Sticky Wage Models and Labor Supply Constraints*

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Abstract

In New Keynesian models with sticky wages, the quantity of labor is solely determined by the demand side. Unions with monopsony power set the wage above what it takes to make agents work. If wages are sticky, however, a change of circumstances may make the demand for labor higher than agents' willingness to work. Because of the simplicity of log-linearization, the literature implicitly assumes that the markup of wages over the willingness to work is large enough to ensure that workers always comply with the quantity of labor demanded. In this paper, we explore the extent to which this is the case and find that workers are required to work against their will about 10% of the time. Moreover, when we use, as proposed by traditional theory (Drèze (1975)), the minimum of the demand and supply of labor instead of the demand-determined quantity, we find that the typical parametrization yields a variance of hours around 25% lower (depending on the particular model). The Dreze equilibrium is hard to compute, yet we find that a boundedly rational use of log-linearization methods gives a good approximation. We estimate the Dreze equilibrium for a version of the Christiano, Eichenbaum, and Evans (2005) environment and find that it yields answers that are sharply different from the demand-determined allocation: neutral productivity shocks go from accounting for 13% of the variance of employment to accounting for 70% of the variance of employment, to the detriment of the role played by investment specific technology shocks and monetary shocks. We conclude that when working with sticky wage economies, the appropriate procedure is to estimate the model using the approximated Dreze equilibrium.

Keywords: Sticky wages, New Keynesian model, Dreze equilibrium, Monetary policy

JEL classifications: E20, E32, E37, E52

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1 Introduction

In New Keynesian models with sticky wages, the quantity of labor is solely determined by the demand side, implicitly assuming that households are always willing to work at whatever rate is specified. In these environments, unions with monopsony power set wages above agents' marginal willingness to work, which provides a cushion that accommodates the effects of various shocks on the demand and supply of labor. If the shocks are small, the cushion is sufficient to guarantee that households will comply with the quantity of labor required. In this paper, we document that the cushion is too small: demand-determined labor often implies that agents are working against their will. Consequently, we propose an alternative determination of quantities that is consistent with standard theory (Drèze (1975)) where the amount of labor is the minimum of labor demand and labor supply. We then proceed to characterize a tractable approximation to the Dreze equilibrium and estimate a version of the Christiano, Eichenbaum, and Evans (2005) environment where agents do not work against their will. We find that the economic properties of the economy are quite different from those that result from estimating the model under the demand-determined solution. The relative importance of the various shocks changes, with neutral technology shocks accounting for 70% of the variance of employment. Moreover, neutral technology shocks tend to have larger but less persistent innovations in general, a necessary feature to induce workers to contribute more labor despite low wages. We conclude that the demand-determined solution, although convenient, gives answers that are too strongly shaped by agents working against their will and that a suitable approximation to the Dreze equilibrium should be used instead.

Modern computational tools such as Dynare have made it very easy to pose and estimate macroeconomic models by means of log-linearization and Bayesian estimation methods. New Keynesian models with sticky wages where the quantity of labor is demand determined are well suited for log-linearization which undoubtedly contributes to their popularity.[1] If wages are sticky, however, a change of circumstances may induce the demand for labor to become higher than agents' willingness to work. The literature ignores this possibility and implicitly assumes that the markup of wages over agents' willingness to work is large enough to accommodate the changed circumstances. But is it? This is the question that we address in this paper.

What is the equilibrium condition when prices are not market clearing? Drèze (1975), following the notion of disequilibrium modeling posed by Barro and Grossman (1971) and Malinvaud (1977), poses that the amount traded is the minimum of the quantities supplied and demanded and that the agents are aware of the limitation in the availability of the trades. The Dreze equilibrium is the approach that we follow in this paper.

Models that study the quantity of labor in the economy sometimes look at employment and sometimes at total hours worked. Although workers do not have to work for a wage that is lower than their reservation

[1] Similar reasoning could be made about New Keynesian models with sticky prices.

wage, the argument can be made that workers may have to work longer hours than desired if they want to keep their jobs. For this reason, we look not only at the standard New Keynesian models concerned with total hours (see Smets and Wouters (2007) or Christiano, Eichenbaum, and Evans (2005)), but also at those New Keynesian models that are explicitly concerned with understanding movements in employment (Galí, Smets, and Wouters (2011)), where it is clear that workers should not work if they do not want to.

To see whether some agents are working against their will in demand-determined allocations, we start our analysis by first substituting the demand-determined quantity of labor that the log-linearization procedure delivers with the minimum of the quantity of labor demanded and the quantity that agents would like to work. Because various wages are coexisting at any point in time, depending on the exact period when the wage was last set, the calculation of how much labor agents want to provide is not trivial. This is not a Dreze equilibrium, since agents made their decisions based on the demand-determined quantity of labor, but it does give us a preliminary account of the extent to which the demand-determined allocation is consistent with agents not working against their will. We refer to this quantity as *voluntary ex post aggregate labor*. We compare the properties of the demand-determined quantity of labor with those of the voluntary ex post aggregate labor on two of the most standard models of the New Keynesian literature: Smets and Wouters (2007) and Galí, Smets, and Wouters (2011). We find that the properties are quite different. In particular, we find that about 10% of the workers in the demand-determined economy are working more than desired. Moreover, the variance of the voluntary ex post aggregate labor is around 25% lower than that of the demand-determined quantity of labor, although it varies across specific models, ranging from 12% to 35%.

Next, we use global methods to solve for the Dreze equilibrium of a New Keynesian model where all agents, firms, unions, and households take into consideration that labor is determined by the minimum of supply and demand when they make decisions. Note that the Dreze equilibrium introduces the labor supply constraint, which is occasionally binding, and this constraint makes the model nonlinear and hence ill-suited for log-linearization. Computing the Dreze equilibrium in standard medium-size dynamic stochastic general equilibrium (DSGE) models presents two main difficulties. First, Calvo-style sticky prices and sticky wages introduce the price distribution and wage distribution as state variables. While under the assumption that labor is demand-determined, we can collapse the whole distribution to a single number. This is not the case when we want to compute the Dreze equilibrium where we have to know what each group of workers, characterized by their own fixed wage, wants to do. This feature dramatically increases the number of state variables, because no matter what approximation scheme we use, we have to keep track of what all groups of workers are being paid. Second, medium-size New Keynesian models have a large number of other state variables because of adjustment costs, backward indexation, various shocks, and the like.

For these reasons, we solve for the Dreze equilibrium in a relatively simple New Keynesian model with Taylor-type staggered wage contracts that stretches the class of economies that we can solve exactly. We find that employment in the Dreze equilibrium behaves similarly to what we found in the medium-size

New Keynesian models: the variance of employment is about 25% smaller than under demand-determined employment.

We also propose a suitable simple approximated solution to the Dreze equilibrium, which requires much less computation cost and can be applied to medium-size DSGE models. As in our calculation of the voluntary ex post aggregate labor, we also employ the log-linearized solution of the demand-determined allocation and then impose the ex post labor supply constraint. But unlike in the construction of the voluntary ex post aggregate labor, the approximated Dreze equilibrium reconstructs all the main aggregate variables recursively, including capital, output, interest rate, and so on. Comparing this approximation with the exact Dreze equilibrium, we find that the allocations are very similar. As a result, we argue that the approximated Dreze equilibrium can be used to address questions in medium-size DSGE models where computing the exact Dreze equilibrium is infeasible.

Finally, as an application, we estimate the Dreze equilibrium in a version of the Christiano, Eichenbaum, and Evans (2005) and Altig, Christiano, Eichenbaum, and Linde (2011) model. We find that the unwillingness of agents in the Dreze equilibrium to contribute more labor leads to very different estimates, so that other pieces of the model can create the observed fluctuations. Notably, although the demand-determined solution implies a small role for the neutral technology shock in accounting for employment volatility (13% of the variance), the Dreze equilibrium estimates point to this shock in accounting for 70% of the labor variance, reducing the role of monetary shocks and investment-specific technology shock, and keeping it more in line with traditional real business cycle models. The Dreze equilibrium estimates of the process for the neutral technology shock become more volatile and less persistent, which simultaneously makes agents more responsive to shocks and less likely to violate the labor supply constraint. Finally, the Dreze equilibrium estimates imply higher wage rigidity, indicating that these rigidities play an important role in generating employment volatility even under the Dreze equilibrium.

Our conclusions are clear: using demand-determined labor in economies with sticky wages yields answers that are quite different from those obtained by what we think is a more reasonable equilibrium condition, the Dreze equilibrium. We find that in general, labor moves about 25% less for a given parametrization, with the actual outcomes varying depending on features of the economies that we detail later on. We also find that an approximated solution, by imposing an ex post labor supply constraint on an economy solved by log-linearization, yields properties very similar to those of the Dreze equilibrium. We conclude that demand determination in the labor market should be substituted by the minimum of the demand and supply of labor, and that the use of the approximated Dreze allocation is both easy to implement and close enough to the true Dreze equilibrium to be used.

The central notion that we highlight in this paper is that agents should not work against their will, and that the labor supply constraint should be thought of as a participation constraint. A similar idea has already been explored in the literature. Hall (2005) develops a search model with sticky wages to account for the observed employment fluctuations. The wage is reset only if it hits the boundary of the bargaining set which is between the minimum wage acceptable to the worker and the maximum wage acceptable to the employer. The workers' participation constraint has to be respected. In a similar fashion, Gertler, Sala, and Trigari (2008) and Gertler and Trigari (2009) explore a search model with Calvo-style sticky wages, and whether the bargaining set is violated or not is checked ex-post. It is generally true that the bargaining set is large enough to accommodate Calvo-type sticky wages when agents are only subject to aggregate shocks, but it remains a question whether the bargaining set is large enough when agents also face idiosyncratic shocks. Recently, Christiano, Eichenbaum, and Trabandt (2013) develop a quantitative model in which the wage is determined by alternating-offers bargaining, a variant of Hall and Milgrom (2008). The alternating-offer-bargaining mechanism introduces wage inertia endogenously, and is free of the concern on violating the participation constraint. Our paper focuses on the willingness of agents to work, but a similar argument can also be made on the willingness of firms to produce goods at a fixed price. For example, Corsetti and Pesenti (2005) emphasize that firms should only produce if the ex-post price markup is larger than one. Bills (2004) and Alessandria, Kaboski, and Midrigan (2010) consider firms' inventory stockout problem, where firms' sales have to be the minimum of the goods demanded and their existing inventory.

We discuss the implicit assumption made in New Keynesian models when there is trade at non-market-clearing prices in Section 2. We proceed to explore in Section 3 the extent to which agents work against their will—what we jocularly label as slavery—in standard New Keynesian models (versions of Smets and Wouters (2007), and Galí, Smets, and Wouters (2011)) and conclude that it happens too often to simply ignore. Section 4 discusses what we think is the appropriate equilibrium concept, the Dreze equilibrium (Drèze (1975)), and compares its properties with those of the demand-determined allocation used in New Keynesian models and with those of an approximation to the Dreze equilibrium in various economies that we can solve. We then proceed to estimate a version of Christiano, Eichenbaum, and Evans (2005) using the approximated Dreze equilibrium and we show that we obtain quite different estimates than those obtained when using demand-determined allocations in Section 5. Section 6 concludes by arguing that the approximation to the Dreze equilibrium should be used in lieu of the demand-determined equilibrium when studying environments with sticky wages. Various Appendices complete the paper: Appendix A adds the details of the specification of the Smets and Wouters (2007) model; Appendix B discusses our approach to deal with the wage markup shocks; Appendix C discusses the problems associated to solving for an equilibrium with occasionally binding constraints when the constraint is a function of the state rather than predetermined; Appendix D provides the details of the global approximation to the Dreze Equilibrium in a relatively simple economy; Appendix E provides the details of the estimation of Altig, Christiano, Eichenbaum, and Linde (2011).

2 The Labor Market in New Keynesian Models

We pose a typical New Keynesian model with sticky wages, first introduced by Erceg, Henderson, and Levin (2000). There is a continuum of differentiated labor varieties n_i, $i \in [0,1]$, which firms combine into a final labor input n for production using a Dixit-Stiglitz aggregator with elasticity of substitution ϵ_w:

$$n = \left[\int n_i^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}. \tag{1}$$

Wage w_i is set by unions that are specific to each labor variety i. Firms take all wages as given. Given wages and total employment n, cost minimization yields a set of demand schedules for each labor variety i that is

$$n_i = \left(\frac{w_i}{w} \right)^{-\epsilon_w} n, \tag{2}$$

where w is an aggregate wage index $w = \left[\int w_i^{1-\epsilon_w} di \right]^{\frac{1}{1-\epsilon_w}}$ that satisfies $\int w_i \, n_i \, di = wn$.

A representative household consists of a continuum of workers, each one with different labor variety i that enjoys the same consumption level. The household's utility is given by

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \int_i v(n_{i,t}) di \right) \right\}. \tag{3}$$

The union sets the wage to maximize agents' utility. The opportunity to reset the wage occurs with probability $1 - \theta_w$ (à la Calvo) every period. The union's problem is

$$\max_{w^*_{i,t}} \quad E_t \left\{ \sum_{k=0}^{\infty} (\beta \theta_w)^k \left[u'(c_{t+k}) \frac{w^*_{i,t}}{p_{t+k}} n_{i,t+k} - v(n_{i,t+k}) \right] \right\}, \qquad \text{subject to} \tag{4}$$

$$n_{i,t+k} = \left(\frac{w^*_{i,t}}{w_{t+k}} \right)^{-\epsilon_w} n_{t+k}. \tag{5}$$

The first order condition is

$$E_t \left\{ \sum_{k=0}^{\infty} (\beta \theta_w)^k \left[n_{i,t+k} u'(c_{t+k}) \left(\frac{w^*_{i,t}}{p_{t+k}} - \frac{\epsilon_w}{\epsilon_w - 1} \frac{v'(n_{i,t+k})}{u'(c_{t+k})} \right) \right] \right\} = 0. \tag{6}$$

Implicit behind these equations is the fact that firms can acquire any quantity that they want of all labor varieties, which implies that workers comply. Note that the worker is not choosing how much to work. If it did, it would choose ℓ_i to equate the real wage to the marginal rate of substitution (the standard intratemporal Euler condition):

$$\frac{w_{i,t}}{p_t} = \frac{v'(\ell_{i,t})}{u'(c_t)}. \tag{7}$$

We refer to the ℓ_i that solves equation (7) as the optimal labor supply under wage w_i.

In the absence of wage rigidity ($\theta_w = 0$), the union sets the wage every period and condition (6) becomes

$$\frac{w^*_{i,t}}{p_t} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{v'(n_{i,t})}{u'(c_t)}. \tag{8}$$

Marginal revenue equals the marginal rate of substitution, or using standard parlance, the real wage is set to equal the marginal rate of substitution multiplied by the wage markup $\frac{\epsilon_w}{\epsilon_w-1}$. The standard values of the elasticity of substitution ensure that $\ell_i \geq n_i$, agents would like to work more than the quantity chosen by firms, and the determination of the equilibrium quantity of labor via the quantity demanded is justified.

Under wage stickiness, however, the wage set by equation (6) may imply an optimal supply of labor $\ell_{i,t} < n_{i,t}$. In this case, the assumption that labor is demand determined implies that workers are working against their will (i.e., slavery).

What is the correct notion of equilibrium in the context of a non-market-clearing price? Drèze (1975), following the disequilibrium models of Barro and Grossman (1971) and Malinvaud (1977), argued that it should be the minimum of supply and demand: trades should be voluntary. This is the notion that we follow in this paper.

But is there anything really inappropriate about posing a model where agents work more than desired? Labor varies because of both changes in hours per worker and changes in the number of workers. The argument could be made that workers may not be free to choose the number of hours that they work without losing their jobs, and therefore our notion that workers should not work against their will only applies to the extensive margin. in that case, it is only when dealing with the extensive margin that the argument that the correct equilibrium condition is the minimum of the quantity supplied and the quantity demanded is really strong.

A recent wave of New Keynesian models (Galí (2011) and Galí, Smets, and Wouters (2011)) have incorporated unemployment by looking explicitly at changes in the extensive margin. In these models, households have a continuum of workers represented by the unit square and indexed by a pair $(i, j) \in [0, 1] \times [0, 1]$. The i-dimension represents the type of labor service, while the j-dimension determines the worker's disutility from work, which equals j^γ if it is employed and zero if unemployed or outside the labor force. The household's utility is now given by

$$E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \int_i \int_0^{n_{i,t}} j^\gamma \, dj \, di \right) = E_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \int_i \frac{n_{i,t}^{1+\gamma}}{1+\gamma} di \right). \tag{9}$$

An individual worker (i, j) takes the household's consumption level and the labor market conditions as given and will find it optimal to participate in the labor market if and only if

$$u'_{ct}\frac{w_{i,t}}{p_t} \geq j^{\gamma}. \tag{10}$$

Hence, the measure of workers in sector i who want to work is ℓ_i, which solves[2]

$$u'_{ct}\frac{w_{i,t}}{p_t} = \ell_{i,t}^{\gamma}. \tag{11}$$

We could (as Galí (2011) and Galí, Smets, and Wouters (2011) do) define the unemployment rate as $u_t = \ell_t - n_t$. Moreover, in the absence of wage rigidities or in a steady state, the natural rate of unemployment rate u^n and the union's market power are linked by

$$\frac{1}{\epsilon_w - 1} \approx \gamma u^n. \tag{12}$$

In these models, labor supply is defined by the number of agents willing to work. When labor demand exceeds labor supply (i.e., when $n_i > \ell_i$), some agents are required to work against their will (hence our use of the term *slavery*). More dramatically, if labor demand exceeds the total population, $n_i > 1$, firms would be hiring workers that do not exist. It is in this type of model where our argument that the appropriate equilibrium is the Dreze equilibrium is the strongest, and we explore the behavior of economies of this type.

3 Are Agents Working against Their Will in Popular New Keynesian Models?

We now turn to the quantitative exploration of the extent to which agents work against their will by comparing the properties of employment in our versions of the standard Smets and Wouters (2007) and Galí, Smets, and Wouters (2011) environments with the level of employment in those economies that would be the minimum of supply and demand. Smets and Wouters (2007) is in a way the standard New Keynesian model, and that is why we look at it. Because its notion of labor is total hours and therefore our argument for the use of the Dreze equilibrium rather than demand-determined labor is weaker, we also look at Galí, Smets, and Wouters (2011). Obviously, such levels of employment are not those that result in a Dreze equilibrium, which requires that agents are aware both of the equilibrium condition and of adjustments in the other model variables. But before we do this analysis, we need to discuss two important issues. The first issue is how to construct the series that yield workers' desired labor supply (Section 3.1). This is not a trivial issue, because at any point in time, economies with wage rigidities have a large number of different wages. Each one of those different wages affect a different group of workers, so there is a different supply of labor for each of those wages. The second issue is in which ways our versions of the

[2] Note that when the labor disutility function is $v(n) = \frac{n^{1+\gamma}}{1+\gamma}$, then equation (11) coincides with equation (7).

standard New Keynesian models differ from the originals (Section 3.2). After discussing the properties of both economies (Sections 3.3 and 3.4), we explore the implications for the analysis of monetary policy (the main purpose of these models) in Section 3.5.

3.1 The Determination of the Worker's Desired Labor Supply

Evaluating the desired labor supply of workers requires that we keep track of not only the aggregate wage index of the economy but also the wages for all labor varieties i. Fortunately, this can be done by noting that all labor varieties that set the wage in a given period choose the same wage. We describe our procedure in three steps.

Step 1: Construct the cross-sectional wage distribution The measure of workers that can reset their wages in the current period is $\mu_0 = 1 - \theta_w$, while the measure of workers with wage reset τ periods before is $\mu_\tau = (1-\theta_w)\theta_w^\tau, \quad \tau = 0, 1, 2, ...$, which for τ large enough, μ_τ becomes negligible.

The aggregate wage index evolves according to

$$w_t = \left[\int w_{i,t}^{1-\epsilon_w} di\right]^{\frac{1}{1-\epsilon_w}} = \left[\theta_w (w_{t-1})^{1-\epsilon_w} + (1-\theta_w)(w_t^*)^{1-\epsilon_w}\right]^{\frac{1}{1-\epsilon_w}}, \tag{13}$$

which can be readily solved for $\{w_t^*\}$. The wages prevailing in period t are then $\{w_{t-\tau}^*\}$, with corresponding measure $\mu_\tau, \tau = 0, 1, 2, ...$

Step 2: Construct cross-sectional labor Demand and labor Supply Given aggregate employment $\{n_t\}$, the labor demand for workers with wage rate $w_{t-\tau}^*$ is

$$n_{\tau,t} = \left(\frac{w_{t-\tau}^*}{w_t}\right)^{-\epsilon_w} n_t. \tag{14}$$

For workers with wage rate $w_{t-\tau}^*$, the optimal number of hours they want to supply or the size of the labor force (depending on the interpretation) is given by the $\ell_{\tau,t}$ that solves

$$\frac{w_{t-\tau}^*}{p_t} = \frac{v'(\ell_{\tau,t})}{u'(c_t)}. \tag{15}$$

Aggregating both series over cohorts, we obtain aggregate demand for labor n_t and aggregate supply of labor ℓ_t.

Step 3: Construct aggregate employment Aggregate demand of labor, which in demand-determined equilibria is aggregate employment, is obtained by simple aggregation of workers with different wages[3]

$$n_t = \left[\int_{\tau=0}^{\infty} \mu_\tau, n_{\tau,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}. \tag{16}$$

If no workers were to work against their will, aggregate employment would be the sum of the minimum of supply and demand at each wage, which is

$$e_t^p = \left[\sum_{k=0}^{\infty} \mu_\tau \left(\min \left\{ n_{\tau,t}, \ell_{\tau,t} \right\} \right)^{\frac{\epsilon_w - 1}{\epsilon_w}} \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{17}$$

where e_t^p is employment, and we use the superscript p to denote that it is an ex post quantity. We call e_t^p voluntary ex-post aggregate labor. We want to emphasize that e_t^p is not an equilibrium object, both because when making decisions, neither firms nor unions or workers take this factor into consideration, and because the implied path of consumption, investment, and capital is that associated with the demand-determined allocation. However, it allows us to check whether the labor supply constraint is an issue or not. If $n_{\tau,t} < \ell_{\tau,t}$ all the time, then $n_t = e_t^p$ and it is correct to use demand-determined labor. If instead, $n_{\tau,t} > \ell_{\tau,t}$ happens frequently and the difference between $n_{\tau,t}$ and $\ell_{\tau,t}$ is large, then n_t will be substantially different from e_t^p and the answers obtained by using demand-determined quantities of labor are questionable.

3.2 Adjustments to the Smets and Wouters (2007) and Galí, Smets, and Wouters (2011) Models

The Smets and Wouters (2007) model lacks a straight identification of the steady state markup and sets its value to 50%, which implies an elasticity of substitution $\epsilon_w = 3$, a much larger value than both the empirical estimates and the number used in the New Keynesian literature.[4] We reestimate the Smets-Wouters model, setting the mean wage markup to values more in accordance with the recent literature, ranging from 6% to 18%.

Smets and Wouters (2007) pose seven shocks: total factor productivity, investment-specific technology, monetary policy, wage markup, price markup, government spending, and risk premium. The variance of the wage markup shock reported is 25.87%, quite a large value[5] that implies that the wage markup itself

[3]Under log-linearization, aggregate employment is not exactly the same as what equation (16) implies, but the difference is negligible.

[4]Lewis (1986) surveys the literature on the wage premium for workers in a union, which corresponds to the wage markup in the model, and the value is between 10% to 20%. The mean wage markup is 5% in Christiano, Eichenbaum, and Evans (2005) and in Altig, Christiano, Eichenbaum, and Linde (2011), 15% in Chari, Kehoe, and McGrattan (2002), and 20% in Levin, Onatski, Williams, and Williams (2006).

[5]That this is a huge value is also emphasized by Chari, Kehoe, and McGrattan (2009).

sometimes turns out to be negative, which does not make economic sense even if it can be handled by the log-linearization of the solution under demand-determined labor. Moreover, as we will see later, the approximation to the Dreze equilibrium requires positive wage markups to calculate the wage distribution for different sectors. Consequently, when we simulate the model, we abstract from this shock.

We make two further modifications to Smets and Wouters (2007), in line with most of the recent literature: the utility function is additively separable in consumption and leisure, and the aggregator of labor inputs is the standard Dixit-Stiglitz aggregator. The details of the model specification and estimation of the Smets and Wouters (2007) model are in Appendix A.

Galí, Smets, and Wouters (2011) set the steady state wage markup level to 18%, which via equation (12) implies a Frisch elasticity of 0.25, a very low value, much lower than macroeconomists and even most microeconomists would use (see Chetty, Guren, Manoli, and Weber (2011) for a recent discussion). Accordingly, and as for the Smets and Wouters (2007) model, we explore lower values for the steady state markup ranging from 6% to 18%, which imply much more reasonable Frisch elasticities.

Although the Galí, Smets, and Wouters (2011) estimate of the standard deviation of the wage markup shock is much smaller than that of Smets and Wouters (2007) (0.04 versus 0.25), we still obtain frequently negative wage markups. Again, we simulate the model without the wage markup shock.

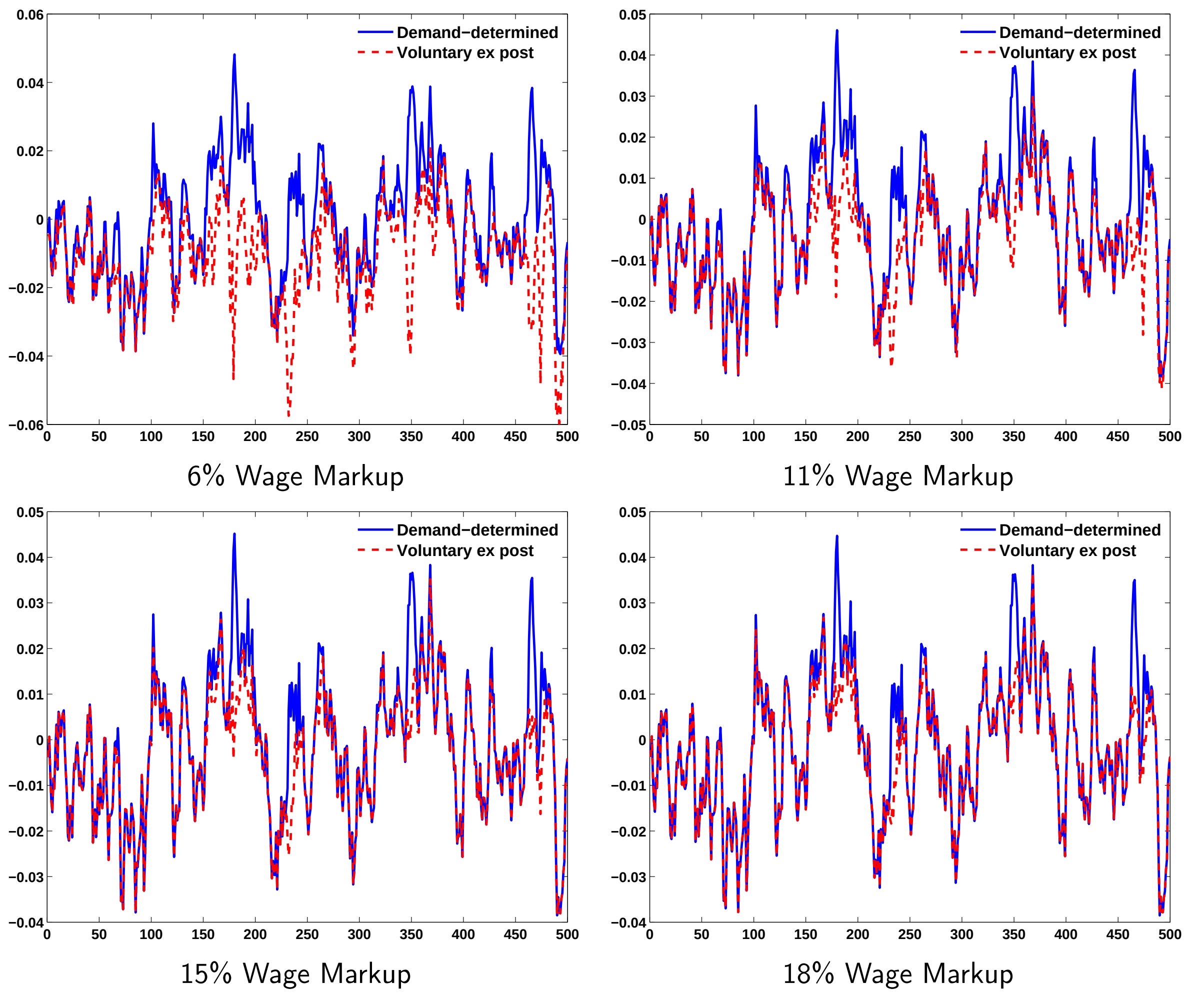
Our choices should be thought of as a conservative benchmark. We have assessed economies with a positive standard deviation of the wage mark-up shock (0.015) (see Appendix B). In this case, the violation of the labor supply constraint becomes more severe.

3.3 Analysis of the Smets and Wouters (2007) Model

Figure 1 displays sample paths of the aggregate employment obtained by demand-determined labor (n_t) and by the voluntary ex post labor e_t^p constructed in the way discussed in Section 3.1 for different wage markups. Note that demand-determined labor is always greater than the voluntary ex post aggregate employment by construction. The difference between these two series is noticeable. Both series coincide in recessions, but the voluntary ex post labor does not expand as much as the demand-determined labor in expansions. Also, the smaller the wage markup, the larger the differences between these two series. The intuition is clear: the larger the markup, the more distance between the wage chosen by the union and workers' actual willingness to work.

Table 1 summarizes the relevant statistics to compare both labor series. Average voluntary ex post labor is about 1% lower than demand-determined labor in the economy with a 6% wage markup, and about 0.15% lower with a 18% wage markup.

Figure 1: Demand-Determined and Voluntary Ex Post Labor in the Smets and Wouters (2007) Model



Column 2 of Table 1 lists the average excess of labor demand over labor supply (the extent of slavery). For a 6% wage markup case, about 27% of work is done against workers' will. For the 18% wage markup case, this fraction is lower: 3.8%. As one could have conjectured, the lower the markup, the larger the difference between average labor of both series and the larger the fraction of labor performed against workers' will. We consider these differences to be large.

As a result of these differences, the business cycle statistics are also quite different. Table 1 also displays the variance and correlation with demand-determined output of the two labor series. The variance of voluntary ex post labor is about 20% to 30% smaller than that of demand-determined labor. The voluntary ex post aggregate labor mutes some booms, and its variance is smaller. However, the relation between the frequency of binding of the labor supply constraint in the two series is not monotonic. The reason is that for a small wage markup, voluntary ex post labor sometimes shrinks to the point of moving in the

Table 1: Demand-Determined and Voluntary ex post Aggregate Labor in the Smets and Wouters (2007) Model

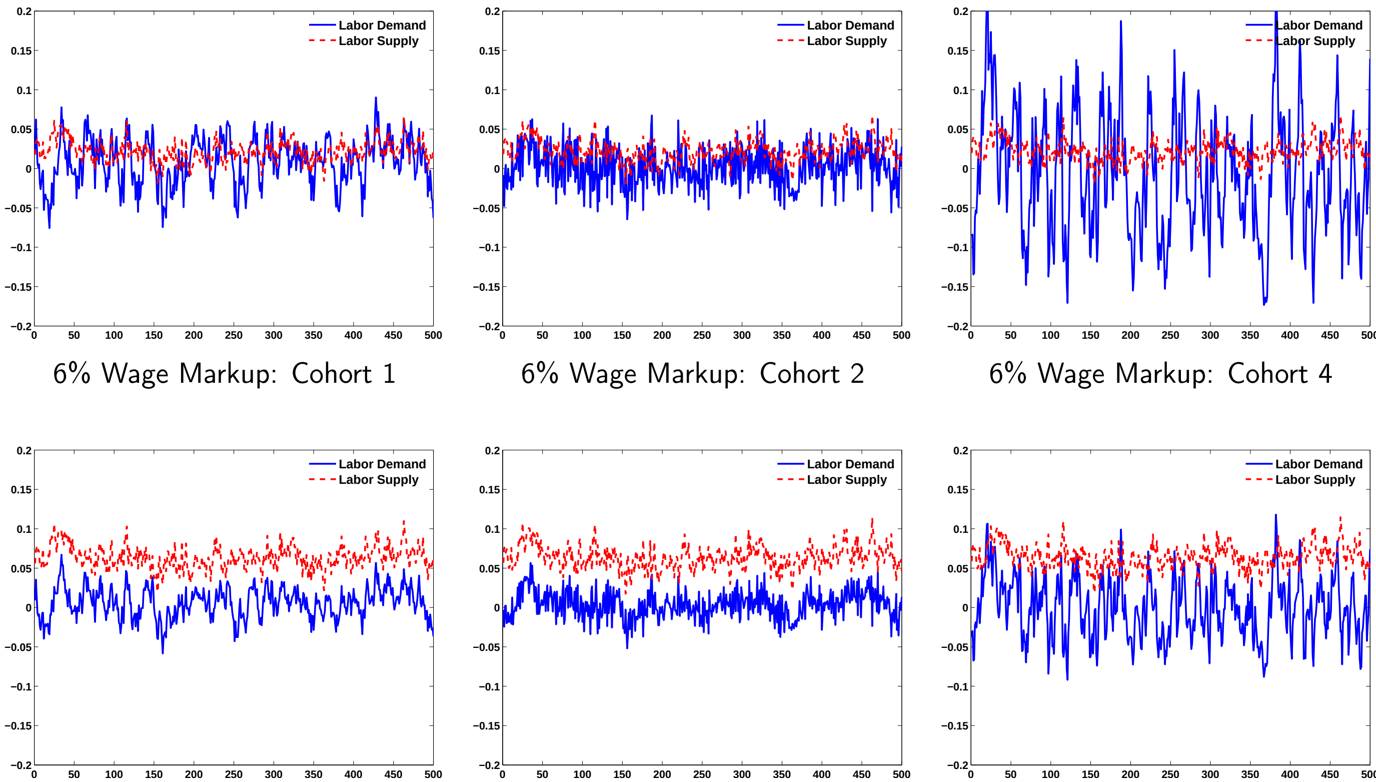
Wage markup: 6%	Mean	Binding Freq.	Var	Corr(N,Y)
Demand-determined labor	—	27.04	1.02	0.80
Voluntary ex post labor	-1.04	—	0.89	0.34
Wage markup: 11%	**Mean**	**Binding Freq.**	**Var**	**Corr(N,Y)**
Demand-determined labor	—	10.67	0.98	0.79
Voluntary ex post labor	-0.42	—	0.67	0.58
Wage markup: 15%	**Mean**	**Binding Freq.**	**Var**	**Corr(N,Y)**
Demand-determined labor	—	5.73	0.97	0.79
Voluntary ex post labor	-0.23	—	0.71	0.68
Wage markup: 18%	**Mean**	**Binding Freq.**	**Var**	**Corr(N,Y)**
Demand-determined labor	—	3.85	0.96	0.79
Voluntary ex post labor	-0.15	—	0.77	0.72

Notes: Numbers are in percentages except for the correlation. All the variables are logged and HP filtered. In the simulation, we include all the shocks except for the wage markup shock.

opposite direction, and hence what is an expansion under demand-determined labor is a recession in terms of voluntary ex post labor (note the much lower correlation with output).

To better understand the properties of the two aggregate labor series, Figure 2 shows labor supply and demand for specific wage cohorts that have some interesting properties. As expected, labor supply is usually (but not always) above labor demand, especially in the high wage markup economy. The variance of labor demand is larger, especially the longer the time that the wage was set. These properties are responsible for the higher volatility of the demand-determined labor without having a monotonic relation with the markup. Another perspective on the behavior of the two labor series is shown in Figure 3 which displays the densities of labor demand and supply across periods and cohorts of the last wage change. With a smaller wage markup, the means of labor demand and supply are closer to each other, but labor demand is much more dispersed.

Figure 2: Labor Demand $n_{\tau,t}$ and Labor Supply $\ell_{\tau,t}$ for Various Cohorts in Smets and Wouters (2007)



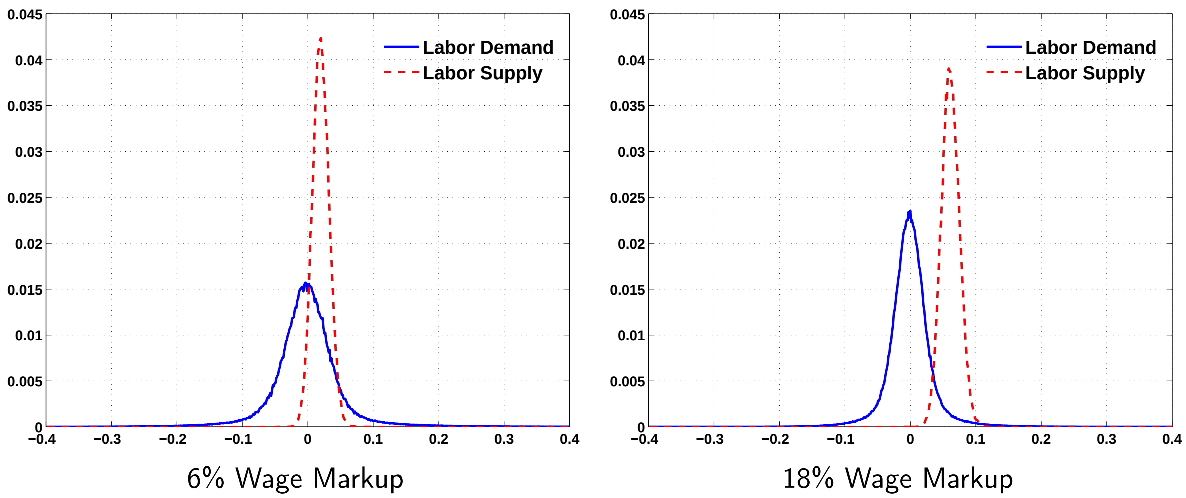
6% Wage Markup: Cohort 1 | 6% Wage Markup: Cohort 2 | 6% Wage Markup: Cohort 4

18% Wage Markup: Cohort 1 | 18% Wage Markup: Cohort 2 | 18% Wage Markup: Cohort 4

3.4 Analysis of the Galí, Smets, and Wouters (2011) Model

Galí, Smets, and Wouters (2011) explicitly refer to employment rather than to total hours, and here the notion of slavery, or working against agents' will, is more incisive. Figure 4 displays some typical sample paths of this economy. Again, we see that the two labor series often differ. In Table 2 we see that average voluntary ex post employment is half a percentage point less than the demand-determined employment and that the frequency of binding is less than 10% in all cases. Yet, the business cycle statistics are quite different for both employment series. In fact, in the lowest wage markup economy, the variance of voluntary ex post employment is larger than the demand-determined employment (and the correlation is lower). For higher wage markup economies, the relative size of the variances is reverted and the correlation with output is higher for the demand-determined employment. These features indicate that both employment measures often move in opposite directions. Note that in the Galí, Smets, and Wouters (2011) model, equation (12) determines a relation between the wage markup and the Frisch elasticity that keeps the average gap between employment demand and employment supply roughly constant across the various

Figure 3: Smets and Wouters (2007) Model: Density of Demand and Supply Panel



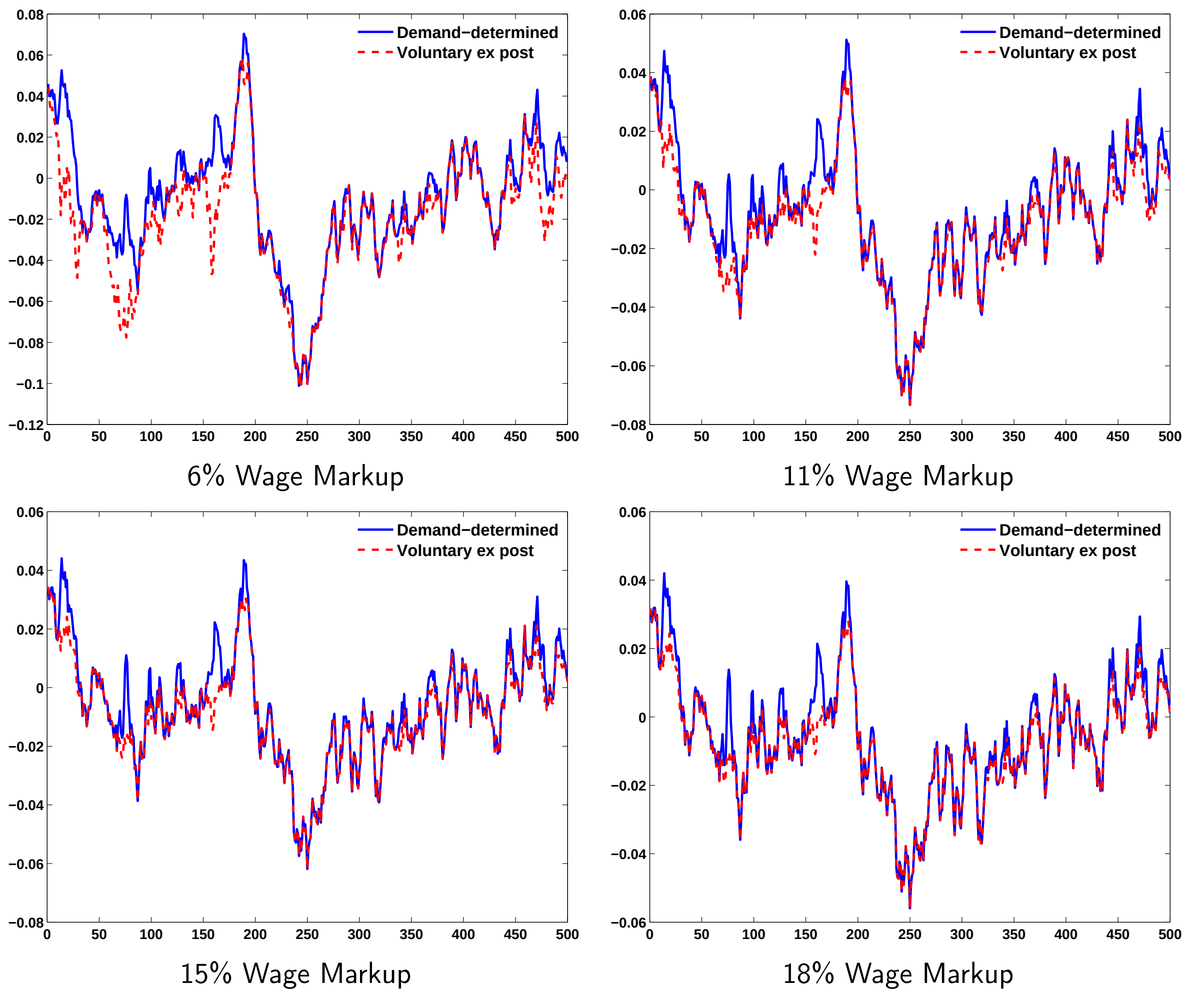
markups.

Figure 5 shows the employment supplied by and the employment demanded of workers whose wages are set in various periods. As in Smets and Wouters (2007), employment supply is almost always, though not always, above employment demand. Again, we also see that the employment demand for workers whose wages were set a long time ago is more volatile than for workers who have their wages newly reset. The volatility of employment demand is larger than for low wage markup economies. Figure 6 shows the densities of employment demand and employment across time and cohorts of wage duration. We see that the differences in the means of both labor demand and labor supply are similar for both wage markups. However, in the low wage markup economy, the dispersion of both series is larger.

3.5 Monetary Policy in the Smets and Wouters (2007) and Galí, Smets, and Wouters (2011) Models

Perhaps the most important use of models with wage rigidities is to explore the role of monetary policy, so we need to assess whether it matters that labor is demand determined or not. To do this, we look at impulse response functions. These instruments of analysis are unit independent in log-linear models or in models that are almost log-linear. This is not the case for the voluntary ex post quantities. If the size of the shock is very small, the cushion of the monopoly power of the union is more than enough to accommodate the extra demand for labor without violating agents' willingness to work. A much larger shock is much more difficult to accommodate. Accordingly we display impulse responses for one, two, and

Figure 4: Demand Determined and Voluntary ex post Employment in Galí, Smets, and Wouters (2011) Model



three standard deviations of a positive expansionary shock (the effects are not symmetric).

In the Smets and Wouters (2007) economy (Figure 7) with a small markup, the voluntary ex post labor implies smaller expansions for small expansionary shocks and even recessions for larger shocks. With a larger markup, normal expansionary shocks are accommodated perfectly, but as the shocks get larger, workers are no longer willing to work as much as is demanded of them. In all cases, the response of voluntary ex post labor is is nonlinear.

The differences between the impulse responses of both employment series are smaller in the Galí, Smets, and Wouters (2011) model, as Figure 8 shows. Still, large expansionary shocks induce a much more restrained response in agents' willingness to work.

Table 2: Demand determined and Voluntary ex post Employment in the Galí, Smets, and Wouters (2011) Model

Wage markup: 6%	Mean	Binding Freq	Var	Corr(N,Y)
Demand Determined Employment	—	9.68	0.86	0.84
Voluntary ex post Employment	-0.49	—	0.93	0.68
Wage markup: 11%	**Mean**	**Binding Freq**	**Var**	**Corr(N,Y)**
Demand Determined Employment	—	5.05	0.61	0.79
Voluntary ex post Employment	-0.21	—	0.48	0.71
Wage markup: 15%	**Mean**	**Binding Freq**	**Var**	**Corr(N,Y)**
Demand Determined Employment	—	4.20	0.54	0.76
Voluntary ex post Employment	-0.15	—	0.41	0.70
Wage markup: 18%	**Mean**	**Binding Freq**	**Var**	**Corr(N,Y)**
Demand Determined Employment	—	3.86	0.51	0.75
Voluntary ex post Employment	-0.13	—	0.39	0.70

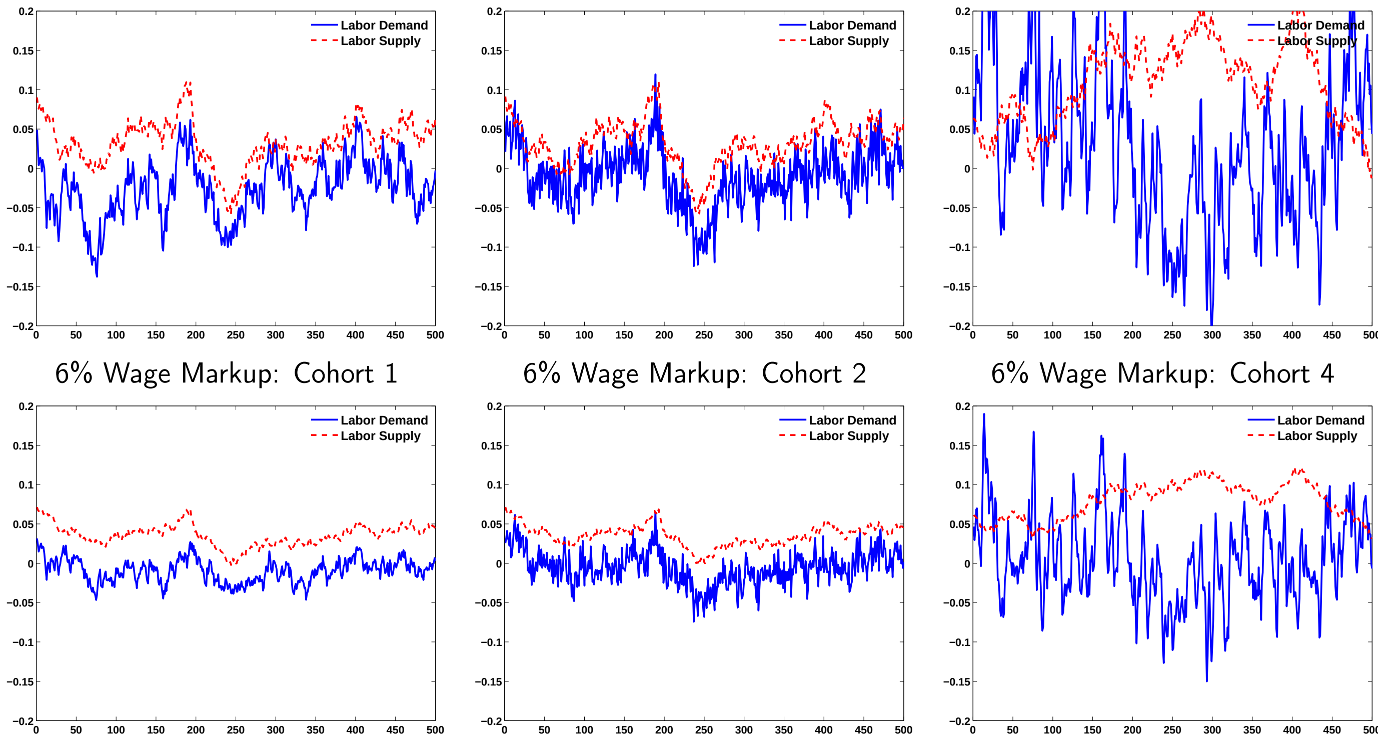
Notes: Numbers are in percentages except for the correlation with output. All the variables are logged and HP filtered. In the simulation, we include all the shocks except for the wage markup shock.

4 Computing the Dreze Equilibrium

So far, we have made the case that the use of demand-determined labor as the equilibrium condition is inappropriate because households want to work less quite often: the minimum of the amount of labor demanded and supplied (as standard theory considers the appropriate equilibrium condition) behaves very differently from the amount of labor demanded. However, the series that we have constructed (voluntary ex post labor) is not an equilibrium because it is constructed along a path defined by the demand-determined labor and its associated series of output consumption, investment, prices, wages, and so on. Moreover, the forecasts of agents are those of the demand-determined allocation. We need to compute the Dreze equilibrium explicitly.

Unfortunately, log-linearization cannot be used to solve for the Dreze equilibrium. Global methods are needed given that the equilibrium condition is based on the min operator. Recent developments in computational economics that allow us to deal effectively with corner solutions (Guerrieri and Iacoviello, 2015)

Figure 5: Galí, Smets, and Wouters (2011) Model: Labor Demand vs Labor Supply



6% Wage Markup: Cohort 1 | 6% Wage Markup: Cohort 2 | 6% Wage Markup: Cohort 4

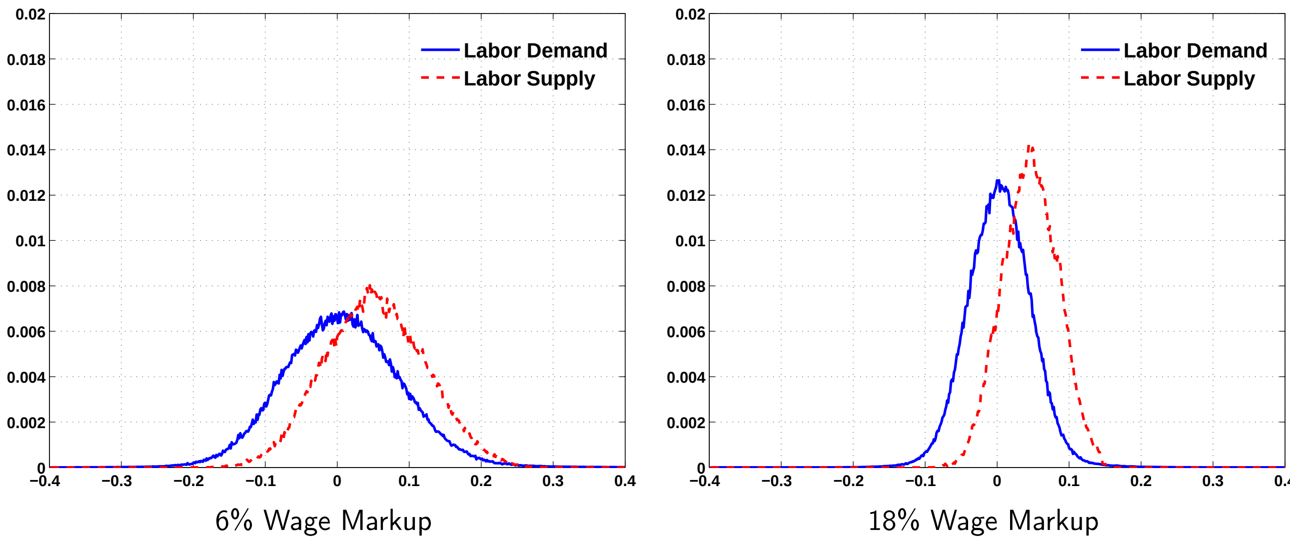
18% Wage Markup: Cohort 1 | 18% Wage Markup: Cohort 2 | 18% Wage Markup: Cohort 4

cannot be applied either because these methods require that the corners or temporarily binding constraints are predetermined (like the zero bound of nominal interest rates or the lower and upper bound of hours worked), whereas in our economies, the min operator applies to two endogenous variables. A detailed illustration of this issue can be found in Appendix C. Moreover, the number of state variables is effectively infinite because the whole set of existing wages is part of the state vector (even if truncating we would still need many state variables). Global methods can only be used with a limited number of variables, which presents a problem.

Our strategy here is to explore the properties of the Dreze equilibrium in an economy that we can solve with global methods (a simplified Galí, Smets, and Wouters (2011) economy with staggered wages à la Taylor), and to compare its solution with a suitable simple approximation (effectively one where we forgo agents' expectations but maintain some properties of the log-linearized solution and feasibility). To the extent that the global solution and the approximation are quite similar, we argue for the use of the approximated solution to the Dreze equilibrium when using New Keynesian macro models with wage staggering.

We first describe the simple model with staggered wage contracts in Section 4.1 and then describe an approximation to its solution that uses as a basis a log-linear approximation to the demand-determined

Figure 6: Galí, Smets, and Wouters (2011) Model: Density of Demand and Supply Panel



equilibrium of the same economy in Section 4.2. We compare the quantitative properties of both objects in Section 4.3.

4.1 The Dreze Equilibrium in a Simple RBC Economy with Staggered Wage Contracts

Consider an infinitely lived stochastic growth monetary economy where the members of a representative household are indexed by (i, j), where i stands for the sector or type of intermediate labor services and j stands for the level of disutility from labor. The household's utility is given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi \int_i \int_0^{e_{i,t}} j^{\gamma} \, dj \, di \right) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - \phi \int_i \frac{e_{i,t}^{1+\gamma}}{1+\gamma} di \right). \tag{18}$$

Households take prices and firms' profits as given, and their budget constraint is

$$p_t \left[c_t + k_{t+1} - (1-\delta) k_t \right] + \frac{1}{R_t} b_{t+1} = r_t^k k_t + \int_i w_{i,t} e_{i,t} di + b_t + \Pi_t. \tag{19}$$

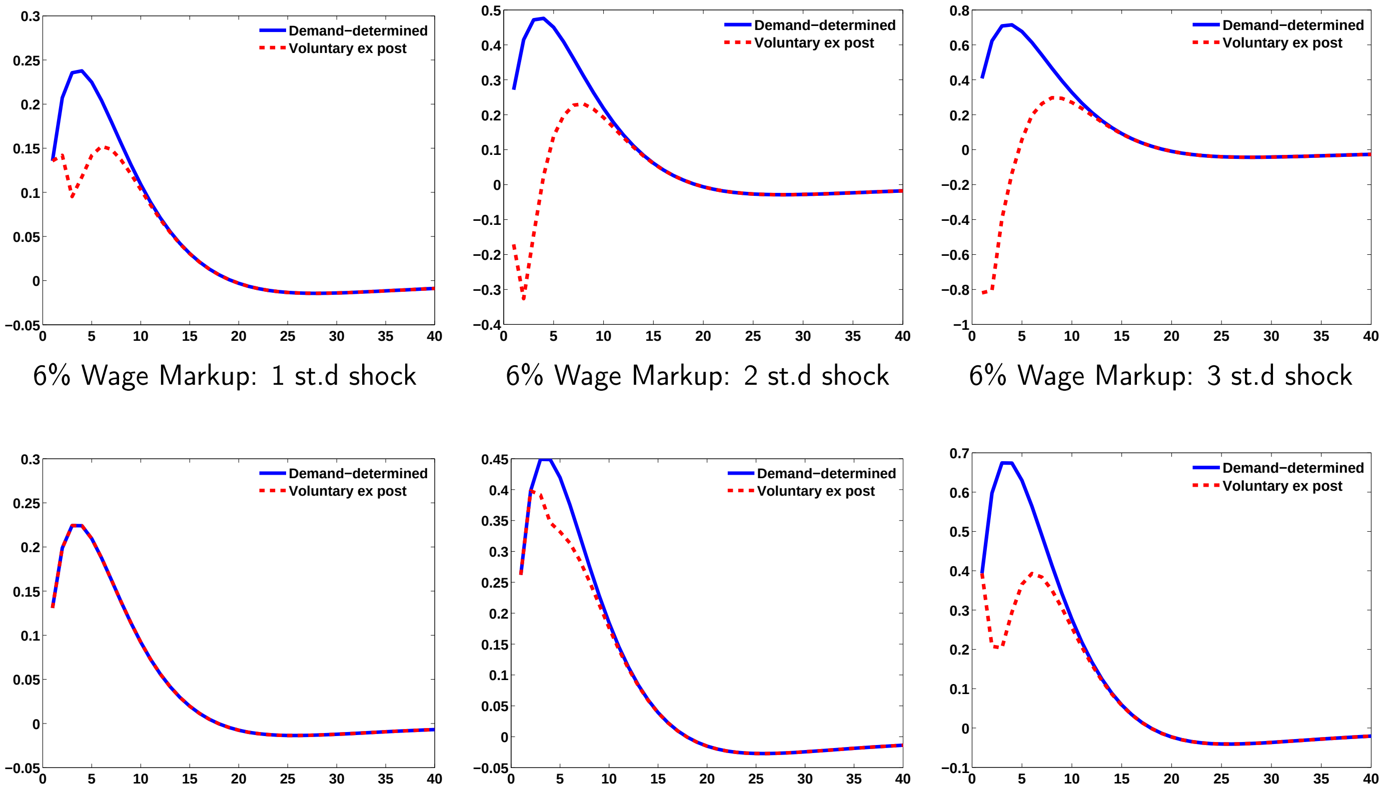
Firms are competitive with production technology

$$y_t = z_t k_t^{\alpha} e_t^{1-\alpha}, \tag{20}$$

where e_t is the final labor used in production aggregated via a Dixit-Stiglitz technology

$$e_t = \left[\int e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{21}$$

Figure 7: Monetary Policy Shock: Smets and Wouters (2007)



6% Wage Markup: 1 st.d shock | 6% Wage Markup: 2 st.d shock | 6% Wage Markup: 3 st.d shock

11% Wage Markup: 1 st.d shock | 11% Wage Markup: 2 st.d shock | 11% Wage Markup: 3 st.d shock

and total factor productivity (TFP) follows an AR(1) process, $\log z_t = \rho_z \log z_{t-1} + \zeta_t^z, \quad \zeta_t^z \sim \mathbb{N}(0, \sigma_z^2)$.

Firms take the price of the final good, the capital rental rate, and the wage rate as given and solve

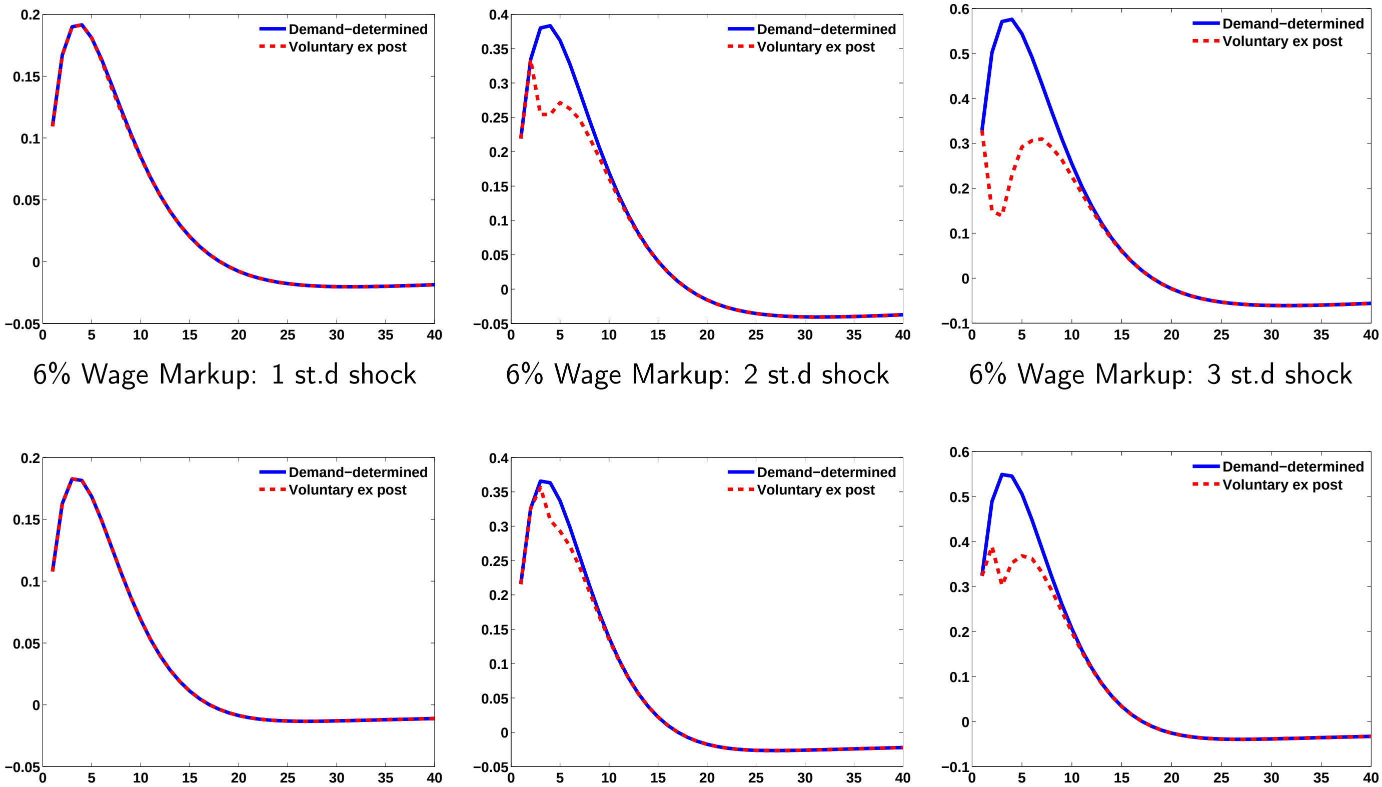
$$\max_{k_t, e_t, e_{i,t}} \; p_t z_t k_t^{\alpha} e_t^{1-\alpha} - r_t^k k_t \quad - \quad \int w_{i,t} \, e_{i,t} \, di \qquad \text{subject to} \tag{22}$$

$$e_t \quad = \quad \left[\int e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \, di \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{23}$$

$$e_{i,t} \quad \leq \quad \Phi(w_{i,t}), \tag{24}$$

where $\Phi(w_{i,t})$ is the maximum amount of labor the firm can demand for intermediate labor i under the wage rate $w_{i,t}$ (what we referred to as ℓ_i earlier). In standard New Keynesian models, the last constraint

Figure 8: Monetary Policy Shock: Galí, Smets, and Wouters (2011)



6% Wage Markup: 1 st.d shock | 6% Wage Markup: 2 st.d shock | 6% Wage Markup: 3 st.d shock

11% Wage Markup: 1 st.d shock | 11% Wage Markup: 2 st.d shock | 11% Wage Markup: 3 st.d shock

is absent. The solution to the firms' problems satisfies

$$\frac{r_t^k}{p_t} = \alpha\ z_t\ k_t^{\alpha-1}\ e_t^{1-\alpha}, \tag{25}$$

$$e_{i,t} = \min\left\{\left[\frac{w_{i,t}}{(1-\alpha)z_t k_t^{\alpha} n_t^{-\alpha} p_t}\right]^{-\epsilon_w} e_t,\ \Phi(w_{i,t})\right\}, \tag{26}$$

$$e_t = \left[\int e_{i,t}^{\frac{\epsilon_w-1}{\epsilon_w}}\, di\right]^{\frac{\epsilon_w}{\epsilon_w-1}}. \tag{27}$$

For later use, let $n_t = \Psi(w_{i,t},)$ denote the desired (not the actual, since it ignores the constraint) labor demand:

$$\Psi_t(w_{i,t}) = \left[\frac{w_{i,t}}{(1-\alpha)z_t k_t^{\alpha} e_t^{-\alpha} p_t}\right]^{-\epsilon_w} e_t. \tag{28}$$

Needless to say, both $\Phi(w_{i,t})$ and $\Psi(w_{i,t})$ depend on the aggregate states. To simplify the exposition, we do not make this dependency explicit here.

In this economy, there is a continuum of labor unions, each setting the wage of the type of labor that they represent, that maximize households' welfare given the behavior of all other parts of the economy. Workers cannot be made to work against their will, and the union takes into account that there is an upper bound on the amount of labor services that will be provided in their sector. The union chooses a nominal wage that will be effective for T^w periods:

$$\max_{w_t^*} \ \mathbb{E}_t \left\{ \sum_{k=0}^{T^w-1} \beta^k u'(c_{t+k}) \frac{w_t^*}{p_{t+k}} e_{i,t+k} - \frac{e_{i,t+k}^{1+\gamma}}{1+\gamma} \right\} \tag{29}$$

subject to

$$e_{i,t+k} = \min \left\{ \left(\frac{u'(c_{t+k}) w_t^*}{\phi p_{t+k}} \right)^{\frac{1}{\gamma}}, \Psi_{t+k}(w_t^*) \right\}, \tag{30}$$

where $\Psi_{t+k}(\cdot)$ is the desired labor demand from the firm's side and $\Phi(w_{i,t})$ is given by

$$\Phi(w_{i,t}) = \left(\frac{u'(c_{t+k}) w_{i,t}}{\phi p_t} \right)^{\frac{1}{\gamma}}. \tag{31}$$

In the standard model, the constraint for the union is simply

$$e_{i,t+k} = \Psi_{t+k}\left(w_t^*\right). \tag{32}$$

Note the three objects that we have defined: labor supply in variety i,

$$\ell_{i,t} = \Phi(w_{i,t}) = \left(\frac{u'(c_{t+k}) w_{i,t}}{\phi p_t} \right)^{\frac{1}{\gamma}}, \tag{33}$$

labor demand in variety i,

$$n_{i,t} = \Psi(w_{i,t}) = \left[\frac{w_{i,t}}{(1-\alpha) z_t k_t^{\alpha} e_t^{-\alpha} p_t} \right]^{-\epsilon_w} e_t, \tag{34}$$

and employment in variety i,

$$e_{i,t} = \min\{\ell_{i,t}, n_{i,t}\}. \tag{35}$$

Following Galí, Smets, and Wouters (2011), the unemployment rate in sector i (the economy-wide counterpart is immediate) is

$$u_{i,t} = \log \ell_{i,t} - \log e_{i,t}, \tag{36}$$

To complete the model, we include a simple Taylor type monetary policy rule:

$$\log R_t = \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t, \tag{37}$$

where $\pi_t = \log \frac{p_t}{p_{t-1}}$ and y^* is the steady state output level. The shock to the monetary policy rule follows an AR(1) process,

$$\eta_t = \rho_m \eta_{t-1} + \zeta_t^m, \quad \zeta_t^m \sim \mathbb{N}(0, \sigma_m^2). \tag{38}$$

The details of the numerical solution via global methods of this economy can be found in Appendix D.

4.2 An Approximation to the Dreze Equilibrium

Our approxmation of the Dreze equilibrium consists of four logical steps.

Step 1: Log-linearize and solve the demand-determined equilibrium This is a standard step. It is important to obtain the decision rules, not just a simulation.

Step 2: Recursively construct a voluntary ex post measure of labor This step is what we described in Section 3.1. The key difference is that there we use the sequence of capital stocks yielded by the demand-determined equilibrium, which may not be feasible. Thus, at this stage we construct a measure of the voluntary employment one period at a time, denoted as e_t^a. In this step we keep track of historical wages, w_t^a, which also include the information about the cross-sectional wage distribution.

Step 3: Recursively construct the main aggregate variables Here we use the employment in period t, e_t^a, and the previous period series of capital k_t^a to calculate output y_t^a (which is also used to construct the output gap). We then use the same policy function as in the demand-determined equilibrium to determine the newly set wage and price. This is an approximation, since in the true Dreze economy, agents will take into account the possibility that the labor supply constraint may be binding. The interest rate R_t^a is set by using the reconstructed output gap. This part is mechanical.

Step 4: Determine consumption, investment, and next period capital This step is not mechanical. We have considered two possibilities: use the same consumption-to-output ratio or the same consumption of the demand-determined solution (investment is set residually to satisfy the resource constraint). We finally chose the same consumption because choosing the consumption-to-output ratio sometimes leads to countercyclical consumption. More specifically, in the demand-determined economy, after a positive technology shock, the consumption-output ratio is below the steady state level because agents understand that it is better to increase investment to take advantage of the temporary high productivity. In the Dreze equilibrium, however, the response of labor is much more subdued with the same positive technology shock, which may lead to a much smaller expansion. If we used the low consumption-to-output ratio of the demand-determined allocation, there would be a recession rather than an expansion.

4.3 A Comparison between the Dreze Equilibrium and Its Approximation in the Simple Economy

We now specify the simple model quantitatively (see Table 3) and solve for the Dreze equilibrium and for its approximation. The model has a large number of state variables because of the staggered wage contract (see Appendix D for more details). The model period is a quarter, the annual interest rate is 4%, and the implied Frisch elasticity is 0.75 ($\frac{1}{\gamma}$), similar to estimates in Heathcote, Storesletten, and Violante (2010). The labor share is 0.64, and the capital depreciation rate is 0.08 annually. The process for the TFP shock is similar to the one used in Ríos-Rull and Santaeulàlia-Llopis (2010). The monetary policy rule is the same as in Christiano, Eichenbaum, and Rebelo (2011). The persistence of the monetary shock is 0.5, the same as in Galí (2008). We set the standard deviation of the innovation to the monetary shock to be 0.004. As discussed earlier, the most important parameter is ϵ_w, which determines the wage markup. The one we use here implies a 10% wage markup. If we apply the logic of equation (12), our choice of ϵ_w and γ leads to a 6% average unemployment rate. We choose the duration of the wage contract to be four model periods, or one year.

Table 3: Baseline Parameters

β	σ	γ	ϕ_π	ϕ_y	α	δ	ϵ_w	ρ_z	σ_z	ρ_m	σ_m
0.99	1.0	1.5	1.5	0.0	0.36	0.02	11.0	0.95	0.006	0.50	0.004

We report in Table 4 the properties of employment in the simple economy using one shock at a time. We compare the statistics generated by the Dreze equilibrium and by its approximation. For further comparability, we also include those of the voluntary ex post employment and the demand-determined solution.

The global solution, its approximation, and even the voluntary ex post employment series are all quite similar.[6] The demand-determined allocation, however, is much more volatile than the others. In this economy, more than 10% of workers contribute labor against their will. Figures 9 and 10 also show the similarity between the Dreze equilibrium and the approximated Dreze. Table 5 displays the business cycle statistics that show the same patterns for the other main macro variables.

We conclude that the approximation to the Dreze equilibrium built via log-linearization of the demand-determined solution and the recursive imposition of the minimum of the amount of labor supplied and

[6] Recall that the voluntary ex post employment series is obtained by imposing on the demand-determined employment the minimum of the supply and demand for labor without concern for the feasibility of its associated output and capital paths.

Figure 9: Employment in the Simple Economy with TFP Shocks

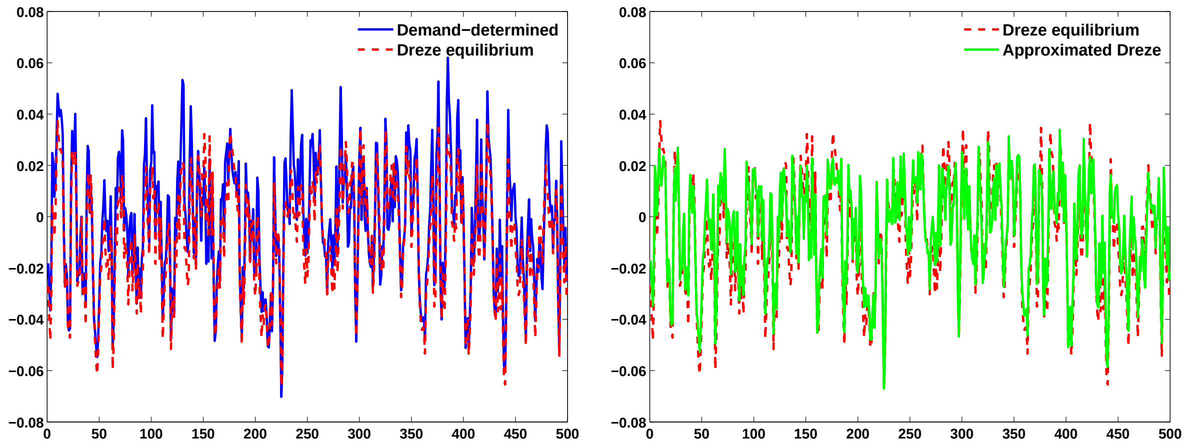
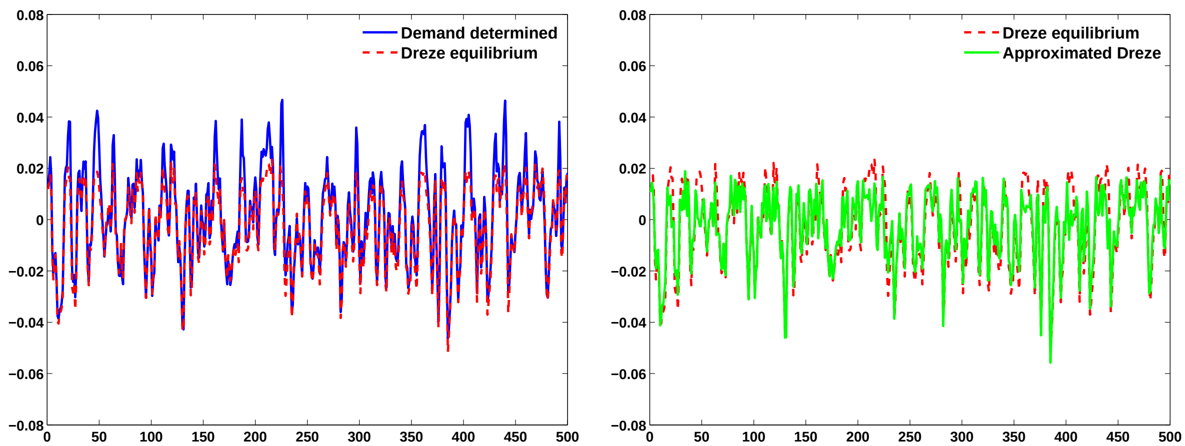


Figure 10: Employment in the Simple Economy with Monetary Shocks



demanded is a good approximation to a global solution of the Dreze equilibrium where the condition that labor is the minimum of the amount supplied and demanded is imposed ex ante.

5 An application of the Dreze Equilibrium: the Estimation of the Christiano-Eichenbaum-Evans Economy

So far, we have argued that in New Keynesian models with sticky wages, the use of demand-determined labor yields allocations that are very different from those that the same parameterized model yields when labor is determined by the Dreze equilibrium where labor is the minimum of the amount supplied and the amount demanded. But this is not what really matters; perhaps different values of parameters yield similar properties between the two ways of determining the quantity of labor, and hence the answers that we obtain are the same. To settle this issue, we have to estimate the models under both types of labor

Table 4: Properties of Employment in the Simple Model for Various Solutions

TFP Shock				
Solution Method	Mean	Var	Corr(N,Y)	Prob of binding
Dreze equilibrium (global solution)	-0.10	2.67	0.93	—
Approximation to Dreze equilibrium	-0.29	2.53	0.94	—
Voluntary ex post employment	-0.33	2.60	0.88	—
Demand-determined employment	—	3.79	0.96	0.11
Monetary Policy Shock				
Solution Method	Mean	Var	Corr(N,Y)	Prob of binding
Dreze Equilibrium (global solution)	-0.52	1.60	0.94	—
Approximation to Dreze equilibrium	-0.41	1.29	0.99	—
Voluntary ex post employment	-0.45	1.38	0.77	—
Demand-determined employment	—	2.26	1.00	0.12

Notes: The numbers for variances are in percentages. All the variables are logged and HP filtered.

determination.

The estimation of both Smets and Wouters (2007) and Galí, Smets, and Wouters (2011) uses modern Bayesian methods that rely on the linearity of the model. Although demand-determined models are not linear, they are very well approximated by log-linear approximations and hence are extremely well suited for Bayesian or maximum likelihood estimation. The combination of the linearity and the Gaussian shock structure permits a relatively easy mapping from model parameters to its implied likelihood. The key feature of the Dreze equilibrium is its nonlinear nature, which unfortunately prevents us from applying the standard linear Kalman filter technique in evaluating the model's likelihood. The alternative nonlinear Kalman filter requires large computational power, which is only feasible for models with a very small number of state variables (two or three according to Aruoba, Bocola, and Schorfheide (2013)) and is not (yet) feasible for medium-scale DSGE models.

We can, however, estimate the Dreze equilibrium in the other central model in the New Keynesian literature: that of Christiano, Eichenbaum, and Evans (2005) and Altig, Christiano, Eichenbaum, and Linde (2011). These two papers estimate a medium-scale DSGE model by matching the impulse responses of various

Table 5: Business Cycle Statistics for the Simple Model with Various Solutions

	TFP Shock			Monetary Policy Shock		
	Dreze Equilibrium	Approximated Dreze Equil.	Demand Determined	Dreze Equilibrium	Approximated Dreze Equil.	Demand Determined
		Variance			*Variance*	
Output	3.12	2.95	3.90	0.65	0.52	0.92
Employment	2.67	2.53	3.79	1.60	1.29	2.26
Consumption	0.17	0.17	0.19	0.01	0.01	0.02
Investment	42.82	38.10	52.11	10.12	7.96	13.95
		Correlation with output			*Correlation with output*	
Employment	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.61	0.57	0.62	0.61	0.57	0.62
Investment	1.00	0.99	1.00	1.00	0.99	1.00

Notes: The numbers for variances are in percentages. All the variables are logged and HP filtered.

variables to different shocks. The impulse responses are recovered from the estimation of a certain structural vector auto regression (VAR) model. The parameters of the model are chosen in such a way that the model's impulse responses to the structural shocks match their counterpart estimated from the data. In particular, three structural shocks are considered: a monetary shock, a neutral technology shock, and an embodied investment technology shock. The estimation method is generalized method of moments (GMM), which only requires the impulse response of the model.[7] Because the likelihood of the model is not required, we can apply this estimation method to the Dreze equilibrium. The details of the model can be found in Appendix E.

Table 6 shows the properties of the estimates of the Dreze equilibrium and of the demand-determined allocation in the Altig, Christiano, Eichenbaum, and Linde (2011) model. Our interpretation of these very different sets of estimates is that the unwillingness of households in the Dreze equilibrium to work a lot under some circumstances requires that other pieces of the model have to do a lot more work to create the observed fluctuations. This is accomplished in a particular way, according to the estimates:

[7] The weighting matrix of GMM is diagonal with the inverse of the standard deviations of the impulse responses estimated in the structural VAR.

1. The neutral technology shock is dramatically affected. This shock is now both much more volatile and less persistent: the overall variance of the neutral technology shock is 0.039 in the Dreze equilibrium relative to 0.024 in the demand-determined allocation. A larger shock induces more circumstances to which households respond. A less persistent shock makes households more engaged in responding to the innovation of the shock, since households are less likely to have the same opportunities in the future. For firms, which are the most important driving force in the demand-driven allocation, persistence shocks are more important because their rationale for hiring workers involves investing more, which requires that good opportunities are present in the near future.

2. There is more wage and price rigidity. The lower response of employment in the Dreze equilibrium also requires, perhaps a bit counterintuitively, larger rigidities in the model to generate more fluctuations. This is true both for wages, where the Dreze equilibrium is imposed, and for prices, where it is not.

3. Two other pieces of the model are now larger. Variable capital utilization has a larger value, as does the investment adjustment cost parameter. Still, these two parameters are somewhat imprecisely estimated and we should not insist on them.

4. The effects of the shocks on monetary policy are adjusted. This is a minor, technical, and relatively unimportant change.

Table 7 shows what the different solutions yield for each of set of estimates obtained. The left panel of the table shows the effects of the processes estimated via the demand-determined solution on employment when we look both at the demand-determined solution and at the approximated Dreze equilibrium. The right panel shows the effects of the processes estimated with the approximated Dreze equilibrium when we both look at the demand determined solution and at the approximated Dreze equilibrium. The numbers in boldface are the properties of the economies when they are used to estimate the parameters. This table shows some other important features of the differences between the demand-determined solution and the approximated Dreze equilibrium:

5. That the estimates of the Dreze equilibrium increase the role of the neutral technology shock can be seen in the higher variance of employment that both solutions display with these estimates relative to the demand-determined estimates. The contribution of the neutral technology shock to the variance of employment goes from 13% to 71%, that of the investment or embodied technology shock goes from 49% to 16%, and that of the monetary shocks goes from 33% to 15%.[8]

[8] As the quick-witted reader may have noticed, the contributions of the orthogonal shocks to the variance of employment add up to slightly above 100%. The reason for this is the nonlinear nature of the model. Fortunately for our analysis, the differences are quite small, and the contribution of each individual shock when the others are shut out gives a good picture of their overall contribution.

6. Under both sets of estimates, the variance of employment is much larger in the approximated Dreze equilibrium. The unwillingness of households to work under many circumstances generates recessions that are not present in the demand-determined solution.

7. The contribution of the other shocks is smaller for the Dreze equilibrium estimates, especially for the demand-determined solution. This is the result both of the different estimates for the processes of the shocks and of the values of other parameters estimated, such as the investment adjustment cost.

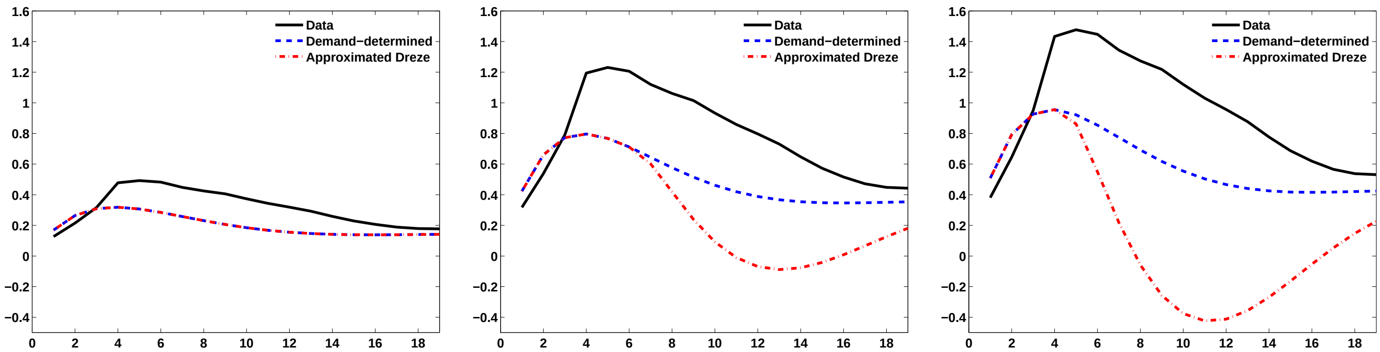
5.1 Robustness of the Findings

We now turn to two considerations about the robustness of our findings: the extent to which the nonlinearity of the Dreze equilibrium solution makes the estimates dependent on how to specify the impulse response function (what size shocks) and the specification of what wage markup to use (because the larger the markup, the larger the model's buffer to accommodate shocks without households becoming unwilling to work).

An important difference between the demand-determined solution and the Dreze equilibrium is that the latter is nonlinear with respect to the shocks. In the demand-determined solution, when the size of the shocks increases, the impulse response increases proportionally. As a result, when matching the empirical impulse response functions, setting the size of the shock to any value leads to the same estimates. In contrast, in the Dreze equilibrium (and in its approximation), the impulse response function is not proportional to the size of the shock. Moreover, even the sign of the impulse response can be reversed as the size of the shock increases. The intuition is simple. If the size of the shock is small, labor demand is typically smaller than the labor supply because of the wage markup, and the economy behaves just like the demand-determined one. As the shock becomes larger, labor demand can be larger than labor supply, and the labor supply constraint starts to be binding.

We have seen how in our baseline estimation with a 5% wage markup, even a one standard deviation shock induces the labor supply constraint to be binding. It may be that for larger wage markups, a one standard deviation shock may not be able to trigger the labor supply constraint. To explore this issue, Figure 11 shows the impulse responses to different sizes of the neutral technology shock where the wage markup is 9% (the use of the larger markup makes the differential response due to nonlinearities of the different size shocks more acute). The estimates are from the demand-determined solution, and they are, of course, invariant to the size of the shock used in the specification of the impulse response function. We can see clearly that as the size of the shock increases, the impulse response of the demand-determined solution adjusts proportionally, whereas the Dreze equilibrium implies a much smaller expansion than the demand-determined solution and may even turn the expansion into a recession within a couple of years. If we estimate the approximated Dreze model using the one standard deviation shock, we would not be able to capture this nonlinearity, and hence we would fail to match the actual response of the economy when the size of the shock is relatively large. When we estimate the approximated Dreze equilibrium, instead,

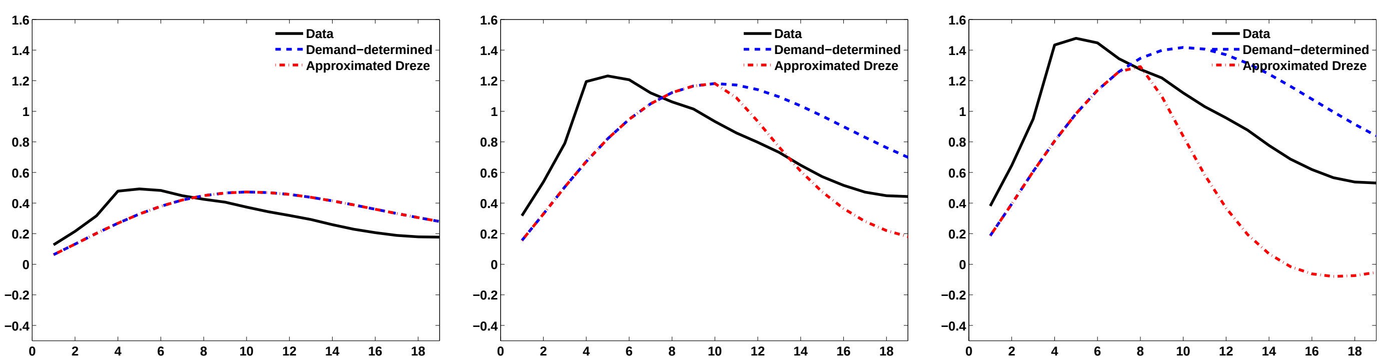
Figure 11: IRF of Labor to Neutral Tech. Shock: Estimation with Demand-Determined Model, 9% Markup



1 std shock 2.5 std shock 3 std shock

we estimate this economy with a shock of three times standard deviations. Figure 12 shows the impulse response of the Dreze equilibrium estimates using different shock sizes to specify the impulse response. Now the estimates have changed and with them the shape of the impulse responses. We see how the impulse response of the Dreze equilibrium changes shape in its attempt to trace that of the data. The dive into recession that resulted two years after a shock of three standard deviations from the demand-determined estimates is now delayed and reduced by the different neutral technology shock, one that is larger and more temporary.

Figure 12: IRF of Labor to Neutral Tech. Shock: Estimation with Approximated Dreze Equilibrium, 9% Markup



1 std shock 2.5 std shock 3 std shock

Table 8 shows the estimates for models with different wage markups and different sizes of the shock. We see again a high standard deviation and a low autocorrelation of the neutral technology shock as well as a larger wage and price rigidity.

In terms of the role of various shocks, we also find that the neutral technology shock becomes more important in accounting for the employment volatility. As shown in Table 9, with a 7% wage markup the contribution of neutral technology shock increases from 13% to 50% when using the approximated Dreze equilibrium. Table 10 displays the results with a 9% wage markup. Again, the contribution of the neutral technology shock is much larger, increasing this time from 13% to 37%.

We conclude that using larger markups and larger shocks to specify which impulse responses the model attempts to replicate generates patterns similar to those in our baseline specification of the Altig, Christiano, Eichenbaum, and Linde (2011) model.

6 Conclusion

In this paper, we have explored what happens in the canonical New Keynesian models when the demand-determined solution for labor is replaced by the more theoretically sound Dreze equilibrium. That is, in these models, "slavery" is abolished and the quantity traded is determined by the short end of the market, in this case by the minimum of the quantity of labor that firms want to hire at the prevailing, staggered wages, and the amount of labor that agents are willing to provide at those wages.

We have shown that the differences are large. Typically between 5% and 25% of the labor force is working against agents' will during any given period in a demand-determined solution. Comparing the demand-determined solution with the Dreze equilibrium in standard models, we see substantially different employment volatilities, typically larger in the demand-determined models.

More importantly, when we estimate the Dreze equilibrium, it yields answers that are substantially different from those provided by the demand-determined solution estimates: in the context of the Christiano, Eichenbaum, and Evans (2005) and Altig, Christiano, Eichenbaum, and Linde (2011) economy, the role of technology shocks rises from 13% to 70%, these shocks become larger and less persistent, and the estimates of the rigidities become larger.

We conclude by encouraging researchers to use the Dreze equilibrium (or an approximation of it) to eliminate the feature that agents work against their will, a notion that we find inappropriate. In this paper, we explain how, and why it matters.

Table 6: Estimated Parameter Values with 5% Wage Markup Using One Standard Deviation Impulse Response Functions

	Demand determined	Approximated Dreze
Std of neutral technology shock, $\sigma_{\mu z}$	0.068 (0.046)	0.140 (0.089)
Autocor neutral technology shock, ρ_{μ_z}	0.902 (0.102)	0.697 (0.240)
Std of monetary shock, σ_M	0.331 (0.084)	0.325 (0.078)
Autocor monetary policy shock, ρ_M	-0.037 (0.111)	-0.040 (0.130)
Std of embodied technology shock, $\sigma_{\mu\Upsilon}$	0.303 (0.042)	0.286 (0.046)
Autocor embodied technology shock, $\rho_{\mu\Upsilon}$	0.241 (0.224)	0.318 (0.176)
Wage rigidity, ξ_w	0.722 (0.123)	0.825 (0.043)
Price rigidity, γ	0.040 (0.029)	0.054 (0.039)
Variable capital utilization, σ_a	1.995 (2.222)	4.564 (7.070)
Investment adjustment cost, S''	3.281 (2.038)	4.752 (2.378)
Interest semi-elasticity of money demand, ϵ	0.808 (0.208)	0.779 (0.193)
Habit formation, b	0.706 (0.045)	0.698 (0.058)
Effects of neutral technology shock on policy, ρ_{xz}	0.343 (0.266)	0.195 (0.480)
Effects of embodied technology shock on policy, $\rho_{x\Upsilon}$	0.824 (0.154)	0.832 (0.132)
Scaling factor of neutral technology shock, c_z	2.997 (2.310)	1.027 (0.749)
Scaling factor of neutral technology shock, c_z^p	1.327 (1.381)	0.665 (0.650)
Scaling factor of embodied technology shock, c_Υ^p	0.135 (0.244)	0.107 (0.268)
Scaling factor of embodied technology shock, c_Υ	0.246 (0.244)	0.305 (0.266)

Table 7: Effects on Labor of the Shocks of Each Set of Parameters over Each Solution Concept

	Estimated with Demand Determined				Estimated with Approximated Dreze: 1 std shock			
Neutral technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	**0.18**	**0.87**	**15.09**	—	0.24	0.97	19.03
Approximated Dreze	-1.57	1.16	0.96	—	**-2.59**	**1.41**	**0.95**	—
Investment technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	–	**0.67**	**0.99**	**6.22**	–	0.52	0.99	7.89
Approximated Dreze	-0.42	0.34	0.98	—	**-0.55**	**0.32**	**0.99**	—
Monetary shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	**0.46**	**1.00**	**2.56**	—	0.33	1.00	1.15
Approximated Dreze	-0.07	0.33	0.99	—	**-0.01**	**0.30**	**1.00**	—
All shocks	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	**1.38**	**0.96**	**18.83**	—	1.15	0.95	22.63
Approximated Dreze	-2.28	2.06	0.98	—	**-3.41**	**1.99**	**0.96**	—

Notes: Numbers are in percentages except for the correlation with output.

Table 8: Estimated Parameter Values for the Dreze Equilibria under Various Markups and Shock Sizes

	Demand Determined	Dreze, 5% markup		Dreze, 7% markup		Dreze, 9% markup	
		1 std	1.5 std	1.5 std	2 std	2.5 std	3 std
$\sigma_{\mu z}$	0.068 (0.046)	0.140 (0.089)	0.110 (0.000)	0.119 (0.063)	0.148 (0.113)	0.126 (0.069)	0.148 (0.130)
ρ_{μ_z}	0.902 (0.102)	0.697 (0.240)	0.579 (0.002)	0.790 (0.152)	0.625 (0.358)	0.750 (0.183)	0.631 (0.407)
σ_M	0.331 (0.084)	0.325 (0.078)	0.319 (0.074)	0.327 (0.078)	0.316 (0.082)	0.329 (0.080)	0.319 (0.078)
ρ_M	-0.037 (0.111)	-0.040 (0.130)	-0.078 (0.121)	-0.013 (0.098)	-0.016 (0.165)	-0.035 (0.117)	-0.031 (0.118)
$\sigma_{\mu\Upsilon}$	0.303 (0.042)	0.286 (0.046)	0.287 (0.046)	0.296 (0.046)	0.296 (0.045)	0.297 (0.045)	0.297 (0.045)
ρ_{μ_Υ}	0.241 (0.224)	0.318 (0.176)	0.344 (0.377)	0.286 (0.190)	0.282 (0.189)	0.284 (0.190)	0.282 (0.190)
ξ_w	0.722 (0.123)	0.825 (0.043)	0.801 (0.135)	0.841 (0.038)	0.854 (0.073)	0.858 (0.057)	0.870 (0.084)
γ	0.040 (0.029)	0.054 (0.039)	0.103 (0.144)	0.058 (0.038)	0.060 (0.058)	0.057 (0.040)	0.059 (0.056)
σ_a	1.995 (2.222)	4.564 (7.070)	0.932 (0.834)	3.918 (4.510)	4.000 (4.997)	3.946 (4.787)	4.190 (5.610)
S''	3.281 (2.038)	4.275 (2.378)	3.246 (2.030)	4.509 (2.505)	4.041 (2.393)	4.283 (2.391)	4.017 (2.423)
ϵ	0.808 (0.208)	0.779 (0.193)	0.722 (0.170)	0.796 (0.197)	0.754 (0.200)	0.791 (0.199)	0.762 (0.189)
b	0.706 (0.045)	0.698 (0.058)	0.719 (0.078)	0.705 (0.056)	0.698 (0.065)	0.702 (0.058)	0.697 (0.064)
ρ_{xz}	0.343 (0.266)	0.195 (0.480)	0.130 (0.553)	0.285 (0.428)	0.235 (0.419)	0.260 (0.430)	0.220 (0.457)
$\rho_{x\Upsilon}$	0.824 (0.154)	0.832 (0.132)	0.882 (0.066)	0.830 (0.128)	0.832 (0.133)	0.830 (0.131)	0.831 (0.135)
c_z	2.997 (2.310)	1.027 (0.749)	1.008 (0.704)	1.354 (0.858)	0.937 (0.789)	1.225 (0.785)	0.949 (0.913)
c_z^p	1.327 (1.381)	0.665 (0.650)	0.715 (0.724)	0.638 (0.556)	0.454 (0.375)	0.596 (0.492)	0.501 (0.479)
c_Υ^p	0.135 (0.238)	0.107 (0.244)	0.110 (0.270)	0.167 (0.242)	0.124 (0.240)	0.153 (0.241)	0.133 (0.241)
c_Υ	0.246 (0.244)	0.305 (0.266)	0.318 (0.276)	0.240 (0.265)	0.276 (0.261)	0.250 (0.263)	0.267 (0.261)

Table 9: Effects on Labor of Shocks over Each Solution Concept: 7% Markup, 2 Std Shock

	Estimated with Demand Determined				Estimated with Approximated Dreze: 1 std shock			
Neutral technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.18	0.87	7.72	—	0.26	0.97	10.10
Approximated	-0.90	0.52	0.92	—	-1.37	0.55	0.91	—
Investment technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	–	0.67	0.99	3.24	–	0.51	0.99	4.13
Approximated	-0.18	0.43	0.98	—	-0.27	0.38	0.99	—
Monetary shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.46	1.00	0.63	—	0.33	1.00	0.07
Approximated	-0.01	0.43	0.99	—	0.00	0.33	1.00	—
All shocks	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	1.38	0.96	10.96	—	1.17	0.96	13.54
Approximated	-1.41	1.26	0.96	—	-2.00	1.11	0.94	—

Notes: Numbers are in percentages except for the correlation with output.

Table 10: Effects on Labor of Shocks over Each Solution Concept: 9% Markup, 3 Std Shock

	Estimated with Demand Determined				Estimated with Approximated Dreze: 1 std shock			
Neutral technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.18	0.87	4.50	—	0.26	0.97	6.67
Approximated	-0.55	0.30	0.87	—	-0.92	0.34	0.89	—
Investment technology shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	–	0.67	0.99	1.52	–	0.51	0.99	2.08
Approximated	-0.07	0.54	0.99	—	-0.12	0.44	0.99	—
Monetary shock	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	0.46	1.00	0.09	—	0.33	1.00	0.00
Approximated	0.00	0.46	1.00	—	0.00	0.33	1.00	—
All shocks	Mean	Var	Corr(N,Y)	Binding Prob	Mean	Var	Corr(N,Y)	Binding Prob
Demand determined	—	1.38	0.96	6.98	—	1.17	0.95	9.28
Approximated	-0.92	1.02	0.95	—	-1.38	0.92	0.93	—

Notes: Numbers are in percentages except for the correlation with output.

References

Alessandria, G., J. P. Kaboski, and V. Midrigan (2010): "Inventories, Lumpy Trade, and Large Devaluations," *American Economic Review*, 100(5), 2304–39.

Altig, D., L. J. Christiano, M. Eichenbaum, and J. Linde (2011): "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," *Review of Economic Dynamics*, 14(2), 225–247.

Aruoba, S. B., L. Bocola, and F. Schorfheide (2013): "Assessing DSGE Model Nonlinearities," Working Paper 19693, National Bureau of Economic Research.

Barro, R. J., and H. I. Grossman (1971): "A General Disequilibrium Model of Income and Employment," *American Economic Review*, 61(1), 82–93.

Bills, M. (2004): "Studying Price Markups from Stockout Behavior," Manuscript, University of Rochester, http://www.econ.rochester.edu/people/BilsPapers/st1.pdf.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2002): "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?," *Review of Economic Studies*, 69(3), 533–563.

Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2009): "New Keynesian Models: Not Yet Useful for Policy Analysis," *American Economic Journal: Macroeconomics*, 1(1), 242–266.

Chetty, R., A. Guren, D. S. Manoli, and A. Weber (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins," *American Economic Review: Papers and Proceedings*, 101(3), 471–475.

Christiano, L., M. Eichenbaum, and S. Rebelo (2011): "When Is the Government Spending Multiplier Large?," *Journal of Political Economy*, 119(1), 78–121.

Christiano, L. J., M. Eichenbaum, and C. L. Evans (2005): "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," *Journal of Political Economy*, 113(1), 1–45.

Christiano, L. J., M. S. Eichenbaum, and M. Trabandt (2013): "Unemployment and Business Cycles," Mimeo, Northwestern University.

Corsetti, G., and P. Pesenti (2005): "International dimensions of optimal monetary policy," *Journal of Monetary Economics*, 52(2), 281–305.

Drèze, J. H. (1975): "Existence of an Exchange Equilibrium under Price Rigidities," *International Economic Review*, 16(2), 301–320.

Erceg, C. J., D. W. Henderson, and A. T. Levin (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of Monetary Economics*, 46(2), 281–313.

Galí, J. (2008): *Monetary Policy, Inflation, and the Business Cycle: An Introduction to the New Keynesian Framework*. Princeton University Press.

——— (2011): "The Return of the Wage Phillips Curve," *Journal of the European Economic Association*, 9(3), 436–461.

Galí, J., F. Smets, and R. Wouters (2011): "Unemployment in an Estimated New Keynesian Model," *NBER Macroeconomics Annual*, pp. 329–360.

Gertler, M., L. Sala, and A. Trigari (2008): "An Estimated Monetary DSGE Model with Unemployment and Staggered Nominal Wage Bargaining," *Journal of Money, Credit and Banking*, 40(8), 1713–1764.

Gertler, M., and A. Trigari (2009): "Unemployment Fluctuations with Staggered Nash Wage Bargaining," *Journal of Political Economy*, 117(1), 38–86.

Guerrieri, L., and M. Iacoviello (2015): "Occbin: A Toolkit to Solve Models with Occasionally Binding Constraints Easily," *Journal of Monetary Economics*, 70, 22–38.

Hall, R. (2005): "Employment Fluctuations with Equilibrium Wage Stickiness," *American Economic Review*, 95(1), 50–65.

Hall, R. E., and P. R. Milgrom (2008): "The Limited Influence of Unemployment on the Wage Bargain," *American Economic Review*, 98(4), 1653–1674.

Heathcote, J., K. Storesletten, and G. L. Violante (2010): "The Macroeconomic Implications of Rising Wage Inequality in the United States," *Journal of Political Economy*, 118(4), 681–722.

Levin, A. T., A. Onatski, J. Williams, and N. M. Williams (2006): "Monetary Policy Under Uncertainty in Micro-Founded Macroeconometric Models," in *NBER Macroeconomics Annual 2005*, ed. by M. Gertler, and K. Rogoff, vol. 20, pp. 229–312. MIT Press.

Lewis, H. G. (1986): "Union Relative Wage Effects," in *Handbook of Labor Economics*, ed. by O. Ashenfelter, and R. Layard, vol. 2, chap. 20, pp. 1139–1181. North-Holland.

Malinvaud, E. (1977): *The Theory of Unemployment Reconsidered*, Yrjö Jahnsson Lectures. Wiley.

Ríos-Rull, J.-V., and R. Santaeulàlia-Llopis (2010): "Redistributive Shocks and Productivity Shocks," *Journal of Monetary Economics*, 57(8), 931–948.

Smets, F., and R. Wouters (2007): "Shocks and Frictions in US Business Cycles: a Bayesian DSGE Approach," *American Economic Review*, 97(3), 586–606.

Appendix

A Details of the Estimation of the Smets and Wouters (2007)

The system of the log-linearized equations for estimation and simulation are the following (the only changes from Smets and Wouters (2007) are in equations (A-2) and (A-10)):

$$y_t = c_y c_t + i_y i_t + k_y \bar{r}^k u_t + \epsilon_t^g \tag{A-1}$$

$$c_t = \frac{h/\zeta}{1+h/\zeta} c_{t-1} + \frac{1}{1+h/\zeta} \mathbb{E}_t c_{t+1} - \frac{1-h/\zeta}{1+h/\zeta} (r_t - E_t \pi_{t+1} + \epsilon_t^b) \tag{A-2}$$

$$i_t = \frac{1}{1+\beta} \left(i_{t-1} + \beta \mathbb{E}_t i_{t+1} + \frac{1}{\zeta^2 \varphi} q_t \right) + \epsilon_t^i \tag{A-3}$$

$$q_t = \frac{\bar{r}^k}{1-\delta+\bar{r}^k} r_{t+1}^k + \frac{1-\delta}{1-\delta+\bar{r}^k} \mathbb{E}_t q_{t+1} - (r_t - \mathbb{E}_t \pi_{t+1} + \epsilon_t^b) \tag{A-4}$$

$$y_t = \Phi \left(\alpha (k_{t-1} + u_t) + (1-\alpha) \ell_t + \epsilon_t^a \right) \tag{A-5}$$

$$r_t^k = \frac{1-\psi}{\psi} u_t \tag{A-6}$$

$$k_t = \left(1 - \frac{1-\delta}{\zeta} \right) k_{t-1} + \frac{1-\delta}{\zeta} i_t + \epsilon_t^i \tag{A-7}$$

$$\pi_t = \frac{1}{1+\beta \iota_p} \left(\iota_p \pi_{t-1} + \beta \mathbb{E}_t \pi_{t+1} + \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p((\Phi-1)\varepsilon_p+1)} (\alpha(k_{t-1} + u_t - \ell_t) - w_t + \epsilon_t^a) \right) + \epsilon_t^p \tag{A-8}$$

$$r_t^k = w_t + \ell_t - k_{t-1} - u_t \tag{A-9}$$

$$\begin{aligned} w_t = \frac{1}{1+\beta} \Bigg\{ & \beta w_{t+1} + w_{t-1} + \beta \pi_{t+1} - (1+\beta\iota_w)\pi_{t-1} - \iota_w \pi_{t-1} \\ & + \frac{(1-\beta\xi_w)(1-\xi_w)}{(1+\gamma\epsilon_w)\xi_w} \left(\gamma \ell_t + \frac{1}{1-h/\zeta} c_t - \frac{h/\zeta}{(1-h/\zeta} c_{t-1} - w_t \right) \Bigg\} + \epsilon_t^w \end{aligned} \tag{A-10}$$

$$r_t = \rho r_{t-1} + (1-\rho)(r_\pi \pi_t + r_y (y_t - y_t^p)) + r_{\Delta y}(y_t - y_t^p - (y_{t-1} - y_{t-1}^p)) + \epsilon_t^m \tag{A-11}$$

In this system, there are seven shocks, $\{\epsilon_t^a, \epsilon_t^i, \epsilon_t^b, \epsilon_t^p, \epsilon_t^w, \epsilon_t^g, \epsilon_t^m\}$, which are shocks to TFP, investment technology, risk premium, price markup, wage markup, government spending, and monetary policy. Some of the shocks are rescaled to facilitate estimation. The processes for various shocks are

$$\epsilon_t^g = \rho_g \epsilon_{t-1}^g + \eta_t^g + \rho_{ga} \eta_t^a \tag{A-12}$$
$$\epsilon_t^b = \rho_b \epsilon_{t-1}^b + \eta_t^b \tag{A-13}$$
$$\epsilon_t^i = \rho_i \epsilon_{t-1}^i + \eta_t^i \tag{A-14}$$
$$\epsilon_t^a = \rho_a \epsilon_{t-1}^a + \eta_t^a \tag{A-15}$$
$$\epsilon_t^m = \rho_m \epsilon_{t-1}^m + \eta_t^m \tag{A-16}$$
$$\epsilon_t^p = \rho_p \epsilon_{t-1}^p + \eta_t^p - \mu_p \eta_{t-1}^p \tag{A-17}$$
$$\epsilon_t^w = \rho_w \epsilon_{t-1}^w + \eta_t^w - \mu_w \eta_{t-1}^w \tag{A-18}$$

As in Smets and Wouters (2007), we fixed a few parameters, which are the depreciation rate $\delta = 0.025$, the average government spending to output ratio $g_y = 0.18$, the Kimball aggregator parameter $\varepsilon_p = 10$, and we use log utility function for consumption. Although Smets and Wouters (2007) set the wage markup to 50%, we estimate the model for various other wage markup levels and report them in Table A-1. The process for the shocks is reported in Table A-2.

The implied true wage markup shock $\widehat{\epsilon}_t^w$ is calculated based on equation (A-10) and is given by

$$\widehat{\epsilon}_t^w = \frac{(1+\beta)(1+\gamma\epsilon_w)\xi_w}{(1-\beta\xi_w)(1-\xi_w)} \epsilon_t^w. \tag{A-19}$$

Table A-1: Smets and Wouters (2007) Model: Structural Parameters

Wage markup, $\frac{\epsilon_w}{\epsilon_w - 1}$	6%	11%	15%	18%
Investment adjustment cost, φ	5.412	5.426	5.430	5.432
Habit formation, h	0.780	0.785	0.787	0.788
Wage Calvo probability, ξ_w	0.539	0.583	0.605	0.617
Inverse Frsich elasticity, γ	2.856	2.802	2.772	2.754
Price Calvo probability, ξ_p	0.677	0.665	0.659	0.656
Wage indexation, ι_w	0.574	0.584	0.589	0.591
Price indexation, ι_p	0.213	0.216	0.218	0.219
Utilization elasticity, ψ	0.536	0.537	0.538	0.538
Fixed cost, Φ	1.556	1.560	1.562	1.563
Share of capital in production, α	0.189	0.190	0.190	0.191
Monetary policy for inflation, r_π	1.008	1.008	1.008	1.008
Monetary policy for output gap, r_y	0.070	0.070	0.071	0.071
Monetary policy persistency, ρ	0.811	0.807	0.805	0.804
Monetary policy for output gap change, $r_{\Delta y}$	0.232	0.229	0.227	0.226
Trend, ζ	1.004	1.004	1.004	1.004
Discount rate, β	0.998	0.998	0.998	0.998

Notes: The specification of the prior distribution is the same as in Smets and Wouters (2007). We only report the posterior modes of the structural parameters in this table for various wage markup levels. The posterior means are not included here but are available upon request. When we simulate the model, we use the posterior modes of the structural parameters.

Table A-2: Smets and Wouters (2007) Model: Persistence and Standard Deviation of Various Shocks

Wage markup, $\frac{\epsilon_w}{\epsilon_w - 1}$	6%	11%	15%	18%
Government spending shock, ϵ^g	0.52	0.52	0.52	0.52
Government spending shock, ρ_g	0.97	0.97	0.97	0.97
Government spending shock, ρ_{ga}	0.54	0.54	0.54	0.54
Risk premium shock, ϵ^b	0.22	0.22	0.23	0.23
Risk premium shock, ρ_b	0.37	0.34	0.33	0.33
Investment shock, ϵ^i	0.38	0.39	0.39	0.39
Investment shock, ρ_i	0.77	0.76	0.75	0.75
TFP shock, ϵ^a	0.46	0.46	0.46	0.46
TFP shock, ρ_a	0.95	0.95	0.95	0.95
Monetary policy shock, ϵ^m	0.52	0.52	0.52	0.52
Monetary policy shock, ρ_m	0.13	0.13	0.14	0.14
Price markup shock, ϵ^p	0.14	0.14	0.14	0.14
Price markup shock, ρ_p	0.90	0.91	0.91	0.91
Price markup shock, μ_p	0.75	0.75	0.75	0.75
Wage markup shock, ϵ^w	0.25	0.25	0.25	0.25
Wage markup shock, ρ_w	0.97	0.98	0.98	0.98
Wage markup shock, μ_w	0.92	0.91	0.91	0.90
Implied true wage markup shock, $\widehat{\epsilon}^w$	64.73	48.47	42.54	39.64

Notes: The specification of the prior distribution is the same as in Smets and Wouters (2007). We only report the posterior modes of the shock processes in this table for various wage markup levels. The posterior means are not included here but are available upon request. When we simulate the model, we use the posterior modes of the shock processes.

B Shocks to Wage markups

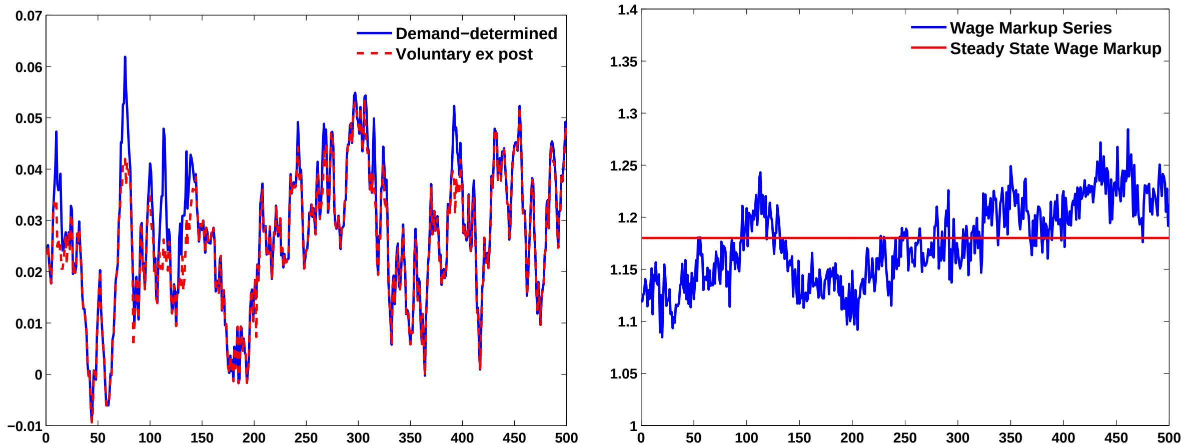
We explore the Galí, Smets, and Wouters (2011) model with shocks to the wage markup. To ensure that the wage markup is always positive, which is necessary to construct the voluntary ex post employment series, we simulate the model with standard deviations of the wage markup shock ranging from 0.0 to 0.015. Table A-3 reports the findings. The larger the variance of the wage markup shock, the lower is mean employment and the larger the frequency of the violation of the labor supply constraint in the demand-determined allocation. The variance of the voluntary ex post employment also increases with the volatility of the wage markup shock. The reason is that in periods when the value of the wage markup is low, the variance of cross-sectional labor demand becomes larger and the level of labor supply is lower, which makes the labor supply constraint be more likely to be violated. Figure A-1 shows a high wage markup sample where the demand-determined and the voluntary ex post employment series are similar to each other. However, in Figure A-2 which shows a sample with a sequence of low wage markup shocks, the differences between the two series are apparent: the voluntary ex post aggregate employment displays large reductions. We conclude that abstracting from wage markup shocks in the baseline analysis of the Galí, Smets, and Wouters (2011) model, if anything, tends to bias toward making the demand-determined allocation look more similar to the Dreze equilibrium than it really is.

Table A-3: Galí, Smets, and Wouters (2011) Model with voluntary Ex Post Aggregate Employment

St.d of Wage Markup Shock: 0.000	Mean	Var	Corr(N,Y)	Binding Freq
Employment w/o constraint	—	0.51	0.75	3.86
Employment w/ voluntary ex post aggregate employment	-0.13	0.39	0.70	—
St.d of Wage Markup Shock: 0.005	**Mean**	**Var**	**Corr(N,Y)**	**Binding Freq**
Employment w/o constraint	—	0.51	0.75	3.90
Employment w/ voluntary ex post aggregate employment	-0.13	0.39	0.70	—
St.d of Wage Markup Shock: 0.010	**Mean**	**Var**	**Corr(N,Y)**	**Binding Freq**
Employment w/o constraint	—	0.51	0.75	4.94
Employment w/ voluntary ex post aggregate employment	-0.14	0.41	0.69	—
St.d of Wage Markup Shock: 0.015	**Mean**	**Var**	**Corr(N,Y)**	**Binding Freq**
Employment w/o constraint	—	0.51	0.75	7.21
Employment w/ voluntary ex post aggregate employment	-0.20	0.51	0.65	—

Notes: Numbers are in percentages except for the correlation with output. All the variables are logged and HP filtered. The standard deviation of the wage markup shock is 0.04 in Galí, Smets, and Wouters (2011).

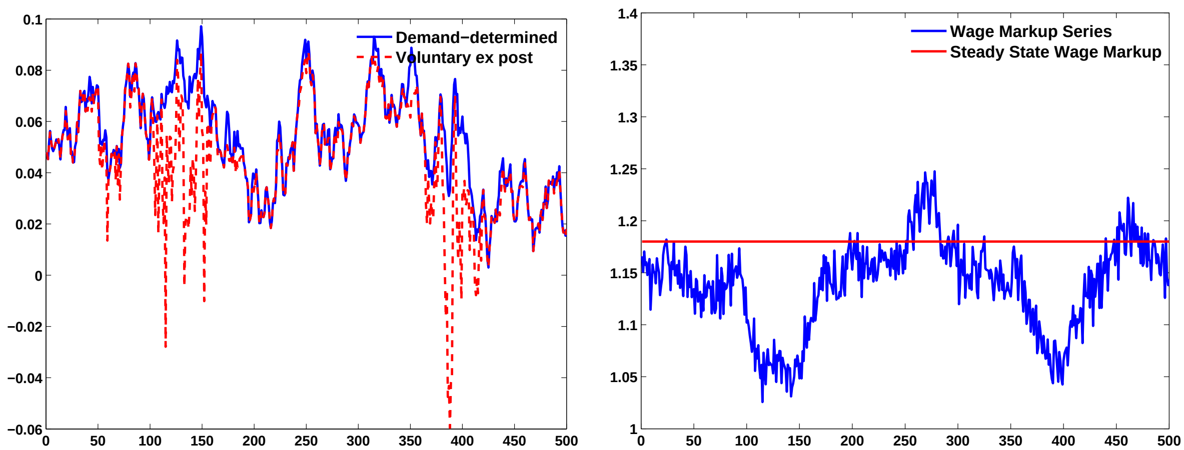
Figure A-1: Galí, Smets, and Wouters (2011) Model with Wage Markup Shock: High Wage Markup Sample



Aggregate Employment

Wage Markup Series

Figure A-2: Galí, Smets, and Wouters (2011) Model with Wage Markup Shock: Low Wage Markup Sample



Aggregate Employment

Wage Markup Series

C Note on Solving the Occasionally Binding Constraint Problem

Guerrieri and Iacoviello (2015) develop a Dynare toolkit that can solve DSGE models with occasionally binding constraints. However, this method cannot be applied in our model for the following reasons.

Guerrieri and Iacoviello (2015)'s method can handle problems with exogenous binding constraints such as non-negative investment, a zero-bound on the nominal interest rate, an exogenous borrowing constraint, and so on. In these cases, one can neatly partition the problem into two regions: in the first region, the constraint is not binding and one can use the first order condition to characterize the solution. In the second region, the constraint is binding and one can simply set the variable to equal the constraint (for example, let the nominal interest rate be zero).

The problem in this paper is more involved. When the labor supply constraint is not binding, employment equals the labor demand, and the first order condition can be applied as in the standard New Keynesian literature. When the labor supply constraint is binding, different from the examples listed earlier, the labor supply constraint is not an exogenous constraint because the level of the labor supply is endogenously determined. What makes this case even worse is that when the labor supply constraint is binding, the union's problem is not concave, which implies that we cannot use either the first order condition or some exogenous value to determine the optimal wage (and hence employment).

To further illustrate the issue, consider the following example. A union needs to set the optimal wage for two periods, and it takes aggregate states as given. For simplicity, we abstract from price change and uncertainty:

$$\max_{w} \ U = u'(c_1)wn_1 - \frac{n_1^{1+\gamma}}{1+\gamma} + \beta\left(u'(c_2)wn_2 - \frac{n_2^{1+\gamma}}{1+\gamma}\right) \tag{A-20}$$

subject to

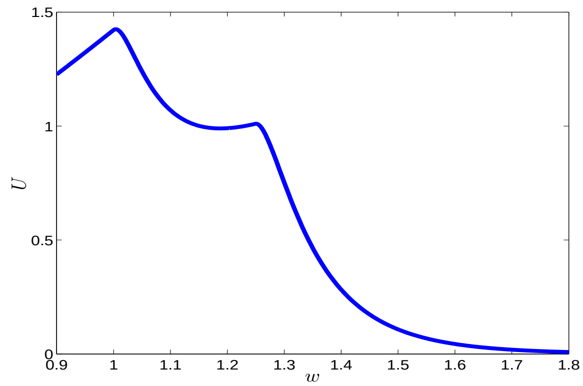
$$n_1 = \min\left\{\left(u'(c_1)w\right)^{\frac{1}{\gamma}}, \left(\frac{w}{\overline{w}_1}\right)^{-\epsilon_w}\overline{n}_1\right\}, \tag{A-21}$$

$$n_2 = \min\left\{\left(u'(c_2)w\right)^{\frac{1}{\gamma}}, \left(\frac{w}{\overline{w}_2}\right)^{-\epsilon_w}\overline{n}_2\right\}. \tag{A-22}$$

The following figure shows how the objective U changes with the choice w for a particular parametrization. It is obvious that the objective function is not concave, and the first order condition cannot be used to solve this problem.

Finally, the computation cost to apply Guerrieri and Iacoviello (2015)'s method is not greatly affected by the number of state variables in the model but is increasing fast with the number of occasionally binding constraints. In Calvo-type sticky wage models, the number of occasionally binding constraints is

Figure A-3: Illustration of the Union's Problem



infinite because there are infinitely many cohorts. In our simple economy with Taylor-type staggered wage contracts, there are four occasionally binding constraints, which is still a relatively large number.

To summarize, in our case, there is no simple way to decide the optimal level of wage and employment when labor supply is binding. Therefore, we solve the Dreze equilibrium using a global method.

D Details of the Computation of the Simple Economy

We use a policy function iteration method to obtain the numerical solution. The system of equations that characterizes the solution is

$$c_t^{-\sigma} = \beta \mathbb{E}_t \left[c_{t+1}^{-\sigma} \frac{R_t}{\pi_{t+1}} \right] \tag{A-23}$$

$$c_t^{-\sigma} = \beta \mathbb{E}_t \left[c_{t+1}^{-\sigma} \frac{1 + r_{t+1}^k - \delta}{\pi_{t+1}} \right] \tag{A-24}$$

$$\frac{r_t^k}{p_t} = \alpha \; z_t \; k_t^{\alpha-1} \; e_t^{1-\alpha}, \tag{A-25}$$

$$\log R_t = \log \frac{1}{\beta} + \phi_\pi \pi_t + \phi_y \log \frac{y_t}{y^*} + \eta_t, \tag{A-26}$$

$$c_t + k_{t+1} = y_t + (1-\delta) k_t \tag{A-27}$$

$$y_t = z_t \; k_t^{\alpha} \; e_t^{1-\alpha}, \tag{A-28}$$

$$e_t = \left[\sum_{i=0}^{T_w} e_{i,t}^{\frac{\epsilon_w - 1}{\epsilon_w}} \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \tag{A-29}$$

$$e_{i,t} = \min \left\{ \left[\frac{w_{t-i}^*}{(1-\alpha) z_t k_t^{\alpha} e_t^{-\alpha} p_t} \right]^{-\epsilon_w} e_t, \; \left(\frac{u'(c_t) w_{t-i}^*}{\phi p_t} \right)^{\frac{1}{\gamma}} \right\}. \tag{A-30}$$

In addition, the optimization problem (29) to (30) is also part of the system.

There are two differences between the demand-determined economy and the Dreze equilibrium: first, the employment is determined by the minimum of the demand and supply in the Dreze equilibrium (see equation (A-30)), whereas in the demand-determined economy, employment always equals to labor demand. Second, the choice of the optimal nominal wage, w_t^*, cannot be characterized by a simple first order condition because of the potential binding labor supply constraint. In the computation, we have to use a global search method to find the optimal wage choice.

We look for policy functions for $\{k_{t+1}, c_t, y_t, e_{i,t}, e_t, w_t^*, \pi_t, R_t\}$. The state variables at period t include the following: the current technology shock z_t or the monetary shock η_t, the capital stock k_t, and the wages set in the previous three periods $\left\{ \frac{w_{t-1}^*}{p_{t-1}}, \frac{w_{t-2}^*}{p_{t-1}}, \frac{w_{t-3}^*}{p_{t-1}} \right\}$. The real wage in period t can be obtained by dividing the current inflation rate:

$$\frac{w_{t-i}^*}{p_t} = \frac{w_{t-i}^*}{p_{t-1}} \frac{1}{\pi_t} = \frac{w_{t-i}^*}{p_{t-1}} \frac{p_{t-1}}{p_t}. \tag{A-31}$$

E Details of the Estimation of Altig, Christiano, Eichenbaum, and Linde (2011)

The system of the log-linearized equations for estimation and simulation is the following:

$$\mathbb{E}\left[\widehat{\lambda}_{z^*,t+1} - \frac{1}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} - \widehat{\mu}_{zt+1} + \frac{\rho\widehat{\rho}_{t+1} + (1-\delta)\widehat{\mu}_{t+1}}{1-\delta+\rho}\middle|\Omega_t^p\right] = 0 \tag{A-32}$$

$$\mathbb{E}\Bigg\{S''(\mu_\Upsilon\mu_{z^*})^2\left[\widehat{i}_t - \widehat{i}_{t-1} + \widehat{\mu}_{\Upsilon t} + \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t} + \widehat{\mu}_{zt}\right] -$$

$$\beta S''(\mu_\Upsilon\mu_{z^*})^2\left[\widehat{i}_{t+1} - \widehat{i}_t + \widehat{\mu}_{\Upsilon t+1} + \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} + \widehat{\mu}_{zt+1}\right] - \widehat{\mu}_t\Bigg|\Omega_t^p\Bigg\} = 0 \tag{A-33}$$

$$\frac{\nu R}{\nu R + 1 - \nu}\widehat{R}_t + \widehat{w}_t + \frac{1}{1-\alpha}\left(\frac{y}{y+\phi}\widehat{y}_t - \widehat{k}_t + \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t} + \widehat{\mu}_{zt} + \widehat{\mu}\Upsilon t\right) - \widehat{\rho}_t - \frac{1}{1-\alpha}\widehat{u}_t = 0 \tag{A-34}$$

$$[\mu_\Upsilon\mu_{z^*} - (1-\delta)]\widehat{i}_t - \left\{\mu_\Upsilon\mu_{z^*}\widehat{k}_{t+1} - (1-\delta)\left[\widehat{k}_t - \frac{1}{1-\alpha}\widehat{\mu}_{\Upsilon t} - \widehat{\mu}_{zt}\right]\right\} = 0 \tag{A-35}$$

$$\mathbb{E}[\beta(\widehat{\pi}_{t+1} - \pi_t) - \gamma\widehat{s}_t - (\widehat{\pi}_t - \pi_{t-1})|\Omega_t^p] = 0 \tag{A-36}$$

$$\widehat{c}_t - \frac{R}{(R-1)(2+\sigma_\eta)}\widehat{R}_t - \widehat{q}_t = 0 \tag{A-37}$$

$$\mathbb{E}\Bigg\{-\left(\frac{1}{c(1-b\mu_{z^*}^{-1})}\right)^2\left[c\widehat{c}_t - \frac{bc}{\mu_{z^*}}\widehat{c}_{t-1} + \frac{bc}{\mu_{z^*}}\left(\frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t} + \mu_{zt}\right)\right] +$$

$$\beta\left(\frac{1}{c(1-b\mu_{z^*}^{-1})}\right)^2\left[c\widehat{c}_{t+1} - \frac{bc}{\mu_{z^*}}\widehat{c}_t + \frac{bc}{\mu_{z^*}}\left(\frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} + \mu_{zt+1}\right)\right] -$$

$$\lambda_{z^*}[(1+\eta(V)) + \eta'(V)V]\widehat{\lambda}_{z^*t} - \lambda_{z^*}\left[2 + \frac{\eta''(V)V}{\eta'(V)}\right]\eta'(V)V(\widehat{c}_t - \widehat{q}_t)\Bigg|\Omega_t^p\Bigg\} = 0 \tag{A-38}$$

$$\mathbb{E}\left[-\lambda_{z^*t} + \lambda_{z^*t+1} + \widehat{R}_{t+1} - \widehat{\pi}_{t+1} - \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} - \widehat{\mu}_{zt+1}\middle|\Omega_t^p\right] = 0 \tag{A-39}$$

$$\frac{\xi_w(\lambda_w\sigma_L - (1-\lambda_w))}{(1-\xi_w)(1-\beta\xi_w)}\Bigg\{\widehat{w}_{t-1} + \left[-\frac{1+\beta\xi_w^2}{\xi_w} + \sigma_L\lambda_w\frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w(\lambda_w\sigma_L - (1-\lambda_w))}\right]\widehat{w}_t + \beta\widehat{w}_{t+1} + \widehat{\pi}_{t-1} + \widehat{\pi}_t + \beta\pi_{t+1} +$$

$$\frac{(1-\xi_w)(1-\beta\xi_w)(1-\lambda_w)}{\xi_w(\lambda_w\sigma_L - (1-\lambda_w))}(-\sigma_L\widehat{h}_t + \widehat{\lambda}_{z^*t}) - (1-\vartheta)\widehat{\mu}_{zt} + \beta(1-\vartheta)\frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t+1} + \beta(1-\vartheta)\widehat{\mu}_{zt+1}\Bigg\} = 0 \tag{A-40}$$

$$(1+\eta)c\widehat{c}_t + \eta'\frac{c^2}{q}(\widehat{c}_t - \widehat{q}_t) + i\widehat{i}_t - (y+\phi)\left[\alpha\left(\widehat{u}_t\widehat{k}_t - \frac{1}{1-\alpha}\widehat{\mu}_{\Upsilon t} - \widehat{\mu}_{zt}\right) + (1-\alpha)\widehat{h}_t\right] + \rho\frac{k}{\mu_{z^*}\mu_\Upsilon}\widehat{u}_t = 0 \tag{A-41}$$

$$\widehat{w}_t + \widehat{h}_t - \frac{xm(\widehat{x}_t + \widehat{m}_t) - q\widehat{q}_t}{xm - q} = 0 \tag{A-42}$$

$$\widehat{x}_{zt} + \widehat{x}_{\Upsilon t} + \widehat{x}_{Mt} - \widehat{x}_t = 0 \tag{A-43}$$

$$\widehat{x}_{t-1} - \widehat{\pi}_t - \frac{\alpha}{1-\alpha}\widehat{\mu}_{\Upsilon t} - \widehat{\mu}_{zt} + \widehat{m}_{t-1} - \widehat{m}_t = 0 \tag{A-44}$$

$$y\widehat{y}_t - (y+\phi)\left[\alpha\left(\widehat{u}_t + \widehat{k}_t - \frac{1}{1-\alpha}\widehat{\mu}_{\Upsilon t} - \widehat{\mu}_{zt}\right) + (1-\alpha)\widehat{h}_t\right] + \rho\frac{k}{\mu_{z^*}\mu_\Upsilon}\widehat{u}_t = 0 \tag{A-45}$$

$$\mathbb{E}\left[\widehat{u}_t - \frac{1}{\sigma_a}\widehat{\rho}_t\middle|\Omega_t^p\right] = 0 \tag{A-46}$$

In this system, there are three shocks, $\{\epsilon_{Mt}, \epsilon_{\mu_z t}, \epsilon_{\mu_\Upsilon t}\}$, which are shocks to monetary policy, neutral technology, and embodied investment technology. The processes for various shocks are

$$\widehat{\mu}_{zt} = \rho_{\mu z}\widehat{\mu}_{zt-1} + \epsilon_{\mu_z t} \tag{A-47}$$

$$\widehat{\mu}_{\Upsilon t} = \rho_{\mu\Upsilon}\widehat{\mu}_{\Upsilon t-1} + \epsilon_{\mu_\Upsilon t} \tag{A-48}$$

$$\widehat{x}_{Mt} = \rho_M \widehat{x}_{Mt-1} + \epsilon_{Mt} \tag{A-49}$$

$$\widehat{x}_{zt} = \rho_{xz}\widehat{x}_{zt-1} + c_z^p \epsilon_{\mu_z t-1} + c_z \epsilon_{\mu_z t} \tag{A-50}$$

$$\widehat{x}_{\Upsilon t} = \rho_{x\Upsilon}\widehat{x}_{\Upsilon t-1} + c_\Upsilon^p \epsilon_{\mu_\Upsilon t-1} + c_\Upsilon \epsilon_{\mu_\Upsilon t} \tag{A-51}$$

In the estimation, we choose the parameters to minimize the distance between model-implied impulse responses and their data counterparts, as in Altig, Christiano, Eichenbaum, and Linde (2011).