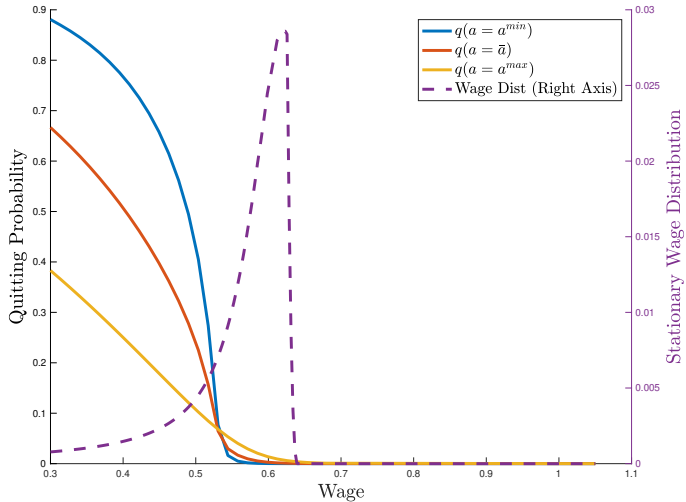
$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[ \left( \frac{1-\delta}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left( \prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- Except that now the probability of keeping a worker after  $j$  periods is

$$\ell^j(w) = 1 - \int h(w; a) q[g^{e \cdot j}(a, w), w] dx^u(a) - \int h(w; a) s[w; g^{e \cdot j}(a, w)] \left[ \int \hat{h}[\tilde{w}; g^{e \cdot j}(a, w), w] \xi \phi^h(\tilde{w}) d(\tilde{w}) \right] dx^u(a)$$

# OJS QUITTING PROBABILITIES, VARIOUS WEALTHS & WAGE DENSITY



- The rich pursue often other activities (leisure?)

# Outside the Labor Force

---

# OUTSIDE THE LABOR FORCE MODEL: TIME-LINE

- 1 Workers enter period with or without a job:  $V^e, V^u$ .
- 2 In the beginning of the period non Workers get a shock to the utility of either searching or not searching. They then choose whether to sit out and not search or to search. It is an extreme value shock. Workers get a utility injection equal to the expected utility of the maximum of those two shocks to get no bias in the value of working versus not.
- 3 Production & Consumption:
- 4 Exogenous Separation
- 5 Quitting? Searching? Neither?:
- 6 Search
- 7  $\hat{V}^u(a'), \{\Omega^j(w)\}$  are determined with respect to this stage.
- 8 Match



## VARIOUS ECONOMIES WITH ADDED LIFE CYCLE (LIVE 50 YEARS)

- Provides a mechanism for having poor agents
- Right now we have Four Economies
  - ① Only Exogenous Quitting
  - ② Endogenous Quitting
  - ③ Exogenous Quitting with On-the-job Search
  - ④ Endogenous Quitting and On-the-job Search
  - ⑤ ... and some agents do not want to work
- Today we will only look at the Economy with Endogenous quitting and On-the-Job-Search (4)

# Quantitative Analysis: Steady States

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## PARAMETER VALUES

	Definition	Value in Yearly Units
$r$	interest rate	3%
$K$	fixed capital required	3
$\delta^f$	firm destruction rate	2.88%
$\delta^k$	capital maintenance rate	6.38%
$\delta^h$	total worker quitting rate	8.56%
$c^v$	job posting cost	0.03
$y$	productivity on the job	1
$b/w$	productivity at home	0.4
$\sigma$	risk aversion	2
Matching function	$m = \chi u^\eta v^{1-\eta}$ , non-OJS	$\chi = 0.15, \eta = 0.62$
	$m = \chi u^\eta v^{1-\eta}$ , OJS	$\chi = 0.3, \eta = 0.5$

- We also explore a lower on the job search economy ( $\theta$ ) high value of leisure economy  $b/w \sim 0.75$

# STEADY STATE ALLOCATIONS IN YEARLY UNITS: ENDOG QUILTS & OJS

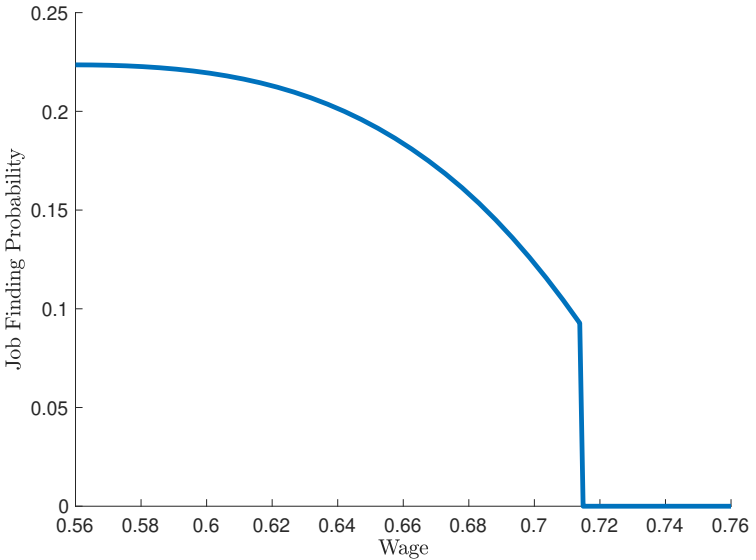
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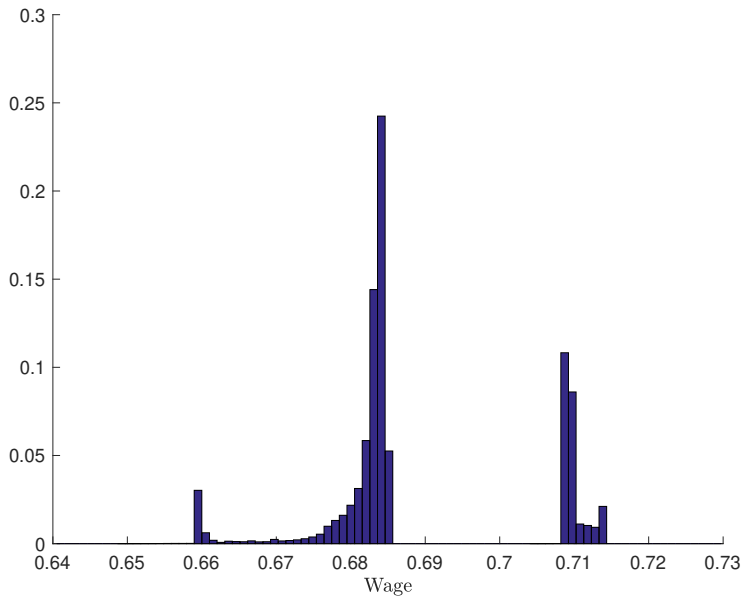
interest rate	0.030
avg consumption	0.651
avg wage	0.689
avg wealth	3.041
stock market value	2.953
avg labor income	0.654
consumption to wealth ratio	0.225
labor income to wealth ratio	0.215
quit ratio	0.090
unemployment rate	0.097
job losers	0.117
wage of newly hired unemp	0.677
std consumption	0.011
std wage	0.002
std wealth	3.606
mean-min consumption	2.051
mean-min wage	1.058
UE transition	0.125
total vacancy	0.578
avg unemp duration	0.773
avg emp duration	7.228
avg job duration	1.898
OJS move rate	0.395

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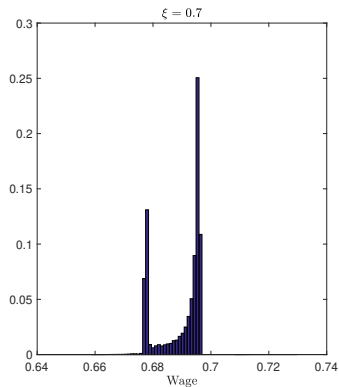
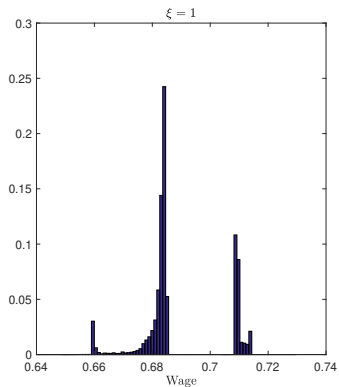
# JOB FINDING PROBABILITY CURVES



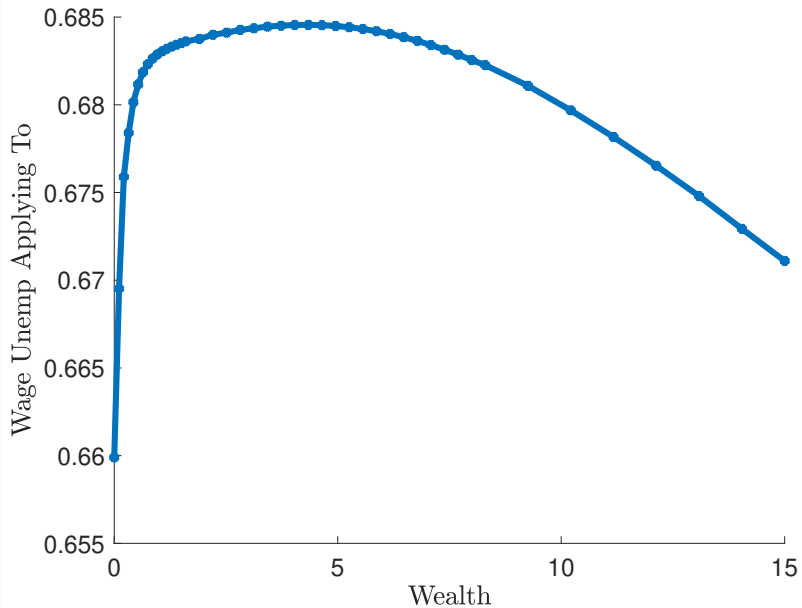
## WAGE DISTRIBUTIONS: BASELINE



# WAGE DISTRIBUTIONS: COMPARING WITH LOWER OJS

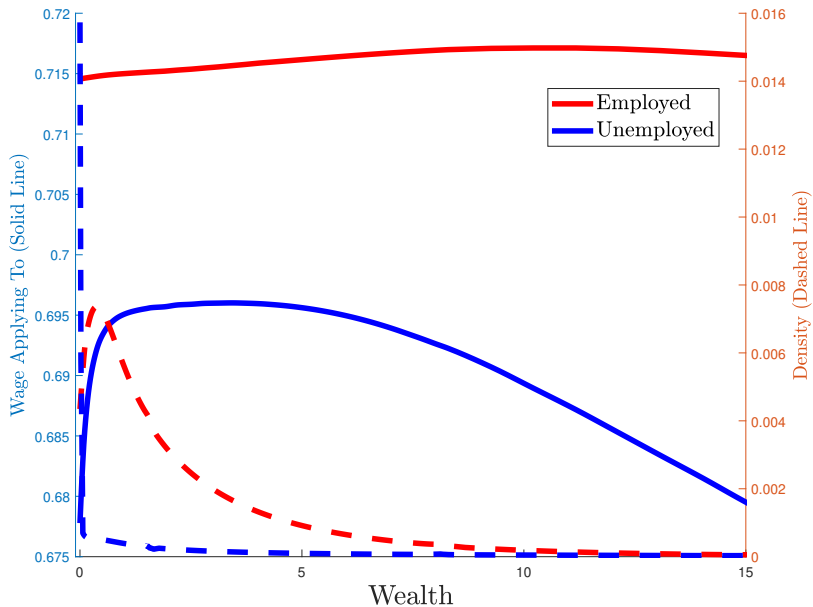


# WAGE APPLICATIONS OF THE UNEMPLOYED BY WEALTH





# WAGE APPLICATIONS OF $U$ AND $\bar{w}$ AND DENSITIES OF ALL



# Aggregate Fluctuations

---

# INTRODUCE AGGREGATE SHOCKS

- We examine the model responses to two type of shocks
  - ① Productivity shocks  $z_t$ :  $\text{Output} = \text{EmpRate} \times (1 + z_t)$
  - ② Firm destruction shocks  $d_t$ :  $\text{Firm Destruction Rate} = \delta^f \times (1 - d_t)$
- We introduce a wage peg assumption:
  - To allow the wage of an already formed job match to respond to  $z_t$  shocks directly (by 50%) (but not to  $d_t$  shocks)
  - If wages were completely rigid there would be massive quits: counterfactual.

# BASELINE: IRF TO Z SHOCK

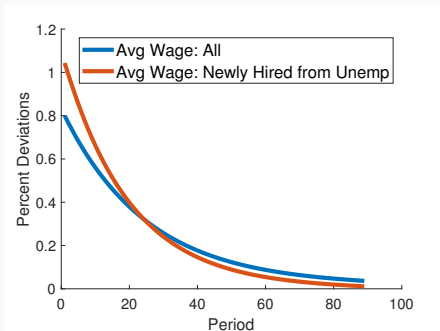


Figure 1: Wages

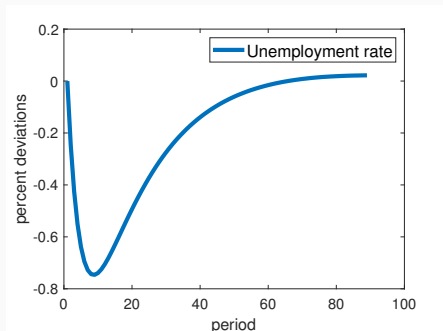


Figure 2: Unemployment Rate

- Responsive new wage (directed search) and average wage (wage peg)
- Non-trivial response of unemployment

## BASELINE: IRF TO Z SHOCK

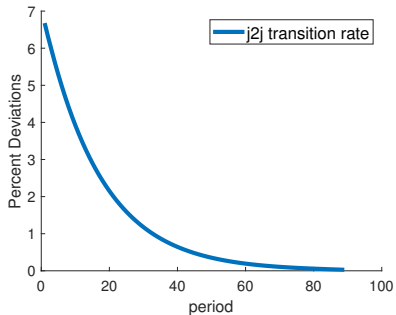


Figure 3: J2J transitions

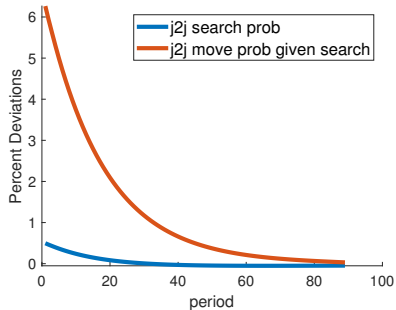


Figure 4: J2J search & JFP

- Too much responsive j2j transitions
- Due to improved job finding probability, not more searchers

## BASELINE: IRF TO $d$ SHOCK

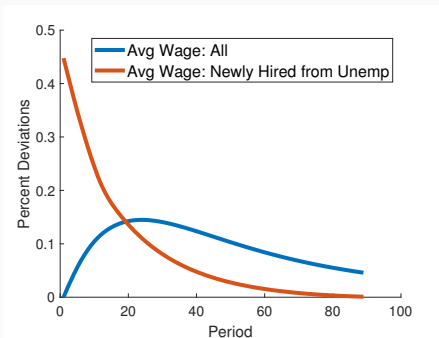


Figure 5: Wages

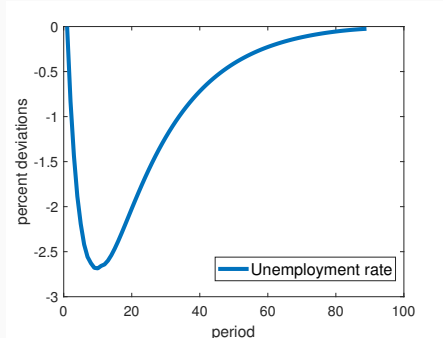


Figure 6: Unemployment Rate

- 1% delta shock = 0.36 base points
- Large response of wage and unemployment to the delta shock
- Note wage is not pegged to the delta shock

## BASELINE: IRF TO $d$ SHOCK

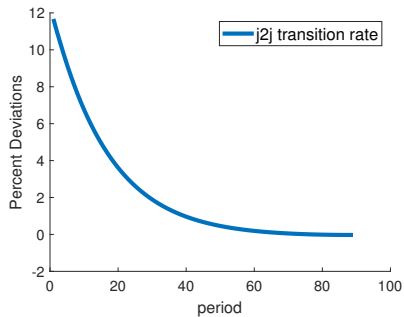


Figure 7: J2J transitions

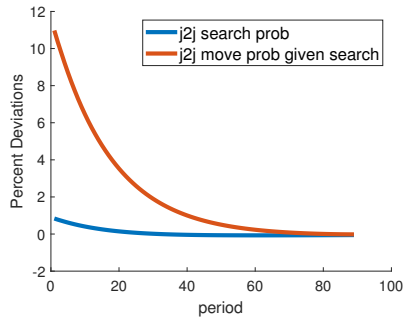


Figure 8: J2J search & JFP

- But too much volatility for job-to-job transitions

## SUMMARY, ON-THE-JOB SEARCH AND QUILTS

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- Pro-cyclical average wages, new wages, and employment, quitting, and job-to-job transitions
- Clear responses of new wages and employment
- Quitting mildly responds to both shocks
- Job-to-job transitions move too much with both shocks



# ASSESSING PERFORMANCE IN TERMS OF STANDARD HP-FILTERED 2ND MOMENTS

---

- 1st order data moments are from standard database: CPS, JOLTS, LEHD and NIPA.
- 2nd order data moments are from Haefke, Sonntag, and Van Rens (2013), Campolmi and Gnocchi (2016), Brown et al. (2017) and Fujita and Nakajima (2016).

- **Only Productivity Shock:**  $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	0.51	0.44-0.84
New Wage	0.95	0.68-1.09
Unemployment	0.35	4.84
Quits + OJS moves	8.94	4.2
OJS moves	10.66	4.62

**Table 1:** Standard Deviation Relative to Output: Only Productivity Shock

- Unemployment moves too little and Quits and OJS moves too much

## PRODUCTIVITY SHOCK: CORRELATION

- Only Productivity Shock:  $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	1.00	0.24-0.37
New Wage	1.00	0.79-0.83
Unemployment	-0.48	-0.85
Quits + OJS moves	0.99	0.85
OJS moves	0.99	0.70

**Table 2:** Correlation with Contemporary Output: Only Productivity Shock

- Correlations are on the spot

	Model	Data
Output	1	1
Average Wage	0.09	0.44-0.84
New Wage	2.02	0.68-1.09
Unemployment	4.70	4.84
Quits + OJS moves	41.66	4.2
OJS moves	49.36	4.62

**Table 3:** Standard Deviation Relative to Output: Only Delta Shock

- Now Unemployment is good but moves are excessive
- Note that relative to output, productivity is very important so employment cannot do that much, but this shock makes employment the only culprit so it has to move a lot

- Only Delta Shock:  $\rho = 0.95$

	Model	Data
Output	1	1
Average Wage	0.13	0.24-0.37
New Wage	0.31	0.79-0.83
Unemployment	-0.99	-0.85
Quits + OJS moves	0.40	0.85
OJS moves	0.42	0.70

**Table 4:** Correlation with Contemporary Output: Only Delta Shock

## BOTH SHOCKS: RELATIVE VOLATILITY VERY CORRELATED

- Interact productivity shock and delta shock
  - High Correlation of shocks = 0.95
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.49	0.44-0.84
New Wage	1.38	0.68-1.09
Unemployment	3.02	4.84
Quits + OJS moves	25.77	4.2
OJS moves	30.53	4.62

**Table 5:** Standard Deviation Relative to Output: Both Shocks

## BOTH SHOCKS: CORRELATION

- Interact productivity shock and delta shock
  - High Correlation of shocks = 0.95
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.77	0.24-0.37
New Wage	0.50	0.79-0.83
Unemployment	-0.37	-0.85
Quits + OJS moves	0.28	0.85
OJS moves	0.29	0.70

**Table 6:** Correlation with Contemporary Output: Both Shocks

## BOTH SHOCKS: **RELATIVE VOLATILITY** UNCORRELATED

- Interact productivity shock and delta shock
  - Low Correlation of shocks = 0
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.40	0.44-0.84
New Wage	1.35	0.68-1.09
Unemployment	2.59	4.84
Quits + OJS moves	23.98	4.2
OJS moves	28.45	4.62

**Table 7:** Standard Deviation Relative to Output: Both Shocks



## BOTH SHOCKS: CORRELATION UNCORRELATED

- Interact productivity shock and delta shock
  - Relative Std of shocks: each shock contributes roughly equal to output volatility

	Model	Data
Output	1	1
Average Wage	0.82	0.24-0.37
New Wage	0.62	0.79-0.83
Unemployment	-0.61	-0.85
Quits + OJS moves	0.47	0.85
OJS moves	0.48	0.70

**Table 8:** Correlation with Contemporary Output: Both Shocks

# Clumsy Experiments & Extensions

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- Now we move to some experiments/extensions to evaluate the business cycle performance of the model
- We look at the following
  - An extension to allow for **different matching function elasticities** for UE and EE moves ( $\eta^u \neq \eta^e$ ).
  - On top of that an economy with **higher  $b$**  (from 0.3 to 0.5-0.6) that illuminates the Shimer/Hagedorn-Manowski debate.

- For all the above exercises we find that the volatility of  $j2j$  transition rate is a magnitude larger than unemployment rate
- In the data unemployment rate is as volatile as (or even more volatile than) the  $j2j$  transition rate.
- Difficult to deliver this in the model from aggregate shocks affecting jobs at all wage levels
  - The percentage changes of firm value, vacancy filling probability and job finding probability are similar at all wage levels
  - Thus as a stock, the response of unemployment would thus be a magnitude smaller than the  $j2j$  transition rate (a flow)

## HETERO- $\eta$ ECONOMY: MOTIVATION

- Two potential fix
  - Make the firm value at high wages more volatile  $\Rightarrow$  hard since high-wage matches feature low profits
  - Make the job finding probability of the employed less responsive to the same percentage change in the firm value  $\Rightarrow$  curvature in the matching function controls this
- Motivated by this, we will allow  $\eta$  in the matching function  $m = \chi u^\eta v^{1-\eta}$  to be low in UE moves but high in EE moves
  - $\psi^h(w) = \chi \left( \frac{\chi}{\psi^f(w)} \right)^{\frac{1-\eta}{\eta}} \Rightarrow \ln \psi^h(w) = \frac{1}{\eta} \ln \chi - \frac{1-\eta}{\eta} \ln \psi^f(w)$
  - Higher  $\eta \Rightarrow$  smaller response of  $\psi^h(w)$  to  $\psi^f(w)$
- Lower  $\eta^u$  from 0.5 to 0.35 and raise  $\eta^e$  from 0.5 to 0.75

# HETERO- $\eta$ ECONOMY: IRF TO $z$ SHOCK

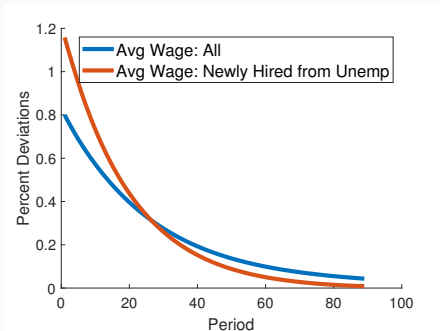


Figure 9: Wages

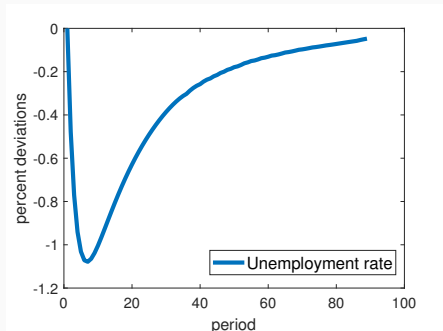


Figure 10: Unemployment Rate

- Similar wage response
- More responsive unemployment (still not enough)

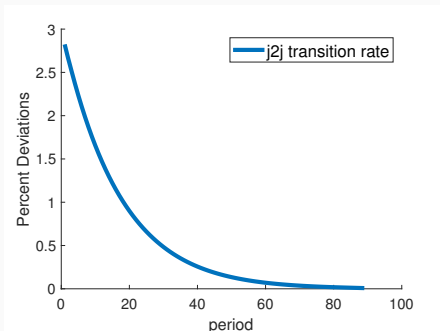


Figure 11: J2J transitions

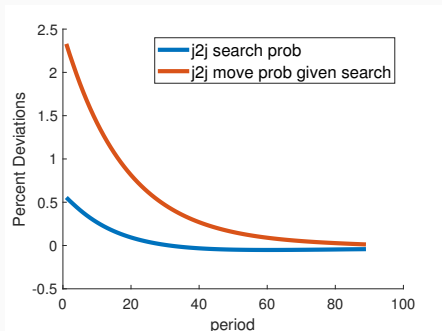


Figure 12: J2J search & JFP

- Response of J2J transition is mitigated
- Due to less responsive job finding probability for the employed workers

## HETERO- $\eta$ ECONOMY: MODEL STATISTICS

	$\eta^e = \eta^u = 0.5$			$\eta^e = 0.75, \eta^u = 0.35$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1.00
Avg Wage	0.690	0.51	1.00	0.688	0.53	1.00
New Wage	0.689	0.95	1.00	0.654	0.92	1.00
Unemp Rate	10.6%	0.35	-0.48	7.7%	0.78	-0.84
Quits+J2J moves	38.4%	8.94	0.99	34.9%	1.42	1.00
J2J moves	29.2%	10.66	0.99	26.9%	1.98	1.00

**Table 9:** Productivity Shock ( $\rho = 0.95$ )

- Allowing for different matching functions for UE and EE moves greatly reduce the gap of volatility between unemployment and j2j transitions
- But they both show insufficient volatility compared to output, in response to the productivity shock



## HETERO- $\eta$ ECONOMY: MODEL STATISTICS

	$\eta^e = \eta^u = 0.5$			$\eta^e = 0.75, \eta^u = 0.35$		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.15	0.13	0.688	0.45	0.47
New Wage	0.689	2.02	0.31	0.654	2.40	0.73
Unemp Rate	10.6%	4.55	-0.99	7.7%	9.37	-0.99
Quits+J2J moves	38.4%	42.41	0.40	34.9%	11.65	0.70
J2J moves	29.2%	49.40	0.42	26.9%	15.55	0.70

**Table 10:** Delta Shock ( $\rho = 0.95$ )

- Allowing for different matching functions for UE and EE moves has similar effect on reduce volatility gap between unemployment and j2j transitions
- Unemployment is much more volatile compared to output in response to the delta shock, because the delta shock only affects total output through employment

## HETERO- $\eta$ ECONOMY: MODEL STATISTICS

- Two ways to aggregate shocks

	shock corr = 0		shock corr = 0.95	
	Std	corr	Std	corr
output	1.00	1.00	1.00	1.00
avg wage	0.48	0.91	0.41	0.94
new wage	1.20	0.80	1.34	0.96
unemployment	3.70	-0.52	3.30	-0.91
quits + j2j movers	4.88	0.60	5.01	0.94
J2J movers	6.50	0.62	6.68	0.96

**Table 11:** Both Shocks ( $\eta^e = 0.75, \eta^u = 0.35, \rho = 0.95$ )

- By allowing for two types of shocks, and different matching functions for UE and EE moves, the model delivers a pretty good match to the data

- The non-market value  $b$  is well recognized to be a key driver of the unemployment volatility (Hagedorn and Manovski, 2008).
- We now raise  $b$  from 0.3 (Shimer, 2005) to 0.5-0.6 (near the upper limit of our model) in the hetero- $\eta$  economy.

# HIGH- $b$ (0.5) & HETERO- $\eta$ ECONOMY: IRF TO $z$ SHOCK

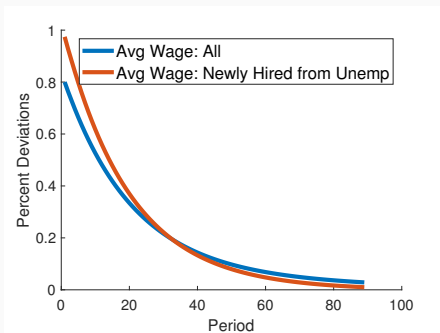


Figure 13: Wage

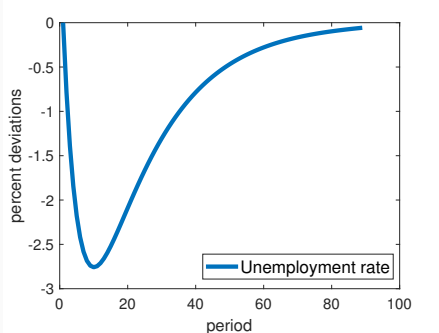


Figure 14: Unemployment Rate

- New wages are a bit less responsive
- Unemployment drops up to 2.7% for 1% increase in productivity

# HIGH- $b$ (0.5) & HETERO- $\eta$ ECONOMY: IRF TO $z$ SHOCK

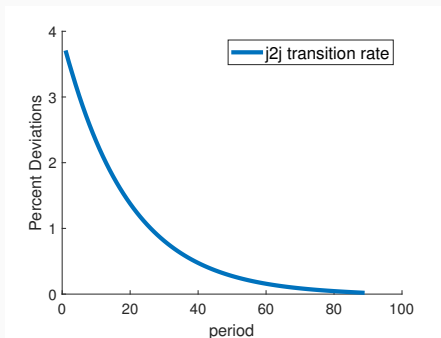


Figure 15: J2J transitions

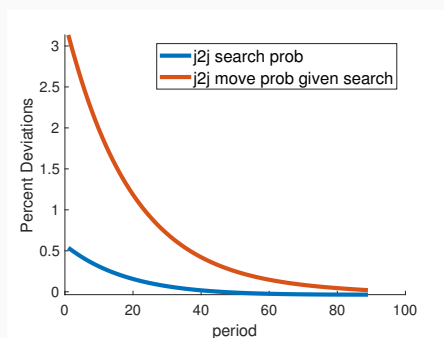


Figure 16: J2J search & JFP

- Response of J2J transition is further mitigated

## HIGH- $b$ (0.5) & HETERO- $\eta$ ECONOMY: MODEL STATISTICS

	Benchmark			New		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.665	0.66	0.98
New Wage	0.689	0.95	1.00	0.656	0.80	0.97
Unemp Rate	10.6%	0.35	-0.48	9.4%	1.24	-0.83
Unemp Rate (normalized)	10.6%	0.35	-0.48	10.6%	1.21	-0.82
Quits+J2J moves	38.4%	8.94	0.99	37.7%	2.32	0.98
J2J moves	29.2%	10.66	0.99	28.7%	3.05	0.98

**Table 12:** High- $b$  & Hetero- $\eta$ : **Productivity** Shock ( $\rho = 0.95$ )

- New:  $b = 0.5$ ,  $\eta^u = 0.35$ ,  $\eta^e = 0.75$ . Benchmark:  $b = 0.3$ ,  $\eta^u = \eta^e = 0.5$
- All together, these extensions lead to 3.5 times more unemployment volatility, and shrink OJS move volatility to less than 30% the original level.

# HIGH- $b$ (0.6) & HETERO- $\eta$ ECONOMY: IRF TO $z$ SHOCK

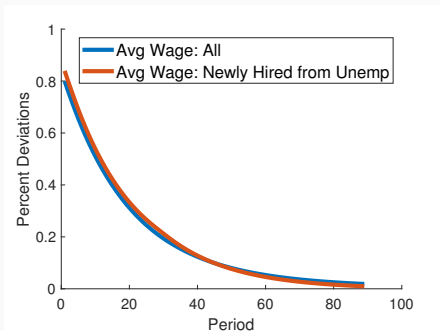


Figure 17: Wage

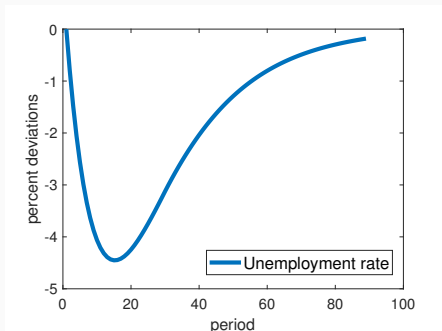


Figure 18: Unemployment Rate

- $b$  to the limit: new wage only slightly more responsive than the average
- Unemployment drops up to 4.5% for 1% increase in productivity

## HIGH- $b$ (0.6) & HETERO- $\eta$ ECONOMY: IRF TO $z$ SHOCK

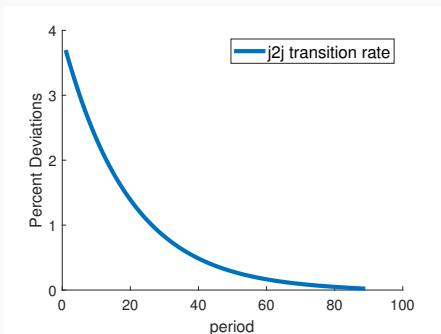


Figure 19: J2J transitions

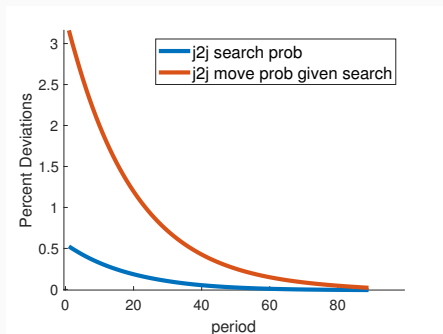


Figure 20: J2J search & JFP

- Response of J2J transition is the same magnitude as unemployment (like in the data)



## High- $b$ (0.6) & Hetero- $\eta$ Economy: Model Statistics

	Benchmark			New		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.665	0.44	0.98
New Wage	0.689	0.95	1.00	0.656	0.47	0.97
Unemp Rate	10.6%	0.35	-0.48	20.9%	1.25	-0.83
Unemp Rate (normalized)	10.6%	0.35	-0.48	10.6%	2.46	-0.84
Quits+J2J Moves	38.4%	8.94	0.99	37.7%	1.54	0.98
J2J Moves	29.2%	10.66	0.99	28.7%	2.06	0.98

Table 13: High- $b$  & Hetero- $\eta$  Economy: **Productivity** Shock ( $\rho = 0.95$ )

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# CONCLUSIONS I

- Develop tools to get a joint theory of wages, employment and wealth that marry the two main branches of modern macro:
  - ① Aiyagari models (output, consumption, investment, interest rates)
  - ② Labor search models with job creation, turnover, wage determination, flows between employment, unemployment and outside the labor force.
  - ③ Add tools from Empirical Micro to generate quits
- Useful for business cycle analysis: We are getting procyclical
  - Quits
  - Employment
  - Investment and Consumption
  - Wages
- On the Job Search seems to Magnify Fluctuation a lot

- Exciting set of continuation projects:
  - ① Incorporate the movements outside of the labor force.
  - ② Endogenous Search intensity on the part of firms
  - ③ Aiming Shocks to soften correlation between wages and wealth
  - ④ Efficiency Wages: Endogenous Productivity (firms use different technologies with different costs of idleness)
  - ⑤ Move towards more sophisticated household structures (more life cycle movements, multiperson households).

- The number of vacancies posted is chosen by firms
- Easy to implement
- Slightly Different steady state

## FREE ENTRY WITH VARIABLE RECRUITING INTENSITY

- Let  $v(\bar{c})$  be a technology to post vacancies where  $\bar{c}$  is the cost paid.
- Then the free entry condition requires that for all offered wages

$$0 = \max_{\bar{c}} \left\{ v(\bar{c}) \psi^f[\theta(w)] \frac{\Omega(w)}{1+r} + [1 - v(\bar{c}) \psi^f[\theta(w)]] \frac{\bar{k}(1 - \delta_k)}{1+r} - \bar{c} - \bar{k} \right\},$$

- With FOC given by

$$v_{\bar{c}}(\bar{c}) \left\{ \psi^f[\theta(w)] \left[ \frac{\Omega(w)}{1+r} - \frac{\bar{k}(1 - \delta_k)}{1+r} \right] \right\} = 1,$$

## HOW TO MAKE IT CONSISTENT WITH THE CURRENT STEADY STATE

- If  $v(\bar{c}) = \frac{v_1 \bar{c}^2}{2} + v_2 \bar{c}$ , we have

$$(v_1 \bar{c} + v_2) \left\{ \psi^f[\theta(w)] \left[ \frac{\Omega(w)}{1+r} - \frac{\bar{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

- By Choosing  $v$  so that for the numbers that have now

$$\frac{v_1 \bar{c}^2}{2} + v_2 \bar{c} \left[ \psi^f[\theta(w)] \frac{\Omega(w)}{1+r} + \left[ 1 - \frac{v_1 \bar{c}^2}{2} - v_2 \bar{c} \right] \psi^f[\theta(w)] \frac{\bar{k}(1-\delta_k)}{1+r} \right] = \bar{c} + \bar{k},$$

- Solving for  $\{v_1, v_2\}$  that satisfy both equations given our choice of  $\bar{c}$  we are done

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