Monetary Policy Via Interest on Reserves

Jose-Victor Rios-Rull, Joao Ritto, Tamon Takamura and Yaz Terajima
Penn, UCL, CAERP — Penn — Bank of Canada — Bank of Canada

In Costa Rica
March 2024

1 Introduction

- In a Growth Model without Money a Necessary Banking Sector (with zero measure banks)
- A Central Bank Controls the Interest Rate on Reserves
- Banks Respond by changing Loans and Deposits
- The Private Sector Follows
- Can Monetary Policy Matter?
- Bianchi and Bigio (2022) meets Ríos-Rull et al. (2023).

2 The Banks

What is a Bank?

- A costly to create firm with curvature in the objective function.
- It can borrow cheap and short *deposits* with adjustment costs.
- It can lend expensive and risky and long *loans* with adjustment costs.
- It can hold reserves at a Central Bank determined rate
- It is subject to
  - Shocks that destroy loans
  - Shocks that reshuffle deposits
  - It can go out of business
  - It has to satisfy
    - Reserve Requirements
    - It may have Capital Requirements
  - It has a somewhat unreliable access to an interbank credit market

1. Bank starts period with depositis d loans ℓ and liquid assets, a.
2. Bank chooses new deposits d^n, new loans ℓ^n, dividends c and liquid assets m (that will be eventually allocated to either reserves r or government bonds b^g), subject to a capital requirement
3. Bank is subject to idiosyncratic withdrawal shock to deposits δ^d (positive or negative with negative mean) and random loan destructtion δ^l.
4. Bankruptcy happens if shocks large enoug.
5. Bank chooses allocation of liquidity between bonds and reserves
6. Bank may borrow or lend in the overnight interbank market b^{on}
7. Bank may borrow from the discount window b^{dw}
8. Bank must satisfy reserve requirement (or $r > 0$ if there is no requirement)
9. Bank pays back overnight and discount window borrowing
10. Bank receives payment on loans principal and interest

Bank Problem, Stage 2 : Portfolio decision

$$V(a,\ell,d) = \max_{c^b\geq 0, \ell^n \geq 0, m, d^n} \left\{ u^b(c^b) + \beta \mathbb{E}_\delta \max \left\{ \Phi \left(m'(\delta^d, \delta^l), \ell'(\delta^l), d'(\delta^d) \right) \right\} \right\}$$

s.t.

$$(BC) \qquad c^b + \ell^n + \varphi^\ell \left(\frac{\ell^n}{\ell} \right) \ell^n + \varphi^d \left(\frac{d^n}{d} \right) d^n + m \leq a + d^n$$

$$(KR) \qquad \frac{\ell + a - c^b - \varphi^\ell \left(\frac{\ell^n}{\ell} \right) \ell^n}{\alpha^r (\ell + \ell^n) + \alpha^s \ m} \geq \theta$$

$$(ED) \qquad d' = (1+\delta^d) \ [d + d^n]$$

$$(ER) \qquad m' = m + \delta^d \ [d + d^n]$$

$$(EL) \qquad \ell' = (1-\delta^\ell) \ (1-\lambda) \ [\ell + \ell^n]$$

$$\varphi^j(0) = 0, \varphi_1^j > 0, \varphi_2^j > 0, \quad j = \ell, d.$$

$$\Phi\left(m', \ell', d'\right) = \max_{b^b, b^{dw} \geq \mathbf{0}, b^g \geq \mathbf{0}, R} \left\{V\left(a', \ell', d'\right)\right\}$$

s.t.

$$(TR) \qquad R = m - b^g + b^b + b^{dw}$$

$$(RR) \qquad R \geq \kappa d$$

$$(TA) \qquad a' = R(1 + i^R) + b^g(1 + i^g) + (1 - \delta^\ell)(\lambda + i^\ell)\ell -$$

$$- d(1 + i^d) - b^b(1 + i^b) - b^{dw}(1 + i^{dw})$$

- Let $\chi(m, \ell, d, \delta)$ be the function that determines the profit/loss in stage 2 of the problem as a function of shocks:

$$\chi(m, \ell, d, \delta) = \begin{cases} \chi^{+} s, & s \geq 0 \\ \chi^{-} s, & s < 0 \end{cases}$$

$$s = m - d[(1 + \delta^{d}) - \delta^{d}]$$

$$\chi^{+} = (i^{g} - i^{R}) \text{ and } = \Psi^{+}(i^{b} - i^{R})$$

$$\chi^{-} = \Psi^{-}(i^{b} - i^{R}) + (1 - \Psi^{-})(i^{dw} - i^{R})$$

V: THIS NEEDS WORK

$$V(a,\ell,d) = \max_{c^b \geq 0, \ell^n \geq 0, m, d} \left\{ u^b(c^b) + \beta\, \mathbb{E}_\delta \max\left\{ V\left(a', \ell', d'\right), 0 \right\} \right\}$$

s.t. (BC)

$$c^b + \ell^n + \varphi^\ell\left(\frac{\ell^n}{\ell}\right)\ell^n + \varphi^d\left(\frac{d^n}{d}\right)\ell^n + m \leq a + d^n$$

(KR)

$$\frac{\ell + a - c^b - \varphi^\ell\left(\frac{\ell^n}{\ell}\right)\ell^n}{\alpha^r(\ell + \ell^n) + \alpha^s\ m} \geq \theta$$

(TL)

$$\ell' = (1-\delta^\ell)(1-\lambda)\,\ell + \ell^n$$

(TA)

$$a' = m(1+i^R) - d(1+i^d) + \chi(m,d,\delta^d) + (1-\delta^\ell)(\lambda + i^\ell)\,\ell$$

- The value function $V(a, \ell, d)$ satisfies homogeneity of degree $1 - \gamma$, so that $V(\alpha a, \alpha \ell, \alpha d) = \alpha^{1-\gamma} V(a, \ell, d)$, if $\ell, a, d > 0$. This means that $V(a, \ell, d) = V\left(\frac{a}{\ell}, 1, \frac{d}{\ell}\right) \ell^{1-\gamma}$

- So we translate the problem into one where the states are liquid assets per unit loan and deposits per unit loans, $\omega_a = a/\ell$, $\omega_d = d/\ell$.

- Not useful for too big to fail (no notion of systemic). All banks have zero measure.

- We proceed by guess and verify, this is, that there exist a function Ω, such that

$$\Omega(\omega_a, \omega_d)\ \ell^{1-\gamma} = V(\ell \omega_a, \ell, \ell \omega_d),$$

Define: $\overline{c} = \frac{c}{\ell}$, $\overline{\ell}^n = \frac{\ell^n}{\ell}$, $\overline{m} = \frac{m}{\ell}$, $\overline{d} = \frac{d}{\ell}$.

$$\Omega(\omega_a, \omega_d) = \max_{\overline{c}\geq \mathbf{0}, \overline{\ell}^n \geq \mathbf{0}, \overline{m}, \overline{d}^n \geq \mathbf{0}} \left\{ u^b(\overline{c}) + \beta \mathbb{E} \max \left\{ \Omega(\omega_a', \omega_d') \left((1-\delta^\ell)(1-\lambda) + \overline{\ell}^n \right)^{\mathbf{1}-\gamma}, 0 \right\} \right\}$$

s.t. (BC)

$$\overline{c} + \overline{\ell}^n + \varphi^\ell \left(\overline{\ell}^n \right) \overline{\ell}^n + \overline{m} + \varphi^d \left(\frac{\overline{d}^n}{\omega_d} \right) \overline{d}^n \leq \omega_a + \overline{d}^n$$

(KR)

$$\frac{1 + \omega_a - \overline{c} - \varphi^\ell \left(\overline{\ell}^n \right) \overline{\ell}^n - \varphi^d \left(\frac{\overline{d}^n}{\omega_d} \right) \overline{d}^n}{\alpha^r (1 + \overline{\ell}^n) + \alpha^s \, \overline{m}} \geq \theta$$

(TA)

$$\omega_a' = \frac{\overline{m}(1+i^R) - (1+i^d)(\omega_d + \overline{d}^n) + \chi(\overline{m}, \omega_d + \overline{d}^n, \delta^d) + (1-\delta^\ell)(\lambda + i^\ell)}{(1-\delta^\ell)(1-\lambda) + \overline{\ell}^n}$$

(TD)

$$\omega_d' = \frac{(\omega_d + \overline{d}^n)(1+\delta^d)}{(1-\delta^\ell)(1-\lambda) + \overline{\ell}^n}$$

$$\Omega\left(\frac{a}{\ell},\frac{d}{\ell}\right)\ell^{1-\gamma}=\max_{c\geq 0,\ell^n\geq 0,m,d^n\geq 0}\left\{\frac{c^{1-\gamma}}{1-\gamma}+\beta\mathbb{E}\max\left\{\Omega\left(\frac{a'}{\ell'},\frac{d'}{\ell'}\right)\overbrace{\left((1-\delta^\ell)(1-\lambda)\,\ell+\ell^n\right)^{1-\gamma}}^{(\ell')^{1-\gamma}},0\right\}\right\}$$

$$\text{s.t.}\quad (BC)\qquad c+\ell^n+\varphi^\ell\left(\frac{\ell^n}{\ell}\right)\ell^n+\varphi^d\left(\frac{d^n}{d}\right)d^n+m\leq a+d^n$$

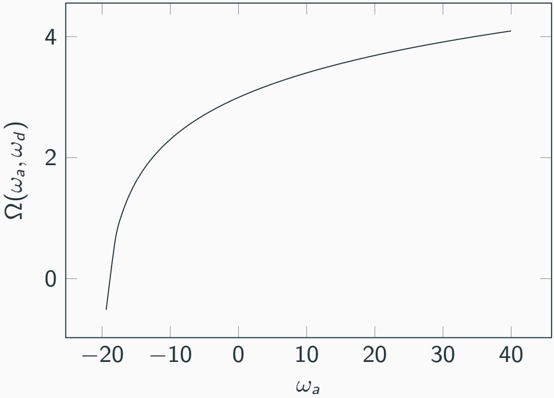
$$(KR)\qquad \frac{\ell+a-c--\varphi^\ell\left(\frac{\ell^n}{\ell}\right)\ell^n-\varphi^d\left(\frac{d^n}{d}\right)d^n}{\alpha^r(\ell+\ell^n)+\alpha^s\ m}\geq\theta$$

$$(TA)\qquad a'=\frac{m(1+i^R)-(1+i^d)(d+d^n)+\chi(m,d+d^n,\delta^d)+(1-\delta^\ell)(\lambda+i^\ell)\ell}{(1-\delta^\ell)(1-\lambda)\ell+\ell^n}$$

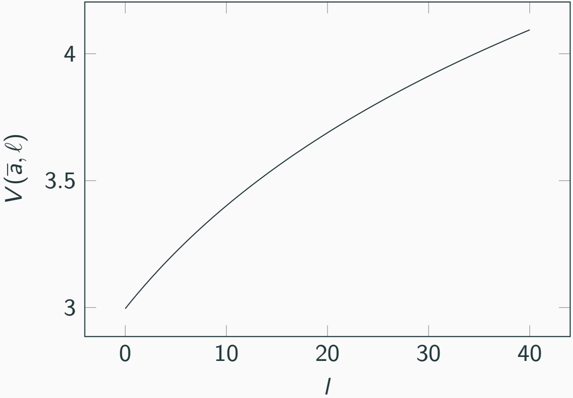
$$(TD)\qquad d'=\frac{(d+d^n)(1+\delta^d)}{(1-\delta^\ell)(1-\lambda)\,\ell+\ell^n}$$

One expects $\Omega(\omega_a, \omega_d)$ to have the following shape:

1. Sliced for given ω_d is:



2. Sliced for given ω_a, V: Should be decreasing:



Banks exit when $\Omega(\omega'_a, \omega'_d) < 0$ (having negative loans is unlikely to happen). The set at which the bank does not exit is given by:

$$\omega' = \frac{\overline{m}(1+i^R) - \overline{d}(1+i^d) + \chi(\overline{m}, \overline{d}, \delta_i^d) + (1-\delta_i^\ell)(\lambda + i^\ell)}{(1-\delta_i^\ell)(1-\lambda) \ + \overline{\ell}^n} \geq \underline{\omega}^C$$

This determines a set for δ, which we denominate $C(\overline{c}, \overline{\ell}^n, \overline{m}, \overline{d})$.

Can write continuation value as:

$$\int_{\delta \in C(\overline{c}, \overline{\ell}^n, \overline{m}, \overline{d})} \Omega(\omega') \left(\frac{[(1-\delta_i^\ell)(1-\lambda) \ + \overline{\ell}^n]}{1+\pi'} \right)^{1-\gamma} dF(\delta)$$

Why would a bank ever exit? There is the capital requirement, but if we assume banks do not need to exit because of it, it is only optimal to exit when probability of being able to pay one's liabilities goes to 0. If we want more exit, we could use a utility operational cost. For the linearity to be preserved, this should take the form $\phi\ell^{1-\gamma}$, so that now the bank gets a flow value:

$$u(c) - \phi\ell^{1-\gamma}$$

To be able to satisfy the (KR) constraint, a bank must have:

$$\frac{1+\omega}{\alpha^r+\alpha^s\omega} > \theta \implies \omega > -\frac{\theta\alpha^r - 1}{\theta\alpha^s - 1} \equiv \underline{\omega}^{KR}$$

Are there banks in this situation? Yes, if $\underline{\omega}^{KR} > \underline{\omega}^{C}$.

- A Stationary Equilibrium consists of value function Ω, and policy functions, prices q^g, q^d, i^ℓ, q^{on}, $\overline{q}^b$, tightness of interbank market ξ, inflation rate π, a mass of entrants m^E, and government policies $(q^r, q^{dw}, R, B, g_N, T)$ such that:

  1. The value and policy functions solve the bank's Bellman equation
  2. The Government's intertemporal budget constraint is satisfied
  3. The tightness in the interbank market is compatible with the aggregate surplus and deficit in the interbank market, as determined by policy functions and the idiosyncratic shock process
  4. Inflation equals the growth rate of nominal liabilities: $g_N = \pi$
  5. All markets clear
  6. The free entry condition is satisfied

1. Define policies (q^r, q^{dw}, R, g_N)
2. Compute inflation $\pi = g_N$
3. Guess q^d, i^ℓ, $\overline{q}^b$, tightness ξ and mass of entrants m^E
4. Compute q^{on} using the expression from bargaining
5. Compute q^g using the no-arbitrage condition
6. Solve bank problem
7. Find stationary distribution of banks in ω
8. Use loan market clearing condition to compute real loans (Firm block)
9. Compute aggregate demand for government liabilities, demand for deposits, and net interbank loans
10. Compute aggregate surplus and deficit of reserves
11. Compute household supply of deposits
12. Compute growth of real loans
13. Check market clearing in the deposit market, tightness condition, interbank market loan clearing, that real loans are stationary and free entry condition
14. Compute demand for discount window loans
15. Compute government bonds
16. Compute taxes

3 Other model blocks

- There is a Rep hhold
    - It owns a Mutual Fund that yields dividends
    - It gets utility from deposits
    - It holds bonds (risk free in St St, not necessarily so outside)
    - Some of its members work
- Many *Putty Clay* firms
    - Start up with bank loans. Become equity firms after Calvo shock.
    - All proceeds go to Mutual Funds
- A Banking Industry.
    - Individual Banks make Loans to firms with maturity λ
    - Borrow and issue deposits
    - Startup costs paid by Mutual Funds with difficulty (via func u^b)
- Mutual Funds
    - Manage Loan firms
    - Own Equity firms
    - Open and own banks with transfer difficulties

Prices, Aggregate Variables, and Other Objects

- Prices
  - Interest rate q for bonds: Safe
  - Interest rate r^ℓ for loans: Unsafe
  - Interest rate for deposits q^D Safe because insured by Gov.
  - Wage function $w(k, C)$ (I am using a guess and verify based on logs)
- Quantities
  - Employment, and Number of Firms/Plants N
  - Capital per Plant K
  - Output, Cons, Inv, $C + \delta NK = Y = NAK^\alpha -$ Intermediate Inputs
  - Loans $L = (1 - \lambda)NK$ V: (Double check, but similar formula)
  - Deposits D
  - Bonds B
  - Taxes, Banks Loses T
- Other Elements
  - A Banking Industry with a measure of banks x, new entrants m^E, and dividends C^b
  - Mutual funds that manage/own all firms

- The definition is exactly like the one in the other paper. But for our purposes we need to link it with the rest of the model.
- We proceed by specifying what are inputs to the banks
- Given safe interest rate, $1/q$, deposit rate $1/q^d$, loan rate r^ℓ and cost of entry κ^{Eb}, it yields
  - A measure of Banks over their states x, including entrants m^E, and fraction of loans in hands of failing banks d^B.
  - Total Quantity of Bonds B
  - Total Quantity of Deposits D
  - Total Dividends C^b
  - Total Loses T to be covered by government
  - Total resources needed by new entrants $m^E \kappa^{Eb}$

- Under Free Entry, One-Worker Putty-Clay Plants arise: $y = A\ k^{\alpha}$.
- Firms get destroyed with probability δ. From the point of view of banks $\delta \sim \gamma_{\delta}$, with mean δ_1.
- Financed with Bank loans of stochastic maturity λ. Upon arrival of Maturity, becomes Equity firm. Mutual Fund pays loan
- All cash flows of firms end up in Mutual Funds.
- Extensive margin: There are N^n new firms each period.
- Intensive margin: Each period firms invest k units.
- Total amount of new loans is $L^n = k\ N^n$.
- Employment or the number of plants is

$$N' = (1 - \delta_1)N + N^n.$$

- Output is

$$Y' = (1 - \delta_1)Y + N^n\ A\ k^{\alpha}.$$

- Firms must borrow 100% of their investment k from a bank.

- If the Bank does not fail (prob $1 - d^B$), then with probability $1 - \lambda$, the firm continues to be debt-financed and pays interest kr^ℓ; with probability λ, a loan terminates. With probability γ, the firm chooses refinancing by banks. Otherwise, the mutual fund pays $(1 + r^\ell)k$ at the beginning of next period, and the firm becomes an Equity firm.

- If the bank fails (prob d^B), we assume that the loan also terminates with prob γ and the Mutual pays the government $k(1 + r^\ell + \zeta^F)$. V: What happens with prob $(1 - \lambda)$?
- d^B is the endogenous fraction of loans held by defaulting banks:

$$d^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell)\in D_i} \ell \; dm_i(a,\ell)}{\sum_{i=1}^{N_\xi} \int \ell \; dm_i(a,\ell)}$$

- Given capital k, the maintenance cost δ_2, interest rate r^ℓ, wage $w(k)$, and the repayment cost ζ^F when banks default, the value of a loan firm is

$$\begin{aligned}\Pi^0(k, r^\ell) = Ak^\alpha - w(k) - (r^\ell + \delta_2)k + (1 - d^B)(1 - \lambda)q(1 - \delta_1)\Pi^0(k, r^\ell)\\ +q\,(1 - \delta_1)\left\{\lambda(1 - d^B) + d^B)\right\}(1 - \gamma)\Pi^0(k)\\ +q(1 - \delta_1)\left[\lambda(1 - d^B) + d^B\right]\gamma\left[-k + \Pi^1(k)\right] - q(1 - \delta_1)d^B\gamma\zeta^F k\end{aligned}$$

- The value of an equity firm is

$$\Pi^1(k) = Ak^\alpha - w(k) - \delta_2 k + q(1 - \delta_1)\Pi^1(k)$$

- Letting $R(k) = Ak^\alpha - w(k)$, $\Pi^0 < \Pi^1$ due to loan repayment costs:

$$\begin{aligned}\Pi^1(k) &= \frac{R(k) - \delta_2 k}{1 - q(1 - \delta_1)}\\ \Pi^0(k, r^\ell) &= \frac{R(k) - \delta_2 k}{1 - q(1 - \delta_1)} - \frac{r^\ell + q(1 - \delta_1)\gamma\left[\lambda(1 - d^B) + d^B + d^B\zeta^f\right]}{1 - q(1 - \delta_1)\left[1 - \gamma\left\{\lambda(1 - d^B) + d^B\right\}\right]}k\end{aligned}$$

- Given the expected value, a firm chooses the size of capital:

$$k^* = \arg\max_k \left\{ q\, \Pi^0(k, r^\ell) - \kappa^{Ef} \right\}$$

- With FOC

$$k^* = \left\{ \frac{(1-\mu)\alpha A}{\frac{r^\ell + q(1-\delta_1)\gamma\left[\lambda(1-d^B)+d^B+d^B\zeta^f\right][1-q(1-\delta_1)]}{1-q(1-\delta_1)\left[1-\gamma\left\{\lambda(1-d^B)+d^B\right\}\right]} + \delta_2} \right\}^{\frac{1}{1-\alpha}}$$

- Firms enter until profits are zero:

$$\kappa^{E,f} = q\Pi^0(k^*; r^\ell)$$

Outcome of Investment Decisions

- Given r^ℓ, q, d^B, L^n, δ_1 and wage function $w(k)$

- Pose parameters of firm problem: δ_2, A, α, μ, $\overline{b}$

- Yields k, w, N, new firms $\delta_1 N$, that satisfy

  1. Wage equation

  2. FOC of firms

  3. Zero Profit Condition

  4. Feasibility: $Y = A\ N\ k^\alpha = C + I +$ costs of starting firms and operating banks

  5. $I = (\delta_1 + \delta_2)kN$

- Households own Mutual Funds which in turn own firms and banks, but do not trade its shares, just passively receive its dividends.
- Mutual Funds create banks and receive its dividends. Even though, banks assess the dividends according to function $u^b()$. Its cash flow is

$$\pi^b = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D_i} c^{i,b}(a,\ell) dm^i(a,\ell) + (c^{E,b} - \kappa^{E,b}) m^E$$

- Mutual Funds manage Loan-firms and own Equity Firms:

$$\begin{aligned}
\pi^f &= Y - \mu Y - (1-\mu)\bar{b}N - r^\ell K^0 \\
&\quad -(1-d^B)\lambda K^0 - d^B(1+\zeta^F)K^0 - \kappa^{E,f} N^n \\
&= \int_{k,r^\ell} \left[R^0(k,r^\ell) - kr^\ell - (1-d^B)\lambda k - d^B(1+\zeta^F)k \right] dm^0(k,r^\ell) \\
&\quad + \int_k R^1(k) dm^1(k) - \kappa^{E,f} N^n
\end{aligned}$$

- By Aggregation we get Profits to be Distributed to Households. It needs

1. New Banks Creation
2. Profits and loses from Banks C^b
3. Cash Flow net of Interest from Loan firms (not zero because of fixed costs)
4. Loan Repayment
5. Profits from Equity Firms

- A bargaining process between the firm and the worker. V: (We may change this to get more wage rigidity and avoid the Shymer puzzle)

- The bargaining process is repeated every period and if unsuccesfull neither firm nor worker can partner with anybody else within a period. We assume that the financial obligations to the bank by the firm do not disappear. Let μ be the bargaining weight of the worker and $\overline{b}$ is workers' outside option. Then, we have

$$w^0(k) = w^1(k) = \mu\, A\, k^\alpha + (1-\mu)\overline{b}$$

- Total (per capita) Labor Income paid in the Economy are

$$W\ N = N \int \left[\mu\, A\, k^\alpha + (1-\mu)\overline{b}\right]\ di = \mu Y + (1-\mu)\overline{b} N$$

$$v(a) = \max_{c,b',d'} \quad u(c,d') + \beta v(a') \qquad \text{s.t.}$$

$$\begin{aligned} c + q^d d' + qb' &= a + W\,N + (1-N)\bar{b} + \pi^f + \pi^B - T \\ a' &= d' + b' \end{aligned}$$

where T is the taxes needed to pay for bank losses. FOCs:

$$\begin{aligned} u_c &= \frac{\beta}{q} u_c' \\ u_d &= q^d u_c - \beta\, u_c' \end{aligned}$$

The cost of deposit insurance is the amount of deposits that defaulting banks owe minus liquidated capital.

$$T = \sum_{i=1}^{N_\xi} \xi^{i,d} \int_{(a,\ell)\in D} dm^i(a,\ell) - K^0 d^B (1-\zeta^B)$$

where ζ^B is the fraction that the government is unable to recover during the liquidation process.

- Given safe interest rate, $1/q$, deposit rate $1/q^d$, Taxes T, wages W, Profits Π, and Bonds B, Employment N we obtain

1. Consumption C

2. Deposits D

Deposits

$$D' = \sum_{i=1}^{N_\xi} \xi_i^d \int_{(a,\ell)\notin D_i} dm^{bi}(a,\ell) + \xi^{dE}\, m^E$$

Bonds

$$qB' = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D_i} q^{ib}\left(\ell, \ell^{in}(a,\ell), b^{i'}(a,\ell)\right)\, b^{i'}(a,\ell) dm^i(a,\ell) + q^{Eb} b'^E m^E$$

New loans

$$k^* N^n + (1-\gamma)\left\{\lambda(1-d^B) + d^B\right\} K^0 = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D_i} \ell_i^n(a,\ell) dm_i(a,\ell) + \ell_E^n m_E$$

Goods

$$\begin{aligned} Y = {} & C + kN^n + \delta_2 kN + \\ & + \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D_i} \xi_i^n\left(\ell_i^n(a,\ell)\right) dm_i(a,\ell) + \xi_E^n\left(\ell_E^n\right) && \text{(Bank loan issuance costs)} \\ & + \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D_i} \xi_b\left(b_i'(a,\ell)\right) dm_i(a,\ell) + \xi_E^b\left(b_E'\right) && \text{(Bank bond issuance costs)} \\ & + \kappa_E^b m_E + \kappa_E^f N^n && \text{(Entry costs)} \\ & + d^B(\zeta^B + \zeta^F) K^0 && \text{(Bank default costs)} \end{aligned}$$

Households: $u(C, D, N) = log(C) + \eta^D log(D)$,

$$q = \beta \tag{1}$$

$$\frac{\eta^D C}{D} = q^d - \beta \tag{2}$$

Firms:

$$k^* = \left\{ \frac{(1-\mu)\alpha A}{\frac{\left[r^\ell + q(1-\delta_1)\gamma\left\{\lambda(1-d^B)+d^B+d^B\zeta^f\right\}\right][1-q(1-\delta_1)]}{1-q(1-\delta_1)\left[1-\gamma\left\{\lambda(1-d^B)+d^B\right\}\right]} + \delta_2} \right\}^{\frac{1}{1-\alpha}} \tag{3}$$

$$\kappa^{Ef} = q\,\Pi^0 \tag{4}$$

$$\Pi^0 = \frac{(1-\mu)\left(A(k^*)^\alpha - \overline{b}\right) - \delta_2 k^*}{1-q(1-\delta_1)} - \frac{r^\ell + q(1-\delta_1)\gamma\left[\lambda(1-d^B) + d^B + d^B\zeta^f\right]}{1-q(1-\delta_1)\left[1-\gamma\left\{\lambda(1-d^B)+d^B\right\}\right]} k^* \tag{5}$$

Wages:

$$w = \mu A(k^*)^\alpha + (1-\mu)\overline{b} \tag{6}$$

Banks:

$$\kappa^{E,b} = W^E(a^E, \ell^E) \tag{7}$$

$$d^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell)\in D^i} \ell \; dm^i(a,\ell)}{\sum_{i=1}^{N_\xi} \int \ell \; dm^i(a,\ell)} \tag{8}$$

Market clearing conditions:

$$D = \sum_{i=1}^{N_\xi} \xi^{i,d} \int_{(a,\ell)\notin D_i} dm^i(a,\ell) + \xi^{E,d}\, m^E \tag{9}$$

$$qB = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D^i} q^{i,b}\left(\ell, \ell^{i,n}(a,\ell), b'^i(a,\ell)\right)\, b'^i(a,\ell) dm^i(a,\ell) + q^{E,b} b'^E m^E \tag{10}$$

$$k^* N^n + (1-\gamma)\left\{\lambda(1-d^B) + d^B\right\} K^{\mathbf{0}} = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D^i} \ell^{i,n}(a,\ell) dm^i(a,\ell) + \ell^{E,n} m^E \tag{11}$$

$$\begin{aligned} Y = C + k^* N^n + \delta_2 k N &+ \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D^i} \xi^{i,n}\left(\ell^{i,n}(a,\ell)\right) dm^i(a,\ell) + \xi^{E,n}\left(\ell^{E,n}\right) m^E \\ &+ \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin D^i} \xi^{i,b}\left(b'^i(a,\ell)\right) dm^i(a,\ell) + \xi^{E,b}\left(b'^E\right) m^E \\ &+ \kappa^{E,b} m^E + \kappa^{E,f} N^n + d^B(\zeta^B + \zeta^F) K^{\mathbf{0}} \end{aligned} \tag{12}$$

Laws of motion:

$$Y = \frac{A(k^*)^\alpha N^n}{\delta_1} \tag{13}$$

$$N = \frac{N^n}{\delta_1} \tag{14}$$

$$K^0 = \frac{k^* N^n}{1 - (1-\delta_1)\left[1 - \gamma\left\{\lambda(1-d^B) + d^B\right\}\right]} \tag{15}$$

Aggregate endogenous variables:

$C, D, B, k^*, K^0, N^n, N, Y, d^B, m(a,\ell), m^E, q, q^d, r^\ell, w$

Parameters:

HHs: $\beta,\ \mu,\ \overline{b},\ \eta^D$

Firms: $\alpha,\ A,\ \kappa^{E,f},\ \zeta^F$

Banks: $u^b(),\ \beta^B,\ \lambda,\ \xi^{i,d},\ \xi^{i,n},\ \xi^{i,b},\ F(\delta'),\ \kappa^{E,b},\ \zeta^B$

- Set Parameters of Banking: $u^b(), \beta^B, \lambda, \xi^{i,d}, \xi^{i,n}, \xi^{i,b}, F(\delta')$ and prices r^ℓ, q^d, q.V: (may come back to this)

- Compute the banking industry equilibrium. Get loans L, deposits D bank dividends C^b, losses T, resources for new entrants $m^E \kappa^{Eb}$.

- Set HH preference parameters $\beta, \overline{b}$, η_D, and the bargaining power μ so that they are consistent with q, the observed consumption-to-deposit ratio and the labor share of 2/3.

- Set Technology A, α as well as δ_2 and ζ^F to solve the firms' problem. Given α and δ_2, adjust A to make sure that all markets clear.
  V: (I think that λ doesn't matter much so we should set this to get the equity/debt ratio of the nonfinancial sector and a normalize)

- Generate key moments of interest.

- We target labor share and the outside option for workers $\overline{b} = \phi_b w$:

$$LS = \mu + (1-\mu)\frac{\overline{b}N}{Y} = \mu + (1-\mu)\frac{\phi_b w}{A(k^*)^\alpha}$$
$$w = \mu A(k^*)^\alpha + (1-\mu)\overline{b} = \mu A(k^*)^\alpha + (1-\mu)\overline{\phi}_b w$$

- Solving the two conditions simultaneously,

$$\mu = \frac{(1-\phi_b)LS}{1-\phi_b LS}$$
$$w(k^*) = \frac{\mu}{1-(1-\mu)\phi_b}A(k^*)^\alpha$$
$$\overline{b} = \phi_b w(k^*)$$

- $LS = 2/3$ and $\phi_b = 0.9$ imply $\mu = 1/6$.

- $\beta = q$ by (1)
- $N^n = \delta_1 \overline{N}$ by (14), where $\overline{N} = 0.9$.
- The banking industry equilibrium gives L^n: back out k^* from (11).
- Set A so that the loan demand (3) is equal to the loan supply.
- $Y = Ak^* \overline{N}$ and $I = (\delta_1 + \delta_2) k^* N$.
- Compute K^0 from (15)
- C is determined as a residual in (12)

- For simplicity, we ignore various intermediate costs for now

- Consumption-deposit ratio:

$$\frac{C}{D} = \frac{C}{L^n}\frac{L^n}{D} = \frac{Y - I}{k^*\delta_1 \overline{N}\left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]}\right]}\frac{L^n}{D}$$

$$= \frac{A(k^*)^{\alpha-1} - \delta_1 - \delta_2}{\delta_1\left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]}\right]}\frac{L^n}{D}$$

$$= \left[\frac{1}{\frac{K}{Y}\delta_1\left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]}\right]} - \frac{\delta_1 + \delta_2}{\delta_1\left[1 + \frac{(1-\gamma)\{\lambda(1-d^B)+d^B\}}{1-(1-\delta_1)[1-\gamma\{\lambda(1-d^B)+d^B\}]}\right]}\right]\frac{L^n}{D}$$

- With $K/Y = 3$, $L^n/D = 0.9$, $\delta_1 = 0.02$, $\delta_2 = 0.08$, $\gamma = \lambda = 0.5$, $d^B = 0$, consumption-deposit ratio is about 5.4.

4 Equilibrium in Terms of Sequences

$$V_t^i(a,\ell) = \max\left\{0, W_t^i(a,\ell)\right\}$$

$$W_t^i(a,\ell) = \max_{\ell^n \geq 0, c \geq 0, b',} u^b(c^b) + \beta^b \sum_i \Gamma_{i,i'} \sum_{\delta'} \left\{\pi_t(\delta') V_{t+1}^{i'}[a'(\delta'), \ell'(\delta')]\right\} \text{ s.t.}$$

$$\begin{aligned}
&(TL) && \ell' = (1-\lambda)\,(1-\delta')\,\ell + (1-\bar{\delta})\ell^n \\
&(TA) && a' = (\lambda + r_t^\ell)(1-\delta')\ell + \lambda(1-\bar{\delta})\ell^n - \xi^{i,d} - b' \\
&(BC) && c^b + \ell^n + \xi^{i,n}(\ell^n) + \xi^{i,b}(b') \leq a + q_t^{i,b}(\ell,\ell^n,b')b' + q_t^d \xi^{i,d} \\
&(KR) && \frac{\ell^n + \ell - q_t^d \xi^{i,d} - q_t^{i,b}(\ell,\ell^n,b')b'}{\omega_t^r(n+\ell) + \omega_t^s\, 1_{b'<0} b' q_t^{i,b}(\ell,\ell^n,b')} \geq \theta_t
\end{aligned}$$

π_t is an exogenous aggregate shock.

θ_t is exogenous. A feedback rule to be considered in the next step.

- Entry condition:

$$W_t^E(a^E, \ell^E) = u^b(\kappa^{E,b}) \tag{16}$$

- A fraction of loans destroyed by bank default:

$$d_{t-1}^B = \frac{\sum_{i=1}^{N_\xi} \int_{(a,\ell)\in M_t^i} \ell \; dm_{t-1}^i(a,\ell)}{\sum_{i=1}^{N_\xi} \int \ell \; dm_{t-1}^i(a,\ell)} \tag{17}$$

- The value is

$$\Pi^1_t(k) = A_t k^\alpha - w_t(k) - \delta_2 k + q_t(1-\delta)\Pi^1_{t+1}(k) \tag{18}$$

- The wage is given by

$$w_t(k) = \mu A_t k^\alpha + (1-\mu)\overline{b} \tag{19}$$

- The value of bank-financed firm is

$$\begin{aligned}\Pi^0_t(k) = Ak^\alpha - w(k) - (r^\ell_t + \delta_2)k + (1-d^B_t)(1-\lambda)q_t(1-\delta_1)\Pi^0_{t+1}(k) \\ + q_t\,(1-\delta_1)\left\{\lambda(1-d^B_t) + d^B_t\right\}(1-\gamma)\Pi^0_{t+1}(k) \\ + q_t(1-\delta_1)\left[\lambda(1-d^B_t) + d^B_t\right]\gamma\left[-k + \Pi^1_{t+1}(k)\right] \\ - q_t(1-\delta_1)d^B_t\gamma\zeta^F k \end{aligned} \tag{20}$$

- Given q_t and Π^0_{t+1}, entrants choose k^*_t:

$$k^*_t = \arg\max_k \left\{q_t\Pi^0_{t+1}(k) - \kappa^{E,f}\right\} \tag{21}$$

- Entry occurs until firms break even *ex-ante*

$$q_t\Pi^0_{t+1}(k^*_t) = \kappa^{E,f} \tag{22}$$

- Aggregate output:

$$Y_t = A_t(k_t^*)^\alpha N_t^n + (1-\delta_1)Y_{t-1} \tag{23}$$

- Aggregate investment:

$$I_t = k_t^* N_t^n + \delta_2 K_{t-1} \tag{24}$$

- Aggregate capital:

$$K_t = k_t^* N_t^n + (1-\delta_1)K_{t-1} \tag{25}$$

- Aggregate capital held by bank-financed firms:

$$\begin{aligned} K_t^0 &= k_t^* N_t^n \\ &\quad + \left[(1-d_{t-1}^B)(1-\lambda) + (1-\gamma)\left\{\lambda(1-d_{t-1}^B) + d_{t-1}^B\right\}\right](1-\delta_1)K_{t-1}^0 \end{aligned} \tag{26}$$

- Consumption Euler equation:

$$u_{c,t} = \beta \frac{u_{c,t+1}}{q_t} \tag{27}$$

- Consumption-deposit marginal condition:

$$u_{d,t} = q_t^d u_{c,t} - \beta u_{c,t+1} \tag{28}$$

$$D_t = \sum_{i=1}^{N_\xi} \xi^{i,d} \int_{(a,\ell)\notin M^i_{t-1}} dm^i_{t-1}(a,\ell) + \xi^{E,d}\, m^E_t \tag{29}$$

$$\begin{aligned} k^*_t N^n_t + (1-\gamma)\left\{\lambda(1-d^B_{t-1}) + d^B_{t-1}\right\} K^0_{t-1} \\ = \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin M^i_t} \ell^{i,n}_t(a,\ell) dm^i_{t-1}(a,\ell) + \ell^{E,n}_t m^E_t \end{aligned} \tag{30}$$

$$\begin{aligned} Y_t = C_t + k^*_t N^n_t + \delta_2 K_{t-1} \\ + \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin M^i_t} \xi^{i,n}\left(\ell^{i,n}_t(a,\ell)\right) dm^i_{t-1}(a,\ell) + \xi^{E,n}\left(\ell^{E,n}_t\right) m^E_t \\ + \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin M^i_t} \xi^{i,b}\left(b'^{\,i}_t(a,\ell)\right) dm^i_{t-1}(a,\ell) + \xi^{E,b}\left(b'^{\,E}_t\right) m^E_t \end{aligned} \tag{31}$$

- Aggregate prices: r_t^ℓ, q_t, q_t^d
- Endogenous aggregate states: Y_{t-1}, K_{t-1}, K_{t-1}^0, $m_{t-1}^i(a,\ell)$, d_{t-1}^B
- Other endogenous aggregate variables: I_t, C_t, D_t, N_t^n, k_t^*, m_t^E
- Banking industry decisions:
$\{c_t^{i,b}(a,\ell), \ell_t^{i,n}(a,\ell), b_t^{\prime i}(a,\ell), M_t^i, c_t^E, \ell_t^{E,n}, b_t^{\prime E}, q_t^{i,b}(\ell,\ell^n,b')\}$
- Exogenous aggregate variables: θ_t, A_t, π_t
- B_t can be computed once we know the equilibrium path.

- The economy is in steady state in $t = 1$ and $t \geq T$
- Banks' problem do not depend on endogenous aggregate quantities, but firms' problem depend on d_t^B. [This isn't the case if a policy rule reacts to, say, aggregate output. But, we can still use what we do here to generate an initial guess.]
- Firm-entry conditions determine r_t^ℓ, given q_t: This process is inexpensive, as opposed to finding q_t^d given q_t and r_t^ℓ from the bank-entry condition
- Thus, our approach is to guess $\{q_t\}_{t=1}^T$, $\{q_t^d\}_{t=1}^T$, $\{d_t^B\}_{t=1}^T$, $\{m_t^E\}_{t=1}^T$ and $\{N_t^n\}_{t=1}^T$, and gradually adjust these objects to meet market-clearing conditions

Guess $\{q_t, q_t^d, d_t^B\}_{t=1}^{T-1}$ and start with V_T, Π_T^0 and Π_T^1. For $t = T-1, \ldots, 2$,

1. Given r_t^ℓ, q_t, d_t^B, Π_{t+1}^0 and Π_{t+1}^1, compute firms' value functions, (18) and (20), where r_t^ℓ is pinned down by the entry condition (22) given q_{t-1}:

$$q_{t-1}\Pi_t^0(k_{t-1}^*; r_t^\ell) = \kappa^{E,f}$$
$$k_{t-1}^* = \arg\max_k \Pi_t^0(k; r_t^\ell)$$

2. Solve the bank's problem given q_t^d, r_t^ℓ, q_t and V_{t+1}

3. Using (21), compute k_t^* given q_t and Π_{t+1}^0

4. Using (27) and (28), compute C_t and D_t given q_t and q_t^d

In each h-th iteration, do the following for $t = 2, \ldots, T-1$, given $Y_{\mathbf{1}}$, $K_{\mathbf{1}}$, $K^{\mathbf{0}}_{\mathbf{1}}$, the decision rules of HHs, banks and firms, and $\{m_t^{E,(h)}, N_t^{n,(h)}\}_{t=\mathbf{1}}^{T}$:

1. Aggregate banks' decisions using $m^i_{t-\mathbf{1}}(a,\ell)$

2. Aggregate output: $Y_t = A_t(k_t^*)^{\alpha} N_t^{n,(h)} + (1-\delta_{\mathbf{1}})Y_{t-\mathbf{1}}$
3. Using the goods MCC (31), compute $N_t^{n,*}$:

$$\begin{aligned} Y_t =& C_t + k_t^* N_t^{n,*} + \delta_{\mathbf{2}} K_{t-\mathbf{1}} + \text{loan issuance costs given } m_t^{E,(i)} \\ &+ \text{WSF issuance costs given } m_t^{E,(i)} \end{aligned}$$

4. Given $N_t^{n,*}$, compute $m_t^{E,*}$ using the loan MCC (30):

$$\begin{aligned} k_t^* N_t^{n,*} &+ (1-\gamma)\left\{\lambda(1-d^B_{t-\mathbf{1}}) + d^B_{t-\mathbf{1}}\right\} K^{\mathbf{0}}_{t-\mathbf{1}} \\ &= \sum_{i=\mathbf{1}}^{N_\xi} \int_{(a,\ell)\notin M_t^i} \ell_t^{i,n}(a,\ell) dm^i_{t-\mathbf{1}}(a,\ell) + \ell_t^{E,n} m_t^{E,*} \end{aligned}$$

5. Update the distribution of banks $m_t^i(a,\ell)$, based on banks' decisions and $m^i_{t-\mathbf{1}}(a,\ell)$: we can get $d_t^{B,*}$ in this process

- The deposit MCC (29) implies excess demand for deposits:

$$X_t^d = \sum_i \xi^{i,d} \int_{(a,\ell)\notin M_t^i} dm_{t-1}^i(a,\ell) + \xi^{E,d} m_t^{E,(h)} - D_t \tag{32}$$

- For $\lambda^d < 0$, the updating algorithm for q_t^d is:

$$q_t^{d,(h+1)} = (1 + \lambda^d X_t^d) q_t^{d,(h)} \tag{33}$$

- An intuition here is to make deposit more expensive when its demand exceeds supply

- From (16), the excess bank-entry condition is:

$$X_t^v = W_t^E(a^E, \ell^E) - \kappa^{E,b} \tag{34}$$

- For $\lambda^v < 0$, the updating algorithm for q_t is:

$$q_t^{(h+1)} = (1 + \lambda^v X_t^d) q_t^{(h)} \tag{35}$$

- An intuition here is to make an entry more costly when the net value of entry is positive

- Updating of d_t^B:

$$d_t^{B,(h+1)} = \gamma^q d_t^{B,*} + (1-\gamma^q) d_t^{B,(h)}$$

- Updating of the measure of bank and firm entry :

$$m_t^{E,(h+1)} = \gamma^m m_t^{E,*} + (1-\gamma^m) m_t^{E,(h)}$$

$$k_t^* N_t^{n,(h+1)} + (1-\gamma)\left\{\lambda(1-d_{t-1}^{B,(h+1)}) + d_{t-1}^{B,(h+1)}\right\} K_{t-1}^{0,(h+1)}$$

$$= \sum_{i=1}^{N_\xi} \int_{(a,\ell)\notin M_t^i} \ell_t^{i,n}(a,\ell) dm_{t-1}^i(a,\ell) + \ell_t^{E,n} m_t^{E,(h+1)}$$

5 Bianchi and Bigio model

1. Stage 1: Portfolio decision
    1.1 Bank starts period with equity e
    1.2 Portfolio decision taking interest rates as given subject to capital requirement
        - Liabilities: Deposits d
        - Assets: Loans ℓ, Government bonds b^g, CB Reserves r
2. Idiosyncratic shocks to deposits (withdrawals) occur, δ_i^d
3. Stage 2: Balancing reserves
    3.1 Bank needs to transfer reserves when withdrawal occurs
    3.2 Bank can exchange bonds for reserves, borrow or lend in the interbank market (b^{on}) and borrow from discount window b^{dw}
    3.3 Bank must satisfy reserve requirement (or $r > 0$ if there is no requirement)

$$W^0(e) = \max_{\ell \geq 0, c^b \geq 0, b^g, r, d} \left\{ u^b(c^b) + \sum_{(\delta_i^d)} \pi(\delta_i^d)\, W^1 \left(\ell, b^g, \widehat{r}(\delta_i^d), \widehat{d}(\delta_i^d) \right) \right\} \text{ s.t.}$$

$$(BC) \qquad c^b + q^\ell \ell + q^g b^g + q^r r \leq e + q^d d$$

$$(KR) \qquad \frac{e - c^b}{q^\ell \ell + q^g b^g + q^r r} \geq \theta$$

$$(ED) \qquad \widehat{d} = (1 + \delta_i^d)\, d$$

$$(ER) \qquad \widehat{r} = r + \delta_i^d d$$

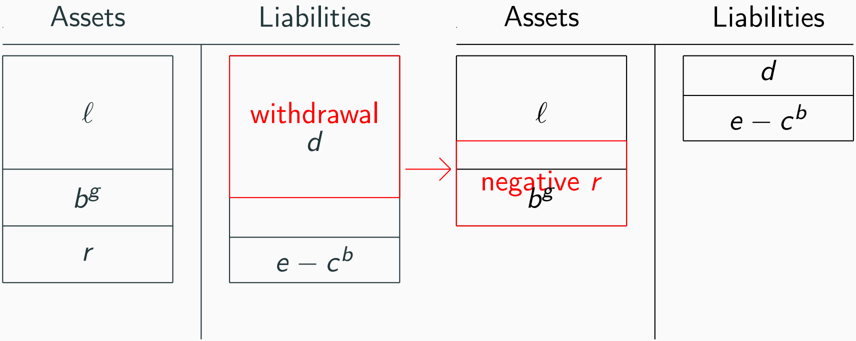
- Bank's initial choice of liquidity (gov bonds and reserves) is not pinned down separately. Banks only pick $\tilde{r} = q^r r + q^g b^g$

- This is because in the next stage (after withdrawal shocks), banks can again exchange government bonds for reserves in a frictionless market

- Because of a no-arbitrage condition, equilibrium prices at which reserves and government bonds are traded in stage 2 is the same as in stage 1.

Bank Problem, Stage 1 Balance Sheet

Assets	Liabilities
ℓ	d
b^g	
r	$e - c^b$


Assets
Liabilities
ℓ
b^g
r
withdrawal
d
$e - c^b$
Assets
Liabilities
ℓ
negative r
b^g
d
$e - c^b$

Bank Problem, Stage 2: Balancing reserves

$$W^1(\ell, b^g, r, d) = \max_{b^{on}, b^{dw}, \widehat{b}^g} \left\{ \beta \mathbb{E} W^0 \left(\frac{e'}{1+\pi'} \right) \right\} \text{ s.t.}$$

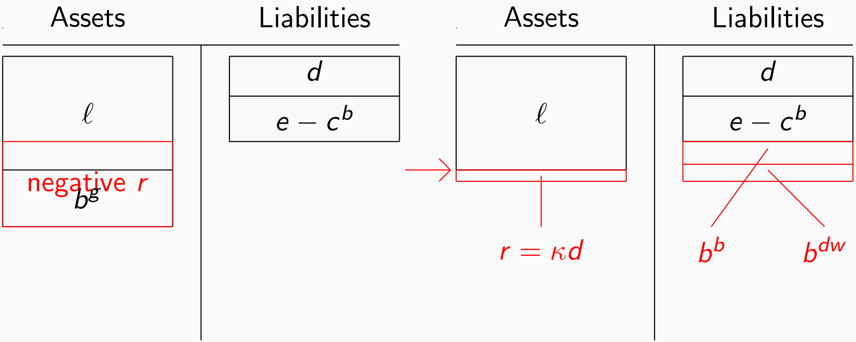
$$(TR) \qquad q^r \widehat{r} = q^r r - q^g (\widehat{b}^g - b^g) + q^{on} b^{on} + q^{dw} b^{dw}$$

$$(RR) \qquad \widehat{r} \geq \kappa d$$

$$(TE) \qquad e' = (\widehat{r} + \widehat{b}^g + \ell - d - b^{on} - b^{dw})(1-\tau)$$

- Banks with scarce reserves need to borrow and can get a share of their needs in the interbank market. The remaining it gets from the discount window (justified by search frictions in the interbank market)
- Share of funds one can borrow in interbank market is a function of tightness
- Banks with excess reserves want to lend in interbank market and manage to lend a share of it. The remaining funds are kept as reserves.
- Share of funds one can lend in interbank market is a function of tightness
- Tightness given by S^+/S^- where S^+ is the aggregate reserve surplus of banks trying to lend and S^- is the aggregate deficit of banks trying to borrow.
- $S^+ - S^- = R - \kappa D$. That is, aggregate surplus subtracted of the aggregate deficit should equal aggregate excess reserves relative to the aggregate reserve requirement

- Let $\tilde{r} = q^r r + q^g b^g$
- If $\tilde{r}/d < \kappa(1+\delta_i^d)q^r - \delta_i^d$
  1. The bank has insufficient reserves after withdrawals
  2. $\widehat{r} = \kappa d \implies q^{on} b^{on} + q^{dw} b^{dw} = d[\kappa(1+\delta_i^d)q^r - \delta_i^d] - \tilde{r}$
  3. $q^{on} b^{on} \propto (q^{on} b^{on} + q^{dw} b^{dw})$
- If $\tilde{r}/d > \kappa(1+\delta_i^d)q^r - \delta_i^d$
  1. The bank has excess reserves after withdrawals
  2. Indifferent between trying to lend excess funds in interbank market or buying bonds
  3. $q^{on} b^{on} \propto \tilde{r} - d[\kappa(1+\delta_i^d)q^r - \delta_i^d]$


Assets
Liabilities
ℓ
negative r
b^g
d
$e - c^b$
Assets
Liabilities
ℓ
d
$e - c^b$
$r = \kappa d$
b^b
b^{dw}

- Central Bank choice variables:

  1. Monetary policy:
     - Interest on reserves, q^r
     - Aggregate reserves vs bonds (OMOs). Pick R and B^g subject to $q^r R + q^g B^g = \overline{D}$
     - Interest on discount window, q^{dw}
     - Rate of growth of nominal liabilities (Bianchi and Bigio assume this is chosen by the monetary authority and the fiscal authority accommodates). In Stationary Equilibrium, this amounts to picking the inflation rate π.
  2. Financial stability policy:
     - Reserve requirement, κ
     - Capital requirement, θ

- $q^r > q^g > q^{on} > q^\ell > q^{dw}$. The interest rate on reserves and the interest rate on the discount window form a corridor. All other interest rates (except the interest rate on deposits which can be below the interest rate on reserves) should be in this corridor.

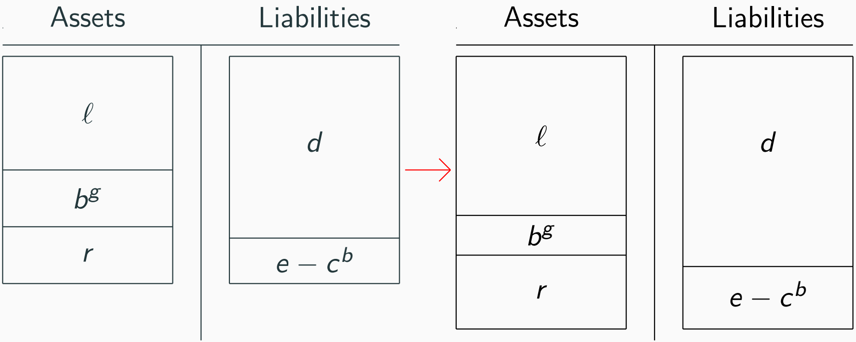
- Banks with excess reserves are indifferent between bonds and going into the interbank market means:

$$\frac{1}{q^g} = \Psi \frac{1}{q^{on}} + (1 - \Psi) \frac{1}{q^r}$$

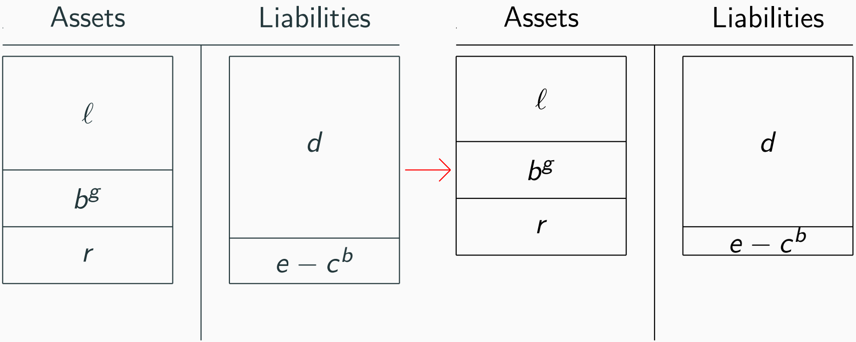
where Ψ is the portion of funds the bank is able to lend in the frictional interbank market. It is a function of the tightness of this market.

1. Open market operation of buying government debt with reserves (outstanding government bonds are lower but there are more reserves)
    - The abundance of reserves pushes the return on government bonds closer to the interest on reserves
    - In the interbank market there are less banks with scarce reserves and more with excess, changing tightness, and pushing the interest rate on these loans down, also closer to the interest on excess reserves
    - This also pushes the interest rate on loans down as banks try to lend more of the reserves (banks initially distribute less dividends to overcome capital constraint and equity is larger in SE)
2. Increasing the interest on reserves (with fixed growth of nominal liabilities this amounts to a change in the real rate)
    - Interest rate on bonds and interbank market goes up too
    - Interest rate on loans also goes up and loans decrease (banks will consume more and decrease SE equity)

OMO: Bank Problem, Stage 1 Balance sheet



Decrease q^r: Bank Problem, Stage 1 Balance sheet



- The effects of a MIT shock in the form of an open market operation or a temporary increase in the nominal interest rate on reserves has the same comparative statics as the ones described above for the Stationary Equilibrium.
- If the nominal interest rate on reserves is temporarily increased:

  1. The nominal interest rate on bonds and the interbank market increases
  2. Banks want to lend less and decrease equity by giving higher dividends
  3. The decrease in loans decreases deposits on aggregate
  4. The decrease in deposits makes reserves more abundant, decreasing the gap between the interbank market rate and the rate on reserves (and same for government bonds), as well as the quantities traded in the interbank market and borrowed in the discount window

- Policy choices: interest on reserves q^r and discount window q^{dw}, inflation rate π, and aggregate reserves R.
- A Stationary Equilibrium consists of value function, policy functions, prices q^g, q^d, q^ℓ, q^{on}, aggregate bonds B, taxes τ, tightness of interbank market ξ, such that:
    1. The value and policy functions solve the bank's Bellman equation
    2. The Government's intertemporal budget constraint is satisfied
    3. The tightness in the interbank market is compatible with the aggregate surplus and deficit in the interbank market, as determined by policy functions and the idiosyncratic shock process
    4. The government assets market, deposits market, and loans market clear

1. Guess q^d, q^ℓ and tightness ξ
2. Compute q^{on} using the expression from bargaining
3. Compute q^g using the no-arbitrage condition,
4. Compute bank problem's solution (portfolio weights)
5. Use loan market clearing condition to compute real equity
6. Compute aggregate demand for government liabilities, loan supply and demand for deposits
7. Compute aggregate surplus and deficit of reserves
8. Compute household supply of deposits
9. Compute growth of equity
10. Check market clearing in the deposit market, tightness condition and that real equity is stationary
11. Compute demand for discount window loans
12. Compute government bonds
13. Compute taxes

1. ...
2. Condition (8) in Bianchi and Bigio:

$$1/q^{on} = 1/q^{r} + (1 - \phi(\xi))(1/q^{dw} - 1/q^{r})$$

3.

$$\frac{1}{q^{g}} = \Psi^{+}(\xi)\frac{1}{q^{on}} + (1 - \Psi^{+}(\xi))\frac{1}{q^{r}}$$

4. See Bianchi and Bigio
5. (4) yields $\overline{d}$, $\bar{l}$, $\overline{a}$, $\overline{c}$. Using this, loan supply is given by:

$$E(1 - \overline{c})\bar{l}$$

Find the E at which loan market is in equilibrium

6.

$$B + R = E(1 - \overline{c})\overline{a}$$

$$L^S = E(1 - \overline{c})\overline{l}$$

$$D^D = E(1 - \overline{c})\overline{d}$$

7. Use the threshold at which a bank becomes deficitary vs surplus and compute these two objects using an integral
8. Exogenous expression
9. Given by:

$$\left[1 + \left(\frac{1}{q^l(1+\pi)} - 1\right)\overline{l} - \left(\frac{1}{q^d(1+\pi)} - 1\right)\overline{d}\right](1 - \overline{c})$$

10. Tightness condition:

11. $W = (1 - \Psi^{-}(\xi))S^{-}$

12. $B = (B + R) - R$

13. $\tau = (1 - \overline{c})[(i^{r} - \pi)\overline{r} + (i^{g} - \pi)\overline{b} - (i^{dw} - i^{r})\overline{w}]$

6 Adding loan risk

What to add and what it delivers:

1. Long-term loans
   - In response to a shock that affects the bank's balance sheet, the bank's reduction in loans may have to be slow over time. Internal propagation of shocks
2. Adjustment costs for loans
   - Also generates a slow response of loans for loan increases. Represents the cost of finding/evaluating prospective borrowers
3. Aggregate shock to loans
4. Idiosyncratic shocks to loans
5. Bank exit and entry

- We want to write the model such that the bank's problem has two individual state variables: (a/ℓ) and ℓ, but all policy functions are linear in ℓ, so that the problem can be solved with a single state variable, a/ℓ.

1. Bank starts period with loans ℓ and liquid assets, a.
2. Bank chooses portfolio, subject to the capital requirement
   - Liabilities: Deposits d
   - Assets: new loans ℓ^n, government bonds b^g, reserves r
3. Bank is subject to idiosyncratic withdrawal shock to deposits δ_i^d and idiosyncratic shock to loans δ_i^ℓ.
4. Exit decision
5. Bank can exchange bonds for reserves
6. Bank can borrow or lend in the overnight interbank market b^{on} and borrow from the discount window b^{dw}
7. Bank must satisfy reserve requirement (or $r > 0$ if there is no requirement)
8. Bank pays back overnight and discount window borrowing
9. Bank receives payment on loans principal and interest

- Let m denote liquid assets. In the first stage of the problem, the bank does not care about the distinction between reserves and bonds (because these can be exchanged after the shock is realized).

$$V^0(a,\ell) = \max_{c^b \geq 0, \ell^n \geq 0, m, d} \left\{ u^b(c^b) + \beta \mathbb{E}_\delta \max \left\{ W^0 \left(\widehat{\ell}(\delta_i^\ell), \widehat{m}(\delta_i^d, \delta_i^\ell), \widehat{d}(\delta_i^d) \right), 0 \right\} \right\} \text{ s.t.}$$

$$(BC) \qquad c^b + \ell^n + \varphi^\ell \left(\frac{\ell^n}{\ell} \right) \ell^n + m - d \leq a$$

$$(KR) \qquad \frac{\ell + a - c^b - \varphi^\ell \left(\frac{\ell^n}{\ell} \right) \ell^n}{\alpha^r (\ell + \ell^n) + \alpha^s \, m} \geq \theta$$

$$(ED) \qquad \widehat{d} = (1 + \delta_i^d) \, d$$

$$(ER) \qquad \widehat{m} = m + \delta_i^d d$$

$$(EL) \qquad \widehat{\ell} = (1 - \delta_i^\ell)(1 - \lambda)\ell + \ell^n$$

$$W^0(\ell, m, d) = \max_{b^b, b^{dw} \geq 0, b^g \geq 0, R} \left\{ V^0 \left(\frac{a'}{1+\pi'}, \frac{\ell'}{1+\pi'} \right) \right\} \text{ s.t.}$$

$$(TR) \qquad R = m - b^g + b^b + b^{dw}$$

$$(RR) \qquad R \geq \kappa d$$

$$(TL) \qquad \ell' = \ell$$

$$(TA) \; a' = R(1+i^R) + b^g(1+i^g) - d(1+i^d) - b^b(1+i^b) - b^{dw}(1+i^{dw}) +$$

$$+ (1-\delta_i^\ell)(\lambda + i^\ell)\ell$$

- Let $\chi(m, d, \delta_i^d)$ be the function that determines the profit/loss in stage 2 of the problem as a function of withdrawal shocks:

$$\chi(m, d, \delta_i^d) = \begin{cases} \chi^+ s, & s \geq 0 \\ \chi^- s, & s < 0 \end{cases}$$

$$s = m - d[(1 + \delta_i^d) - \delta_i^d]$$

$$\chi^+ = (i^g - i^R) \text{ and } = \Psi^+(i^b - i^R)$$

$$\chi^- = \Psi^-(i^b - i^R) + (1 - \Psi^-)(i^{dw} - i^R)$$

$$V^0(a,\ell) = \max_{c^b\geq 0, \ell^n \geq 0, m, d} \left\{ u^b(c^b) + \beta \mathbb{E} \max \left\{ V^0\left(\frac{a'}{1+\pi'}, \frac{\ell'}{1+\pi'}\right), 0 \right\} \right\} \text{ s.t.}$$

$$(BC) \qquad c^b + \ell^n + \varphi^\ell\left(\frac{\ell^n}{\ell}\right)\ell^n + m - d \leq a$$

$$(KR) \qquad \frac{\ell + a - c^b - \varphi^\ell\left(\frac{\ell^n}{\ell}\right)\ell^n}{\alpha^r(\ell+\ell^n) + \alpha^s\ m} \geq \theta$$

$$(TL) \qquad \ell' = (1-\delta_i^\ell)(1-\lambda)\,\ell + \ell^n$$

$$(TA) \qquad a' = m(1+i^R) - d(1+i^d) + \chi(m,d,\delta_i^d) + (1-\delta_i^\ell)(\lambda + i^\ell)\,\ell$$

Define $a/\ell = \omega$. Then $a = \omega\ell$ and the problem can be rewritten as:

$$V(\omega, \ell) = \max_{c^b \geq 0, \ell^n \geq 0, m, d} \left\{ u^b(c^b) + \beta \mathbb{E} \max \left\{ V\left(\omega', \frac{\ell'}{1+\pi'}\right), 0 \right\} \right\} \text{ s.t.}$$

$$(BC) \qquad c^b + \ell^n + \varphi^\ell \left(\frac{\ell^n}{\ell}\right) \ell^n + m - d \leq \omega\ell$$

$$(KR) \qquad \frac{\ell(1+\omega) - c^b - \varphi^\ell \left(\frac{\ell^n}{\ell}\right) \ell^n}{\alpha^r(\ell + \ell^n) + \alpha^s\ m} \geq \theta$$

$$(TL) \qquad \ell' = (1-\delta_i^\ell)(1-\lambda)\ \ell + \ell^n$$

$$(TA)\ \omega' = \frac{m(1+i^R) - d(1+i^d) + \chi(m, d, \delta_i^d) + (1-\delta_i^\ell)(\lambda + i^\ell)\ \ell}{(1-\delta_i^\ell)(1-\lambda)\ \ell + \ell^n}$$

Guess $V(\omega, \ell) = \Omega(\omega)\ell^{1-\gamma}$, $\ell > 0$. And define: $\overline{c} = c/\ell$, $\overline{\ell}^n = \ell^n/\ell$, and same for $\overline{m}$, $\overline{d}$.

$$V(\omega,\ell) = \max_{\overline{c}\geq 0, \overline{\ell}^n \geq 0, \overline{m}, \overline{d}} \left\{ \frac{\overline{c}^{1-\gamma}}{1-\gamma}\ell^{1-\gamma} + \beta\mathbb{E}\max\left\{ V\left(\omega', \frac{\ell'}{1+\pi'}\right), 0\right\}\right\} \text{ s.t.}$$

$$(BC) \qquad \left(\overline{c} + \overline{\ell}^n + \varphi^\ell\left(\overline{\ell}^n\right)\overline{\ell}^n + \overline{m} - \overline{d}\right)\ell \leq \omega\ell$$

$$(KR) \qquad \frac{\left(1 + \omega - \overline{c} - \varphi^\ell\left(\overline{\ell}^n\right)\overline{\ell}^n\right)\ell}{\left(\alpha^r(1+\overline{\ell}^m) + \alpha^s\,\overline{m}\right)\ell} \geq \theta$$

$$(TL) \qquad \ell' = \left((1-\delta_i^\ell)(1-\lambda) \; + \overline{\ell}^n\right)\ell$$

$$(TA) \; \omega' = \frac{\left(\overline{m}(1+i^R) - \overline{d}(1+i^d) + \chi(\overline{m}, \overline{d}, \delta_i^d) + (1-\delta_i^\ell)(\lambda + i^\ell)\right)\ell}{\left((1-\delta_i^\ell)(1-\lambda) \; + \overline{\ell}^n\right)\ell}$$

1. Plugging ℓ' into the Bellman equation:

$$\Omega(\omega)\ell^{1-\gamma} = \max_{c^b \geq 0, \ell^n \geq 0, m, b^b, d} \left\{ \frac{\overline{c}^{1-\gamma}}{1-\gamma}\ell^{1-\gamma} + \beta \mathbb{E} \max \left\{ \Omega(\omega') \left(\frac{[(1-\delta_i^\ell)(1-\lambda) \ + \overline{\ell}^n]\ell}{1+\pi'} \right)^{1-\gamma}, 0 \right\} \right\}$$

2. One can factor out $\ell^{1-\gamma}$ and get:

$$\Omega(\omega)\ell^{1-\gamma} = \max_{\ell^n \geq 0, c^b \geq 0, m, b^b, d} \left\{ \frac{\overline{c}^{1-\gamma}\ell^{1-\gamma}}{1-\gamma} + \beta\ell^{1-\gamma} \mathbb{E} \max \left\{ \Omega(\omega') \left(\frac{[(1-\delta_i^\ell)(1-\lambda) \ + \overline{\ell}^n]}{1+\pi'} \right)^{1-\gamma}, 0 \right\} \right\}$$

1. Cut $\ell^{1-\gamma}$ from both sides:

$$\Omega(\omega) = \max_{c^b \geq 0, \ell^n \geq 0, m, d} \left\{ \frac{\overline{c}^{1-\gamma}}{1-\gamma} + \beta \right.$$
$$\left. \mathbb{E} \max \left\{ \Omega(\omega') \left(\frac{[(1-\lambda)(1-\delta^\ell + \delta_i^\ell) \ + \overline{\ell}^n]}{1+\pi'} \right)^{1-\gamma}, 0 \right\} \right\}$$

$$\Omega(\omega) = \max_{\overline{c}\geq 0, \overline{\ell}^n \geq 0, \overline{m}, \overline{d}} \left\{ \frac{\overline{c}^{1-\gamma}}{1-\gamma} + \beta \mathbb{E} \max \left\{ \Omega(\omega') \left(\frac{[(1-\delta_i^\ell)(1-\lambda) \ + \overline{\ell}^n]}{1+\pi'} \right)^{1-\gamma}, 0 \right\} \right\} \text{ s.t.}$$

$$(BC) \qquad \overline{c} + \overline{\ell}^n + \varphi^\ell \left(\overline{\ell}^n \right) \overline{\ell}^n + \overline{m} - \overline{d} \leq \omega$$

$$(KR) \qquad \frac{1 + \omega - \overline{c} - \varphi^\ell \left(\overline{\ell}^n \right) \overline{\ell}^n}{\alpha^r (1 + \overline{\ell}^m) + \alpha^s \ \overline{m}} \geq \theta$$

$$(TA) \ \omega' = \frac{\overline{m}(1+i^R) - \overline{d}(1+i^d) + \chi(\overline{m}, \overline{d}, \delta_i^d) + (1-\delta_i^\ell)(\lambda + i^\ell) \ ]}{(1-\delta_i^\ell)(1-\lambda) \ + \overline{\ell}^n}$$

The way we defined Ω and did our normalizations causes problems for $\ell = 0$. But one find this as:

$$V^0(a,0) = \max_{c^b \geq 0, \ell^n \geq 0, m, d} \left\{ u^b(c^b) + \beta \mathbb{E} \max \left\{ \Omega(\omega') \left(\frac{\ell'}{1+\pi'} \right)^{1-\gamma}, 0 \right\} \right\} \text{ s.t.}$$

$$(BC) \qquad c^b + \ell^n + \varphi^\ell \left(\frac{\ell^n}{\ell} \right) \ell^n + m - d \leq a$$

$$(KR) \qquad \frac{0 + a - c^b - \varphi^\ell \left(\frac{\ell^n}{\ell} \right) \ell^n}{\alpha^r (0 + \ell^n) + \alpha^s \ m} \geq \theta$$

$$(TL) \qquad \ell' = (1-\lambda)(1-\delta_i^\ell)\, 0 + \ell^n$$

$$(TA) \ \omega' = \frac{m\,(1+i^R) - d(1+i^d) + \chi(m,d,\delta_i^d)}{(1-\delta_i^\ell)(1-\lambda)0 \ + \ell^n}$$

Normalize with respect to a, given that $a > 0$...

Guess: $V^0(a,0) = V^0(1,0)a^{1-\gamma}$, $a > 0$. Define $\tilde{c} = c/a$ and $\tilde{m} = m/(a(1-\tilde{c}))$ and so on...

$$V^0(1,0) = \max_{\tilde{c}\geq 0}\left\{\frac{\tilde{c}^{1-\gamma}}{1-\gamma} + \beta(1-\tilde{c})^{1-\gamma}\right.$$

$$\left.\max_{\tilde{\ell}^n\geq 0,\tilde{m},\tilde{b}^b,\tilde{d}} \mathbb{E}\max\left\{\Omega(\omega')\left(\frac{\tilde{\ell}^n}{1+\pi'}\right)^{1-\gamma},0\right\}\right\} \text{ s.t.}$$

$$(BC) \qquad \tilde{\ell}^n + \varphi^\ell\left(\frac{\ell^n}{\ell}\right)\tilde{\ell}^n + \tilde{m} - \tilde{d} \leq 1$$

$$(KR) \qquad \frac{0 + 1 - \varphi^\ell\left(\frac{\ell^n}{\ell}\right)\tilde{\ell}^n}{\alpha^r(0+\tilde{\ell}^n) + \alpha^s\ \tilde{m}} \geq \theta$$

$$(TA) \qquad \omega' = \frac{\tilde{m}(1+i^R) - \tilde{d}(1+i^d) + \chi(\tilde{m},\tilde{d},\delta_i^d)}{\tilde{\ell}^n}$$

The value function $V^0(a, \ell)$ satisfies homogeneity of degree $1 - \gamma$, so that $V^0(\alpha a, \alpha \ell) = \alpha^{1-\gamma} V^0(a, \ell)$, in the following domain: $\ell > 0 \vee a > 0$. This means the following are true in that domain:

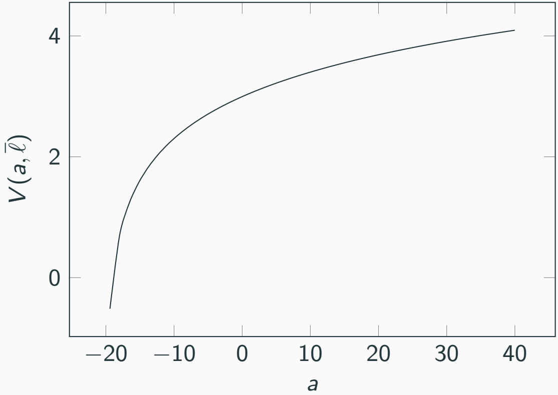
$$V^0(a, \ell) = V^0(a/\ell, 1)\ell^{1-\gamma}$$

$$V^0(a, \ell) = V^0(1, \ell/a)a^{1-\gamma}$$

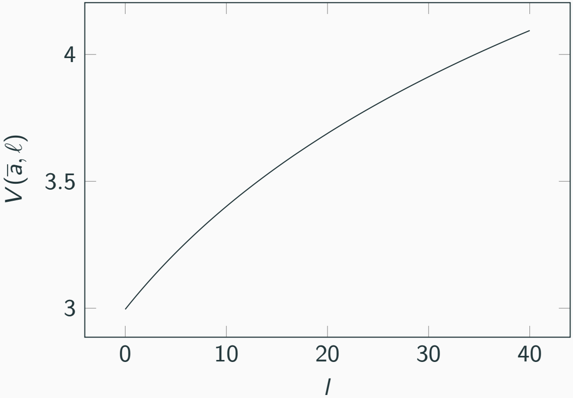
Importantly, homogeneity does not hold when $\ell = 0$ and $a < 0$. In this case, doubling a should decrease the value of the bank.

One expects $V(a, \ell)$ to have the following shape:

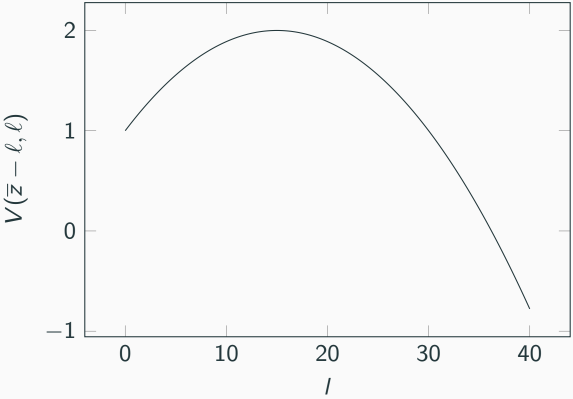
1. Sliced for given $\bar{\ell}$, $V(a, \bar{\ell})$ is:



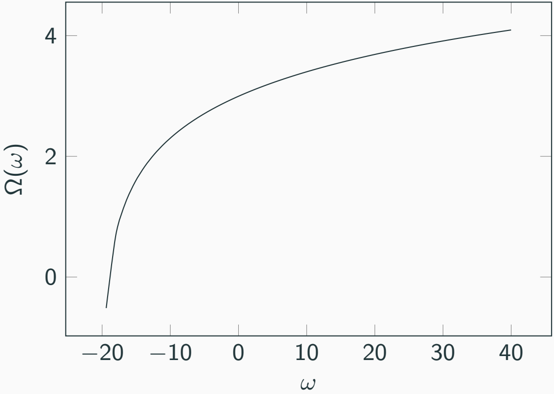
2. Sliced for given $\overline{a}$, $V(\overline{a}, \ell)$ is:



3. Consider the following slicing: $a + \ell = \overline{z} \implies a = \overline{z} - \ell$. Then, as we increase ℓ we are reducing a. Hence, we are comparing whether it is better to have an additional dollar as loans or as cash. Sliced for given $\overline{z}$, $V(\overline{z} - \ell, \ell)$ is:



We guessed: $V(a, \ell) = \Omega(a/\ell)\ell^{1-\gamma}$. Notice that for $\ell = 1$, $V(a, 1) = \Omega(a/1)$. This is the meaning of our Ω, it is the sliced value function of a bank with $\ell = 1$. Hence:



One dollar that enters as a bank, has individual state variables $(a = 1, \ell = 0)$. The cost is κ^E.

The free entry condition is:

$$V^0(1,0) = \kappa^E$$

Bianchi, J. and S. Bigio (2022): "Banks, Liquidity Management, and Monetary Policy," ***Econometrica*****, 90, 391–454.**

Ríos-Rull, J.-V., T. Takamura, and Y. Terajima (2023): "Banking Dynamics, Market Discipline and Capital Regulations," Unpublished Manuscript, Bank of Canada.