Credit Lines*

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Abstract

This paper develops a new theory of long term unsecured credit contracts that matches the data in a variety of dimensions. Credit lines are long term relations between lending firms and households that pre-specify a credit limit and interest rate in each period. Households can unilaterally default as they do in the U.S. Bankruptcy code, and switch credit lines. Lending firms are required not to demand repayment of current loans, but otherwise cannot commit not to increase interest rates or to change credit limits. The existence of search type frictions allow for the lack of commitment to matter. We characterize and solve and for equilibria, finding the resulting set of contracts as well as the distribution of households over interest rates, credit limits and wealth. A version of this model calibrated to the U.S. economy replicates the main properties of unsecured lending as well as the typical lending contracts in the U.S. We use the theory to study the new regulatory rules in the U.S. credit card market which require a stronger commitment from lending firms not to raise interest rates discretionally. This results in tighter limit but lower interest rates, reduced indebtedness and lower default. Typically, but not for all households, the new policy improves welfare.

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1 Introduction

Lending contracts in the U.S. do not live in a vacuum but are shaped by a set of regulations of which bankruptcy provisions and Regulation Z are the central ingredients. The regulations specify the conditions under which debt can be discharged and also the degree of commitment that lending institutions have to abide to. In this paper we take these regulations to heart together with the notion that contracting is costly both to borrowers and lenders[1], and we develop a theory of unsecured credit where borrowing contracts take the form of credit lines that specify credit limits and interest rates. As in the U.S., one of the terms of a contract may be changed by the lending firms and households can switch lenders. Our theory reproduces the main features of the U.S. unsecured lending environment, including the patterns of switching, bankruptcy, interest rates, credit limits and debt levels across households.

We use the theory that we develop to study the effects of the new regulatory rules in the U.S. credit card market. The Federal Reserve has recently approved fundamental changes in the industry with the aim of curbing so called 'deceptive practices' on the part of issuers.[2] Prominent among these changes is the banning of interest rate hikes on existing credit card balances.[3] The Fed and consumer groups welcome this shift as they expect it to benefit consumers by protecting them against surprise and abusive changes in conditions. The banking industry have expressed concerns that this will hurt consumers by reducing competition and access to credit. The unexplored question that this paper addresses is how will consumers be affected in the end by the restrictions on pricing introduced?

In order to address this question effectively, we first develop a suitable analytical framework capable of generating loan contracts between lending firms and households that extend over many periods under the current U.S. legal system and the limited commitment of the parties that it entails. With such a model in place we are in a position to assess the enacted policy change and its welfare implications. We characterize the main properties of the equilibrium of the model and we further specify it to map it to the U.S. economy. We then perform a policy change like the one currently under way where banks can no longer change the interest rates of credit lines and we study the implications of the change and its welfare properties.

We find that the new policy will result in a lower need of precautionary savings, with less

[1] Due to either search frictions or transactions costs.

[2] The Board voted the new regulation on December 18, 2008. These rules were amended and renamed to the Credit Card Accountability Responsibility and Disclosure Act of 2009 or the Credit CARD Act of 2009. President Obama signed the legislation on May 22, 2009, making it part of the Truth in Lending Act. These rules are in effect already (June 2010).

[3] Except under limited circumstances associated to lack of payments.

switching, more borrowing but less default. We also find that almost all households prefer the new policy making it a good idea overall.

The analytical framework that we develop uses the basic equilibrium model of consumer idiosyncratic risk and incomplete markets used in much quantitative macroeconomics (e.g., Bewley (1986), İmrohoroğlu (1989), Huggett (1993), Aiyagari (1994)). The model introduces three fundamental assumptions. First, borrowing is not restricted to have to be conducted by means of one period debt contracts only, thus allowing for lasting credit lines. Second, within a given credit line, a lender can change the debt limit at any time but may or may not deviate from the interest rate initially signed depending on the regulatory setting. A form of partial commitment intends to capture the shift in the U.S. consumer protection law. Third, indebted households are not committed to honour their debts and can exercise the option to declare bankruptcy in a way that encompasses the main provisions of Chapter 7 of the U.S. bankruptcy code.

In the benchmark version of the model, there is no commitment and lending firms can alter the terms of the contract – both the interest rate and the limit – at the beginning of any period. In this setting, a household holding one credit card knows its associated current limit and interest rate, but can only make a (rational) guess about how these terms will evolve in subsequent periods. Switching to a different credit card is possible but costly for the household. Any of the credit lines available for a consumer to switch to specifies the consumer's current class type and – assuming initial full information – the size of the initial loan. Subsequently, the household can borrow any amount up to the debt limit set by the bank, switch to another line or declare default.[4] Note that, given the switching cost, consumers may in general wish to hold on to the present credit card even if their personal circumstances change. Credit lines are issued by competitive financial intermediaries or lending firms, the second group of actors in the economy. A bank launching a line offers the initial terms of the contract. In any subsequent period, and as long as the line survives, the bank sets the credit limit and interest rate at will under incomplete information about the card holder's current earnings.

There is a large set of potential types of credit lines. The characteristics of the credit lines that are effectively traded must be determined in equilibrium by the requirements that the lending firms' rules for debt limit and interest rates are time-consistent and that there is no profitable entry of other types of contracts. This calls for studying the behavior of households and existing lending firms to deviations in the credit limit and interest rate. The stationary distribution of the model delivers profiles of consumption, wealth/debt, credit card limits and interest rates, default

[4]Notice that the properties of the contracts of our model agree with the typical credit card contracts. The only issue is exclusivity, but this is implicit in the existing arrangements. Lending firms know the asset position of its customers every few months and rearrange the terms of the contract accordingly.

and switching across households.

We study two quantitative version of this no-commitment benchmark economy. The parameters of the model are calibrated to match U.S. aggregate statistics for the distributions of wealth and earnings, the default and write-off rates on unsecured debt, and the credit limits and interest rates by earnings groups. Within this setting, lending firms do often raise interest rates after the initial period and credit limits respond to changes in the borrower's observable characteristics. High-earnings households enjoy disproportionately larger limits, borrow less frequently but, when they do, incur higher debts. Many households revolve credit repeatedly within a given contract, but about two thirds of borrowers are switching to a new contract, typically one with a looser limit but a higher interest rate.

In order to study the regulatory changes in the U.S., we also consider a version of the model where lending firms are not allowed to change the interest rate at their discretion. There is commitment to stick to the initially agreed price. In this case, compared to the no-commitment benchmark, there is even a larger set of potential types of contracts.In an equilibrium with free-entry, there must be no profitable entry of other types of contracts. In the quantitative setting, the adoption of such price commitment causes changes in the credit market. Generally, lending firms tighten debt limits but charge lower interest rates. This changes is sharper among high-earnings households. In terms of aggregates, borrowing decreases and wealth increases, the risk of default and the frequency of contract switching both decline, and average consumption rises.

There is an emerging literature analyzing bankruptcy and credit in quantitative general equilibrium models. In Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) there is no switching cost and credit conditions adjust instantaneously. So it considers one-period contracts, where the interest adjusts with the size of the loan, and there is no room for pre-approved credit limits. Livshits, MacGee, and Tertilt (2007b) and Livshits, MacGee, and Tertilt (2007a) and Athreya and Simpson (2006) share similar features. In contrast, the present paper has the realistic feature that, due to the cost of switching, households can keep the same credit conditions for loans of varying size and over changing personal circumstances. Mateos-Planas (2007) studies the determination of an endogenous credit limit that is binding for some households in a model where lending firms confront an adverse selection problem since the information about the characteristics of individuals is limited. It has one-period contracts with credit limits where the interest does not depend on the amount borrowed but, for tractability reasons, constraints the range of contracts that can arise. It cannot be used to study variability in these conditions. The present paper instead delivers long-lasting contracts and a non-trivial distribution of credit limits and interest rates. A new strand of the literature (Chatterjee, Corbae, Rios-Rull, and Dempsey (2018)) attempts to provide a theory of credit without exogenous punishment. In independent work, Drozd and Nosal (2007) have posed a model with credit lines and default in an environment with search type frictions. They also have multiperiod contracts, but full commitment and constant conditions within any given contract. Tam (2011) assumes commitment not to change prices and an exogenous contract length. Therefore none of these existing works accord with the essential features of the U.S. credit market nor can be used to address the changes in the level of commitment brought about by the recent reform. Notice that none of these existing models can be used to address changes that affect the level of commitment in the lenders' policies.

[[There is also an important literature on endogenous borrowing constraints and contracts, which includes Kehoe and Levine (1993), Wang (1995), Kocherlakota (1996), Cole and Kocherlakota (2001), and Hopenhayn and Werning (2008). The main differences with the present paper are ...]]

We interpret the legal environment as on where banks cannot commit to certain features of the contracts that they have with borrowers. This is not the same as interpreting that the current law allows the banks to choose a contract with or without commitment to future terms, but that with the previous legislation the bank could always ex-post change the terms of the contract even if they had promised not to do so. A rationale for this is that it can always be expressed as a function of a change in the circumstances of either the economy or the borrower, and that these are too complicated to be fully prespecified in a contract.

Section 2 discusses both the regulatory environment and properties of the existing contracts that we are relevant for our study. Section 3 puts forward the model for the households and the financial intermediaries and describes the decisions of these agents. Section 4 defines the equilibrium and discusses some issues related to its characterization and computation. Section 5 maps the theory (without commitment) to the data to obtain the benchmark quantitative model, and assesses its implications against evidence on debt, credit limits and interest rates across households. Section 6 describes the behavior of the economy with commitment. Section 7 explores and evaluates quantitatively the effects of the proposed policy change, both for observable variables and for welfare. Section 8 concludes.

2 Facts and institutions

We use the theory to study the effects of the new regulatory rules in the U.S. credit card market. The CARD Act 2009 culminates the Federal Reserve's earlier steps towards curbing so called 'deceptive practices' in the industry. The Board of Governors of the Federal Reserve System voted the new regulation on December 18, 2008. These rules were amended and renamed into the Credit Card Accountability Responsibility and Disclosure Act of 2009 or the Credit CARD Act of 2009.

President Obama signed the legislation on May 22, 2009, amending and extending the provisions in the Truth in Lending Act 1968. These rules are in effect already (June 2010). Prominent among these changes is the banning of interest rate increases above the agreed terms. Exception is only made for the cases of consumer's violation of the contract's agreement, indexed or scheduled changes, or a decreased rate being returned to a preexisting rate.[5] Therefore, the law now prevents in effect any-time any-reason repricing practices. The unexplored question is how consumers will be affected in the end by these restrictions on pricing.

Our analytical approach permits to address this question in a rigorous manner for the first time. It encompasses the fundamental characteristics of the credit card market and of the shift in the environment occurring as a consequence of the reform. Regarding general features, the model contains a number of realistic elements. First, like the open ended credit arrangement in the U.S. market, contracts in the model provide lines of revolving credit. That is, creditors cannot demand full repayment of the outstanding balance. We specifically assume that they can only demand the payment of interest.[6] Second, subject to that condition, banks can change the limit. The regulation remains muted over this, as its focus is on pricing. Third, households can decide to switch to a different contract or to declare bankruptcy.[7]

Regarding the essential elements of the regulatory setting, the benchmark model without commitment corresponds to the pre-reform environment in the U.S. Intermediaries in this model are free to reset the interest rate at any time, either at a higher or at a lower level. Before the reform, the environment in the U.S. was one were lenders would exercise effectively unconstrained discretion in changing the interest rate. The superseded Regulation Z, in place at the time, dealt primarily with information and transparency matters, with only weak provisions regarding changes in lending terms.[8] In effect, it is the perception of widespread recourse to interest rate hikes that spurred the Fed's most recent regulatory initiative.

A form of partial commitment intends to capture the recent regulatory shift in the U.S. The commitment version of the model is one where lenders face an additional restriction in that they are not allowed to increase the interest rate above the initially signed rate. Lenders can still reduce

[5]Section 171, part a, of the Act states the principle. Part b contemplates exceptions when interest rates may be increased. Section 101 establishes that such increases must be given an extended 45-day notice and cannot apply to existing balances. Therefore, only new balances can be repriced, and then only in these exceptional circumstances.

[6]An issue is whether the restriction is on debt or the amount lent. We will discuss this later in the context of the model.

[7]We assume exclusivity though.

[8]Regulation Z implemented the Truth in Lending Act (Chapter 1 of the Consumer Protection Act). It prescribes full disclosure of price terms. This does not rule out specified changes in interest rates on "existing" balances when circumstances change: end of teasing period; penalty rates (when late payments, exceeding limit, bouncing checks); tracking general index. (X: Lo puedo mirar con mas detenimiento por si acaso)

it and then increase it back. This an accurate representation of section 171 of the reform that bans interest rate rises, and contains the provision that banks may however return to it after a past reduction.

There is some discussion on whether this applies to new or existing balances. In the paper, the regulation prevents increases in both existing and new balances. As mentioned in the opening paragraph of this section, the actual regulation contemplates special circumstances where increases may happen but only affecting existing balances. These circumstances are abstracted from in the model so it is accurate that the ban on interest rates rises affects existing balances as well as new ones.

Also note that in the model any initial interest is permitted, and the restriction is only on changes on that initial rate. Likewise, the U.S. regulation does not include price controls or rate caps.

We use the theory to study the effects of the new regulatory rules in the U.S. credit card market. The CARD Act 2009 culminates the Federal Reserve earlier steps towards changes aimed at curbing so called 'deceptive practices' in the industry.[9] Prominent among these changes is the banning of interest rate increases above the agreed terms. Exception is made for the cases of consumer's violation of the contract's agreement, indexed or scheduled changes, or a decreased rate being returned to a preexisting rate.[10] Therefore, the law now prevents in effect any-time any-reason repricing practices.[11]

The Fed and consumer groups welcome this shift as they expect it to benefit consumers by protecting them against surprise and abusive changes in conditions. The banking industry have expressed concerns that this will hurt consumers by reducing competition and access to credit. The unexplored question that this paper addresses is how consumers will be affected in the end by the restrictions on pricing introduced.

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[10]Section 171, part a, of the Act states the principle. Part b contemplates exceptions when interests rate may increase. Section 101 establishes that such increases must be give an extended 45-day notice and cannot apply to existing balances. X: Esto es: Only new balances can be repriced, but then only in the exceptional cases.

[11]But it does not include price controls or rate caps.X: Creo que Tam se equivoca

3 The model

The environment poses uninsurable earnings risk à la Bewley-Imrohoroglu-Huggett-Aiyagari, and a U.S.-type household bankruptcy law for unsecured debts as in Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) and Livshits, MacGee, and Tertilt (2007b). In addition, both lenders and households incur costs of setting up unsecured credit contracts due to transaction or search frictions, and the lender must allow for debt to be rolled over. Thus these contracts may last for more than one period. The contracts provide credit lines, that is, they offer credit up to a certain limit at a certain interest rate.[12]

Neither lenders nor households can commit to future behavior beyond that defined by the legal system. Indebted households can either default and they suffer the cost of a punishment and the exclusion from future credit, or pay back and keep open the line of credit, or switch to another lender and enjoy better terms. Banks can change in each period the terms – interest rates and credit limits – that they offer to households in an attempt to maximize profits.

We look for Markov perfect equilibria that only have payoff relevant state variables. The characterization of equilibria requires not only a description of households' behavior along the equilibrium path (Section 3.1) but also the characterization of their behavior under temporary deviations in the behavior of the lenders (Section 3.2), since this will be a crucial ingredient for the decisions faced by the lenders (Section 3.3).[13]

3.1 Households

There are many infinitely-lived households with standard preferences over consumption streams and for whom certain actions will result in utility loses.

Endowments. Each period, households have a Markovian, stochastic endowment of labor earnings $e \in \{e^1, \cdots, e^{N_e}\} = E$ with transition matrix $\Gamma_{e,e'}$.

Assets. The household's asset position, $y \in \mathcal{Y}$, is publicly observed. Positive assets yield a risk free rate of return, that we describe by using q^*, the one-period-ahead, or discounted, price of the asset. In fact, to avoid confusion through this Section we refer to the discounted price of an asset

[12] X: (This footnote goes to Intro) This type of contract is different from the one in the unsecured-credit literature which offer one interest rate for each size of debt. We think our assumption is realistic and also more congruent with the existence of revolving debt. Menu-pricing would render ineffective the condition that the household must allowed to roll over outstanding debt: The lender could always offer a zero price for that level of debt while offering a positive price for some smaller level of debt. Moreover, as we will discuss later, the results will probably be robust.

[13] This lack of commitment introduces certain features in this environment which are similar to those in the optimal time-consistent policy literature (see Klein, Krusell, and Ríos-Rull (2008) for instance).

instead of its interest rate. If the household has an active contract, or if it acquires a new contract with a lending firm, they may take on a negative asset position with an associated discounted price and debt limit that is specified by the terms of the contract. For mathematical and computational simplicity the set $\mathcal{Y}$ is discrete.

Contracts. A new contract $\theta \in \Theta$ specifies the initial amount that the household borrows, $y_0'^{\theta} \leq 0$, and its discounted price q_0^{θ}. The contract also specifies whatever observable characteristics of the household at the time of signing have predictive power about its future behavior which in this model is just earnings e_0^{θ}.

After the initial signing period, the contract becomes a credit line. This is, it specifies a credit limit and a discounted price for every possible asset position and earnings of the borrower, which we denote $b^{\theta}(e, y)$ and $q^{\theta}(e, y)$. These two variables are the only ones publicly observable, and therefore, the only ones that can affect the terms of the contract. Earnings affect forecasts of future earnings and, together with the asset position, determines the incentives to default, switch, or pay back. The set Θ of available contracts is an equilibrium object to be determined below.

Credit history. Households have a credit history flag that we denote with $h \in \{0, 1\}$, indicating good and bad credit respectively. A household with $h = 0$ changes to $h' = 1$ next period upon defaulting, or more precisely, filing for bankruptcy. A household with bad credit cannot borrow, which we write as the imposition to hold the $\theta = 0$ contract. A household with $h = 1$ changes to $h' = 0$ with constant probability δ, a parameter that controls the expected length of the punishment.

Idiosyncratic utility costs. Filing for bankruptcy entails a utility cost χ^d. Engaging in contracting or recontracting (switching in our own jargon) also carries a utility cost, χ^s. Both utility costs are independent, identically distributed random variables with continuous support and distributions $F^s(\chi^s)$ and $F^d(\chi^d)$. Both utility costs are privately observed, with χ^s only observed after the household has decided not to default. The technical purpose of these random cost shocks is to impart continuity in the profit functions of the lenders.

States. To summarize, households are characterized by both public and privately observed variables. The publicly observed variables at the beginning of a period are its earnings, e, its asset position, y, its credit history h, and its credit line or contract held $\theta \in \Theta$, with $\theta = 0$ meaning no contract. The privately observed variable is the utility cost of defaulting, χ^d, known to the household at the beginning of the period.

Household's choices, constraints, and preferences. Households make three types of interrelated choices: whether to file for bankruptcy (or default) $d \in \{0, 1\}$ when the household is in debt; whether to sign a new contract (or to switch) $s \in \{0, 1\}$, and if so, which one $\theta' \in \Theta$, when the household has good credit record $h = 0$; and how much to save or borrow, y'. When the household defaults it can neither save (a bankruptcy code restriction) nor borrow. These assumptions imply that we can write the budget constraint in a compact, if cumbersome, manner as

$$c + \left\{ q^* 1_{y' \geq 0} + \left[s\ q_0^{\theta'}\ 1_{y'=y_0'^{\theta'}} + (1-s)\, q^{\theta}(e, y) \right] 1_{y'<0} \right\} y'(1-d) = (1-d)\ y + e, \qquad (1)$$

where the three terms that multiply the assets to be held the following period include the relevant discounted prices when the household is saving, switching, or borrowing via an existing credit line. The right hand side is cash in hand, assets plus income.

Households have standard preferences over streams of consumption, with per period utility function $u(c)$ and discount rate β. Consequently, we write the per period total utility taking into account the possible utility costs associated with either defaults or switches as

$$u(c) - \chi^s\ s - \chi^d\ d. \qquad (2)$$

The decision problem. It is convenient to model the three decisions of the household sequentially. First, the bankruptcy choice, followed by the switching choice and, finally, the asset position choice. As it is customary, the analysis proceeds backwards until we solve for the value function at the beginning of the period before the default cost becomes known, $v(y, e, \theta, h)$.

In the third stage, default and switching decisions have already been determined. The third stage involves various possible cases. If the household had defaulted, $d = 1$, its consumption is its earnings, $c = e$, and $h' = 1$, $\theta' = 0$, and $y' = 0$. Its corresponding value in this stage, v^1, is a function only of its earnings and the cost of defaulting, for which we write

$$v^1(e, \chi^d) = u(e) - \chi^d + \beta \sum_{e'} \ \Gamma_{e,e'}\ \ v(0, e', 0, 1). \qquad (3)$$

If the household had switched, $s = 1$ (which requires that it had not defaulted), it keeps the good credit, and borrows as specified by the new chosen contract. The corresponding value v^s is

$$v^s(y, e, \chi^s) = \max_{\theta' \in \{\Theta:\ e_0^{\theta} = e\}} \ u\left(y + e - q_0^{\theta'} y_0'^{\theta'}\right) - \chi^s + \beta \sum_{e'} \Gamma_{e,e'}\ \ v(y_0'^{\theta'}, e', \theta', 0), \qquad (4)$$

with the optimal choice of contract yielding the policy rule $\theta'^s(y, e)$. If the household neither defaulted nor switched, then it chooses how much to borrow subject to the debt limit and price that is specified in its current contract. The value v^n for a household with good credit is

$$v^n(y,e,\theta,0) = \max_{y' \geq b^\theta(e,y)} \; u\left(y + e - \left[q^*\, 1_{y'\geq 0} + q^\theta(e,y)\, 1_{y'<0}\right]\, y'\right) + \beta \sum_{e'} \Gamma_{e,e'} \;\; v\left(y', e', \theta, 0\right). \quad (5)$$

Finally, if the household had started the period with bad credit, its choice in the third stage accounts for the possible reinstatement of good credit, yielding

$$v(y,e,0,1) = \max_{y'\geq 0} \;\; u\left(y + e - q^*\, y'\right) + \beta \sum_{e'} \Gamma_{e,e'} \;\; \left[\delta\, v\left(y', e', 0, 0\right) + (1-\delta) v\left(y', e', 0, 1\right)\right]. \quad (6)$$

We write the solution for households with both good and bad credit as $y'^n(y,e,\theta,h)$.

In the second stage, having decided not to default, the household with good credit $h = 0$ compares the values of switching, $v^s(y,e,\chi^s)$, or not switching, $v^n(y,e,\theta,0)$. There is a critical value of the switching cost such that those with a lower value do switch, and those with a higher value do not. Switching costs are distributed according to F_s, so the probability of switching before drawing the switching cost is

$$p^s(y,e,\theta) = \int 1_{[v^s(y,e,\chi^s) > v^n(y,e,\theta,0)]} \;\; dF^s, \quad (7)$$

which yields the value of not defaulting

$$v^0(y,e,\theta) = [1 - p^s(y,e,\theta)]\; v^n(y,e,\theta,0) + \int 1_{[v^s(y,e,\chi^s)>v^n(y,e,\theta,0)]} v^s(y,e,\chi^s)\; dF^s. \quad (8)$$

In the first stage, only households with good credit can default. The household compares the value of not defaulting $v^0(y,e,\theta)$ and the value of defaulting $v^1(e,\chi^d)$. Given the distribution of the default costs, these actions yield a probability of defaulting as a function of the household's initial state

$$p^d(y,e,\theta) = \int 1_{[v^1(e,\chi^d) > v^0(y,e,\theta)]} \;\; dF^d. \quad (9)$$

The ex-ante value function of a household with good credit is

$$v(y, e, \theta, 0) = \left[1 - p^d(y, e, \theta)\right]\ v^0(y, e, \theta) + \int 1_{[v^1(,e,\chi^d) > v^0(y,e,\theta)]} v^1(e, \chi^d)\ dF^d. \quad (10)$$

The decision rules $\{y'^n, p^s, \theta'^s, p^d\}$ operate along the equilibrium path. We now turn to rules that apply outside of the equilibrium path.

3.2 Households out of the equilibrium path

The behavior of the household that we have described is not sufficient to characterize equilibrium. The reason is that the lending firms can change the terms at will due to the lack of commitment and, consequently, will evaluate their profits for all feasible credit limits and discount prices. Lenders have therefore to assess what households *would* do if the terms of the contract were different from those prescribed by the equilibrium rules (Section 3.2.1). On the other hand, because of free entry, we also have to assess what the response of households would be to prospective entry of contracts that are *not* in the equilibrium set of contracts (Section 3.2.2).

3.2.1 The impact of deviations in debt limits and prices of an existing contract

We consider the problem of households with a contract that offers arbitrary lending terms, credit limit b and discounted price q, in the current period and reverts later to the terms specified by the contract. We split again this problem into three stages. We use tildes to denote this situation. The analysis uses the value function developed in (6) and (10), v, that yields the continuation value after a deviation is over.

In the third stage, deviation terms b and q are relevant for a non–defaulter non–switcher, and yield a value to the household

$$\widetilde{v}^n(y, e, \theta, 0, b, q) = \max_{y' \geq b} u\left(y + e - [q^*\ 1_{y' \geq 0} + q\ 1_{y'<0}]\ y'\right) + \beta \sum_{e'} \Gamma_{e,e'}\ v(y', e', \theta, 0), \quad (11)$$

with solution $\widetilde{y}'^n(y, e, \theta, 0, b, q)$.[14] In the second stage, the decision whether to switch results in the probability of switching

$$\widetilde{p}^s(y, e, \theta, b, q) = \int 1_{[v^s(y,e,\chi^s) > \widetilde{v}^n(y,e,\theta,0,b,q)]}\ dF^s, \quad (12)$$

[14] Notice that the relevant argument is now cash in hand $y + e$, since the credit terms are not dependent on y separately.

and an associated value of not defaulting

$$\widetilde{v}^0(y, e, \theta, b, q) = [1 - \widetilde{p}^s(y, e, \theta, b, q)]\, \widetilde{v}^n(y, e, \theta, 0, b, q) + \int 1_{[v^s(y,e,\chi^s) > \widetilde{v}^n(y,e,\theta,0,b,q)]} v^s(y, e, \chi^s)\ dF^s. \tag{13}$$

In the first stage, default decisions result in the default probability

$$\widetilde{p}^d(y, e, \theta, b, q) = \int 1_{[v^1(e,\chi^d) > \widetilde{v}^0(y,e,\theta,b,q)]}\ dF^d. \tag{14}$$

We refer to the tilde decision rules $\{\widetilde{y}'^n, \widetilde{p}^s, \widetilde{p}^d\}$ as *within contract deviation decision rules*.

Characterizing the consumer behaviour given contracts in Θ amounts to solving functional equations (3- 10) for the problem of the household as well as functional equations (11- 14) for the household problem under deviations. Note that the behaviour of the household as implied by equations (3- 10) is the same as that in (11- 14) when we impose in the latter that the credit limit and discounted price satisfy $b = b^\theta(e, y)$ and $q = q^\theta(e, y)$.[15]

3.2.2 The impact of deviation in contracts

So far, the set of available contracts Θ has been taken as given. However, in order to determine this set, we need to understand the household's response to the one-off introduction of a contract not in this set, $\omega \notin \Theta$. The new value functions are denoted with the hat notation.

Consider a household holding contract $\omega \notin \Theta$ in a period where this contract is no longer available for other households to acquire. It has to account for the continuing use of contract ω with its associated discounted price and credit limit functions $\widehat{b}^\omega$ and $\widehat{q}^\omega$, as well as possible one-period deviations from those terms within the contract. The value of not switching is calculated similarly to (11) as $\widehat{v}^n(y + e, e, \omega, 0, b, q)$ yielding the policy function $\widehat{y}'^n(y + e, e, \omega, 0, b, q)$. One can now use expressions analogous to (13), (12) and (14) to calculate no-default values $\widehat{v}^0(y + e, e, \omega, b, q)$, switching rules $\widehat{p}^s(y + e, e, \omega, b, q)$ and default rule $\widehat{p}^d(y + e, e, \omega, b, q)$. The continuation value in the new contract $\widehat{v}(y + e, e, \omega, h)$ is found by evaluating an expression similar to (10), having used the bank policy rules $\widehat{b}^\omega(y, e)$ and $\widehat{q}^\omega(y, e)$ as the arguments b and q in $\widehat{p}^d(y + e, e, \omega, b, q)$.

Consider now a household holding a contract on the existing available set $\theta \in \Theta$ in the period when it is possible to sign the deviation contract ω. The availability of ω may have an effect on

[15] The analysis could have been done entirely in terms of the tilde functions, but we have kept the distinction between equilibrium and out-of-equilibrium rules for ease of exposition.

the choice of the optimal new contract and the value of switching which we can now express as

$$\widehat{v}^{s,\omega}(y,e,\chi^s) = \max\left\{ \max_{\theta'\in\{\Theta:e=e_0^{\theta'}\}} u\left(y+e-q^{\theta'}y'^{\theta'}\right) + \beta\sum_{e'}\Gamma_{e,e'}\, v\left(y'^{\theta'},e',\theta',0\right), \right.$$
$$\left. u\left(y+e-q^{\omega}y'^{\omega}\right) + \beta\sum_{e'}\Gamma_{e,e'}\,\widehat{v}\left(y'^{\omega},e',\omega\right) \right\} - \chi^s \quad (15)$$

which yields a new rule for the preferred contract $\widehat{\theta}'^{s,\omega}(y,e)$ and where $\widehat{v}$ is the continuation value within the deviation contract introduced above. Note that the $\widetilde{v}^n$ for existing $\theta\in\Theta$ and v^1 in (11) and (3) remain unaffected. Therefore one can now use expressions analogous to (12), (13), and (14) to calculate no-default values $\widehat{v}^{0,\omega}(y,e,\theta,b,q)$, switching probabilities $\widehat{p}^{s,\omega}(y,e,\theta,b,q)$ and default probabilities $\widehat{p}^{d,\omega}(y,e,\theta,b,q)$ for an existing contract θ in response to a plausible entry of contract $\omega\notin\Theta$. Determining whether this household would leave θ to take up ω requires evaluating these decision rules at the terms offered by the current contract.

3.3 Lenders

Financial intermediaries, or lenders, issue contracts to households at a fixed initial cost π and with the condition that the borrower is allowed to roll over outstanding balances. There is also a variable intermediation cost that determines a risk free spread λ. We start by describing the value of a given contract for an arbitrary level of lending and debt price. Then we turn to how, in the absence of commitment, the current value of the bank depends on its choices given their impact on default, survival and the amount lent. We then study lenders' behavior when there is entry of a deviation contract not in the equilibrium set.

3.3.1 The value of a contract

Denote by $\Psi(y',e,\theta,q)$ the value for a lender of holding a contract $\theta\in\Theta$ with a household with earnings e and lending debt $-y'<0$ at discounted price q. It has two components. The first is the current cash-flow from the current loan. Using the household's probability of defaulting p^d in (9), it reads

$$m(y',e,\theta,q) \equiv \left[q^*\sum_{e',e'}\Gamma_{e,e'}\left[1-p^d(y',e',\theta)-\lambda\right]-q\right](-y')1_{y'<0}.$$

This cash flow depends on the profit margin - i.e., interest spread and the expected default risk - and loan size $(-y')$. The second part is the value on the possible future loans conditional on the contract surviving in the sense that the borrower does not switch to another line or default. It thus depends on predicted future patterns for borrowing as well as default and switching. These predictions are based on the decision rule y'^n and on the probabilities of defaulting and switching p^d and p^s, which embody the the rules that will govern the future terms of credit, b^θ and q^θ. The present value is the sum of the two components, which can be expressed recursively as follows:

$$\Psi(y', e, \theta, q) = m(y', e, \theta, q) + q^* \sum_{e'} \Gamma_{e,e'} \left[1 - p^d(y', e', \theta)\right] \left[1 - p^s(y', e', \theta)\right] \Psi\left[y'^n(e', y', \theta, 0), e', \theta, q^\theta(e', y')\right]. \quad (16)$$

Clearly the initial value of a new contract θ is given by $\Psi(y_0'^\theta, e_0^\theta, \theta, q_0^\theta)$. If there were commitment to the rules for the limits and prices, b^θ and q^θ, for the duration of the contract, the only decision by the bank would be whether to offer a particular type of credit line θ and this initial present value would be all the information needed.

3.3.2 The continuation problem of the lender

Because there is no commitment, after the first period of a contract $\theta \in \Theta$, the bank has the option to change the credit limit and price in any period. The lender observes the household's earnings e and current financial position y, and evaluates the implications of various values for the limit b and price q in that particular period using the household's within-contract deviation (or tilde) decision rules $\{\widetilde{y}', \widetilde{p}^s, \widetilde{p}^d, \widetilde{\theta}'^s\}$ developed in Section 3.2.1. The bank will be assuming that, in the future, contracts will adhere to the equilibrium rules b^θ and q^θ. The value of the existing contract θ dealing with a household with y and e, for a limit b and price q is:

$$W(e, y, \theta, b, q) = [1 - \widetilde{p}^d(y, e, \theta, b, q)]\,[1 - \widetilde{p}^s(y, e, \theta, b, q)] \Psi[\widetilde{y}'^n(y, e, \theta, 0, b, q), e, \theta, q] - y\,[1 - \widetilde{p}^d(y, e, \theta, b, q)]. \quad (17)$$

By changing b and q, the lender can affect the value of the current contract through the repayment of outstanding debt, the survival of the contract, and the level of lending.

There are various exogenous restrictions imposed on the behavior of lenders. The first is that the bank must allow debt to be rolled over or

$$b \leq y. \quad (18)$$

As for prices, we consider two regimes. The benchmark one is the case without commitment where q can be set freely. The other case, representative of the recent regulatory change, prevents the bank from reducing the discounted price above the initial one or

$$q \geq q_0^\theta. \tag{19}$$

The bank faces several trade-offs in the problem of maximizing W. Choosing terms that are too stringent (low limit, low discounted price) may induce some borrowers to default, other households may choose to switch to better terms, while others may stay paying more interest but perhaps on smaller loans. Choosing terms that are too loose may reduce current default but attract more borrowing which may prove costly in the future when households have an opportunity to default on them. It is a careful balancing of all these margins that determines the choice of banks.

In the absence of commitment, each period the lender, or rather branch manager, maximizes $W(e, y, \theta, b, q)$ in (17) by choosing b and q subject to (18). Under the new regulation the branch manager has the added constraint (19) when choosing credit limits and discounted prices. A solution to either of these problems is a pair of functions of (e, y, θ). Clearly, a time consistent contract θ is one where this solution coincides with the rules of the contract or,

$$\{b^\theta(e, y), q^\theta(e, y)\} = \arg\max_{b,q} W(y, e, \theta, b, q) \tag{20}$$

subject to (18) and, if pertinent, (19). Only time consistent contracts can be equilibrium contracts so we restrict our attention to them.

3.3.3 Lenders and deviation contracts

The intermediaries described above take as given the households' decision rules associated with the given set of available contracts Θ. However, with free entry in banking, banks must account for the consequences of potential new entrants on their customers' behavior. Consider now a contract ω not in Θ that becomes temporarily available. The response of a household holding a contract $\theta \in \Theta$ is described by the policy rules for default $\widehat{p}^{d,\omega}(y, e, \theta, b, q)$ and switching $\widehat{p}^{s,\omega}(y, e, \theta, b, q)$ discussed above in 3.2.2, and the unchanged rule for the asset-position $\widetilde{y}'^n$.

Therefore, the one-off introduction of ω triggers a response of the existing contract θ given by a limit b and price q that maximize the function

$$\begin{aligned}&[1 - \widehat{p}^{d,\omega}(y, e, \theta, b, q)]\,[1 - \widehat{p}^{s,\omega}(+y, e, \theta, b, q)] \\ &\qquad \Psi[\widetilde{y}'^n(y, e, \theta, 0, b, q), e, \theta, q] - y\,(1 - \widehat{p}^{d,\omega}(y, e, \theta, b, q)),\end{aligned} \tag{21}$$

which yields the existing lender's reaction $\widehat{b}^{\theta,\omega}(y, e)$ and $\widehat{q}^{\theta,\omega}(y, e)$.

For the deviation contract $\omega \notin \Theta$, if the contract becomes active, its value depends on the household policy rules within this line, $\widehat{p}^d(y, e, \omega, b, q)$, $\widehat{y}'^n(y, e, \omega, 0, b, q)$, and $\widehat{p}^s(y, e, \omega, b, s)$ discussed above in section 3.2.2. For a bank holding a deviation contract ω and lending $-y'$ to a household of type e, the present value can be expressed recursively as $\widehat{\Psi}(y', e, \omega, q)$ in a way similar to (16). This new entrant will also have the opportunity to adjust its debt limit and price at its discretion. The value of the lender with contract ω for an arbitrary limit b and price q is:

$$[1 - \widehat{p}^d(y, e, \omega, b, q)]\, [1 - \widehat{p}^s(y, e, \omega, b, q)] \\ \widehat{\Psi}[\widehat{y}'^n(y, e, \omega, 0, b, q), e, \omega, q] + (-y)\, [1 - \widehat{p}^d(y, e, \omega, b, q)]. \quad (22)$$

Maximization yields the new deviation lender's policies $\widehat{b}^\omega(y, e)$ and $\widehat{q}^\omega(y, e)$.

4 Equilibrium: definition, characterization and computation

In this section we define an equilibrium, highlight some generic properties, and discuss computational issues. We will end with a description of the implications that are consistently found for the behavior of households and lenders.

4.1 Definition

The definition of equilibrium in this economy is more involved than in standard environments. In addition to the conditions of households optimizing and free entry in banking, we have to characterize time-consistent lenders' decisions and the set of equilibrium contracts Θ that emerges in equilibrium. It is now when the work in Section 3 pays off.

Definition 1. *An equilibrium consists of a set Θ of available contracts – including the initial price q_0^θ and value $y_0'^\theta$, and continuation rules $q^\theta(e, y)$ and $b^\theta(e, y)$ for each contract θ – households' equilibrium decision rules $y'^n(y, e, \theta, h)$, $p^d(y, e, \theta)$, $p^s(y, e, \theta)$, and $\theta'^s(y, e)$, and households' within-contract deviation rules $\widetilde{y}'^n(y, e, \theta, 0, b, q)$, $\widetilde{p}^d(y, e, \theta, b, q)$, $\widetilde{p}^s(y, e, \theta, b, q)$ such that:*

1. *Household's optimization. Given the available contracts $\theta \in \Theta$, the households' equilibrium decision rules correspond to the solution of* (3) - (10), *and the within-contract deviation rules solve (11-14)*

2. *Time consistency of bank's policies. Given the available contracts Θ and the household's decision rules, the debt-limit and price rules in a contract θ, q^θ and b^θ, are the solution to* (20) *subject to* (18) *and, if applicable because of commitment,* (19).

3. *Free entry. Given the household's equilibrium decision rules, for a contract θ in Θ the initial price q_0^θ is such that the bank's value in* (16) *satisfies the zero profit condition initially*

$$\Psi(y_0'^\theta, e_0^\theta, \theta, q_0^\theta) - \pi = 0, \qquad \forall \theta \in \Theta. \tag{23}$$

4. *Unprofitable entry of other contracts. Consider a deviation credit line $\omega \notin \Theta$. Consider also the household's decision rules within this deviation contract $\widehat{p}^d(y, e, \omega, b, q)$, $\widehat{y}'(y, e, \omega, 0, b, , q)$, and $\widehat{p}^s(y, e, \omega, b, q)$, as well as the household's decision rules for existing contracts $\theta \in \Theta$ when ω becomes available $\widehat{p}^{d,\omega}(y, e, \theta, b, q)$, $\widehat{p}^{s,\omega}(y, e, \theta, b, q)$, and and $\widehat{\theta}^{\prime s,\omega}(y, e)$. Consider also the policies of the lender in the deviation contract $\widehat{b}^\omega(y, e)$ and $\widehat{q}^\omega(y, e)$, and the polices of existing contracts $\theta \in \Theta$ when ω becomes available $\widehat{b}^{\theta,\omega}(y, e)$ and $\widehat{q}^{\theta,\omega}(y, e)$. The following conditions cannot be all satisfied:*

   (a) *Optimal household reaction: Given Θ and the value functions solving* (3) - (10) *and (11-14), and the deviation lender's policies $\widehat{b}^\omega$ and $\widehat{q}^\omega$, the household's decision rules within the deviation contract are determined as in section 3.2.2.*

   (b) *Profitable entry: Given the household's decision rules within the deviation contract, the deviation contract value discussed in section 3.3.3 is such that $\widehat{\Psi}(y_0'^\omega, e_0^\omega, \omega, q_0^\omega) - \pi \geq 0$.*

   (c) *Entrant's time consistency. Given the household's decision rules within the deviation contract, the deviation contract rules $\widehat{b}^\omega$ and $\widehat{q}^\omega$ maximize the value in* (22) *from section 3.3.3 subject to* (18) *and, if applicable,* (19)*.*

   (d) *Some households choose it. Given the household's decision rules on existing contracts θ when the deviation contract ω becomes available, it is the case that*

   $$\widehat{\theta}^{\prime s,\omega}(y, e) = \omega \ \text{ and } \ \widehat{p}^{s,\omega}(y, e, \theta, \widehat{b}^{\theta,\omega}(y, e), \widehat{q}^{\theta,\omega}(y, e)) > 0$$

   for some (y, e, θ), with $\theta \in \Theta$ and given the existing contracts' reaction $\widehat{b}^{\theta,\omega}(y, e)$ and $\widehat{q}^{\theta,\omega}(y, e)$ that maximize (21).

Some remarks about this definition are in order.

Remark 1. *This definition allows for contracts that are available but not used. So the equilibrium set Θ can be quite large. The definition allows for the existence of a proper subset of Θ that is also an equilibrium without any change in the allocation. We could have added the requirement that set Θ should not be a subset of any other equilibrium set. We do not think that this additional requirement adds anything of substance.*

Remark 2. *The asset set in this a definition is implicitly taken to be a compact set (see Section 3.1). This specification is not a restrictive one. In economies of this type $\beta < q^*$ guarantees the existence* *of an upper bound to asset holdings. So the upper bound is not a problem. The lower bound of assets is not a problem either. There is a maximum amount of debt that can be paid for with positive probability. This means that more debt than that is only consistent with $q = 0$. So we could use such a bound. Typically, though it is sufficient to pose a level of debt for which the rate of return is high but not infinity and that prevents households from borrowing.*

Remark 3. *Note that we have not imposed any condition on the distribution of agents. This is because the environment is essentially a small open economy (or storage economy) and the risk free rate is given. To completely characterize the economy, we need as initial condition a measure of households over states, that we denote x, and a Markovian process on the states to construct an updating operator for the distribution of agents. Moreover, in this economy standard conditions about the persistence properties for earnings, Γ, guarantee that there is a unique stationary distribution x^* and that such distribution is the limit of any initial one. These are all standard operations for our environment and we spare the reader the details. See Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007), for example, for details.*

4.2 Characterization of equilibrium

Our equilibrium concept involves decision rules and value functions that are stationary. However, in order to study existence, it is convenient to consider an extended version of the economy with decision rules, value functions and set of contracts that can be time-dependent. For that economy, existence of an equilibrium can be established using a sequential argument as follows. Consider a version of the economy where, after finite period T, all continuation value functions, decision rules, for both households and lenders, and the set of contracts are arbitrarily fixed, in a not necessarily stationary manner. Solving all problems backwards, the equilibrium conditions are satisfied up to period T from standard arguments.[16] By induction, the set of objects that are equilibria for T contains those that are equilibria for $T + 1$. This establishes existence for the limit of the sequence of finite economies.

Proposition 1. *An equilibrium exists for the economy with time-dependent rules.*

In all cases we have studied numerically we have found a stationary equilibrium (i.e. Markovian) which, given the discreteness of the set of possible asset holdings, amounts to a proof for an open set of economies.

Equilibrium conditions do not constraint much the qualitative patterns of the discount price of loans on new contracts. Although the increased probability of default of loans with larger promised

[16] See Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) for such arguments when loan sizes are discrete.

repayments will tend to reduce their price, the relation between the size and price of debt may be non monotonic. In fact, for some range of loan sizes, the price may be increasing in loan size because the fixed positive cost of issuing a contract ($\pi > 0$) has to be recovered. Regarding the relation to the risk-free price, even contracts that generate some positive continuation value can clearly issue an initial discounted price q_0^ω below the risk-free price q^*, but only if $\pi > 0$ and greater than future surpluses. In general, however, contracts can also feature a higher initial price (i.e., a signing bonus).[17] To summarize, the discounted price of a new contract may be above or below the risk-free price and need not be monotonic in the level of debt.

Turning to the terms of existing old contracts, in the benchmark model without commitment the continuation terms of a contract credit limits and an interest rates that maximize (17) but which are not constrained in any way by the history of the contract. Only the effective amount of debt outstanding and the earnings of the borrower matter. We have the following implications.

Proposition 2. *In the economy without commitment, the banks' continuation decision rules are identical across contracts. As a consequence, the households' decision whether to switch is only based on current borrowing conditions, and not on which contract is being used. Moreover, each pair of initial debt and income type will be associated with only one type of new contract.*

These properties will not hold in the economy with commitment where the lender cannot reduce the price below the initial one and, therefore, the initial terms of the contract will matter in the future.

Existing contracts are different from new contracts for various reasons. Consider a household with a large debt that receives a low income realization. New contracts will offer draconian terms to this household since it wants to borrow a large amount. On the other hand, for the bank in the existing contract this situation may cause a current loss. The bank holding the existing contracts is restricted by the rollover constraint to keep lending but, in addition and more importantly, the bank may wish to offer terms that keep the household from defaulting in the hope that future good realizations will compensate the current loses. In other situations, the bank can make larger profits from the existing contract by charging the household a high interest with low expected default. New contracts might therefore attract these households but the existence of frictions in the environment creates a wedge that the bank with the current contract exploits by offering harsher terms. Summarizing we have

[17] To see these assertions, it suffices to look at situations where there is no immediate default or switching. From (23) with (16), the initial price q_0^ω of a contract equals the risk-free price, plus the continuation value minus the setting up cost as proportions of the debt size. Using self explanatory short-hand notation, $q_0^\theta = q^* + [q^* E\Psi' - \pi]/(-y_0'^\theta)$.

Proposition 3. *The terms offered by a bank to its existing customer will generically differ from those offered by a new contract. There may be tougher terms as lenders holding existing contracts exploit the contracting frictions, but also softer terms for highly indebted households because of the lender's attempt to limit the risk of default loses.*

Recall that switching costs χ^s have continuous support and are privately observed.[18] Existing banks will seek a solution to the trade-off between losing customers and lowering profits per continuing customer. The lender, not knowing ex-ante the household's cost, may find it optimal to trade some probability of switching for a higher value of the contract. For that number to be strictly positive, the best choice for the bank has to yield a continuation value of profits larger than π. Otherwise households will have no incentive to find a different new contract with a higher value of profits π, and will either stick with the current contract or default. We thus have

Remark 4. *Switching occurs only in situations where current lenders issue loans with a value in excess of the setting up cost* π*.*

The equilibrium of this economy even with zero transaction and switching costs is not the same than in Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) or Livshits, MacGee, and Tertilt (2007c) where only one-period contracts can be traded. There are two reasons for this: that in this paper the lender can change the odds of the household defaulting by extending further credit, which was assumed away in previous literature, and that the rollover constraint effectively links the actions of lenders across different periods imposing on them certain obligations towards the future. Summing up, we have

Remark 5. *Even if transaction and switching costs are zero, this environment is not equivalent to one with one-period contracts.*

4.3 Computation

We discretize the set of possible asset holdings, which has two convenient implications: that the household solves a discrete choice problem, which is particularly advantageous given that the original problem was not convex (because of the default choice) and that the equilibrium objects, being functions of assets, become vectors. Obviously, prices themselves are not discretized. The utility costs shocks play a role by helping make profit functions smooth functions of prices.

[18] If the switching cost was a fixed known value, switching would be an all-or-nothing outcome. If, in addition, there was no rollover constraint, one can argue that the lender would always choose to keep the customer in. Our assumption of a switching cost that is random and private information would therefore be necessary in that case for positive switching to emerge at all. When we add the rollover constraint, however, this result cannot be established as easily.

Proposition 2 simplifies things considerably for the economy without commitment as it states that for each pair of initial debt and income type there is a unique contract. Let us denote the set of such pairs $\bar{\Theta}$, with each of its generic elements associated with a contract θ. For each θ, we need to compute the initial q_0^θ, and continuation discounted price functions $q^\theta(e, y)$ and credit limits $b^\theta(e, y)$. We look for outcomes that guarantee that households and lenders maximize and that yield zero profits. We call such an object a $\bar{\Theta}$-equilibrium. Note that not all contracts in $\bar{\Theta}$ will be taken up by households, but if they are they yield zero profits. Proposition 2 tells us that once we find a $\bar{\Theta}$-equilibrium we are done for the economy without commitment, since there is no need to study point 4 of the equilibrium definition.

Finding a $\bar{\Theta}$-equilibrium is a large but manageable problem (if there are N_y asset positions and N_e earnings, this amounts to $N_e^2 \times N_y^2 + N_e \times N_y$ discounted prices and $N_e^2 \times N_y^2$ credit limits) which seeks a fixed point that we have solved by backward iterations. Section A in the Appendix describes steps of our computational procedures to find a $\bar{\Theta}$-equilibrium. In all cases we have found a $\bar{\Theta}$-equilibrium without problem.

For the economy with commitment, we proceed by a guess-and-verify strategy. We start by looking for a suitable candidate for $\bar{\Theta}$ which, like the one in the economy without commitment, only has one contract per each initial earnings class and loan size (e, y'). Once we find it, we are not done yet as we do not have a counterpart to Proposition 2 for this economy. The question is whether there could be a one-shot entry of a different contract ω with different conditions for the same initial e and y' as some contract in the $\bar{\Theta}$-equilibrium. Specifically, we need to see whether ω would make non-negative profits *and* attract any borrowers, as in point 4 of the equilibrium definition.

If ω had the same initial price as θ, it would have to specify different continuation policies. Such a possibility would include something like, for instance, a contract with a tighter credit limit but a higher discounted price. The non-existence of such a contract is trivial to verify. For each type of customer, there is (generically) a unique optimal discounted price per credit limit and, given this, there will be a unique credit limit that maximizes profits. In sum, for a given initial price, the optimal continuation policies are unique. Therefore, we can focus on deviation contracts that offer a different initial price or $q_0^\omega \neq q_0^\theta$.

Because of commitment, the initial price influences the continuation policies of the contract. Specifically, a higher initial price for ω restricts more the future actions of the bank but also makes households more attached to the contract. We then have to verify whether for different discount prices, the different continuation time-consistent credit policies in ω yield non-negative profits and positive consumer take-up. We verify these conditions numerically by construction of a fine grid of alternative discounted prices for each contract of the $\bar{\Theta}$-equilibrium. In all cases we found no viable deviation away from the $\bar{\Theta}$-equilibrium.[19] We always found that contracts with a higher initial price generate negative value for the lender; those with a lower initial price fail to attract any borrower.

5 The benchmark model economy

We now turn to a particular specification of the model without commitment in order to capture the U.S. economy before the recent credit card reform. We specify parameters for preferences and technology so that the economy is representative of the 80% non-retired wealth-poorest U.S. households. The reason for this is that we want to concentrate on the population who uses non secured credit. The concern for the richest households adds severe computational costs without adding anything of substance as the set of possible asset holdings become very large.

Also due to the difficulty of computing equilibria we use a very stark set of possible earnings. To ensure that our findings are not dependent on the particular choices that we made we have computed many economies that differ in subtle details. We choose to report two of them.

Some parameters are set directly. We pose a yearly risk free interest rate of 3 per cent and a standard risk aversion of 2. Bankruptcy leads to an average exclusion period of 2 years. T

Whether parameters are set in order to get a good approximation to empirical observations. We seek to match aggregate measures of unsecured debt and wealth in the Survey of Consumer Finances 2007. We focus on non-retired households belonging to the bottom 80 per cent networth quantile. Consistently with the model's focus on unsecured debt, negative net worth is use to measure debt and identify indebtedness in the data. We target an average ratio of net worth to income of about 1.98, a proportion of households with unsecured debt of 12 per cent, and a debt to income ratio of 0.048. As for default, we target two thirds of one percent of households defaulting. In the SCF data, the corresponding number for the chosen sample is about 1.4 per cent but we

[19] Although the direct effects work against the feasibility of deviations, there are other subtler effects. Suppose a deviation contract starting with a higher price, that is $q_0^\omega > q_0^\theta$. In the initial period, this deviation contract ω obviously makes lower profits than the existing θ; in subsequent periods, in ω optimal price policies will be more restricted. These effects tend to make negative ex-ante expected profits for deviation contracts. However, because of the commitment, the contract is more attractive to existing customers and the bank could extract some extra rent. Consider now a deviation contract starting with a lower price, that is $q_0^\omega < q_0^\theta$. The initial terms of ω are less favorable for the borrower than those of θ. Therefore the deviation contract ω can be taken up by borrowers only if its continuation policies offer more attractive terms than those of θ. The price choices for the bank under the deviation ω are less restricted than under the existing θ. The direct effect would be to make the household worse off. However, the extra freedom to lower the discounted price might make the bank willing to offer a looser credit limit which could benefit the household. The viability of deviations will also be limited by the off-equilibrium reaction of existing banks' policies to entry. In practice, it is therefore sufficient to rule out deviations given the equilibrium policies in existing contracts.

are only interested in those that default due to voluntary debts which reduces this number quite substantially.[20] (X: Argue further?) We also aim at a number of contract switchers equivalent to about 1.5 per cent of households. The 2006-2009 data on credit card usage from the Consumer Finance Monthly supports this choice.[21] Regarding interest rates, the average credit card rate in the SCF is about 13.74 per cent.

For the process of individual earnings, we consider only two states. We choose set the conditional density so that there is public information.[22] Based on the synthetic income variable in the survey, we target a variance of the log of income of about 0.76.

Besides these targets, we also also seek to avoid unreasonably extreme values for realized interest rates. A low persistence of the bad income shock and a low switching cost matters to avoid extremely high interest rates charged on new and existing contracts. Lowering the switching cost also helps avoid unreasonably large initial discounts on small loans.

The actual parameters that define this baseline economy are in Table 1 while the list of targets and model outcomes are in Table **??**. Note that we have to compromise on the wealth target but are able to get a very acceptable approximation to the remaining targets. (X: Can we think of a better story to tell?) Given the parsimonious parameterization of the model, this is quite remarkable.

[20]See Chatterjee, Corbae, Nakajima, and Ríos-Rull (2007) for an extended discussion of this issue.

[21]The CFM is an ongoing national survey carried out by the Center of Human Resources Research, Ohio State University. As reported in Abdelrahman (2011), the sample contains 12,926 observations, with 186 consisting of switchers.

[22]We will present results for a version with private information later in the Appendix. (X: But we have not done it yet!)

Table 1: **Parameter Values and Targets in the Model Economies**

Description	Parameter	Value
Risk aversion	σ	2.
Risk-free interest	$\frac{1}{q_0} - 1$	1.03
Prob. clear history	δ	0.5
Discount rate	β	0.92
Low Endowment		1/6
Fixed intermed cost	π	0.01
Default cost: Mean	χ^d	1.5
Var	ξ_d	1.2
Switching cost: Mean	χ^s	0.3
Var	ξ_s	0.075

		Economy		
		iid Ec	Pers Ec	V Pers Ec
		$\Gamma_{1,1} = .5$	$\Gamma_{1,1} = .6$	$\Gamma_{1,1} = .5$
Variable	Target	$\Gamma_{2,2} = .5$	$\Gamma_{2,2} = .6$	$\Gamma_{2,2} = .9$
Default rate %	0.66	0.70	0.69	
Switching rate %	1.5	1.4	1.2	
Percentage in debt	0.12	0.10	0.08	
Debt to income	0.048	0.47	0.039	
Wealth to income	2.0	1.45	1.92	
Aver. interest rate %	14.	15.	17.	
var log inc	0.8	0.8	0.8	

The main properties of this economy for the distribution of credit, average debt prices, and size of debt per loan are in Table 2. All of the borrowers have low income; those with high income will never borrow regardless of heir current liabilities. The 4 per cent interest rate on new contracts is about 13 per percentage points lower than the 17 per cent on existing contracts, pointing to the power that banks command over households due to the switching costs. We see also that on average loan size increases along the life of a contract. A substantial 83 per cent of the households have a credit contract open, an indication of the recurrence of the bad state.

Table 2: **Main Credit Statistics** (not normalized)

Mass distribution:	
Mass with a contract	0.868
Mass no contract but clean	0.118
Mass borrowing	0.0974
Mass borrowing new contracts	0.0144
Mass borrowing continuing contracts	0.0829
Loan size:	
All contracts	0.264
New contracts	0.202
Continuing contracts	0.275
Loan price:	
All contracts	0.870
New contracts	0.944
Continuing contracts	0.857

6 Behavior without commitment

We describe the properties of the model based on the above baseline numerical setting. Nonetheless they will be qualitatively valid for the wide variety of examples that we have studied. In all these economies there are households with good and bad income realizations, with good and bad credit histories, holding or not a contract, and with different levels of assets or debts.

Households with bad credit have defaulted in the recent past and lack currently the option to borrow. Their behavior is governed by the standard self-insurance motive. Households with good credit have the option to borrow which lowers the value of accumulating a buffer stock of assets for self insurance. Regardless of their credit status, households finds themselves in positive asset positions most of the time. Figure 1 displays the wealth distribution for the population and for the subgroup without a credit contract.

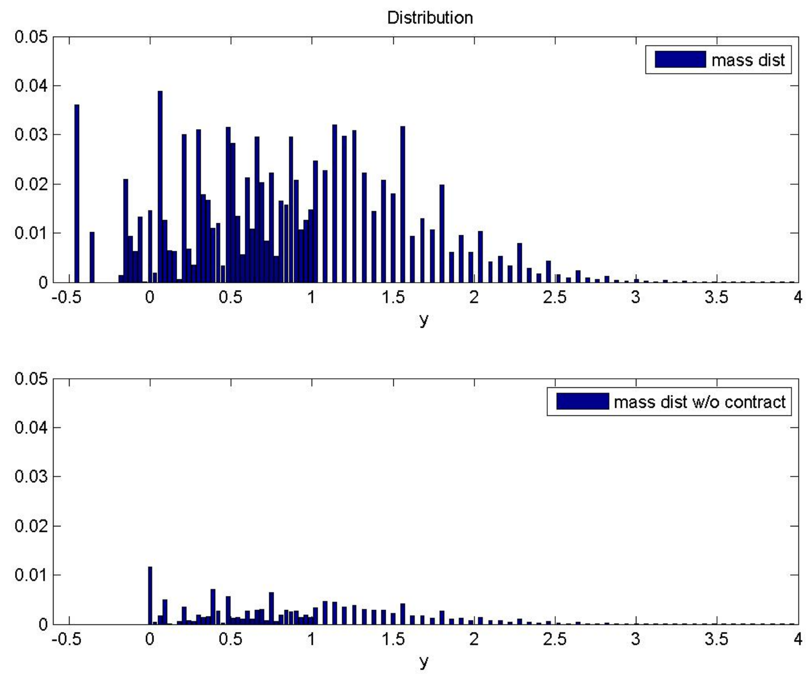

Figure 1: Wealth distribution

When facing a bad earnings state, households with good credit reduce their asset position and may decide to borrow. This happens in different ways depending on whether the household already holds a contract or not.

A households without a contract needs to acquire one. Competitive lenders supplying new contracts may offer attractive initial discounted prices. Since the bank will exploit to its advantage the switching costs of the households in the future, by the zero profit condition, a typical new contract involves a form of signing bonus or teaser rate. Then banks suffer immediate loses on new contracts which, given time, they expect to recoup.[23] The top part of Figure 2 displays the price of debt in new contracts (i.e., ** in the model's notation), featuring low interest rates for initial debts of moderate size. The profile of expected initial profit for the lender in the bottom part shows a short run loss for moderate debts below 0.25. Most contracts, above 90 per cent (X: Need to check exact number!), belong in this range.

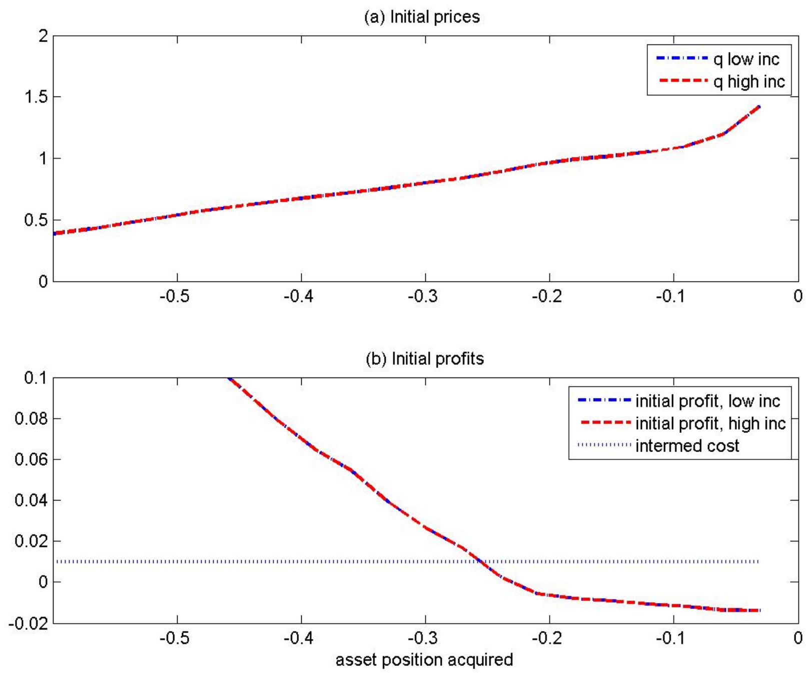

Figure 2: (a) Initial prices. (b) lender's profits on initial loan.

If instead the household already has a contract, she can choose to switch over to a new one in the terms just described, or she can continue using the one contract currently held. In the latter case, the incumbent lender may be able to raise the interest rate and extract some of the surplus because the household would have to incur costs if it were to switch. Figure 3 (a) shows the interest rate on a continuing contract (i.e., ** in notation) as a function of the low-income household's current asset position; section (b) displays the corresponding debt limit and the optimal decision rule for the asset position next period. Part (a) also contains the interest rate if the household were to switch and borrow the same amount of debt on the new contract. We can see that continuing contracts have a higher interest for positions below a certain level of debt. In this sense, this economy features interest rate hikes.

When banks ponder which terms to apply to existing customers they are conditioned by the

[23]That the lines for low and high income coincide is due to the iid assumption for the income process.

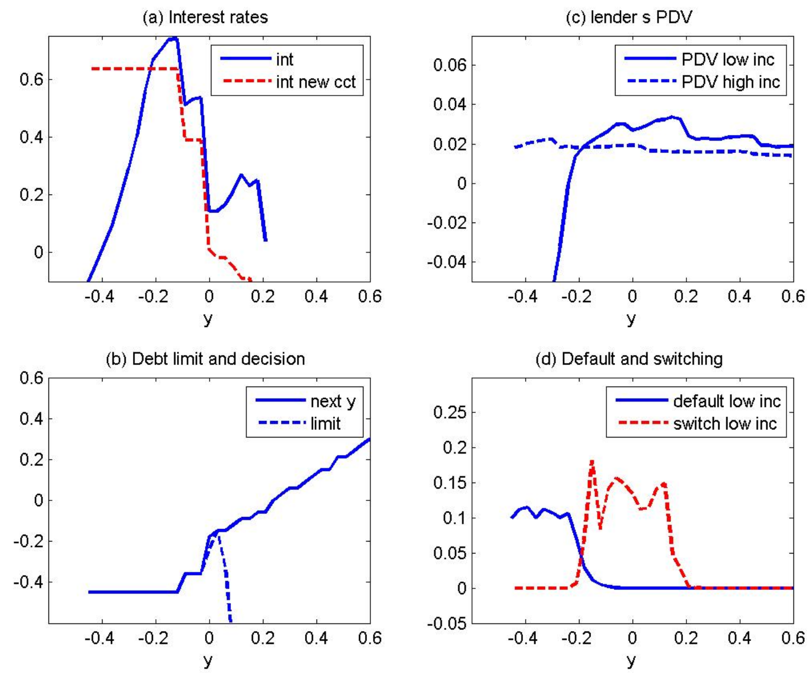

Figure 3: Continuing contracts. Low income household in (a), (b) and (d)

household's willingness to default or to switch to another contract. Low income households with either low assets or moderate debt want to increase their debt. The main concern restraining the bank's policy is their incentive to switch since, at these positions, there are attractively priced contracts available in the market. As it turns out, it is households in this group that are the main source of profits to the lender.[24] Figure 3 (c) shows the present discounted value of a lender in a contract as a function of the household's current position. The largest values occur in this mid range of asset positions. They coincide with the situations where the lender can apply high interest rates seen in panel (a). These individuals, with low income but not too much debt, are very likely to pay back. Moreover, they are willing to issue a lower discounted price to avoid switching costs to some extent. Figure 3 (d) shows that switching occurs among this type of individuals, which the lender optimally chooses to tolerate to some degree. Default is minimal in these states.

On the other hand, low income households with high debts are certainly not tempted by switching as nobody else would lend them any funds. They are more inclined to default though. Banks will find it hard to raise profits in these type of situations. Figure 3 (c) shows the value of the contract dropping sharply there. The lender has to be generous to low-income highly-indebted

[24] In some parameterizations high income households borrow if highly indebted. Since they want to reduce the debt slowly, lenders extract profits as well in these situations.

households and, as seen in Figure 3 (a) and (b), it will either extend further credit or roll over existing debts at higher discounted prices. The reason is clearly that low-income households are in no position to repay the debts and any harsh terms will induce large numbers of bankruptcies, an outcome that the bank wants to contain. Figure 3 (d) shows that substantial default occurs at the upper end range of debt.

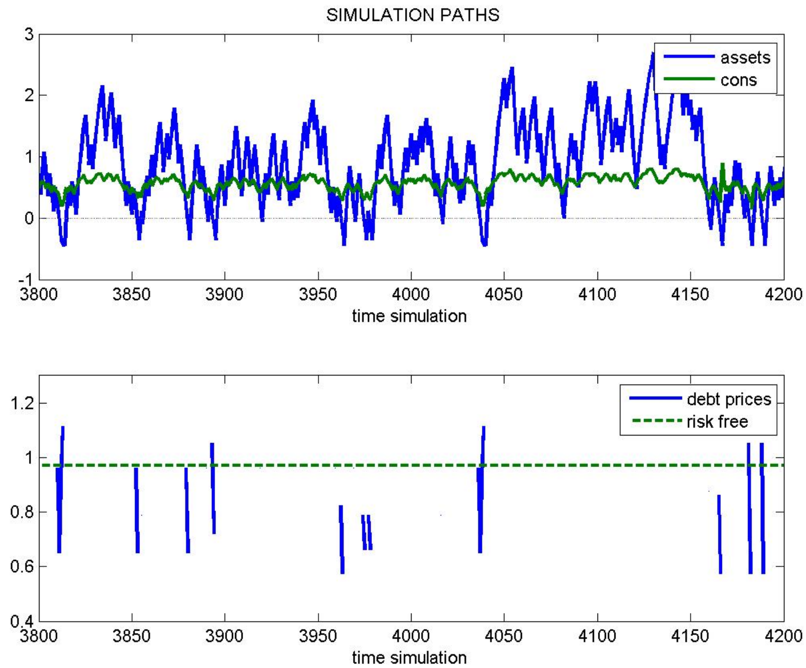

Figure 4: Consumption, asset position and prices.

We end documenting the typical situations households find themselves in based on simulations in Figure 4. We can also have a closer look at particular events and describe the corresponding default and switching choices. Figure 5 contains various examples of sequences of events where, after having borrowed in period -1, the household decides whether to switch to a new contract in period 0 and, subsequently, decides about switching and defaulting. Default happens after persistent bad luck. It also shows prices, decreasing first and then possibly increasing as debts build up. (X: Look for better examples; explain better)

7 Commitment and policy reform

The previous analysis reveals that, in the absence of commitment in the pricing policy, existing banks often use their ability to increase the interest rate on an existing credit line. The recent regulatory change in the U.S. restricts severely this type of practice. In this section we set out to

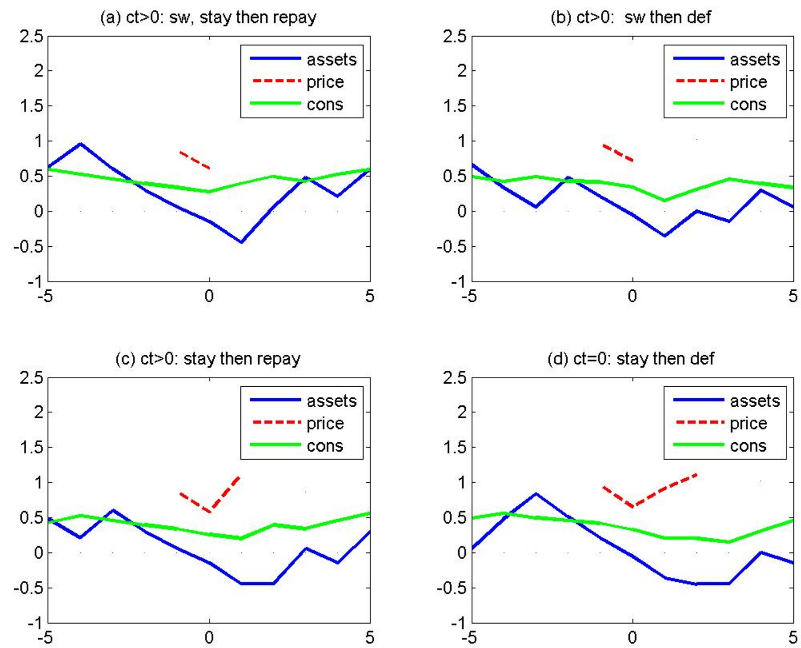

Figure 5: Consumption, asset position and prices. Starts borrowing in period -1. Described sequences starting at 0.

explore the implications of this restriction. We interpret the policy change as a commitment not to increase the interest rate on an existing contract. Under the new policy, banks will therefore not be able to issue low rates only for the initial periods, and they will have to resort more often to tightening the debt limits when managing credit lines.

7.1 Steady states

We start our analysis by considering the aggregate steady-state outcomes under the new policy in Table 3. The reform causes a reduction in the default rate and a rise in the switching rate. Aggregate measures of wealth, debt ad indebtedness change only very slightly with commitment. There is some reduction in the proportion of population in debt. More visibly, commitment implies a higher average lending interest rate. Table 4 provides greater details on the change in credit conditions. There is a compositional shift of borrowers towards new contracts and away from existing contracts. The average size of loans and the average interest rate both increase with commitment. These average changes reflect the response of the credit conditions on new contracts; on continuing contracts, in fact, both the interest rates and the size of loans taken are lower with commitment.

Table 3: **Aggregates with commitment**

Variable	Post reform	Pre reform
Default rate	0.0066	0.0070
Switching rate	0.0186	0.0144
Percentage in debt	0.094	0.097
Debt to income	0.046	0.047
Wealth to income	1.483	1.484
Aver. interest rate	16.32	14.96

Table 4: **Main Credit Statistics post reform** (not normalized)

	post-reform	pre-reform
Mass distribution:		
Mass with a contract	0.873	0.868
Mass no contract but clean	0.114	0.118
Mass borrowing	0.0944	0.0974
Mass borrowing new contracts	0.0186	0.0144
Mass borrowing continuing contracts	0.0758	0.0829
Loan size:		
All contracts	0.268	0.264
New contracts	0.301	0.202
Continuing contracts	0.260	0.275
Loan price:		
All contracts	0.860	0.870
New contracts	0.766	0.944
Continuing contracts	0.883	0.857

7.2 Contracts

Let us consider new contracts first. Under the new commitment policy, households face an equilibrium menu with predominantly higher interest rates since teaser rates are no longer an option. Figure 6 shows the schedule of rates for new contracts with and without the commitment policy. For large initial loans, commitment does not change much the interest rate since for those loan sizes the restriction not to raise interest rates is not binding.[25]

We now turn to consider the properties of continuing contracts. Under commitment, continuing contracts are influenced by their initial interest rate which, as just discussed, is directly associated with its initial size. As it turns out, there are accordingly two sharply differentiated types of active contracts which we discuss separately.

There are contracts with low initial interest and debt level. Under commitment, the low interest

[25]Commitment can even reduces the starting interest since, given the higher initial higher initial rates on most regular new available contracts, contracts commanding a loose interest ceiling can extract more surplus from the household during the life of the contract.

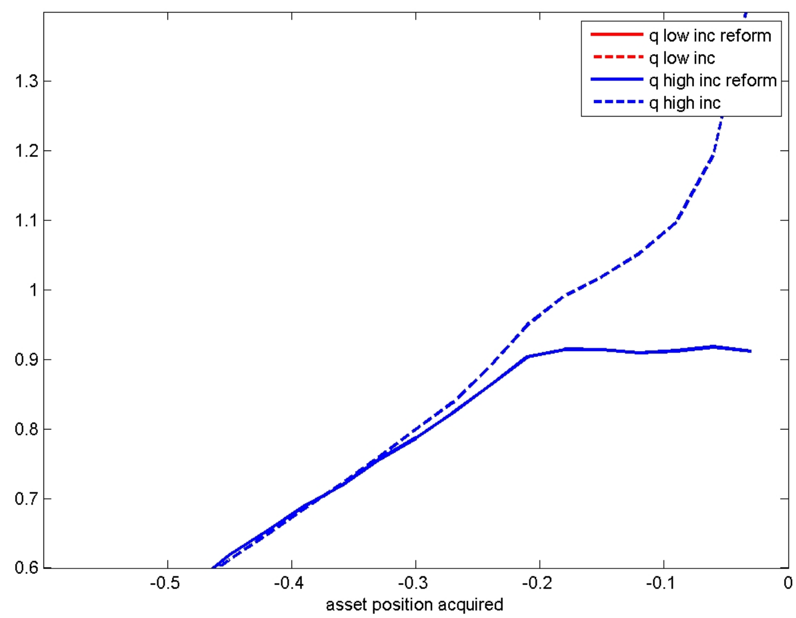

Figure 6: Initial prices. Commitment (solid) and no commitment (dashed).

ceiling is binding very often and results in bank policies with interest rates much lower than in the economy without commitment. Credit limits are generally much stricter with commitment which, compared to the environment without commitment, is also associated with larger number of borrowers switching to new contracts. Figure 7 shows the interest-rate and limit decision rules for a representative contract in this class under commitment alongside the rules corresponding to a contract in the no commitment case. It also shows the profile of household default and switching. The last section displays the consumption-equivalent welfare gains to commitment for households in this contract. In contrast with the no commitment case, the tighter limit under commitments provokes the exit - mostly through switching but also via default- of all households before they run up high debts. There are welfare gains to having commitment for the household in this contract, except when the tighter limit bites the most.[26]

The second group of contracts under commitment have larger starting debt and higher interest rate ceiling. The differences with the no-commitment case in pricing and limit policies remain but are smaller than in the low-interest contracts. Switching and default patterns with and without commitment are similar. Welfare gains, while smaller, still dominate in this type of contracts.

[26]The loss at the highest debt position can be explained easily: the contract has a higher asset value for the household under commitment which the lender exploits.

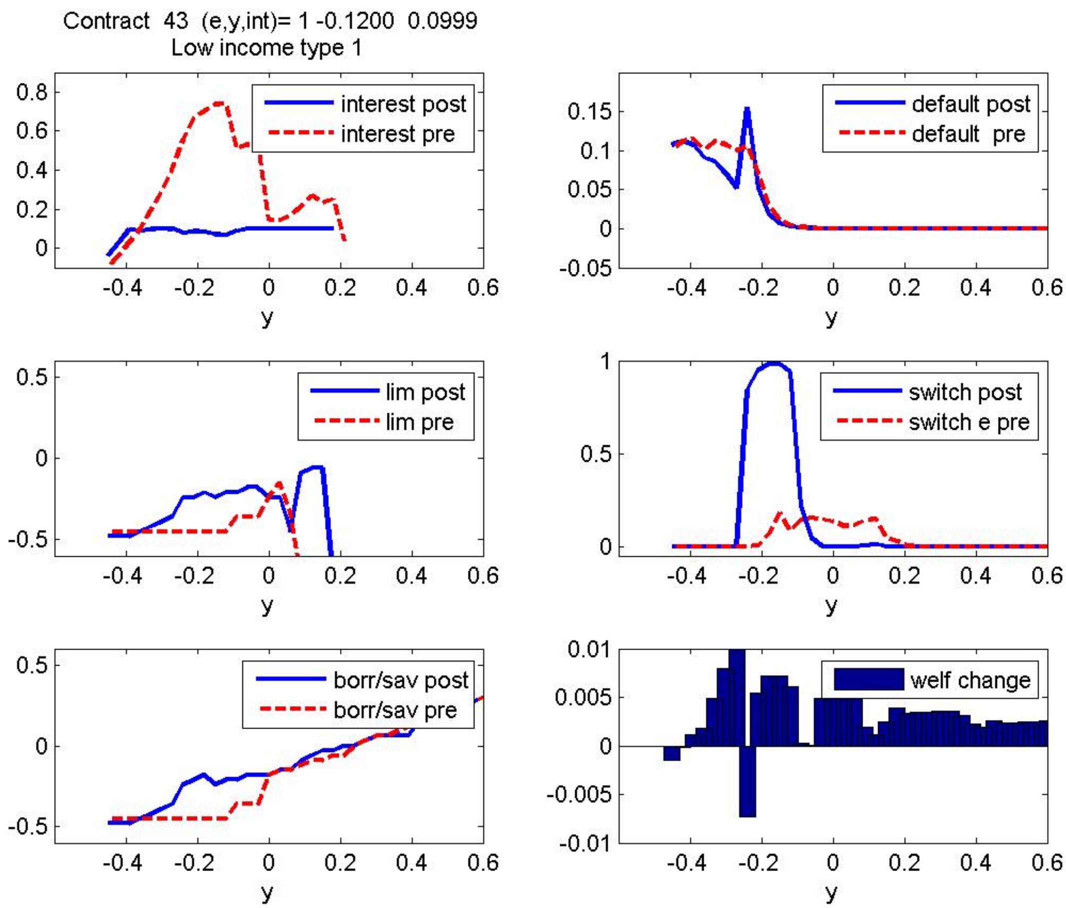

Figure 7: Low-interest contracts. Low income. Commitment versus no-commitment. This is contract $\theta = 43$ such that, under commitment, $(e^{\theta}, q^{\theta}, y'^{\theta}) = (1, 0.909, -0.12)$.

Figure 8 shows the characteristics of one such contract. The contract has more similarities with the no-commitment contract, including the fact that it allows the household to run up large debts.[27]

7.3 Welfare

We want to asses the welfare implications of commitment. We have already shown in Figure 7 and 8 that welfare changes for low earning households, while predominantly positive, vary across contracts at different asset positions. Therefore in order to make a precise evaluation we will need to account for the distribution of households across contracts and asset positions, and also income.

[27]Notice that the contract under commitment may even charge a higher interest rate for some moderate loans since the outside option of a new contract for the household is more costly than without commitment. This accounts for the welfare loss shown for households at at those positions.

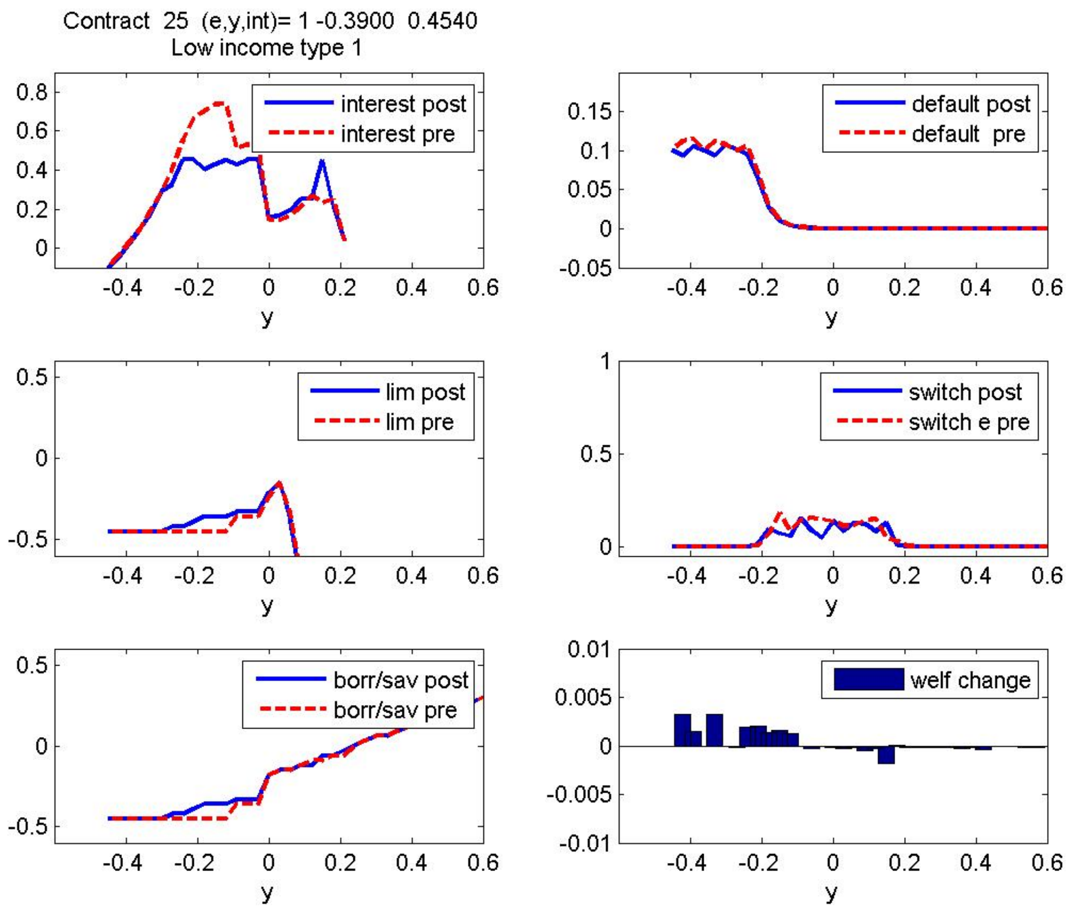

Figure 8: High-interest contracts. Low income. Commitment versus no-commitment. This is contract $\theta = 25$ such that, under commitment, $(e^{\theta}, q^{\theta}, y'^{\theta}) = (1, 0.688, -0.39)$.

We accordingly consider two alternative welfare evaluations, one starting from the no-commitment pre-reform conditions and the other starting from the post-reform commitment conditions. We begin with the first approach.

7.3.1 Introducing commitment

Introducing commitment over a baseline economy that does not have commitment requires laying out specific assumptions about how this reform deals with pre-existing contracts, which were signed under no commitment. We choose to assume that an existing contract will become constrained by an interest ceiling equal to the starting interest rate of a new contract with initial size equal to the initial size of this pre-existing contract. That is, it is given the same interest that it would command if it were to be issued as a new contract when the reform is in place.

To appreciate the significance of this, notice that Figure 10 shows that the starting distribution of households is more concentrated on contracts with relatively small debt which, as seen in Figure 6, will experience a relatively large rise in the initial interest rate with the reform. So our assumption will tend to give a conservative measure of the gains of the reform compared to the alternative of subjecting pre-existing contracts to their historical initial rate.

The aggregate welfare effect for broad groups of introducing commitment in this way is shown in the first column of Table 5. Commitment improves overall welfare by 0.12 per cent per capita. This means **$ per capita or about ** billion dollars, or about ** the cost of business cycles found by Lucas. Looking in greater detail, the households with low income improve the most, doing so by 0.15 per cent of their consumption. The gains for indebted households are of 18 per cent. Households benefit mainly if they hold a contract. Those without a contract loose out with the reform.

!!(merely reflect the !!benefits accruing in the prospect of holding a contract in the !!future.)

Figure 9 part (b) shows the changes in welfare for the different wealth and income groups existing before the reform. Most types gain on average. The largest gains, which are experienced by unemployed households in debt, are between 0.57 and 0.62 per cent. Some households with moderate assets also see substantial increases in welfare, between 0.3 and 0.4 per cent. Smaller gains are concentrated at the tails of the wealth distribution. The bottom panel (c) shows loses of small size for households not holding initially a contract.

The overall gains follow from the effects of the reform on contracts discussed earlier in section **??**. It keeps interest rates from rising too sharply although, on the other hand, it may cause tighter debt conditions. On net it results in welfare gains.

Table 5: **Welfare effect of commitment (% change)**

	direct	reverse	uniform profits	prop profits
all	+0.121	+0.052	+0.038	+0.041
low-income	+0.147	+0.072	+0.056	+0.060
high-income	+0.092	+0.030	+0.018	+0.020
indebted	+0.181	+0.136	+0.120	+0.136
card holders	+0.141	+0.061	+0.048	+0.050
non card holders	-0.013	-0.012	-0.028	-0.024

7.3.2 Keeping commitment

Our alternative approach to assessing the reform starts from the stationary allocation after the reform with commitment. It evaluates the welfare gain to keeping this policy in place instead of reverting to the no-commitment regime. By design, this exercise avoids the need to make

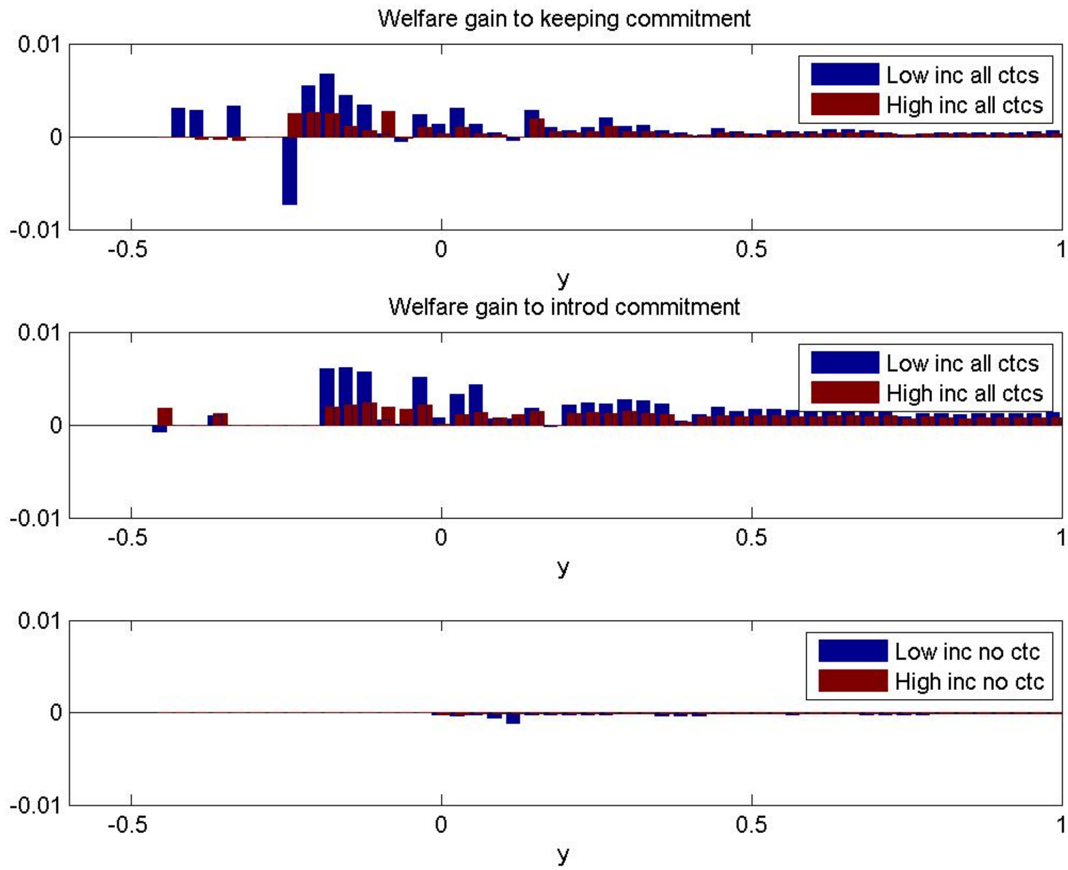

Figure 9: Welfare effect to commitment reform. Weighted average across existing households and contracts. (a.) Gain to not reversing the reform. (b.) Gain to introducing reform. Assumes initial price after reform determines interest ceilings on existing contracts starting with same $(y'^{\theta}, e^{\theta})$.

assumptions on how the transition is implemented since it simply lifts the pricing constraints uniformly across all contracts.

The second column of Table 5 reports gains for broad groups of households. They are quantitatively smaller than with the first approach. Figure 9 part (a) shows again the effects at the various income and debt positions within the stationary distribution of the economy with commitment.

The differences stem from the different initial composition of the population. Figure 10 shows that after the reform, households are more concentrated at contracts with larger initial debts which, as argued in subsection **??**, are those where welfare gains to commitment are smaller.

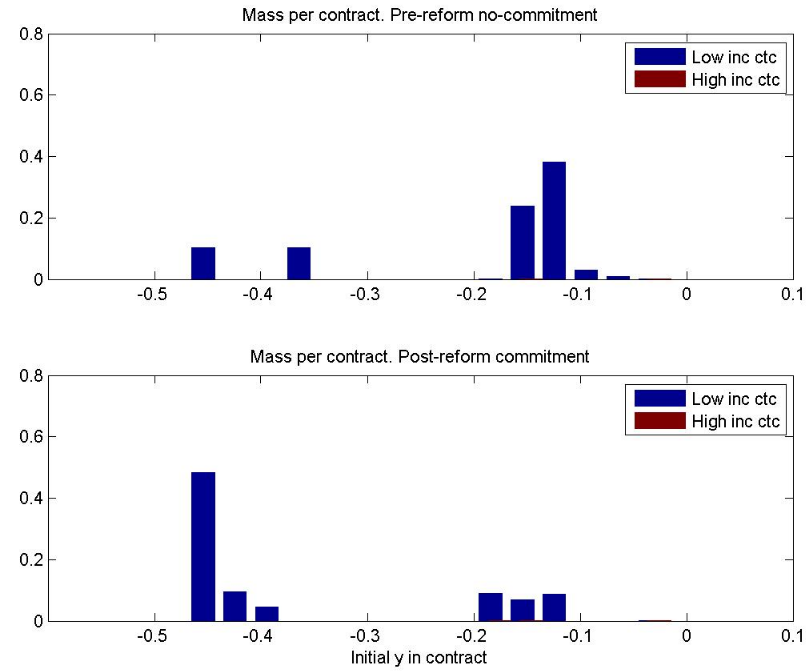

Figure 10: Distribution of households across contracts by initial debt, pre- and post-reform.

7.3.3 Welfare and banks' profits

Banks incur costs every time they open new contracts and do not recover those costs immediately, but over the life of the contract. An equilibrium condition requires that every new contract has zero value ex-ante. In this model without bank-level uncertainty, a bank is a collection of contracts that exactly matches its assets and liabilities within a given policy regime. As such, the residual value of the bank after paying its debts is zero and the issue of who owns the banks is moot. However, a change in policy regime alters the terms for contracts already in existence and causes a transitory change in their net value.

Consider specifically the experiment that keeps the reform in section 7.3.2. Lifting the prohibition to increase interest rates generates a rent for the banks. In our analysis, it implies an increase in the value of existing banks of 3 per cent. The question of who owns the banks becomes now important.

In the model there is not a clear way to allocate ownership and hence in Section 7.3.2 we have assumed that the revaluation of banking assets has no direct impact on households, as if banks were owned by foreign residents. This assumption clearly exaggerates the value of commitment. We now include this increased value of the banks in the calculations of welfare for domestic households.

Let us assume first that ownership is uniformly distributed across all households and the capital gains are transferred as a lump sum to each household as if banks were public. The increased value of the banks is given by

$$s = \int_{\omega>0} \left[\Psi^{no-commit}(e, y+s, \omega) \; - \Psi^{commit}(e, y, \omega)\right] \; x^{commit}(e, y, \omega, h=0) \; \mathtt{d}e \; \mathtt{d}y \; \mathtt{d}\omega.$$

Note how we use the transfer itself to compute the gain in the value when it is owned by households. The gain in welfare to keeping commitment so measured is shown in the third column of Table 5, to be compared with those that abstract from the bank's profits in the second column. The gains of commitment are preserved but become smaller for all the groups shown.

Another possible way of handling the newly acquired bank profits is to impute them proportionally to the household's positive assets. The relevant proportion s now solves

$$\begin{aligned} s \int_{y>0} y \, x^{commit}(e, y, \omega, h) \, \mathtt{d}e \; \mathtt{d}y \; \mathtt{d}\omega \; \mathtt{d}h = \\ \int_{\omega>0} \left[\Psi^{no-commit}(e, y(1+s), \omega) - \Psi^{commit}(e, y, \omega)\right] x^{commit}(e, y, \omega, h=0) \, \mathtt{d}e \; \mathtt{d}y \; \mathtt{d}\omega. \end{aligned}$$

The gains in welfare are shown in the fourth column of Table 5. Commitment improves welfare but less than when ignoring bank's profits. The effect on indebted households remains unchanged as they do not receive any transfer.

8 Conclusion

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A Some Issues in Computation

The model economy without commitment to the interest rate is well behaved. Discrete choice models have a problem of continuity of decision rules and/or value functions. Often, continuity is ensured by posing i.i.d. shocks to the decision makers. This is what we did with the household decisions. The lending firms do not have this problem as their choice of the interest rate is a continuous one which ensures that the profit functions are continuous functions of the interest rates.

The model economy that implements the new policy does not have this feature because the choice of the lending firms towards existing contracts is a discrete one described by

$$\max_{b \geq y} \widetilde{\Psi}(e, y, \theta, b; q^\theta, b^*) = -y \; 1_{y<0} \; \sum_{\varepsilon,\chi} F_e(\varepsilon, \chi) \; [1 - \widetilde{d}(e, y, \theta, \varepsilon, \chi, b)] +$$
$$\sum_{\varepsilon} F_e(\varepsilon, \chi) \; \widetilde{g}(e, y, \theta, \varepsilon, \chi, b) \; q^\theta \; \widetilde{y}'^n(e, y, \varepsilon, w, 0, b) \; 1_{\widetilde{y}'^n(e,y,\varepsilon,w,0,b)<0}$$
$$+ q^* \sum_{\varepsilon} F_e(\varepsilon, \chi) \; \widetilde{g}(e, y, \theta, \varepsilon, \chi, b) \; \sum_{e'} \Gamma_{e,e'} \; \Psi(e', \widetilde{y}'^n(e, y, \varepsilon, w, 0, b), \theta) \quad (24)$$

where we have written the semicolon to indicate objects that matter, that the bank takes as given but that are constant which include the policy function b^* followed by next period's bank manager and the contracts interest rate q^θ. Notice also that we have dropped the interest rate out of the problem of the house since under this policy it is not relevant.

A time consistent policy for bank managers is a fixed point: the solution to this maximization problem is a function that sets the credit limit as a function of $\{e, y, \theta\}$ that should be equal to function b^θ. The discreteness of the choice of credit limit may induce the inexistence of such a fixed and successive iterations may, and sometimes do, generate an endless cycle.

To avoid this problem we introduce a small change in the problem of the bank. Such a problem is a minimal cost in getting the desired outcome. It will only be triggered when the problem enters a cycle which for now has been sufficient.

Let b_1 and b_2 be a pair of solutions such that when bank managers take as given one policy, then they choose the other and viceversa. Let the associated values be Ψ_1 and Ψ_2. Once we identify one such loop define

$$\overline{\Psi}(p, \Psi_1, \Psi_2) = p \; \Psi_1 + (1 - p) \; \Psi_2 \quad (25)$$

$$\overline{\Psi}_1(p,\Psi_1,\Psi_2) = -y\ 1_{y<0} \sum_{\varepsilon,\chi} F_e(\varepsilon,\chi)\ [1-\widetilde{d}(e,y,\theta,\varepsilon,\chi,b_1)] + \tag{26}$$
$$\sum_{\varepsilon} F_e(\varepsilon,\chi)\ \widetilde{g}(e,y,\theta,\varepsilon,\chi,b_1)\ q^\theta\ \widetilde{y}'^n(e,y,\varepsilon,w,0,b_1)\ 1_{\widetilde{y}'^n(e,y,\varepsilon,w,0,b_1)<0}$$
$$+q^* \sum_{\varepsilon} F_e(\varepsilon,\chi)\ \widetilde{g}(e,y,\theta,\varepsilon,\chi,b_1)\ \sum_{e'} \Gamma_{e,e'}\ \overline{\Psi}(p)(e',\widetilde{y}'^n(e,y,\varepsilon,w,0,b_1),\theta)$$
$$\overline{\Psi}_2(p,\Psi_1\Psi_2) = -y\ 1_{y<0} \sum_{\varepsilon,\chi} F_e(\varepsilon,\chi)\ [1-\widetilde{d}(e,y,\theta,\varepsilon,\chi,b_2)] + \tag{27}$$
$$\sum_{\varepsilon} F_e(\varepsilon,\chi)\ \widetilde{g}(e,y,\theta,\varepsilon,\chi,b_2)\ q^\theta\ \widetilde{y}'^n(e,y,\varepsilon,w,0,b_2)\ 1_{\widetilde{y}'^n(e,y,\varepsilon,w,0,b_2)<0}$$
$$+q^* \sum_{\varepsilon} F_e(\varepsilon,\chi)\ \widetilde{g}(e,y,\theta,\varepsilon,\chi,b_2)\ \sum_{e'} \Gamma_{e,e'}\ \overline{\Psi}(p)(e',\widetilde{y}'^n(e,y,\varepsilon,w,0,b_2),\theta)$$
$$\overline{\overline{\Psi}}(\overline{p},p,\Psi_1,\Psi_2) = \overline{p}\ \Psi_1 + (1-\overline{p})\ \Psi_2) \tag{28}$$

Then we can solve the problem

Where we can interpret ζe^2 as the cost of exerting effort choice e for a suitably chosen ζ. We are satisfied when we obtain a fixed point this is a solution $\overline{p}$ that is equal to p. We think of this as a heuristic approach to solve the fixed point problem of the branch bank managers.

Remark 1 This case has only occurred less than *xxxx* percent of the time.

Remark 2 The solution, while not independent of ζ yields statistics that are essentially identic for all ζ. The case reported is valid for environments where the effort cost is *xxxx* percent of profits.

Remark 3 There is a residual slight inconsistency for the households that do not internalize the randomness in the continuation policies of the bank. Switching their beliefs between the two policies does not change the outcome.