On the Quantitative Importance of Market Completeness

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Abstract

This paper quantitatively compares allocations in calibrated large overlapping generations growth economies that only differ in their market structures for insuring against aggregate risk. The findings are that the equilibrium behavior of aggregate variables is very similar across all market structures. There are only minor differences: the ratio of the volatility of aggregate consumption relative to that of investment is higher in complete market economies, due partly to the higher volatility of consumption of older people. It was also found that the risk premia in complete markets economies is basically zero. Therefore, we can conclude that we can for the most part abstract from the issue of whether there exist markets for aggregate risk.

Key Words: Market Completeness; Dynamic General Equilibrium; Risk Premium

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1 Introduction

In a well known piece that criticizes the use of the representative agent stochastic growth model for the study of business cycles, Summers (1986) states that those models are "stochastic Robinson Crusoes". To avoid this criticism, it is argued that the sole agent is in fact representative of society as a whole, capable of aggregating the behavior of large numbers of economic agents. For this to be true, all those economic agents should have their marginal rates of substitution equated both across time and states of nature. Equalization can be achieved either through market mechanisms by having complete markets in the sense of Arrow-Debreu (time zero markets) or in the sense of Arrow (by having a full set of one period ahead securities), or by some other type of social institutions.[1]

In order to legitimately use the representative agent growth models for quantitative research this issue has to dealt with. There are two main ways of doing so. One is to look at individual consumption data and see whether agents do in fact equate their marginal rates of substitution. The other is to construct artificial economies with heterogeneity of agents built in it and compare their equilibrium allocation across market structures. If for a variety of questions the answers obtained by these models do not depend on the market structure, we could conclude that the representative agent growth model is a reasonable abstraction for the questions in hand.

This paper follows the second approach. It constructs an heterogeneous agents growth model that can be used to study real business cycles. It compares the equilibrium allocations of a variety of market structures. They differ in whether agents can or cannot write contracts contingent on the main source of uncertainty in real business cycles: productivity shocks. The models are of the overlapping generations variety with agents living a large number of periods to be taken to be years, that can be calibrated to microeconomic and demographic evidence and to key growth ratios of the U.S. These economies are basically stochastic versions of the economies first used by Auerbach & Kotlikoff (1987) in their study of fiscal policy. There is an important rationale to use overlapping generations models to address this issue. Agents have different age-specific productivities, this means that the aggregate productivity levels of the economy are more important at the peak of their earning potentials, giving agents of different ages a rationale to insure each other.[2]

[1]For example, Phelan & Townsend (1991) and Townsend (1987) explore how Pareto optimal allocations can be achieved in a variety of environments.

[2]In an infinitely lived multiagent economy with agents only varying in wealth and earning ability, constant relative risk aversion utility implies that all agents want to share the same proportion of the aggregate risk,

The key findings are that economies differing the in set of markets available to insure away aggregate uncertainty have very similar equilibrium allocations. For the type of questions typically explored in the business cycle literature, the answers are basically the same across market structures. There are, however, some small differences. The relative volatility of aggregate consumption with respect to investment is higher in complete market economies. This seems to imply that investment is less of a buffer of productivity shocks when there are markets that allow to insure away such uncertainty. Another difference pertains the relative behavior of the volatility of consumption across age groups. Older agents have higher volatility in complete markets environments while the opposite happens for young agents. The opposite happens, for leisure. This seems to indicate that there is some intergenerational insurance going on in complete markets with older agents absorbing some risks. These differences of volatilities in consumption and leisure translate into differences in the behavior of asset holdings.

In some of the economies explored in this research a portfolio with a sure non-stochastic return can be constructed from the contingent securities. The risk premium observed is virtually zero.

There is some other work that explores numerically the properties of economies with a variety of market structures. İmrohoroğlu (1989), Hansen & İmrohoroğlu (1992), Díaz-Giménez & Prescott (1989), and Díaz-Giménez (1990), study the properties of a variety of economies where assets are held for precautionary motives. None of these economies have capital and agents are hit by idiosyncratic shocks that cannot be insured away. They find that the inexistence of markets for commodities contingent on the realization of some idiosyncratic component has quantitatively important consequences for the allocations implied by the model, although the impossibility of society-wide variations in investment as a buffer mechanism makes it hard to compare these models with standard real business cycles models.

The other approach to explore the empirical relevance of the restrictions implied by market completeness is to empirically determine whether people do in fact equate marginal rates of substitution across time and states of nature. This has been done by a variety of researchers that test some restrictions implied by such equalization. In this fashion, Altug & Miller (1990) pose and estimate a model of individual choice of consumption and labor supply and test the restrictions that are implied by the complete markets hypothesis. Their results are that they cannot reject the hypothesis that markets for aggregate uncertainty are com-

and therefore that accumulating physical capital is sufficient to implement Pareto optimal allocations.

plete. They specifically test the equalization of marginal rates of substitutions across agents for aggregate shocks while allowing them to have idiosyncratic shocks. Mace (1991) tests the constraints imposed on the behavior of consumption by the complete market hypothesis with some specific forms of homothetic preferences: exponential utility (which requires that first differences of consumption, net of preference shocks, get equated across individuals) and power utility (growth rates of consumption are equalized across individuals). Her results are that complete markets are consistent with exponential utility. Cochrane (1991) tests whether individuals' consumption is affected by some observable idiosyncratic variables such as involuntary job loss, long (more than a hundred days) illnesses, short illnesses, spells of unemployment, involuntary move, and loss of work due to strike, finding that full insurance can be rejected for the first two variables. Hayashi, Altonji, & Kotlikoff (1993) examine the role of risk sharing between unrelated households from the same families using the PSID data set. They reject all forms of efficient risk sharing, even after allowing for nonseparibilities between consumption and leisure.

This paper is by no means unique in its study of the nature of market completeness in overlapping generations models. In deterministic settings, Marshall, Sonstelie, & Gilles (1987) show that money does not play a role of completing markets, but that its introduction is equivalent to a transfer of wealth in a complete market setting. In stochastic settings, Wright (1987) shows that if we interpret the model by indexing agents by both the period that they are born and the history prior to their birth, an Arrow-Debreu equilibrium can be implemented as a sequence of market equilibrium. Baxter (1989) shows that if Wright's interpretation of what agents are is not followed, money does not complete markets, but it alters the patterns of intertemporal trade and interagent risk sharing. Baxter (1989) is also able to compute several parametric versions of the basic overlapping generations model. In Baxter's computational exercises, the object is to see how the introduction of money effectively allows agents to implement intertemporal risk sharing. Note that the fact that they do not overlap for more than one period prevents agents from using other means to implement intergenerational risk sharing.

Numerical computations of multiagent economies with market structures have been used to study the nature of the risk premium puzzle. Marcet & Singleton (1990), Ketterer & Marcet (1989), Telmer (1992), Lucas (1991), explore the asset market implications of the inexistence of complete markets for infinitely lived two agents economies with each of them facing large income risks. Trades on a long lived asset is used to smooth consumption over time. Under the parametrization chosen by these authors, the average wealth to consumption ratio is around 10 in their economies. This allows agents to almost implement the first best by using the long lived asset as a buffer. These findings are then similar to those in this paper, even though their economies are very restrictive. Not surprisingly either is the fact that the described research does not find evidence of a sizable risk premium in their economies[3].

This paper also relates to the theoretic literature on general equilibrium in incomplete markets. This line of work has long been interested in the welfare properties of the allocations achieved when some markets are missing (see Geanakoplos (1990) for an excellent review). Their results are that generically equilibrium allocations are constrained inefficient in the sense of Diamond (1965). This means that among the allocations that are not only feasible, but can be achieved via market interactions with the existing assets, there are some that Pareto dominate the equilibrium allocations, see Geanakoplos, Magill, Quinzii, & Dreze (1990) for a proof in economies with production). But their results give no indication of what is the magnitude of these inefficiencies. This paper indicates that they are very small when there is a durable asset, and when the markets that are missing are those for trading commodities indexed by aggregate changes in productivity.

Section 2 describes the model, while section 3 defines equilibrium in a way suited for its computation. Section 4 describes the calibration procedures, and gives an outline of the different economies varying in market structure that are studied. The findings and analysis of the experiments are reported in Section 5, while section 6 concludes. The computational procedures are described in the Appendix.

2 The model

The economy is of the overlapping generations type where agents live a maximum of I periods. The probability of surviving between age i and age $i+1$ is s_i. Therefore, the unconditional probability of being alive at age i is $s^i = \prod_{j=1}^{i-1} s_j$. The population grows at rate λ and its distribution across ages is assumed to correspond to the stable population[4], where the ratios between age groups are constant over time. This implies that, after normalization, μ_{it}, the measure of agents of age i in period t, is constant over time, with $\mu_1 = 1$, which allows us to drop the time subscripts. To differentiate the current period from next period, the latter values are denoted by primes. Agents are endowed with one unit of time per period that can be enjoyed as leisure or can be used as an input to produce, jointly with capital, a consumption good. One unit of time of an age i agent can be transformed into ϵ_i units of

[3]Marcet & Singleton (1990) get a 3% risk premium but they attribute it to their specification for the process of the shock, and not necessarily to the structure of trades.

[4]The age structure of the population that results if initial fertility rates and death rates remain constant for a very large period of time.

labor input, where $\epsilon = \{\epsilon_1, \ldots, \epsilon_I\}$ is a vector of exogenously given parameters. Note that the fact that these parameters are different for different ages imply that productivity shocks affect different types of agents differently providing a rationale for the trade of risks. For instance, agents at the peak of their productivity have more to gain or lose than those that are retired.

Preferences are of the standard type in the real business cycle literature, i.e. expected discounted utility of a time separable, twice continuously differentiable, strictly concave, utility function of leisure and a consumption good (where the expectation takes into account the fact that agents might die before the last possible period of their lives):

$$E\{\sum_{i=1}^{I} \beta^i s^i U(c_i, l_i)\} \tag{1}$$

There is a neoclassical production function, $f(K, N)$, that is affected by a multiplicative shock $z \in Z = \{z_1, z_2, \ldots, z_{n_z}\}$. This shock follows a Markov process with transition matrix Γ and it is observed at the beginning of the period. The fact that Z is a finite set guarantees the computability of the prices for Arrow securities since discrete realizations of the shock imply that complete markets can be achieved with only a finite number of securities. Output can be used either for consumption or investment. Capital depreciates at rate δ, and what remains after depreciation can also be used for consumption within the same period. With these properties of the production process, there is a maximum sustainable capital stock $\bar{\kappa}$, and a maximum feasible consumption $\bar{c}$.

We want to explore the relevance of complete markets, so we will study versions of these economies that only differ in the market structure. A useful way of formulating the market structure is to allow trade in securities that deliver one unit of the capital good next period. At the beginning of the period the shock is observed, then the contracts for the securities are honored, and, finally, production takes place. This choice of timing of delivery has the nice property that the price of a bundle of securities that deliver one unit for sure upon the realization of next period's shock, has to be 1. Otherwise agents could arbitrage since they can store the good. A general way of setting up different degrees of market completeness is to allow for trade on securities contingent on the elements of $\mathcal{F} = \{\varphi_1, \varphi_2, \ldots, \varphi_{n_\varphi}\}$, a partition of Z, with $n_\varphi \leq n_z$, as for all $n \in \{1, \ldots, n_z\}$, there exists $m \in \{1, \ldots, n_\varphi\}$, such that $z_n \in \varphi_m$. When $\mathcal{F} = \{Z\}$, savings can only be made uncontingent, resulting in an extreme case of market incompleteness as it relates to aggregate productivity shocks. When $\mathcal{F} = \{z_1, z_2, \ldots, z_{n_z}\}$, the full set of Arrow securities can be traded.

There are also annuities markets. In these markets, agents enter into financial arrangements that specify what happens in the event of early death in the part of the agents. The only role that these markets play is to increase the return on savings by one over the probability of surviving until next period. Perhaps the simplest of these financial arrangements is to have all agents of the same cohort signing a contract in which survivors share the assets (or debts) of the agents that die.

In this manner, next period's assets holdings (debts) are this period's savings (borrowings) divided by the probability of surviving. This implies that agents face the following budget constraints (for convenience, the time subscripts are dropped):

$$a_1 = 0, \tag{2}$$

$$a_i R + W(1-l_i)\epsilon_i = \sum_{\varphi \in \mathcal{F}} b_i^{\varphi} q^{\varphi} + c_i, \tag{3}$$

$$a'_{i+1}(\varphi) = b_i^{\varphi}/s_i, \tag{4}$$

$$b_{I+1,}^{\varphi} \geq 0, \qquad \text{for } \varphi \in \mathcal{F}, \quad i = 1, \ldots, I. \tag{5}$$

where a_i is the net wealth accumulated so far by an age i agent, b_i^{φ} constitutes the amount of state contingent capital goods bought by agents of age i, q^{φ} is the price of the state contingent assets, R is the return on assets holdings, and W is the spot price of one unit of labor input in terms of the consumption good.

The aggregate feasibility constraint becomes:

$$\sum_i \mu_i (b_i^{\varphi} + c_i) \leq z f(K, N) + (1-\delta)K, \qquad \text{for all } \varphi \in \mathcal{F}, \text{ all } z \in Z. \tag{6}$$

Note that the feasibility constraint imposes that the aggregate amount of security holdings has a common upper bound that is always binding as agents are maximizing. In other words, next period's aggregate capital stock cannot be made contingent on tomorrows shock, although its distribution can. This is just another way of stating the market clearing condition for the Arrow securities. In the feasibility constraint, aggregate inputs are respectively,

$$N = \sum_i \mu_i \, (1-l_i)\epsilon_i, \qquad \text{and, } K = \sum_i \mu_i \, a_i. \tag{7}$$

3 Equilibrium

We define equilibrium recursively. This is a construct that allows one to find equilibrium allocations without keeping track of the whole history of the economy, and follows the spirit of Prescott & Mehra (1980). It is a generalization of the definition of equilibrium in Ríos-Rull (1992). In order to define it, we have to define what the state variables are at both the economy wide level and at the agent's level. The state of the economy is characterized by the economy wide shock $z \in Z$, and by the distribution of assets holdings by agents in each age group, $k \in \mathcal{A} \subset \mathbf{R}^{\mathbf{I}}$. The state variables for any given agent are its own asset holdings $a \in \mathcal{A}_i \subset \mathbf{R}$, and the economy wide state (k, z). Note that since all agents of the same age share the same strictly concave utility function and convex choice set, they do the same thing in equilibrium, $a_i = k_i$. Therefore, the aggregate laws of motion for the economy only depend on the decision rules of the agents and on the process for the shock, and, since this follows a Markov process, its current value is all the information needed to know tomorrow's distribution. With the type of securities chosen, next period's state k', depends on next period's shock, z'. Then, $k'(z') = b^{\varphi(z')}$, corrected by the probabilities of surviving, where $\varphi(z')$ is the element of $\mathcal{F}$ that contains z'. The value functions involved in the definition are indexed by the age of the agent and their value is expected remaining utility. Formally, the definition is as follows:

Definition 1 *A Recursive Stochastic Competitive Equilibrium is:*

(i) A set of decision rules $b_i^{\varphi}(a, k, z)$, $c_i(a, k, z)$, $l_i(a, k, z)$, for all $i = 1, .., I$, $\varphi \in \mathcal{F}$, $a_i \in \mathcal{A}_i$, $k \in \mathcal{A}$, $z \in Z$.

(ii) A set of pricing functions $W(k, z)$, $R(k, z)$, $q^{\varphi}(k, z)$, for all $\varphi \in \mathcal{F}$, $k \in \mathcal{A}$, $z \in Z$.

(iii) A set of value functions $v_i(a, k, z)$, for all $i = 1, .., I$, $a_i \in \mathcal{A}_i$, $k \in \mathcal{A}$, $z \in Z$.

(iv) A Law of Motion for the capital stocks $k'_{i+1} = g_i^{\varphi(z')}(k, z)$, for all $i = 1, .., I-1$, $k \in \mathcal{A}$, $z \in Z$, $z' \in Z$.

(v) A pair of functions for aggregate variables $K(k, z)$, $N(k, z)$, for all $k \in \mathcal{A}$, $z \in Z$.

Such that:

a) The allocation is feasible, i.e., for all k, z, and all φ,

$$\sum_i \left(b_i^{\varphi}(k_i, k, z) + c_i(k_i, k, z)\right)\mu_i = zf[K(k, z), N(k, z)] + (1-\delta)\sum_i k_i \mu_i. \tag{8}$$

b) Factor prices are competitive, i.e., they are the marginal productivities:

$$\begin{aligned} W(k, z) &= zf_2[K(k, z), N(k, z)], && (9) \\ R(k, z) &= 1 - \delta + zf_1[K(k, z), N(k, z)]. && (10) \end{aligned}$$

c) Given the law of motion for capital stocks, the price functions and the transition for z, decision rules for age i agents solve their maximization problem:

$$\{b_i^{\varphi}(a, k, z), c_i(a, k, z), l_i(a, k, z)\} \in$$

$$\underset{b^\varphi, c, l}{\text{ARGMAX}} \qquad U(c,l) + \beta s_i E\{v_{i+1}(b^{\varphi(z')}/s_i, g^{z'}(k,z), z')|z\} \tag{11}$$

$$\text{st: } aR(k,z) + (1-l)\epsilon_i W(k,z) = c + \sum_{\varphi \in \mathcal{F}} b^\varphi q^\varphi(k,z).$$

d) The value functions are generated by the policy functions and $v_{I+1} = 0$, and the budget constraint is satisfied:

$$\begin{aligned} v_i(a,k,z) = U(c_i(a,k,z), l_i(a,k,z)) + \\ \beta s_i E\Big\{v_{i+1}(b_i^{\varphi(z')}(a,k,z)/s_i, g^{z'}(k,z), z')|z\Big\}, \end{aligned} \tag{12}$$

$$\begin{aligned} aR(k,z) + [1 - l_i(a,k,z)]W(k,z)\epsilon_i = \\ c_i(a,k,z) + \sum_{\varphi \in \mathcal{F}} b_i^\varphi(a,k,z) q^\varphi(k,z). \end{aligned} \tag{13}$$

e) The law of motion of the capital stocks is generated by the decision rules of the agents:

$$b_i^{\varphi(z')}(k_i,k,z)/s_i = g_{i+1}^{\varphi(z')}(k,z), \qquad \text{for all } z' \in Z, \text{ all } i = 1,..,I-1. \tag{14}$$

f) Aggregate functions K and N are generated by aggregation and the decision rules of the agents:

$$K(k,z) = \sum_i \mu_i \, k_i, \tag{15}$$

$$N(k,z) = \sum_i \mu_i \, [1 - l_i(k_i,k,z)] \, \epsilon_i. \tag{16}$$

g) There are no arbitrage opportunities:

$$\sum_{\varphi \in \mathcal{F}} q^\varphi(k,z) = 1., \text{ or } q(k,z) \in \Delta^{n_\varphi}, \text{ for all } k \in \mathcal{A}, z \in Z. \tag{17}$$

h) Securities markets clear:

$$\sum_i \mu_i b^\varphi(k_i,k,z) = \sum_i \mu_i b^{\varphi'}(k_i,k,z), \text{ for all } \varphi, \varphi' \in \mathcal{F}. \tag{18}$$

Note that the last condition is already embodied in the first, as aggregate savings cannot depend on next period's realization of the shock.

In Ríos-Rull (1992), there is a proof of existence of equilibrium for complete markets economies that is not based in the second welfare theorem. This proof is not for the recursive notion defined above, but for a sequence of markets notion of equilibrium.

The equilibrium with complete markets can be implemented into an Arrow-Debreu economy by using a Wright (1987) type of construction where agents born under different histories are considered different agents. The sequences of equilibrium prices of goods are constructed from the prices of the Arrow securities and the rates of return. To be able to do this, the price sequence constructed has to belong to L_1, and this can be accomplished if expected rates of return at every node are bigger than one (see Zilcha (1990). The simulations of the complete market economy run tend to indicate that this is the case, allowing for a tentative labeling of the complete market equilibrium allocation as Pareto Optimal. Note that this would not be the case if, as Baxter (1989) does, we considered that agents are indexed by date of birth but not by history. In this case agents would like to insure themselves against events that occur previous to their birth which cannot be achieved with the recursive market structure posed.

4 Parametrization of the Model Economies

Functional forms have to be chosen for both utility and production functions. The utility function of an agent born in period t is assumed to be: $E_t\{\frac{1}{1-\gamma}\sum_{i=1}^{I}\beta^i s^i (c_{i,t+i-1}^{\theta}\, l_{i,t+i-1}^{1-\theta})^{1-\gamma}\}$.

The reasons for this choice are both that this function implies that the level of leisure is independent of productivity, and that the parameters needed for its calibration, β, θ, and γ, have been extensively studied in the literature. The constancy over time of the factor shares of income recommends a Cobb-Douglas production function where non-depreciated capital can be used for consumption:

A transformation of the variables is done in order to allow for both productivity and population growth. Both types of growth are assumed to be at a constant rate. Population growth only affects the age distribution of agents, with $\mu_{i+1} = \mu_i s_i/(1+\lambda_\mu)$, λ_μ being the rate of population growth, while productivity growth affects both the discount factor and the transformation of today's savings into tomorrow's assets holdings. The transformed discount rate is $\hat{\beta} = \beta(1+\lambda_y)^{\theta(1-\gamma)}$. The rate of growth of productivity λ_y, is 1.83%, the rate of growth of per capita consumption for the last century, and the rate of population growth is 1.24%, the average U.S. rate of growth of population for the 1933-1986 period.

A summary description of the values of the parameters chosen is[5]:

γ: (risk aversion parameter or inverse of intertemporal elasticity of substitution). A value of 4 is used as in Auerbach & Kotlikoff (1987). This choice is a compromise between

[5]For a detailed discussion see Ríos-Rull (1992).

different studies (Ghez & Becker (1975), MaCurdy (1981)), that both use a life cycle model and explicitly account for leisure. Mehra & Prescott (1985) and Prescott (1986) discuss other estimates in the literature and conclude that a reasonable number is not too far from 1 for preferences without leisure. This value is similar to the value of 4 with leisure as the coefficient of consumption is .33. Note also that the models they use have quarters as periods, and we should expect that the substitution possibilities decrease with the length of the period.

θ: (coefficient of leisure versus consumption). The elasticity of substitution between consumption and leisure is taken to be 1 given the stability of average yearly hours worked since World War II. For θ, I follow Prescott (1986), and choose a value of .33.

α: (labor share). The value of this parameter depends on the treatment of consumer durables, proprietors income, and the attribution of the difference between National Income and GNP to the different factors of production. See Christiano (1988) for a discussion. Following Prescott (1986) the value of this parameter is set to 0.64, which is very similar to the value of .66 used by Christiano.

δ: (depreciation rate). It is taken to be 10%, as a reasonable approximation to capture not only the depreciation of capital, but also that of consumer durables, which, typically, depreciate faster.

ϵ: (age specific endowment of efficiency units). These weights are obtained from CPS data. The procedure is to compute averages for the wages of each age group for the 1979-1987 period. The average wages over this period become the efficiency weights for each age group.

β: (discount factor). The value used is 1.011, a point estimate from Hurd (1989) that accounts for mortality.

Z and Γ: (the process on the technology shock). The values chosen are:

$$Z = \begin{pmatrix} 0.966 & 1. & 1.034 \end{pmatrix}$$

$$\Gamma = \begin{pmatrix} .823 & .169 & .008 \\ .084 & .832 & .084 \\ .008 & .169 & .823 \end{pmatrix}$$

The reason is to obtain values for the variance and autocovariance equal to those in Prescott (1986), where he discusses the process for the Solow residual. Matching those two parameters pin down a 2×2 matrix. It was expanded to 3×3 using the procedure described in Rebelo & Roewenhorst (1988).

Early retirement is imposed on the agents. The algorithm is not able to handle economies where the age of retirement is a function of the state, but it is able to find early retirement of the agents in steady state. Since this does not always happen with the parameterization chosen, the model is implicitly assuming the existence of a compulsory retirement law. Retirement is imposed at age 67, which in terms of the 55 period lived agents in the model is age 48.

The probabilities of surviving at each age are taken from the 1985 vital statistics of the U.S. population. The demographic structure becomes a stable population where the age distribution is the one that would be achieved if mortality rates remain at the 1985 levels for a long period of time and the rate of growth of population remains constant at the chosen value.

With this parameterization, that includes 3 possible realizations of the shock, four possible market structures are studied. The first structure considered has complete markets in the sense that trade can be made contingent on all possible realizations of next period's shock. The next structure considered does not allow for contingent trading, and agents can only save to protect themselves against future shocks. The last two structures considered include partial contingent trading. One considers trades contingent on whether the best state occurs or not, while the other allows for trade contingent on the bad state occurring. In terms of the partition $\mathcal{F}$ defined in section 2, we have for the economies described that $\mathcal{F}$ is respectively $\{z_1, z_2, z_3\}$, $\{Z\}$, $\{z_1, \{z_2, z_3\}\}$, and $\{\{z_1, z_2\}, z_3\}$.

Table 1, reports the steady state values of key variables. Note that the rate of return of capital is 6.1%, and a capital output ratio of 2.2, not very different from the standard calibration of the growth model where the discount rate is chosen to imply a rate of return of 4%. The calibration chosen is capable of generating enough capital.[6] Keys for this success are not only the discount rate, but also the pattern of the age specific endowments of efficiency units of labor. The success is more important if we note that the depreciation rate chosen might be too high. This model matches quite well the growth ratios that are used to calibrate the models in the real business cycle literature. The steady state version of the model also agrees with key life-cycle properties in the data. Figures 1, and 2 described the profile of total and per capita household consumption by age of the household head.[7] Both figures

[6] In their original work Auerbach & Kotlikoff (1987) abstracted from depreciation in order to increase steady state capital.

[7] This data is from the Consumer Expenditure Survey for 1990. Some of these statistics do not exactly match the Bureau of Labor Statistics published aggregates because some editing has been undertaken at the University of Pennsylvania. The affected variables are rent, lodging away from home, ground rent on residence and vacation home and expenses for college students. Additionally, consumption has been adjusted

show an inverse U shape. The figures are also pretty rugged. This could be very much due to sample error. By grouping people into 5 year age groups one could obtain a much smoother series, while keeping the shape of an inverse U. Figures 3, and 4 describe the age profile of weeks worked per year and the number of total hours worked.[8]

Figures 5, 6 report the age specific consumption, and leisure for the simulated economies (they are practically identical to the steady state). Three key properties to note from these graphs. First, both consumption and leisure have the right type of shapes (inverse U for consumption, and U for leisure). Second, the increasing nature of both consumption and leisure are due to the fact that the interest rate is higher that the rate of time preference. Third, the curves have kinks. The first two of these properties are present in the data, specially if we consider per capita consumption which is the concept to which consumption in the model corresponds as the models abstract from family considerations. Regarding leisure, note that early retirement in the data corresponds in the model to the increase leisure that happens before retirement. The kinks that these curves have are due to the fact that the labor productivity profile have not been smoothed; they have been obtained from data that aggregate age groups producing long periods with constant labor productivity separated by very sharp changes.

Each experiment is very time consuming, taking between 2 and 3 days on a Sun Sparc station 1. For this reason, there is no attempt to study the behavior of these economies for many other values of the parameters. Several experiments with economies where agents live a very small number of periods were carried. They are characterized by having a much larger volatility of the productivity shock. In Ríos-Rull (1992), several experiments for different values of the preference and technology parameters were run for complete market economies. Most of the answers were similar across the different specifications.

5 Findings

5.1 Comparison of Allocations across Different Market Structures

As stated, Table 1 reports the deterministic steady state values of some key variables. They include the values for output, capital-output ratio, consumption-investment ratio, the rate of return of capital, utility of a newborn, and consumption, leisure, and asset holdings for

for imputed rent of owner occupied housing.

[8]The source for this data is also the Consumer Expenditure Survey for 1990.

selected ages. The latter are consumption and leisure of the youngest, consumption of the oldest, and asset holdings of age 2, the youngest age at which non-zero holdings are possible, and of age 42, the age at which asset holdings peak. The aggregate variables are chosen for their intrinsic importance, the age-specific ones as generic examples of properties of the equilibrium allocation. Given the large number of agents, there is a very large number of time series produced by these artificial economies. Reasons of space determine that only a few of those time series can be reported. The same reasons of space apply for not reporting of equilibrium decision rules of these economies.[9]

Tables 3, 4, 5, and 6 report the averages and standard deviations of sample averages and sample percentage standard deviations from trend for each model economy (to compute sample second moments, the series were detrended using the Hodrick-Prescott filter with a parameter value of 100). They are the average values of 200 realizations of length 200 periods. The realizations of the economies are separated by 300 periods. The sequences of shocks are identical across economies. These tables report the standard statistics used to answer questions in business cycles. In order to get a sense of how these stochastic properties compare to the U.S., Table 2 reports the second moments for the key aggregate variables in the data. It can be noted that these models give the same type of answers to standard business cycle questions than representative agent models[10]. We can see that, as in more standard real business cycle models, productivity shocks account for roughly three quarters of total output volatility. Since in the models, periods are considered to be years, the actual value of the statistics is different than for quarters, for example it changes the nature of the phase shift of output and capital. The model compares very well in this respect with the data specially if we think (see Table 2) that the right timing of capital in the data to compare with the model's notion of initial capital is mid year capital.

Before more careful analysis of the properties of these tables we should note that, as we should have perhaps expected, the values of the semicomplete markets economies, Tables 5, and 6, lie in between those of the complete (CM), Table 3, and those of the incomplete markets economy (IM), Table 4, so I will concentrate in exploring these two extreme cases.

The first thing to note is how similar are the sample averages of the first moments of all aggregate variables, they are almost identical for all variables. These tables, however do not

[9]Reporting just the aggregate law of motion for asset holdings, g, for one of the economies involves reporting $54 \times 55 \times 3 \times 3 = 26,730$ numbers.

[10]See Ríos-Rull (1992) for a detailed study of the business cycle properties of large overlapping generations economies.

give an adequate sense of how similar are the allocations across market structures since they report the average and percentage standard deviations of sample statistics, and there might be some patterns that are lost in the aggregation process. In order to further analyze the similarities across economies of the sample statistics, we will construct some measures that compare individual statistics and not just their averages. In this fashion, let $\{x^m_{j,n}\}^{m \in M}_{j \in J, n \in N}$ be a sample statistic, where the j stands for statistic (say, average output, or average first period consumption), the m for market structure, and the n for the specific sample. If the allocations of the different economies are very similar, then $\{x^m_{j,n} - x^{m'}_{j,n}\}$ would be close to zero. Furthermore, the correlations between pairs of $\{x^m_{j,n}, x^{m'}_{j,n}\}$ across samples would be close to one.

Table 7 reports the percentage standard deviations of the differences of sample averages across economies. In particular, the formula for the statistic computed is $\frac{\sqrt{\frac{\sum_n (x^m_{jn} - x^{m'}_{jn})^2}{N}}}{(\bar{x}^m_j - \bar{x}^{m'}_j)/2} 100$. Where $\bar{x}^m_j$ is the average value of x^m_{jn}. They are very close to zero, with, perhaps, minor exception of assets held by age 2 agents. Table 8 reports the correlations between the $(x^m_{j,n}, x^{m'}_{j,n})$ for all market structures, for selected variables. All these correlations are almost 1, with, again, the exception of age 2 assets that is a mere .2, implying substantially different patterns for this statistic across economies.

Another way of seeing these properties is through Figures 8, 9, 10, that plot a set of 200 sample averages for CM, and IM for aggregate output, consumption of the youngest, and asset holdings of age 2. The statistics plotted follow the same order as the sample realizations. They are paired because they correspond to the same values of the exogenous shocks. Figures 8, and 9 seem to include only one series. They do not. If we look carefully, we will notice that in observation 117 there is a slight difference, implying the existence of two almost perfectly overlapping series. Figure 10, does not have this feature. Both series of assets holdings of age 2 are very different, recall that their correlation is .2.

These last two tables and the Figures show that such statistics are in fact very similar across samples, with the exception of age 2 asset holdings.

Note that utility of the newborn is slightly higher in the complete market economy. However, the size of the standard deviations shows that the differences are indeed small. One could still argue that the fact that utility is higher on average in the complete market case could be directly due to the completeness of markets. This is not necessarily the case. The fact that agents wealth at birth is given by the present value of their future labor earnings, implies that the amount of capital in the economy at the moment of their birth is important. Consequently, the higher utility of the complete market economy might be accounted for by its higher average capital stock.

Regarding the second moments of the model economies, the second column of Tables 3 through 6 report the sample averages of the percentage standard deviations of key variables. Note that the standard deviation of real output is practically identical across economies: IM's value is 99.7% of that of CM. Some differences start to arise regarding the volatility of the components of output. Consumption is about 4.5% more volatile in the complete market economy while investment is 2.4% less volatile. These differences in the volatility of consumption can be traced to specific age groups. Figure 11 plots the profile of age specific standard deviations of consumption for CM and IM. In both types of economies the patterns are very similar in the middle periods of life, with a very slightly higher standard deviation in the IM in early years. The picture changes dramatically as agents approach retirement and reduce their time spent in market activities. At these ages, agents in CM have a much larger volatility of consumption than their counterparts in IM, up to 3 times bigger in the last period of life. These type of behavior has its counterpart in the volatility of leisure, and it can be seen in Figure 12 that plots the profile for the volatility of leisure. The behavior here is the opposite. Young agents have a higher volatility of leisure in CM than in IM (opposite to consumption at those ages). Old agents before retirement, have a higher volatility of hours worked in IM than in CM, again the opposite relative volatility than for consumption. This interaction between the volatilities of consumption and leisure is expected as preferences are nonseparable and marginal utilities of consumption are affected by the level of leisure, or in other words there is a composite good of both consumption and leisure that which is the object that agents want to smooth. The opportunities to do so do depend on the market structure. Figure 13 plots the profile for the volatility of assets holdings for agents older than 30 years of age (since these are percentage standard deviations, assets of younger ages are very small, and even negative). They are consistent with the previous observations about consumption and leisure. After agents retire, variations in leisure cannot be used to smooth the composite good and, the higher volatility of consumption of retirees in CM translates into higher volatility of asset holdings at those ages.

The higher volatility of consumption for the older people in the complete markets world translates into a higher volatility of aggregate consumption, and lower aggregate volatility of investment. A way to interpret this is to think of aggregate investment as an economy-wide buffer for fluctuations in output. If the economy is efficient in allocating consumption risk between different agents, the role of investment as a buffering mechanism becomes a lot less important compared with its role in less efficient economies, economies without complete markets.

The autocorrelation properties of variables associated with real allocations are very similar across market structures. Less so the asset positions of agents.

The key thing to note is that any question regarding the properties of these models, specially their business cycles properties will obtain basically the same answer. The differences found in the relative volatility of consumption and investment are small. This, I think, justifies the use of the representative agent growth for the study of business cycles. The explicit account of different market structures with different possibilities for insuring against aggregate uncertainty does not change the answers obtained in any siginificant manner.

In order to further explore the differential properties of market structures, several experiments with agents living a small number of periods with a much larger variance of the shock were conducted. Agents live only 5 periods, and they spend the last period in retirement, so the interpretation of periods as years can no longer be made. The productivity shock that hits this economy takes two values of 1.2 and .8, generating a very large standard deviation, about 8 times bigger than the one used in the previous examples. The autocorrelation of this shock is the same that that of the large economies.

Comparing the properties of these two small economies, averages are very similar while there are some differences in the standard deviations across both markets structures. The volatility of output in the incomplete market economy is 2.1% higher, that of consumption 2.3% higher and that of investment 7.2% higher. Hours, however, are a lot more volatile in the complete markets economy (this is possible because both the volatility of hours is small in both economies, and because capital is more volatile in the incomplete market economy).

Compared with the previously studied large calibrated economies, the same pattern occurs for the behavior of the workforce: the labor input (and aggregate hours too) is more volatile in CM rather than IM, although in the large economies the difference is tiny. The relative size of output volatilities is, however, reversed, but a key property remains: the relative volatility of consumption with respect to investment is .46 in the small complete markets economy and .43 in the small incomplete market economy. In the large economies, these ratios are .25 for CM, and .23 for IM.

Figures 14, and 15, report the crossectional profile by age groups of the standard deviation of consumption and leisure. Two properties appear. First, the much higher volatility of hours in the complete market world is replicated at every age. Second, and, perhaps more interestingly, the relative volatility of consumption by age groups changes in the same fashion as it did for the large, calibrated economies. For the younger agents is higher in the incomplete markets world, while the contrary happens in the complete markets world. This seems to indicate that young agents are effectively more risk averse than older ones, since when given the opportunity of insuring each other older people will increase their volatility while younger people will reduce it. This property was also present in the large economies previously studied.

Incomplete market economies have a higher relative volatility of investment in terms of consumption. The economics of this lie on the fact that with incomplete markets, investment acts more as a buffer, since complete markets provide a mechanism to appropriately allocate risks among agents.

5.2 Other Findings: The Risk Premium in these Economies

Recall that complete market economies allow for the pricing of all securities. In particular, the return of riskless bonds can be obtained by constructing an appropriate bundle of Arrow securities. Table 3 shows the value of both the economy wide rate of return and the risk free rate of return. They are almost identical. They are even closer than in the calibrated economy of Mehra & Prescott (1985). The reason for this lies probably in the fact that in their work and in most standard representative agent pricing models, the riskless security is not traded, so its price reflects the economy wide attitude towards risk, while in the economies with complete markets studied here, there is active trade of securities, and the risk is allocated efficiently towards those agents that mind risk less In the complete market version of the small economy with large productivity shocks, the risk premium is bigger, the difference being about 3.2% of the value of the return, compared to .032% in the large complete market economy. Still a lot smaller than the observed values even though the small economy that generated it is a few times more volatile than the U.S. economy. This seems to indicate that the risk premium puzzle will not disappear by just considering heterogeneous agents environments since in them risk can be efficiently allocated[11].

6 Conclusion

In this paper experiments were done to address the issue of how important is the market structure for aggregate uncertainty for the equilibrium allocations. The issue was posed within an overlapping generations environment that gives an important rationale for heterogeneity across agents that makes them differently affected by productivity shocks. The findings are that the behavior of aggregate variables is very similar across different market structures. In complete market economies the relative volatility of consumption relative to

[11] There is a recent literature Lucas (1991), Marcet & Singleton (1990), and Telmer (1992) that have posed models with two agents and where individual endowment processes are not perfectly correlated. They explore the nature of the equilibrium allocations and prices under various market structures that include trading on some assets and find that they are very similar to the ones obtained in complete market settings, effectively reaching to the same conclusion that this paper does.

that of investment is slightly higher than in incomplete market economies, seeming to indicate that there is more of a role for investment to act as a buffer when agents do not have opportunities to insure each other. Older agents in complete market economies have higher volatilities of consumption that their counterparts in incomplete market economies, while the opposite occurs for leisure. Young agents have more similar behavior across economies, with volatility of consumption being a little bit higher in incomplete markets environments, and that of leisure higher in complete markets ones.This indicates that given the opportunity to sign insurance contracts, older agents tend to absorb risks from younger agents.

This type of economies also allow to study issues about the nature of the price of risk in contexts where it is actively traded. It was found that the risk premium is virtually zero.

The type of exercises done might also be used to study issues concerning the volume of trade and its relation to economy wide fluctuations. This kind of models can also be used to study how optimal portfolios relate to age.

Overall, the allocations are very similar and all of the typical questions asked in the real business cycle obtain very much the same answer across the different market structures. These findings, I think, help in legitimizing the use of representative agent models to address some issues, as the implicit assumption of market completeness for aggregate uncertainty does not seem to affect possible answers that these models can give.

APPENDIX

Computation of the Equilibrium

The computational problem is solved by linear quadratic methods. The current return of the agents is approximated by a quadratic function (a matrix), and iterations on the value functions of the agents (that are quadratic) are performed. For this, we define an operator that maps functions from the space V where the value functions are defined into itself. This space consists of $n_z(I+1)$ matrices of dimension $(I+2)\times(I+2)$, as value functions are quadratic forms indexed by both age and current state of nature. Note that we are searching for the law of motion of the economy, and this is a function $k'(z') = g_z^{\varphi(z')}(k)$, that gives the asset distribution next period when today's distribution of assets is k, this period's shock is z, and next period's is z'.

Let R_{iz} be the quadratic approximation to the current return of an agent of age i, i.e. $R_{iz}(a,k,N,q^\varphi,b^\varphi,l) = U(c,l)$, with consumption being substituted using the budget constraint, and where factor prices are substituted by the expressions for marginal productivities which depend on (K,N,z). We can think of the R_{iz} as quadratic forms that map $\mathcal{A}_i \times \mathcal{A} \times [0, \sum_i \mu_i\epsilon_i] \times \Delta^{n_\varphi} \times \mathcal{B}_{i+1} \times [0,1]$ into the real line. Here, Δ^{n_φ} is the n_φ-dimensional simplex, the space where prices for securities lie, and $\mathcal{B}_i$, the set of possible security holdings.

The value functions of the agents are found by successive approximations using the following operator $T : V \to V$:

$$
\begin{aligned}
(T\,v)_{iz}(a,k) = \; max_{l,b^\varphi} \qquad & R_{iz}(a,k,N,q^\varphi,b^\varphi,l) + \beta s_i \sum_{z'\in Z} v_{i+1,z'}\Big(\frac{\mathrm{b}^{\varphi(\mathrm{z}')}}{\mathrm{s_i}}, \mathrm{k}'(\varphi(\mathrm{z}'))\Big)\Gamma(\mathrm{z},\mathrm{z}') \\
k'(\varphi) \;&=\; g_z^\varphi(k;v) \\
q^\varphi \;&=\; q_z^\varphi(k;v) \\
N \;&=\; N_z(k;v)
\end{aligned}
$$

The use of this operator involves simultaneously determining functions $g_z^{z'}(k;v)$, $q_z^\varphi(k;v)$, and $N_z(k;v)$. For this reason they are indexed by v. The maximizing agents take them as given, but they have to be generated by their decision rules obtained in the maximization problem, and by the market clearing and no arbitrage conditions. $(T\,v)_{I+1}$ is set identically to zero.

It is easy to note that by construction a fixed point of this operator of this operator is a recursive competitive equilibrium. In practice after 120 iterations convergence was obtained. I describe now in detail all the steps involved in the computation, so that implementing the algorithm can become easy for the interested reader.

- *Step 1.* Compute a linear quadratic approximation for the current return for each age and state of nature. This implies obtaining a set of $n_z I$ matrices R_{iz} of dimension $(I + 4 + 2\varphi)$, that are set to be a quadratic form applied to a vector of the form $(1, a, k, N, q^\varphi, b^\varphi, l)$. To obtain them, for each i and z compute a linear quadratic approximation to the utility function at the point implied by setting l to its steady state value and the consumption obtained by using the budget constrained set at the following values. Set k, N to their steady state values and compute for each z the implied factor prices, set a and b^φ for all φ, equal to the steady state level of assets for age i, and set the security prices, q^φ, to the conditional probabilities of future states, $\varphi \in \mathcal{F}$. At this time, the matrices v are initialized. My option was to set all $(I+2) \times (I+2)$ elements of each of the $n_z(I+1)$ matrices to zero.

- *Step 2.* Solve the above maximization problem abstracting from the constraints. This produces compute optimal linear decision rules for state contingent savings, $b^\varphi_{i,z}(a, k, N, q^\varphi, k'(\varphi))$, and for leisure, $l_{i,z}(a, k, N, q^\varphi, k'(\varphi))$. They consist of $n_z I$ vectors of dimension $I + 3 + n_\varphi(I+1)$ for the leisure decision, and $n_z I n_\varphi$ vectors of the same dimension for contingent savings. (Note that as always there is one more dimension that variables as the optimal decision rules are not linear but afin, and the first element of the vector can be associated to a constant).

- *Step 3.* Impose on the individual decision rules, the representative agent conditions that state that in equilibrium, $a_i = k_i$, and $b^\varphi_i = k'_{i+1}(\varphi)/s_i$, and aggregate over agents to obtain the following:

  - n_z linear functions for aggregate labor input of the form $\hat{N}_z(k, N, q^\varphi, k'(\varphi)) = \sum_i \mu_i \epsilon_i \left[1 - l_{iz}(k_i, k, N, q^\varphi, k'(\varphi)\right]$.
  - $(n_\varphi - 1)$ market clearing conditions $\sum_i \mu_i k_i'^{\varphi_1} = \sum_i \mu_i k_i'^{\varphi_j}$, for all $j \in \{2, \ldots, n_\varphi\}$.
  - $n_\varphi I n_z$ age specific demand equations for state contingent securities $\hat{b}^\varphi_{i,z}(k, N, q^\varphi, k'(\varphi)) = b^\varphi_{i,z}(k_i, k, N, q^\varphi, k'(\varphi))$.

  We can write now these equations together with the no arbitrage condition (that prices for securities have to add up to one) in matrix form:

$$
\begin{pmatrix} N \\ q^{\varphi_1} \\ \vdots \\ q^{\varphi_{n_\varphi}} \\ k_2'^{\varphi_1} \\ k_3'^{\varphi_1} \\ \vdots \\ k_I'^{\varphi_1} \\ \vdots \\ k_I'^{\varphi_{n_\varphi}} \end{pmatrix}
=
\begin{pmatrix} \hat{N}_z \\ 0 \\ \vdots \\ 0 \\ \hat{b}^{\varphi_1}_{1,z}/s_1 \\ \hat{b}^{\varphi_1}_{2,z}/s_2 \\ \vdots \\ \hat{b}^{\varphi_1}_{I-1,z}/s_{I-1} \\ \vdots \\ \hat{b}^{\varphi_{n_\varphi}}_{I-1,z}/s_{I-1} \\ 0 \end{pmatrix}
\begin{pmatrix} 1 \\ k \end{pmatrix}
+
\begin{pmatrix}
 & & & \tilde{N}_z & \\
0 & \dots & \mu & -\mu & 0 \\
 & & & \dots & \\
0 & \dots & \mu & 0 & -\mu \\
 & & & \tilde{b}^{\varphi_1}_{1,z}/s_1 & \\
 & & & \tilde{b}^{\varphi_1}_{2,z}/s_2 & \\
 & & & \vdots & \\
 & & & \tilde{b}^{\varphi_1}_{I-1,z}/s_{I-1} & \\
 & & & \vdots & \\
 & & & \tilde{b}^{\varphi_{n_\varphi}}_{I-1,z}/s_{I-1} & \\
0 & 1 & 0 & \dots &
\end{pmatrix}
\begin{pmatrix} N \\ q^{\varphi_1} \\ \vdots \\ q^{\varphi_{n_\varphi}} \\ k_2'^{\varphi_1} \\ k_3'^{\varphi_1} \\ \vdots \\ k_I'^{\varphi_1} \\ \vdots \\ k_I'^{\varphi_{n_\varphi}} \end{pmatrix}
$$

Where the $\hat{}$ are used for the $I+1$ first elements of the equations, the ones associated with the constant and the vector of asset holdings k, and the $\tilde{}$, are used for the rest $(1 + n_\varphi(I+1))$ elements.

- *Step 4.* The above matrix equation has to hold for every k, in every state z, so denoting the first matrix $\hat{\mathbf{A}}_\mathbf{z}$, and the second $\tilde{\mathbf{A}}_\mathbf{z}$, we solve for $\mathbf{B}_\mathbf{z} = (\mathbf{I} - \tilde{\mathbf{A}}_\mathbf{z})^{-1}\hat{\mathbf{A}}_\mathbf{z}$. This delivers aggregate labor input, securities prices, and security holdings as functions of the state:

$$
\begin{pmatrix} N \\ q^{\varphi} \\ k'^{\varphi_1} \\ \vdots \\ k'^{\varphi_{n_\varphi}} \end{pmatrix} = \mathbf{B}_\mathbf{z} \begin{pmatrix} 1 \\ k \end{pmatrix}
$$

- *Step 5.* Substitute these functions in the first order conditions to obtain decision rules.

- *Step 6.* Substitute these functions, and decision rules in return functions R_{iz}, and value functions v_{iz}, and update, to obtain (Tv).

Note that this operator is not necessarily well defined, as $(\mathbf{I} - \tilde{\mathbf{A}})^{-1}$ need not exist. There are no guarantees of convergence, although, in every case that was used, it did.

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Table 1: Steady State Values
Agents live from 20 to 74 years of age and retire at 67

Variables x	Steady State Values
Capital Output Ratio	2.241
Consumption Investment Ratio	2.409
Output	21.322
Aggregate Consumption	15.067
Aggregate Investment	6.255
Rate of Return	6.067
Total Hours	11.055
Labor Input	13.544
Physical Capital	47.774
Consumption of Age 1	0.271
Leisure of Age 1	0.701
Assets of Age 2	-0.036
Assets of Age 42	3.032
Consumption of Age 55	0.474
Utility of Newborn	-74.208

Table 2: Cyclical Behavior of the U.S. Economy
Deviations From Trend of Key Variables 1956-1986

		Correlations with Output		
Variables x	% St Dev	x_{t-1}	x_t	x_{t+1}
Gross National Product	2.23	0.50	1.00	0.51
Aggregate Consumption	1.69	0.62	0.86	0.34
Aggregate Investment	8.66	0.46	0.87	0.14
Total Hours Worked	1.88	0.44	0.94	0.52
Labor Input	1.72	0.46	0.93	0.46
Physical Capital	1.30			
Beginning of Year		-0.46	-0.24	0.35
Middle of Year		-0.38	0.07	0.59

Source: Ríos-Rull (1992).
The series for capital covers the period 1954-83.

Table 3: Complete Market Economy
Agents live from 20 to 74 years of age and retire at 67
Averages and Deviations From Trend of Key Variables
200 Observations per Sample, 200 Samples

			Correlations with output		
Variables x	Average	% St Dv	x_{t-1}	x_t	x_{t+1}
Output	21.283	1.651	.466	1.00	.467
	(.276)	(.002)	(.058)	(.000)	(.058)
Aggregate Consumption	15.048	0.894	.396	.978	.571
	(.164)	(.001)	(.054)	(.003)	(.057)
Aggregate Investment	6.235	3.556	.499	.992	.394
	(.112)	(.003)	(.059)	(.001)	(.057)
Rate of Return	6.098	29.119	.552	.898	.165
	(.085)	(.027)	(.059)	(.014)	(.047)
Risk Free Rate of Return	6.096	19.414	.556	.847	.088
	(.104)	(.018)	(.058)	(.021)	(.044)
Total Hours	11.048	.549	.522	.973	.325
	(.016)	(.001)	(.060)	(.004)	(.054)
Labor Input	13.536	.543	.523	.973	.325
	(.019)	(.001)	(.060)	(.004)	(.054)
Physical Capital	47.628	0.791	-.291	.031	.595
	(.857)	(.001)	(.058)	(.046)	(.008)
Consumption of Age 1	0.271	0.897	.398	.980	.569
	(.003)	(.001)	(.054)	(.002)	(.057)
Leisure of Age 1	0.702	0.249	-.518	-.977	-.333
	(.001)	(.000)	(.060)	(.003)	(.055)
Assets of Age 2	-0.037	0.430	-.218	-.252	.704
	(.000)	(.001)	(.066)	(.053)	(.024)
Assets of Age 42	3.036	0.590	-.289	.038	.582
	(.044)	(.001)	(.038)	(.046)	(.014)
Consumption of Age 55	0.480	1.157	.309	.827	.578
	(.005)	(.001)	(.065)	(.026)	(.067)
Utility of Newborn	-74.410	27.903	.450	.999	.494
	(.249)	(.036)	(.057)	(.000)	(.059)

Table 4: Incomplete Market Economy
Agents live from 20 to 74 years of age and retire at 67
Averages and Deviations From Trend of Key Variables
200 Observations per Sample, 200 Samples

			Correlations with output		
Variables x	Average	% St Dv	x_{t-1}	x_t	x_{t+1}
Output	21.280	1.646	.466	1.000	.467
	(.280)	(.002)	(.058)	(.000)	(.058)
Aggregate Consumption	15.046	.855	.388	.974	.579
	(.164)	(.001)	(.053)	(.004)	(.056)
Aggregate Investment	6.234	3.643	.500	.992	.393
	(.116)	(.003)	(.059)	(.001)	(.057)
Rate of Return	6.101	29.168	.552	.892	.156
	(.092)	(.028)	(.059)	(.015)	(.046)
Total Hours	11.048	.541	.523	.971	.316
	(.016)	(.001)	(.060)	(.004)	(.054)
Labor Input	13.536	.536	.523	.971	.318
	(.019)	(.001)	(.060)	(.004)	(.054)
Physical Capital	47.618	.812	-.288	.033	.596
	(.887)	(.001)	(.059)	(.046)	(.008)
Consumption of Age 1	0.271	.914	.398	.980	.570
	(.003)	(.001)	(.054)	(.002)	(.057)
Leisure of Age 1	0.702	0.234	-.521	-.974	-.325
	(.001)	(.000)	(.060)	(.003)	(.054)
Assets of Age 2	-0.037	0.151	.227	.536	.905
	(.000)	(.000)	(.084)	(.060)	(.007)
Assets of Age 42	3.035	.583	-.277	046	.608
	(.042)	(.001)	(.060)	(.047)	(.011)
Consumption of Age 55	0.480	.326	.296	.861	.703
	(.003)	(.000)	(.069)	(.017)	(.042)
Average Utility	-74.431	28.380	.451	.999	.494
	(.256)	(.027)	(.057)	(.000)	(.059)

Table 5: Semicomplete Market Economy
Securities Contingent on Whether the GOOD State Occurs
Agents live from 20 to 74 years of age and retire at 67
Averages and Deviations From Trend of Key Variables
200 Observations per Sample, 200 Samples

			Correlations with output		
Variables x	Average	% St Dv	x_{t-1}	x_t	x_{t+1}
Output	21.283	1.649	.466	1.000	.467
	(.278)	(.002)	(.058)	(.000)	(.058)
Aggregate Consumption	15.048	.877	.393	.976	.575
	(.164)	(.001)	(.054)	(.005)	(.057)
Aggregate Investment	6.235	3.598	.499	.992	.393
	(.114)	(.003)	(.059)	(.001)	(.057)
Rate of Return	6.098	29.147	.552	.896	.162
	(.087)	(.028)	(.-59)	(.015)	(.047)
Total Hours	11.048	.546	.523	.972	.322
	(.016)	(.001)	(.060)	(.004)	(.054)
Labor Input	13.536	.541	.523	.972	.322
	(.019)	(.001)	(.060)	(.004)	(.054)
Physical Capital	47.629	.801	-.290	.031	.595
	(.869)	(.001)	(.058)	(.046)	(.008)
Consumption of Age 1	0.271	.903	.398	.980	.570
	(.003)	(.001)	(.054)	(.002)	(.057)
Leisure of Age	0.702	0.242	-.530	-.976	-.330
	(.001)	(.000)	(.060)	(.003)	(.054)
Assets of Age 2	-0.037	0.318	-.087	-.031	.710
	(.000)	(.001)	(.091)	(.121)	(.044)
Assets of Age 42	3.036	.585	-.282	.044	.597
	(.043)	(.001)	(.059)	(.046)	(.013)
Consumption of Age 55	0.480	.896	.333	.791	.492
	(.005)	(.001)	(.073)	(.036)	(.066)
Average Utility	-74.428	28.136	.451	.999	.494
	(.252)	(.027)	(.057)	(.000)	(.059)

Table 6: Semicomplete Market Economy
Securities Contingent on Whether the BAD State Occurs
Agents live from 20 to 74 years of age and retire at 67
Averages and Deviations From Trend of Key Variables
200 Observations per Sample, 200 Samples

			Correlations with output		
Variables x	Average	% St Dv	x_{t-1}	x_t	x_{t+1}
Output	21.283	1.648	.464	1.000	.467
	(.278)	(.002)	(.058)	(.000)	(.058)
Aggregate Consumption	15.048	.876	.392	.976	.575
	(.164)	(.001)	(.053)	(.003)	(.057)
Aggregate Investment	6.235	3.593	.499	.992	.393
	(.114)	(.003)	(.059)	(.001)	(.057)
Rate of Return	6.098	29.194	.552	.895	.161
	(.087)	(.027)	(.059)	(.014)	(.047)
Total Hours	11.048	.545	.523	.972	.321
	(.016)	(.001)	(.060)	(.004)	(.054)
Labor Input	13.536	.539	.523	.972	.322
	(.019)	(.001)	(.060)	(.004)	(.054)
Physical Capital	47.629	.801	-.290	.032	.596
	(.869)	(.001)	(.059)	(.046)	(.009)
Consumption of Age 1	0.271	.908	.399	.980	.494
	(.003)	(.001)	(.054)	(.002)	(.057)
Leisure of Age 1	0.702	0.242	-.519	-.974	-.330
	(.001)	(.000)	(.060)	(.003)	(.055)
Assets of Age 2	-0.037	0.348	-.124	-.094	.681
	(.000)	(.000)	(.069)	(.072)	(.034)
Assets of Age 42	3.036	.589	-.285	.040	.591
	(.043)	(.001)	(.059)	(.047)	(.014)
Consumption of Age 55	0.480	.837	.336	.817	.506
	(.005)	(.001)	(.060)	(.034)	(.073)
Average Utility	-74.428	28.107	.451	.999	.494
	(.252)	(.026)	(.057)	(.000)	(.059)

Table 7: Standard Deviations of the Differences of Sample Standard Deviations as a Percentage of the Average Value

	Differences Between Economies *					
Statistic	CM-IM	CM-GM	CM-BM	IM-GM	IM-BM	GM-BM
Output	0.084	0.012	0.015	0.083	0.082	0.020
Aggregate Consumption	0.077	0.003	0.004	0.076	0.076	0.005
Aggregate Investment	0.115	0.042	0.047	0.107	0.104	0.064
Rate of Return	0.247	0.077	0.091	0.233	0.223	0.121
Total Hours	0.008	0.005	0.004	0.007	0.007	0.006
Labor Input	0.008	0.005	0.004	0.006	0.007	0.006
Physical Capital	0.170	0.042	0.047	0.164	0.161	0.064
Consumption of Age 1	0.085	0.024	0.025	0.081	0.080	0.035
Leisure of Age 1	0.011	0.006	0.007	0.008	0.008	0.010
Assets of Age 2	0.519	0.313	0.346	0.398	0.358	0.475
Utility of Newborn	0.025	0.006	0.008	0.024	0.023	0.010

* The notation used is GM for the economy that has securities contingent on the good state ocurring, BM for that with the bad state occurring. CM and IM refer respectively, as in the text, to the complete and the imcomplete markets economies.

Table 8: Correlations Between Sample Standard Deviations

	Economies					
Statistics	CM-IM	CM-GM	CM-BM	IM-GM	IM-BM	GM-BM
Output	0.998	1.000	1.000	0.998	0.998	1.000
Aggregate Consumption	0.998	1.000	1.000	0.998	0.998	1.000
Aggregate Investment	0.999	1.000	1.000	0.998	0.999	0.999
Rate of Return	0.989	0.999	0.999	0.989	0.990	0.997
Total Hours	0.999	1.000	1.000	0.999	0.999	0.999
Labor Input	0.999	1.000	1.000	0.999	0.999	0.999
Physical Capital	0.996	1.000	1.000	0.996	0.996	0.999
Consumption of Age 1	0.997	1.000	1.000	0.997	0.997	1.000
Leisure of Age 1	0.999	0.998	0.998	0.998	0.997	0.993
Assets of Age 2	0.189	0.431	0.593	0.602	0.712	0.215
Utility of Newborn	0.998	1.000	1.000	0.998	0.998	1.000

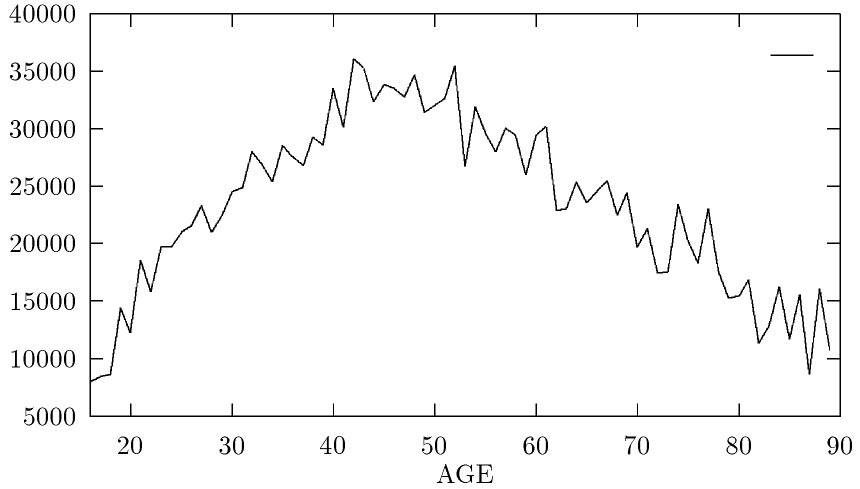

Figure 1: Average Household Consumption by Age in Current Dollars, Source: Consumption Expenditure Survey, 1990.

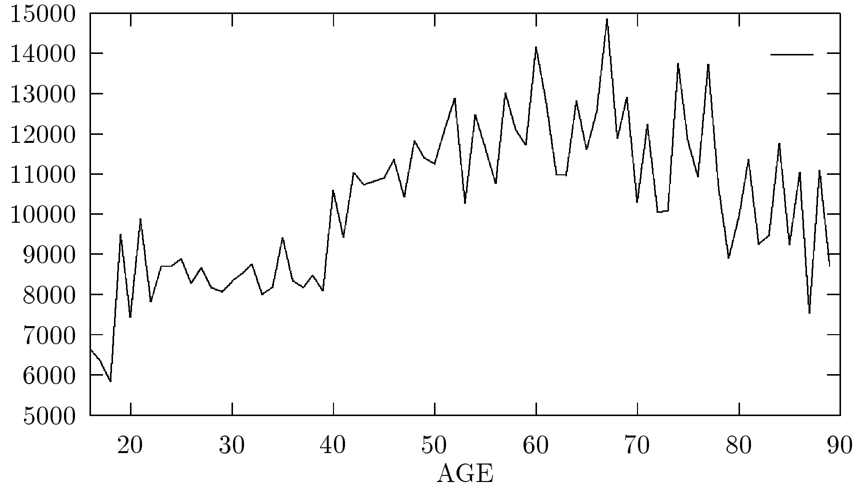

Figure 2: Average Per Capita Consumption by Age of Household in Current Dollars, Source: Consumption Expenditure Survey, 1990.

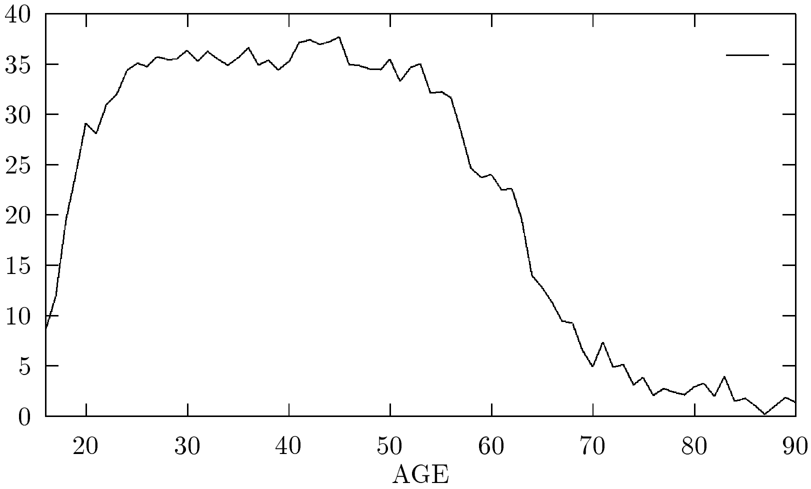

Figure 3: Average Yearly Weeks Worked by Age, Source: Consumption Expenditure Survey, 1990.

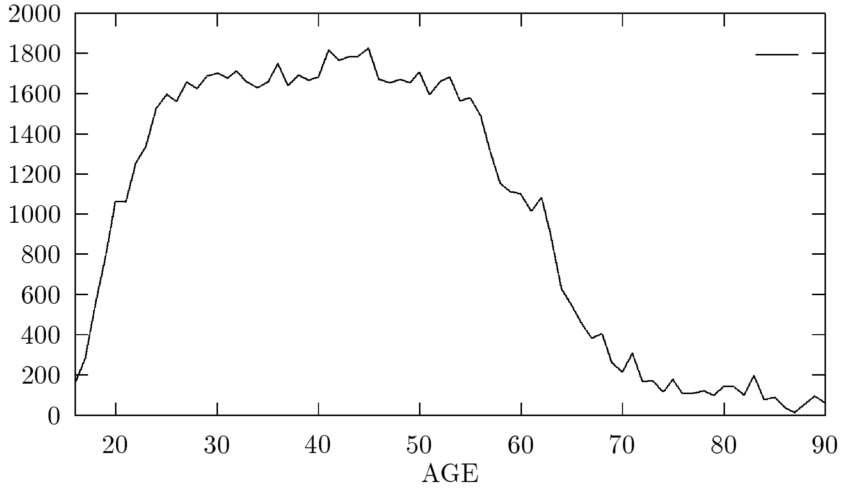

Figure 4: Average Total Hours Worked by Age, Source: Consumption Expenditure Survey, 1990.

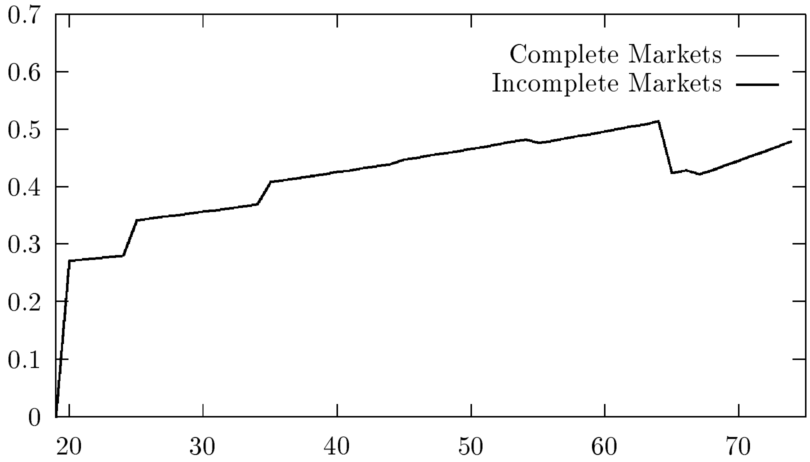

Figure 5: Average Consumption by Age

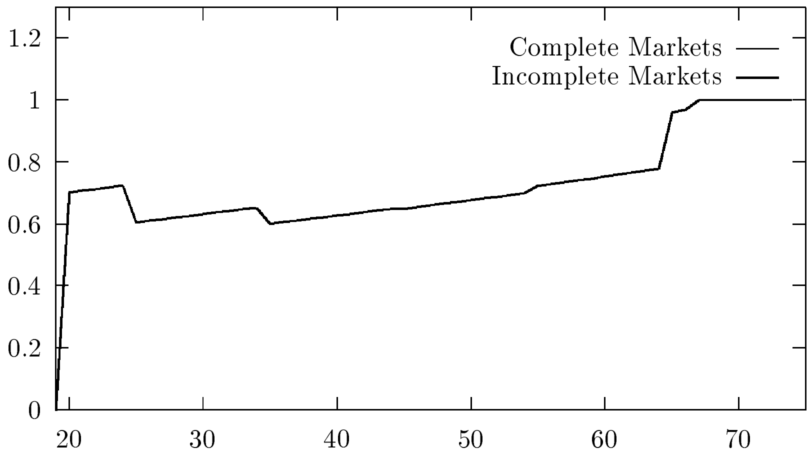

Figure 6: Average Leisure by Age

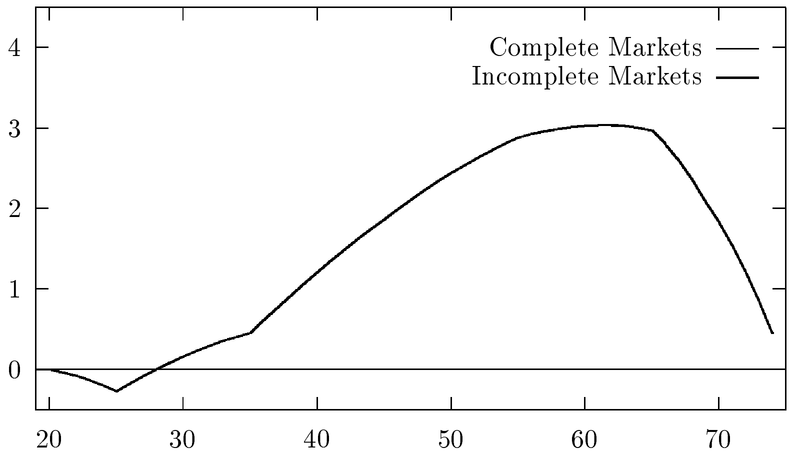

Figure 7: Average Assets by Age

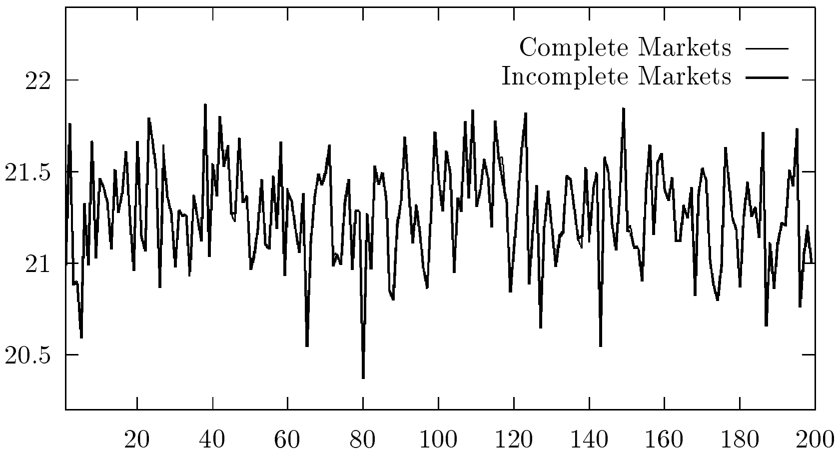

Figure 8: Sample Averages of Aggregate Output

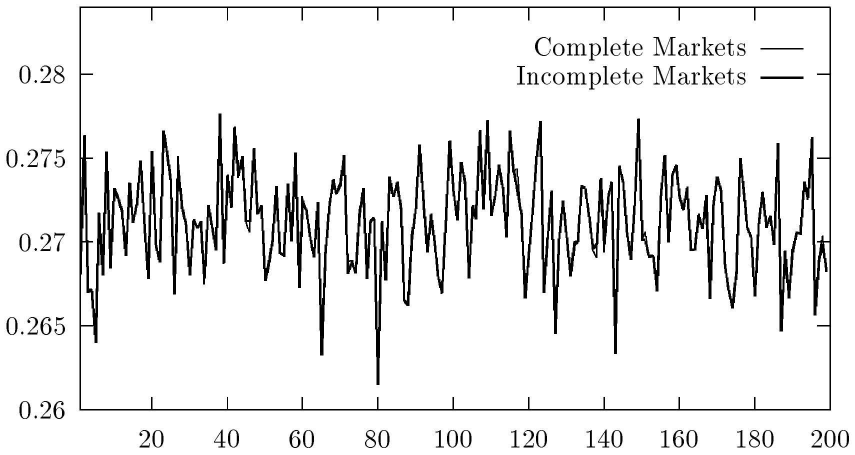

Figure 9: Sample Averages of Age 1 Consumption

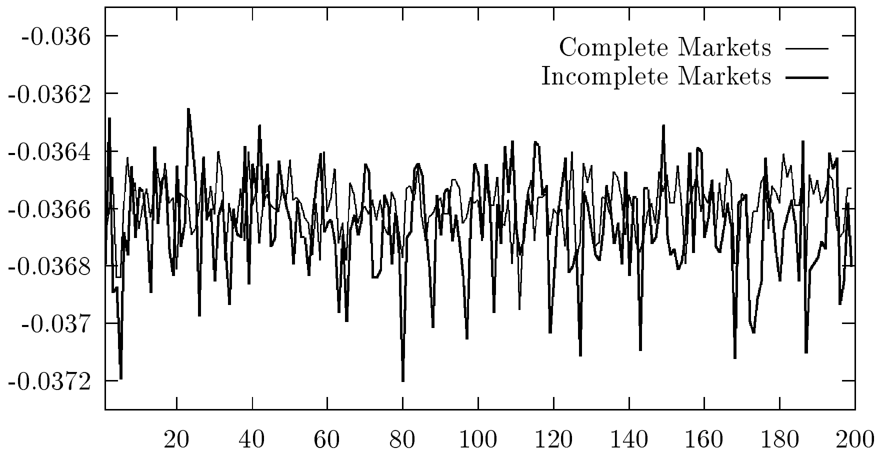

Figure 10: Sample Averages of Age 2 Assets

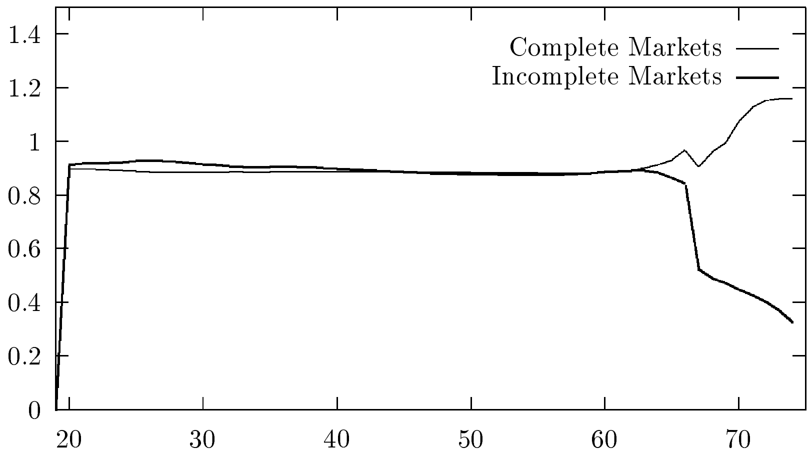

Figure 11: Percentage Standard Deviation of Consumption by Age

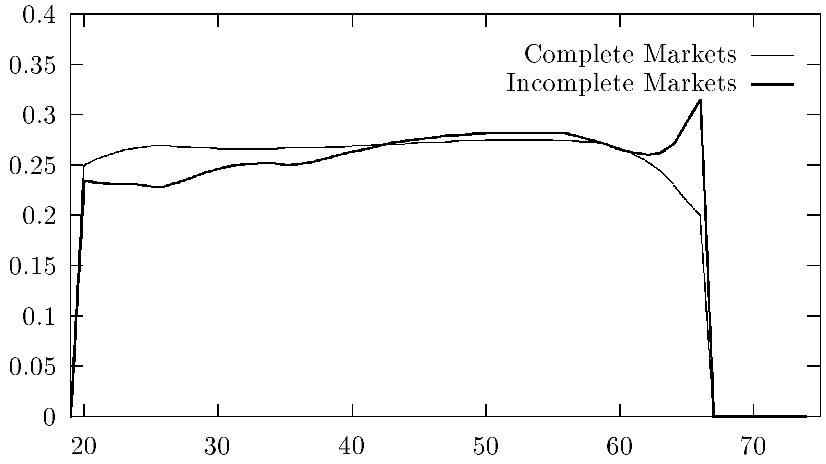

Figure 12: Percentage Standard Deviation of Leisure by Age

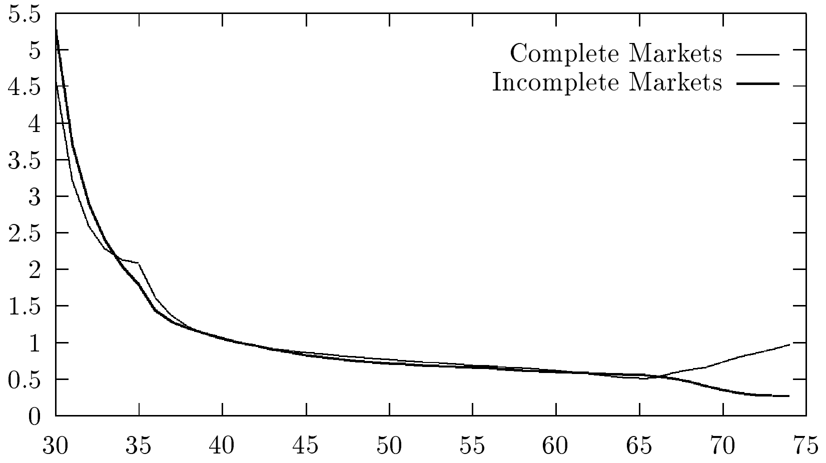

Figure 13: Percentage Standard Deviation of Asset Holdings by Age

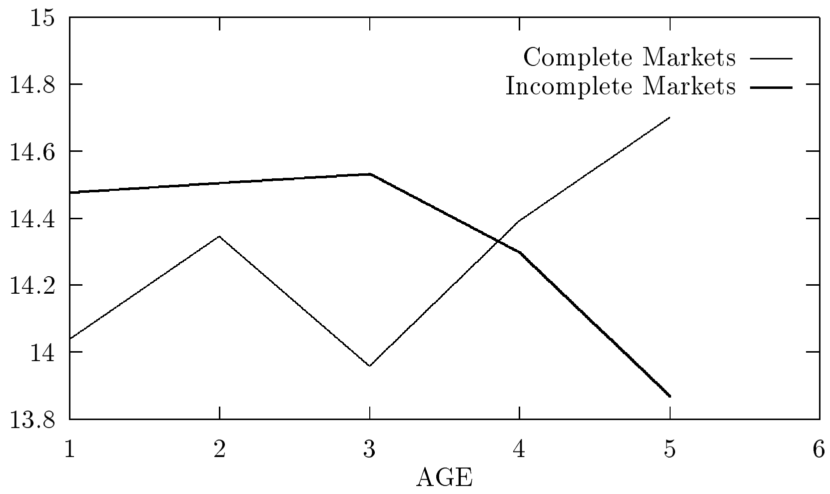

Figure 14: Percentage Standard Deviation of Consumption by Age for the Five period Economy with Large shocks

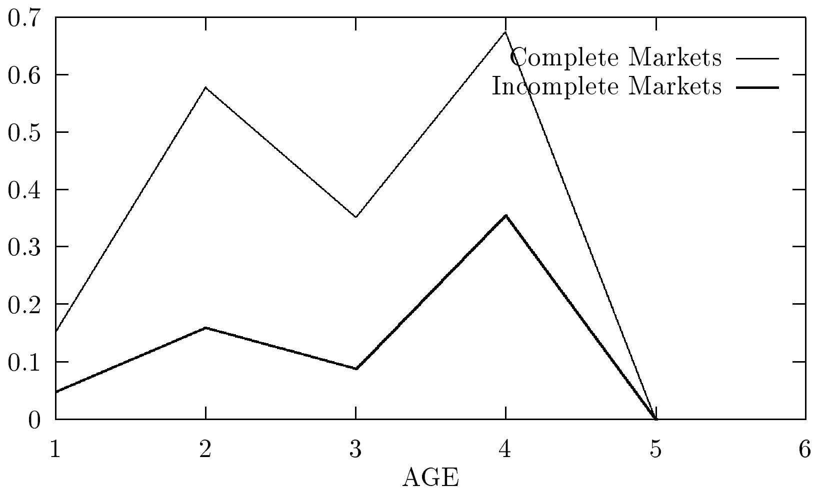

Figure 15: Percentage Standard Deviation of Leisure by Age for the Five period Economy with Large shocks