

# Tightening Financial Frictions on Households, Recessions, and Price Reallocations\*

Zhen Huo

University of Minnesota

Federal Reserve Bank of Minneapolis

José-Víctor Ríos-Rull

University of Minnesota

Federal Reserve Bank of Minneapolis

CAERP, CEPR, NBER

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## Abstract

We explore the effects of financial shocks in heterogeneous agent economies with aggregate savings and with frictions in some consumption markets, where demand contributes to productivity. Households of various wealth and earnings levels search for goods at different intensities and pay different prices in differently crowded markets. Increases in savings arising from a financial shock that tightens the borrowing limit trigger a recession via two channels: 1) the reduction in the consumption of goods that are subject to search frictions reduces productivity and output; 2) because the poorest households are more affected by the shock, consumption tilts toward the richest households, causing an additional reduction in output and productivity. We model fixed prices in a competitive search environment and show how price rigidities dramatically exacerbate the recession.

Keywords: Credit crunch, Endogenous productivity, Price dispersion, Household heterogeneity

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\*Corresponding author, Ríos-Rull, [vr0j@umn.edu](mailto:vr0j@umn.edu). Ríos-Rull thanks the National Science Foundation for Grant SES-1156228. We are thankful for discussions with Yan Bai, Kjetil Storesletten, and Nir Jaimovich. We also thank Sebastian di Tella for his discussion at the St. Louis Review of Economic Dynamics Special Issue Conference which greatly improved the paper, and to the organizers of the conference and editors of the special issue, Mark Gertler and Steve Williamson. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

# 1 Introduction

In this paper, we build a model in which financial shocks to households generate a recession. Despite the popular press's common attempt to link the Great Recession with households' financial difficulties, the attempt to build general equilibrium models that are capable of establishing such a link has met with little success. Many papers, however, model a recession generated from increased financial frictions on the firm's side (mostly for reasons of the [Bernanke and Gertler \(1989\)](#) type). In this paper, we show how an environment with goods market frictions displays a recession after an increase in household financial distress. The theoretical contribution of this paper is to highlight the mechanism that starts with increased financial frictions, proceeds to reductions in consumption, and ends with reductions in productivity and employment. This entire process takes place without the need to resort to households' inability to save or to the existence of multiple equilibria.

In our economy, the onset of a financial crisis transpires in the following way. After a time characterized by a financial bonanza, many households are simultaneously holding little wealth and consuming large amounts of goods. A financial shock requires some households to reduce their consumption in order to satisfy the tighter financial borrowing constraint. More important, all households may want to reduce consumption as they find their current financial holdings insufficient for bearing the daily vicissitudes of life. In our model, these vicissitudes take the form of idiosyncratic earnings shocks. Hence, all households find themselves attempting to reduce their consumption and increase their savings. Although in the new steady state, consumption and output will eventually surpass that in the initial state—households will end up becoming richer—the economy experiences a recession during the early stages of the transition. Standard models do not have this property, however. Instead, employment expands immediately after the financial shock and the increase in savings. The onset of a recession requires two conditions. First, there must be some goods that cannot readily be used for saving; otherwise, the output of those goods would be saved, and a recession would never occur. The second and more interesting condition is that a drop in the prices of these goods is not sufficient to maintain their level of production.

The gist of our contribution is to show how search frictions in some consumption markets suffice to generate a recession. We postulate the existence of some consumption goods—for convenience, we refer to these consumption goods as services—that cannot be accumulated and which require the active engagement of consumers to be acquired. Specifically, the search friction requires that services must be searched for and found before they can be purchased. The drop in consumption that accompanies the financial shock translates to a drop not only in the price of services but

also in the quantity purchased, because households spend less time searching for services. Hence, a recession results. The recession occurs with a reduction in labor and productivity. Note that this occurrence does not reflect measurement error in the sense of random deviation from true productivity. Instead, it is the result of applying NIPA procedures to our economy, because those procedures ignore the role of households in accessing final goods.

In our environment, households' attempts to save more directly induce a reduction in output arising from reductions in search. An additional channel also reduces consumption: the financial shock not only reduces overall consumption but also tilts it toward the wealthiest households, because the poorest households are the ones that are closest to their borrowing limits. Rich households exert less search effort per unit of consumption than poor households, thereby further reducing productivity and output.

The economy that we pose can be readily compared with the U.S. economy in all its relevant aspects, especially with respect to its income and wealth dispersion. When the financial shock hits the economy, a recession ensues. The recession is, however, relatively mild, because the reduction in the prices of services makes it attractive for the richest households to increase their consumption of services, thereby limiting the seriousness of the recession. For this reason, we explore the implications of price rigidity in this economy.

The analysis of price rigidity of goods in heterogeneous agent environments with competitive search is, to the best of our knowledge, a completely new feature, and we deem this feature to be an important contribution in and of itself. We model price rigidity as nine months of no price changes before a shift occurs toward a flexible price regime, and we find that in such an environment, the recession is about 14 times larger than it would be in an environment with flexible prices. Nonetheless, we find that to properly account for an event as large as the Great Recession, we need a much stronger saving motive for the richest households. We believe that the missing ingredient is the inclusion of housing, since housing prices are negatively affected by the financial shock. We have begun work on this topic, with encouraging preliminary results (see [Huo and Ríos-Rull \(2014\)](#)).

Our model is a two-good heterogeneous agent version of the [Bai, Ríos-Rull, and Storesletten \(2011\)](#) search frictions environment in which there is a positive marginal need for labor when a match occurs. In our model, services cannot be used directly for savings purposes—a feature that ensures that the desire to save does not generate an expansion. The environment is built on top of a structure of the [Bewley \(1986\)](#)-[Imrohoroglu \(1989\)](#)-[Huggett \(1993\)](#)-[Aiyagari \(1994\)](#) type, with

many agents and incomplete markets (how else can one talk of financial frictions seriously?). In these economies, households face uninsurable idiosyncratic risk, and they protect themselves from this risk through wealth accumulation. In particular, we are more interested in getting the total amount of wealth right than in providing an endogenous interest rate, so in this respect our work is closest in spirit to that of [Imrohoroglu \(1989\)](#).

In our environment, a market friction on services is managed via competitive search. The ability to acquire services in cheap, albeit inconvenient, markets provides households with another channel for self-insurance. In the steady state, the wealth-poor or income-poor households purchase fewer services than richer households. More important, they do so at a lower price, at the cost of exerting more search effort, which is in line with the empirical findings of [Broda, Leibtag, and Weinstein \(2009\)](#) and [Kaplan and Menzio \(2013b\)](#).

In the recession generated by increased financial tightness, most households cut their services consumption, and they do so by exerting less effort into search. The aggregate economy hence operates at a lower capacity. We modify the utility function used in [Bai, Ríos-Rull, and Storesletten \(2011\)](#) to ensure that the response of search efforts to a negative wealth effect makes productivity procyclical. Meanwhile, households also choose to go to markets with lower prices, and as a result we observe a decline in the average prices of services. Depending on their asset position, the reactions of households to the financial shock vary quite a bit. Consequently, another contribution of this paper is to provide strong predictions about the distribution of both the amount of household expenditures and the prices faced (chosen, in fact) by different types of consumers.

This paper is closely related to the literature that attributes the Great Recession to household financial distress. [Guerrieri and Lorenzoni \(2011\)](#) study the effects of a reduction in the borrowing limit, in a way that is very similar to our baseline economy. Their basic setup is a [Huggett \(1993\)](#) economy in which households borrow from each other and aggregate wealth is zero. A tightening of the borrowing constraint induces the poorest households to increase their work effort and savings. To clear the bond market, the interest rate must drop dramatically, because even the richer agents have an incentive to increase work and savings at the old interest rate. The incentives of the rich to save are smaller, since the rich are further away from the borrowing limit. The reduction in the interest rate induces households to delay gratification, to the extent that the rich may end up consuming more and working less while the poor consume less and work more. The overall effect in the economy that [Guerrieri and Lorenzoni \(2011\)](#) parameterize is that output declines because of the reduction in labor of the very high-skilled workers. Total working hours, however, increase because most households work more. Clearly, this outcome is not what we have in mind when we

think of recessions associated with reductions in both output and hours.

Another important paper in this literature is one by [Midrigan and Philippon \(2011\)](#), who consider an environment with two types of agents, rich and poor. Both types are liquidity constrained, but only the poor are credit constrained. A shock to the collateral constraint for liquidity significantly reduces aggregate demand if the rich cannot quickly convert credit into liquidity. To prevent households from working harder or moving to a tradable sector that is capable of accommodating the lack of demand, the authors assume not only labor reallocation costs, but also wage rigidity. A shock to the collateral constraint for credit has very small effects because the unconstrained households substantially increase consumption. Both [Guerrieri and Lorenzoni \(2011\)](#) and [Midrigan and Philippon \(2011\)](#) face substantial difficulty in generating a recession because the immediate effect of the financial constraint is to induce households to work harder.

In [Huo and Ríos-Rull \(2013\)](#) impoverishment –or, in general, any attempt to save– generates a recession because households contribute to productivity. The gist of the mechanism is the search for varieties of nontradable consumption goods in an environment with frictions. A reduction in consumption is implemented via a reduction in the quantity of varieties and in the amount consumed of each variety, which is what is responsible for the reduction in productivity and employment. Their environment is built on top of a two-sector growth model in which increasing capacity for tradable goods production is very costly. The authors apply the model to account for the recession in Southern Europe and find that the mechanism is quantitatively sound.

[Eggertsson and Krugman \(2012\)](#) consider an environment with sticky prices and a Taylor type of monetary policy rule. When borrowers are suddenly forced to reduce their nominal debt, the depressed demand puts downward pressure on the interest rate. In this environment, there are no real means to save. To generate a recession, nominal rigidities must be present and the zero bound must be binding. [Justiniano, Primiceri, and Tambalotti \(2013\)](#) incorporate mortgage choices and the housing investment sector into an otherwise standard dynamic general equilibrium model. They assume that a tightened collateral constraint applies only to new mortgages, not to existing mortgages. Quantitatively, a shock to the collateral constraint has only limited effects on aggregate activities, because most households can maintain their old mortgage plan. Also, savers and borrowers move in the opposite direction, and they cancel out in the aggregate.

Our paper complements this body of work in a number of ways. First, we show that goods markets frictions are an important factor that contributes to the decline in aggregate output. As in [Huo and Ríos-Rull \(2013\)](#), a decline in household demand translates into a decline in productivity even under

competitive search. Second, how a household is affected by a financial shock crucially depends on its asset position. The effects on aggregate activity are determined by the relative size of different household groups. Except for [Guerrieri and Lorenzoni \(2011\)](#), in which aggregate wealth is zero, most papers assume that there are only two types of households, savers and borrowers. To properly investigate the effects of financial shock, it is important that the model produces the right wealth distribution. Our model economy is calibrated to U.S. income and wealth distributions, and we show how the reaction of households is very different depending on household's assets and earnings. Third, consumption inequality changes during the recession and exacerbates the recession. The poor have stronger incentives than the rich to cut consumption expenditures, leading to an increased cross-sectional variance of consumption. Because rich households have a higher saving rate than poor household's, the increase in the variance of consumption provides an additional channel through which to reduce output. Fourth, unlike [Guerrieri and Lorenzoni \(2011\)](#) or [Eggertsson and Krugman \(2012\)](#) where a reduction in consumption demand translates into a reduction in output because there is no saving technology, the recession in our model does not require this artificial assumption. Households cut their consumption of services, which is responsible for the recession, but at the same time they increase their savings in numeraire goods. Fifth, our results do not rely on nominal rigidities. Despite a large drop in the prices of services, a loss in output still occurs. We also explore the role of price rigidity in our model economy and find that it can greatly enlarge the magnitude of a recession.

Our paper is also closely related to the literature on goods market frictions. Our approach to modeling the services market frictions builds on [Bai, Ríos-Rull, and Storesletten \(2011\)](#). In addition to posing a model with heterogeneous agents, we modify the utility function used in that paper to allow for search efforts and expenditures to be positively correlated both cross-sectionally and in the business cycle.

[Alessandria \(2009\)](#) and [Kaplan and Menzio \(2013a\)](#) also study the role of goods market frictions in business cycle analysis. Our results resemble theirs in the sense that households buy cheaper goods when they are poor or while in a recession. The most crucial difference is that in both [Alessandria \(2009\)](#) and [Kaplan and Menzio \(2013a\)](#), the sellers' occupation rate is independent of the households' search effort, whereas the rate is positively correlated with households' efforts in our model. [Petrosky-Nadeau and Wasmer \(2011\)](#) study the interaction between goods market frictions and labor market frictions. They show that search in the goods market can greatly amplify and propagate the effects of technology shocks on labor markets. Unlike in our paper, in which consumers search every period in spot markets, in [Petrosky-Nadeau and Wasmer \(2011\)](#), consumers

and firms form long-term relationships and consumers search for goods only after their match ends exogenously. Another difference is that in their model, the price is determined via Nash bargaining with a random firm, whereas in our model, consumers actively choose their optimal price, which leads to a nondegenerate price distribution. [Michaillat and Saez \(2013\)](#) use an environment with variable occupation, but they treat price as an exogenous parameter, and market tightness is the variable responsible for market clearing. Ultimately, they think of their environment as a textbook illustration of the notion that search frictions in goods may affect the macroeconomy. Unlike their paper, both price and market tightness are determined endogenously in our model, and the price dispersion has rich implications.

Our paper also provides a framework in which to study competitive search with price rigidity. Pioneered by [Moen \(1997\)](#), competitive search protocol has been widely adopted in the search literature, but little is known about the properties when they are combined with price rigidity. [Michaillat and Saez \(2013\)](#) have fixed prices in their economy, but trading is via bilateral bargaining and there is no nondegenerate price distribution. In our model, the equilibrium involves solving for a fixed point in a multidimensional functional space or finding the sequences of market tightness for a continuum of markets each period, rather than a sequence of equal profit conditions.

The rest of the paper is organized as follows. Section 2 describes the theory and poses the baseline economy. Section 3 analyzes the role of searching and frictions in ensuring that the economy displays the properties in which we are interested. It provides an analytical characterization of the effects that we are studying. Section 4 poses a quantitative version of this model that relies heavily on the properties of the U.S. economy and provides a quantitative assessment of the effects of a financial shock to the borrowing limit. Section 5 studies the recession induced by a financial shock. Section 6 studies the version of this economy with price rigidity and displays its properties. Section 7 concludes.

## 2 The Economy

The economy runs endlessly, and in each period there are two types of goods. The first type of good can be used either for consumption or for storage with net return  $r$ . For simplicity (to avoid cumbersome but vacuous market clearing conditions in the loan market), we assume that each household can hold negative amounts of this good up to an amount  $\underline{a}$ . Negative storage holdings can be thought of as loans, and  $\underline{a}$  as the credit limit. We use this good as the numeraire and refer to goods of this type as *numeraire goods* or, simply, as goods. The second type of good can be

easily described if we think of it as services, although we have in mind a much larger class of goods. As such, these goods can be used only for consumption and not for saving. They are traded in decentralized markets and are subject to additional frictions, which we will describe in detail. As in [Trejos and Wright \(1995\)](#), households have to consume the services produced by others but not by themselves. Because it is subject to market frictions we refer to this type of good as the *market good* or, simply, as services.

There is a continuum of households, and each household owns a measure of locations. Each location can either be active or not. Every period, a measure  $y_s$  of these locations is active. The number of active locations is part of the random endowment of the household. To be operated, an active location requires  $\epsilon < 1$  units of labor from the household. If matched with a purchasing shopper, an additional  $1 - \epsilon$  units of labor are required to produce one unit of the service. If the active location is not matched with a shopper, no output occurs. In addition, a household also receives  $y_c$  units of the numeraire goods. The endowments of both types of locations are perfectly correlated,  $y = \{y_c, y_s\} = \{\kappa y_s^i(1 + \eta), y_s^i(1 + \eta)\}$  and the stochastic process that drives it has two parts, a Markovian part with transition  $\Pi_{i,i'}$ , where  $i \in I$  is one of finitely many types, and an independently and identically distributed (i.i.d.) part  $\eta$  with a continuous distribution function  $F^\eta(\eta|i)$  and large, continuous support. This specification has the feature that provides arbitrary persistence in income, a large variance, a small state space, and a smooth distribution of households over resources.

## 2.1 Directed Search for Services

Different markets are indexed by their price and market tightness  $(p, q)$ , where market tightness is defined as the ratio of the measure of locations to the measure of shoppers. By sending a shopper, or *shopping unit*, to market  $(p, q)$ , the household expects to meet a location with probability  $\Psi^d(q)$  at price  $p$ . Not all markets are active. In fact, households understand that there is an equilibrium-determined expected revenue for sellers,  $\zeta = p \Psi^f(q)$ , that active markets have to satisfy. Searching is costly for households, and we use preferences that not only yield a unique choice of which market to go to, but also expect that wealthy households search in markets with higher price and shorter lines, while poor households search in more crowded markets but pay less. This case, we believe, is the natural one to explore.

Households choose not only where to shop but also where to send their locations. By sending a location to market  $(p, q)$ , the household expects to meet a shopper with probability  $\Psi^f(q)$ , and the expected revenue is  $\zeta = p \Psi^f(q)$ .

## 2.2 Preferences and State Variables

Households live forever, discount the future at rate  $\beta$ , and are expected utility maximizers. They have preferences over numeraire goods  $c$ , market goods  $s$ , (or goods and services), and shopping disutility  $d$ , represented by a period utility function  $u(c, s, d)$ . A household is characterized by  $(i, y, a)$ , where  $a$  is the household's asset position. Let  $x(i, y, a)$  be the measure over households' types. The total number of active locations that yield services is

$$T_s = \int y_s dx(i, y, a), \quad (1)$$

whereas the total endowment of numeraire goods is

$$Y_c = \int y_c dx(i, y, a). \quad (2)$$

In a steady state, no change in aggregate state variables takes place, so we need only be concerned with individual state variables.

## 2.3 Households' Problem

To present the household problem, we introduce the following notations. Let the measurable space for the market index  $(p, q)$  be  $(\mathbb{R}_+^2, \mathcal{B}^2)$ , where  $\mathcal{B}^2$  denotes the Borel sets in  $\mathbb{R}_+^2$ . Let  $B(\zeta)$  be the set of market indexes in which the expected revenue of a location is smaller than or equal to  $\zeta$ :

$$B(\zeta) = \{(p, q) \in \mathbb{R}_+^2 : p\Psi^f(q) \leq \zeta\}. \quad (3)$$

Using asterisks to describe the price and market tightness of the market in which the household shops, we can write the recursive problem of the household as

$$V(i, y, a) = \max_{\substack{a', c, s, d, \\ p^*, q^*, t}} u(c, s, d) + \beta \sum_{i'} \Pi_{i, i'} \int V(i', y'(i', \eta'), a') F(d\eta' | i') \quad (4)$$

subject to

$$p^* s + c + a' \geq (1 + r) a + \zeta y_s + y_c, \quad (5)$$

$$s = d \Psi^d(q^*), \quad (6)$$

$$\zeta \leq p^* \Psi^f(q^*), \quad (7)$$

$$a' \geq \underline{a}, \quad (8)$$

$$y_s \geq \int_{B(\zeta)} t(dp, dq), \quad (9)$$

where  $y'(i', \eta') = \{\kappa y_s^{i'}(1 + \eta'), y_c^{i'}(1 + \eta')\}$ . The household's budget constraint is (5). The search friction requires that services must be found, which is constraint (6). To guarantee that locations are sent to market  $(p^*, q^*)$ , condition (7) has to hold. Condition (8) is an ad hoc borrowing limit that the financial shocks will hit. Finally, condition (9) makes it explicit that the household allocates its measure of locations to active markets, where  $t$  is the measure of households' active locations over various markets (a measure over  $\mathcal{B}^2$ ). Even though households are indifferent toward sending their locations to different markets, we pose the condition here because it sharpens the definition of equilibrium.

The first order conditions can be written as

$$u_s = p^* u_c + \frac{-u_d}{\Psi^d(q^*)}, \quad (10)$$

$$s u_c = d \frac{\Psi_q^d(q^*)}{\Psi_q^f(q^*) p^*} \frac{\Psi^f(q^*)}{\Psi^d(q^*)} u_d, \quad (11)$$

$$u_c \geq \beta (1 + r) \sum_{i'} \Pi_{i, i'} \int u_c' F(d\eta' | i'). \quad (12)$$

In equation (10), the term  $\frac{-u_d}{\Psi^d(q)}$  captures the additional search disutility associated with acquiring services. Equation (11) displays the optimality condition associated with the choice of which market to go to. Its left-hand side shows the marginal gain of going to a market with a cheaper service, while the right-hand side shows the marginal cost of the additional search that is required, given the market-determined expected revenue of the sellers. When the borrowing constraint is not binding, the intertemporal Euler equation (12) holds with equality.

## 2.4 Equilibrium

Finding the equilibrium in our model economy requires various service markets to clear simultaneously. Define  $H$  as the measure of locations over the set of market index  $\mathcal{B}^2$ . For any  $B \in \mathcal{B}^2$ ,  $H(B)$  is derived by aggregating households' choices on where to send their locations:

$$H(B) = \int t(B; i, y, a) dx(i, y, a). \quad (13)$$

In a similar fashion, denote  $M$  as the measure of households' search efforts over the market indexes. For any  $B \in \mathcal{B}^2$ ,

$$M(B) = \int d(i, y, a) 1_{\{(p^*(i, y, a), q^*(i, y, a)) \in B\}} dx(i, y, a). \quad (14)$$

In the service goods market,  $H$  measures the supply of the locations in certain markets, while  $M$  measures the demand (total search efforts) for locations.<sup>1</sup>

The measure over households' type  $x$  may not be absolutely continuous because of the mass point at the borrowing limit. As a result, the measure of locations  $H$  may not be absolutely continuous. Let  $L$  denote the support of measure  $H$ , which is the set of active markets. By Lebesgue's decomposition theorem, we can partition  $L$  into two disjoint sets,<sup>2</sup>

$$L = L_{ac} \cup L_{pp}. \quad (15)$$

$H$  and  $M$  are absolutely continuous on  $L_{ac}$  and permit density functions  $h(p, q)$  and  $m(p, q)$ . The set  $L_{pp}$  contains all the mass points.

A steady-state equilibrium is a set of decision rules and values for the households  $(s, c, d, p^*, q^*, a', t, V)$  as functions of individual state variables  $(i, y, a)$ ; expected revenue of active locations  $\zeta$ , the measures  $H, M$  and their associated density functions, the set of active locations  $L$ , and a stationary distribution of households  $x$ , such that

1. Given aggregate variable  $\zeta$ , households solve their problem.
2. The measures  $H$  and  $M$  are derived from (13) and (14), respectively.

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<sup>1</sup>We thank the referee for pointing out some mistakes in defining the measures in the original version of this paper and for the suggestions on how to correct them.

<sup>2</sup>Strictly speaking, the decomposition may also contain a continuous singular part, but we assume that it does not happen in our case.

3. All active locations yield the same expected revenue,  $\zeta = p \Psi^f(q)$ , for all  $(p, q) \in L$ .
4. All active markets fulfill their expected tightness. This is for  $(p, q) \in L$ .

$$q = \begin{cases} \frac{h(p,q)}{m(p,q)}, & \text{if } (p, q) \in L_{ac}, \\ \frac{H[(p,q)]}{M[(p,q)]}, & \text{if } (p, q) \in L_{pp}. \end{cases} \quad (16)$$

5. The measure  $x$  is stationary and is updated by households' choices and the process for the endowment shocks.

In equilibrium, market tightness  $q$  is increasing in the price  $p$ . For all the active markets  $(p, q) \in L$ , we can also write a function  $q = g(p)$  that tells us which market tightness is associated with which price. In the baseline model, the market tightness function  $g(p)$  can simply be obtained by solving the equilibrium condition

$$\zeta = p \Psi^f[g(p)]. \quad (17)$$

This condition, however, will not hold when the price of a location cannot be changed freely, which is the case we will discuss in the next section.

### 2.4.1 Out of Steady-State Equilibrium

We analyze an unforeseen but gradual change to the borrowing limit  $\underline{a}$ . That is, while in a steady state associated with  $\underline{a}$ , households learn that there will be a new set of borrowing limits  $\{\underline{a}_t\}$ , with  $\underline{a}_t > \underline{a}_{t-1}$ , for  $t < T$  and  $\underline{a}_t = 0$  for  $t \geq T$ . If the tightening of the borrowing limit were instantaneous, some households would be facing an empty choice set, requiring some form of bankruptcy, a situation that we abstract from in the model.

We carry out an alternative interpretation of the exercise that does not require the slightly oxymoronic notion of unexpected shock. In this interpretation, we start the economy with an arbitrary wealth distribution (which, in our case, is the steady state with the loose borrowing constraint) and trace the equilibrium given the particular path for the borrowing constraint. Whatever the interpretation, the associated definition of equilibrium consists of sequences of decision rules and aggregate variables, after an initial condition,  $x_0$ . The equilibrium also entails a sequence of expected revenues for each period  $\{\zeta_t\}$  that makes the actions of shoppers and buyers compatible in every period. Formally, the elements of the equilibrium are the same as in the steady state, except for the fact that all objects are indexed by  $t$ , so we do not repeat the definition here.

Even though this economy is a storage economy, the computation of the equilibrium is quite involved in the sense that it requires that we obtain not only a sequence of decision rules that can be solved for backwards, but also the calculation of a fixed point in sequence space: the sequence of decision rules for all agents' types, the sequence of expected revenues  $\zeta_t$ , the sequence of measures  $H_t$  and  $M_t$ , and the sequence of active markets or loci of  $\{L_t\}$  that households choose.

## 2.4.2 Out of Steady-State Equilibrium with Fixed Prices

Given the role of prices in this economy and the existence of a large body of work in macroeconomics that places price rigidity as the key mechanism in shaping economic outcomes, a natural step is to explore how price-adjusting difficulties shape the recession. The typical forms of price rigidity in the literature (Calvo or Taylor adjustments) may have a minor impact on the outcome because the case may be that only a few firms need to adjust their prices in order to achieve the same outcome as in the case with total price flexibility.<sup>3</sup> To avoid this possibility while still exploring price rigidity, we pose a version of our model in which all locations have to maintain the same price for a certain number of periods  $\lambda$ .

We also analyze the effects of fixed prices in the context of an economy that faces an unforeseen but gradual change to the borrowing limit. In this case, the economy has the property that all locations cannot change their prices for  $\lambda$  periods. During those periods, all the adjustment is done by shoppers that reallocate their shopping efforts by looking for the most convenient location. During the periods in which prices are fixed, the equal profit condition across locations does not hold. This creates an indeterminacy in how the income of locations is distributed. We assume that all households choose the same allocation of locations, and hence they have the same income per location. That is, for all  $(i, y, a)$  and  $B \in \mathcal{B}^2$ ,  $t(B; i, y, a) = y_s t(B)$ .

Given the definition of  $H$  and  $M$  in (13) and (14), we denote their marginal measures over price as  $H^p$  and  $M^p$ . For any set  $A \in \mathcal{B}$ , the marginal measures in the initial steady state are given by

$$H_0^p(A) = H_0(A \times \mathbb{R}_+), \quad (18)$$

$$M_0^p(A) = M_0(A \times \mathbb{R}_+). \quad (19)$$

Let  $L_0^p$  be the set of prices with which the markers are active in the initial steady state. Recall that we can decompose  $L_0$  into the absolutely continuous part  $L_{ac,0}$  and the mass point part  $L_{pp,0}$ , and we can similarly decompose  $L_0^p$  into  $L_{ac,0}^p$  and  $L_{pp,0}^p$ . The measures  $H_0^p$  and  $M_0^p$  will allow density

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<sup>3</sup>For reasons similar to those in [Head, Liu, Menzio, and Wright \(2012\)](#).

functions  $h_0^p(p)$  and  $m_0^p(p)$  on  $L_{ac,0}^p$ .

During the first  $\lambda$  periods when prices are fixed, the measure of locations over prices is fixed as well:

$$H_t^p(A) = H_0^p(A), \quad (20)$$

$$L_t^p = L_0^p. \quad (21)$$

However, households can change their search efforts and their targeted markets, which implies that  $M_t$  and  $M_t^p$  will change on transition.

The household problem for  $t \leq \lambda$  becomes

$$V_t(i, y, a) = \max_{\substack{a', c, s, \\ d, p^*, q^*}} u(c, s, d) + \beta \sum_{i'} \Pi_{i,i'} \int V_{t+1}(i', y'(i', \eta'), a') F(d\eta' | i'), \quad (22)$$

subject to

$$p^* s + c + a' \geq (1 + r) a + \zeta_t y_s + y_c, \quad (23)$$

$$s = d \Psi^d(q^*), \quad (24)$$

$$q^* = g_t(p^*), \quad (25)$$

$$a' \geq \underline{a}_t, \quad (26)$$

$$p^* \in L_0^p. \quad (27)$$

For  $t > \lambda$ , the household problem is the one discussed in the previous subsection. A few comments should be made here. First, the value  $\zeta_t$  is the equilibrium value of aggregate profits determined every period. Second, the household is free to choose any location provided that it is available. Although with flexible prices the set of available locations had to satisfy condition (7), which was characterized by one number  $\zeta_t$ , these locations are now described by function  $g_t$ , which associates market tightness with each price and is an equilibrium-determined object. So for the first  $\lambda$  periods that prices are fixed two equilibrium objects have to be determined: revenues per location  $\zeta_t$  and market tightness per price  $g_t(p)$ . The formal equilibrium conditions that these two objects have to satisfy are

$$g_t(p) = \begin{cases} \frac{h_0^p(p)}{m_0^p(p)}, & \text{if } (p, q) \in L_{ac,0}^p, \\ \frac{H_0^p[(p,q)]}{M_0^p[(p,q)]}, & \text{if } (p, q) \in L_{pp,0}^p, \end{cases} \quad (28)$$

$$\zeta_t = \int p \Psi^f[g_t(p)] dh_0^p(p). \quad (29)$$

This introduces features that are quite theoretically interesting and computationally challenging. Particularly, we need to find the market tightness functions  $g_t(p)$  in equation (28), which involves solving for a fixed point in a multidimensional functional space.

### 3 Analysis of the Role of Searching

Before moving onto full-blown quantitative analysis, we can increase our understanding of the role of searching by directly analyzing the problem. The first order conditions (10-12) are quite informative. We see that both shopping disutility and search frictions are needed to achieve less than full utilization of services (our stand-in for full employment). Consider an economy in which there is no shopping disutility,  $u_d = 0$ , but there are search frictions,  $s = d \psi^d(q)$ . Then, households provide an infinite amount of shopping effort,  $d$ , to overcome the search friction. Moreover, they go to the cheapest market, the one in which firms are guaranteed to find a shopper with probability one, effectively using all services  $S = T_s$ . In this case, condition (10) becomes  $p = \frac{u_s}{u_c}$ , and condition (11) disappears, resulting in a two-good version of the standard incomplete markets economy with storage. Alternatively, consider an economy with shopping disutility but no search frictions. In it, condition (10) becomes  $p = \frac{u_s + u_d}{u_c}$ , with  $u_d < 0$ , reducing the equilibrium price of services. In either of these economies, an attempt to save more by cutting consumption has no bearing on the amount of output. Consumption of the good goes down and savings go up, while consumption of services remains constant. The relative price of goods versus services adjusts, typically lowering the price of services.

To further characterize the role of searching theoretically, a useful approach is to consider a static version of this model in which households may differ in their income and hence in their spending. In the following problem, we write the explicit dependence of the choices on the households' income and on the equilibrium value of the expected revenue from locations:

$$V[y; \zeta] = \max_{p, q, c, s, d} u(c, s, d), \quad (30)$$

subject to

$$ps + c = y, \quad (31)$$

$$s = d\Psi^d(q), \quad (32)$$

$$\zeta = p\Psi^f(q). \quad (33)$$

To simplify the algebra, consider a Cobb-Douglas matching function, so  $\Psi^d(q) = q^{1-\mu}$  and  $\Psi^f(q) = q^{-\mu}$ , yielding simple first order conditions,

$$pu_c = (1 - \mu)u_s, \quad (34)$$

$$\mu q^{1-\mu} u_s = -u_d. \quad (35)$$

Now, consider a utility function of the form  $u(c, s, d) = c^\alpha - \xi \frac{d^{1+\gamma}}{1+\gamma} = c^\alpha s^{1-\alpha} - \xi \frac{d^{1+\gamma}}{1+\gamma}$ ; that is, goods and services combine in a Cobb-Douglas manner, and search effort carries a disutility with a constant Frisch elasticity equal to  $1/\gamma$ . Using the budget constraint, we obtain the following expressions for goods and services:

$$c = \frac{\alpha}{\alpha + (1 - \mu)(1 - \alpha)} y, \quad (36)$$

$$ps = \frac{(1 - \mu)(1 - \alpha)}{\alpha + (1 - \mu)(1 - \alpha)} y. \quad (37)$$

Note that in this case, the shares of expenditures in both types of goods are constant and affected by the elasticity of the matching function. Our functional form choices allow us to get closed-form solutions for the choices of price and market tightness using first order condition (35) and noting that the equal profit condition across active locations is  $\zeta = pq^{-\mu}$ . We have

$$\log q = \log \kappa_q + \frac{\gamma}{1 + \gamma - \mu(1 - \alpha)} \log y, \quad (38)$$

$$\log p = \log \kappa_p + \left[ \frac{\gamma}{1 + \gamma - \mu(1 - \alpha)} \right]^\mu \log y, \quad (39)$$

$$\log s = \log \kappa_s + \left( 1 - \frac{\mu\gamma}{1 + \gamma - \mu(1 - \alpha)} \right) \log y, \quad (40)$$

where  $\kappa_q = \frac{\xi}{\zeta^{\alpha+\gamma\mu(1-\alpha)}} \left[ \frac{(1-\mu)(1-\alpha)}{\alpha+(1-\mu)(1-\alpha)} \right]^\gamma$ ,  $\kappa_p = \xi \kappa_q^\mu$ , and  $\kappa_s = \frac{(1-\mu)(1-\alpha)}{\alpha+(1-\mu)(1-\alpha)} \frac{1}{\zeta} \kappa_s^{-\mu}$ , which are the same for all households (but will not be over time). The term  $\frac{\gamma}{1+\gamma-\mu(1-\alpha)}$  is the elasticity of  $q$  with respect to  $y$ . Notice that both  $\alpha, \mu \in (0, 1)$  because they are coefficients of Cobb-Douglas

functions and that  $\gamma \geq 0$ , which is the requisite for concavity. These parameter values imply the following:

1. When  $\gamma = 0$ , preferences are linear in the search disutility, and all households go to the same market and pay the same price regardless of their income. Clearly, households differ in the amount of services that they consume.
2. When  $\gamma > 0$ , price and market tightness are affected by households' income or expenditure; that is, richer agents will go to less crowded but more expensive markets. As a result, the ratio of goods to services consumed increases with income. Moreover, this case is more likely the larger the curvature of the search disutility, so as  $\gamma \rightarrow \infty$ , agents differ on the price but they get the same service and the same amount of search.
3. Note that the decision rules are strictly concave, which has implications for the role played by the amount of inequality. Imagine, for instance, two economies with the same average income. In the first economy, income is equally distributed, whereas in the other economy, income differs across agents. If  $\gamma = 0$ , then total services and all other aggregate variables are the same. However, if  $\gamma > 0$ , the unequal income economy has lower total services, higher average prices, and lower average search, and hence lower productivity, thereby inducing a negative relation between inequality and total consumption. This cross-sectional feature is very relevant for the analysis below, because a tightening of the budget constraint not only reduces total expenditure in consumption, but also does so more for the poor than for the rich. This result not only lowers average expenditure but also increases its variance, providing an additional force with which to reduce output and productivity in the service sector.
4. If we consider an aggregate reduction in income (relevant to the analysis below), a slight complication arises from the dual facts that expected profits,  $\zeta$  enter expressions (38-40) and reduce agents' income. The role of  $\zeta$  in the demand functions is the same for all agents, thereby reducing all variables in the same proportion. This is not the case regarding its role in shaping household income. However, in our experiments, the cross-sectional characterization that we have outlined is the same as the behavior of the economy over time.
5. For the more general case in which the functional form that governs the relation between goods and services has constant elasticity of substitution rather than being Cobb-Douglas, that is,  $c_A = \left( (1 - \omega)c^{\frac{\eta-1}{\eta}} + \omega s^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$ , we cannot obtain a closed-form solution to the agents' problem, but the same general properties occur. Figure 1 displays the choices of

prices and services for households as a function of income for a variety of values of the elasticity of substitution between goods and services ( $\eta$ ) and the Frisch elasticity of the search disutility. As we can see, for  $\gamma = 0$ , higher values of  $\eta$  make the relation of services to income less sensitive. The interesting case is when  $\gamma > 0$ . In this case, the higher the elasticity of substitution, the lower the response of prices and services to income. Moreover, as we can see, the relation between income and the amount of services is always concave, confirming the role of increased inequality in contributing to the reduction in output after the tightening of the borrowing constraint.

[Figure 1 about here.]

## 4 Quantitative Assessment of a Financial Shock

We now turn to specify a baseline economy by mapping the model to U.S. data. We then pose a shock to the financial friction that makes households' access to credit more difficult.

### 4.1 Specification of the Baseline Economy

We now discuss functional forms, targets for the parameter values in an initial steady state, and some accounting issues.

#### 4.1.1 Functional Forms and Parameters

**Preferences** The aggregate consumption bundle is valued via an Armington aggregator of goods and services,

$$c_A = \left( (1 - \omega)c^{\frac{\eta-1}{\eta}} + \omega s^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \quad (41)$$

where  $\eta$  is the elasticity of substitution between the two types of consumption and  $\omega$  is the bias toward services. We adopt GHH ([Greenwood, Hercowitz, and Huffman \(1988\)](#)) preferences between consumption and shopping effort, which is sufficient to guarantee that both variables move in the same direction in response to changes in savings:

$$u(c, s, d) = \frac{1}{1 - \sigma} \left( c_A - \xi_d \frac{d^{1+\gamma}}{1 + \gamma} \right)^{1 - \sigma}, \quad (42)$$

where  $\sigma$  determines the risk aversion,  $\xi_d$  determines average shopping effort, and  $\gamma$  determines how much more effort households want to exert as they become richer. The other preference parameter

is the discount factor  $\beta$ .

**Matching** A Cobb-Douglas matching function is the most widely used. However, in this matching function the probability of finding a location or a shopper can be greater than 1. Consequently, we use the matching function suggested by [den Haan, Ramey, and Watson \(2000\)](#), which does not have this drawback:

$$M(D, T) = \frac{DT}{(D^\mu + T^\mu)^{\frac{1}{\mu}}}. \quad (43)$$

The probabilities of finding a location and a shopper are

$$\Psi^d(q) = (1 + q^{-\mu})^{-\frac{1}{\mu}}, \quad (44)$$

$$\Psi^f(q) = (1 + q^\mu)^{-\frac{1}{\mu}}. \quad (45)$$

**Assets** Two parameters related to households' asset holdings: the first one is the return to storage  $r$ , and the second is the borrowing limit  $\underline{a}$ .

#### 4.1.2 Targets and Values

The model period is a quarter. We separate the parameters into two groups: those in the first group (shown in [Table 1](#)) are determined exogenously, and those in the second group (shown in [Tables 2 and 3](#)) are jointly determined by solving a large system: the equations require that the steady-state model statistics equal the targets, and the parameters are the unknowns.

We set the risk aversion at 2 and the rate of return to storage (in annual terms) at 4%. To determine the elasticity of substitution between market goods and numeraire goods, we follow the trade literature on the elasticity of substitution between nontradable goods and tradable goods. We choose the benchmark value used in [Bianchi \(2011\)](#), 0.83, which is also similar to the estimate in [Heathcote and Perri \(2002\)](#). We choose a curvature of the searching disutility,  $\frac{1}{\gamma} = 0.60$ , in order to have a value that is consistent with the Frisch elasticity of labor.

Parameter  $\epsilon$  has no role whatsoever in shaping the allocation. It only determines the units of the steady-state amount of labor. This parameter will play a role, however, down the road when we decompose decreases in output into decrease in productivity and decrease in labor. We can set the parameter so that decomposition will be whatever we want, so in a sense, we learn little to nothing from this parameter. It does, however, allow us to make the case that the structure that we pose is flexible enough to allow us to discuss decreases in both labor and productivity over a

recession. We choose the parameter so that changes in output are due in equal measure to changes in productivity and changes in labor.

[Table 1 about here.]

We now turn to the parameters that require solving the model to determine their value; that is, we estimate them using the exact method of moments. As is customary, we associate the parameters with the target that provides the most intuition for its value, but all parameters are determined jointly. We choose the discount factor  $\beta$  to target a wealth-to-income ratio of 4.0. The borrowing limit  $\underline{a}$  is such that the ratio of negative net worth to total wealth is 0.25% (calculated from 2007 Survey of Consumer Finances [SCF] data). Two parameters are related to the goods market frictions: the units of search costs  $\xi_d$  (which could have been posed in the matching function without loss of generality) and the elasticity of the matching function  $\mu$ . We choose them so that the service occupation rate is 81% and the standard deviation of prices is 10%. According to [Kaplan and Menzio \(2013a\)](#), the standard deviation of prices for the same good ranges from 20% to 30%. Among the dispersion, one-third can be attributed to store differences. In our model, in different markets, shoppers exert different efforts to obtain the same good, which we interpret as only store differences. The average occupancy rate reported by the Federal Reserve Board is 81%. We target a service-to-total-output ratio of 67%, which pin downs  $\omega$ . We also target that the endowment for numeraire goods accounts for 15% of the total output, which (assuming as we do that both endowments are perfectly correlated) determines the relative units of the two endowments,  $y_c = \alpha y_s$ .

[Table 2 about here.]

We calibrate the endowment process to capture the earnings and wealth distribution in the United States. We use four discrete endowment levels, that is,  $y_s \in \{y_{s,1}, y_{s,2}, y_{s,3}, y_{s,4}\}$ . We interpret the first three endowments as earnings for the majority of households, with type 1 being poor, type 2 being average and type 3 being rich. The fourth endowment level is intended to capture the super-rich households in the economy, as in [Castañeda, Díaz-Giménez, and Ríos-Rull \(2003\)](#) or [Díaz, Pijoan-Mas, and Ríos-Rull \(2003\)](#). The transition probability and the levels of the first three states are calibrated to approximate an AR(1) process using the method by [Tauchen \(1986\)](#). Following [Nakajima \(2012\)](#) and [Domeij and Heathcote \(2004\)](#), we set the persistence for the endowment process at 0.91 and the standard deviation for the innovation term at 0.20. We assume that the

first three types of households have the same probability of becoming type 4 households who are super rich,  $\pi_{entry} = \Pi_{i,4}, i \in \{1, 2, 3\}$ , and that type 4 households return to one of the first three states with the same probability  $\pi_{exit} = \Pi_{4,i}, i \in \{1, 2, 3\}$ . We calibrate the rest of the parameters to match the earnings Gini index, 0.64, the wealth Gini index, 0.82 (calculated from 2007 SCF data), and the fact that the richest 10% of the population own 70% of total wealth. Figure 2 shows how the Lorenz curves for wealth in the model and the United States compare. The transition probability and endowment levels are shown in Table 3. To smooth out the distribution of assets, we also include a type-dependent transitory income shock  $\eta$ . We assume that this i.i.d. shock follows a uniform distribution, that is,  $F(\eta|i) = \frac{\eta - \underline{\eta}_i}{\bar{\eta}_i - \underline{\eta}_i}$ . The variance of the i.i.d. shock does not affect our quantitative results, and we choose  $\bar{\eta}_i = 0.03, \underline{\eta}_i = -0.03$  for all  $i$ .

[Figure 2 about here.]

[Table 3 about here.]

### 4.1.3 Measurement Issues

In the baseline model economy, total output is

$$Y = \int_0^{T_s} p_i \Psi^f(q_i) di + \int y_c dx(i, y, a) + r \int a dx(i, y, a) = \zeta T_s + Y_c + rA, \quad (46)$$

where  $A$  is the aggregate wealth in the economy,  $A = \int a dx(i, y, a)$ .

The distribution of prices changes across periods, so we need to set a price index to compare output over time. Let  $P_0 : [0, T_s] \rightarrow R_+$  be the function that indicates the price of location  $i \in [0, T_s]$  in the initial steady state, where we rank the locations by their prices in an increasing order, that is,  $P_0(i) \leq P_0(j)$  if  $i < j$ .

We then use the following base year prices to define real output in different periods:

$$Y_t = \int_0^{T_s} P_0(i) \Psi^f(q_{it}) di + \int y_c(i, y, a) dx_t(i, y, a) + r \int a dx_t(i, y, a). \quad (47)$$

Note that  $\Psi^f(q_{it}) = \frac{\zeta_t}{p_{it}}$  is the occupation probability for location  $i$  in period  $t$ , where the ranking of the locations is according to its price under the new distribution. This definition of output only reflects the change of average occupation rates at a location.

## 4.2 The Steady State of the Baseline Economy

To highlight how market frictions affect households' decisions, Figure 3 plots the choice of price and market tightness depending on the wealth level of households for each earnings group. Households with more wealth or more income buy more services and exert more effort, but their shopping relative to the services they consume is lower compared with poorer households. More specifically, rich households choose a market with a higher price and higher market tightness, where each shopping utility has a higher probability of finding the locations that deliver services, and hence where the amount of utilities required to find each location is lower. As shown in [Broda, Leibtag, and Weinstein \(2009\)](#), households with lower income systemically pay a lower price for the same goods. Specifically, the average prices paid by the four income types are 0.86, 0.87, 0.90, and 1.19. [Kaplan and Menzio \(2013a\)](#) find that households with unemployed working-age members, an indirect measure of earnings, pay 1% to 5% less than those without unemployed members. Based on these findings, we conclude that our model is able to replicate the salient features of price dispersion.

[Figure 3 about here.]

[Aguiar and Hurst \(2005\)](#) and [Aguiar and Hurst \(2007\)](#) document that retired and unemployed people tend to spend more shopping time. [Aguiar, Hurst, and Karabarbounis \(2013\)](#) show that during recessions, consumers spend 7% more shopping time on average. At first glance, this finding may seem to contradict our model prediction that rich households exert more shopping effort than poorer households. In our model, the search effort is the disutility associated with consumption, such as waiting for a restaurant table or booking hotels online. The search effort is different from the shopping time spent on finding a cheaper price, such as searching for coupons or sales, which is what [Aguiar and Hurst \(2005\)](#) and [Aguiar and Hurst \(2007\)](#) tend to capture. Whether consumers actually show up in restaurants or theatres affects the occupation rate of the economy, and in recessions, firms operate with a lower capacity. As shown in [Aguiar and Hurst \(2005\)](#), retired and unemployed people do eat at restaurants less frequently than working people. We therefore conclude that an enhanced interpretation of shopping, rather than just bargain hunting, makes our model look more like the data. In the baseline economy, about 35% of households hold negative or zero net worth, which we can interpret as borrowing from richer households. Of these households, 7% are actually "in the corner", meaning that they are borrowing-constrained households. Note that in the data, 15% of households have zero or negative wealth. We made a choice to target total

negative wealth rather than the number of households holding negative wealth. In this economy, both statistics cannot be achieved simultaneously because there is only one asset.

### **4.3 The Long Term Implications of a Financial Shock: A Steady State without Borrowing**

This paper explores the breakdown of borrowing possibilities, which can naturally be implemented by setting the lower bound of assets to 0, which carries the interpretation of households being able to save but not borrow.

In a steady state with a strict inability to borrow, aggregate wealth is higher than in a steady state when borrowing is feasible due to the precautionary saving motive. Table 4 shows that total wealth is 1.7% higher for the baseline economy. Without borrowing possibilities, households are richer and, because of the additional interest on their assets, output is higher. Households consume more of both goods and services. The additional consumption of goods equals the additional interest obtained from higher wealth. Services expand because of the higher income, and as we discussed in Section 3, with the preferences that we specify, households increase their consumption of services, searching more for them and paying higher prices. However, services increase less than goods. As Table 4 shows, the increase in goods consumption is much higher than that of services. That the matching function has constant returns to scale and that the supply of services locations is constant together imply that search effort has to increase more than services. The substantial increase in the average price is due to the competition among households for low market tightness.

[Table 4 about here.]

This table has two additional points of interest. The first is that a reduction in wealth inequality occurs, given that the poorest people have a stronger incentive to save more—a finding that shows up in the lower Gini index. A reduction in the dispersion of prices that households choose to pay also occurs. Again, as we discussed in Section 3, there is an additional effect on economic activity via the variance of wealth. In this case, the reduction in wealth dispersion further contributes to increased output. Table 4 shows this result by comparing the services of the average household with average services and showing how this ratio decreases. The second point is the decomposition of the increase in output into labor and productivity. As we described above, this decomposition is arbitrary and we have chosen it to be equal during the first period of the transition. Interestingly, when comparing steady states, we see that the increased output mostly arises from increases in

productivity, simply because across steady states, the main difference in output is in goods, not services.

To summarize, the main feature of the comparison between steady states is that households in the economy with a tighter borrowing constraint consume more and the economy operates at a higher occupation rate. This feature is standard when comparing steady states of economies that differ in total wealth. So, how can a recession be the result of a tightening of the borrowing constraint? The answer is that the process of adjusting to higher wealth entails the onset of a recession along the transition, during the increased saving stages. Having completed our quantitative assessment of the effects of a shock to the borrowing limit, we now turn to analyzing the transition following the financial shock.

## 5 A Recession after the Financial Shock

As we discussed, we explore a transition to a steady state without borrowing where the financial tightening happens gradually. We assume that  $\underline{a}$  moves monotonically over six periods (18 months) from its original value at the beginning to 0, as shown in Figure 4.

[Figure 4 about here.]

### 5.1 Aggregate Variables in the Recession

Figure 5 displays the transition paths of the main aggregate variables during a recession. Output is less than its initial value for three years and then bypasses its original level. Households start saving as soon as the borrowing constraint begins to tighten because they see that the constraint is going to become even tighter over the next few periods. The increase in savings is quite steady and is still going on after 10 years. Households reduce their consumption of goods, which is necessary to increase their assets. After 10 years, households' consumption is still below their original level even though their interest income is now higher than before the budget constraint tightened.

Central to this paper is that the reduction in the consumption of services lasts for five years. Although average prices decrease, so does aggregate search effort, which is ultimately what is responsible for the reduction in output. The reduction stems from the two effects that we discussed above: the reduction in search effort that all households implement when reducing their consumption expenditures, which we termed the direct effect, and the indirect or general equilibrium effect resulting from the fact that the reduction in consumption is larger for poorer households—those

more directly affected by the tightening of the borrowing constraint. These poor households are willing to bear more searching disutility and choose markets with very low prices. On the other hand, rich households, while also wanting to increase savings, have a weaker desire than the poor to do so, resulting in an increase in price dispersion. This finding is in line with the empirical finding that worse economic conditions enlarge the price dispersion ([Kaplan and Menzio \(2013a\)](#) document that the price dispersion is larger in areas with a higher unemployment rate). We can see this result in the last two panels of [Figure 5](#), which show an increase in price dispersion during the recession and an increase in the ratio of the services consumed by an average agent relative to the average amount of services in the economy. That this last ratio increases indicates that the cross-sectional changes in the economy contribute directly toward magnifying the recession.

[Figure 5 about here.]

### 5.1.1 The Distribution of Prices

To illustrate the extent of price adjustments in the recession, the first panel of [Figure 6](#) displays the measure of prices across locations, in the steady state and during the first period of the transition. The jagged nature of the distribution is due to the existence of four types, three of them with little wealth dispersion and each of those types choosing markets very similar to each other. The market tightness of locations—what households really care about most—is very smooth, however, as the second panel of [Figure 6](#) shows.

[Figure 6 about here.]

### 5.1.2 Movements in Labor and Productivity

[Figure 7](#) shows the decomposition of the decrease in output into a decrease in labor and productivity. Recall that we are essentially free to determine the labor requirements of the service industry. As such, we set them so that during the first period of the recessions, the contributions of labor and productivity are equal to each other. What is interesting is that the passage of time induces a faster increase in the role of productivity rather than the role of labor. Consumption of goods is growing because the role of accumulated wealth, and we have assumed no labor in the returns to storage.

[Figure 7 about here.]

## 5.2 Cross-Sectional Patterns in the Recession

[Figure 8 about here.]

To gain a better understanding of the mechanism that reduces the output of services, a useful approach is to investigate the cross-sectional response of households. Figure 8 shows that response of households with different earnings and wealth displays significant heterogeneity. As we have discussed, all households reduce their consumption of goods in response to the adverse financial shock. However, only households of types 1 and 2, which together account for almost 75% of the population, actually increase their wealth, and types 3 and 4 initially reduce their wealth, although they eventually end up with higher wealth. Why is this the case? The recession gives them an opportunity to increase their consumption of services because of the large decline in prices that they face. Although the total amount of search that households engage in moves in the same direction as their consumption of services, this is not the case for the finding probability of those services. In the most revealing panel of Figure 5, we can see that types 1 and 2 go to more crowded markets in their attempt to bear the adverse effect of the financial shock, in a way consistent with the findings of [Kaplan and Menzio \(2013a\)](#). Households of types 3 and 4 see the recession as a bonanza, and they not only enjoy higher services consumption at cheaper prices but also go to less crowded markets. Even with these conflicting patterns across the cross-sectional distribution of income and wealth, the overall response of the economy is to enter a recession with an overall reduction in the prices of services as well as a reduction in aggregate search behavior.

To summarize, a tightening of the borrowing limit induces a recession arising from weaker aggregate demand. Total output decreases along with total search effort, productivity, and labor. The recession is not very large because large reductions are made in the prices of services—reductions large enough to induce some households to increase their consumption of services even though they are also reducing their consumption of goods.

## 6 Price Rigidity in a Search Economy

Given the prominent role of price dispersion in our economy, we want to know what constitutes the role of price rigidity in our economy. Unfortunately, in the baseline economy above, because of the existence of multiple types, the distribution of locations over prices is very jagged, as we showed in Figure 6, which has prevented us from finding the equilibrium sequence of market tightness functions numerically. We have opted to turn to a simplified economy in which there is only one

type of agents who receive i.i.d. income shocks. In this economy, the distribution of locations over prices is much smoother, as the first panel of Figure 9 shows, allowing us to solve the equilibrium in the rigid price economy. Because of the lack of multiple types, this simple economy fails to capture the rich income and wealth heterogeneity present in the baseline economy. Still, for our purpose, which is to explore the extent to which price rigidity amplifies the recession, the simplified economy is extremely useful. The recession induced in this simple economy by the financial shock with flexible prices resembles the associated recession in the baseline economy. The magnitude of the recession becomes much larger after incorporating price rigidity.

[Figure 9 about here.]

In the simple economy, most parameters are the same as in the baseline economy. The main difference lies in the income process. Here, we assume a relatively large standard deviation of the i.i.d. shock, 0.56 (relative to the same mean as in the baseline), so that we can generate a sizeable amount of wealth dispersion with one type of households. Because some agents' income can be very low, the tightening of the credit limit will occur more slowly in the simple economy in order to avoid having agents with an empty budget set. In particular, the tightening lasts 20 periods. When we pose the fixed price version, we assume that the prices of all locations are fixed for three periods after the financial shock, that is,  $\lambda = 3$ , which corresponds to three quarters and is flexible thereafter.

Table 5 shows the comparison between the initial steady state and the steady state after the financial shock in the simple and baseline economies. Although the increase in wealth is larger in the simple economy because wealth is lower in the original steady state, the rest of the variables behave quite similarly: they all move in the same direction, and the magnitudes are close. In particular, the change in the ratio of services purchased by the average household relative to the total amount of services purchased is the same. The simple economy has a smaller increase in services and a smaller increase in the final average prices of services indicating that, if anything, rigid prices are less important than they are in the baseline economy.

[Table 5 about here.]

Recall that during the transition, in periods  $t \leq \lambda$ , the prices of all locations are fixed. As described in Section 2.4.1, the revenue in each location is

$$\zeta_t = \int p \Psi^f [g_t(p)] dh_0^p(p), \quad (48)$$

where  $h_0^p(\cdot)$  is the measure of locations over prices in the initial steady state. Since prices are fixed, the change in  $\zeta_t$  reflects the change in the utilization rates in each location. In contrast, in the economies with flexible prices,  $\zeta_t$  is the same for all locations with different prices, that is,  $\zeta_t = p\Psi^f(q)$  for all active  $(p, q)$  markets. In aggregate,

$$\zeta_t = \int p\Psi^f[g_t(p)] dh_t^p(p) = \int \zeta_t dh_t^p(p). \quad (49)$$

The change in  $\zeta_t$  reflects both the change in the utilization rate and the change in prices. In a recession with flexible prices, the decrease in average prices is much larger than the decrease in the utilization rate during the recession, which dominates the change in  $\zeta_t$ .

After the financial shock, the poor households (those close to the borrowing limit) have to cut their expenditures. When prices are flexible and there are no search frictions, prices will drop to the extent that agents will consume the same amount of services as before. When prices are flexible and there are search frictions, both prices and the quantity of services will decline, but the magnitude of the drop in services will be limited, since agents can switch to cheaper markets. When prices are rigid and there are search frictions, the drop in the quantity of services will be much larger, since poor agents can no longer switch to cheaper markets. We now turn to the quantitative performance of the model with rigid prices.

## 6.1 Transition Dynamics in the Rigid Price Model

Table 6 shows the direct effect of the financial shock in the main aggregate in the simple economy with flexible and fixed prices. The recession is smaller in the simple economy because not many households have low wealth. The key point here is that the impact of price rigidity is to essentially make the recession 14 times larger. Interestingly, the decline in goods consumption in the two economies is similar, but the decline in services is much more pronounced in the rigid price economy. To understand this difference, note that the average price drops by 2.4% in the flexible price economy, but it basically remains constant in the rigid price economy. Also, the increase in price dispersion is only noticeable in the flexible price model. The inability of the fixed price economy to reduce prices by 2.4% and increase price dispersion is what induces the reduction in the consumption of services.

[Table 6 about here.]

[Figure 10 about here.]

Figure 10 displays the impulse response of the financial shock in both the flexible price economy (solid blue line) and the rigid price economy (dotted red line). We see a much sharper recession with fixed prices. Consumption of services drops much more dramatically in the rigid price economy. Consumption of goods declines by a similar magnitude in these two economies. The flexible price economy provides households with an opportunity to increase their consumption of services arising from the lower prices, thereby inducing households to cut their consumption of goods (recall that its reduction does not contribute to the recession because goods that are not consumed are saved). After three quarters of price rigidity, both economies look very similar except for one detail: the lower consumption of services in the rigid price economy has induced additional savings, and hence the ulterior path of the economy makes the recession slightly smaller with rigid prices but only after prices become flexible. Interestingly, in the recession of the rigid price economy, the drop in labor plays a larger role than the drop of productivity.

The differential performance of both economies can also be seen in the second panel of Figure 9. Here, the tightness function,  $g(p)$ , is displayed, and it provides further information about the change in search behavior across the various prices in the economy. With rigid prices, market tightness changes the most in the highest prices, because households not only search less but also move to cheaper markets. In markets with low prices, the drop in services is compensated by an inflow of households that are looking for cheaper, even if more crowded, markets. In the flexible price economy, the main role in the changes is that of price reductions rather than reallocation of households.

## 6.2 Heterogeneous Response to the Financial Shock

[Figure 11 about here.]

Figure 11 shows the reactions of agents organized by their wealth, partitioned into 20 quintiles. The poorest households are the ones that are hit the worst after a financial shock. With respect to goods, both economies affect the poor similarly, where the rich reduce their consumption more in the rigid price economy. In both economies, as agents become rich, their reaction to the shock becomes weaker. In contrast, in the flexible price economy, rich people actually increase their services consumption, since prices drop significantly. With rigid prices, rich households can no longer take advantage of cheaper services, and, as a result, they also reduce their consumption of services and their search effort, the latter to a lesser extent compared with poor agents. When prices are flexible, rich households and poor households move in the opposite direction and cancel

each other out in aggregate. When prices are rigid, the rich lose the possibility of picking up the slack. The poor and the rich move uniformly, which exacerbates the recession.

## 7 Conclusion

In this paper, we have shown how an adverse financial shock can generate a recession in heterogeneous agent economies with aggregate savings and with frictions in some goods markets, where demand contributes to productivity. In these economies, households of different wealth and earnings search for goods in differently crowded markets and pay different prices. Increases in savings arising from financial shocks trigger a recession via the reduction in search effort required for consumption. In an economy with storage and an ad hoc borrowing limit, a tightening of the borrowing limit generates a mild decline in aggregate output because few households are badly affected. The households that are more affected by the shock are too poor to matter for aggregate output, and the rich can pick up some of the slack. If the prices of services cannot adjust, the recession will be much more severe. We expect that a recession like the Great Recession requires a financial shock capable of both negatively affecting larger groups of agents and doing so with more severity. We think that this would require the existence of other assets with prices that may drop. Preliminary findings in [Huo and Ríos-Rull \(2014\)](#) show that this direction is a promising one to pursue.

## References

- AGUIAR, M., AND E. HURST (2005): "Consumption versus Expenditure," *Journal of Political Economy*, 113(5), 919–948.
- (2007): "Life-Cycle Prices and Production," *American Economic Review*, 97(5), 1533–1559.
- AGUIAR, M., E. HURST, AND L. KARABARBOUNIS (2013): "Time Use during the Great Recession," *American Economic Review*, 103(5), 1664–1696.
- AIYAGARI, S. R. (1994): "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, 109(3), 659–684.
- ALESSANDRIA, G. (2009): "Consumer Search, Price Dispersion, and International Relative Price Fluctuations," *International Economic Review*, 50(3), 803–829.
- BAI, Y., J.-V. RÍOS-RULL, AND K. STORESLETTEN (2011): "Demand Shocks as Productivity Shocks," manuscript, Federal Reserve Bank of Minneapolis.
- BERNANKE, B., AND M. GERTLER (1989): "Agency Costs, Net Worth, and Business Fluctuations," *American Economic Review*, 79(1), 14–31.
- BEWLEY, T. (1986): "Stationary Monetary Equilibrium with a Continuum of Independently Fluctuating Consumers," in *Contributions to Mathematical Economics in Honor of Gérard Debreu*, ed. by W. Hildenbrand, and A. Mas-Colell. North Holland, Amsterdam.
- BIANCHI, J. (2011): "Overborrowing and Systemic Externalities in the Business Cycle," *American Economic Review*, 101(7), 3400–3426.
- BRODA, C., E. LEIBTAG, AND D. E. WEINSTEIN (2009): "The Role of Prices in Measuring the Poor's Living Standards," *Journal of Economic Perspectives*, 23(2), 77–97.
- CASTAÑEDA, A., J. DÍAZ-GIMÉNEZ, AND J.-V. RÍOS-RULL (2003): "Accounting for the U.S. Earnings and Wealth Inequality," *Journal of Political Economy*, 111(4), 818–857.
- DEN HAAN, W. J., G. RAMEY, AND J. WATSON (2000): "Job Destruction and Propagation of Shocks," *American Economic Review*, 90(3), 482–498.

- DÍAZ, A., J. PIJOAN-MAS, AND J.-V. RÍOS-RULL (2003): "Precautionary Savings and Wealth Distribution under Habit Formation Preferences," *Journal of Monetary Economics*, 50(6), 1257–1291.
- DOMELI, D., AND J. HEATHCOTE (2004): "On the Distributional Effects of Reducing Capital Taxes," *International Economic Review*, 45(2), 523–554.
- EGGERTSSON, G. B., AND P. KRUGMAN (2012): "Debt, Deleveraging, and the Liquidity Trap: A Fisher-Minsky-Koo Approach," *Quarterly Journal of Economics*, 127(3), 1469–1513.
- GREENWOOD, J., Z. HERCOWITZ, AND G. W. HUFFMAN (1988): "Investment, Capacity Utilization and the Real Business Cycle," *American Economic Review*, 78(3), 402–417.
- GUERRIERI, V., AND G. LORENZONI (2011): "Credit Crises, Precautionary Savings, and the Liquidity Trap," Working Paper 17583, National Bureau of Economic Research.
- HEAD, A., L. Q. LIU, G. MENZIO, AND R. WRIGHT (2012): "Sticky Prices: A New Monetarist Approach," *Journal of the European Economic Association*, 10(5), 939–973.
- HEATHCOTE, J., AND F. PERRI (2002): "Financial Autarky and International Business Cycles," *Journal of Monetary Economics*, 49(3), 601–627.
- HUGGETT, M. (1993): "The Risk-Free Rate in Heterogeneous-Agent, Incomplete-Insurance Economies," *Journal of Economic Dynamics and Control*, 17(5), 953–969.
- HUO, Z., AND J.-V. RÍOS-RULL (2013): "Paradox of Thrift Recessions," Research Department Staff Report 490, Federal Reserve Bank of Minneapolis.
- (2014): "Financial Frictions, Asset Prices, and the Great Recession," manuscript, Federal Reserve Bank of Minneapolis.
- IMROHOROĞLU, A. (1989): "Cost of Business Cycles with Indivisibilities and Liquidity Constraints," *Journal of Political Economy*, 97(6), 1364–1383.
- JUSTINIANO, A., G. E. PRIMICERI, AND A. TAMBALOTTI (2013): "Household Leveraging and Deleveraging," Working Paper 18941, National Bureau of Economic Research.
- KAPLAN, G., AND G. MENZIO (2013a): "Deconstructing Price Dispersion," manuscript.
- (2013b): "Shopping Externalities and Self-Fulfilling Unemployment Fluctuations," Working Paper 18777, National Bureau of Economic Research.

- MICHAILLAT, P., AND E. SAEZ (2013): "A Model of Aggregate Demand and Unemployment," Working Paper 18826, National Bureau of Economic Research.
- MIDRIGAN, V., AND T. PHILIPPON (2011): "Household Leverage and the Recession," Working Paper FIN-11-038, New York University.
- MOEN, E. R. (1997): "Competitive Search Equilibrium," *Journal of Political Economy*, 105(2), 385–411.
- NAKAJIMA, M. (2012): "Business Cycles in the Equilibrium Model of Labor Market Search and Self-Insurance," *International Economic Review*, 53(2), 399–432.
- PETROSKY-NADEAU, N., AND E. WASMER (2011): "Macroeconomic Dynamics in a Model of Goods, Labor and Credit Market Frictions," GSIA Working Paper 2011-E6, Carnegie Mellon University.
- TAUCHEN, G. (1986): "Finite State Markov-Chain Approximations to Univariate and Vector Autoregressions," *Economics Letters*, 20(2), 177–181.
- TREJOS, A., AND R. WRIGHT (1995): "Search, Bargaining, Money, and Prices," *Journal of Political Economy*, 103(1), 118–141.

Figure 1: Prices and services choices as a function of income for various combinations of elasticity of substitution between goods and services ( $\eta$ ) and Frisch elasticity of search effort  $\frac{1}{\gamma}$ .

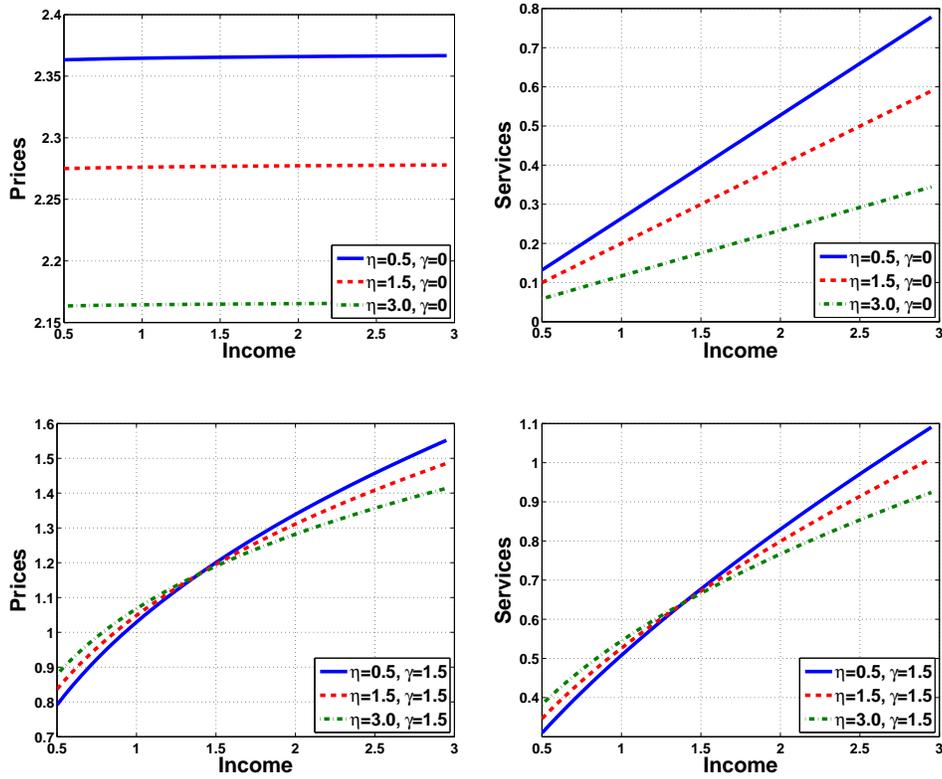


Figure 2: Wealth Distribution and Lorenz Curve

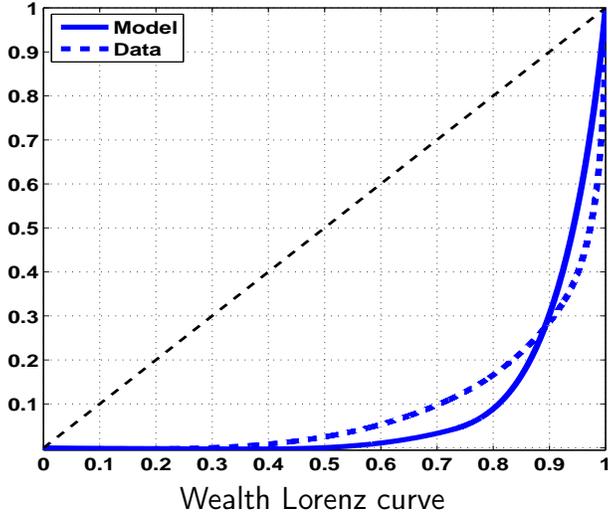


Figure 3: Market Choice and Shopping Effort as a Function of Wealth

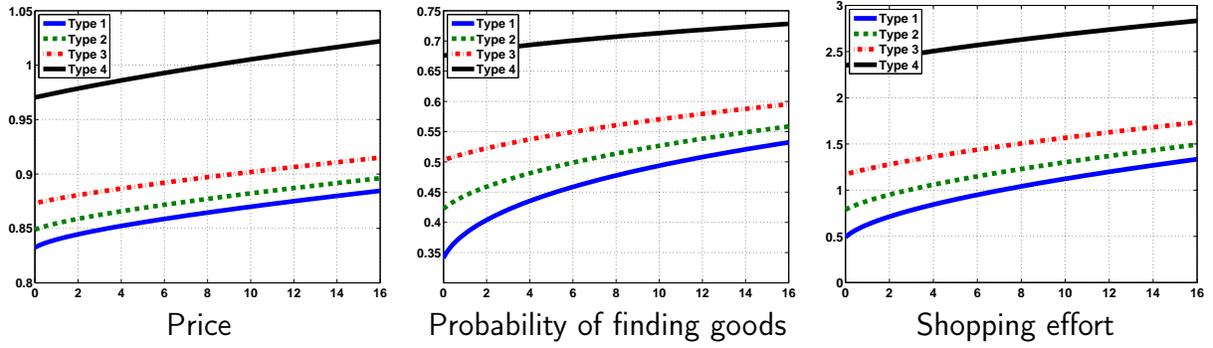


Figure 4: Transition of the Borrowing Limit  $\underline{a}$

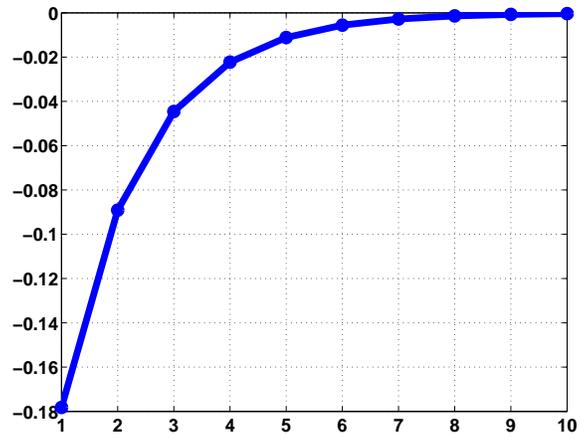


Figure 5: Aggregate Economy Response: Shock to Borrowing Limit

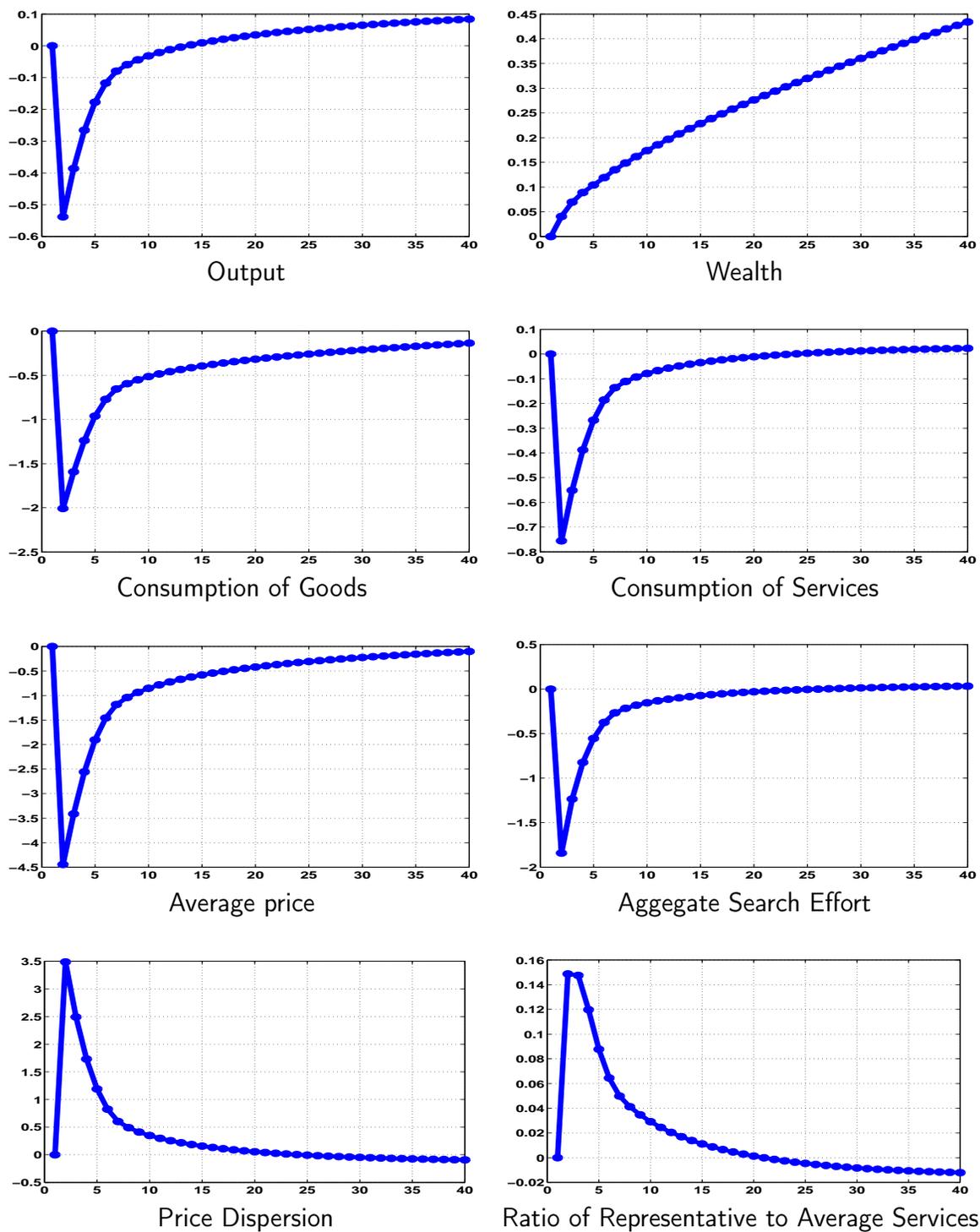


Figure 6: Change in the Distribution of Prices and Market Tightness

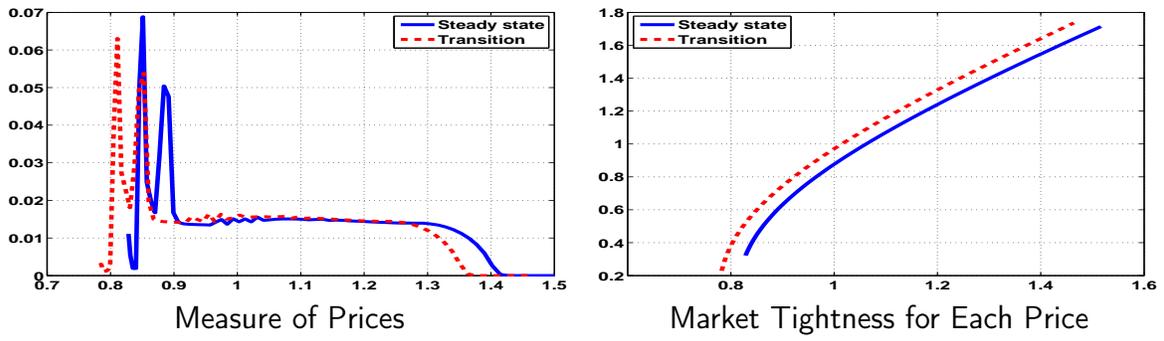


Figure 7: Transition of Labor and Productivity

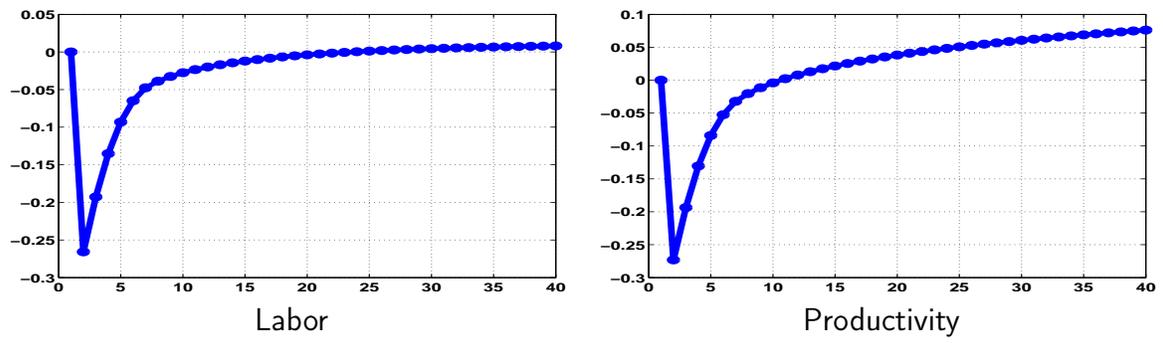


Figure 8: Cross-Sectional Response: Shock to Borrowing Limit

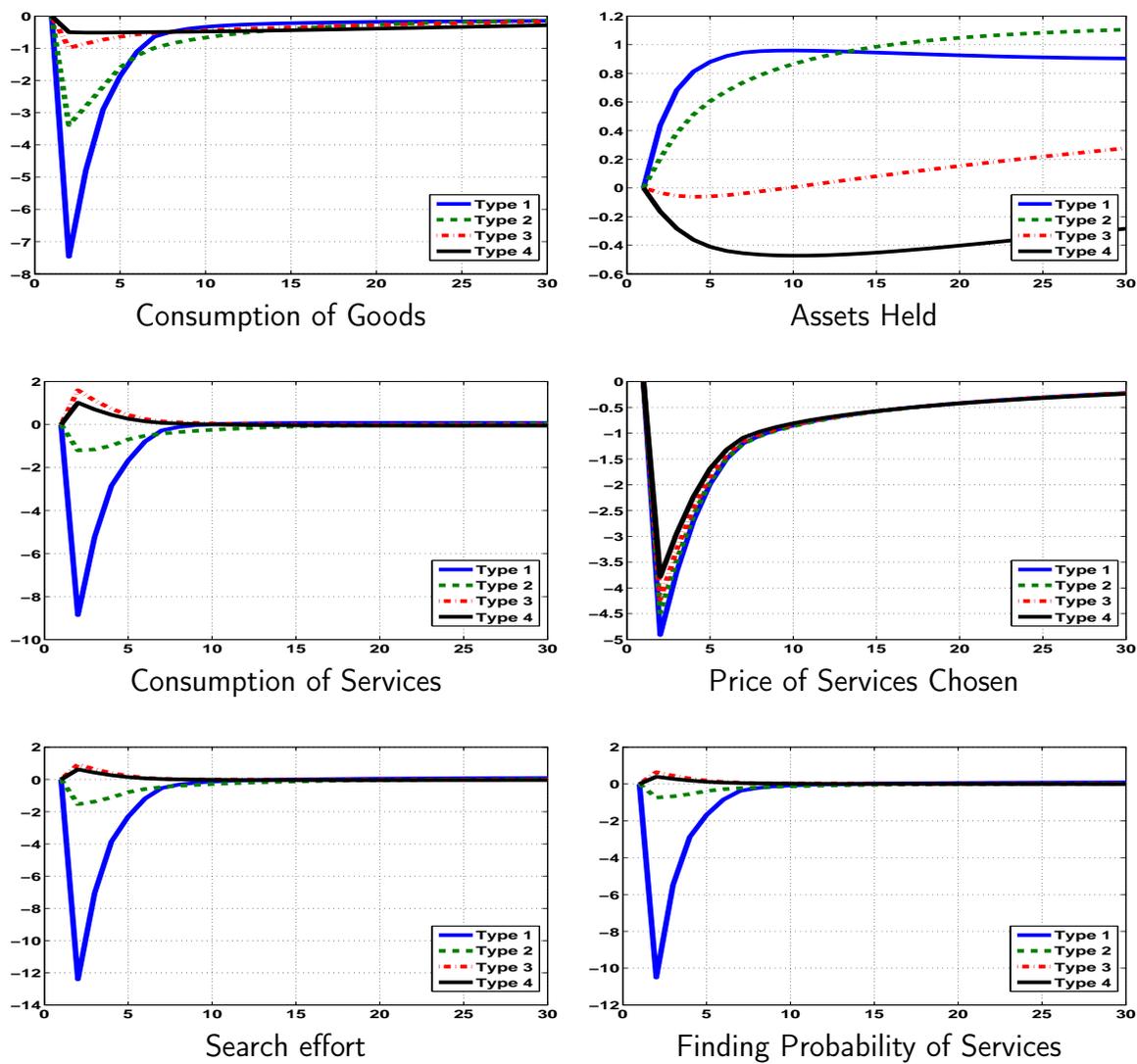


Figure 9: Change in the Distribution of Prices and Market Tightness in the Simple Economy

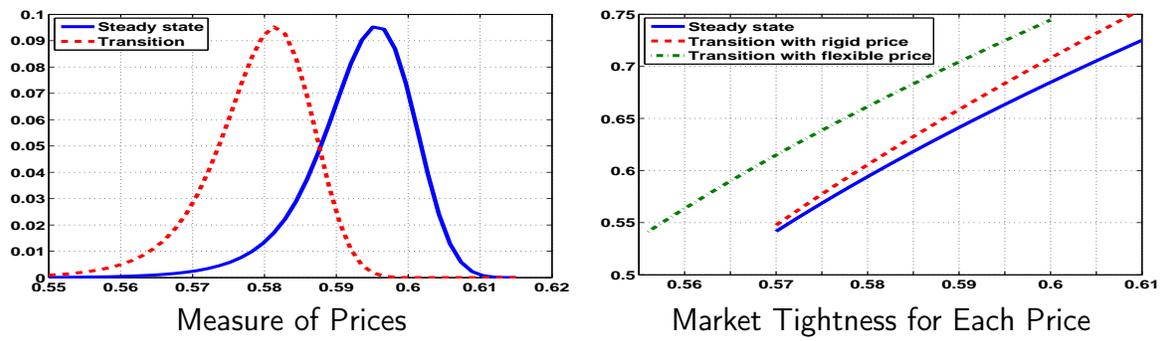


Figure 10: Aggregate Economy Response: with Price Rigidity

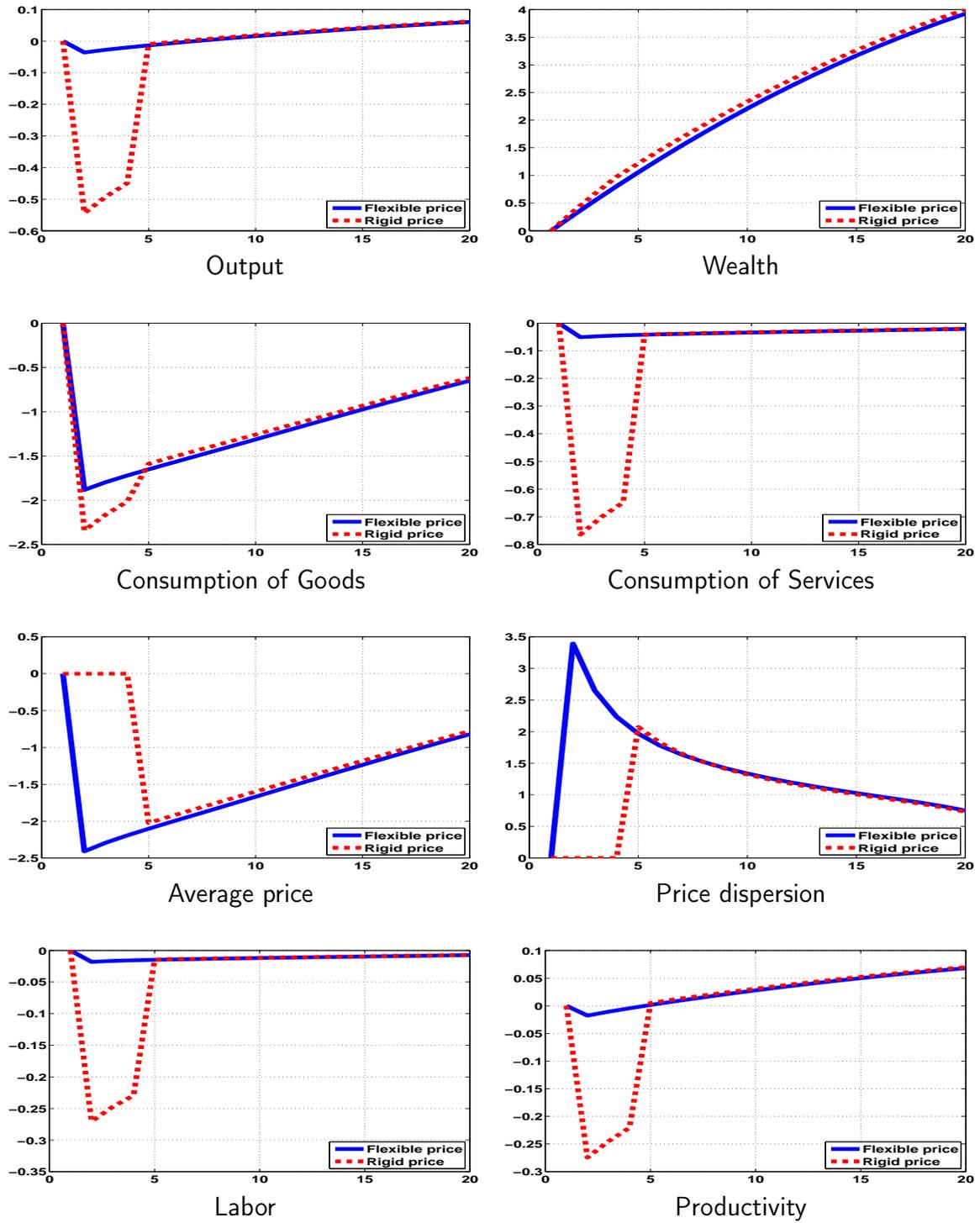


Figure 11: Cross-Sectional Response: Cross-Sectional Changes in Consumption

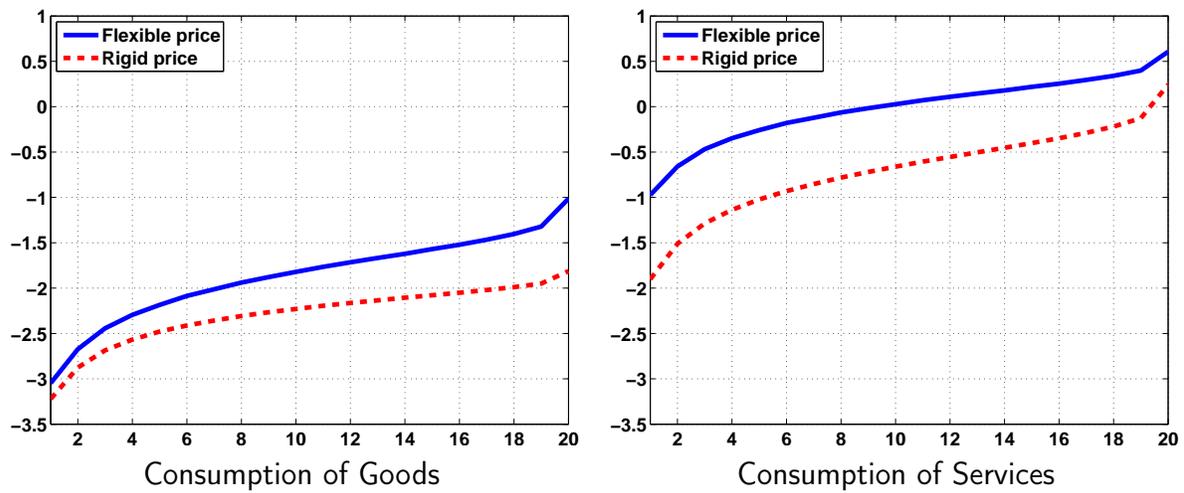


Table 1: Exogenously Determined Parameters of the Baseline Economy

Parameter	Value
Risk aversion, $\sigma$	2.0
Return to storage (annual), $r$	4%
Elasticity of substitution between tradables and nontradables, $\eta$	0.83
Frisch elasticity of substitution of search effort $1/\gamma$	0.60
Fixed labor to keep a location open, $\epsilon$	0.60

Table 2: Steady-State Targets and Associated Parameters of the Baseline Economy

Parameter	Value	Target	Value	Model
$\beta$	0.96	Wealth-to-output ratio	4.00	3.99
$\underline{a}$	0.18	Negative-net-worth-to-wealth ratio	0.25%	0.24%
$\mu$	2.58	Service occupation ratio	0.81	0.81
$\xi_d$	0.04	St.d of price dispersion	0.10	0.10
$\omega$	0.82	Services-to-output ratio	0.67	0.67
$\alpha$	0.20	Numeraire-endowments-to-output ratio	0.15	0.16

Table 3: Parameters Related to the Endowment Process in the Baseline Economy

Parameter	Value	Target	Value	Model
$y_{s,4}$	8.813	Wealth held by top 10%	0.70	0.70
$y_{s,1}$	0.145	Total number of locations, $T_s$	1.00	1.00
$\Pi_{1,4}$	0.002	Income Gini index	0.64	0.65
$\Pi_{4,1}$	0.011	Wealth Gini index	0.82	0.83
$\Pi_{1,1}$	0.964	Persistence, $\rho_s$	0.91	0.91
$\Pi_{2,2}$	0.976	St.d of innovation, $\sigma_s$	0.20	0.20
$\Pi_{3,2}$	0.033	<a href="#">Tauchen (1986)</a> method	—	—
$\Pi_{1,2}$	0.033	$\Pi_{1,2} = \Pi_{3,2}$	—	—
$\Pi_{1,3}$	0.000	$\sum_{i=1}^3 \Pi_{1,i} = 1 - \Pi_{1,4}$	—	—
$\Pi_{2,1}$	0.011	$\Pi_{2,1} = \Pi_{2,3}$	—	—
$\Pi_{2,3}$	0.011	$\sum_{i=1}^3 \Pi_{2,i} = 1 - \Pi_{1,4}$	—	—
$\Pi_{3,1}$	0.000	$\Pi_{3,1} = \Pi_{1,3}$	—	—
$\Pi_{3,3}$	0.964	$\sum_{i=1}^3 \Pi_{3,i} = 1 - \Pi_{1,4}$	—	—
$\Pi_{2,4}$	0.002	$\Pi_{2,4} = \Pi_{1,4}$	—	—
$\Pi_{3,4}$	0.002	$\Pi_{3,4} = \Pi_{1,4}$	—	—
$\Pi_{4,2}$	0.011	$\Pi_{4,2} = \Pi_{4,1}$	—	—
$\Pi_{4,3}$	0.011	$\Pi_{4,3} = \Pi_{4,1}$	—	—
$\Pi_{4,4}$	0.967	$\sum_{i=1}^4 \Pi_{4,i} = 1$	—	—
$y_{s,2}$	0.362	$\log y_{s,2} = 0.5(\log y_{s,3} + \log y_{s,1})$	—	—
$y_{s,3}$	0.908	$\log y_{s,3} - \log y_{s,1} = 2\sigma_y$	—	—

Table 4: Steady-State Comparison

	Percentage change
Wealth	1.672
Output	0.298
Goods consumption	0.815
Services consumption	0.043
Search effort	0.094
Average price	1.131
Gini index for wealth	-0.843
St. dev. of price dispersion	-0.188
Services of average household relative to average services	-0.013
Labor	0.015
Productivity	0.282

Table 5: Steady State Comparison for Simple Economy

	Percentage change	
	Simple economy	Baseline
Wealth	6.505	1.672
Output	0.135	0.298
Goods consumption	0.511	0.815
Services consumption	0.009	0.043
Search effort	0.035	0.094
Average price	0.652	1.131
Gini index for wealth	-5.663	-0.843
St. dev. of price dispersion	-0.175	-0.188
Services of average household relative to average services	-0.001	-0.013
Labor	0.003	0.015
Productivity	0.132	0.282

Table 6: Changes in Aggregate Variables  
as a Result of the Financial Shock

	Flexible Prices	Rigid Prices
Output	-0.04	-0.54
Goods consumption	-1.88	-2.34
Services consumption	-0.05	-0.76
Average price	-2.41	0.00
St. dev. of price dispersion	3.39	0.00
Labor	-0.02	-0.29
Productivity	-0.02	-0.26