WORKING IN THE MARKET, WORKING AT HOME, AND THE ACQUISITION OF SKILLS: A GENERAL EQUILIBRIUM APPROACH

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The following features of labor market variables can be found in the data.

Fact 1: Over time, productivity and education have increased substantially, while hours per worker have declined but (due to an increase in participation) total hours worked per worked per capita have stayed about the same.
Fact 2: In a cross section, higher-wage individuals have more years of schooling, more hours worked in the market, fewer hours worked in the home, and a lower variance of market hours.
Fact 3: Over the life cycle, older individuals have higher wages, more hours worked in the market, more hours worked in the home, a lower variance of market hours, and almost the same number of years of education that younger agents.
Fact 4: Over the business cycle, older individuals have higher wages and a lower variance of market hours.

In this paper, a computable general equilibrium growth model is constructed that attempts to simultaneously account for these four facts. Its main features include the possibility of skills acquisition through schooling and the presence of home production. The model has the key property of allowing otherwise identical agents willingly to take different actions in equilibrium, actions that are associated with very different intertemporal profiles of wages and hours worked. Active intertemporal trading between types occurs as agents who choose to remain unskilled have a comparative advantage of working when they are young, while agents who become skilled have that advantage later in life. This feature generates longer hours worked in the market by skilled agents than unskilled ones. Unskilled agents also specialize in home production. Since the model is of the overlapping generations type, it generates predictions about the behavior of different age groups and about the behavior of its time series. The technology is stochastic which provides implications about the cyclical behavior of the economy.

The first three properties of the data described are reconciled here by recognizing the important role of home production, or non-market activities, and by the intertemporal comparative advantage generated by education. At any point in time, there are young and old agents. Among the latter there are skilled and unskilled. The skilled agents have an intertemporal comparative comparative advantage of working in market production and they do not participate in home production. They are both older and have higher wages, which accounts for facts 2 and 3. Since these properties are independent of preferences, fact 1 can be accounted for by preferences that make leisure independent of real wages. We also investigate the properties of the model regarding issues in fact 4.

A nonstationary version of the model is also used to illustrate some issues about long term growth. In it, economic development is seen not only as perpetual productivity growth, but also, over time, a market technology that improves relative to the home. This implies that in equilibrium there is an endogenous increase of schooling over time, together with a shift of effort from the home to the market. This is accomplished without any relative improvement of the skilled technology versus the unskilled one. It is due purely to the relative improvement of the market technologies with respect to the home ones. Models of this type might be used to account for the increase in formal education that accompanies economic growth.

Since the seminal work of Gary S. Becker (1964), a considerable literature on the determinants of the accumulation process of human capital has been developed. Among the mechanisms through which human capital increases are schooling and on-the-job training. Becker (1965) also stressed the role of nonmarket goods as major determinants of household choices (see Richard B. Freeman, [1986], and Reuben Gronau, [1986], for surveys of these topics). Most of the work in this tradition is concerned with the determinants of individual behavior and it carefully encompasses the possibility of heterogeneous behavior. However, the environments in which households make decisions are not generally seen in the context of an equilibrium model (exceptions to this are Sumru Altug and Robert Miller, [1990], Joseph V. Hotz and Robert Miller [1989] and Kenneth Wolpin [1984]).

Macroeconomists are concerned with these issues in very specific, albeit different, ways. Human capital is seen either as an exogenous pattern of skills distribution (as in Finn E. Kydland and Edward C. Prescott [1991], Kydland [1984], and Gary D. Hansen [1986]), or as a variable that delivers sustained growth (like in Robert E. Lucas [1988]). Home production has been explicitly treated as something different than leisure only recently in real business cycle theory (Jeremy Greenwood and Zvi Hercowitz [1991], Jess Benhabib, Richard Rogerson and Randy Wright [1991]). Typically, these models are fully developed general equilibrium models, with carefully specified preferences, endowments, technologies, and informational structures. Complexities of the equilibrium usually dictate that they construct representative agent economies, leaving no scope to the possibility of heterogeneous behavior which is the focus of the current analysis.

In Section 2 we review the major empirical regularities concerning hours and wages (and to some extent education, skills, and work at home) from the different perspectives referred to above: long term trends, cyclical, cross section, and life cycle. Section 3 develops the basic model. Section 4 constructs a stationary example. Its equilibrium is computed, and its properties are compared with the data. Section 5 shows the nonstationary example and its equilibrium is also computed and its characteristics analyzed. Section 6 concludes the paper.

1 Empirical Regularities

In this section we review some major facts on the relation between hours and wages from four different points of view: long term trends, cross section, age profile, and cyclical.

1.1 Long Term Trends

Over the past hundred years or so, yearly hours worked by workers have fallen considerably (see Angus Maddison [1982, 1987]). Table 1 shows some summary statistics reported by Maddison [1987] about long term behavior of hours worked productivity and education level. The data for hours worked in the early periods are mostly from factory workers, which perhaps tends to overestimate work effort. For males the labor force participation rate has declined, with most of this reduction being the result of earlier retirement and of increased schooling (see John Pencavel [1986]). Females, however, although participating in the decline of yearly hours per worker, have increased their participation rates to the extent that hours worked per woman of working age have gone up considerably (see Mark R. Killingsworth and James J. Heckman [1986]). Overall, annual hours per person of working age either have gone down a little or stayed stable (especially for post-war U.S. data, see Gilbert R. Ghez and Gary S. Becker [1975]). In this fashion, Maddison (1987, Table 19), also reports average rates of growth of population, employment and hours per employee for the periods 1913-50, 1950-73, 1973-84. If we compute from his data the rate of growth of hours worked per person since 1950, we obtain a $-.23$ percent for the period 1950-73 and .34 percent for the last eleven years of the sample, which seems to indicate that the reduction of hours per worker has been offset to some extent by the increase of the active population. Needless to say, there has been over the same period a substantial increase in real wages, and also a considerable increase in education levels.

1.2 Cross section

When we talk about cross sectional behavior of certain variables we have to define precisely what we mean. Although stock variables have a precise definition on their value at a point in time, flow variables do not. For example, when we look at the relation between hourly wages and hours worked we have to be precise about what type of wages and especially, hours we are looking at. It can be hours in a week, year, or lifetime. In this research we are interested in the relation between hours and ages for different types of individuals differing in age and skills. At this point we try to get away from intertemporal substitution effects, so what we consider is the longest period possible. The Panel Study of Income Dynamics (PSID) reports on yearly variables for a sample of families (the version of the PSID data used excludes the people in the additional sample of low income families drawn from the Survey of Economic Opportunity). We have used the 1969-82 sample, end have taken the subsample that has reported positive hours worked in the market (but less than 4380 to exclude outliers as this number implies working 12 hours a day every day of the year) for at least 4 years. It was also required at least one year of reports in both education, number of years of formal education, and homework, number of hours spent in home production activities, but only if less than 3000, also to exclude outliers. This is done for individuals.[1]For each person we have computed average number of annual hours worked, average hourly wage earned, standard deviation of hours worked, years of education achieved and average hours of homework. Then we classify individuals into five wage groups and take averages over individuals within each wage group.

The brackets used for each wage group are [0,2), [2,2.8), [2.8,3.8), [3.8,5.3) and 5.3 and more 1969 U.S. dollars. These brackets were chosen because they include similar numbers of people. As Table 2 shows, individuals in the higher wage groups have[2]:

- higher annual hours worked in the market,
- higher achieved education level,
- lower annual hours in home production,
- lower standard deviation of annual hours worked in the market.

1.3 Age Profile

The model that will be presented is a two period overlapping generations model. Since we want to compare its predictions with the data, a decision has to be made about which ages represent the young and which the old. We break the population in the PSID sample into two age groups, 16-27 and 28-59. This choice is arbitrary, but attempts to capture as young people the age groups that might be involved in schooling, and as old the age groups that are not involved in schooling and have not retired yet. Table 3 reports the same variables used for the analysis of cross sectional data, namely age-group averages of average wage, average annual hours, personal standard deviation of hours, education level and annual hours of homework. As this table shows, younger people work less hours, have a smaller wage, a slightly higher standard deviation of hours, and very similar values for education (although the young have their education level underrepresented since the education process may not be complete). Younger agents also show lower values for homework than their elders, reflecting, perhaps, more child-rearing by the latter.

1.4 Cyclical

"It is a well-known fact that unskilled workers in the United States labor force exhibit greater employment fluctuation over the business cycle than do skilled workers," (Finn E. Kydland [1984], p.176). This has been stated by using sex-age groups as proxies for skills (Jacob Mincer [1966], and Kim B. Clark and Laurence H. Summers [1981]). For example, in Table 4, the percentage standard deviation of total hours worked by age group is reported. The four age groups have considerably different average wages which is a measure of skills. The volatility of an index of the labor input that weights hours according to skill levels constructed from age and sex groups is about 91.5 percent of that of total hours in yearly terms (José-Víctor Ríos-Rull [1992]) or about 94 percent in quarterly terms, (Hansen [1986]. This type of behavior has also been documented by looking at particular industries (railroads in Sherwin Rosen [1968]) or by using education (Kydland [1984]). Kydland and Prescott (1988) have constructed a labor input index where skilled hours are weighted more than unskilled ones using also data from the PSID samples. Their findings include the fact that their labor input index is only 72 percent as volatile as aggregate hours. Table 2 showed the same pattern where the indicator for skills is average wage for each person. People here have been grouped into five wage, and hence, skills groups. All these findings tend to show that unskilled hours are more volatile than skilled ones as they carry less weight in measures of the labor input than in series of total hours worked.

With regard to the comovements of hours and GNP through different skills groups, Kydland and Prescot (1988) report similar correlations of their two measures (0.75 for the labor input, and 0.80 for aggregate hours). Ríos-Rull (1992) also finds similar behavior: 0.93 for the labor input and 0.94 for aggregate hours.

2 The Model

There are two-period-lived overlapping generations with preferences defined over three goods, $\{c, z, \ell\}$. Good c can be produced only in the market, as say cars, good z can be produced in the market or at home, as say getting the house clean or filling in IRS forms, and ℓ is leisure. Good c can be produced by skilled or unskilled labor, where skilled labor constitutes old workers who have invested a fraction of time γ in schooling when young[3]. Good z can be produced by skilled or unskilled labor in the market and also by home production.

Technologies are linear and subject to stochastic shocks. One unit of unskilled labor produces φ_t units of good c, while one unit of skilled labor produces π_t. Good z can be produced in the market by both types of skills, and it can also be produced at home, where all agents have the same ability which is better than the unskilled technology. Units are chosen so that one unit of unskilled time delivers $(1-\delta)\varphi_t$ units of z, one unit of skilled time delivers $(1-\delta)\pi_t$ units with $0 < \delta < 1$, while at home one unit of time of the prospective consumer regardless of his skills is transformed into φ_t units of z. The units of z produced at home cannot transferred. Finally, time becomes leisure on a one to one basis. All the goods are perfectly divisible and there are no storage possibilities.

The productivity level, φ_t, is of the form $\varphi_t = \theta_t \varphi_{t-1}$, φ_0 given, where θ_t is a Markov chain with support in $\Theta = \{\theta^1, \ldots, \theta^M\}$ and transition matrix Γ. Furthermore, $\theta^m > 1$ for all $m = 1, \ldots, M$, which gives perpetual growth. Productivity of skilled labor, π_t, is of the form $\pi_t \;\; = \;\; \varphi_{t-1}\pi$, where $\pi >$ $max_{1 \leq m \leq M} \quad \{(1-\delta)^{-1}\theta^m\}$. This ensures that working at home is a worse alternative for skilled agents than working in the market and acquiring good z. Agents of generation t are born after θ_t is realized. Note that productivity of unskilled labor in t is stochastic for agents born in $t-1$, while productivity of skilled labor is not. We can think of the process of learning at t as the development of a technology that arrived in period t and is used at $t+1$ in the manner specified in the previous period. The unskilled technologies at $t+1$ are, however, new ones. The reason to do this is to allow agents to do risk sharing in relevant ways. Given the type of preferences that will be used in the examples (Cobb-Douglas), if both types of agents were hit with the same multiplicative shock to productivity, the allocation they would achieve through trades in contingent markets would be essentially the same as the one achieved without those markets as these preferences have constant risk aversion and the shocks are multiplicative.

There are complete markets. This can be implemented through Arrow Securities. Since agents live for two periods and we are only interested in non-monetary equilibria, all trades occur between members of the same generation[4]. To characterize this economy we can start by looking at the properties of another economy $\epsilon(\varphi_t)$ that consists only of the individuals born at t, when the productivity level is φ_t.

A young agent chooses education, which influences his skill level when old. Let $e = 1$ if he chooses to become skilled and $e = 0$ otherwise. We can renormalize the price of consumption to 1 every period. The young agent chooses $c, z, \ell, h, \mathbf{s}, e \in \{0, 1\}$ to solve:

$$\begin{aligned}
&\max \quad E_0 \left\{ U\Big(c(0), z(0)) + a(0)\varphi(0), \ell(0), c(\theta), z(\theta) + a(\theta)\varphi(\theta), \ell(\theta) \Big) \right\} \\
(1)& \\
&s.t. \quad c(0) + p(0)z(0) - \mathbf{s}{\cdot}\mathbf{q} \leq w^0(0)h(0), \\
&\qquad c(\theta) + p(\theta)z(\theta) \leq w^e(\theta)h(\theta) + s(\theta), \qquad \text{for all } \theta, \\
&\qquad \ell(0) = 1 - h(0) - a(0) - e\gamma, \\
&\qquad \ell(\theta) = 1 - a(\theta) - h(\theta), \qquad \text{for all } \theta.
\end{aligned}$$

The index (0) denotes the value of a variable when young; (θ) denotes its value when old and the random state is θ; c, z and ℓ are consumption of the goods and leisure; a is time spent in home production; and φ is the productivity parameter. Notice that total consumption of the second good is the sum of of that purchased in the market, z, plus that produced at home $a\varphi$. In the budget constraints, p is the price of good z bought on the market, in terms of the market good. $\mathbf{s} = \{s(\theta_1), ..., s(\theta_M)\}$ is a vector of quantities of Arrow securities, where each component $s(\theta)$ pays off one unit if the state is θ and zero otherwise, $\mathbf{q}$ is the securities price vector, and w^e is the wage if a person with skill level $e \in \{0, 1\}$. Thus, in the first periods the wage is w^0, and labor income can be spent on c and z or invested in $\mathbf{s}$. In the second period, the wage is w^0 or w^1, depending on e, and labor income plus securities income is spent on c, and z. Finally, the time constraints say that leisure is total time net of (market plus home) work, and net of time spent in skills investment. γ is the amount of time devoted to skills investment in the first period, it only ocurrs if $e = 1$.

Note that the agent can compare the maximized expected value of the problem with $e = 1$ and $e = 0$, and will choose whatever value of e yields higher utility. If we let μ be the fraction of a given generation that chooses $e = 0$, then we have the equilibrium conditions:

$$
\begin{aligned}
0 < \mu < 1 &\Rightarrow u^0 = u^1 \\
\mu = 1 &\Rightarrow u^0 \geq u^1 \\
\mu = 0 &\Rightarrow u^0 \leq u^1
\end{aligned}
\tag{2}
$$

The next thing to do is to note that the linear production functions pin down the skilled wage, the unskilled wage, and the price of z (at each date in each state), hence we could substitute them directly into the consumer problem. In particular, in every state, $p(.) = 1 - \delta$, $w^0(.) = \varphi(.)$, and $w^1(\theta) = \pi\, \varphi(0)$ for all θ. So the only prices that have to be determined are those of the Arrow Securities.

Whenever both types coexist, there are certain properties of the choices that stand out. Agents that choose to be skilled do not produce good z at home when old. The unskilled ones never acquire it through the market.

In the economy considered with initial level of productivity φ_t, the probability distribution of next period's states is given by the transition matrix Γ and by the θ_t associated with φ_t.

An allocation is a tuple $(x^0, \mathbf{s}^0, x^1, \mathbf{s}^1, \mu)$, $\mu \in [0,1]$. x^0 is the allocation for unskilled agents which includes the time worked both at home and in the market and the set of goods acquired, and $\mathbf{s}^0$ are their securities holdings, while $(x^1, \mathbf{s}^1)$ are for the skilled, and μ is the proportion of the unskilled. It is feasible if it satisfies the following aggregate technological constraints:

$$\begin{aligned} \mu[c^0(0) + \frac{z^0(0)}{(1-\delta)}] + (1-\mu)[c^1(0) + \frac{z^1(0)}{(1-\delta)}] \leq \\ [\mu h^0(0) + (1-\mu)h^1(0)]\varphi, \end{aligned} \tag{3}$$

and for all θ,

$$\begin{aligned} \mu[c^0(\theta) + \frac{z^0(\theta)}{(1-\delta)}] + (1-\mu)[c^1(\theta) + \frac{z^1(\theta)}{(1-\delta)}] \leq \\ \mu h^0(\theta)\varphi\theta + (1-\mu)h^1(\theta)\varphi\pi. \end{aligned} \tag{4}$$

We proceed now to define a competitive equilibrium for economy $\epsilon(\varphi_t)$

Definition 1 *A Competitive Equilibrium for $\epsilon(\varphi_t)$ is a feasible allocation $(x^0, \mathbf{s}^0, x^1, \mathbf{s}^1, \mu)$ and prices $p, w^0, w^1, \mathbf{q}$ such that: x^0 solves (1) with the restriction that $e = 0$, x^1 solves (1), with the restriction $e = 1$ conditions (2) are satisfied and the market for Arrow securities clears: i.e.*

$$\mu \mathbf{s}^0(\theta) + (1-\mu)\mathbf{s}^1(\theta) = 0 \tag{5}$$

To show that genericly an equilibrium exists, we define $\epsilon^\mu(\varphi_t)$ as an economy where everything is the same as in $\epsilon(\varphi_t)$ except that in the former a proportion μ is unskilled and $(1-\mu)$ skilled, in other words, the choice is no longer available. Werner Hildenbrand (1974) shows that for "most" μ the set of equilibrium prices is finite and constitute a continuous correspondence of μ, of which a continuous function can be selected. This makes utility of both types also continuous on μ. The relation between u^1 and u^0 can only be of three types. Either $u^1 \geq u^0$ for all μ in which case $\mu = 0$ is an equilibrium, or $u^0 \geq u^1$ for all μ making $\mu = 1$ an equilibrium, or $u^0 \geq u^1$, for some but not all μ, in which case the intermediate value theorem guarantees the existence of a μ that equates the utility of both types.

After analyzing the one generation only economy $\epsilon(\varphi_t)$, we proceed to recover the original economy, ϵ, with overlapping generations. Since agents from different generations are never together for more than one date-event, equilibrium for the overlapping generations economy ϵ amounts to a sequence of equilibria for each of the $\epsilon(\varphi_t)$ one generation only, two period economies. Formally:

Definition 2 *A Competitive Equilibrium for ϵ is a set of stochastic processes $\{x_t^0, \mathbf{s}_t^0, x_t^1, \mathbf{s}_t^1, \mu_t, \mathbf{q}_t, p_t, w_t^e\}$, measurable with respect to $h_t \equiv \{\varphi_0, \theta_1.\theta_2, \ldots \theta_t\}$, where μ_t has support on $[0,1]$, such that for all h_t the t component of the stochastic processes, is an equilibrium for $\epsilon(\varphi_t)$.*

Existence of equilibrium for ϵ amounts, then, to existence of equilibrium for each $\epsilon(\varphi_t)$. Monetary equilibria arise in economies where the nonmonetary equilibrium allocation is such that a set of Pareto improving transfers from the young to the old is feasible. This can only happen when the prices of future consumption in terms of current consumption is too high. Therefore, nonexistence of monetary equilibria amounts to have non-monetary equilibria with appropriate bounds on intertemporal contingent prices. It is argued in the Appendix that a sufficient set of those bounds, are that for all $\epsilon(\varphi_t)$, all m:

$$\mathbf{q}_{\mathbf{h_t}}(\theta^m) < \frac{\Gamma(\theta^m|\theta_t)}{\bar{\theta}} \tag{6}$$

Where $\bar{\theta} = \max\{\theta^1, \ldots, \theta^M\}$[5]. If condition (6) is satisfied, and, in addition, for each $\epsilon(\varphi_t)$ there is a unique Competitive Equilibrium, then the equilibrium for ϵ would be unique, as we can construct it from the $\epsilon(\varphi_t)$ equilibria, and we have ruled out the existence of monetary equilibria. This property is very helpful for computing the equilibrium since it allows us to find the equilibria for each $\epsilon(\varphi_t)$ breaking up the problem into a much simpler collection of problems. The equilibrium is not necessarily stationary even after transforming goods c and z to take account for growth because because, in general, allocations depend on the levels of c and z in non linear ways.

In the following section we display a stationary example of an economy of the type described. Its equilibrium is computed, and its statistical properties analyzed, especially in relation to the facts described above regarding the behavior of wages and hours.

3 A Stochastic Stationary Economy

As stated, an equilibrium for the economy ϵ is a set of stochastic processes mapping histories into allocations and prices. The class of economies under study presents perpetual productivity growth preventing the existence of stationary equilibrium. In the first example we display preferences that have the property that a certain well defined transformation of the equilibrium allocations is stationary.

Consider the Cobb-Douglas utility function,

$$U[c(0), z(0), \ell(0), c(1), z(1), \ell(1)] = c(0)^{\alpha_c} z(0)^{\alpha_z} \ell(0)^{\alpha_\ell} c(1)^{\alpha_c} z(1)^{\alpha_z} \ell(1)^{\alpha_\ell},$$

where indices 0 and 1 refer to the young and the old, respectively. For this type of preferences, the share of income spent on each good is independent of relative prices.

As noted, the choices of the agents born after a specific history up to time t depend on the productivity level at t and on the probability distribution of shocks at $t+1$, and since the shocks follow a Markov process, the distribution is characterized by the period t shock θ_t. So the equilibrium allocation in this date-event depends only on the pair (φ_t, θ_t). With Cobb-Douglas preferences, however, the level of productivity is not important. We consider the following transformation:

$$\hat{c}_t^s(0) = \frac{c_t^s(0)}{\varphi_{t-2}}$$

$$\hat{c}_t^s(m) = \frac{c_t^s(m)}{\varphi_{t-1}}$$
$$\hat{z}_t^u(m) = \frac{z_t^u(m)}{\varphi_{t-1}} \qquad \text{for all } 1 \leq m \leq M$$

where $c_t^s(0)$ is consumption when young of an agent born in period t, and $c_t^s(m)$, and $z_t^u(m)$ refer to the consumption in period $t+1$ of an agent born in period t if the shock is $\theta_{t+1} = \theta(m)$. Note that consumption allocations when young are divided by φ_{t-2} and when old by φ_{t-1}. The reason for this is that in every period young and old people coexist consuming c and z and that we want units of goods consumed by both age groups to be the same.

With these preferences, the equilibrium is unique for each of the $\epsilon(\varphi_t)$ associated economies. Therefore, the economy we are interested in that includes all generations has also a unique equilibrium. This economy is stationary. With these preferences, the transformed economy is independent of the productivity level, and the equilibrium allocation only depends on the conditional distribution of the one period ahead shock that was in place at the time when the agents that are alive made their decisions. Then, the state of the economy is $\{\theta, \theta'\}$.

It is well-known that sample moments converge to the moments of the ergodic distribution. Therefore, computing the latter serves to characterize the long-run statistical properties of an economy. This is what we do in the example: the statistics reported are those of the ergodic distribution.

The values of the preference parameters for this economy are $\alpha_c = .01$, $\alpha_z = .005$, and $\alpha_\ell = .03$. The relative values of these coefficients are chosen so that the average amount of time devoted to leisure is around two-thirds of total time, while the ratio of hours worked in the market to hours worked at home is about three to one[6]. Juster and Stafford (1991) report that, in 1981, U.S. males used 65.5 percent of their time in either leisure or personal care (which includes sleep), while the value for females is 67.6 percent. The ratio reported by Juster and Stafford between hours worked in the market and hours worked at home varies a lot across sexes, being 3.2 for males, and .8 for females. Given the prominent role of females in child rearing, and the fact that such a consideration is absent in our paper, the parameters were chosen to imply allocations closer to the males. The sum of the coefficients of the utility function is less than .1, which makes the utility of the composite good not very different from a logarithmic utility, a typical benchmark case.

Regarding γ, the amount of time needed when young to become skilled, the choice was .1 of total available time. Juster and Stafford report that U.S. high school students spend 30.0 hours a week doing school work, while Japanese spend 60.4. There are no data on American college students, but the Japanese ones spend 41.2 hours a week. All these numbers are considerably bigger than that used in our model. In the model, however, agents live only two periods. For this reason, a smaller amount of time devoted to learning captures the fact that formal education takes less than half the productive life of a person.

We choose $\delta = .1$, where $(1 - \delta)$ is the ratio of the productivity of producing good z through the market technologies and the productivity when making it at home. This number is not important for our findings. As long as it is positive, skilled agents will always acquire good z through the market. It affects the effective gap between skilled and unskilled workers. The value we choose is a compromise. It takes into account partially the tax advantages of not acquiring the good through the market (which are much bigger than 10 percent), and the fact that in the data not all home production ceases for skilled agents.

For π, the ratio between productivities of technologies when used by skilled agents and when used by unskilled, we choose a value of 1.47. This gives a ratio between the wages of the two skilled groups of around 40 percent, comparable to any two contiguous wage groups in Table 2.

There are two possible shocks $\bar{\theta}$ and $\underline{\theta}$ with values of 1.06 and 1.04 and the transition matrix Γ is symmetric with a diagonal value of .52. This implies a small autocorrelation of the shock. This is important as periods are a large fraction of the life of an agent, much larger than a year.

With this structure, states are $\{\bar{\theta}, \bar{\theta}\}, \{\bar{\theta}, \underline{\theta}\}, \{\underline{\theta}, \bar{\theta}\}$, and $\{\underline{\theta}, \underline{\theta}\}$ in this order, where the first element is the previous period shock. This implies that good and bad shocks are equally common, and that the economy has two consecutive identical shocks 52 percent of the time while 48 percent of the time there is a switch.

With this parameterization some general properties of the allocation are that unskilled agents have less leisure than skilled ones when young. The opposite happens when old: skilled agents work longer. Comparing their consumption allocation, it has to be noted that skilled agents consume more of the market good than unskilled agents both when young and old. Of course, given that the skills level choice is voluntary, both types of agents have the same expected utility.

We now turn to discuss how the model behaves in relation to the facts discussed above, by looking at the relevant set of statistics.

3.1 Long Term Trend

The model is stationary in the time allocation variables, hence it agrees with the behavior of U.S. postwar data. This property is not surprising given the nature of the preferences used. But the low frequency behavior of leisure is actually the only important property of the model that depends on preferences, so a negative relation between the real wage and the number of hours worked could be obtained with different preferences without changing other properties of the model. This would complicate, a lot, the computation of equilibria, since the allocation would become nonstationary.

3.2 Cross-section

With respect to the cross-sectional performance of the model, the fact that skilled agents get good z through the market and unskilled agents make it at home guarantees that, in every state, skilled agents work longer hours in the market and shorter hours (zero, in fact) at home than the unskilled. This is true for every type of unskilled agent. The fact that to become skilled requires formal education guarantees that skilled agents have higher education levels than the unskilled. Also, note that since we count among the unskilled the young agents that will be skilled when old, this group includes three types of agents, two of which have a comparative advantage to enjoy leisure in that period, providing yet another reason for the difference in hours worked among the two groups. Finally the standard deviation of contingent hours in the model refers to the contingent hours worked when old by agents of each type. This is an individual statistic, it is the average ex-ante standard deviation of hours that agents will have when old. All this is reported in Table 5, where averages have been taken over the value of individual variables for the ergodic distribution. As noted, the model reproduces the behavior of the data with respect to all variables.

3.3 Age Profile

Table 6 shows the age group averages of the five variables we have been studying. First, note that older agents work more hours in the market, but less at home. Overall, however, older agents work a little longer (the reason for this is the learning process when young). Their real wage is higher and their education level is similar. All this agrees with the behavior of the data (although the way we calculated education for young people in the PSID data set underestimates the length of their education). Since there is only one observation per agent within each age group, the standard deviation of personal hours is not a meaningful variable. Young agents have only one realization of their hours worked, making its standard deviation zero. For old agents, the standard deviation of the contingent hours they choose to work is reported. Given the length of the period relative to the life of agents, this variable may not be very meaningful. Young agents do more homework.

Most of the properties regarding long term trend, cross section, and age profile, do not depend strongly on the specific values of the parameters that are chosen. These properties are shared by most models that have equilibria with all agents working in all periods and states of nature, as is the case with the parameters used.

3.4 Cyclical

We define as the model's cyclical behavior the values of the standard deviations of the variables and of the correlations between them with respect to the ergodic distribution. Given that agents live only for two periods, the model's cyclical behavior does not relate very closely standard notions of cyclical behavior in the data. Table 7 shows the first and second ergodic moments of the most relevant variables. Note that these are economy wide variables, and they are statistics computed from aggregate data, that include the behavior of all agents. Regarding the variances, we see that the model fails to reproduce the higher variance of the unskilled hours as compared to the skilled ones that we find in the data. This is partly due to changes in the distribution of agent types. Note that GNP varies much more than the other variables and, if we adjust GNP to include homework, their difference further increases. This behavior is the opposite to the one predicted by Jess Benhabib, Richard Rogerson and Randy Wright [1991] where homework plays the role of a complement to the market but without perfectly correlated stochastic perturbations.

The model also presents correlations between the variables as shown in Table 8. Note that skilled and unskilled hours move in opposite directions. This is because skilled technologies are nonstochastic and there is risk sharing. Hours move with GNP; this is due to unskilled hours. Skilled hours are countercyclical. This is partly because of the risk sharing and partly because good shocks are positively autocorrelated, and when there is a good shock more agents choose to become unskilled since their prospects for next period are better. All this is also true when we include homework in hours, in GNP, or in both. This agrees with the notion found in the data that unskilled hours are more closely related to GNP than are skilled ones. The negative correlation between GNP and skilled hours does not, however, agree with the data. Overall, given that agents live only two periods in this model, changes in the distribution of types play a role that is too important when comparing the model to the cyclical frequencies in the data.

4 A Nonstationary Economy

The second example displayed is different. It introduces a simple form of nonstationarity without using a utility function where the choice of leisure depends on the productivity level. The ratio between the productivity in the market technology to produce good z and the productivity at home grows with time. The rationale for this is that technical development is not only a process that keeps producing more quantities with less inputs, but also that new products are introduced, products that match better the agents' needs, so that the market technologies become closer to the home one to produce good z. This will make the alternative of becoming skilled more attractive over time, and, hence, provide a rationale for the secular increase in schooling observed.

The economy is in this case deterministic with preferences given by:

$$U\,(\,c(0), z(0), \ell(0), c(1), z(1), \ell(1)\,) = c(0)^{\alpha c} z(0)^{\alpha z} \ell(0)^{\alpha \ell}$$
$$+c(1)^{\alpha c} z(1)^{\alpha z} \ell(1)^{\alpha \ell}$$

This utility function is different than the one used in the previous example. The degree of curvature (the sum of the $\alpha' s$) determines also the degree of intertemporal substitution and not only the risk aversion.

For these preferences, corner solutions in the sense that the workload might be zero at some age, can arise, allowing CE and the equilibrium allocation that would be obtained if lotteries could be traded to differ. Another reason to use this type of preferences is that it is of the form most widely used in growth models.

Everything else is as in the first example. As stated, the market technology improves over time. The ratio of productivity between market and home technologies becomes $(1 - \delta_t)$, where $\delta_t = \delta^{1+t/10}$, and t is time.

The properties of the equilibrium vary with the values of the parameters. In general, however, the fraction of agents that becomes skilled grows, as expected given the process of the market technology for good z. Together with this increase in the proportion of skilled agents, there is a rise in interest rates (recall that this is the ratio of the price of unskilled time of two consecutive periods), and thus, a concentration of the workload in the early age.

Table 9 contains the equilibrium distribution of agent types for an economy of the type described. The values of the parameters are 1.05 for θ, the rate of growth, φ_0, the productivity level at period 0, is $.334411 \times 10^{-08}$.

The preference parameters $\alpha_c, \alpha_z, \alpha_c$ are .01, .005, and .03, respectively. The parameter δ that regulates the value of the productivity ratio between the market technology and the home technology to produce good z is $.3\pi$, the productivity premium of the skilled technology is 1.522, and γ, the amount of time required to learn the skill, is .15. As these values imply, the unskilled choice is less attractive as time passes.

In period 1, all agents are unskilled; the allocation that gives both types equal utility is unfeasible (proportion of unskilled higher than one). In period 3 equilibrium is such that both types coexist. In this period, the intertemporal price of unskilled labor lies in between those that would prevail in all skilled or all unskilled economies. The value of this intertemporal price is such that skilled agents do not work at all when young. This prevents marginal rates of substitution between consumption and leisure to be equated across both types of agents. It is in here where the absence of lotteries becomes crucial as a lottery equilibrium would be characterized by marginal rates of substitution being equated across agents, while it is not necessarily the case that the total utility of both types are equated. In this case total, utility in the non-lottery equilibrium would be lower than in the lottery one, as the latter would implement the allocation that solves for the maximum aggregate utility with everybody being weighted equally.

As $(1-\delta_t)$ grows, so does the intertemporal ratio of unskilled wages, inducing agents to move their workload from their old to their young age until, eventually, skilled agents start working when young (Period 13). As $(1-\delta_t)$ gets closer to 1, the equilibrium allocation converges to a stationary value. In the last period reported, 171, the allocation is such that the precision of the computer is unable to detect any further changes.

5 Conclusion

This paper has developed a simple model in which ex-ante homogeneous agents become heterogeneous in equilibrium due to specialization in the process of skills acquisition, indicating that equilibrium induced heterogeneity may be important when considering heterogeneous behavior. It has been used to address the joint behavior of wages and hours from different points of view: long-term trend, cross-sectional, age-profile and cyclical behavior.

The model economy of Section 4 has proved successful in replicating some properties of the data. Cross-sectionally, the success is important, matching the relative size of all the variables we have considered. With regard to the long term trends, the model is designed so that working time remained stationary. The reason to do this (and also to keep stationary the aggregate education levels) is to be able to use a stationary equilibrium with a unique ergodic set from which high frequency statistics can be derived. As Section 5 shows, non-stationary examples with secular increases in education levels can easily be constructed. In the same fashion, preferences with the property that leisure grows with the real wage could be chosen to match the long run decrease in hours documented by Maddison (1982, 1987). With respect to the age profile, the model does well in wages and time worked but fails with respect to the standard deviation of hours and homework. The length of the period and the fact that the model abstracts from child-bearing considerations seem to be crucial issues in this respect. Finally, at the cyclical level, the model does well along certain lines (correlation between hours and GNP for example) and not so well along others (negative correlation of skilled hours and GNP, excessive volatility of skilled hours), mainly, perhaps, as a result of the excessive importance of the changes in the distribution of types across the different states of the model economy.

This paper has shown how a simple model can be constructed both to generate multidimensional predictions in directions consistent with the way data are collected and organized, and to allow for endogenous partitions of the population resulting in equilibrium induced heterogeneity. Structures of this type may be used to address questions regarding the role of heterogeneity, both ex ante and that induced in equilibrium, in the determination of aggregate variables.

Appendix

In order to see why condition (6) prevents the existence of a monetary equilibrium, assume that one exists with real price of money given by $p_{h_t} > 0$, and let the nominal amount be normalized to be one dollar. Then, arbitrage requires $1 = \sum_m \frac{p_{\{h_t,\theta^m\}}}{p_{h_t}} q_{h_t}(\theta^m)$. Rearranging we get $\bar{\theta} p_{h_t} < \sum_m p_{\{h_t,\theta^m\}} \Gamma(\theta^m|\theta_t)$. This can be interpreteted as saying that the conditional expectation of the value of money next period is bigger than today's value of money augmented by the biggest growth rate of the economy. It is easy to see then that this implies that there exists τ, h_τ such that p_{h_τ} is bigger than the total possible output of the economy, contradicting the existence of a monetary equilibria. See Rodolfo Manuelli (1990) and Rao S. Aiyagari and Dan Peled (1991) for related issues on monetary equilibria of stochastic overlapping generation economies.

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Footnotes

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[1] For details on how the family data was transformed into individual data and how the hourly wage series was constructed, see Kydland and Prescott (1988).

[2] Within sex groups, women are more concentrated in the lower wage groups (working a small number of hours) and men are concentrated in the higher wage groups (and working more hours). The key properties of the data also hold within age groups. In this fashion, hours worked per year for males range between 1909 and 2089, while for females the range is from 831 to 1379 (when we partition both populations in five equally sized wage groups each). The monotonic relation between hours and wages is only violated by group 4 of females who work 1411 hours on average. The behavior of all

other variables within the same sex group has the same properties that the aggregate, except for the volatility of hours worked by the second wage group of females that is less than 1.% higher than the lowest wage group.

[3] This introduces a nonconvexity in the choice of the agents, possibly leaving scope for Richard Rogerson's (1988) style lotteries. We will not consider this possibility. In the stationary versions of the model there is no role for them as the choice of the agents is interior in the sense that the non-lottery equilibrium has agents equating marginal rates of substitution. This is exactly the same condition that comes out from the social planner problem that finds the allocation can be implemented through the use of lotteries, implying, therefore, that both contractual arrangements provide the same allocation.

[4] Note that the only intergenerational trades that can occur involve the exchange of goods of the same period, and that the allocations achieved through these trades can also be achieved through trades between individuals of the same generation.

[5] Note that the previously established continuity a.e. of the set of prices as a function of μ implies that it is enough that this condition holds for $\epsilon^0(\varphi_t)$, and $\epsilon^1(\varphi_t)$.

[6] Table 7 reports the average values for some of these key variables.

Table 1: Long Term Behavior of Wages and Hours

	Year			
	1870	1913	1950	1984
Hours Worked per Worker per Year[a]	2,964	2,605	1,867	1,623
Labor Productivity Level (GDP per Hour Worked)[b]	1.92	4.49	10.93	21.31
Average Years of Formal Education of the 15-64 Age Group[c]	-	6.93	9.46	12.52

[a] Source: Maddison (1987), Table A-9.

[b] Source: Maddison (1987), Table A-5.

[c] Source: Maddison (1987), Table A-12.

Table 2: Wage Groups Averages of Personal Variables

	Wages				
Averages Within Each Age Group of	1	2	3	4	5
Average Hourly Real Wage	1.48	2.37	3.28	4.46	7.24
Average Annual Hours Worked	1112	1556	1795	1920	2009
Standard Deviation of Hours	579	529	479	415	341
Years of Education	11.18	11.97	12.73	13.00	14.3
Average Annual Hours of Homework	648	582	465	350	261

Table 3: Age Group Averages of Personal Variables

	Age Group	
Averages Within	Young	Old
Each Age Group of	(16-27)	(28-59)
Average Annual Hours	1624	1735
Average Hourly Real Wage	2.7	3.8
Standard Deviation of Hours	477	416
Years of Education	12.98	12.66
Average Annual Hours of Homework	672	733

Table 4: Average Percentage Standard Deviation of Aggregate Hours by Skilled Level as Measured through Age Groups 1956-1986[a]

	Age Group			
	16-24	25-44	45-64	65&+
Average Measure of Skills	0.67	1.25	1.36	0.89
Average % Standard Deviation of Total Hours Worked	3.94	1.83	1.43	3.22

[a] Source: Ríos-Rull (1992)

Table 5: Cross-section Values of the

Model Economy for the Personal Variables Studied

Averages within Each Wage Group	Unskilled	Skilled
Average Real Wage	1.103	1.545
Average Time Worked	0.172	0.391
Standard Deviation of Contingent Hours	0.00043	0.00038
Average Education	0.026	0.10
Average Time of Homework	0.1313	0.0

Table 6: Averages for Personal Variables

within Age Group

	Age Group	
Averages Within Each Age Group	Young	Old
Average Time Worked	0.209	0.262
Average Real Wage	1.103	1.545
Standard Deviation of Contingent Hours	0.0	0.00038
Education Achieved	0.059	0.059
Average Homework	0.105	0.051

Table 7: Ergodic Moments of the Model Economy

	Averages	Percentage Standard Deviation
Unskilled Hours per Unskilled Worker	0.172	0.0878
Same with Home Production	0.284	0.049
Skilled Hours per Skilled Worker	0.391	0.097
Skilled Hours per Capita	0.081	0.235
Unskilled Hours per Capita	0.121	0.116
Unskilled Hours per Capita(with Home Prod)	0.201	0.085
GNP	0.633	1.058
GNP (Adjusted for Home Production)	0.817	1.126
Total Hours	0.472	0.089
Unskilled Hours	0.243	0.119
Skilled Hours	0.223	0.157
Total Hours Including Home Production	0.631	0.063

Table 8: Correlations of the Model Economy

Skilled-Unskilled Hours	-0.15
Hours-GNP	0.32
Unskilled Hours-GNP	0.97
Skilled Hours-GNP	-0.40
Hours-GNP Adjusted for Home Prod	0.39
Unskilled Hours-GNP Adjusted for Home Prod	0.98
Skilled Hours-GNP Adjusted for Home Prod	-0.34
Skilled-Unskilled Hours (with Home Hours)	-0.35
Hours(with Home Hours)-GNP	0.51
Unskilled Hours(with Home Hours)-GNP	1.00
Hours(with Home Hours)-GNP Adjusted for Home Prod	0.57
Unskilled Hours(with Home Hours)-GNP Adjusted for Home Prod	1.00

Table 9: A Non Stationary Model Economy

	Period			
	1	3	13	171
Proportion of Unskilled	1.	0.992	0.840	0.786
Interest Rate Measured as Normalized Ratio between Wages of the Unskilled in Consecutive Periods if				
All Unskilled	0.999	0.992	0.992	0.992
All Skilled	1.68	1.68	1.68	1.68
Both Types Coexist	0.96	1.00	1.09	1.13