A Quantitative Theory of the Credit Score*

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This draft: February 2023

Abstract

What is the role of credit scores in credit markets? We argue that it is, in part, the market's assessment of a person's unobservable type, which here we take to be patience. We postulate a model of persistent hidden types where observable actions shape the public assessment of a person's type via Bayesian updating. We show how dynamic reputation can incentivize repayment. Importantly, we show how an economy with credit scores implements the same equilibrium allocation. We estimate the model using both credit market data and the evolution of individuals' credit scores. We conduct counterfactuals to assess how more or less information used in scoring individuals affects outcomes and welfare. If tracking of individual credit actions is outlawed, poor young adults of low type benefit from subsidization by high types despite facing higher interest rates arising from lower dynamic incentives to repay.

Keywords: Credit Scores, Unsecured Consumer Credit, Bankruptcy, Persistent Hidden Information.

JEL Classification Numbers: D82, E21, G51.

*This project has been a long time in the making. An earlier version circulated under the title "Credit Scoring and the Competitive Pricing of Default Risk." We thank our editor Chad Jones and three referees for very helpful comments on an earlier version of this paper and Isaiah Andrews as well as Tsung-Hsien Li and Jan Sun for this version. We thank all the economists who have had an important input into the final product: Murat Tasci, Pablo D'Erasmo, Daphne Chen, Jake Zhao, and Kuan Liu. We thank Hongchao Zhang for kindly sharing with us his DFBOLS Fortran code. We thank Cole Drier and Michael Slonkosky for research assistance. We also thank the many seminar and conference participants who commented on earlier versions of the paper. Finally, Corbae and Ríos-Rull wish to thank the National Science Foundation for support under grants SES-0751380 and SES-0351451. The views expressed in this paper are those of the authors and do not necessarily reflect views of the Federal Reserve Bank of Philadelphia or of the Federal Reserve System.

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1 Introduction

Credit scores are a fundamental ingredient of a borrower's access to credit. In the United States, credit bureaus and credit rating agencies serve this function for individual borrowers. Similar agencies exist in many other countries. Credit scores affect borrowing terms and change with credit use and repayments. Despite their widespread use in actual credit markets, credit scores are conspicuously absent from standard quantitative models of consumer default, which are typically more concerned with allocations than the contractual arrangements that generate them.

In this paper we provide a theory of the joint behavior of unsecured credit and credit scores which accounts for both allocations and arrangements. Reputations are formed in the presence of hidden information about a persistent, credit-relevant individual characteristic, which we take to be patience. The incentive to maintain a good reputation plays a central role, shaping borrowing and saving behavior over individuals' lifetimes.

Our theory is founded on the premise that an individual's true propensity to repay — i.e., the individual's true *type* — is hidden from her creditors, and it is the presence of this *persistent hidden information* that makes an individual's history of actions relevant for lenders. Our theory is dynamic: at any point in time, lenders use a person's observable history of actions to perform a Bayesian update of her type; individuals understand this and choose actions mindful of the consequence any action has on the future beliefs of lenders. A loss of reputation, rather than stigma or exogenous exclusion from future borrowing is the only dynamic punishment from default. Specifically, an individual's credit score falls upon default and she subsequently faces worse borrowing terms. Our theory is competitive: information available to any lender is available to all lenders and there is free entry into the business of lending. Finally, our theory respects a key feature of the institutional arrangement under which unsecured consumer credit is extended in the United States: at some monetary cost, individuals can choose to have their debts discharged via Chapter 7 bankruptcy.

We make several contributions. First, we extend the theory of unsecured credit to accommodate persistent hidden information about individual types. Our model environment is rich enough to cover four of the five characteristics lenders use to assess creditworthiness: character (reflected in credit history), capacity (reflected in debt-to-income ratio), capital (wealth), and conditions (amount of the loan).[1] Competition drives lending contracts to be indexed by all observable borrower and loan characteristics relevant for predicting the probability of default on a loan. When there is hidden information, a new individual characteristic becomes relevant: a borrower's type probability vector — in the terminology

[1] The fifth, collateral, is not relevant for unsecured credit. See https://www.investopedia.com/terms/f/five-c-credit.asp.

of this paper, the borrower's *type score* — indicating the probability that a person is of each of the different types existing in the economy. The Bayesian update of an individual's type score conditions on all relevant observables: the individual's current type score, her current net wealth, all the relevant information to forecast future earnings, and, of course, her current action (save, borrow or default). One way to interpret the large number of conditioning variables is that the lender is using "big data." Our framework easily encompasses "small data" cases in which lenders observe only some strict subset of actions (an instance of a"small data" world is explored in Section 6.2).

Second, after proving an equilibrium with type scores exists, we show that a market arrangement which uses credit scores replicates the same equilibrium allocation without any reference to type scores. Specifically, we use the type score to define a *credit score* – an object that yields a ranking of individuals with regard to their probability of default on a particular contract. Such an ordinal ranking is widely used by credit bureaus. We provide an easily verifiable sufficient condition such that the equilibrium under the arrangement that uses credit scores to index contracts has the same allocation as the equilibrium of our baseline economy with type scores. Just as agents take prices as given in standard competitive equilibrium models, in our equilibrium with credit scores individuals and lenders take credit-score-dependent prices and the distribution of future credit scores conditional on their current state and actions as given; they do not need to know what is behind such functions, just that they exist. In doing so, we provide a theory of the credit score itself and of how it evolves over time in response to fundamentals. In this context, we take to heart that the actual market arrangement is a form of data and our equivalence result allows for the use of such data for empirical purposes.

Third, we take our model to the data, estimating preference parameters (specifically a stochastic process for unobservable discount factors) from the joint behavior of credit scores over an individual's lifetime and aggregate credit market moments.[2] It is here that our decision to model age variation in the evolution of earnings and hidden characteristics pays off. For these estimates, we verify that the sufficient condition which guarantees equivalence between the type score economy and credit score economy holds. We find what we believe are important properties of the U.S. population with regard to (hidden) patience as revealed by the properties of the credit market: (i) the difference in discount factors between patient and impatient people is 13% annually; (ii) slightly less than one third of people are born patient, but the share of patient people rises with age; and (iii) patience is persistent but not permanent (the transitions between types occur with an average duration of between 4 and 5 years). Random changes in unobserved type (i.e., in patience) along with transitory variation in unobserved

[2] There is an extensive empirical literature finding evidence of adverse selection in credit markets which includes Ausubel (1999), Agarwal et al. (2010), Einav et al. (2013), and Hertzberg et al. (2018). Related empirical papers which study credit scoring and default include Albanesi et al. (2022) and Albanesi and Vamossy (2019).

shocks to earnings and extreme value shocks to utility, prevent fast learning about an individual's type. As we quantify later in section 5.3 on selection and reputation effects, the 13% difference in unobservable discount factors makes it costly for low discount factor (i.e. high risk) types to mimic the asset market behavior of high discount factor (i.e. low risk) types. Thus, our estimates suggest there is sufficient scope for signalling (and separation). The force for separation induces our estimates of the variance of the transitory shocks to preferences, especially for the default decision, to be relatively high. These two countervailing forces lead us into a sweet spot of hidden information consistent with the mean and standard deviation of rankings of credit scores across the age profiles in the data.

Fourth, we use our estimates to explore the role of hidden information in the U.S. unsecured credit market. We start by considering a policy counterfactual in which lenders are prohibited from keeping track of the history of an individual's asset market actions but can condition on the observable length of individuals' credit history (effectively their age). In this case, impatient types are pooled with patient types without having to bear the costs of imitating them in order to obtain better borrowing terms. Since young adults wish to borrow against their higher expected future income, and most start their adult life impatient, the policy has the possibility of improving the welfare of those young adults. However, the policy removes the incentives to maintain a good reputation which leads to individuals facing higher interest rate offerings. We find the negative incentive effects roughly offset the potential pooling benefits except for young, poor impatient adults who are made substantially better off.

Our second counterfactual considers an economy in which one's type is perfectly observable. The findings are intuitive. Since the impatient are known in this economy, they face a more adverse situation; their interest rates are higher and they borrow less. The opposite is true for the patient. As people age, this knowledge becomes less relevant because people accumulate precautionary balances and rarely borrow. The benefits to the patient outweigh the costs to the impatient resulting in a relatively large welfare improvement associated with full information. Furthermore, individual-level allocations in the full information economy are quite different from individual-level allocations in the base economy, showing that our baseline economy is still far from being a full information economy.

Fifth, we make several methodological contributions. We combine both screening and dynamic signaling where these screening and signaling opportunities are constrained by noise which we introduce via extreme value shocks.[3] The shocks ensure that beliefs held by lenders following any feasible action

[3] The microeconomic literature classifies hidden knowledge/adverse selection models as "screening" or "signaling" models (Riley (2001)). As in screening models, in our paper lenders offer loans distinguished by loan characteristics (size of the loan) and observable personal characteristics (income, previous history) that give ample scope for separation (if such separation is desirable from an individual point of view and can be sustained in equilibrium). And, as in signaling models, there are actions that an individual can take (e.g. saving) that have no effect on the payoff to any lender but which convey valuable information to them. In the use of history to condition prices, our model shares a connection to the microeconomic

are determined in equilibrium (reminiscent of Selten (1975) and Myerson (1978)).[4] The shocks also cloud inference about unobservable type; different types may choose the same action analogous to a semi-separating or partial pooling equilibrium. Finally, the shocks eliminate multiplicity of equilibria that can arise in signalling games from variation in off-the-equilibrium-path beliefs and provide tractability.

Sixth, we extend quantitative theory models of default with full information, like that in Livshits et al. (2007) and Chatterjee et al. (2007), to include hidden information which requires us to index the pricing of credit to the market assessment of individual types.[5,6] While a credit score (the probability of repayment on a given size loan) can be constructed in a full information model like those above, the history of past asset market actions plays no role in that construction. Here, in the presence of hidden information, past asset market actions are informative about an individual's unobservable characteristics that are correlated with their repayment probability encapsulated in a credit score. Related quantitative theory papers with hidden information applied to consumer default include Athreya et al. (2012) and Exler et al. (2021).[7] The former paper makes an anonymous markets assumption where only current asset choices are observed but no prior information about an individual's asset market behavior can be used to infer their unobservable type to price credit while the latter paper makes assumptions on types that effectively eliminates the adverse selection problem for lenders. Closely related quantitative theory papers with hidden information applied to sovereign default include D'erasmo (2011) and Fourakis (2021).

A number of papers have examined the role of improvements in information technology on credit access. In these papers, the technology is a noisy signal of a borrower's true characteristics and an improvement in technology is an increase in signal precision. These include Narajabad (2012), Livshits

literature that studies the conditioning of prices on customers' purchase histories (see, for instance, Acquisti and Varian (2005)).

[4]Even if there was no hidden information and no Bayesian updating of beliefs, a continuous support of shocks would be needed to ensure the existence of a *pure* strategy equilibrium; otherwise existence would require that people be allowed to play mixed strategies. Despite individuals playing pure strategies, the shocks ensure different types may choose the same action analogous to a "semi-separating" equilibrium. Further, the assumption that the shocks are drawn from a Type 1 extreme value distribution delivers the tractability as in Rust (1987).

[5]In full information environments, the observation that the competitive pricing of defaultable debt requires indexing the price of the loan to some observable characteristics like its size appeared in a clear form in Jaffee and Russell (1976) and Eaton and Gersovitz (1981). A large literature on quantitative models of defaultable consumer and sovereign debt now exists (see Exler and Tertilt (2020) and Aguiar et al. (2016) for recent surveys).

[6]Gale (1992) and Dubey and Geanokoplos (2002) prove existence of competitive equilibrium in environments with hidden information. In contrast to us, they adopt the *anonymous markets assumption* of classical GE theory which does not permit prices to depend on personal characteristics of buyers or sellers (such as a credit score). Prescott and Townsend (1984) characterize constrained efficient allocation in an adverse selection environment but show that there is no natural decentralization of it via a price system. Guerreri et al. (2010) prove existence and uniqueness of separating equilibria in static adverse selection models by expanding the contract space to include competitive search over submarkets which helps sustain separation. Our framework expands the contract space to include dynamic type scores which are used to help separate borrowers.

[7]Other related papers which include an information problem of some sort are Luo (2017), Kovrijnykh et al. (2019), Nelson (2022), and Blattner et al. (2022).

et al. (2016), Drozd and Serrano-Padial (2017), and Sanchez (2018). For instance, Narajabad (2012) examines the polar cases where the credit market lacks information on borrower's riskiness and rating technologies do not work well resulting in a pooling equilibrium versus the case where there is sufficient information to separate borrowers according to their unobservable cost of default. Livshits et al. (2016) consider a simple asymmetric information model with costly contracting where borrowers know their types but uninformed lenders receive a noisy signal of a borrower's type. As signal precision improves, the level of partial pooling of borrowers in a given contract falls.

While previous quantitative theory models imposed exogenous punishment, we incorporate dynamic reputation as a means of disciplining borrowers along the lines of Diamond (1989) and more recently Elul and Gottardi (2015). Our reputational environment, where everyone optimizes but people have hidden knowledge about their preferences, is closely linked to repeated games with incomplete information (see Peski (2014) for a discussion of this literature). Reputation in debt markets in which one player is a commitment type have been recently studied by Amador and Phelan (2021). The fact that reputation in one market may discipline behavior in another market has been considered in Cole and Kehoe (1998), Chatterjee et al. (2008), Corbae and Glover (2018), and Braxton et al. (2020).

Section 2 describes our baseline economy with hidden information. Section 3 describes the equilibrium problems faced by our agents. Section 4 describes how we map the model to data and Section 5 studies the properties of the estimated model. Section 6 compares our baseline economy to alternative economies with different information structures. Section 7 concludes. There is an accompanying online appendix, where we provide additional theoretical (Appendix A) and computational (Appendix B) results, description of data (Appendix C), and an extension of the baseline model to delinquency (Appendix D).

2 Environment

We pose a model of perpetual youth as in Blanchard (1985) and Yaari (1965) with constant population. Agents die with with probability $1-\rho$ at the end of the period and those who die are replaced by newborns so that there is always a unit measure of agents. An individual's persistent log earnings, denoted $e_t \in \mathcal{E} = \{e_1, e_2, ..., e_E\} \subset \mathbb{R}_{++}$, are exogenously drawn from a stationary finite state Markov process $Q^e(e_{t+1}|e_t)$. In addition, there are purely transitory (log) earnings, denoted $z_t \in \mathcal{Z} = \{z_1, z_2, \dots, z_Z\} \subset \mathbb{R}_{++}$, which are exogenously drawn from a stationary probability distribution $H(z_t)$. All earnings draws are independent across individuals and we denote individual total earnings as

$y_t(e_t, z_t) = \exp(e_t + z_t)$. A newborn's earnings are drawn from initial distribution F_e.[8]

At time t individuals can choose assets $a_{t+1} \in \mathcal{A} = \{a_1, a_2, ..., a_N\} \subset \mathbb{R}$, where $a_1 < a_2 < ... < a_N$, at discount price q_t determined in a competitive market. We assume the finite set $\mathcal{A}$ includes 0 with $a_1 < 0$ and $a_N > 0$. If an agent holds debt (i.e. $a_t < 0$), she can choose whether or not to file for bankruptcy $d_t \in \mathcal{D} = \{0, 1\}$. If she files (i.e. $d_t = 1$), then in the period of filing she cannot borrow or save (i.e. $a_{t+1} = 0$) and her earnings net of the costs of bankruptcy become $y_t(e_t, z_t)(1 - \kappa_1) - \kappa$ where $\kappa > 0$ is the bankruptcy filing fee and $\kappa_1 \in (0, 1)$ proxies for the negative consequences of bankruptcy on one's earnings.[9]

In each period t, the individual values consumption c_t using a utility function $u(c_t) : \mathbb{R}_{++} \rightarrow \mathbb{R}$ which is is continuous, increasing, and concave. At time t, an individual discounts her future utility at rate $\beta_t \in \mathcal{B} = \{\beta_1, \beta_2, ..., \beta_B\}$ if she survives. Her discount factor varies stochastically over time drawn from a finite state Markov process $Q^\beta(\beta_{t+1}|\beta_t)$. The β_t are drawn independently across individuals and are unobservable to others. We call $\beta_t \in [0, 1)$ a household's *type*.

In addition, households receive action-specific, additively separable extreme value preference shocks which enter households' flow utility each period. The first set of shocks attach to the bankruptcy/no bankruptcy choice ($d_t \in \{0, 1\}$) and is therefore drawn only by borrowers:

$$\nu_t = (\nu_t^{d=0}, \nu_t^{d=1}). \tag{1}$$

The second set of shocks of length N attaches to a_{t+1} choices in the event of no default:

$$\epsilon_t = (\epsilon_t^{a_1}, \epsilon_t^{a_2}, ..., \epsilon_t^{a_N}). \tag{2}$$

The vectors ν_t and ϵ_t are drawn independently across individuals. Each element ν_t^d and $\epsilon_t^{a_n}$ is drawn from type I extreme value distributions F_ν and F_ϵ with scale parameters α and λ, respectively.[10]

Intermediaries can observe individuals' persistent earnings (i.e. e_t) and asset market behavior (i.e. a_t, d_t, and a_{t+1}), but cannot observe their preferences (i.e. ν_t, ϵ_t, and β_t) nor the transitory component of earnings (i.e. z_t). Since ν_t, ϵ_t and z_t are i.i.d. over time and individuals, nothing can be learned about their future values from their current values. In contrast, since β_t is drawn from a persistent

[8] By setting F_e to the degenerate distribution that has all mass on the lowest persistent earnings level, we can parsimoniously achieve a rising earnings profile over an individual's working life as an approximation to the full life cycle model in Livshits et al. (2007).

[9] See Corbae and Glover (2018) for a model in which a poor credit record adversely affects an individual's earnings.

[10] For reasons that we explain when computing the model in Section 4.1, we permit the location parameters of the extreme value shocks to depend on the options available to households.

Markov process, the probability distribution of its future values depends on its current (unobservable) value. We denote the creditor's probability assessment that an individual is of type β_i at the beginning of period t before any actions are taken as $s_t(\beta_i) = \Pr(\beta_t = \beta_i)$. We call $s_t = (s_t(\beta_1), ..., s_t(\beta_B))$ an individual's *type score* with $\sum_{i=1}^{B} s_t(\beta_i) = 1$.[11]

Given an individual's observable characteristics $\omega_t = (e_t, a_t, s_t)$ as well as their credit market actions (d_t, a_{t+1}), financial intermediaries revise their assessments of an individual's type from s_t via Bayes' rule.[12] We denote this update as $\psi_t^{(d_t,a_{t+1})}(\omega_t) \in [0,1]^B$. As a result of this assessment, the prices faced by an individual in the credit market will also depend on her observable state and her credit market actions. Thus, we denote the price function for an individual with observable characteristics ω_t who chooses assets a_{t+1} by $q_t^{a_{t+1}}(\omega_t)$. The arguments of q_t influence the price because they can directly affect the likelihood of repayment on a loan (as in standard debt and default models) and indirectly by revealing information about the individual's current type (this is encoded in the update $\psi_t^{(0,a_{t+1})}(\omega_t)$).[13] Importantly, note that all the *future* implications of current credit market actions are encapsulated in the update $\psi_t^{(d_t,a_{t+1})}$. In particular, the only punishment to bankruptcy in future periods (aside from those that follow from the requirement that saving is not permitted in the filing period) is the possible loss of reputation stemming from intermediaries' adverse assessments of her unobservable type.

For technical reasons, we assume $s_t \in \mathcal{S}$, a finite subset of $[0,1]^B$. This assumption makes it possible to apply standard methods to prove existence of equilibrium. Since the posterior ψ_t may not lie on one of the finite points in $\mathcal{S}$, we assign it randomly to nearby points in $\mathcal{S}$. We denote the probability mass function implied by our random assignment as $Q^s(s_{t+1}|\psi_t)$.[14] We can show that:

Lemma 1. *There exists an assignment rule satisfying: (i)* $\mathbb{E}_{s_{t+1}\in\mathcal{S}}\,[Q^s(s_{t+1}|\psi_t)] = \psi_t$ *(i.e. consistency), (ii) the variance of the approximation error (i.e. of* s_{t+1} *from* ψ_t*) is arbitrarily small, and (iii)* $Q^s(s_{t+1}|\psi_t)$ *is continuous in* ψ_t*.*

Definition 1. The *timing* in any given period is as follows:

1. All individuals (survivors and newborns) begin with the vector (β_t, e_t, a_t, s_t) and receive a transitory earnings shock z_t.

[11] Of course the framework is rich enough to add more unobservables. For instance, if the persistent component of earnings are unobservable, then $s_t = (s_t(\beta_1, e_1), s_t(\beta_1, e_2), \dots, s_t(\beta_B, e_E))$.

[12] As in the original econometric use of extreme value shocks in the discrete choice literature, (ν_t, ϵ_t) provide a parsimonious way to capture how a type scorer may observe different choices (d_t, a_{t+1}) by two individuals in the same observable starting state (ω_t) due to, for instance, unobserved preference heterogeneity.

[13] Note that in the absence of hidden information regarding type, the pricing function would be independent of a_t (as is the case in Chatterjee et al. (2007)) since past debts have no bearing on the repayment probability of newly incurred debt.

[14] This rule is specified in equation (28) in Online Appendix A. There is nothing of substance in this randomization over contiguous elements of $\mathcal{S}$ since $\mathcal{S}$ is finely gridded when we estimate the model.

2. Individuals who have $a < 0$ receive the random utility vector ν_t and decide whether to file for bankruptcy ($d_t = 1$) or not ($d_t = 0$).

3. Individuals who have not filed for bankruptcy receive the random utility vector ϵ_t and choose a feasible action given prices $q_t^{a_{t+1}}(\omega_t)$.

4. Based on each individual's actions (d_t, a_{t+1}) and observable characteristics ω_t, intermediaries revise their assessments of an individual's type via Bayes' rule, updating s_t to ψ_t.

   (a) Individuals who survive draw beginning-of-next-period realizations of β_{t+1} and e_{t+1} from the exogenous transition functions $Q^\beta(\cdot|\beta_t)$ and $Q^e(\cdot|e_t)$. The beginning of next period type score s_{t+1} is drawn from the probability mass function $Q^s(\cdot|\psi_t)$.

   (b) Newborns begin life with β_{t+1} drawn from initial distribution F_β, earnings class e_{t+1} drawn from initial distribution F_e, zero assets, and a type score s_{t+1} equal to F_β for consistency. We assume $F_\beta \in \mathcal{S}$.

3 Equilibrium

3.1 Individuals' problem

Let x_t be denoted x and x_{t+1} be denoted x'. Denote the part of the state space observable to creditors by $\Omega = \{\mathcal{E} \times \mathcal{A} \times \mathcal{S}\}$ with typical element ω. An individual takes as given:

- the price function $q^{a'}(\omega) : \mathcal{A} \times \Omega \to [0, 1]$
- the type scoring functions $\psi^{(0,a')}(\omega) : \mathcal{A} \times \Omega \to [0, 1]^B$ and $\psi^{(1,0)}(\omega) : \Omega \to [0, 1]^B$ which perform Bayesian updating of an individual's type based on all observables following asset choice and bankruptcy, respectively.

For ease of notation, we will denote the triplet of functions $\{q^{a'}(\omega), \psi^{(0,a')}(\omega), \psi^{(1,0)}(\omega)\}$ by $f \in F$, where $F = \{(f_1, f_2, f_3)|\ f_1 : \mathcal{A} \times \Omega \to [0, 1]\,, f_2 : \mathcal{A} \times \Omega \to [0, 1]^B \text{ and } f_3 : \Omega \to [0, 1]^B\}$.

Definition 2. Given (z, ω) and $f \in F$, the *set of feasible actions* is a finite set $\mathcal{F}(z, \omega|f)$ that contains all actions (d, a') such that consumption $c^{(d,a')}(z, \omega|f)$ is strictly positive where:

$$c^{(d,a')}(z, \omega|f) = \begin{cases} y(e(\omega), z) + a(\omega) - q^{a'}(\omega) \cdot a' & \text{if } (d, a') = (0, a') \\ y(e(\omega), z)(1 - \kappa_1) - \kappa & \text{if } a(\omega) < 0 \text{ and } (d, a') = (1, 0) \end{cases} \tag{3}$$

where we use $a(\omega)$, $e(\omega)$ and $s(\omega)$ to denote the corresponding elements of ω.

Assumption 1. $y(e_1, z_1) + \min\{a_1, -\kappa - \kappa_1 y(e_1, z_1)\} > 0.$

We make this assumption to ensure that it is always feasible for an indebted individual to file for bankruptcy and always feasible for her to pay back her debt.

We work backwards from an individual's state at stage 3 in timing. Given the functions f, for $(d, a') \in \mathcal{F}(z, \omega|f)$ we denote the *conditional value function*

$$\begin{aligned} v^{(d,a')}(\beta, z, \omega|f) &= u\left(c^{(d,a')}(z, \omega|f)\right) \\ &+\beta\rho \cdot \sum_{(\beta', z', e', s')} Q^{\beta}(\beta'|\beta)\ Q^{e}(e'|e)\ H(z')\ Q^{s}(s'|\psi^{(d,a')}(\omega))\ W(\beta', z', \omega'|f) \end{aligned} \tag{4}$$

where the *expected value function* W integrates the value function over ν and is defined below.

The value function $V^{ND}(\epsilon, \beta, z, \omega|f) : \mathbb{R}^N \times \mathcal{B} \times \mathcal{Z} \times \Omega \to \mathbb{R}$ for an individual who chooses not to file for bankruptcy at stage 2 is then given by

$$V^{ND}(\epsilon, \beta, z, \omega|f) = \max_{(0,a') \in \mathcal{F}(z,\omega|f)} v^{(0,a')}(\beta, z, \omega|f) + \epsilon^{a'} \tag{5}$$

$$W^{ND}(\beta, z, \omega|f) = \int V^{ND}(\epsilon, \beta, z, \omega|f)\ \mathrm{d}F_{\epsilon}(\epsilon). \tag{6}$$

Given the sequential nature of choices in our timing, the value function at stage 2 is then given by

$$V(\nu, \beta, z, \omega|f) = \begin{cases} W^{ND}(\beta, z, \omega|f) & \text{if } a(\omega) \geq 0 \\ \max\left\{v^{(1,0)}(\beta, z, \omega|f) + \nu^{D}, W^{ND}(\beta, z, \omega|f) + \nu^{ND}\right\} & \text{if } a(\omega) < 0. \end{cases} \tag{7}$$

W^{ND} shows up because ϵ has not yet been drawn at stage 2. Finally, as promised we have

$$W(\beta, z, \omega|f) = \int V(\nu, \beta, z, \omega|f)\mathrm{d}\ F_{\nu}(\nu). \tag{8}$$

Given that ν and ϵ are drawn from type I extreme value distributions, there are simple closed form solutions for choice probabilities. Conditional on not filing for bankruptcy, let $\tilde{\sigma}^{(0,a')}(\beta, z, \omega|f)$ be the probability that the individual in state (β, z, ω) chooses action $a' \in \mathcal{F}(z, \omega|f)$:

$$\tilde{\sigma}^{(0,a')}(\beta, z, \omega|f) = \begin{cases} \dfrac{\exp\left\{\frac{v^{(0,a')}(\beta,z,\omega|f)}{\lambda}\right\}}{\sum_{(0,\hat{a}') \in \mathcal{F}(z,\omega|f)} \exp\left\{\frac{v^{(0,\hat{a}')}(\beta,z,\omega|f)}{\lambda}\right\}} & \text{for } a' \in \mathcal{F}(z, \omega|f) \\ 0 & \text{for } a' \notin \mathcal{F}(z, \omega|f). \end{cases} \tag{9}$$

Note that infeasible actions are assigned zero probability. Similarly, the probability of bankruptcy for an individual with debt $(a(\omega) < 0)$ is

$$\sigma^{(1,0)}(\beta, z, \omega|f) = \frac{\exp\left\{\frac{v^{(1,0)}(\beta,z,\omega|f)}{\alpha}\right\}}{\exp\left\{\frac{v^{(1,0)}(\beta,z,\omega|f)}{\alpha}\right\} + \exp\left\{\frac{W^{ND}(\beta,z,\omega|f)}{\alpha}\right\}}. \tag{10}$$

Then, given that ν and ϵ are independent, asset choice probabilities are given by

$$\sigma^{(0,a')}(\beta, z, \omega|f) = \tilde{\sigma}^{(0,a')}(\beta, z, \omega|f)\left(1 - \sigma^{(1,0)}(\beta, z, \omega|f)\right) \tag{11}$$

noting that an individual with $a(\omega) \geq 0$ has $\sigma^{(1,0)}(\beta, z, \omega|f) = 0$ by definition. Furthermore, these choice probability expressions imply simple expressions for W^{ND} in (6) and W in (8). Specifically,

$$W^{ND}(\beta, z, \omega|f) = \lambda \ln\left(\sum_{(0,a')\in\mathcal{F}(z,\omega|f)} \exp\left\{\frac{v^{(0,a')}(\beta, z, \omega|f)}{\lambda}\right\}\right) + \lambda\gamma_E + \bar{\epsilon}(\mathcal{A}) \tag{12}$$

and

$$W(\beta, z, \omega|f) = \begin{cases} W^{ND} & \text{if } a(\omega) \geq 0 \\ \alpha \ln\left(\exp\left\{\frac{v^{(1,0)}(\beta,z,\omega|f)}{\alpha}\right\} + \exp\left\{\frac{W^{ND}(\beta,z,\omega|f)}{\alpha}\right\}\right) + \alpha\gamma_E + \bar{\nu}(\mathcal{D}) & \text{if } a(\omega) < 0 \end{cases}, \tag{13}$$

where γ_E is the Euler–Mascheroni constant.

In Online Appendix A.2 we prove:

Theorem 1. *Given f, there exists a unique solution $W(\beta, z, \omega|f)$ to the individual's decision problem in (3)-(8).*

3.2 Intermediaries' problem

Competitive intermediaries with deep pockets have access to an international credit market where they can borrow or lend at the risk-free interest rate $r \geq 0$. Any given intermediary takes prices q and scoring function ψ (i.e. f) as given. We assume that losses and gains resulting from individuals' deaths accrue to the financial intermediary effectively implementing an annuity contract. The profit $\pi^{a'}(\omega|f)$ on a contract of size a' with agents with observables ω is:

$$\pi^{a'}(\omega|f) = \begin{cases} \rho \cdot \frac{p^{a'}(\omega|f)\cdot(-a')}{1+r} - q^{a'}(\omega)\cdot(-a') & \text{if } a' < 0 \\ q^{a'} \cdot a' - \rho \cdot \frac{a'}{1+r} & \text{if } a' \geq 0, \end{cases} \tag{14}$$

where the probability of repayment on a contract of size a' made to individuals with observable characteristics ω is $p^{a'}(\omega|f) : (\mathbb{R}_{--} \cap \mathcal{A}) \times \Omega \to [0,1]$. Given perfect competition and constant returns to scale in lending, if a solution to the intermediary's problem exists, then optimization by the intermediary implies zero profits for strictly positive measures of contracts issued or

$$q^{a'}(\omega|f) = \begin{cases} \frac{\rho \cdot p^{a'}(\omega|f)}{1+r} & \text{if } a' < 0 \\ \frac{\rho}{1+r} & \text{if } a' \geq 0. \end{cases} \tag{15}$$

Assessing an individual's probability $p^{a'}(\omega|f)$ of repaying a debt next period given her current observable characteristics ω given unobservable (β, ϵ, z), takes two steps:

1. Assess the probability that an individual in state ω who takes action (d, a') will be of unobservable type β' next period via Bayes rule (the type scoring function $\psi_{\beta'}^{(d,a')}(\omega)$).
2. For each possible future unobservable type β', compute the individual's probability of future repayment conditional on being that type and transitions over observable characteristics and then compute the weighted sum over future types to obtain p.

Starting with step 1, an individual's probability of being type $(\beta'_1, ..., \beta'_B)$ next period is given by the type scoring function $\psi^{(d,a')}(\omega) = \left(\psi_{\beta'_1}^{(d,a')}(\omega), ..., \psi_{\beta'_B}^{(d,a')}(\omega)\right)$, where

$$\psi_{\beta'}^{(d,a')}(\omega|f) = \begin{cases} \sum_{\beta} Q^{\beta}(\beta'|\beta) \cdot \frac{\sum_z \sigma^{(d,a')}(\beta,z,\omega|f)\cdot H(z)\cdot s(\beta)}{\sum_{\hat{\beta},z} \sigma^{(d,a')}(\hat{\beta},z,\omega|f)\cdot H(z)\cdot s(\hat{\beta})} & \text{for } (d,a') \in \mathcal{F}(z,\omega|f) \\ \sum_{\beta} Q^{\beta}(\beta'|\beta) \cdot s(\beta) \text{ for } (d,a') \notin \mathcal{F}(z,\omega|f). \end{cases} \tag{16}$$

Note that in (16), the assessment uses Bayes' rule to assign the probability of an individual in state ω taking a feasible action (d, a') being of type β' next period. By (9) and (10), the probability of choosing any $(d, a') \in \mathcal{F}(z, \omega|f)$ is strictly positive for every β'. Hence, $\psi_{\beta'}^{(d,a')}(\omega|f)$ is well-defined in (16) for all feasible actions. Thus, since every feasible action is chosen with some probability due to the presence of extreme value shocks, we avoid having to assign off-the-equilibrium path beliefs for feasible actions. For completeness, without loss of generality, the bottom branch of (16) handles the case of infeasible actions. Turning to step 2, given observable state ω, we obtain the probability of repayment the intermediary uses for pricing debt (i.e. for $a' < 0$) via:

$$p^{a'}(\omega|f) = \sum_{\beta',z',e',s'} H(z') \cdot Q^e(e'|e) \cdot Q^s(s'|\psi^{(0,a')}(\omega|f)) \cdot s'(\beta') \cdot \left(1 - \sigma^{(1,0)}(\beta', z', e', a', s'|f)\right). \quad (17)$$

3.3 Evolution

Let $\mu(\beta, z, \omega|f)$ denote the beginning-of-period measure of individuals in state (β, z, ω) for a given f. Then, the cross-sectional distribution evolves according to

$$\mu'(\beta', z', \omega'|f) = \sum_{\beta,z,\omega} T(\beta', z', \omega'|\beta, z, \omega; f) \cdot \mu(\beta, z, \omega|f), \quad (18)$$

where the transition function is

$$\begin{aligned} T(\beta', z', \omega'; \beta, z, \omega|f) \quad = \quad & \rho \cdot Q^\beta(\beta'|\beta) \cdot H(z') \cdot Q^e(e'|e) \cdot \sigma^{(d,a')}(\beta, z, \omega|f) \cdot Q^s(s'|\psi^{(d,a')}(\omega|f)) \\ & +(1-\rho) \cdot F_\beta(\beta') \cdot H(z') \cdot F_e(e') \cdot \mathbf{1}_{\{a'=0\}} \cdot \mathbf{1}_{\{s'=F_\beta\}}. \end{aligned} \quad (19)$$

The first line in equation (19) is the probability of a survivor transitioning to (β', z', e', a', s') while the second line is the probability that a newborn arrives in state (β', z', e', a', s').

An invariant distribution is a fixed point $\overline{\mu}(\cdot|f) = T\overline{\mu}(\cdot|f)$. In Online Appendix A.3 we prove:

Lemma 2. *There exists a unique invariant distribution $\overline{\mu}(\cdot|f)$ and $\{\mu_0 T^n\}$ converges to $\bar{\mu}(\cdot|f)$ at a geometric rate for any initial distribution μ_0.*

Note that although the invariant distribution is critical for computing cross-sectional moments used to map the model to the data, none of the other equilibrium objects (i.e. the set of functions f, the value function V or the decision rule σ) takes μ as an argument. This simplifies the model and eases the computational burden, but is not necessary. Other specifications in which knowledge of the distribution is required are possible, but we do not consider these in the baseline model.

3.4 Existence

We can now give the definition of a stationary recursive competitive equilibrium.

Definition 3. A stationary *Recursive Equilibrium* is a pricing function q^*, a type scoring function ψ^*, a choice probability function σ^*, and a steady state distribution $\overline{\mu}^*$ such that:

(i). Optimality: $\sigma^{(d,a')*}(\beta, z, \omega|f^*)$ satisfies (9) and (10) for all $(\beta, z, \omega) \in \mathcal{B} \times \mathcal{Z} \times \Omega$ and $(d, a') \in \mathcal{F}(z, \omega|f^*)$,

(ii). Zero Profits: $q^{a'*}(\omega|f^*)$ satisfies (15) with equality for all $\omega \in \Omega$ with $p^{a'*}(\omega|f^*)$ satisfying (17) for all $\omega \in \Omega$,

(iii). Bayesian Updating: $\psi_{\beta'}^{(d,a')*}(\omega|f^*)$ satisfies (16) for all $(\beta', \omega) \in \mathcal{B} \times \Omega$, and

(iv). Stationary Distribution: $\overline{\mu}^*(\beta, z, \omega|f^*)$ solves (18) for $T(\beta', z', \omega'; \beta, z, \omega|f^*)$.

The key step in proving the existence of a recursive competitive equilibrium is proving that the value function $W(\beta, z, \omega|f)$ is continuous in f. In Online Appendix A.4 we first prove:

Lemma 3. *$W(\beta, z, \omega|f)$ is continuous in f, and for any $(d, a') \in \mathcal{F}(z, \omega|f)$, $\sigma^{(d,a')}(\beta, z, \omega|f)$ is continuous in f.*

Using this result, we then prove

Theorem 2. *There exists a stationary recursive equilibrium.*

3.5 Equivalence to an Economy with Credit Scores

In the economy described thus far, an individual's reputation is her type score. In U.S. credit markets, an important measure of reputation is the *credit score*. A credit score is an index that is positively related to the likelihood of repayment. The goal of this subsection is to show that under certain conditions, the equilibrium described in the previous subsections can be implemented via an arrangement in which lenders use a model equivalent of a credit score to assess the probability of repayment on a loan given other relevant characteristics such as earnings and current assets.

In this paper, we formalize the notion that a credit score depicts a consumer's creditworthiness by defining it to be the probability of repayment on a loan of some standard size $a' = \bar{a} < 0$. According to the timing in Definition 1 part 4(a), since $\omega = (e, a, s)$ is known at the end of $t-1$, an individual's credit score can be calculated at the end of period $t-1$ to be $m = p^{\bar{a}}(e, a, s)$ using equation (17). The credit score of newborns who arrive at the end of period $t-1$ is $m = p^{\bar{a}}(e_1, 0, F_\beta)$.

In the financial arrangement with credit scores, an individual in (observable) state $\hat{\omega} = (e, a, m)$ takes as given a pricing function $q^{a'}(\hat{\omega})$ and a credit-score transition function $Q_m^{(d,a')}(m'|e', \hat{\omega})$ which tells her the probability distribution of her future credit score conditional on her current observable state, current actions and future earnings. Intermediaries take as given the pricing function (which must satisfy the zero profit condition) and the probability of repayment function $p^{a'}(\hat{\omega})$ (which must be consistent with the individual's objective likelihood of repayment). In Online Appendix A.5, we restate the household and financial intermediary problems for this financial arrangement and provide a definition of a Recursive Equilibrium with Credit Scores.

An equivalence between the type-scoring and credit-scoring environments will exist if there is a one-to-one and onto mapping between s and m, holding fixed the other factors that affect credit scores, namely, e and a. Then, wherever s appears in the theoretical model, it can be replaced by m. Thus, the equivalence will hold if the inverse function $(p^{\bar{a}*})^{-1}(e, a, m)$ exists.[15] Now note that since $\mathcal{S}$ is a finite collection of grid points, the occurrence of distinct grid points in $\mathcal{S}$ mapping to precisely the same probability of repayment on $\bar{a}$, given e and a, will be purely coincidental.[16] Thus, barring coincidences, the mapping $m = p^{\bar{a}*}(e, a, s)$ will be one-to-one, and it can be made onto by restricting the range of p to contain only those m that are implied by some $s \in \mathcal{S}$, given e and a. In other words, regardless of the number of types, the finite support of s can be used to encode both an individual's type score and her probability of repayment on $\bar{a}$. In our application, we verify that the one-to-one property holds and $(p^{\bar{a}})^{-1}(e, a, m)$ exists.

In Online Appendix A.5 we prove:

Theorem 3. *Given a Recursive Equilibrium, let $m = p^{\bar{a}*}(e, a, s)$. Suppose that the inverse function $s = (p^{\bar{a}*})^{-1}(e, a, m)$ exists. Then a Recursive Equilibrium with Credit Scores exists in which the choice probabilities $\sigma^{(d,a')*}(\beta, z, e, a, m) = \sigma^{(d,a')*}(\beta, z, e, a, s)$ for $s = (p^{\bar{a}*})^{-1}(e, a, m)$.*

4 Mapping the Model to Data

We now examine the U.S. unsecured credit market through the lens of our model. We rely on the equivalence result between the model with type scores and the model with credit scores described in Theorem 3 when there are two β types ($\beta_H > \beta_L$). Specifically, it allows us to use the model with type scores in order to target the joint behavior of earnings, aggregate credit market moments, and credit rankings over their working age. We then verify that the sufficient condition in Theorem 3 is satisfied for the estimated parameters so that the equivalence result holds.

[15] If the relationship between s and m is not one-to-one, then two individuals with the same e, a, and m choosing the same level of debt could face different prices in the economy with type scores because their s's are different.

[16] To see why, suppose that there are 3 types of individuals and let $s = \{s_1, s_2, s_3\}$ be a specific type score. For concreteness, assume that in the same circumstances type 1's probability of repayment is greater than type 2's, and type 2's is greater than type 3's (i.e., $\sigma^{(1,0)}(\beta_1, z, e, \bar{a}, s|f) < \sigma^{(1,0)}(\beta_2, z, e, \bar{a}, s|f) < \sigma^{(1,0)}(\beta_3, z, e, \bar{a}, s|f)$). If we now consider another $\hat{s}$ where the value of $\hat{s}_1$ is higher than s_1, the probability of repayment on $\bar{a}$ can remain unchanged if $\hat{s}_2$ is lower than s_2 by some specific amount and $\hat{s}_3$ is higher than s_3 by some specific amount. However, it will be a pure coincidence if the exact combination $(\hat{s}_1, \hat{s}_2, \hat{s}_3)$ is an element of $\mathcal{S}$.

In the data, a credit score is an *ordinal* measure of creditworthiness, typically ranging from around 300 to 850, not a direct estimate of the probability of repayment m.[17] To close this gap, we associate with $m = p^{\bar{a}}(\omega)$ a number in the unit interval that gives $p^{\bar{a}}(\omega)$'s position (i.e. ranking) in the overall distribution of $p^{\bar{a}}(\omega)$ in the model economy.

Definition 4. An individual's *credit ranking* in state ω is given by

$$\chi^{\bar{a}}(\omega) = \sum_{\tilde{\omega} \in J^{\bar{a}}(\omega)} \mu(\tilde{\omega}), \tag{20}$$

where $J^{\bar{a}}(\omega) = \{\tilde{\omega} : p^{\bar{a}}(\tilde{\omega}) \leq p^{\bar{a}}(\omega)\}$ and $\mu(\omega) = \sum_{\beta,z} \mu(\beta, z, \omega)$.

Clearly, $\chi^{\bar{a}}(\omega) \in [0, 1]$. We construct the data analogue of $\chi^{\bar{a}}(\omega)$ by associating with each credit score its percentile position in the overall distribution of credit scores.[18] Simply put, after computing credit scores for each individual in the economy we then line them up and associate each individual with its rank in the credit score distribution.

Furthermore, real-world credit scores do not mention any specific level of borrowing $\bar{a}$. One way to interpret this fact is to think that the ranking of individuals with respect to probability of repayment holds for *any* level of debt. For this to be true, we need the following property:

Definition 5. Let $\hat{a} < \bar{a} < 0$. Then, $p^{a'}(\omega)$ *preserves order with respect to* a' if $p^{\bar{a}}(\omega) \geq p^{\bar{a}}(\tilde{\omega})$ if and only if $p^{\hat{a}}(\omega) \geq p^{\hat{a}}(\tilde{\omega})$.

If $p^{a'}(\omega)$ preserves order, then $J^{\hat{a}}(\omega) = J^{\bar{a}}(\omega)$ and $\chi^{\bar{a}}(\omega)$ becomes invariant to the choice of $\bar{a}$. This order preserving property holds for a wide range of debt levels for our estimated model in Section 4.

Credit rankings, earnings and assets all grow with age on average, and we want our model to capture those features. Unfortunately, we do not have access to a panel dataset which contains all these dimensions. So we use a version of simulated method of moments to estimate our model. Specifically, we take some non-controversial information from outside the model: the earnings process, the risk free rate of return, demographics, preferences over risk, a measurement of the costs of bankruptcy filings, and a generic value of debt ($\overline{a}$) to which the credit score is normalized.[19]

Next we obtain a set of data moments that summarize the properties of the unsecured credit market

[17] See, for example, https://www.investopedia.com/terms/c/credit_score.asp.

[18] Thus while our theory is in terms of type scores which we map to credit scores in $[0, 1]$ via Theorem 3, since real world credit scores have no interpretation in the model, we convert them to something interpretable in both the data and the model: credit rankings. Hence we use all three concepts: (1) s; (2) $m = p^{\bar{a}}(e, a, s)$; (3) $\chi^{\bar{a}}(\omega) = \sum_{\tilde{\omega} \in J^{\bar{a}}(\omega)} \mu(\tilde{\omega})$.

[19] We verify that the choice of $\overline{a}$ does not matter provided it is higher than the bankruptcy filing costs.

(bankruptcy filing rates, average interest rates, dispersion of interest ratios, fraction of households in debt, debt to income ratio) and we approximate the behavior of credit scores as a function of age (specifically, affine functions of the mean and the standard deviation of credit scores and the autocorrelation of the annual change in individual scores). One can interpret age as the length of an individual's credit history; agents are "born" with no credit history and the length of their credit history grows with age.

We then proceed to estimate the parameters of interest which are the values of patience for both types, the transition probabilities of types and their frequency at birth, the proportional earnings loss from bankruptcy, as well as measures of noise (the variances of the extreme value shocks) by minimizing the weighted sum of squared differences between the values of the moments in the data and their model counterparts. We have tried various alternative sets of moments with minimal effects on the findings. While earnings, credit and bankruptcy statistics have been used since Chatterjee et al. (2007) and Livshits et al. (2007), credit scores, and their evolution by age, have not. The evolution of credit scores is crucial for understanding the building of a reputation over the early part of the life-cycle.

4.1 Computation

Computation of equilibrium requires solving for two endogenous functions: the bond price function and the type-score updating function. The bond price function is standard in unsecured debt models like Chatterjee et al. (2007), except that the endogenous type score is an additional dimension. The type scoring function is new: individuals take as given how feasible actions change the market's perception of their type that is updated using Bayes law and this perception has to be consistent with the actions taken by both types.

While we introduced extreme value shocks in order to keep the Bayesian posterior well behaved, it can, however, exacerbate grid sensitivity associated with approximating continuous choices.[20] We deal with this by making the location parameters of the extreme value shock associated with each asset choice depend positively on the measure of consumption points that are associated with that action in the individual's feasible set $\mathcal{F}(z, \omega|f)$.[21] We describe adjustments to mitigate grid sensitivity after

[20]As an example, imagine that we are approximating the interval $[0, 2]$ with a discrete grid that is log-spaced. This means that there are more grid points in $[0, 1]$ than in $[1, 2]$. Now assume that the value of the action associated with any grid point i is just $v + \epsilon^i$. Then, any one of these grid points has an equal chance of being selected. But, since there are more points in $[0, 1]$, it is more likely that the choice will be from that interval. In the context of our model, this effect imparts a bias toward actions close to the origin (debt or small levels of assets).

[21]For instance, in the example of footnote 20, the adjustment to the location parameter of the shocks lowers the mean of the extreme value shocks associated with closely-packed choices. The result is that with the adjustment it is equally likely that the best choice is in $[0, 1]$ or $[1, 2]$. This adjustment has implications for savings behavior explored in Briglia et al. (2021). See also subsection B.4.

introducing the functional forms of the shock distributions in the next section.

4.2 Estimation

We use a minimum distance estimator to parameterize the model. We discuss which ex-ante restrictions we specify (Section 4.2.1), the targets that we use and the data from which they come (Section 4.2.2), the estimation strategy (Section 4.2.3), the estimates (Section 4.2.4), and we finish with a discussion of the robustness of our estimates (Section 4.2.5).

4.2.1 Functional Forms and Parameters Chosen Outside the Model

A model period is one year. We take the relevant working life span of people to be 40 years, as the bulk of borrowing is by young people, implying a working age survival probability of 0.975. We choose a CRRA utility function with risk aversion parameter 1.5. We pose a risk free rate of 1%, which implies an effective interest rate on savings of 3.59% in the presence of perfect annuity markets. We take the cost of filing for bankruptcy to be about 1.5% of median earnings taken from Albanesi and Nosal (2018).[22] Since it is a dominant action not to invoke bankruptcy on debts less than the filing cost, we choose $\bar{a}$ (the debt value used to compute the probability of repayment for a credit score) to be 3.5% of median earnings (i.e. well above those costs). Finally we take the earnings class to be the persistent AR1 process estimated by Floden and Lindé (2001) and assume agents are born with the lowest earnings level to replicate the upward earnings path during one's working age.[23] These parameters chosen outside the model are summarized in Table 1.

We parameterize the cumulative distribution function of the type 1 extreme value ν shocks associated with the default choice as:

$$F_\nu(\nu^d;\alpha) = \exp\left\{-\exp\left(-\frac{\nu^d-\bar{\nu}}{\alpha}\right)\right\} \text{ for } d \in \{0,1\}. \tag{21}$$

Given α, we choose the location parameter $\bar{\nu}$ to eliminate the incentive for a household to choose

[22] Albanesi and Nosal (2018) report a filing fee of \$697 in 2005 pre-BAPCA. Median household income in 2004, adjusted for 3.39% inflation between 2004 and 2005, was \$45,837; the ratio of these numbers yields 1.52%.

[23] Recalling total earnings is given by $y_t(e_t, z_t) = \exp(e_t + z_t)$, we approximate the AR1 process by a five-state Markov chain using the Adda and Cooper (2003) method, which yields support $\mathcal{E} = \{-0.71, -0.27, 0.00, 0.27, 0.71\}$, transition matrix $Q^e(e'|e) = \begin{bmatrix} 0.767 & 0.207 & 0.025 & 0.001 & 0.000 \\ 0.207 & 0.496 & 0.253 & 0.043 & 0.001 \\ 0.025 & 0.253 & 0.446 & 0.253 & 0.025 \\ 0.001 & 0.043 & 0.253 & 0.496 & 0.207 \\ 0.000 & 0.001 & 0.025 & 0.207 & 0.767 \end{bmatrix}$, and a transitory component with a three-point uniform distribution on support $\mathcal{Z} = \{-0.25, 0, 0.25\}$.

Table 1: Parameters Chosen Outside the Model

Parameter		Value	Notes
Demographics and preferences			
Survival probability	ρ	0.975	avg. life span 40 years
Risk aversion	γ	1.5	CRRA preferences
Earnings at birth	$\underline{e}$	-0.71	See Footnote 23
Technology			
Risk-free rate (%)	r	1.000	
Bankruptcy filing cost	κ	0.0152	1.5% of median earnings
Debt level for computing credit score	$\bar{a}$	-0.035	2.9% of median earnings
Earnings			
Persistence of log(e)	ρ_e	0.9136	Floden and Lindé (2001)
Variance of innovations to log(e)	ν_e^2	0.0426	Floden and Lindé (2001)
Variance of log(z)	ν_z^2	0.0421	Floden and Lindé (2001)

debt simply in order obtain favorable draws of the extreme value shock associated with the bankruptcy decision.[24] We parameterize the cumulative distribution function of the ϵ shocks associated with asset choices as:

$$F_\epsilon(\epsilon^{a_n};\lambda) = \exp\left\{-\exp\left(-\frac{\epsilon^{a_n} - \overline{\epsilon}^{a_n}(z,\omega)}{\lambda}\right)\right\} \text{ for } n \in \{1,...,\overline{n}(z,\omega)\}. \tag{22}$$

where $\overline{n}(z,\omega)$ is the index of the largest budget feasible a' for an agent with (z,ω). As discussed in Section 4.1, given λ, we specify choice and state-specific means for the ϵ shocks $\overline{\epsilon}^{a_n}(z,\omega)$ to mitigate grid sensitivity.[25] Estimates of the scale parameters α and λ are discussed in Section 4.2.4.

4.2.2 Data and Targets

The set of statistics that we deem important to target pertain to the main aggregate characteristics of the U.S. unsecured credit market: credit usage (the fraction of households in net debt and the debt-to-income ratio, credit terms (average interest rates and their dispersion), and the bankruptcy

[24] We set $\bar{\nu} = -\alpha \cdot (\gamma_E + \ln(|\mathcal{D}|)$ where γ_E is the Euler-Mascheroni constant in equation (35) in Appendix B.1 so that $\mathbb{E}[\max\{\nu^D, \nu^{ND}\}] = 0$. This correction implies that for an indebted household $a(\omega) < 0$ for whom $v^{(1,0)} = W^{ND}$ in (7), the ex-ante value W in (8) is equal to $v^{(1,0)} = W^{ND}$. In other words, the presence of the default/no-default shocks do not add any extra utility in expectation.

[25] Specifically, we set $\overline{\epsilon}^{a_n}(z,\omega|f_j) = -\lambda\gamma_E + \lambda \ln \eta^{a_n}(z,\omega|f_j)$ in equation (37) of Appendix B.1 where $\eta^{a_n}(z,\omega|f_j)$ is the measure of consumption in an agent's budget set accounted for by a given asset choice a_n. This maps our exogenous discrete grid over a' into consumption weights that help correct distortions to individual decision making when adding arbitrary points to the a' grid. In particular, the correction down-weights choices on dense portions of the grid.

rate. Importantly, we are also interested in matching properties of the age profile of credit rankings. In particular, we match the intercept and slope of the mean and standard deviation of credit rankings across the working age profile, as well as the mean of the autocorrelation of credit ranking changes.[26] This amounts to using 10 moments as targets.

To obtain these data targets, we use three primary sources: the Survey of Consumer Finances (SCF), the administrative records of the U.S. Bankruptcy Courts, and the Federal Reserve Bank of New York Consumer Credit Panel/Equifax (FRBNY CCP/Equifax). The first provides information on individual level variation in debt and interest rates, the second provides information on aggregate bankruptcy filing rates, and the last contains individual-level information on credit records from an anonymized panel which provides us with moments on the variation and evolution of credit scores. The credit score measure is the Equifax Risk Score (hereafter Risk Score), which is a proprietary credit score similar to other risk scores used in the industry.

We choose 2004 as our baseline year. This is because this is the latest year for which the bankruptcy filing statistics are unaffected by the Bankruptcy Abuse Prevention and Consumer Protection Act of 2005 (which changed the eligibility requirements for a discharge in ways we do not model in this paper). To align with this choice, we use the 2004 SCF for our credit market moments and the data for 2004 from the CCP for the age profile of credit rankings. For the autocorrelation of year-to-year changes in credit rankings we use CCP data from 2003, 2004 and 2005.

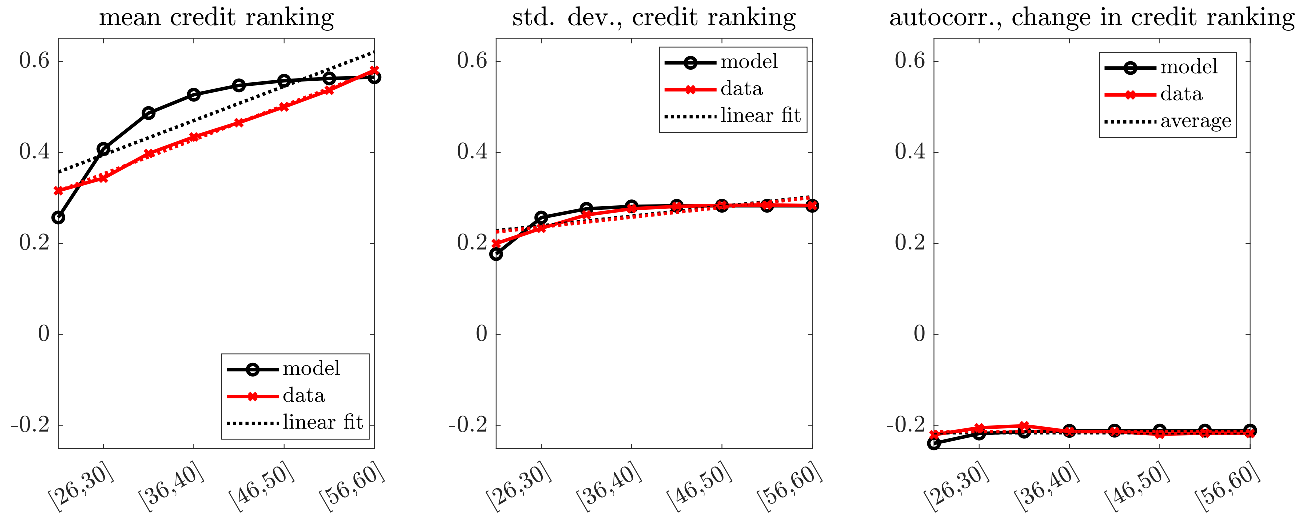
In the SCF, we focus on the subset of households with heads between the ages of 20 and 60 years excluding the top 5% of the wealth distribution, for whom we think our theory is not relevant. The fraction of indebted households is the fraction of such households with negative net worth. The average debt-to-income ratio is the ratio of total unsecured debt of indebted households to 2004 per household U.S. GDP. For the mean and standard deviation of interest rates we used the interest rates reported on unsecured debt by all households with negative net worth.

The bankruptcy rate is the ratio of the total number of nonbusiness Chapter 7 filings in 2004 reported by the U.S. Bankruptcy Courts, scaled by the total number of U.S. households in 2004.

While the previous credit market data targets have been used in numerous quantitative theory papers on bankruptcy, what is novel is our use of the age profile of Risk Score moments. For this we use a 2% sample (approximately 150,000 observations) of the FRBNY CCP/Equifax panel (described in detail in Online Appendix C). With this data, we create credit rankings, defined as the percentile ranking of an

[26] The age profile of the autocorrelation of credit ranking changes showed no significant slope in the data, so we only target the intercept.

Figure 1: Credit Ranking Age Profile: Model vs. Data



Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data, whose construction is detailed in Online Appendix C. The linear approximation to the model-generated credit ranking age profiles uses the regression coefficients from Table 2.

individual's Risk Score relative to the overall sample distribution of Risk Scores, and group individuals between the ages of 21 and 60 in 5 year bins. We then compute the means and standard deviations of credit rankings within each age bin. With these age bin data values, we estimate affine age profiles for means, standard deviations. To compute autocorrelations of year-to-year changes in credit rankings, we create credit rankings for 2003 and 2005 for each individual. We place individuals in the same 5-year age bins and compute the correlation between the change in rankings between 2003 and 2004 and between 2004 and 2005 for each age bin.[27] Figure 1 shows the data and affine approximations to the data as well as the model generated data and approximation.

4.2.3 Estimation Strategy

To estimate the parameters, we proceed with a mixture of simulated method of moments for the aggregate statistics and indirect inference for the affine fit (intercept and slope coefficients) of the age profile of credit score means, standard deviations, and autocorrelations. Our system is overidentified (we have 8 parameters $\theta = (\beta_H, \beta_L, Q^\beta(L'|H), Q^\beta(H'|L), F_{\beta_H}, \kappa_1, \alpha, \lambda)$ and 10 moments (five from the credit market and five of the age profile of credit rankings) so not all moments will be exactly replicated. Specifically, the consistent estimated parameter values in Table 3 solve

$$\widehat{\theta} = \arg\min_{\theta} \widehat{g}'(\theta)\widehat{W}\widehat{g}(\theta) \tag{23}$$

[27]We do not use the slope of the age profile of the autocorrelation of year-to-year credit score changes as a target because it is zero, which makes matching the relative deviation between model and data a problem.

Table 2: Estimation Targets

Moment (%)		Data	Model
Aggregate credit market moments			
Bankruptcy rate	BR	1.00	1.02
Average interest rate	AI	11.9	11.5
Interest rate dispersion	SDI	7.00	7.08
Fraction of HH in debt	FID	7.92	9.16
Debt to income ratio	DTY	0.40	0.26
Credit ranking age profile moments			
Intercept, mean credit ranking	I:MCR	0.278	0.320
Slope, mean credit ranking	S:MCR	0.038	0.038
Intercept, std. dev. credit ranking	I:SDCR	0.215	0.218
Slope, std. dev. credit ranking	S:SDCR	0.011	0.011
Average autocorrelation of credit ranking changes	AUTO	-0.220	-0.215
Sum of squared errors			
Aggregate credit market moments			0.144
Credit ranking age profile moments			0.023
Total			0.167

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. The sum of squared errors are computed in percentage deviation terms to control for relative magnitudes of moments, each receiving equal weight.

where $\widehat{g}(\theta) = (\widehat{S} - S(\theta))$ is the (percentage) difference between data $\widehat{S}$ and model $S(\theta)$ moments and $\widehat{W}$ is a weighting matrix. In our base estimation, we use an identity weighting matrix which we discuss further in Section 4.2.5.[28]

4.2.4 Parameter Estimates

The values of the data moments and their model counterparts, as well as the average mean square errors (both total and for each of the two blocks of moments), are reported in Table 2 and Figure 1.

[28]For more details on computation and estimation see Online Appendix B.1.

Table 3: Parameters Chosen Within the Model

Parameter		Value
Evolution of types		
High discount factor	β_H	0.930
Low discount factor	β_L	0.809
High to low β transition	$Q^\beta(L'\|H)$	0.226
Low to high β transition	$Q^\beta(H'\|L)$	0.205
Fraction high β at birth	G_{β_H}	0.318
Proportional default cost	κ_1	0.067
Extreme value parameters		
Scale in $F_\nu(\nu;\alpha)$	α	0.029
Scale in $F_\epsilon(\epsilon;\lambda)$	λ	0.002

The estimated values for patience are $\beta \in \{0.809, 0.930\}$, so that low types have a 13% lower discount factor than high types.[29] This differential allows reputation acquisition to play a role in equilibrium: types are far enough apart to want to behave differently, but close enough that imitation is not too costly when individuals start with low earnings and zero assets, i.e., when they are young. This is explained in more detail in Section 5.3.

The estimated transition matrix Q^β implies an ergodic distribution featuring 48% high types, but the life-cycle demographic structure implies a slightly lower stationary fraction equal to 47%.[30] There is demographic improvement in the average assessment of an individual's type (and hence creditworthiness) over one's lifetime: agents' initial type scores are consistent with the estimated initial fraction of high types, $F_\beta = 32\%$, with the average assessment updating via $\overline{s}' = \overline{s} \cdot Q^\beta(H|H) + (1-\overline{s}) \cdot Q^\beta(H|L)$ which converges to 48%. This is independent of alternative credit arrangements we consider. The fact that F_β is estimated to be below the stationary fraction of high types is a robust consequence of the rising average credit score over the age profile.

The estimated earnings loss from filing for bankruptcy is 6.7 percent of the persistent component of

[29]Our annual estimates of discount factors translate into quarterly values $\beta_H = 0.982$ and $\beta_L = 0.948$.

[30]The fraction of type H in the stationary distribution (call it μ_H) solves $\mu_H = \rho \cdot \left[(1-Q^\beta(L'|H))\mu_H + Q^\beta(H'|L)(1-\mu_H)\right] + (1-\rho)\cdot F_{\beta_H}$ or $\mu_H = \frac{\rho Q^\beta(H'|L)+(1-\rho)F_{\beta_H}}{1-\rho(1-Q^\beta(L'|H))+\rho Q^\beta(H'|L)}$. For our estimated parameter values in Table 3, $\mu_H = 0.47$. In a cohort, the fraction of type H asymptotes to 0.48 (the value of μ_H corresponding to $\rho = 1$).

earnings. This relatively large estimated cost indicates that there are costs to bankruptcy that are not captured by the loss of reputation in unsecured credit markets only. For example, reputation loss can impact one's job finding prospects, secured borrowing costs like mortgages, and even insurance premia. For reasons of parsimony, these other channels are captured by our estimate of κ_1.

Our estimates of the parameters α and λ of the extreme value distributions imply that there is more noise in the bankruptcy decision than in asset choices, but not so much that fundamentals are overridden (i.e. fundamental heterogeneity in unobservable type and earnings are the key drivers of default and asset choice). One measure of the size of the extreme value shocks is how noisy consumption decisions are; at an individual level, the variance of consumption decisions, conditional on state (β, z, ω), is zero without the extreme value shocks. In our model, the average coefficient of variation of consumption across all agents is only modestly higher at 2.03%.[31] This is especially true for the ϵ shock process associated with the asset choice decision where the share of total borrowing and saving actions by "modal agents" (i.e. those for whom an action in the set under consideration is the mode) is 85.6% and 99.9%, respectively.[32] On the other hand, the default/no-default action is associated with more variability in the shock process ν where the share of modal defaulters is only 5.25%. This can be explained by the fact that our parsimonious ν shock process is capturing other unobservable factors behind the default decision not included in our model (for example, Chatterjee et al. (2007) include other shocks to capture medical expenses and lawsuits which survey respondents cited as reasons for filing for bankruptcy).

4.2.5 Sensitivity Analysis

In lieu of standard errors, we provide a measure of the sensitivity of our parameter estimates to the moments of the data using the local methods in Andrews et al. (2017). Table 4 presents a version of their sensitivity measure Λ applied to classical minimum distance estimation:

$$\Lambda = -\left(G'WG\right)^{-1} G'W \tag{24}$$

where $G = \mathbb{E}\left[\nabla_\theta \widehat{g}(\theta)\right]$ is the 10×8 probability limit of the Jacobian and W is the probability limit of the weighting matrix, which we have simply taken to be the identity matrix. Λ measures how sensitive the parameter estimates in Table 3 are to local perturbations of the data moments. Further, there is a tight connection between Λ and standard errors in GMM/SMM. Specifically, given (24), the limiting

[31] At each point in the state space, we compute the standard deviation and mean of consumption implied by the decision rule σ. We then take the ratio of these numbers at each point and average over the stationary distribution.

[32] See Online Appendix B.5 for a description of these calculations (in particular equation (41)).

Table 4: Sensitivity Analysis: Implied Percentage Change in Parameter given 1% Change in Empirical Moment

	BR	AI	SDI	FID	DTY	I:MCR	S:MCR	I:SDCR	S:SDCR	AUTO
β_H	0.00	-0.08	-0.06	-0.08	0.09	-0.12	-0.12	-0.11	-0.04	0.02
β_L	0.34	-0.11	0.02	-0.04	-0.21	0.51	0.44	0.47	0.13	-0.02
$Q^{\beta}(L'\|H)$	-0.47	0.93	0.36	2.33	-2.67	23.42	14.19	19.71	4.29	0.65
$Q^{\beta}(H'\|L)$	-5.83	6.06	0.54	3.12	2.44	5.57	-0.03	2.60	0.00	0.71
G_{β_H}	-2.49	2.45	0.06	1.24	1.39	-10.66	-0.69	7.17	-0.29	0.39
κ_1	0.17	-0.19	-0.03	-0.06	0.01	-0.64	-0.43	-0.88	-0.13	0.01
α	-0.35	-0.22	-0.03	0.48	0.03	-0.59	-0.43	-0.77	-0.13	0.02
λ	7.22	-6.67	-0.22	-7.07	2.02	-3.32	-3.46	-5.06	-1.03	0.08

Notes: Each entry corresponds to the implied percentage change in the estimated parameter in the row associated with a 1% change in the indicated empirical moment in the column. Abbreviations for moments are available in Table 2. All numbers are reported in percentage points, i.e. the 0.00 (which when expanded is actually 0.0011%) in the top left cell implies that if the bankruptcy rate were 1% higher, our estimate of β_H would increase by 0.0011%, from 0.930 to 0.931. BR: bankruptcy rate; AI: interest rate average; SDI interest rate disperion; FID: fraction of households in net debt; DTY: debt-to-income ratio; I:MCR, S:MCR: intercept and slope of the mean of credit rankings across the age profile; I:SDCR and S:SDCR: intercept and slope of the standard deviation of credit rankings across the age profile; AUTO: mean of the autocorrelation of credit ranking changes.

distribution of the estimates can be written

$$\sqrt{T}\left(\widehat{\theta} - \theta_0\right) \xrightarrow{d} \mathcal{N}\left[0, \Lambda\Omega\Lambda'\right] \tag{25}$$

where $\Omega = \mathbb{E}\left[g(\theta)g(\theta)'\right]$ is the limiting variance-covariance matrix of the data moments, θ_0 is the true parameter value, and T is sample size. For a given Ω, (25) makes clear that small values of Λ are associated with more precise parameter estimates. Since there are several moments where there is effectively no sample variation in the data (i.e. the 2004 bankruptcy rate is the population moment and the credit ranking statistics from the 2004 FRBNY CCP/Equifax essentially comprise the population moment due to its large sample size) and we are drawing from very different data sources, some elements (e.g. zeros) of the estimate of Ω are hard to interpret. Hence, in lieu of standard errors in Table 3, we instead focus on Λ in Table 4.[33]

Looking across the rows of Table 4, there are several parameters (β_H, β_L, κ_1, and α) which appear to be relatively insensitive to a 1% change in our moments.[34] In contrast, parameters like $Q^{\beta}(L'|H)$,

[33]Section B.3 of Appendix B provides a detailed explanation of our implementation of Andrews et al. (2017).

[34]Our sensitivity numbers are not invariant to the scaling of the parameters. The most natural scaling would be with

Table 5: Jacobian Analysis:
(Numerical) Derivative of Target Moments with Respect to Estimated Parameters, $\hat{G}'$

	BR	AI	SDI	FID	DTY	I:MCR	S:MCR	I:SDCR	S:SDCR	AUTO
β_H	-3.03	-19.6	151	-20.8	-0.65	-0.33	0.07	-0.33	0.05	1.02
β_L	-8.80	-30.9	59.7	-64.1	-2.05	-0.27	0.05	-0.32	0.05	2.09
$Q^{\beta}(L'\|H)$	0.81	4.29	-6.47	5.79	0.18	0.06	-0.02	0.07	-0.02	-0.71
$Q^{\beta}(H'\|L)$	-1.11	-6.24	-1.53	-7.59	-0.24	-0.10	0.02	-0.07	0.02	-0.22
G_{β_H}	-0.05	0.11	-2.49	-0.24	-0.01	0.06	-0.01	-0.02	0.00	-0.28
κ_1	7.90	-209	-195	215	10.1	5.74	-1.15	7.83	-1.35	-58.6
α	-24.2	536	-104	-628	-20.6	-2.66	0.54	-3.31	0.54	19.0
λ	-15.5	846	-1802	8.38	-19.5	-5.07	1.09	-6.66	1.36	-25.5

Notes: Each entry is the numerical derivative of the target moment (column) with respect to a 0.1% change in the corresponding parameter (row). BR: bankruptcy rate; AI: interest rate average; SDI interest rate disperion; FID: fraction of households in net debt; DTY: debt-to-income ratio; I:MCR, S:MCR: intercept and slope of the mean of credit rankings across the age profile; I:SDCR and S:SDCR: intercept and slope of the standard deviation of credit rankings across the age profile; AUTO: mean of the autocorrelation of credit ranking changes.

$Q^{\beta}(H'|L)$, and F_{β_H} appear to be sensitive to moments like the intercept of the age-profile of mean credit rankings (I:MCR). This suggests that if we had parameterized the age-profile of credit rankings differently (say with a quadratic instead of affine function), the estimate of these parameters might be affected. The fact that these parameters are all jointly sensitive to the intercept of the mean credit ranking profile is related to the fact that the affine function (intercept and slope) depends on the transition of high types across age bins which depends explicitly on all those parameters (see footnote 30). There also appears to be sensitivity in the estimate of the variance of extreme value shocks associated with asset choices with respect to several of the moments but this may be related to our very small estimate of λ in Table 3.

4.2.6 Jacobian Matrix

The transformation of the estimated sensitivity matrix $\hat{\Lambda}$ presented in Table 4 is useful for thinking about which moments of the data drive our parameter estimates. The estimated Jacobian matrix $\hat{G}$ in Table 5 is an essential input into that analysis (per (24)), while also containing useful information on its own for understanding how model parameters drive model moments.

We start by thinking about implications for the aggregate credit market moments (five leftmost

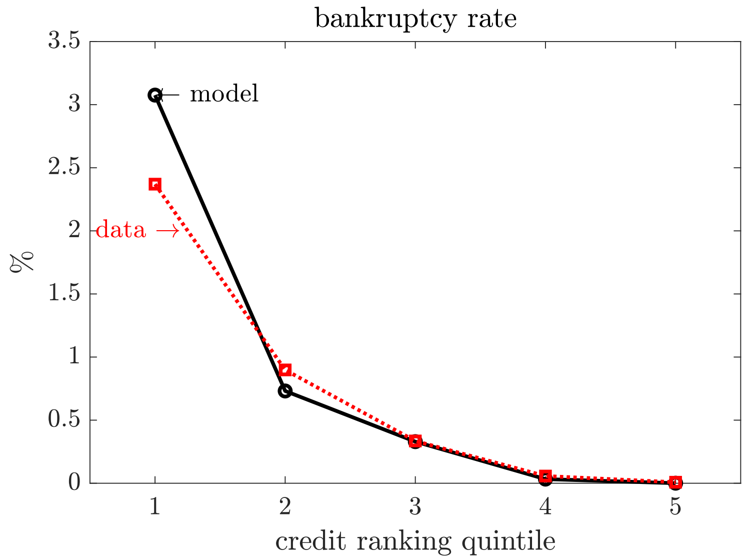
respect to Ω, but we do not pursue this here given the issues with estimation of Ω discussed above.

columns of the table). If either the high or low β type becomes more patient, there is less borrowing (both the fraction in default and the debt to income ratio drop) which leads to less bankruptcy and lower average interest rates. If the transition from type H to L (L to H) rises, leading to more (less) L types on average, there is more (less) borrowing, higher (lower) interest rates, and higher (lower) bankruptcy rates. A higher initial fraction of high types yields less borrowing and fewer bankruptcies. Finally, a higher variable filing cost (i.e. a higher punishment to filing for bankruptcy) yields much lower interest rates, inducing more borrowing and ultimately leading to more bankruptcy in equilibrium.

Increasing either α or λ increases the noisiness of the associated decision: bankruptcy vs. no bankruptcy and the choice of a', respectively. Increasing either parameter clouds lenders' ability to infer types based on actions. Consider bankruptcy (α) first. Since bankruptcy is generally not optimal for most borrowers, raising α increases the rate at which borrowers file all else equal. In equilibrium, though, this leads to a surge in interest rates across all loans and a sharp decline in both the share of borrowers and the overall amount of borrowing, ultimately lowering the bankruptcy rate. Next, consider borrowing and saving (λ). Raising λ increases the likelihood with which households will deviate from their "optimal choices" or, put differently, be "off their Euler equations." The first order effect of this is that households are much more willing to take on small debts which carry non-trivial interest rates due to default risk, driving up the fraction in debt and the average interest rate. At the same time, though, since debt to income falls, the total volume of debt decreases, lowering the bankruptcy rate in equilibrium.

Finally, consider the credit ranking life cycle moments (five rightmost columns of the table). In general, these moments are less sensitive to our target parameters than the aggregate credit market moments because: (i) a sizable portion of the life cycle of credit rankings is driven by the exogenous life cycle of earnings; and (ii) the endogenous upward trend in types is not changed much in a neighborhood of our initial parameters. This second point, in particular, explains the relatively small magnitudes in the first five rows and last five columns of Table 5. Turning to the final three rows, then, we can highlight several intuitive patterns. Increasing the variable filing cost makes bankruptcy less attractive for the young, increasing their credit ranking at birth (i.e. I:MCR increases) but lowering the upward trend in credit ranking (i.e. S:MCR decreases). Since default becomes a clearer signal of type, then, we see a similar pattern for the cross-sectional variation of credit ranking. Raising either α or λ has the opposite effect: by driving up the incentive to file or borrow when young, it lowers credit rankings at birth and increases the upward trend in mean credit rankings and by lowering the informational content of decisions it lowers variation in credit rankings at birth (consistent with more pooling).

Figure 2: Bankruptcy Rate by Credit Ranking Quintiles



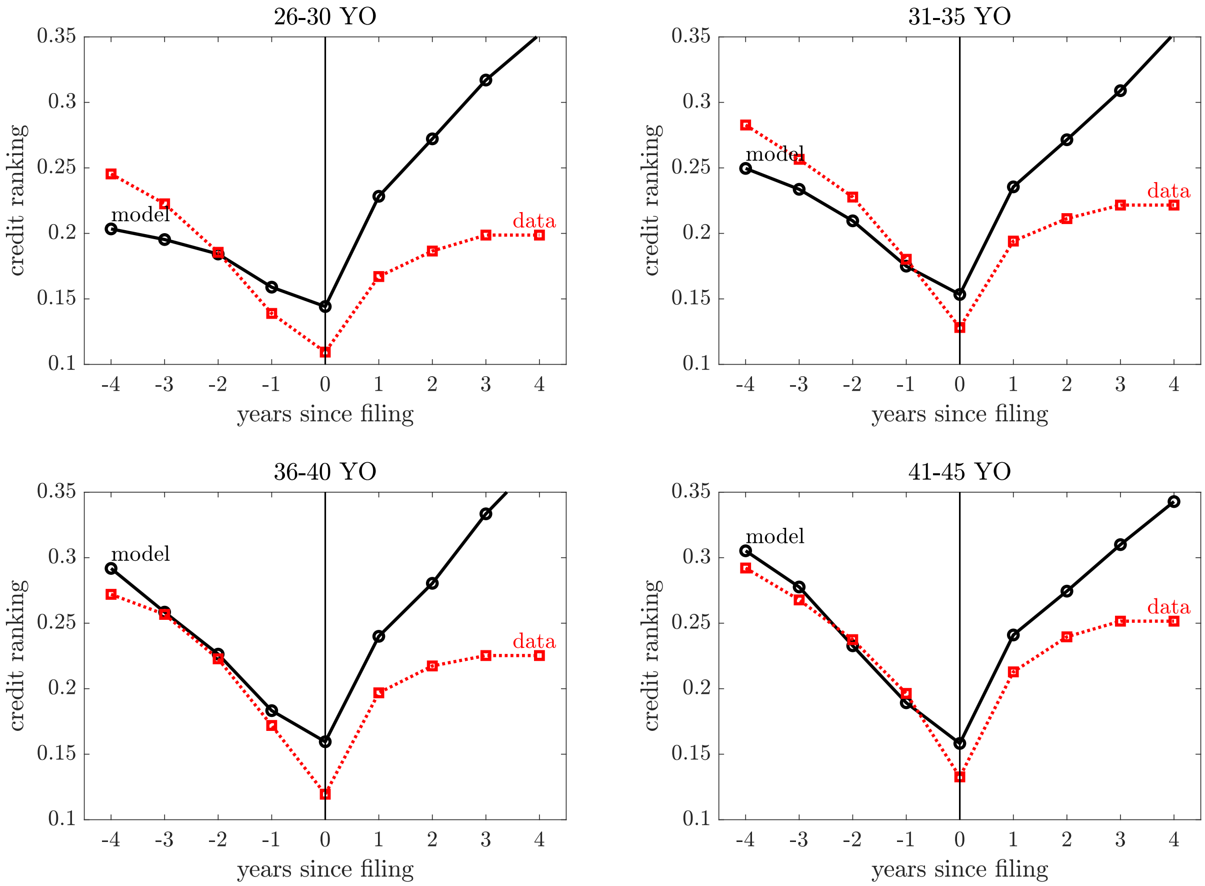
Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data.

4.3 Model Fit: Credit Rankings and Bankruptcy Filing

We next assess how the model performs relative to certain non-targeted properties in the data. Figure 2 shows the non-targeted bankruptcy rate by credit ranking quintiles in the data and in the model. As in the data, the model generates high filing rates among individuals with low credit rankings and low filing rates among individuals with high credit rankings. The model replicates the decreasing pattern in the data.

Another key property of real world risk scores is that they fall upon bankruptcy and mean revert. We illustrate this property for credit rankings in both the data and the model in Figure 3. Specifically, we conduct an event study of the average change in credit rankings around a bankruptcy filing for various age bins. While the model underpredicts the fall in credit rankings for younger cohorts and overpredicts the long run recovery in credit rankings, it does remarkably well in matching the rank at the time of filing and the patterns we see in the data despite not being targeted in our estimation.

Figure 3: Event Study: Credit Rankings around Bankruptcy Filings by Age



Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. Model results are obtained by simulating a panel of 10, 000 individuals for 1, 000 periods and dropping the first 100 periods. Bankruptcies are then isolated, and each data point reported represents the mean of credit rankings across all bankruptcies for the given lead or lag from the date of the bankruptcy (normalized to 0). The results are binned by 5-year age groups consistent with our earlier results.

5 Model Mechanics

5.1 Choice Mechanics

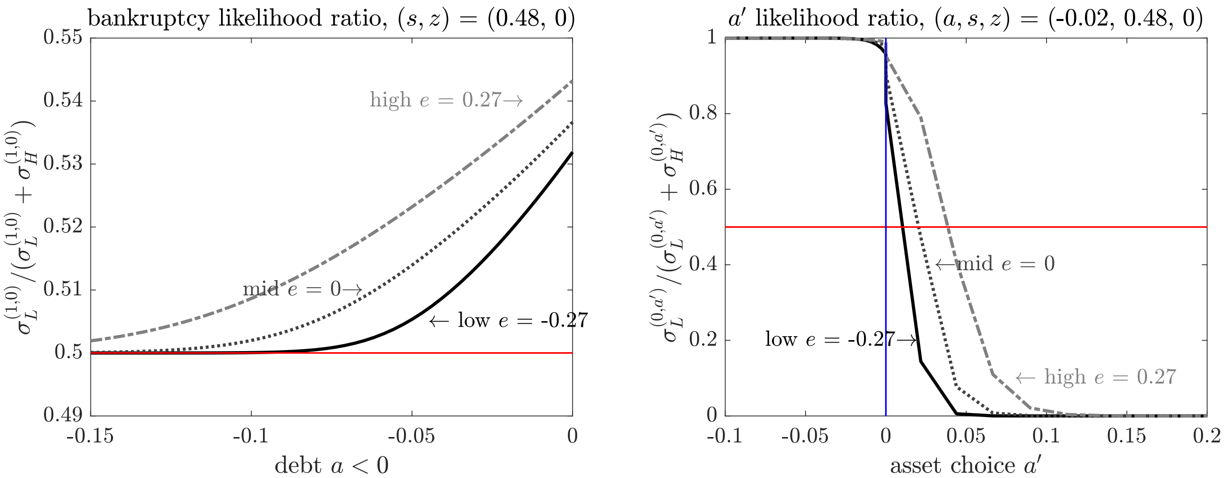
The workings of our model depend on differences in patience among types, as well as their earnings. Figure 4 uses likelihood ratios to illustrate the fact that patience matters in the simple sense that agents of different types take different actions.[35] The left panel shows the bankruptcy likelihood ratio across different persistent earnings levels as a function of debt. While the low type is more likely than

[35] Here we define the likelihood ratio of an action (d, a') as the type β_L choice probability relative to the sum of the two choice probabilities. That is,

$$\frac{\sigma^{(d,a')}(\beta_L, z, \omega)}{\sigma^{(d,a')}(\beta_L, z, \omega) + \sigma^{(d,a')}(\beta_H, z, \omega)}$$

which lies in $[0, 1]$. For an action which is uninformative about an agent's type, this ratio is 0.5.

Figure 4: Likelihood Ratios of Default and Borrowing/Saving Decisions



the high type to file for bankruptcy for all earnings levels, the fact that the likelihood ratio increases in earnings indicates the importance of differences in type. While both types have the same current gain from default, type β_H cares more about the future consequences of a drop in type scores which disincentivizes her from filing for bankruptcy. Further, for any given earnings level, the difference in bankruptcy probability across types decline as debt increases: as debt increases, the current gain from bankruptcy rises enough to offset the future consequences of a drop in one's type score.

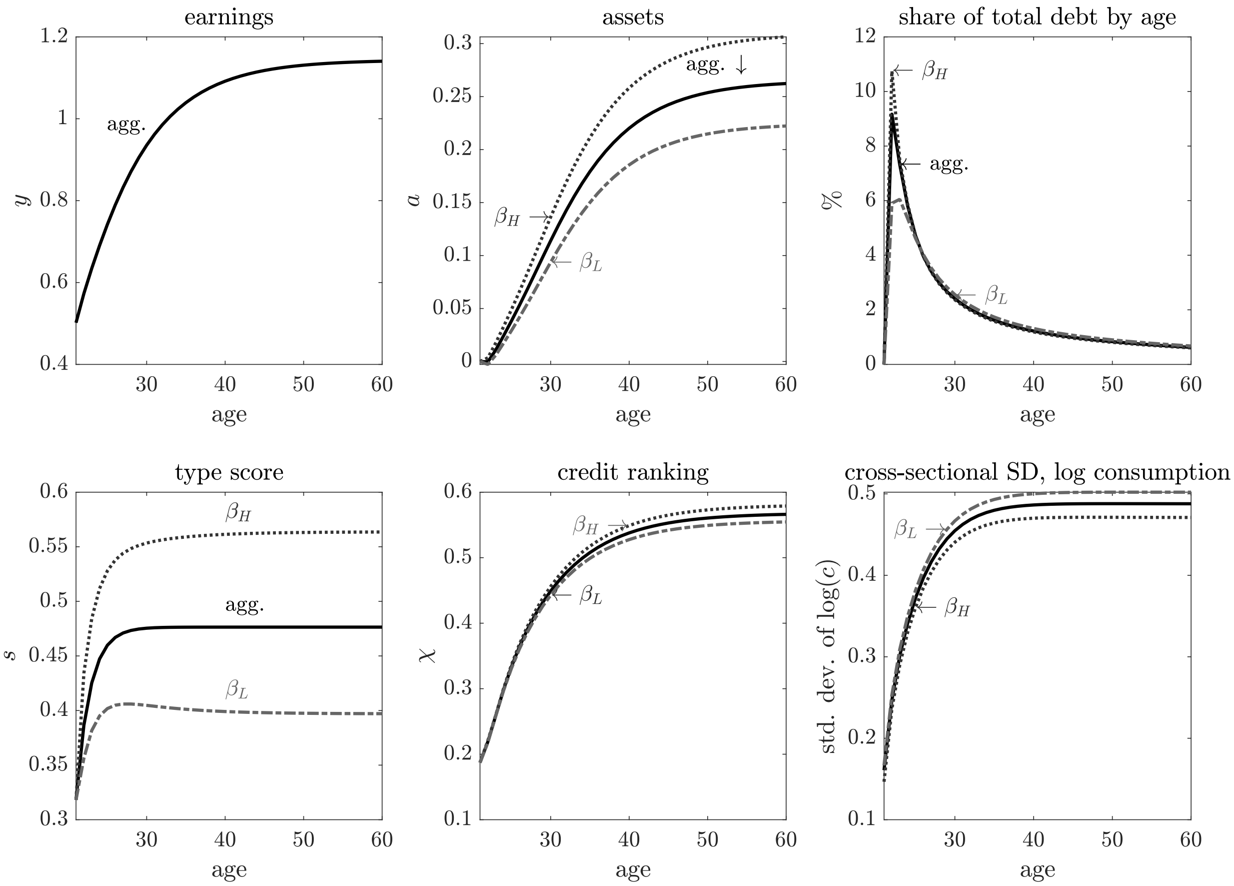
The right panel of Figure 4 shows likelihood ratios associated with asset choices. Type β_L borrows much more frequently than type β_H, regardless of earnings. Regarding savings, type β_L saves more frequently than type β_H (the likelihood ratio is greater than 0.5) for small levels of savings but saves less frequently for higher levels of savings. Furthermore, the level of savings beyond which type β_L saves less frequently than type β_H is increasing in earnings. These properties are consistent with type β_L tending to choose lower values of a' (i.e., higher values of current consumption) than type β_H in the same circumstances.

As is a feature of many default models, the probability of bankruptcy is increasing in debt and decreasing in earnings for those with sufficiently large debt.[36] The latter implies that, all else equal, realizations of an individual's earnings have important implications for credit rankings as evident in equation (17). This highlights the composite nature of credit scores: they depend on earnings, asset positions, and type scores.

While age is not a state variable in the decision problems of individuals and lenders, demographics

[36] We document these facts in the supplementary materials to this article. For very small debt, however, the lowest earners (who have the highest marginal utility of consumption) are least likely to default in order to avoid bearing the costs (κ_0 and $\kappa_1 \times \exp(e)$) of bankruptcy.

Figure 5: Average Moments by Age and Type in Baseline Model



Notes: In each panel, each line corresponds to the average moment indicated at the specified age in the baseline model. For the type-specific measures, the average is computed conditional on type. The share of total debt by age is the share of economy-wide debt across all ages for the indicated type accounted for by agents of that type at that age. For example, high-β 22 year-olds account for 10.6% of the debt held by high β agents.

play a role through how we model the arrival of newborns and the Markov process for hidden type. Specifically, since all newborns begin with the lowest earnings class, our Markov process for earnings implies that earnings are expected to rise through an individual's life as shown in the top left panel of Figure 5. The earnings profile induces an increasing wealth profile in the top center panel. Given the rising earnings profile, the young do the lion's share of borrowing as evident in the top right panel.[37] Our estimates of F_β and Q^β from Section 4 imply that the fraction of type H newborns is lower than the long run fraction of type H. This implies that average type score rises with age according to $\bar{s}' = \bar{s} \cdot Q^\beta(H|H) + (1 - \bar{s}) \cdot Q^\beta(H|L)$, documented in the bottom left panel of Figure 5. The age profile for types scores induces a similar ordering for credit rankings in the bottom left panel. Finally, the bottom right panel shows that the "within-group" variance of consumption is higher for type L

[37] Given that economy-wide debt across all ages by type H is lower than that for type L (i.e. the denominator of the share), type H have a higher share than type L when each are poor, which reverses as type H accumulate more savings through time.

than type H, consistent with more precautionary saving by (top right panel) and greater credit access (bottom center panel) for type H.[38] A notable feature of the bottom right panel of Figure 5 is that the cross-sectional variance of consumption grows in early life, consistent with the empirical evidence in Figure 14 of Heathcote et al. (2010).

5.2 Scoring Mechanics

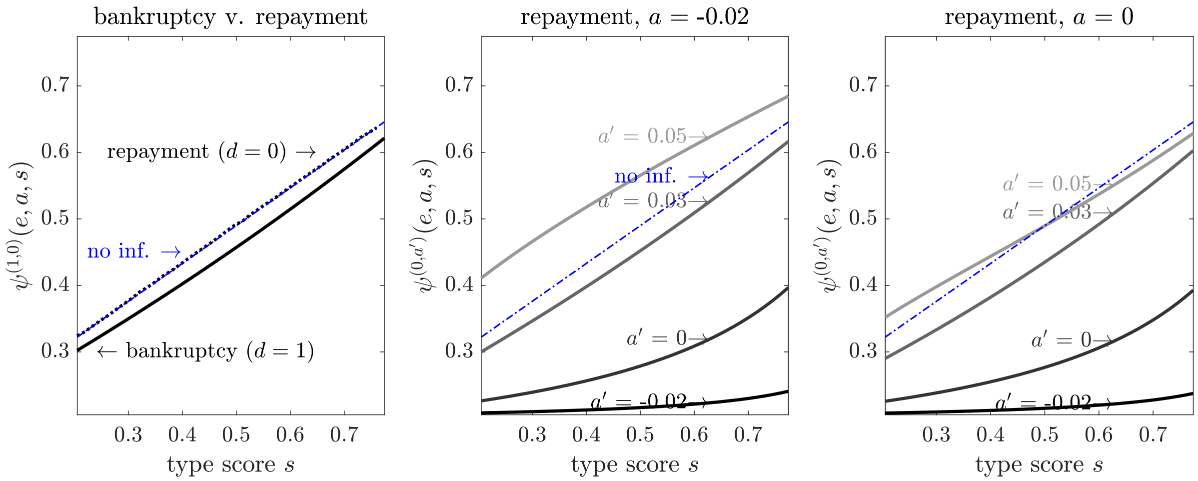
Figure 6 plots the change in the public assessment of an individual's type resulting from Bayesian updating given her current type score and actions (i.e. $\psi^{(d,a')}(e,a,s)$ in (16)). Because our estimates exhibit non-zero off-diagonal elements of Q^{β}, an individual's type can switch from one period to the next even if their action reveals themselves to be one type or another. Thus, the domain of the type scoring function in Figure 6 lies between ($s = 0 + Q^{\beta}(H'|L) = 0.205$ and $s = 1 - Q^{\beta}(L'|H)) = 0.774$). The left plot shows the different updates for bankruptcy filers and non-filers for $a = -0.02$ integrated over earnings and all a' choices in the case of non-filers. It also plots the posterior of an agent's type even if their actions are not observed, which we call "no inference."[39] The mean reversion in type score accounts for why a person's score falls upon repayment if the current score is sufficiently high or rises upon default if it is sufficiently low. Still, it remains true that repaying leads to a higher type score than filing for bankruptcy. Since the ν shock is noisy, the choice to not file does not reveal much; thus, the no bankruptcy line is only imperceptibly higher than no inference line in the left panel of Figure 6.

The center and right plots of Figure 6 show the Bayesian updates that result from different actions taken by an individual either already in debt (center at $a = -0.02$) or with zero assets (right) for the median earner ($e = z = 0$). For an individual already in debt, staying in debt (i.e. $a' = -0.02$) signals the individual is likely to be type β_L as we saw in Figure 4 leading to a drop in their posterior score. As the individual chooses higher a' their posterior rises. It is not until sufficiently high savings choices (e.g. $a' = 0.05$) that there is enough separation to raise an individual's posterior higher than what would be associated with mean reversion only (i.e. no inference). The right panel documents that starting from a higher asset position ($a = 0$), all the assessments shift down; that is, the smaller net change in asset position makes the inference less likely to be a high type.

[38] Krueger and Perri (2006) term "across-group" variation owing to observable differences like education and "within-group" variation the residual which includes idiosyncratic income. Here we are grouping people on observables like age and also unobservables like type.

[39] One might expect to compare the type scoring function to the 45 degree line to see whether the agent's reputation improves or deteriorates. However, given the upward trend in mean type scores implied by the discount factor process, it is more natural to compare to the no inference line.

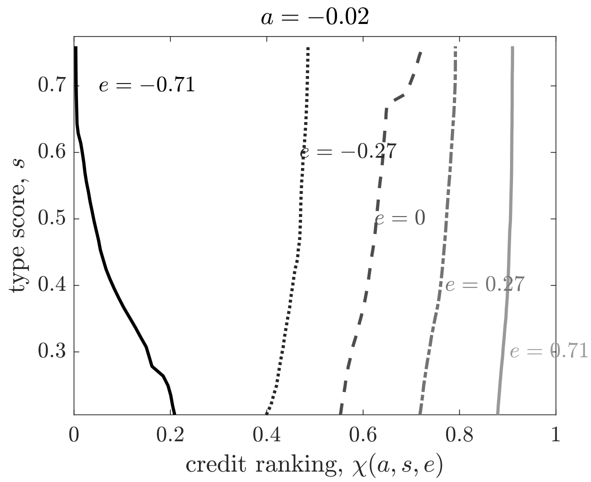
Figure 6: Type Score Responses



Notes: In each panel, each line is the implied type score update (ψ, y-axis) given the current type score (s, x-axis) for the indicated action. For the left panel, the actions considered are repayment and bankruptcy, conditional on having debt $a = -0.02$. For the middle and right panels, we consider 4 non-bankruptcy actions, $a' \in \{-0.02, 0, 0.03, 0.05\}$. This shows how different choices affect the type score update for two different levels of wealth, $a = -0.02$ (middle panel) $a = 0$ (right panel). In each panel, the blue "no inference" line corresponds to the type score update the borrower would receive just based on the upward drift in β over age.

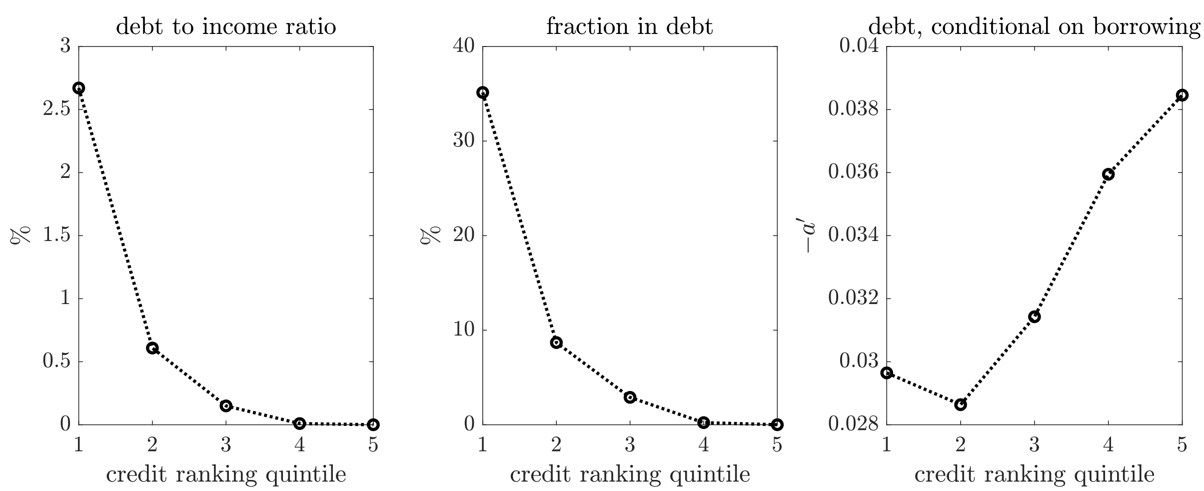
Figure 7 illustrates some important points. First, the figure verifies a form of the sufficient condition (one-to-one mapping between type scores and credit scores conditional on persistent earnings and assets) in Theorem 3 that establishes the equivalance between the fundamental type score equilibrium (RCE) and the credit score equilibrium (RCECS). Specifically, we graph the inverse function since Theorem 3 assumes that the inverse function $s = \left(p^{\bar{a}*}\right)^{-1}(e, a, m)$ exists. This graph is indeed one-to-one, and we have verified that the function is one-to-one conditional on observables across the entire state space. Second it shows the important effect of earnings on credit rankings; higher earnings are correlated with higher rankings. Third, higher beginning of period assets are not necessarily correlated with higher rankings (e.g. for $e = 0.71$, borrowing $\bar{a}$ from a higher initial asset holding lowers one's credit ranking). Finally, while higher type scores are associated with higher credit rankings for most earnings levels, it is not true when earnings are very low. In this case, type β_H actually file for bankruptcy slightly more frequently than type β_L generating the negative relation between type score and credit ranking. At low earnings, it may actually be optimal to borrow rather than default (see Figure 1 in Chatterjee et al. (2007)). In our current case where we have different types, the gain from borrowing is stronger for type β_L than for β_H since they care less about the relative drop in their future type score.

Figure 7: Credit Scoring Function



In Figure 8 we plot the cross-section of debt choices across credit ranking quintiles (behind their aggregate counterparts in Table 2). The figure illustrates that borrowers with low credit rankings are more likely to be in debt and have high debt-to-income ratios. It also illustrates that conditional on actually borrowing, those with high credit rankings tend to borrow more (since they can do so at lower interest rates). These relations about debt, income, and credit status are consistent with the empirical findings in Diaz-Gimenez et al. (2011) [Table 17, p. 19].

Figure 8: Outcomes by Credit Ranking Quintiles



Notes: Average moments are computed as the average conditional on credit ranking quintile. The debt to income ratio is the average of individual debt to individual income across the population. The size of debt conditional on borrowing averages across all choices made by each agent in a given state.

5.3 Selection and Reputation Effects

We now consider how one's current asset choice affects the price they face today via revelation about the individual's unobservable type. If an individual of observable type $\omega = (e, a, s)$ were to borrow a' she would be facing a price that depends on the default probabilities of her type tomorrow $\omega' = (e', a', s')$. Since a' is given at the beginning of the next period and does not affect the probability distribution of e', what matters is how a' affects the update s'. To isolate the contribution of the asset choice on the update, we compare the baseline equilibrium price which depends on the update through $Q^s(s'|\psi^{(d,a')})$ with a price schedule that results from excluding a' from the Bayesian updating formula. This price schedule is now given by $\widetilde{q}^{a'}(a, s, e) = \rho \widetilde{p}^{a'}(a, s, e)/(1 + r)$, where

$$\widetilde{p}^{a'}(a, s, e) = \sum_{\beta', e', z', s'} H(z')\ Q^e(e'|e)\ Q^s(s'(\beta')|\widetilde{s}')\ s'(\beta')\ \left[1 - \sigma^{(1,0)}(\beta', e', z', a', s')\right] \tag{26}$$

where $\widetilde{s}' = s \cdot Q^\beta(H|H) + (1 - s) \cdot Q^\beta(H|L)$ updates the prior s using only Q^β and ignores the information in the borrowing choice a'.

Figure 9 shows the percentage increase in $\widetilde{q}$ relative to the baseline equilibrium price q. Recall from Figure 4 that borrowing reveals oneself to very likely be type β_L and therefore when lenders cannot take this information into account, ignoring what can be learned from selection effects induces the price $\widetilde{q}$ to exceed q. The figure shows that if an individual starts with a low prior s (here we take the 10th percentile $s = 0.21$), the price effect is smaller than if the individual starts with a high prior s (here we take the 90th percentile $s = 0.76$). Further, the positive price effects for the two priors are amplified the more borrowing is undertaken. The latter effect arises since for higher debt levels the pool of borrowers will tend to contain less creditworthy type β_L - that is, more adverse selection.

In addition to the contemporaneous effects discussed above, asset choices can also have long lasting effects. This requires that: (i) prices depend on an individual's current reputation (i.e. type score) for a given current action; and (ii) her choice today affects her future reputation (and hence future prices). Having established the second condition in Figure 6, we next establish the first in Figure 10.

Specifically, Figure 10 plots the percentage change in debt prices that an individual with current type score s can obtain relative to a person with the highest type score $s = 0.77$ for two different debt choices. The fact that both lines are downward sloping establishes that type scores matter; for a given debt choice, a higher s fetches a higher price. The reason is simple; since β_H types repay with higher probability than the β_L types, there is information about the probability of repayment in the current type score (which, in turn, reflects the history of the individual's past actions). The fact that the line in

Figure 9: Static Effect of Borrowing Choice

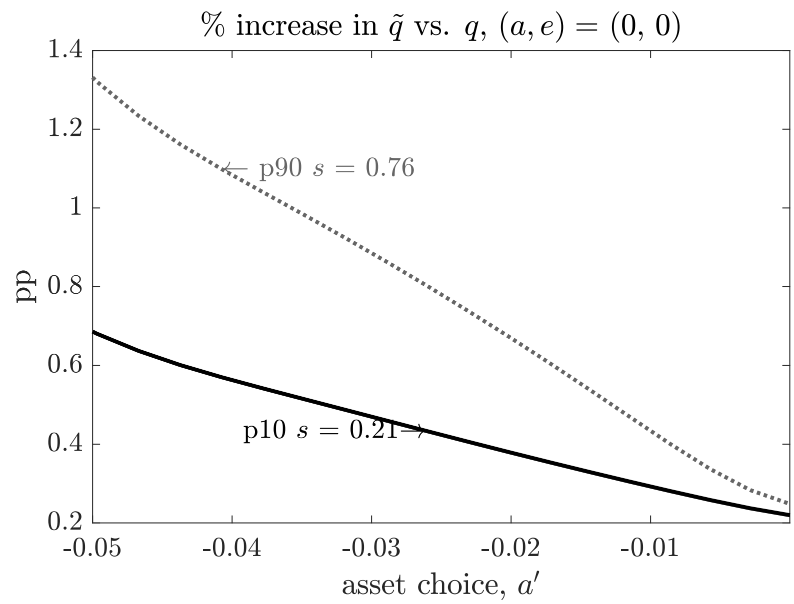
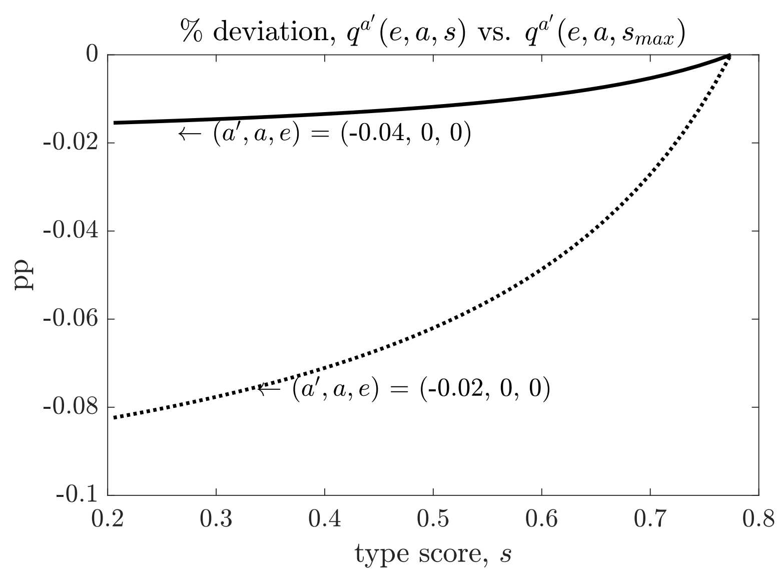


Figure 10 is less steep for large debts is consistent with Figure 4: type β_L are almost perfectly separated from type β_H at those debt levels, and so the prior s does not matter as much for assessing repayment probability.

Figure 10: Reputation and Prices



Finally, we examine signalling costs. In models with hidden types, the "bad" types have an incentive to imitate the "good" types in order to pool with them and obtain better terms of trade, while the "good" types have an incentive to separate themselves from the "bad" types to get even better terms. Here we assess the costliness for an impatient type L to imitate the actions of a patient type H.

Changing one's action has three effects: (i) a change in today's consumption; (ii) a change in tomorrow's net wealth; and (iii) a change in tomorrow's reputation. To explore these three effects for a type L to imitate a type H, we assume the type L follows the choice probability function $\sigma(\beta_H, z, \omega)$ instead of $\sigma(\beta_L, z, \omega)$. One measure of the consumption cost from a type L individual mimicking a type H is the average difference in consumption between H types and L types implied by the differences in their choice probabilities relative to the average consumption of a type L individual. Similar measures can be computed for next period net wealth and credit ranking.

Table 6 provides these calculations for our calibrated parameters (i.e. where $(\beta_H - \beta_L)/\beta_H = 13\%$). The table illustrates an important point. Type L newborns have a much lower consumption loss to mimicking a type H individual than their older counterparts. This is because the imitation costs are increasing in earnings and assets, both of which rise on average through one's life as evident in Figure 5. Since type H choose to save more, this imposes a bigger consumption loss to type L from mimicking later in life. Alternatively, it is easier to mimic when young, as the dispersion in assets and scores are lower in youth. The fact that it is less costly to mimic when young implies there is more pooling among the young and the fact that it is more costly to mimic when old implies there will be more separation among the old. The consequence is that while there is a bigger jump in credit ranking of β_L type when mimicking in old age, it is more costly to do so.

Table 6: Signaling Costs and Benefits

% Average Gain in:	**Consumption ($\widehat{C}$)**	**Wealth ($\widehat{A}$)**	**Credit Ranking($\widehat{\chi}$)**
All	-3.65	3.80	1.31
Newborns	-0.77	0.81	0.37

Notes: The first column measure is $\hat{C} = \frac{\sum_{z,\omega} \mu(\beta_L,z,\omega)\left[\sum_{(d,a')\in F(z,\omega)} \left(\sigma^{(d,a')}(\beta_H,z,\omega) - \sigma^{(d,a')}(\beta_L,z,\omega)\right) c^{(d,a')}(z,\omega)\right]}{\sum_{z,\omega} \mu(\beta_L,z,\omega)\left[\sum_{(d,a')\in F(z,\omega)} \sigma^{(d,a')}(\beta_L,z,\omega) c^{(d,a')}(z,\omega)\right]}$ while the second and third columns substitute $a'(z,\omega)$ and $\chi^{(0,\overline{a})}(\omega)$ for $c^{(d,a')}(z,\omega)$.

6 Impact of Alternative Information Structures

How important is the information structure for allocations? What would happen if society outlawed tracking of individual credit histories and with it the incentives to build a good reputation? Would the credit market shrink dramatically as the usefulness of maintaining a good reputation disappears? These are natural questions that we can answer quantitatively by using our model to compare outcomes in economies that differ from our baseline only in their information structures.

Before we get into the details of our answers to these questions it is important to keep in mind certain features of hidden information that are present in our model. First, because of imperfect separation, low types are subsidized by high types. Second, there are incentives to repay debt and to save more to imitate a high type. Third, there can be important interactions of hidden information across the age profile. Specific to our model, all newborns are low earners and face an (expected) upward sloping age-earnings profile. Thus, newborns and young have *a life-cycle reason to borrow* and so are more impacted by hidden information. Finally, individuals face idiosyncratic earnings shocks against which direct insurance is unavailable. Since borrowing to smooth consumption is costly in all the economies that we explore, all individuals have *a precautionary savings motive*. Differences between the economies imply not only that individuals behave differently on account of the incentives that they face, but also that the equilibrium prices reflect these changes. Accordingly we have to look at both aspects simultaneously.

6.1 Description of alternative economies

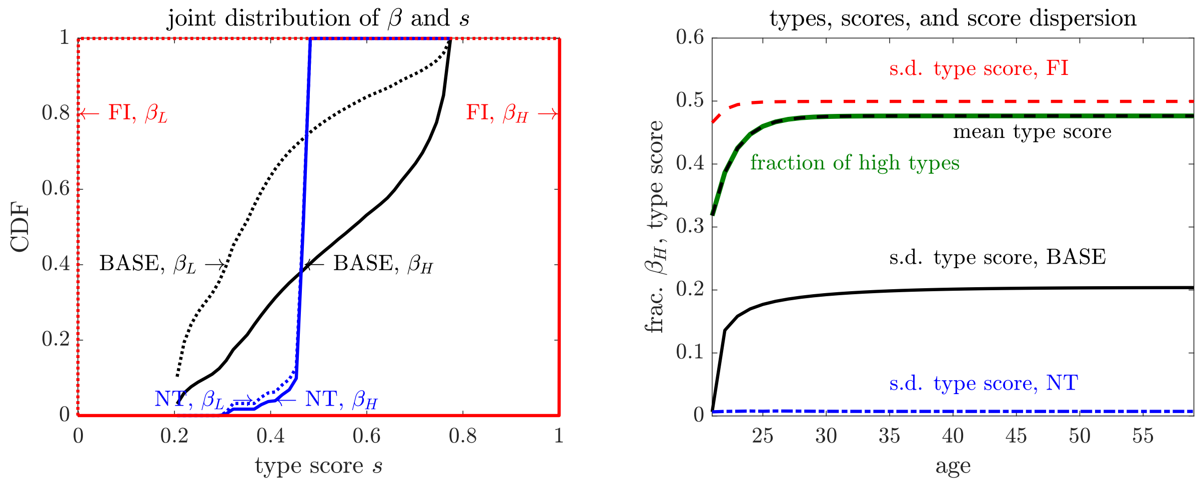
We now consider two stark alternative information structures in which reputation plays a limited role: one where past actions cannot be used to price discriminate but demographic drift can be used to infer type and another where type is public information.[40]

Our first alternative economy poses hidden information as in the baseline model (hereafter termed BASE), but prohibits creditors from using a person's past to price loans. We assume that the length of one's credit history (proxied here by age) is both publicly observable and legally used to price debt. This assumption isolates the role of reputation from the role of the demographic drift in the credit market. In this economy some information about the individual's type is learned contemporaneously from her asset choice, but this information is not carried across periods. We refer to this economy as the *no-tracking* (NT) economy since an individual's assets cannot tracked over time. To be concrete, in this alternative economy individuals' type scores initially equal the fraction of high types among newborns and evolve thereafter according to the demographic drift.[41] This implies a one-to-one mapping between an individual's age and the prior that she is a high type (her type score in this alternative economy). Consistent with no-tracking, lenders are also not allowed to use information about an individual's beginning-of-period asset holdings when pricing loans since this also contains information about her past actions. However, lenders are able to use the current action and the cross-sectional distribution of agents in the NT economy when forming a posterior about the likelihood of repayment necessary to price loans. The NT economy has hidden information and cross-subsidization, but there are no dynamic reputational incentives, as actions cannot be used to impute type as in the BASE economy.

[40] For the formal specification of these alternative economies, see Online Appendix B.6.

[41] As before, the evolution is simply given by $\overline{s}' = \overline{s} \cdot Q^{\beta}(H|H) + (1 - \overline{s}) \cdot Q^{\beta}(H|L)$ with initial condition $\overline{s} = 0.32$.

Figure 11: Evolution of Types and Type scores in Alternative Economies



Notes: The left panel plots the type-specific CDF of type scores in each model economy. Black / blue / red refer to BASE / NT / FI model economies, and solid (dashed) lines refer to high (low) types. The green and black dashed lines of the right panel correspond to the fraction of high types at the indicated age and the mean type score at each age in each economy. The three model variants considered have different type score standard deviations.

Our second alternative features *full information* (hereafter termed FI) where the type is directly observed by lenders that use it to price discriminate. Except for our extreme value shocks, this alternative economy is similar to Chatterjee et al. (2007). There is no need to infer a person's type, and the price for a loan of size a' depends directly on β and all other relevant observables. Importantly, prices do not depend on s nor a because they are not directly payoff relevant. Comparing the FI economy to the BASE reveals the full impact of hidden information: in the FI there is no cross-subsidization nor are there any incentives to imitate or separate.

Figure 11 highlights some of the main differences among the three economies. The left panel shows the CDFs of type scores for each type. This indicates the degree to which information about type is revealed to creditors. In the BASE economy, the L-type CDF rises steeply and fast, indicating that most type L individuals have low scores. In contrast, the H-type CDF rises more gradually, indicating that type H individuals have more dispersed type scores. In the NT economy, type scores (priors) are trapped between 0.32 (score at birth) and 0.48 (score at age infinity). Although people of all types share the same age-specific type score at each age, there are more type L individuals at each age than type H and, consequently, the CDF of low types rises somewhat faster. Most importantly, the CDFs are closest to each other for the NT economy, indicating that less is being learned about an individual's type as she ages, compared to the BASE and FI economies.

The right panel of Figure 11 plots the mean and standard deviation of type scores for each age across the alternative economies. Importantly, the mean type score at each age is correctly assessed in all economies to be equal to the fraction of high types. In the NT economy the standard deviation of type scores at all ages is zero, as nothing is learned.[42] For the FI economy, the dispersion in type scores at any age is the dispersion of types themselves in the economy. In the BASE economy it is increasing at a faster rate than in the FI economy, consistent with learning.

6.2 No-Tracking Economy Results

The key feature of NT is that the only exogenous information (earnings class and age-implied type score) can be used by lenders in the future. Thus there are no incentives to maintain one's reputation in asset markets. This can cause equilibrium price menus to drop, as is evident from the fact that average interest rates rise in Table 7.[43] In response to the rise in interest rates, the fraction of the population in debt falls. Since reputation effects are absent, though, those who choose to borrow are willing to do so at higher interest rates. Consequently, Table 7 shows that in equilibrium both the debt-to-income ratio and average interest rate rise. Higher debt in turn leads to an increase in the bankruptcy rate.

Table 7 also decomposes the aggregate statistics by unobservable type. It shows that type β_H borrowing and default is more sensitive to the change in incentives compared to type β_L. This arises due to the lack of persistent reputational costs incurred through borrowing and default, which were present in the baseline economy.

Regarding welfare, we focus on newborns.[44] Rising interest rates associated with the effect of eliminating incentives that rely on credit histories generally make newborns worse off. Only the type β_L newborns with lowest transitory earnings shock z benefit from the cross-subsidization that comes with no tracking. Since their gain is large (0.089) relative to the small losses in all other cases, the mean overall gain for newborns is positive (0.020).[45]

[42] Strictly speaking, because the evolution of s implied by the Markov type transition function for the NT economy typically does not yield scores which fall on the grid points in $\mathcal{S}$, there is some negligible dispersion in type scores.

[43] For an example of such a price menu, see Figure 13 in Online Appendix B.6.1.

[44] One consequence of focusing on newborns is that we do not need to compute a transition. At the moment of the policy switch, the average asset holdings of older cohorts are potentially different from those of the same age group in the steady state of the NT economy. Hence, even in a small open economy, all except the newborns face a transition of prices as the cross-sectional distribution used to infer future default probabilities evolves to the invariant distribution.

Besides this, there are multiple ways to think of how the switch from the BASE to NT would be implemented for people already alive. One possibility is to immediately outlaw the use of personal asset market history beyond the length of one's credit history (i.e. age). Alternatively, one could treat older individuals just like newborns, using information on their asset holdings and type score for the period of the policy switch but then knowledge about subsequent savings or defaults cannot be used. Hence, rather than make a choice on implementation, we focus on newborns.

[45] Our wealth equivalent welfare measure is standard; details are in the supplementary materials to this article.

Table 7: Comparison of Baseline, No Tracking, and Full Information Economies

economy	No Tracking (NT)			Full Information (FI)		
discount factor type	high	low	all	high	low	all
Panel A: % difference from BASE						
bankruptcy rate	1.40	0.95	1.12	-0.98	0.36	-0.13
average interest rate	2.02	1.02	1.44	-7.15	0.83	-2.52
interest rate dispersion	10.7	0.75	7.57	-5.12	0.71	-2.15
fraction in debt	-0.21	-0.13	-0.15	0.23	-0.10	0.00
debt-to-income ratio	0.45	0.34	0.39	-0.30	0.12	-0.04
Panel B: wealth equivalent welfare measure, newborns (% of mean wealth)						
low z	-0.001	0.089	0.060	0.187	0.089	0.121
median z	-0.000	-0.000	-0.000	0.089	0.044	0.058
high z	-0.000	-0.000	-0.000	0.135	0.089	0.104
mean	-0.001	0.030	0.020	0.137	0.074	0.094

Notes: Each entry in Panel A is the difference, in percentage points of the BASE moment, of the moment in the indicated alternative economy (FI or NT) relative to the BASE economy. Panel B reports the amount of additional wealth an agent would have to be given in the baseline economy in order to be indifferent between being born into the indicated alternative economy in the indicated state and being born in the baseline economy. The units for Panel B are percentages of mean wealth. Table 10 in Appendix C explores how variations in α and λ affect the "all" columns in this table.

6.3 Full Information Results

Under full information (FI), types are observed and cross-subsidization ends, as do incentives for a type β_L to imitate a type β_H. Therefore, equilibrium debt price menus change fundamentally for each type.[46] Specifically, as one might expect, type β_L in the FI economy face lower loan prices (higher interest rates) and type H face higher prices (lower interest rates) than the BASE economy where there is some cross-subsidization. Price differences also change with age. Interestingly, as agents accumulate assets through time, the act of borrowing is assessed to be even more likely to come from a low type so there is little difference between prices in FI and BASE for type β_L but large differences for type β_H. These differences in interest rates faced by the two types are clearly illustrated in Table 7.

In response to the changes in the menu of interest rates, Table 7 documents that the fraction of type H (type L) who borrow rises (falls) as one would expect. While there are also no reputation effects

[46] For an example, see Figure 14 in Appendix B.6.2.

in the FI case, the changes in debt-to-income ratio come about for different reasons for the two types. Debt-to-income for type L increases despite the rise in interest rates because they were holding so little debt in the BASE economy in order to raise their reputation by mimicking high types (i.e. the rise is not that they are holding more debt in FI but they were holding so little debt in BASE). The lower debt-to-income ratio for type H arises from the large increase in $q^{a'}_{BASE}$ which makes it cheaper to achieve a desired inflow for consumption (i.e. since $q^{a'}_{BASE}$ rises, one can achieve the desired inflow $q^{a'}_{BASE}a'$ with a smaller a'). Lower (higher) debt-to-income for type H (L) explains the fall (rise) in bankruptcy across type in Table 7. One important takeaway from the differences across type is that they tend to cancel out in the aggregate, leading to only slight differences in aggregate statistics except for the impact on equilibrium interest rates.

As Table 7 documents, all newborns are better off in the FI economy. While it is clear that the newborn high types would rather live with full information where they do not subsidize the low types, even low types prefer (albeit less so) full information since they transit to type H with a relatively high probability $Q^{\beta}(H'|L) = 0.205$. The aggregate welfare gains from eliminating cross-subsidization are quite high (0.094) in the FI economy relative to the gain (0.020) in the NT economy. Thus our "big data" BASE economy yields welfare properties for newborns which are very close to the "small data" no-tracking economy. This is in contrast to the relatively large welfare gains that can come from eliminating hidden information.

7 Conclusion and Directions for Future Research

In this paper, we present a hidden information model of unsecured consumer credit with risk of default. People are subject to unobserved persistent and transitory shocks, and the history of people's asset market actions helps forecast future defaults. The setup is possibly the simplest environment to quantitatively study the role of credit scores in regulating consumer credit. We showed how this can be done using shocks drawn from an extreme value distribution and recursive updating of beliefs.

Our quantitative model not only accounts for aggregate credit market moments, but also the age profile of credit rankings observed in U.S. data. In this sense, our model provides a quantitative theory of the credit score.

Two implications of our theory are worth highlighting. First, we found that restricting lenders' access to an individual's history of asset market actions (no tracking) leads to an overall welfare gain for young adults. Since the young tend to borrow against their future income, the insurance afforded poor young adults of low type who are cross-subsidized by others in better standing outweighs the costs of higher interest rates associated with negative incentive effects from not having to maintain a good reputation. Our "big data" baseline model suggests that the intratemporal insurance for a subset of the population in a "small data" economy can outweigh the incentive effects worsening intertemporal insurance.

Second, even though our model allowed lenders unrestricted access to the history of all actions relevant for inferring an individual's type, the equilibrium allocations at an individual level remain far removed from those of a full information economy. This stems from the fact that individuals *select* actions that only partially reveal their type, while in the full information economy they get that revelation for free. Despite big differences at the micro level, the macro (aggregate) differences can be small.

For simplicity, we have assumed that the only possible actions for an indebted agent were to either payback its debt completely and choose another asset position facing prices that are based on its observables or to file for bankruptcy at a cost and have its debts discharged. In the real world, indebted agents can also go delinquent. In Online Appendix D, we modify our model along the lines of Athreya et al. (2019) to include a delinquency option and quantitatively assess its implications for credit market outcomes. Notably, the following basic results from the BASE model hold in the extended model: (i) type β_L are more likely to go bankrupt; (ii) each type is more likely to file for bankruptcy at higher levels of debt; and (iii) bankruptcy on average leads to a downward revision of one's type score.

Where next? First, type does not have to correspond to an individual's hidden time preference. Alternatively, it could correspond to hidden ability differences that exogenously affect earnings. Hidden time preference can affect a hidden human capital decision (i.e. moral hazard) to endogenously affect earnings or a variety of other personal traits.

Second, reputation in the unsecured credit market can spill over to other markets, reinforcing reputation effects. A person's reputation (or type score) in the unsecured credit market may have implications for other markets (e.g. insurance, labor, housing) and other interactions (marriage) that are worth exploring. Finally, considering the interaction of financial literacy and imperfect competition in the unsecured consumer credit market are important directions for future research.

References

Acquisti, A. and H. R. Varian (2005): "Conditioning Prices on Purchase History," *Marketing Science*, 24, 367–381.

Adda, J. and R. Cooper (2003): *Dynamic Economics: Quantitative Methods and Applications*, Cambridge, MA: MIT Press.

Agarwal, S., S. Chomsisengphet, and C. Liu (2010): "The Importance of Adverse Selection in the Credit Card Market: Evidence from Randomized Trials of Credit Card Solicitations," *Journal of Money, Credit and Banking*, 42.

Aguiar, M., S. Chatterjee, H. L. Cole, and Z. R. Stangebye (2016): "Quantitative Models of Sovereign Debt Crises," in *Handbook of Macroeconomics, Volume 2B*, ed. by H. Uhlig and J. B. Taylor, Elsevier B.V., 1697–1755.

Albanesi, S., G. DeGiorgi, and J. Nosal (2022): "Credit growth and the financial crisis: A new narrative," *Journal of Monetary Economics*, 132, 118–139.

Albanesi, S. and J. Nosal (2018): "Insolvency after the 2005 Bankruptcy Reform," Tech. Rep. Working Paper No. 24934, National Bureau of Economic Research.

Albanesi, S. and D. F. Vamossy (2019): "Predicting Consumer Default: A Deep Learning Approach," Working Paper No. 26165, National Bureau of Economic Research.

Amador, M. and C. Phelan (2021): "Reputation and Sovereign Default," *Econometrica*, 89, 1979–2010.

Andrews, I., M. Gentzkow, and J. Shapiro (2017): "Measuring the Sensitivity of Parameter Estimates to Estimation Moments," *Quarterly Journal of Economics*, 132, 55–70.

Athreya, K., J. Mustre-del-Rio, and J. M. Sanchez (2019): "The Persistence of Financial Distress," *Review of Financial Studies*, 32, 3851–83.

Athreya, K., X. S. Tam, and E. R. Young (2012): "A Quantitative Theory of Information and Unsecured Credit," *American Economic Journal: Macroeconomics*, 4, 153–183.

Ausubel, L. (1999): "Adverse Selection in the Credit Card Market," Unpublished, University of Maryland.

Blanchard, O. J. (1985): "Debt, deficits, and finite horizons," *Journal of Political Economy*, 93, 223–247.

Blattner, L., J. Hartwig, and S. Nelson (2022): "Information Design in Consumer Credit Markets," Working paper.

Braxton, J. C., K. F. Herkenhoff, and G. M. Phillips (2020): "Can the Unemployed Borrow? Implications for Public Insurance," Working Paper 27026, National Bureau of Economic Research.

Briglia, L., S. Chatterjee, D. Corbae, K. Dempsey, and V. Rios-Rull (2021): "Saving for a Sunny Day: An Alternative Theory of Precautionary Savings," Unpublished.

Chatterjee, S., D. Corbae, M. Nakajima, and J.-V. Rios-Rull (2007): "A Quantitative Theory of Unsecured Consumer Credit with Risk of Default," *Econometrica*, 75, 1525–1589.

Chatterjee, S., D. Corbae, and J.-V. Ríos-Rull (2008): "A Finite-life Private-information Theory of Unsecured Consumer Credit," *Journal of Economic Theory*, 142, 149–177.

Cole, H. and P. Kehoe (1998): "Models of Sovereign Debt: Partial Versus General Reputations," *International Economic Review*, 39, 55–70.

Corbae, D. and A. Glover (2018): "Employer Credit Checks: Poverty Traps versus Matching Efficiency," NBER Working Paper 25005.

D'erasmo, P. (2011): "Government Reputation and Debt Repayment in Emerging Economies," Unpublished.

Diamond, D. W. (1989): "Reputation Acquisition in Debt Markets," *Journal of Political Economy*, 97, 828–862.

Diaz-Gimenez, J., A. Glover, and J.-V. Rios-Rull (2011): "Facts on the Distribution of Earnings, Income, and Wealth in the United States: 2007 Update," *Quarterly Review, Federal Reserve Bank of Minneapolis*, 34, 2–32.

Drozd, L. and R. Serrano-Padial (2017): "Modeling the Revolving Revolution: The Debt Collection Channel," *American Economic Review*, 107, 897–930.

Dubey, P. and J. Geanokoplos (2002): "Competitive Pooling: Rothschild-Stiglitz Reconsidered," *The Quarterly Journal of Economics*, 117, 1529–1570.

Eaton, J. and M. Gersovitz (1981): "Debt with Potential Repudiation: Theoretical and Empirical Analysis," *Review of Economic Studies*, 48, 289–309.

Einav, L., M. Jenkins, and J. Levin (2013): "The Impact of Credit Scoring on Consumer Lending," *The RAND Journal of Economics*, 44, 249–274.

Elul, R. and P. Gottardi (2015): "Bankruptcy: Is it Enough to Forgive or Must We Also Forget," *American Economic Journal: Microeconomics*, 7, 294–338.

Exler, F., I. Livshits, J. MacGee, and M. Tertilt (2021): "Consumer Credit with Over-Optimistic Borrowers," Working Paper 21-42, Federal Reserve Bank of Philadelphia.

Exler, F. and M. Tertilt (2020): "Consumer Debt and Default: A Macro Perspective," *Oxford Research Encyclopedia of Economics and Finance*.

Floden, M. and J. Lindé (2001): "Idiosyncratic Risk in the United States and Sweden: Is There a Role for Government Insurance?" *Review of Economic Dynamics*, 4, 406–437.

Fourakis, S. (2021): "Sovereign Default and Government Reputation," Unpublished.

Gale, D. (1992): "Walrasian Theory of Markets with Adverse Selection," *The Review of Economic Studies*, 59, 229–255.

Guerreri, V., R. Shimer, and R. Wright (2010): "Adverse Selection in Competitive Search," *Econometrica*, 78, 1823–1862.

Heathcote, J., F. Perri, and G. Violante (2010): "Unequal We Stand: An Empirical Analysis of Economic Inequality in the United States, 1967-2006," *Review of Economic Dynamics*, 13, 15–51.

Hertzberg, A., A. Liberman, and D. Paravisini (2018): "Screening on Loan Terms: Evidence from Maturity Choice in Consumer Credit," *The Review of Financial Studies*, 31, 3532–3567.

Jaffee, D. M. and T. Russell (1976): "Imperfect Information, Uncertainty, and Credit Rationing," *The Quarterly Journal of Economics*, 90, 651–666.

Kovrijnykh, N., I. Livshits, and A. Zetlin-Jones (2019): "Building Credit Histories with Heterogeneously Informed Lenders," Working Paper 19-17, Federal Reserve Bank of Philadelphia.

Krueger, D. and F. Perri (2006): "Does Income Inequality Lead to Consumption Inequality ? Evidence and Theory," *Review of Economic Studies*, 73, 163–193.

Livshits, I., J. C. MacGee, and M. Tertilt (2007): "Consumer Bankruptcy: A Fresh Start," *American Economic Review*, 97, 402–418.

——— (2016): "The Democratization of Credit and the Rise in Consumer Bankruptcies," *Review of Economic Studies*, 83, 1673–1710.

Luo, W. (2017): "The Consumer Credit Channel of Monetary Policy," Georgetown University.

Myerson, R. B. (1978): "Refinements of the Nash Equilibrium Concept," *International Journal of Game Theory*, 7, 73–80.

Narajabad, B. N. (2012): "Information Technology and the Rise of Household Bankruptcy," *Review of Economic Dynamics*, 15, 526–550.

Nelson, S. (2022): "Private Information and Price Regulation in the US Credit Card Market," Unpublished.

Peski, M. (2014): "Repeated Games with Incomplete Information and Discounting," *Theoretical Economics*, 9, 651–694.

Prescott, E. C. and R. M. Townsend (1984): "Pareto Optima and Competitive Equilibria with Adverse Selection and Moral Hazard," *Econometrica*, 52, 21–46.

Riley, J. G. (2001): "Silver Signals: Twenty-Five Years of Screening and Signaling," *Journal of Economic Literature*, 39, 432–478.

Rust, J. (1987): "Optimal Replacement of GMC Bus Engines: An Empirical Model of Harold Zurcher," *Econometrica*, 55, 999–1033.

Sanchez, J. M. (2018): "The Information Technology Revolution and the Unsecured Credit Market," *Economic Inquiry*, 56, 914–930.

Selten, R. (1975): "Reexamination of the Perfectness Concept for Equilibrium Points in Extensive Games," *International Journal of Game Theory*, 4, 25–55.

Yaari, M. (1965): "Uncertain lifetime, life insurance, and the theory of the consumer," *Review of Economic Studies*, 32, 137–150.

Online Appendices

A Model Appendix

A.1 Construction of $Q^s(s'|\psi)$ and Proof of Lemma 1

Let $G \equiv \{0, 1/K, 2/K, \dots 1\}$ be a uniform discrete approximation of $[0,1]$. Let $D = 1/K$ denote the distance between adjacent (grid) points of G. Let $\mathcal{S} = \{(s_1, s_2, \dots s_B) | s_i \in G \text{ and } \sum_{i=1}^{B} s_i = 1\}$ be the associated probability simplex.

Lemma A.1. *Let $s_i \in G$ for $i = 1, 2, \dots B-1$. If $\sum_{i=1}^{B-1} s_i < 1$, then $1 - \sum_{i=1}^{B-1} s_i \in G$.*

Proof. $\sum_{i=1}^{B-1} s_i < 1 \Rightarrow \sum_{i=1}^{B-1} (\ell_i/K) < 1 \Rightarrow \sum_{i=1}^{B-1} \ell_i < K$ where the ℓ_i's are integers between 0 and K. Since a sum of integers is an integer and a difference of two integers is also an integer, $K - \sum_{i=1}^{B-1} \ell_i$ is a positive integer and it is less than K. Therefore, by the definition of G, $1 - \sum_{i=1}^{B-1} \ell_i/K \in G$. □

Definition A.1. *All elements of the matrix Q^β are strictly positive.*

Lemma A.2. *Let $\psi = (\psi_1, \psi_2, \dots, \psi_B)$ be any vector of type scores resulting from the Bayesian update. Then, $\psi_i \geq \underline{Q} > 0$.*

Proof. Let $\underline{Q}$ be the smallest element of Q^β. By Assumption 1, $\underline{Q} > 0$.

$$
\begin{aligned}
\psi_i &= \sum_j Q^\beta(i|j) \times \text{ posterior probability of } j|\text{actions} \\
&\geq \sum_j \underline{Q} \times \text{ posterior probability of } j|\text{actions} \;=\; \underline{Q}.
\end{aligned}
$$

The first equality follows from the definition of ψ_i, the inequality follows from Assumption 1 and the last line follows from the fact that the sum of posterior probabilities is 1. □

We now identify the elements of $\mathcal{S}$ that approximate any given type-score vector ψ resulting from the Bayesian update. Let $s_{i,L} = \max_{s \in G} s \leq \psi_i$ and $s_{i,H} = s_{i,L} + D$. Consider the collection of 2^{B-1} vectors:

$$
S_\psi = \left\{ (s_{1,l(1)}, s_{2,l(2)}, \dots, 1 - \sum_{i=1}^{B-1} s_{i,l(i)}) \right\} \text{ where for each } i,\ l(i) \in \{L, H\}
$$

Lemma A.3. *If $D < \underline{Q}/(B-1)$ then $S_\psi \subset \mathcal{S}$.*

Proof. By construction, $s_{i,L} \in G$. Next, observe that $s_{i,L}$ cannot be 1 since that would imply that $\psi_i = 1$ and, therefore, $\psi_{j \neq i} = 0$ in contradiction to Lemma A.2. Therefore, $s_{i,H} = s_{i,L} + D$ must belong in G for all i. To show that $(s_{1,I(1)}, s_{2,I(2)}, \ldots, 1 - \sum_{i=1}^{B-1} s_{i,I(i)})$ belongs in $\mathcal{S}$ it is sufficient to show, by virtue of Lemma A.1, that $\sum_{i=1}^{B-1} s_{i,I(i)} < 1$.

$$\begin{aligned}\sum_{i=1}^{B-1} s_{i,I(i)} &\leq \sum_{i=1}^{B-1} s_{i,H} \\ &\leq \sum_{i=1}^{B-1} (\psi_i + D) \\ &= (1 - \psi_B) + (B-1)D \\ &\leq 1 - \underline{Q} + (B-1)D < 1\end{aligned}$$

The first inequality follows because $s_{i,I(i)} \leq s_{i,H}$. The second inequality follows because $s_{i,L} = s_{i,H} + D$ and $\psi_i \geq s_{i,L}$. The third equality follows because $\sum_{i=1}^{B} \psi_i = 1$. The fourth inequality follows from Lemma A.2 and the final inequality follows from the hypothesis of the lemma. □

By Lemma A.3 we can take S_ψ to be the collection of approximating vectors. Note that for each member of this set, the first $B-1$ components are within $\psi_i \pm D$ so, in this sense, the vectors are close to ψ.

We now determine the probability assigned to each of these vectors. To this end, let

$$p(s_{i,L}) = \frac{s_{i,H} - \psi_i}{D} \text{ and } p(s_{i,H}) = \frac{\psi_i - s_{i,L}}{D} \text{ for } i = 1, 2, 3, \ldots B-1. \tag{27}$$

Since $s_{i,L} \leq \psi_i < s_{i,H}$ and $s_{i,H} - s_{i,L} = D$, $p(s_{i,L})$ and $p(s_{i,H})$ are nonnegative and sum to 1. We set

$$\Pr\left[\left(s_{1,I(1)}, s_{2,I(2)}, s_{3,I(3)}, \ldots, 1 - \sum_{i=1}^{B-1} s_{i,I(i)}\right)\right] = \prod_{i=1}^{B-1} p(s_{i,I(i)}), \quad I(i) \in \{L, H\}, i = 1, 2, \ldots B-1.$$

Then our assignment rule $Q^s(s'|\psi) : \mathcal{S} \to [0,1]$ is given by:

$$Q^s(s'|\psi) = \begin{cases} \prod_{i=1}^{B-1} p(s'_{i,I(i)}) & \text{if } s' \in S_\psi \\ 0 & \text{otherwise.} \end{cases} \tag{28}$$

For this assignment rule, we can prove:

Lemma 1. *(i)* $\sum_{s' \in \mathcal{S}} s'_i Q^s(s'|\psi) = \psi_i, \forall i$ *(consistency), (ii)* $\sum_{s' \in \mathcal{S}} (s'_i - \psi_i)^2 Q^s(s'|\psi) \leq 2(B-1)D^2, \forall i$ *(variance of the approximation error can be made arbitrarily small), and (iii)* $Q^s(s'|\psi)$ *is continuous in*

ψ *(continuity).*

Proof. (i) First, note that $\sum_{s' \in \mathcal{S}} s'_i Q^s(s'|\psi) = \sum_{s' \in S_\psi} s'_i Q^s(s'|\psi)$ since (28) assigns positive probability only to vectors that are in S_ψ. Let $i \in \{1, 2, \dots, B-1\}$. Now, group the collection of vectors in S_ψ into two: In the first group are all vectors for which $s'_i = s_{i,L}$ and in the second group are all vectors for which $s'_i = s_{i,H}$. Denote these groups as S^L_ψ and S^H_ψ. Then,

$$\begin{aligned}
\sum_{s' \in S_\psi} s'_i Q^s(s'|\psi) &= \sum_{s' \in S^L_\psi} s'_i Q^s(s'|\psi) + \sum_{s' \in S^H_\psi} s'_i Q^s(s'|\psi) \\
&= s_{i,L} \sum_{s' \in S^L_\psi} Q^s(s'|\psi) + s_{i,H} \sum_{s' \in S^H_\psi} Q^s(s'|\psi) \\
&= s_{i,L} p(s_{i,L}) + s_{i,H} p(s_{i,H}) = \psi_i.
\end{aligned}$$

The third equality follows from the fact that the first and second sums in the second line are the probabilities of selecting a vector from group L and group H, respectively. Since the assignment of $s_{i,L}$ or $s_{i,H}$ for s'_i is done independently of the assignments to the other $B-2$ components, the probability of selecting a vector in group L is $p(s_{i,L})$ and in group H is $p(s_{i,H})$. The last equality follows from (27).

Next, let $i = B$. Then,

$$\begin{aligned}
\sum_{s' \in S_\psi} s'_B Q^s(s'|\psi) &= \sum_{s' \in S_\psi} [1 - s'_1 - s'_2 - \dots - s'_{B-1}] Q^s(s'|\psi) \\
&= \sum_{s' \in S_\psi} Q^s(s'|\psi) - \sum_{i=1}^{B-1} \sum_{s' \in S_\psi} s'_i Q^s(s'|\psi) \\
&= 1 - \sum_{i=1}^{B-1} \psi_i = \psi_B.
\end{aligned}$$

(ii) Let $i \in \{1, 2, \dots, B-1\}$.

$$\begin{aligned}
\sum_{s' \in S_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi) &= \sum_{s' \in S^L_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi) + \sum_{s' \in S^H_\psi} (s'_i - \psi_i)^2 Q^s(s'|\psi) \\
&= \sum_{s' \in S^L_\psi} (s_{i,L} - \psi_i)^2 Q^s(s'|\psi) + \sum_{s' \in S^H_\psi} (s_{i,H} - \psi_i)^2 Q^s(s'|\psi) \\
&\leq D^2 \sum_{s' \in S^L_\psi} Q^s(s'|\psi) + D^2 \sum_{s' \in S^H_\psi} Q^s(s'|\psi) \\
&= D^2 (p(s_{i,L}) + p(s_{i,H})) = D^2.
\end{aligned}$$

Let $i = B$. Then,

$$\begin{aligned}\sum_{s' \in S_\psi} (s'_B - \psi_B)^2 Q^s(s'|\psi) &= \sum_{s' \in S_\psi} \left(1 - \sum_{i=1}^{B-1} s'_i - 1 + \sum_{i=1}^{B-1} \psi_i\right)^2 Q^s(s'|\psi) \\ &= \sum_{s' \in S_\psi} \left(\sum_{i=1}^{B-1} (s'_i - \psi_i)\right)^2 Q^s(s'|\psi) \\ &= \sum_{i=1}^{B-1} \sum_{s' \in S_\psi} \left(s'_i - \psi_i\right)^2 Q^s(s'|\psi) + \text{ expectations of cross product terms} \\ &\leq (B-1)D^2.\end{aligned}$$

The inequality in the final line follows from the bound on each of the variances and from the fact that the assignments of s'_i for $i \in \{1, 2, \dots, B-1\}$ are independent of each other so that the expectation of all the cross product terms is zero.

(iii) Let ψ_n be a sequence converging to ψ^*. Consider first the case where $\psi_i^* \notin G$. Then, for $n > N$, N sufficiently large, $\psi_i^n \in (s^*_{i,L}, s^*_{i,H})$ and, so,

$$p^n(s_{i,L}) = \frac{s^*_{i,H} - \psi_i^n}{D} \text{ and } p^n(s_{i,H}) = \frac{\psi_i^n - s^*_{i,L}}{D}.$$

It follows that $\lim_{n\to\infty} p^n(s_{i,L}) = p^*(s_{i,L})$ and $\lim_{n\to\infty} p^n(s_{i,H}) = p^*(s_{i,H})$. Next consider the case where $\psi_i^* \in G$. Then, by construction

$$s^*_{i,L} = \psi_i^*, \;\; s^*_{i,H} = s^*_{i,L} + D \text{ and } p^*(s^*_{i,L}) = 1.$$

Then, for $n > N$, N sufficiently large, either $\psi_i^n \in (s^*_{i,L} - D, s^*_{i,L})$ or $\psi_i^n \in (s^*_{i,L}, s^*_{i,L} + D)$. Therefore, $p^n(s^*_{i,L})$ converges to $1 = p^*(s^*_{i,L})$ as ψ_i^n converges to ψ_i^*. □

Note that by reducing the distance D between adjacent points of G, or, equivalently, increasing the number of (uniformly-placed) grid points K approximating the unit interval, the dispersion of s' around ψ can be made arbitrarily small.

A.2 Proof of Theorem 1 (Existence of the Value Function)

Theorem 1. *Given f, there exists a unique solution $W(\beta, z, \omega|f)$ to the decision problem in (3)-(8).*

Proof. The proof relies on the Contraction Mapping Theorem. However, since the extreme value shocks ν and ϵ can take any value on the real line, it is mathematically more convenient to seek a solution to (3), (4), (12), and (13) since the extreme value shocks do not appear in these. Define the operator $(T_f)(W) : \mathbb{R}^{B+Z+|\Omega|} \to \mathbb{R}^{B+Z+|\Omega|}$ as the map that takes a vector W in $\mathbb{R}^{B+Z+|\Omega|}$ and returns a vector $(T_f)(W)$ via (4), (12), and (13) using (3). We may easily verify that T_f satisfies Blackwell's sufficiency condition for a contraction map (with modulus $\beta\rho$). Since $\mathbb{R}^{B+Z+|\Omega|}$ is a complete metric space (with, say, the uniform metric $\rho(W, W') = \max_{1\leq i\leq B+Z+|\Omega|} \|W_i - W'_i\|$), by Theorem 3.2 of Stokey and Lucas Jr. (1989), there exists a unique $W(\beta, z, \omega|f)$ satisfying $(T_f)(W) = W$. □

A.3 Proof of Lemma 2 (Existence of the Invariant Distribution)

Lemma 2. *There exists a unique invariant distribution $\overline{\mu}(\cdot|f)$ and $\{\mu_0 T^n\}$ converges to $\bar{\mu}(\cdot|f)$ at a geometric rate for any initial distribution μ_0.*

Proof. We will use Theorem 11.4 in Stokey and Lucas Jr. (1989) to establish this result. To connect to that theorem, let i be a typical element of the finite state space $\mathcal{B} \times \mathcal{Z} \times \Omega$. Let the transition matrix Π in their theorem correspond to T in (19) and let π_{ij} denote the probability of transitioning to j from i. Further, let $\epsilon_j = \min_i \pi_{ij}$ and $\epsilon = \sum_j \epsilon_j$. Then it is sufficient to establish that $\epsilon > 0$. To this end, consider the state $\hat{j} = (\hat{\beta}, \hat{z}, \hat{e}, 0, F_\beta)$ with the property that $F_\beta(\hat{\beta})H(\hat{z})F_e(\hat{e}) > 0$. Then, (19) implies $\pi_{i\hat{j}} \geq (1-\rho)F_\beta(\hat{\beta})H(\hat{z})F_e(\hat{e}) > 0$ for all i. Hence $\epsilon_{\hat{j}} \geq (1-\rho)F_\beta(\hat{\beta})H(\hat{z})F_e(\hat{e}) > 0$. Since $\epsilon_j \geq 0$ for all other j, it follows that $\epsilon > 0$. □

A.4 Proof of Lemma 3 (Value Continuity) and Theorem 2 (Equilibrium Existence)

The fact that there are zero profits in equilibrium implies $q^{(0,a')}(\omega|f) = \frac{\rho}{1+r}$ for $a' \geq 0$ (i.e. the price on savings is a function only of parameters). In what follows we take $F^* \subset F$ to contain only those f_1 for which $f_1(a', \omega) = \frac{\rho}{1+r}$ for $a' \geq 0$.

Lemma 3. *$W(\beta, z, \omega|f)$ is continuous in f and for any $(d, a') \in \mathcal{F}(z, \omega|f)$, $\sigma^{(d,a')}(\beta, z, \omega|f)$ is continuous in f.*

Proof. We first show that the operator T_f defined in Theorem 1 is continuous in f (meaning that for any given W, small changes in f lead to small changes in $T_f(W)$). Inspection of (6) and (8) shows that this will be true if the conditional value functions $v^{(d,a')}(\beta, z, e, a, s|f)$ in (4) are continuous in f. Let $\bar{f} \in F^*$ and let $(\hat{d}, \hat{a}') \in \mathcal{F}(z, \omega|\bar{f})$. Let $f^n \in F^*$ be a sequence converging to $\bar{f}$. By Assumption

1, $(0,0)$ and $(1,0)$ are feasible choices regardless of the value of any inherited debt (i.e. $a<0$), so all debt choices ($a'<0$) and the default choice belong in $\mathcal{F}(z,\omega|f^n)$. Furthermore, if an asset choice (i.e. $a'\geq 0$) is feasible for $\bar{f}$, that asset choice remains feasible for f^n since the price of any asset is the same in $\bar{f}$ and f^n (namely, $\rho/(1+r)$). Therefore, $(\hat{d},\hat{a}')\in\mathcal{F}(z,\omega|f^n)$ and so $v^{(\hat{d},\hat{a}')}(\beta,z,e,a,s|f^n)$ is well-defined for all n. Observe that f^n affects $v^{(d,a')}(\beta,z,e,a,s|f^n)$ in (4) via how q^n affects the feasible set given in (3) and how ψ^n affects $Q^s(s'|\psi^n)$ in (4). Since $\lim_{n\to\infty}c^{(\hat{d},\hat{a}')}(z,\omega|f^n)=c^{(\hat{d},\hat{a}')}(z,\omega|\bar{f})$, the continuity of u gives $\lim_{n\to\infty}u(c^{(\hat{d},\hat{a}')}(z,\omega|f^n))=u(\lim_{n\to\infty}c^{(\hat{d},\hat{a}')}(z,\omega|f^n))=u(c^{(\hat{d},\hat{a}')})z,\omega|\bar{f}))$. From Lemma 1, $\lim_{n\to\infty}Q^s(s'|\psi^{(d,a')}_{\beta'}(\omega|f^n))=Q^s(s'|\psi^{(d,a')}_{\beta'}(\omega|\bar{f}))$. It follows that $v^{(d,a')}(\beta,z,e,a,s|f)$ is continuous in f and hence $\lim_{n\to\infty}T_{f^n}=T_{\bar{f}}$. Since F is a Banach space and T_f is a contraction map, we may apply Theorem 4.3.6 in Hutson and Pym (1980) to conclude that W is continuous in f. The continuity of $\sigma^{(d,a')}(\beta,z,\omega|f)$ in f follows directly by continuity of σ in W.

Theorem 2. *There exists a stationary recursive competitive equilibrium.*

Proof. The proof of existence uses Brouwer's Fixed Point Theorem (Theorem 17.3 in Stokey and Lucas Jr. (1989)). To connect to that theorem, we reinterpret the function f as a point in a unit (hyper)cube in high-dimensional Euclidean space. To this end, let $\mathcal{G}=\{((d,a'),\beta,z,\omega):(d,a')\in\mathcal{Y},\beta\in\mathcal{B},z\in\mathcal{Z},\omega\in\Omega\}\subset\mathcal{Y}\times\mathcal{B}\times\mathcal{Z}\times\Omega$ where $\mathcal{Y}=\{(d,a'):(d,a')\in\{0\}\times\mathcal{A}\text{ or }(d,a')=(1,0)\}$. Let M and K be the cardinalities of $\mathcal{G}$ and $\mathcal{Y}\setminus\{(1,0)\}$. Then, $f\in F^*$ can be thought of as a vector composed by stacking $q\in[0,1]^K$ and $\psi\in[0,1]^{B\cdot M}$. Then $f\in[0,1]^{K+B\cdot M}$ and $F^*\subset[0,1]^{K+B\cdot M}$. Next, use (15) (with equality) to construct the vector $q^{a'}_{\text{new}}(\omega|f)$ and use (16) to construct the vector $\psi^{(d,a')}_{\text{new}}(\omega|f)$. Then, let J be the mapping

$$f_{\text{new}}\equiv\left(q^{a'}_{\text{new}},\psi^{(0,a')}_{\text{new}},\psi^{(1,0)}_{\text{new}}\right)=J(f):F^*\to F^*.$$

Since $\sigma^{(d,a')}(\beta,z,\omega|f)$ is a continuous function of f (Lemma 3), J is a continuous self-map as (15) and (16) are continuous functions of $\sigma^{(d,a')}(\beta,z,\omega|f)$. And since F^* is a nonempty, closed, bounded and convex subset of a finite-dimensional normed vector space, by Brouwer's FPT there exists $f^*\in F^*$ such that $f^*=J(f^*)$. □

A.5 Equivalence

Given an RCE, let $\mathcal{P}(e,a)=\bigcup_{s\in\mathcal{S}}\{m:m=p^{\bar{a}*}(e,a,s)\}$ and $\hat{\Omega}=\{(e,a,m):(e,a)\in\mathcal{E}\times\mathcal{A}\text{ and }m\in\mathcal{P}(e,a)\}$ with typical element $\hat{\omega}\in\hat{\Omega}$. An individual in state $(\beta,z,\hat{\omega})$ chooses whether to default d and conditional on not defaulting chooses asset a' taking as given

- a price function $q^{a'}(\hat{\omega}) : \mathcal{A} \times \hat{\Omega} \to [0,1]$,
- credit-score transition functions $Q_m^{(0,a')}(m'|e',\hat{\omega}) : \mathcal{P}(e',a') \times \mathcal{D} \times \mathcal{A} \times \mathcal{E} \times \hat{\Omega} \to [0,1]$ and $Q_m^{(1,0)}(m'|e',\hat{\omega}) : \mathcal{P}(e',a') \times \mathcal{D} \times \mathcal{A} \times \mathcal{E} \times \hat{\Omega} \to [0,1]$.

As in (3), this implies that an individual of type β in state $(z,\hat{\omega})$ chooses $(d,a') \in \mathcal{F}(z,\hat{\omega})$ inducing consumption $c^{(d,a')}(z,\hat{\omega})$ satisfying:

$$c^{(d,a')}(z,\hat{\omega}) = \begin{cases} y(e(\hat{\omega}),z) + a(\hat{\omega}) - q^{a'}(\hat{\omega}) \cdot a' & \text{if } (d,a') = (0,a') \\ y(e(\hat{\omega}),z)(1-\kappa_1) - \kappa & \text{if } a < 0 \text{ and } (d,a') = (1,0) \end{cases} \tag{29}$$

For all $(d,a') \in \mathcal{F}(z,\hat{\omega})$, the value functions given by equations (5), (7), (12), and (13) and choice probabilities given by equations (9), (10), and (11) associated with the individual's problem are unchanged in form after substituting $\hat{\omega}$ for ω except for equation (4) now given by:

$$\begin{aligned} v^{(d,a')}(\beta,z,\hat{\omega}) &= u\left(c^{(d,a')}(z,\hat{\omega})\right) \\ &\quad +\beta\rho \cdot \sum_{\beta',z',e',m'} Q^{\beta}(\beta'|\beta) Q^{e}(e'|e) H(z') Q_m^{(d,a')}(m'|e',\hat{\omega}) W(\beta',z',\hat{\omega}). \end{aligned} \tag{30}$$

Intermediaries issue a positive measure of contracts taking the price function $q^{a'}(\hat{\omega})$ and probability of repayment function $p^{a'}(\hat{\omega})$ as given to maximize profits:

$$\pi^{a'}(\hat{\omega}) = \begin{cases} \rho \cdot \frac{p^{a'}(\hat{\omega}) \cdot (-a')}{1+r} - q^{a'}(\hat{\omega}) \cdot (-a') & \text{if } a' < 0 \\ q^{a'} \cdot a' - \rho \cdot \frac{a'}{1+r} & \text{if } a' \geq 0 \end{cases}. \tag{31}$$

If the intermediary issues a strictly positive measure of credit contracts, then zero profits require:

$$q^{a'}(\hat{\omega}) = \begin{cases} \frac{\rho \cdot p^{a'}(\hat{\omega})}{1+r} & \text{if } a' < 0, \\ \frac{\rho}{1+r} & \text{if } a' \geq 0 \end{cases} \tag{32}$$

which is the analogue of (15).

Consistency requires that the probability of repayment satisfy the analog of (17), namely,

$$p^{a'}(\hat{\omega}) = \sum_{\beta',z',e',m'} H(z') \cdot Q^{e}(e'|e) \cdot Q_m^{(d,a')}(m'|e',\hat{\omega}) \cdot M_{\beta'}(\hat{\omega}') \cdot \left(1 - \sigma^{(1,0)}(\beta',z',\hat{\omega}')\right). \tag{33}$$

Here, $M(\hat{\omega}) : \hat{\Omega} \to \mathcal{S}$ where $M(\hat{\omega}) = (M_{\beta_1}(\hat{\omega}), ..., M_{\beta_B}(\hat{\omega}))$ with the function $M_{\beta}(\hat{\omega})$ mapping m to the probability an individual is of a given type β.

The transition function in equation (19) which tracks the probability that an individual in state $(\beta, z, \hat{\omega})$ transitions to state $(\beta', z', \hat{\omega}')$ is now given by:

$$
\begin{aligned}
T(\beta', z', \hat{\omega}'; \beta, z, \hat{\omega}) = & \\
& \rho \cdot Q^{\beta}(\beta'|\beta) \cdot H(z') \cdot Q^{e}(e'|e) \cdot \sigma^{(d,a')}(\beta, z, m) \cdot Q_m^{(d,a')}(m'|e', \hat{\omega}) \\
& + (1-\rho) \cdot F_{\beta}(\beta') \cdot H(z') \cdot F_e(e') \cdot \mathbf{1}_{\{a'=0\}} \cdot \mathbf{1}_{\{m'=p^{\bar{a}*}(e_1, 0, F_{\beta})\}}.
\end{aligned} \tag{34}
$$

We can now give the definition of a stationary recursive competitive equilibrium with credit scores.

Definition 6. Stationary Recursive Competitive Equilibrium with Credit Scores A stationary Recursive Competitive Equilibrium with Credit Scores (RCECS) is a pricing function $q^{a'*}(\hat{\omega})$, a credit-scoring function $Q_m^{(d,a')*}(m'|e', \hat{\omega})$, a choice probability function $\sigma^{(d,a')*}(\beta, z, \hat{\omega})$, a repayment probability function $p^{a'*}(\hat{\omega})$, a credit-score-to-type-probability function $M^*(\hat{\omega})$, and a distribution $\overline{\mu}^*(\hat{\omega})$ such that:

(i). Optimality: Given $q^{a'*}(\hat{\omega})$ and $Q_m^{(d,a')*}(m'|e', \hat{\omega})$, $\sigma^{(d,a')*}(\beta, z, \hat{\omega})$ satisfies (10) and (11) for all $(\beta, z, \hat{\omega}) \in \mathcal{B} \times \mathcal{Z} \times \hat{\Omega}$ and $(d, a') \in \mathcal{F}(z, \hat{\omega})$,

(ii). Zero Profits: Given $Q_m^{(d,a')*}(m'|e', \hat{\omega})$, $M^*(\hat{\omega})$, and $\sigma^{(1,0)*}(\beta, z, \hat{\omega})$, $p^{a'*}(\hat{\omega})$ satisfies (33) for all $\hat{\omega} \in \hat{\Omega}$ and given $p^{a'*}(\hat{\omega})$, $q^{a'*}(\hat{\omega})$ satisfies (32) with equality for all $\hat{\omega} \in \hat{\Omega}$,

(iii). Stationary Distribution: Given $Q_m^{(d,a')*}(m'|e', \hat{\omega})$ and $\sigma^{(d,a')*}(\beta, z, \hat{\omega})$, $\overline{\mu}^*(\beta, z, \hat{\omega})$ is a fixed point of $\mu'(\beta', z', \hat{\omega}') = \sum_{\beta, z, \hat{\omega}} T^*(\beta', z', \hat{\omega}'|\beta, z, \hat{\omega}) \cdot \mu(\beta, z, \hat{\omega})$ for T^* in (34).

Note the difference between the RCE Definition 3 and the RCECS Definition 6: an RCE requires the updating function to be consistent with Bayes Law (in (iii) of Definition 3), while Definition 6 simply postulates the existence of $Q_m^{(d,a')*}$ and M^* and requires that these be consistent with zero profits.

Theorem 3. *Given an RCE, let $m = p^{\bar{a}*}(e, a, s)$. Suppose that the inverse function $s = (p^{\bar{a}*})^{-1}(e, a, m)$ exists. Then an RCECS exists in which the choice probabilities $\sigma^{(d,a')*}(\beta, z, e, a, m) = \sigma^{(d,a')*}(\beta, z, e, a, s)$ for $s = (p^{\bar{a}*})^{-1}(e, a, m)$.*

Proof. Given an RCE and the existence of the inverse function $s = (p^{\bar{a}*})^{-1}(e, a, m)$, set

(a). $M^*(e, a, m) = (p^{\bar{a}*})^{-1}(e, a, m)$

(b). $q^{a'}(e,a,m) = q^{a'*}(e,a,M^*(e,a,m))$,

(c). $Q_m^{(d,a')*}(m' = \tilde{m}|e',e,a,m) = Q^s(M^*(e',a',\tilde{m}))|\psi^{(d,a')*}(e,a,M^*(e,a,m))$, *if* $\tilde{m} \in \mathcal{P}(e',a')$ and 0 otherwise,

(d). $W(\beta,z,e,a,m) = W^*(\beta,z,e,a,M^*(e,a,m))$

By (b) $\mathcal{F}(z,\hat{\omega}) = \mathcal{F}(z,\omega)$ in (29) and (3) and by (c) and (d), $v^{(d,a')}(\beta,z,\hat{\omega})$ in (30) is identical to $v^{(d,a')*}(\beta,z,\omega)$ in (4). Hence $\sigma^{(d,a')*}(\beta,z,\hat{\omega}) = \sigma^{(d,a')*}(\beta,z,\omega)$ satisfying condition (i) in Definition 6. If the choice probabilities are the same, then repayment probabilities in (33) and (17) are the same since $s'(\beta') = M^*_{\beta'}(\hat{\omega}')$ and $Q^*_m = Q^s$, thereby satisfying the requirement on $p^{a^{*}*}(\hat{\omega})$ in (ii) in Definition 6. If the repayment probabilities in (33) and (17) are the same, then prices in (32) and (15) are the same, thus satisfying the requirement on $q^{a'*}(\hat{\omega})$ in (ii) in Definition 6. Since $\sigma^{(d,a')*}(\beta,z,\hat{\omega}) = \sigma^{(d,a')*}(\beta,z,\omega)$ and $Q^*_m = Q^s$, then (34) is the same as (19) so that (iii) in Definition 6 holds. □

B Computational Appendix

B.1 Computational Algorithm for the Baseline Model

In this subsection, we describe the algorithm used to compute the RCE stated in Definition 3. The model is calibrated by using the procedure below to solve the model for a given set of parameters, and then updating parameters to minimize the distance between the model moments and the data moments. This outer minimization is performed using the Nelder-Mead simplex method over hundreds of (randomly-chosen) initial conditions.

1. Specify all grids and parameters. Relevant details:

   (a) asset grid is log-spaced in both directions from 0 with 50 points between $[-0.15, -0.00001]$ and 130 points between $[0, 15]$

   (b) type score grid is linearly-spaced with 40 points between $\min\{G_{\beta_H}, Q^\beta(\beta'_H|\beta_L)\}$ and $Q^\beta(\beta'_H|\beta_H)$.

   (c) equilibrium convergence is on p and ψ functions with gradual updating; since ψ is more sensitive, we use a relaxation parameter of $\theta \in (0,1)$ on p and $\eta\theta$ on ψ for $\eta \in (0,1)$.

   (d) persistent and transitory earnings grids are 5- and 3-point discretizations of the processes in Table 1 respectively, yielding $\mathcal{E} = \{-0.71, -0.27, 0, 0.27, 0.71\}$ and $\mathcal{Z} = \{-0.18, 0, 0.18\}$.

   (e) all newborns have no assets, lowest e, and $s = F_{\beta_H}$. They are distributed across β and z according to F_{β_H} and $H(z)$, respectively.

(f) given α, we set the mean of the ν shocks to be

$$\overline{\nu} = -\alpha(\gamma_E + \ln 2) \implies \mathbb{E}[\max\{\nu_D, \nu_{ND}\}] = 0 \quad (35)$$

(g) compute consumption associated with all non-borrowing actions (since r is exogenous, these don't change iteration to iteration).

i. *Savings*: for each $\omega = (a, s, e)$ and z, compute the consumption associated with each feasible action $a' \geq 0$ such that

$$c^{(0,a')}(z, \omega) = y(e(\omega), z) + a(\omega) - \frac{\rho}{1+r}a' > 0$$

Let $\overline{n}(z, \omega)$ denote the index of the largest budget feasible a' for an agent with (z, ω).

ii. *Default*: define the consumption for a defaulter to be

$$c^{(1,0)}(z, \omega) = y(e(\omega), z)(1 - \kappa_1) - \kappa$$

where κ is a fixed bankruptcy filing cost and κ_1 is a cost that scales with earnings.

2. **Main equilibrium loop.** Every iteration j starts with a value of: (i) $f_j = (q_j^{a'}(\omega), \psi_j^{(0,a')}(\omega), \psi_j^{(1,0)}(\omega))$; and (ii) the (ex-ante) value function $W_j(\beta, z, \omega)$.[47,48]

(a) Compute consumption associated with all $a' < 0$ given current prices:

$$c^{(0,a')}(z, \omega | f_j) = y(e(\omega), z) + a(\omega) - q_j^{a'}(\omega)a'$$

Note that our Assumption 1 implies that all debt choices are always feasible, which is critical for keeping our Bayesian updates well-defined.

(b) Compute mean of extreme value shock associated with each $a' \in \mathcal{F}(z, \omega | f_j)$:

i. For $n = 1, ..., \overline{n}(z, \omega)$, compute

$$\begin{aligned} c^{(0,\hat{a}_n)}(z, \omega | f_j) &= y(e(\omega), z) + a(\omega) - q_j^{\hat{a}_n}(\omega)\hat{a}_n \\ \text{where } \hat{a}_1 &= a_1 \text{ and } \hat{a}_n = a_{n-1} + \frac{a_n - a_{n-1}}{2} \text{ for } n = 2, ..., N. \end{aligned}$$

[47] While we index these functions by $f = (q^{a'}(\omega), \psi^{(0,a')}(\omega), \psi^{(1,0)}(\omega)))$ to maintain consistency with notation in the text, the algorithm actually iterates on $p^{a'}(\omega)$ which directly yields $q^{a'}(\omega)$ via (15).

[48] Since the full information version of the model solves very quickly, for the initial $j = 0$ values, the value functions and loan price schedules provide a good initial guess. For type scores, a consistent initial guess is $\psi^{(d,a')}(e, a, s) = sQ^{\beta}(\beta_H|\beta_H) + (1 - s)Q^{\beta}(\beta_H|\beta_L)$.

and $q^{\hat{a}_n}(\omega)$ is given by the linear interpolation of the q function

$$\begin{aligned} q_j^{\hat{a}_1}(\omega) &= q_j^{a_1}(\omega) \text{ for } a' = a_1, \\ q_j^{\hat{a}_n}(\omega) &= \frac{q_j^{a_{n-1}}(\omega) + q_j^{a_n}(\omega)}{2} \text{ for } n = 2, ..., N. \end{aligned}$$

ii. define the measure of consumption associated with choice $a' = a_n$ as

$$\eta^{a_n}(z, \omega | f_j) = \begin{cases} |c^{(0,\hat{a}_n)}(z, \omega | f_j) - c^{(0,\hat{a}_{n+1})}(z, \omega | f_j)| & \text{for } n = 1, ..., \overline{n}(z, \omega) - 1, \\ |c^{(0,\hat{a}_n)}(z, \omega | f_j) - 0| & \text{for } n = \overline{n}(z, \omega). \end{cases} \tag{36}$$

iii. the mean of ϵ^{a_n} for $n = 1, ..., \overline{n}(z, \omega)$ is taken to be

$$\overline{\epsilon}^{a_n}(z, \omega | f_j) = -\lambda \gamma_E + \lambda \ln \eta^{a_n}(z, \omega | f_j) \tag{37}$$

where λ is the common scale parameter for all shocks.

(c) Iterate to convergence on the value function. Starting with $W_{j,k=1}(\beta, z, \omega) = W_j(\beta, z, \omega)$

i. Compute the conditional value function in (4):

$$\begin{aligned} v_k^{(d,a')}(\beta, z, \omega | f_j) &= u\left(c^{(d,a')}(z, \omega | f_j)\right) + \\ & \beta \rho \cdot \sum_{(\beta', z', e', s')} Q^{\beta}(\beta' | \beta) Q^e(e' | e) H(z') Q^s(s' | \psi_j^{(d,a')}(\omega)) W_{j,k}(\beta', z', \omega') \end{aligned}$$

ii. As in (12), let

$$\begin{aligned} W_k^{ND}(\beta, z, \omega | f_j) &= \mathbb{E}\left[\max_{n=1,\ldots,\overline{n}(z,\omega)} v_k^{(0,a'_n)}(\beta, z, \omega | f_j) + \epsilon^{a'_n}\right] \\ &= \lambda \gamma_E + \lambda \ln\left(\sum_{n=1}^{\overline{n}(z,\omega)} \exp\left(\frac{v_k^{(0,a'_n)}(\beta, z, \omega | f_j) + \overline{\epsilon}^{a'_n}(z, \omega | f_j)}{\lambda}\right)\right) \\ &= \lambda \ln\left(\sum_{n=1}^{\overline{n}(z,\omega)} \eta^{a'_n}(z, \omega | f_j) \exp\left(\frac{v_k^{(0,a'_n)}(\beta, z, \omega | f_j)}{\lambda}\right)\right). \end{aligned} \tag{38}$$

Note that this step applies the definition in (37) from step (2(b)iii).

iii. As in (13), update

$$W_{j,k+1}(\beta, z, \omega) = \begin{cases} W_k^{ND}(\beta, z, \omega | f_j) & \text{if } a(\omega) \geq 0 \\ \mathbb{E}\left[v_k^{(1,0)}(\beta, z, \omega | f_j) + \nu^D, W_k^{ND}(\beta, z, \omega | f_j) + \nu^{ND}\right] & \text{if } a(\omega) < 0 \end{cases}.$$

For the $a(\omega) < 0$ case, using $\overline{v}$ from step (1f) we simply have

$$\begin{aligned} W_{j,k+1}(\beta,z,\omega) &= \alpha\gamma_E + \alpha \ln\left(\exp\left(\frac{W_k^{ND}(\beta,z,\omega|f_j)+\overline{v}}{\alpha}\right) + \exp\left(\frac{v_k^{(1,0)}(\beta,z,\omega|f_j)+\overline{v}}{\alpha}\right)\right) \\ &= -\alpha\ln 2 + \alpha\ln\left(\exp\left(\frac{W_k^{ND}(\beta,z,\omega|f_j)}{\alpha}\right) + \exp\left(\frac{v_k^{(1,0)}(\beta,z,\omega|f_j)}{\alpha}\right)\right). \end{aligned}$$

iv. If $\sup|W_{j,k+1}(\beta,z,\omega) - W_{j,k}(\beta,z,\omega)|$ is less than desired tolerance go to step (2d) otherwise go to step (2c) starting with $W_{j,k+1}(\beta,z,\omega)$.

(d) Compute decision densities:

i. As in (9) in the text, the probability of choosing $a'_n \in \mathcal{F}(z,\omega|f_j)$ conditional on not defaulting is 0 if $a'_n \notin \mathcal{F}(z,\omega|f_j)$, otherwise

$$\tilde{\sigma}^{(0,a'_n)}(\beta,z,\omega|f_j) = \frac{\eta^{a'_n}(z,\omega|f_j)\exp\left(\frac{v^{(0,a'_i)}(\beta,z,\omega|f_j)}{\lambda}\right)}{\sum_{n=1}^{\overline{n}(z,\omega|f_j)} \eta^{a'_n}(z,\omega|f_j)\exp\left(\frac{v^{(0,a'_n)}(\beta,z,\omega|f_j)}{\lambda}\right)} \tag{39}$$

ii. As in (10) in the text, the probability of default for $a(\omega) < 0$ is 0 if $a(\omega) \geq 0$, otherwise

$$\sigma^{(1,0)}(\beta,z,\omega|f_j) = \frac{\exp\left(\frac{v^{(1,0)}(\beta,z,\omega|f_j)}{\alpha}\right)}{\exp\left(\frac{v^{(1,0)}(\beta,z,\omega|f_j)}{\alpha}\right) + \exp\left(\frac{W^{ND}(\beta,z,\omega|f_j)}{\alpha}\right)}$$

iii. combining these, we obtain the unconditional probability

$$\sigma^{(0,a'_n)}(\beta,z,\omega|f_j) = \left(1 - \sigma^{(1,0)}(\beta,z,\omega|f_j)\right)\tilde{\sigma}^{(0,a'_n)}(\beta,z,\omega|f_j).$$

(e) Given the decision probabilities $\sigma^{(1,0)}(\beta,z,\omega|f_j)$ and $\sigma^{(d,a')}(\beta,z,\omega|f_j)$, compute the new set of equilibrium functions, $f_{j+1} = (q^{a'}_{j+1}(\omega), \psi^{(0,a')}_{j+1}(\omega), \psi^{(1,0)}_{j+1}(\omega))$:

i. Compute $\psi^{(0,a')}_{j+1}(\omega)$ and $\psi^{(1,0)}_{j+1}(\omega)$ according to (16).

ii. Compute $q^{a'}_{j+1}(\omega)$ according to (15) using $p^{a'}_{j+1}(\omega)$ in (17) .

(f) Assess equilibrium function convergence in terms of the sup norm metric

$$\max\left\{\sup|\psi^{(d.a')}_{j+1}(\omega) - \psi^{(d.a')}_{j}(\omega)|, \sup|q^{a'}_{j+1}(\omega) - q^{a'}_{j}(\omega)|, \sup|W_{j+1}(\beta,z,\omega) - W_j(\beta,z,\omega)|\right\}$$

If less than tolerance, proceed to step 3; otherwise, start step 2 with f_{j+1} and $W_{j+1}(\beta,z,\omega)$.

3. Compute the stationary distribution.

(a) Given f_j from step (2), compute $\mu_{k+1}(\beta, z, \omega|f_j)$ using the transition operator T in (19) applied to $\mu_k(\beta, z, \omega|f_j)$.

(b) Assess convergence based on the sup norm metric $\sup|\mu_{k+1}(\beta, z, \omega|f_j) - \mu_k(\beta, z, \omega|f_j)|$. If less than tolerance, stop; otherwise, iterate on $\mu_{k+1}(\beta, z, \omega|f_j)$ using T.

B.2 Model moment definitions

The bankruptcy rate is computed as the total fraction of the population who files for bankruptcy within a given period. The probability of a given state is given by $\mu(\cdot)$, and the probability of bankruptcy given a state is $\sigma^{(1,0)}(\cdot)$, and so the aggregate bankruptcy rate is $\sum_{\beta,z,\omega} \sigma^{(1,0)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega)$. By type, we have $\sum_{\omega} \sigma^{(1,0)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega)/\sum_{\hat{\omega}} \mu(\beta, z, \hat{\omega})$. (Analogous type conditions hold for all other moments as well; we omit them here for brevity.) The fraction in debt is the share of the population choosing $a' < 0$ in a given period: $\sum_{\beta,z,\omega,a'<0} \mu(\beta, z, \omega)\sigma^{(0,a')}(\beta, z, \omega)$. The debt to income ratio is the ratio of average debt to average income: $\frac{\sum_{\beta,z,\omega,a'<0} a'\sigma^{(0,a')}(\beta,z,\omega)\mu(\beta,z,\omega)}{\sum_{\beta,z,\omega,a'<0} y(e(\omega),z)\mu(\beta,z,\omega)}$. The average interest paid in the economy is the weighted average of the interest rates paid, $1/q - 1$, over the stationary distribution and decision probabilities: $\sum_{\omega} \overline{\mu}(\omega) \cdot \sum_{\beta,z} \frac{\mu(\beta,z,\omega)}{\sum_{\hat{\beta},\hat{z}} \overline{\mu}(\omega)} \sum_{a'} \frac{\sigma^{(0,a')}(\beta,z,\omega)}{\sum_{\tilde{a}} \sigma^{(0,\tilde{a})}(\beta,z,\omega)} \left(\frac{1}{q^{a'}(\omega)} - 1\right)$ where $\overline{\mu}(\omega) = \sum_{\beta,z} \mu(\beta, z, \omega)$. The standard deviation is the square root of the second moment of this object.

B.3 Sensitivity Analysis: Implementation of Andrews et al. (2017)

We begin by computing the 10×8 Jacobian matrix $\hat{G}$ of the 10-vector of model moments with respect to the 8-vector of internally estimated model parameters. We approximate this matrix by taking numerical derivatives. Using a parameter step size of $\delta_p \cdot \hat{\theta}_p$ for $p = 1, ..., 8$ (i.e. proportional scaling, where δ_p is the proportional increase and $\hat{\theta}_p$ is the estimated parameter), we solve the model for the baseline calibration $\hat{\theta} = \{\hat{\theta}_p\}_{p=1}^{8}$, obtaining moment vector $\hat{m} = \{\hat{m}_n\}_{n=1}^{10}$, and for a sequence of 8 perturbations in with the p-th parameter is increased by the step size. We set $\delta_p = \delta = 0.1\%$ for all p. The entry of the estimated Jacobian matrix $\hat{G}$ corresponding to moment n and parameter p, is then $\hat{g}_{np} = (\hat{m}_{np} - \hat{m}_n)/\delta\hat{\theta}_p$.The transpose of this matrix, $\hat{G}'$ is presented in Table 5.

Given our estimate of $\hat{G}'$, we compute an estimate of Andrews et al. (2017) sensitivity matrix $\hat{\Lambda}$ using equation (24) with the identity weighting matrix $W = I_{10}$. What is presented in Table 4 is not $\hat{\Lambda}$ directly, but a more easily interpretable transformation which we now describe. Our goal is to answer the question: "by what percent would the estimated parameter $\hat{\theta}_p$ change if target moment m_n changed by δ_n percent?" We assume a change in moment m_n of $\delta_n \hat{m}_n$; for ease of exposition, we choose $\delta_n = 1\%$ for all n. Then, the bias in the $\hat{\theta}_p$ associated with the perturbation to moment m_n is $b_{pn} = \hat{\lambda}_{pn}\delta_n\hat{m}_n$, where $\hat{\lambda}_{pn}$ is the corresponding entry of the $\hat{\Lambda}$ matrix. We then report the implied percentage change relative to the estimated parameter, $\hat{\ell}_{pn} = \frac{\hat{\theta}_p - b_{pn}}{\hat{\theta}_p} - 1$. Each cell of Table 4 is the relevant $\hat{\ell}_{pn}$ entry.

B.4 The Role of Extreme Value Preference Shocks

One of the key modifications in our model relative to standard consumer bankruptcy models in macroeconomics is the inclusion of the additive, action-specific preference shocks.[49] The mean of these shocks is adjusted to insure that the utility bonus scales with the measure of feasible consumption rather than the density of the grid used for computation (see Briglia et al. (2021) for details). In contrast, we calibrate the scale parameters α and λ which govern the variance of the default and a' shocks, respectively. How does behavior in the model change with respect to these parameters? In this section we address this question by computing actual decision rules under different parameter combinations and describe the differences graphically.[50]

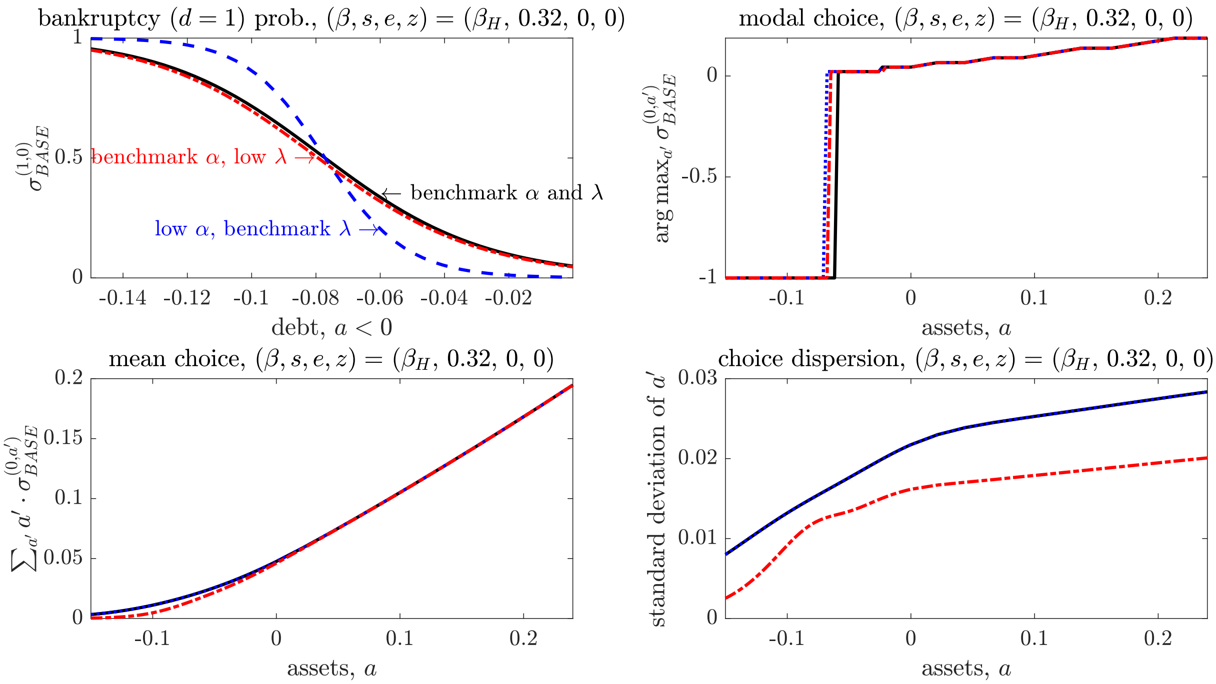
Figure 12 demonstrates the impact of changing α and λ on decisions in our baseline model. Each figure contains three lines, corresponding to: (i) the baseline parameterization of Table 3; (ii) a parameterization with low variance α on the bankruptcy decision in which λ is held fixed; and (iii) a parameterization with low variance λ on the a' decision in which α is held fixed. All figures are presented for an agents with $(\beta, s, e, z) = (\beta_H, F_{\beta_H}, 0, 0)$. In each parameterization, the equilibrium pricing function, and therefore the conditional action values, are held fixed, and so the changes in response shown here can be thought of as partial equilibrium in order to highlight the direct effects on decisions.

Consider first the bankruptcy filing decision. The top left panel shows how this decision varies over a range of levels of debt. By lowering α, the slope of increase in filing probability as the level of indebtedness increases is much sharper than in the baseline parameterization. This is because there is less chance for a high value shock to be realized for an action with lower fundamental value, so the decision rule becomes more centered at the mode for each level of a. By lowering λ, the expected value of repaying increases, and so the bankruptcy filing probability shifts down.

The remaining three figures show how a' decisions are affected by changes in the extreme value parameters. The top right panel depicts the modal decision across each case (with bankruptcy depicted as choosing $a' = -1$ for simplicity). Conditional on repaying, there is little change in the modal decision, but lowering either α or λ makes bankruptcy the modal decision only for larger levels of debt. The bottom left and bottom right panels show the mean and standard deviation of the savings decision rule, conditional on repaying, respectively. Changing α has virtually no effect conditional on repaying. Mean

[49] Dvorkin et al. (2021) have employed extreme value shocks to smooth out decision rules in models of sovereign default.
[50] An analytical approach is contained in the supplementary materials to this article.

Figure 12: Impact of extreme value preference shocks



Notes: "Benchmark" refers to the parameterization of the extreme value shock process from Table 3. Low α (λ) is half the baseline value: $\alpha' = \alpha/2$ ($\lambda' = \lambda/2$). All panels fix the state of an agent at $(\beta, s, e, z) = (\beta_H, F_{\beta_H}, 0, 0)$. In the top right panel, a modal choice of -1 corresponds to bankruptcy.

decisions are nearly linear in wealth for positive a given the low risk aversion, but there is convexity in the decision rule when in debt since default risk changes the return on borrowing relative to saving. Lowering λ lowers both the mean and standard deviation of savings choices, with the latter effect being more pronounced. Finally, we note that these changes in decision rules are similar (holding price and type score functions fixed) in the full information and no tracking economies as well.

B.5 Modal choice metrics

This section describes a series of metrics which quantify the dispersion in decisions implied by extreme value shocks. These results are summarized in Table 8, but we first describe the construction of the metrics. Let $x = (\beta, z, \omega)$ be the state variable of an agent, let $\sigma^{(d,a')}(x)$ denote her decision rule, and let $\mu(x)$ be the stationary distribution over individual states in the baseline economy. We want to get a sense of dispersion around the highest value (or modal) choice, which may be defined as

$$y^*(x) \equiv \arg \max_{(d,a') \in \mathcal{F}(x)} \sigma^{(d,a')}(x).$$

Table 8: Modal Choice Metrics

Action type	share for whom action type is modal (%)	share of total action from modal agents (%)	share of decisions w/in k grid pts. of mode (%) $k=0$	$k=1$	$k=2$
Default	2.72	5.25	-	-	-
Non-Default	99.8	99.8	49.5	83.1	93.3
Borrowing	8.10	86.6	34.7	80.3	93.8
Saving	91.6	99.9	50.8	83.4	93.2

Notes: For the right 3 columns, the share is computed over the population of agents for whom the action type is modal.

Let $\mathcal{Y} \subseteq \{(1,0), \{(0,a')\}|a' \in \mathcal{A}\}$ denote a set of possible actions. The share of agents for whom an action in set $\mathcal{Y}$ is modal is

$$m(\mathcal{Y}) = \sum_x \mu(x) 1\left[y^*(x) \in \mathcal{Y}\right] \tag{40}$$

where $1[S]$ is an indicator function which which takes on the value 1 if S is true. The total mass of agents choosing an action in the set $\mathcal{Y}$ includes those for whom the action is not modal, and so we can compute the share of the actions in this set accounted for by "modal agents," those for whom an action in this set is the mode, via

$$\frac{\sum_{x,(d,a')\in\mathcal{Y}} \mu(x)\sigma^{(d,a')}(x) 1\left[y^*(x) \in \mathcal{Y}\right]}{\sum_{x,(d,a')\in\mathcal{Y}} \mu(x)\sigma^{(d,a')}(x)}. \tag{41}$$

Lastly, for agents whose modal action is not default, we can compute the share of decisions within k grid points of the mode. For a given individual (whose mode is not default), let $i^*(x)$ denote the grid index of the mode $y^*(x)$. Then let a k-band of actions around the mode be defined by

$$\mathcal{Y}_k(x) = \{i^*(x) - k, ..., i^*(x), ..., i^*(x) + k\}.$$

Finally, define the total weight on decisions in the k-band of the mode for agent x via

$$\zeta_k(x) = \frac{\sum_{(0,a')\in\mathcal{Y}_k(x)} \sigma^{(0,a')}(x)}{1 - \sigma^{(1,0)}(x)},$$

where the denominator normalizes to exclude default. We can aggregate over any group of actions $\mathcal{Y}$

$$\overline{\zeta}(\mathcal{Y}_k) = \frac{\sum_{\{x|y^*(x)\in\mathcal{Y}_k\}} \zeta_k(x)\mu(x)}{m(\mathcal{Y})}. \tag{42}$$

B.6 Details of Alternative Economies

B.6.1 No tracking (NT)

The key formal difference in this economy relative to the baseline comes from the separation of the type score updates (which follow individuals) and the static assessment of types (relevant for pricing). An individual's type score updates based only on exogenous transition probabilities, and so there is no incentive to acquire reputation. As a result, s' evolves from s according to $\psi^1_{NT,\beta'}(s) = \sum_\beta Q^\beta(\beta'|\beta)s(\beta)$. In the two-type case we employ in our quantitative model, we have

$$\psi^1_{NT}(s) = sQ^\beta(\beta_H|\beta_H) + (1-s)Q^\beta(\beta_H|\beta_L). \tag{43}$$

In this version of the model, lenders perform **intraperiod** updating of type assessments based on the a' chosen by the borrower. That is, the lenders compute

$$\psi^2_{NT,\beta'}(a', s, e) \equiv \Pr(\beta'|a', s, e) = \sum_\beta Q^\beta(\beta'|\beta)\Pr(\beta|a', s, e).$$

All of the action is in the last term of the expression above, and so we analyze it here:

$$\begin{aligned}\Pr(\beta|a', s, e) = \frac{\Pr(\beta, a', s, e)}{\Pr(a', s, e)} &= \frac{\sum_{z,a}\Pr(\beta, a', s, e, z, a)}{\sum_{\tilde\beta,z,a}\Pr(\tilde\beta, a', s, e, z, a)}\\ &= \frac{\sum_{z,a}\sigma^{(0,a')}(\beta, e, z, a, s)\mu(\beta, e, z, a, s)}{\sum_{\tilde\beta,z,a}\sigma^{(0,a')}(\tilde\beta, e, z, a, s)\mu(\tilde\beta, e, z, a, s)},\end{aligned}$$

where the first line uses Bayes' Rule, the second sums over unobserved idiosyncratic states, and the third once more applies Bayes' Rule via

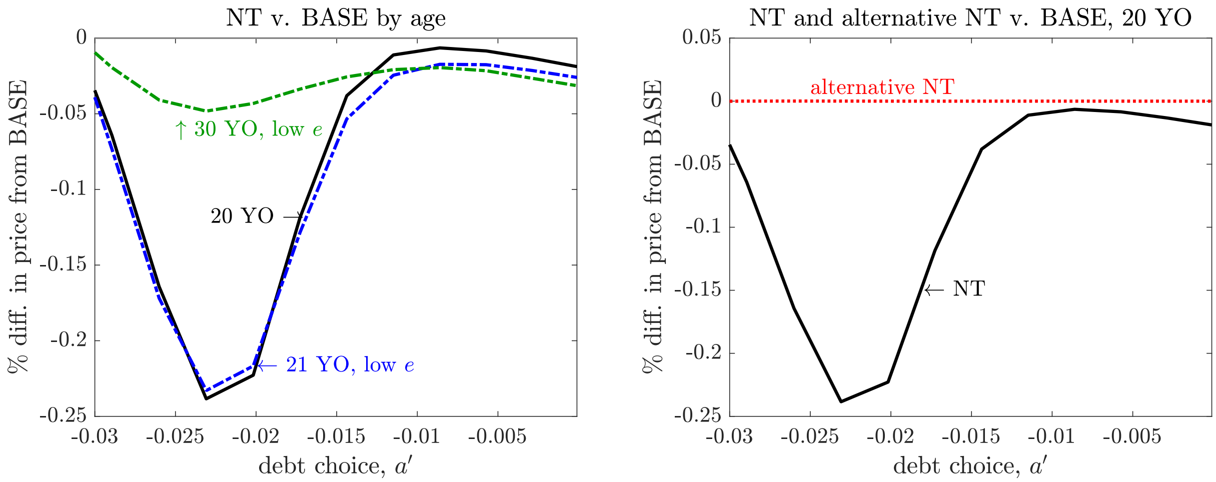
$$\Pr(a', \beta, e, z, a, s) = \Pr(a'|\beta, e, z, a, s)\Pr(\beta, e, z, a, s) = \sigma^{(0,a')}(\beta, e, z, a, s)\mu(\beta, e, z, a, s).$$

Therefore, we obtain

$$\psi^2_{NT,\beta'}(a', s, e) = \sum_\beta Q^\beta(\beta'|\beta)\frac{\sum_{z,a}\sigma^{(0,a')}(\beta, e, z, a, s)\mu(\beta, e, z, a, s)}{\sum_{\tilde\beta,z,a}\sigma^{(0,a')}(\tilde\beta, e, z, a, s)\mu(\tilde\beta, e, z, a, s)}. \tag{44}$$

What the lender must compute is the probability that a' is repaid tomorrow given s, e observed today. For each choice of a', the lender revises the **borrower's assessed type today** via (44). At the same time, though, due to the implicit "anonymity" assumption in this economy, they recognize

Figure 13: Loan Price Comparison Between BASE and NT Economies



Notes: Let s_j denote the average type score for an agent of age j, and let a_j^{NT} be the average wealth of an agent of age j in the NT economy. Each line in each panel represents $100 \cdot (q_{NT}^{a'}(j, e)/q_{BASE}^{a'}(e, a_j^{NT}, s_j) - 1)$. The black lines in the left and right panels are the same by construction. The "alternative" NT line in the right panel replaces σ in equation (44) with σ_{BASE}, the decision rules from the BASE economy. All price schedules are for the lowest $e = -0.71$.

that the **borrower's type score tomorrow** (which is relevant for tomorrow's default decision) will be determined via (43). Therefore, the $p(\cdot)$ function in this economy is

$$\begin{aligned} p(a', s, e) &= \Pr(\text{repay } a'|s, e) \\ &= \frac{\Pr(\text{repay } a', s, e)}{\Pr(s, e)} \\ &= \frac{\sum_{\beta', e', z', s'} \Pr(\text{repay } a'|\beta', e', z', s', a', s, e)\Pr(\beta', e', z', s'|a', s, e)}{\sum_{\beta, a, z} \Pr(\beta, e, z, a, s)} \\ &= \frac{\sum_{\beta', e', z'} \left[1 - \sigma^{(1,0)}(\beta', e', z', a', \psi_{NT}^1(s))\right] \Pr(\beta', e', z'|a', s, e)}{\sum_{\beta, a, z} \mu(\beta, e, z, a, s)} \\ &= \frac{\sum_{\beta, \beta', e', z'} \left[1 - \sigma^{(1,0)}(\beta', e', z', a', \psi_{NT}^1(s))\right] Q^e(e'|e)H(z')Q^\beta(\beta'|\beta)\Pr(\beta|a', s, e)}{\sum_{\beta, a, z} \mu(\beta, e, z, a, s)} \\ &= \psi_{NT}^2(a', s, e) \sum_{e', z'} \left[1 - \sigma^{(1,0)}(\beta_H, e', z', a', \psi_{NT}^1(s))\right] Q^e(e'|e)H(z') \\ &\quad + \left(1 - \psi_{NT}^2(a', s, e)\right) \sum_{e', z'} \left[1 - \sigma^{(1,0)}(\beta_L, e', z', a', \psi_{NT}^1(s))\right] Q^e(e'|e)H(z'), \end{aligned} \tag{45}$$

where the last line once more applies the two-type implementation from our quantitative model.

Figure 13 shows the percentage differences between the price menus faced by some agents in the NT economy relative to the BASE economy in comparable states (specifically the lowest persistent earning state $e = -0.71$ since they are the likely to borrow). For newborns (i.e. 20 year olds in our mapping to the data) the comparison is easy, as all newborns begin life in the same observable state in both economies (and it is common knowledge they do), i.e., they all have zero assets, are in the low earnings class, and are high types with probability 0.32. The price difference comes only from the different probabilities of repayment across the two economies owing to differences in dynamic incentives. Prices are comparable up to a loan size of 0.01 and for larger loans the prices are lower in NT, reflecting higher default probabilities at each loan size. This is due to the lower incentives to repay in the NT economy. These incentive effects, though mitigated, are present even at older ages.

A decomposition exercise In order to highlight the role of dynamic reputational incentives, we construct an alternative price schedule for the NT economy by replacing the decision rule $\sigma(\cdot)$ in the definition of the static inference function $\psi^2_{NT}(\cdot)$ defined in equation (44) with the decision rule from the baseline economy, $\sigma_{BASE}(\cdot)$. Having obtained this alternative $\tilde{\psi}^2_{NT}(\cdot)$, we then compute repayment probabilities (and therefore prices) according to (45) with $\psi^2_{NT}(\cdot)$ replaced by $\tilde{\psi}^2_{NT}(\cdot)$. The alternative price schedule is depicted – relative to the analogous price schedule for the baseline economy – for the youngest cohort in the red dashed line in the right panel of Figure 13.[51] For convenience, we also present the standard NT price schedule (solid black line) in this figure. The alternative price schedule is virtually indistinguishable from the baseline price schedule, while the NT prices differ significantly from the baseline. What drives this? In the alternative, the static inference of type reflects the dynamic reputational incentives of the baseline model by construction. The fact that the alternative and baseline so closely resemble each other while the NT and baseline differ markedly highlights the role of dynamic reputational incentives.

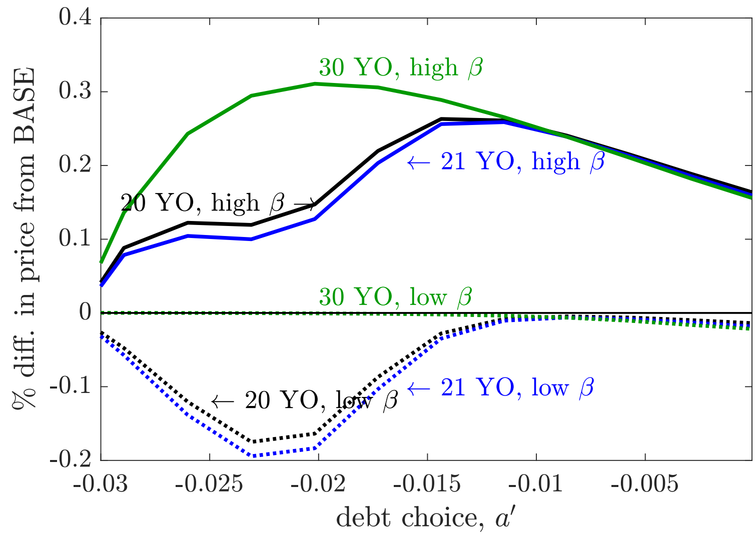
B.6.2 Full information (FI)

Since there is no incentive to infer one's type, there is no type score in this model. Therefore, an agent's full state is (β, e, z, a), and the set of equilibrium functions does not include ψ. For comparability, and since it is purely i.i.d. and contains no information for inference, we maintain the assumption that z is unobservable. Therefore, the lender can observe $\omega_{FI} = (\beta, e, a)$ for each individual.

The household problem and equilibrium stationary distribution are exactly the same as in the main text, with the state variable s removed. The only substantial change is in the pricing and repayment probability equations. The repayment probability function in this case is $p^{(0,a')}_{FI}(\omega_{FI}) = \Pr(\text{repay } a'|\omega_{FI})$. Since ω_{FI} directly includes β and z is i.i.d., there is no further inference to be done. Therefore, a has

[51] We choose the youngest cohort to avoid integrating over a given the assumption across all models that all individuals start with no wealth and the different arguments to the pricing functions in the NT and BASE economies.

Figure 14: Loan Price Comparison Between Full Information and Baseline Economies



Notes: Let s_j and a_j^{FI} denote the average type score for an age-j agent in BASE and FI, respectively. Each line in the figure represents $100 \cdot (q_{FI}^{a'}(\beta, e)/q_{BASE}^{a'}(e, a_j^{FI}, s_j) - 1)$. All price schedules are for the lowest e.

no impact on pricing, and we obtain

$$p_{FI}^{a'}(\beta, e) = \sum_{\beta', e', z'} \left[1 - \sigma_{FI}^{(1,0)}(\beta', e', z', a')\right] Q^{\beta}(\beta'|\beta) Q^{e}(e'|e) H(z'). \tag{46}$$

The loan pricing function, $q_{FI}^{a'}(\beta, e)$, adjusts for the interest rate as in the baseline model.

In Figure 14, we compare the prices that individuals of a given age face in the FI economy with their counterpart in the BASE economy who has the average type score for that age and the FI economy's average asset holdings for that age. Since we use the average type score for a given age, the price comparison does take into account the learning that naturally occurs in the Base economy (i.e. type H (L) have a higher (lower) type score than the average type score for their given age).

As one might expect, Figure 14 shows that for each age, type L in the FI economy face lower loan prices (higher interest rates) and type H face higher prices (lower interest rates) than the BASE economy where there is some cross-subsidization. The figure also shows that these price differences change with age. Recall that current asset holdings do not affect debt prices in the FI model but do in the BASE model. As individuals age and accumulate assets, this has an impact on $q_{BASE}^{(0,a')}$. Any type in the BASE economy who borrows by age 30 having accumulated precautionary assets is very likely assessed to be type β_L. Thus there is not much difference between the economies for a 30 year old type β_L which explains the imperceptible price difference, but since type β_H is pooled with type β_L by their borrowing, hence facing much lower $q_{BASE}^{(0,a')}$, the price difference is magnified.

C Data Appendix

This appendix describes the construction of the data underlying the life cycle credit ranking moments reported in Table 2 and Figures 1, 2, and 3. We begin with a 2 percent random sample of the FRBNY CCP/Equifax anonymized panel containing an individual's birth year and an individual's credit score in each quarter of 2003, 2004, and 2005. The credit score measure is the Equifax Risk Score (hereafter Risk Score), which is a proprietary credit score similar to other risk scores used in the industry. We consider only living individuals who were between the ages of 21 and 60 years in 2004 and had a Risk Score value in each quarter of the three years. This yields our *base sample*.

For this sample, we compute the within-quarter percentile ranking of individual's Risk Scores in each quarter. We call this the individual's *credit ranking* — it is a number that gives the fraction of people who had Risk Scores not exceeding the individual's score in that quarter. We then placed individuals in 5-year age bins according to their age in 2004. We compute the mean and standard deviations of the credit rankings in each bin, averaged over the four quarters of 2004. These moments were used in the regressions that determine the coefficients in the first 4 rows of the middle panel of Table 2. To obtain the autocorrelations, we computed, for each quarter of 2004, the changes in an individual's credit ranking from the same quarters in 2003 and 2005. For each age bin, we then computed the correlation between these pairs of individual changes for each quarter of 2004.

Turning next to the default event study in Figure 3, we first isolated individuals 26 years or older who filed for Chapter 7 bankruptcy in 2004 in our base sample. This yielded our *base sample of bankrupts*. For each individual in this sample, we recorded birth year and Risk Score in the filing quarter and in the 16 quarters preceding and following the filing quarter. We converted each Risk Score into a credit ranking by computing the percentile of each Risk Score in the overall distribution of Risk Scores. We then placed each individual in the appropriate 5-year age bin based on her age in 2004. We computed the average credit ranking (percentile) in each age bin for each of the 33 quarterly observations.

D Delinquency

Default can arise either through delinquency, whereby agents neither repay their debts nor file for bankruptcy, as well as bankruptcy. Here we modify out model to include this option. In our modified model, the unobservable income loss from default can depend on one's type by a factor of proportionality $\tau(\beta)$, where $\tau(\beta_H) \geq \tau(\beta_L) = 1$ so that a default can be weakly more costly for high types than low.[52]

[52] Corbae and Glover (2018) provides an adverse selection labor matching model with pre-employment credit screening which generates a larger income loss for type β_H than type β_L.

Under the former choice, the household's income net of any costs associated with delinquency (which we take to be $y(e,z)(1-\kappa_2 \cdot \tau(\beta))$ with $\kappa_2 < \kappa_1$ so there are lower costs than in bankruptcy) is used for consumption and its obligation next period is the face value of its current debt plus a penalty specified in the contract that we take to be a factor $\eta > 0$ of the debt. Upon becoming delinquent, a household can payback its debts, file for bankruptcy, or become delinquent again. Lenders with delinquent debt are required (by law) to remove (charge-off) such debts from their books which they do by selling delinquent debt to third-party collectors. For simplicity, we assume all delinquent debt is pooled and sold to third party collection agencies at an equilibrium price $\bar{q}_\delta$ to be described below. Buyers of delinquent debt operate at a per unit cost γ and are competitive.

D.1 The household problem

We modify the problem in Section 3.1 by expanding the set $\mathcal{D} = \{0, 1, 2\}$ where $d = 2$ signifies delinquency. Delinquency adds a new option and allows a household to avoid repaying its debt without incurring a bankruptcy fee but saddling it with more debt next period. In this case (3) becomes:

$$c^{(d,a')}(z,\omega|f) = \begin{cases} y(e(\omega),z) + a(\omega) - q^{a'}(\omega) \cdot a' & \text{if } (d,a') = (0,a'), \\ y(e(\omega),z)(1-\kappa_1 \cdot \tau(\beta))) - \kappa & \text{if } a(\omega) < 0 \text{ and } (d,a') = (1,0), \\ y(e(\omega),z)(1-\kappa_2 \cdot \tau(\beta)) & \text{if } a(\omega) < 0 \text{ and } (d,a') = (2, a(\omega)(1+\eta) \geq a_1). \end{cases} \tag{47}$$

The addition is the last line of (47). For any $a(\omega) \in [a_1, 0)$ such that $a(\omega)(1+\eta) < a_1$ (i.e. a delinquency would take the agent past the lowest grid point) we assume the agent cannot go delinquent and must choose either bankruptcy or repayment (both of which are feasible by Assumption 1). These assumptions imply that delinquency can only happen a finite number of times in a row.[53]

Recall that earlier a household first chose whether to file for bankruptcy or not, and if not, how much to save. We now pose that the household chooses whether to default or not *and* the mode of default. If the household chooses to default, it also chooses whether to file for bankruptcy or to become delinquent; if it does not, it chooses how much to save, receiving a vector of shocks ϵ attached to each a' choice exactly as in the baseline model according to (22). To allow for correlation between the shocks associated with the default actions we posit a nested logit structure for the shocks no default / bankruptcy / delinquency shocks. That is, rather than the independent draws from (21) as in the

[53] When $a(\omega)(1+\eta)$ is not on the grid $\mathcal{A}$, similar to what we did with type scores, we distribute $a(\omega)(1+\eta) \in [a_j, a_{j+1}]$ with probability w to a_j and probability $1-w$ to a_{j+1} where $w = (a_{j+1} - a(\omega)(1+\eta))/(a_{j+1} - a_j)$.

baseline, the vector ν is now drawn from

$$F_\nu(\nu) = \exp\left\{-\exp\left(-\frac{\nu^{d=0}-\bar{\nu}}{\alpha}\right) - \left[\exp\left(-\frac{\nu^{d=1}-\bar{\nu}}{\phi\alpha}\right) + \exp\left(-\frac{\nu^{d=2}-\bar{\nu}}{\phi\alpha}\right)\right]^\phi\right\} \tag{48}$$

where the new parameter ϕ specifies the correlation between the shocks associated with bankruptcy ($d = 1$) and delinquency ($d = 2$).[54] The value functions conditional on each one of the choices in the feasible set $\mathcal{F}(z, \omega)$ follow trivially.

D.2 Pricing

All that remains is to determine how lenders price debt given the two types of default. Regulation requires that banks charge off loans that are severely past due.[55] Hence, unlike Athreya et al. (2019) where delinquent debt is held on a lender's balance sheet as long as the individual is delinquent, we assume all delinquent debt is pooled after a period and sold at price $\bar{q}_\delta$ per unit. Competition ensures that debt collectors obtain zero profits net of the transaction (collection) costs to the lending process.

Turning first to the new pricing equation of loans by the financial intermediary, the probability of repayment on a new loan of size a' is altered from that given in equation (17) to

$$\begin{aligned} p^{a'}(\omega) \quad = \quad & \sum_{\beta',z',e',s'} H(z')\cdot Q^e(e'|e)\cdot Q^s(s'(\beta')|\psi_{\beta'}^{(0,a')}(\omega))\cdot s'(\beta') \\ & \cdot\left[1-\sigma^{(1,0)}(\beta',z',e',a',s')-\sigma^{(2,(1+\eta)a')}(\beta',z',e',a',s')\right]. \end{aligned} \tag{49}$$

The probability of delinquency on that new loan is

$$\begin{aligned} \delta^{a'}(\omega) \quad = \quad & \sum_{\beta',z',e',s'} H(z')\cdot Q^e(e'|e)\cdot Q^s(s'(\beta')|\psi_{\beta'}^{(0,a')}(\omega))\cdot s'(\beta') \\ & \cdot\sigma^{(2,(1+\eta)a')}(\beta',z',e',a',s'). \end{aligned} \tag{50}$$

Consequently, the competitive price of a new loan offered by lenders is altered from (15) to

$$q^{a'}(\omega) = \frac{\rho}{(1+r)}\left[p^{a'}(\omega) + \delta^{a'}(\omega)\cdot\bar{q}_\delta\cdot(1+\eta)\right], \tag{51}$$

where the second term on the right is the recovery from selling the delinquent debt to a collector.

[54] The adjustment to kill the bonus associated with debtors' extra options in this setting is now $\bar{\nu} = -\alpha\gamma_E - \alpha\ln(1+2^\phi)$.

[55] From https://en.wikipedia.org/wiki/Charge-off, In the United States, federal regulations require creditors to charge off installment loans after 120 days of delinquency, while revolving credit accounts must be charged-off after 180 days.

Turning next to the value of debt held by a collection agency, the probability of repayment on delinquent debt a of a household in state ω held by a collector is

$$\begin{aligned} p_\delta^{(1+\eta)a}(\omega) &= \sum_{\beta',z',e',s'} H(z') \cdot Q^e(e'|e) \cdot Q^s(s'(\beta')|\psi_{\beta'}^{(2,(1+\eta)a)}(\omega)) \cdot s'(\beta') \\ &\cdot \left[1 - \sigma^{(1,0)}(\beta', z', e', (1+\eta)a, s') - \sigma^{(2,(1+\eta)^2 a)}(\beta', z', e', (1+\eta)a, s')\right] \end{aligned} \tag{52}$$

noting the key differences between (49) and (52) are the type updates $\psi_{\beta'}^{(d,a')}$ and the future debt obligations a'. Equation (52) makes clear that punishment associated with delinquency arises from being saddled with penalties augmenting what is owed and delinquency's impact on type score.

We assume a collector does not need to discharge its own debt holdings if a person becomes delinquent again, but it pays collection costs γ each period. Denoting by $q_\delta^{(1+\eta)a}(\omega)$ the value per unit of delinquent debt a of a person in state ω held by a collector, we have

$$\begin{aligned} q_\delta^{(1+\eta)a}(\omega) &= \frac{\rho}{(1+r)(1+\gamma)}\Bigg[p_\delta^{(1+\eta)a}(\omega) \\ &+ \sum_{\beta',z',e',s'} H(z') \cdot Q^e(e'|e) \cdot Q^s(s'(\beta')|\psi_{\beta'}^{(2,(1+\eta)a)}(\omega)) \cdot s'(\beta') \\ &\cdot \sigma^{(2,(1+\eta)^2 a)}(\beta', z', e', (1+\eta)a, s') \cdot q_\delta^{(1+\eta)^2 a}(e', (1+\eta)a, s') \cdot (1+\eta)\Bigg]. \end{aligned} \tag{53}$$

The zero profit condition for debt collectors is then

$$\bar{q}_\delta = \frac{\sum_{\beta,z,\omega} q_\delta^{(1+\eta)a}(\omega) \cdot a \cdot \sigma^{(2,(1+\eta)a)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega)}{\sum_{\beta,z,\omega} a \cdot \sigma^{(2,(1+\eta)a)}(\beta, z, \omega) \cdot \mu(\beta, z, \omega)}. \tag{54}$$

Substituting (53) into (54) yields γ residually given an observed $\bar{q}_\delta$. We require that $\gamma \geq 0$.

D.3 Parameterization

To illustrate our model with both bankruptcy and delinquency, we supplement the estimated parameters from the BASE model with parameters chosen to approximate certain moments like credit card recovery rates, delinquency rates and penalties, and certain restrictions implied by the model on the data. We set the recovery rate $\overline{q}_\delta$ to 0.22 as in Chatterjee and Gordon (2012). We set the penalty rate in delinquency η to 30% consistent with industry averages.[56] The extreme value parameter $\phi = 0.2$ and variable cost in delinquency $\kappa_2 = 0.03$ are set to be roughly consistent with the bankruptcy and

[56] See for example, https://www.thebalancemoney.com/credit-card-default-and-penalty-rates-explained-960643.

Table 9: Target Moments, Delinquency v Baseline

Moment (%)	Data	Baseline	Delinquency
Aggregate credit market moments			
Bankruptcy rate	1.00	1.02	0.74
Average interest rate	11.9	11.5	16.6
Interest rate dispersion	7.00	7.08	2.76
Fraction of HH in debt	7.92	9.16	11.4
Debt to income ratio	0.40	0.26	0.36
Delinquency rate	1.54	N.A.	1.11
Credit ranking age profile moments			
Intercept, mean credit ranking	0.278	0.325	0.394
Slope, mean credit ranking	0.038	0.037	0.022
Intercept, std. dev. credit ranking	0.215	0.219	0.267
Slope, std. dev. credit ranking	0.011	0.010	0.003
Average autocorrelation of change in credit ranking	-0.220	-0.204	-0.238

Notes: Our model is yearly, so we classify delinquency as for 4 consecutive quarters of delinquency.

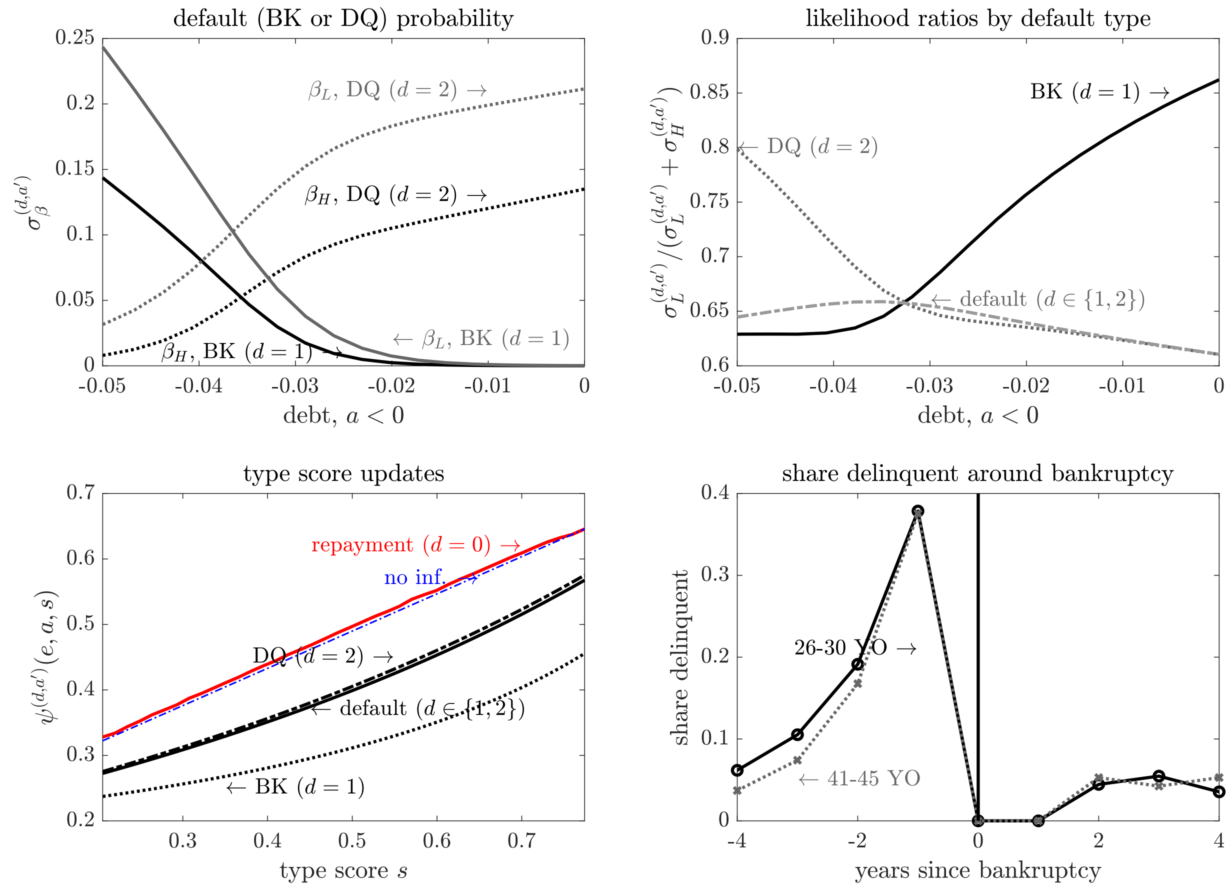
delinquency rate (measured as being delinquent for four quarters in a row consistent with our annual model period). The collection cost $\gamma = 2.07$ satisfies (54) given (53). Finally, the proportional income loss from default $\tau(\beta_H)$ is 25% higher for type β_H than β_L.

D.4 How does a delinquency option change equilibrium outcomes?

In Table 9, we provide the moments from our delinquency extension of the BASE model (adding the 4 quarter delinquency rate that was absent from Table 2). While there are some differences, perhaps the most noteworthy result is that the addition of the delinquency option yields model moments not very different from their data counterparts despite not re-estimating the model.

As in the BASE model, type β_L default (i.e. choose either delinquency or bankruptcy) more than type β_H as evident in the top right panel of Figure 15 since their likelihood ratio for default exceeds 0.5 (similar to the earlier results in Figure 4). The novel aspects stem from the fact that, as evident in the budget sets of equation (47), delinquency provides a low current resource cost way to default at the expense of incurring more future debt and lowering one's future reputation. Since type β_L care less

Figure 15: Bankruptcy and Delinquency Choice Probabilities



Notes: The individual state for the top panels of this figure is $s = 0.48$ and $e = z = 0$. The left panel presents the probability of either type of default for each type while the right panel presents the likelihood ratio for each type of default. The bottom left panel The bottom right panel plots the share of agents from a simulated panel who file for bankruptcy in year 0 who are delinquent in year t. This share is zero in the year of the bankruptcy (declaring bankruptcy precludes delinquency) and the year after (bankruptcy in year 0 implies $a = 0$ in year 1, so delinquency is infeasible).

about the future and more about current consumption than type β_H and have lower costs of default, they are more likely to choose delinquency and bankruptcy. This difference is clearly evident in the top row of Figure 15. Since type β_L is more likely to go delinquent and bankrupt, the bottom left panel of Figure 15 shows that such default decisions lead to a fall in their type scores similar to the earlier results in Figure 6.[57] It also shows that a bankruptcy leads to a bigger downward revision of type score than a delinquency.[58] As one might expect, the bottom right panel of Figure 15 shows that bankruptcies often follow delinquencies; a little more than 20% of the individuals who choose bankruptcy are already delinquent (for a model period of one year).[59]

[57] The bottom left panel also shows that repayment raises one's score relative to the no inference case.

[58] This ordering is sensitive to $\tau(\beta_H)$. A bankruptcy can hurt one's type score less than delinquency for $\tau(\beta_H)$ close to 1.

[59] There is a smaller spike in delinquencies preceding bankruptcy for the older cohort since they have lower debts.

The extended model provides testable predictions. For example, the top left panel shows that both types are more likely to choose delinquency for low debt levels and more likely to choose bankruptcy for high debt levels (for a given earnings and type score). This generates a pattern where for both types as debt grows, they substitute out of delinquency into bankruptcy as a form of default. This is intuitive as the future debt cost of delinquency is more severe with higher debt. If one integrates across all individuals who default, this provides a prediction that those who go bankrupt have higher debt levels than first-time delinquents. Our model generates a 17% higher level of debt held by bankrupts than first time delinquents while the data generates a 37% higher level.[60]

References

Andrews, I., M. Gentzkow, and J. Shapiro (2017): "Measuring the Sensitivity of Parameter Estimates to Estimation Moments," *Quarterly Journal of Economics*, 132, 55–70.

Athreya, K., J. Mustre-del-Rio, and J. M. Sanchez (2019): "The Persistence of Financial Distress," *Review of Financial Studies*, 32, 3851–83.

Briglia, L., S. Chatterjee, D. Corbae, K. Dempsey, and V. Rios-Rull (2021): "Saving for a Sunny Day: An Alternative Theory of Precautionary Savings," Unpublished.

Chatterjee, S. and G. Gordon (2012): "Dealing with Consumer Default: Bankruptcy vs Garnishment," *Journal of Monetary Economics*, 59, S1–S16.

Corbae, D. and A. Glover (2018): "Employer Credit Checks: Poverty Traps versus Matching Efficiency," NBER Working Paper 25005.

Dvorkin, M., J. M. Sanchez, H. Sapriza, and E. Yurdagul (2021): "Sovereign Debt Restructurings," *American Economic Journal: Macroeconomics*, 13, 26–77.

Hutson, V. and J. S. Pym (1980): *Applications of Functional Analysis and Operator Theory*, Academic Press.

Stokey, N. L. and R. E. Lucas Jr. (1989): *Recursive Methods in Economic Dynamics*, Harvard University Press.

[60]The average credit card debt of delinquents (bankrupts) is $5564 ($7625) for a ratio of 1.37 (estimates are for individuals with positive credit card debt in 2004 (authors' calculations using FRBNY CCP/Equifax data)).

Other Supplementary Materials

A Welfare Metric

Table 7 reports welfare using a wealth equivalent measure whose construction we describe in this section. Given that utility flows in our model are derived not only directly from consumption flows but also from the extreme value shocks attached to consumption choices, we are unable to use a standard Lucas consumption equivalent measure.[61] As an alternative, we construct a measure that answers the question: "how much additional wealth must an agent in state (β, z, ω) be given in the BASE economy in order to be indifferent between being born into the BASE economy or into a given alternative (ALT) economy?" Denoting the value functions in the BASE and ALT economies by $W_{\text{BASE}}(\beta, z, \omega)$ and $W_{\text{ALT}}(\beta, z, \omega)$, respectively, we formally solve for a set of numbers $\phi_{\text{ALT}}(\beta, z, \omega)$ that satisfies

$$W_{\text{BASE}}\left(\beta, z, a(\omega) + \phi_{\text{ALT}}(\beta, z, \omega), s(\omega), e(\omega)\right) = W_{\text{ALT}}\left(\beta, z, a(\omega), s(\omega), e(\omega)\right) \tag{55}$$

Note that if the ALT economy is the FI economy, then s is not a state variable. However, ϕ_{ALT} still depends on s since this shifts the value in the BASE economy. The numbers reported in Table 7 are the ratios of the numbers computed via this expression to mean wealth in the BASE economy, $\overline{a} = \sum_a a \cdot \sum_{\beta,z,s,e} \mu(\beta, z, a, s, e)$.

This measure has several attractive properties. First, by varying wealth in the BASE economy only, we avoid any issues related to the fact that in the NT economy there are no agents with the type score of a newborn and debt, since type scores increase monotonically with age and all newborns are born with zero wealth. Second, the units are directly interpretable as physical quantities of wealth rather than utils. Relatedly, the change in wealth is purely discretionary and not imposed to be translated into consumption immediately. Third, it involves only a trivial calculation given the value functions from each equilibrium; the set of numbers ϕ may be solved via simple bisection.

B Deriving the impact of EV parameters

To ease notation in this section, let an agent's entire state be denoted by $x = (\beta, e, z, a, s)$, and the set of feasible actions for that agent be denoted by $\mathcal{F}(x)$. The goal of this section is to show how the choice probability function σ varies with the extreme value scale parameters α and λ. We first cover the repayment ($d = 0$) actions, and then bankruptcy ($d = 1$). To ease computations in this section rather

[61] In particular, the indirect utility function for an agent in state (β, z, ω) may not be written as an appropriately discounted infinite sum of period utility flows.

than compute derivatives with respect to λ or α directly, we will compute them with respect to $1/\lambda$ or $1/\alpha$.[62] Throughout this section, we focus on the first order effects of changes in these parameters by ignoring all derivatives with respect to action-specific value terms, i.e. $\frac{\partial v^{(d,a')}(x)}{\partial \frac{1}{\lambda}}$.

Saving and borrowing actions Equation (9) describes the probability of choosing a feasible action $(0, a')$ conditional on not filing for bankruptcy. Considering first λ,

$$\begin{aligned}
\frac{\partial \tilde{\sigma}^{(0,a')}(x)}{\partial(1/\lambda)} &= \left[\exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\} \frac{v^{(0,a')}(x)}{\lambda} \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\} \right. \\
&\quad \left. - \exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\} \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\} \frac{v^{(0,\tilde{a})}(x)}{\lambda} \right] \Bigg/ \left[\sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\} \right]^2 \\
&= \frac{\exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\}}{\sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\}} \frac{\sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\} \left[v^{(0,a')}(x) - v^{(0,\tilde{a})}(x)\right]}{\sum_{(0,\tilde{a})\in\mathcal{F}(x)} \exp\left\{\frac{v^{(0,\tilde{a})}(x)}{\lambda}\right\}} \\
&= \tilde{\sigma}^{(0,a')}(x) \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) \left[v^{(0,a')}(x) - v^{(0,\tilde{a})}(x)\right].
\end{aligned}$$

We can sign this derivative according to

$$\begin{aligned}
\frac{\partial \tilde{\sigma}^{(0,a')}(x)}{\partial(1/\lambda)} > 0 \quad &\Longleftrightarrow \quad \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) \left[v^{(0,a')}(x) - v^{(0,\tilde{a})}(x)\right] > 0 \\
&\Longleftrightarrow \quad v^{(0,a')}(x) > \sum_{(0,\tilde{a})\in\mathcal{F}(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) v^{(0,\tilde{a})}(x),
\end{aligned}$$

where the second line uses the fact that $\sum_{(0,\tilde{a})\in F(x)} \tilde{\sigma}^{(0,\tilde{a})}(x) = 1$ by construction. Therefore, the probability of choosing $(0, a')$ conditional on not filing for bankruptcy increases in $1/\lambda$ (decreases in λ) if and only if the conditional value of choosing $(0, a')$, $v^{(0,a')}(x)$, exceeds the expected value of choosing from the set of alternative actions $(0, \tilde{a})$ at the current decision rule.

The inclusive value of repaying, $W_{ND}(x)$, takes the familiar log-sum form of (6). Since it will be useful in computing how $\sigma^{(1,0)}(x)$ varies with λ, we compute:

$$\begin{aligned}
\frac{\partial W_{ND}(x)}{\partial(1/\lambda)} &= \lambda \left[\frac{\sum_{(0,a')\in\mathcal{F}(x)} \exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\} v^{(0,a')}(x)}{\sum_{(0,a')\in\mathcal{F}(x)} \exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\}} - \lambda \ln\left(\sum_{(0,a')\in\mathcal{F}(x)} \exp\left\{\frac{v^{(0,a')}(x)}{\lambda}\right\} \right) \right] \\
&= \lambda \left[\sum_{(0,a')\in F(x)} \tilde{\sigma}^{(0,a')}(x) v^{(0,a')}(x) - W_{ND}(x) \right]
\end{aligned}$$

[62] This keeps the analysis clean by avoiding repeated applications of the quotient rule for derivatives to the extent possible.

which is positive if and only if the average action-value weighted by decision probabilities exceeds the value of filing for bankruptcy.

Bankruptcy Equation (10) defines the probability of filing for bankruptcy as a function of the conditional value of filing for bankruptcy and the inclusive value of repaying. We obtain

$$\begin{aligned}
\frac{\partial \sigma^{(1,0)}(x)}{\partial(1/\lambda)} &= -\sigma^{(1,0)}(x)\left(1-\sigma^{(1,0)}(x)\right)\frac{\frac{\partial W_{ND}(x)}{\partial(1/\lambda)}}{\alpha} \\
\frac{\partial \sigma^{(1,0)}(x)}{\partial(1/\alpha)} &= \sigma^{(1,0)}(x)\left(1-\sigma^{(1,0)}(x)\right)\left(v^{(1,0)}(x)-W_{ND}(x)\right)
\end{aligned}$$

The first expression above implies that $\frac{\partial \sigma^{(1,0)}(x)}{\partial(1/\lambda)}$ takes the opposite sign of $\frac{\partial W_{ND}(x)}{\partial(1/\lambda)}$. If raising λ raises $W_{ND}(x)$, it makes repaying more attractive and therefore lowers the probability of filing. The second expressions shows that as α decreases, the probability of filing increases if and only if the conditional value of filing exceeds the inclusive value of repaying.

C Extreme value shocks in the alternative economies

In this section, we explore our alternative economies under different extreme value parameterizations in order to measure how these parameters affect our counterfactuals. We present two robustness exercises relative to our results in Table 7 of Section 6. Specifically, the "benchmark" column in Table 10 simply provides our model moments and welfare measures in the FI and NT economies relative to the BASE economy using our estimated parameters (i.e. replicates Table 7). We compare these same measures under two alternative parameterizations: one in which we raise α by 10% in *both* the base and alternative economies and one in which we similarly raise λ by 10%. The results of this analysis are presented in the remaining columns of Table 10. It should be noted that these 10% increases in α and λ can have large effects on outcomes in the BASE economy (to which we are comparing the alternative outcomes) since the estimated values were chosen to match the data in the benchmark. Furthermore, 10% parametric decreases in α and λ can yield even bigger deviations between model and data moments, since this correspond to significant increases in the informational content of actions, especially for α.

Starting with Panel A for the NT economy in Table 10, we note that the differences across columns are not large relative to the BASE values. This continues to be the case for the FI economy. Panel B shows that the positive welfare mean gain in the NT economy is eliminated as we raise α and λ indicating that the negative effects on dynamic incentives outweigh the static insurance benefits at already high levels of pooling. The welfare numbers in Panel B for the FI economy indicate that there can be larger gains in the presence of higher α and λ since inference is harder in the BASE economy, so that FI revelation of type benefits welfare.

Table 10: Alternative Economies and Extreme Value Parameters

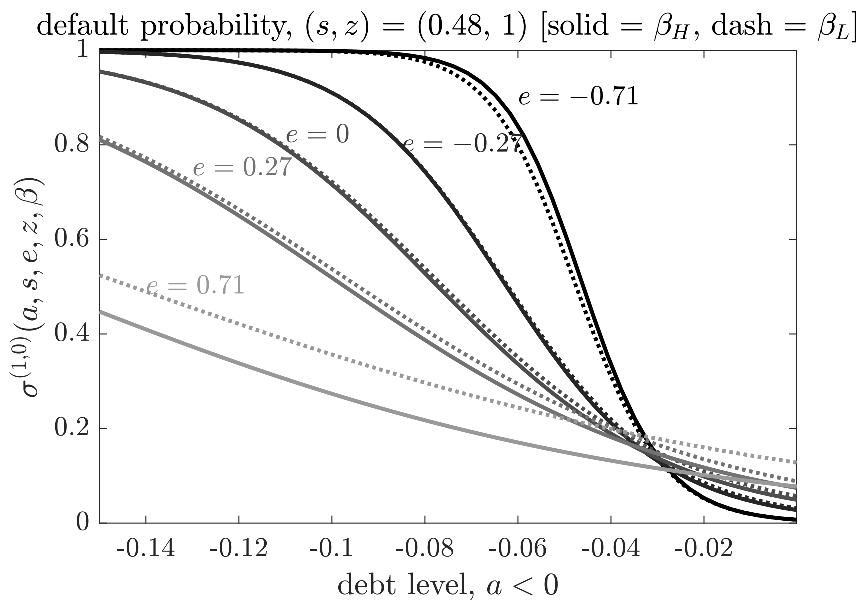
model	**No Tracking (NT)**			**Full Information (FI)**		
parameterization	benchmark	high α	high λ	benchmark	high α	high λ
α value	0.0290	**0.0319**	0.0290	0.0290	**0.0319**	0.0290
λ value	0.0015	0.0015	**0.0017**	0.0015	0.0015	**0.0017**
Panel A: % difference from BASE model with same parameters						
bankruptcy rate	1.12	1.02	1.07	-0.13	0.27	-0.09
average int. rate	1.44	1.49	1.45	-2.52	-2.57	-2.48
int. rate dispersion	7.57	5.28	5.80	-2.15	-1.40	-1.58
fraction in debt	-0.15	-0.23	-0.17	0.00	0.05	0.01
debt to income ratio	0.39	0.35	0.35	-0.04	0.17	-0.02
Panel B: wealth equivalent welfare measure, newborns						
low z	0.060	-0.003	-0.001	0.121	0.547	0.139
median z	-0.000	-0.001	0.000	0.058	0.188	0.064
high z	-0.000	-0.001	0.000	0.104	0.163	0.103
mean	0.020	-0.002	0.000	0.094	0.299	0.102

Notes: Each entry in Panel A is the difference, in percentage points of the BASE moment, of the moment in the indicated alternative economy (FI or NT) relative to the BASE economy for the same parameterization. The high α and high λ parameterizations raise the value of these parameters by 10% in each case. Panel B reports the amount of additional wealth an agent would have to be given in the baseline economy in order to be indifferent between being born into the indicated alternative economy in the indicated state and being born in the baseline economy. The units for Panel B are percentages of mean wealth. The "base" columns for each economy match the "all" columns for each economy from Table 7.

D Other Results

Default probabilities by earnings level Figure 16 illustrates that, as is a feature of many default models, the probability of default is increasing in debt. It is also evident that default probabilities are decreasing in earnings for those with sufficiently large debt, another standard feature of default models. For very small debt, however, the lowest earners (who have the highest marginal utility of consumption) are least likely to default in order to avoid bearing the costs (κ and $\kappa_1 \times \exp(e)$) of default.

Figure 16: Default Probability by Earnings and Type



Credit Access Following Default For all $a' < 0$, define the two price schedules

$$
\begin{aligned}
q_D^{a'}(e,a,s) &\equiv q^{a'}\left(e, 0, \psi^{(1,0)}(e,a,s)\right), \\
q_N^{a'}(e,a,s) &\equiv q^{a'}\left(e, 0, \psi^{(0,0)}(e,a,s)\right),
\end{aligned}
$$

where the former corresponds to default (D) and the latter corresponds to no default (N). In order to compute an "average" effect of defaulting, we can weight the price differences for each action by the stationary distribution of agents who have the option to default. Specifically, define

$$
\overline{\mu}(e,a,s) = \frac{\sum_{\beta,z} \mu(\beta,e,z,a,s)}{\sum_{\beta,z,\tilde{a}<0} \mu(\beta,e,z,\tilde{a},s)} \text{ for all } a < 0.
$$

Then, we can compute the aggregate metrics for each debt choice $a' < 0$

$$
\Delta^q(a') = \sum_{e,a<0,s} \overline{\mu}(e,a,s) \left[\frac{q_N^{a'}(e,a,s)}{q_D^{a'}(e,a,s)} - 1 \right].
$$

E Numbers for Figures 1, 2, and 3

Table 11 reports the mean and standard deviations of the credit rankings in each bin, averaged over the four quarters of 2004. These moments were used in the regressions that determine the coefficients in the first 4 rows of the middle panel of Table 2. The correlations reported in the final column of Table

Table 11: Age Profile of Credit Rankings

Age Bins	Mean, Score Pctl	SD, Score Pctl	Corr($\Delta Pctl_{03}^{04}$, $\Delta Pctl_{04}^{05}$)
21-25 years	0.32	0.20	-0.22
26-30 years	0.35	0.23	-0.18
31-35 years	0.40	0.26	-0.20
36-40 years	0.44	0.28	-0.21
41-45 years	0.47	0.28	-0.21
46-50 years	0.50	0.28	-0.21
51-55 years	0.54	0.28	-0.19
56-60 years	0.58	0.28	-0.20

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. All entries are averages over the four quarters of 2004.

11 are the averages of these correlations over the four quarters of 2004. The final row of the middle panel of Table 2 reports the average over age bins of the correlations in the final column Table 11. The average credit ranking by age bin of people in the base sample of bankrupts reported in Table 12.

Table 12: Default Event Study Data

Years	26-30 years	31-35 years	36-40 years	41-45 years
-4	0.25	0.28	0.27	0.29
-3	0.22	0.26	0.26	0.27
-2	0.19	0.23	0.22	0.24
-1	0.14	0.18	0.17	0.20
0	0.11	0.13	0.12	0.13
1	0.17	0.19	0.20	0.21
2	0.19	0.21	0.22	0.24
3	0.20	0.22	0.23	0.25
4	0.20	0.23	0.22	0.25

Notes: The credit ranking data is based on author calculations using FRBNY CCP/Equifax data. The data presented in this table corresponds to the black lines in Figure 3. Since calculations are performed quarterly, the indicated year is the start of the year; that is, year -4 is the observation preceeding the bankruptcy by 16 quarters.