A Quantitative Theory of Unsecured Consumer Credit with Risk of Default

Satyajit Chatterjee
Federal Reserve Bank of Philadelphia

Dean Corbae
University of Pittsburgh

Makoto Nakajima
University of Pennsylvania

José-Víctor Ríos-Rull
Penn, CAERP

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1. Introduction

•People are entitled to and sometimes do file for bankruptcy.

•We build a model with bankruptcy that looks like the data in other dimensions as well. The model will

- Incorporate the U.S. legal system. (Those that default lose access temporarily to credit markets and incur transaction costs.)

- Include a competitive loan industry with free entry where lenders can offer any menu of loan sizes and borrowing rates.

•Notice that we are not after optimal contracts.

•We use the model to ask quantitative regulatory questions.

•We will use the model to ask fluctuations and monetary policy questions.

Literature

- Athreya (1999). First to study default in an environment similar to ours. Unexploited profits remain.

- Lehnert and Maki (2000). Assumes commitment on the part of lenders. There are periods where firms make negative profits.

- Livshits, MacGee, Tertilt (2002) follows some of our notions.

  There is another literature in endogenous trading arrangements. It ignores U.S. legal code and interprets credit disruptions in the data in a non literal manner.

- Kehoe and Levine (1993,2001), Kocherlakota (1998), Alvarez and Jermann (2000), Kruger and Perri (2000-2001). Lucky borrowers want to default (individual rationality constraint binds in high income states). No household goes bankrupt in equilibrium.

Facts about Bankruptcy

- People default mainly via Chapter 7 of the Bankruptcy Code.

- Under Chapter 7, a person files for bankruptcy; upon successful completion of the process (a very easy thing), the person's assets are liquidated above a certain amount (it varies by state), the household's debts disappear and creditors lose any rights to recover future assets; the household gets to keep its future labor earnings, and it cannot file again for seven years; after ten years, the bad credit history disappears.

- In 1998, 1,007,922 persons filed for bankruptcy under Chapter 7 while 1,379,249 was total filings. Moreover, approximately 90% of Chapter 13 debt was not repaid in 1997 (WEFA data) which makes this legal figure less relevant or, more likely, a prelude to a Chapter 7 filing.

We Interpret Bankruptcy as

• With a good credit history, a household can borrow and file for bankruptcy

- Its debts disappear; its creditors lose any future claims to those debts.

- In the filing period, the household cannot save and must consume its current earnings.

- Its credit history turns bad.

• If it has a bad credit history, the household cannot borrow but can save. It suffers some inconveniences (bonded credit cards) that we model as a proportional γ loss of income. Upon termination of the punishment period, the household's credit history turns good.

•The law only prevents another bankruptcy filing within a 7 year period as well as fixing a 10 year limit to a bad credit history. We *interpret* this to mean that households cannot borrow during that amount of time. We abstract from renegotiation issues.

Preview of Findings

•We restrict somewhat the set of households that our theory looks at and then the model accounts for

1. The U.S. wealth and income distribution
2. U.S. unsecured credit volumes.
3. U.S. bankruptcy rates.

In the model, we explicitly include

1. Preference Shocks (urges to consume) in addition to earnings shocks. These shocks may not be *i.i.d.*

2. Demographics. People that default are younger and poorer than average. The model is explicit about this. Population is exponential.

The Model

•The presentation has a streamlined version of the model to avoid cumbersome notation (no demographics and *i.i.d.* shocks). The paper has those and persistent preference shocks.

•A continuum of ex-ante identical agents with idiosyncratic earnings shocks.

•Free entry in the credit market. Firms operate at zero costs.

•All loans are one–period loans. We abstract from other types of loans.

•Interest rates on loans can only depend on the size of the loan (what matters is whether good or bad credit history). With non i.i.d shocks, their predictive content also affects interest rates.

•The legal system is that of the U.S.

•The risk free rate is exogenous (storage technology or world interest rate).

Households

- Earnings: $e \in E = [\underline{e}, \overline{e}] \subset \mathbb{R}_{++}$, i.i.d. across individuals and time. Continuous cdf $F(e)$ and probability space $(E, \mathcal{B}(E), \mu)$.

- Borrowing/Saving opportunities: Households can hold after interest assets $\ell \in L = \{\ell_{\min}, \cdots, 0, \cdots, \ell_{\max}\}$, a finite set.

- Preferences: $E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \, u(c_t) \right\}$. $u : [0, \ell_{\max} - \ell_{\min} + \overline{e}] \to \mathbb{R}$ concave.

$$A1 : u(\underline{e}) - u(0) \geq \frac{\beta}{1-\beta} \left[u\left(\overline{e} + \ell_{\max} - \ell_{\min}\right) - u\left(\underline{e}\left(1-\gamma\right)\right) \right].$$

- Storage technology on $L^+ = (L \cap \mathbb{R}_+)$, gross return $\frac{1}{\hat{q}}$, where $1 > \hat{q} > \beta$.

- Let $q_\ell \in [0, \widehat{q}]$ be the discounted price of asset position $\ell \in L$ next period (the interest rate r is $1/q - 1$.). The price schedule is $q : L \to [0, \widehat{q}]$. Let Q be the set of possible prices.

Default Options

- Household credit history, $h \in \{0, 1\}$.
- Default decision, $d \in \{0, 1\}$.
- If $h = 0$ (good credit history), choosing $d = 0$, implies a standard problem.
- If $h = 0$ (good credit history), choosing $d = 1$, implies
  - $\ell = 0$ (debt is wiped clean)
  - $\ell' = 0$ (cannot save in same period you default).
- If $h = 1$, (the household has a bad credit history).
  - $\ell' \geq 0$ (cannot borrow).
  - $h' = 0$ with probability $1 - \lambda$.

Budget Sets

Defined on (ℓ, h, d, e, q) in 3 parts.

1. Good Credit History, Don't Default: (can be empty for $e + \ell < 0$ and small q'_ℓ's).

$$B_{\ell,0,0}(e,q) = \{c \in \mathbb{R}_+, \ell' \in L : \quad c + q_{\ell'} \ \ \ell' \leq e + \ell\}.$$

2. Good Credit History, Default:

$$B_{\ell,0,1}(e,q) = \{c \in \mathbb{R}_+, \ell' = 0 : \quad c \leq e\}.$$

3. Bad Credit History:

$$B_{\ell,1,.}(e,q) = \{c \in \mathbb{R}_+, \ell' \in L^+ : \quad c + \hat{q} \ \ \ell' \leq e + \ell\}.$$

Household Problem

•$v_{\ell,h}(e,q)$ is the value function. Let $w_{\ell,h}(q) = \int_E v_{\ell,h}(e,q)\, d\mu$. If $B \neq \emptyset$

$$\chi_{\ell,0,0}(e,q) = \max_{c,a' \in B_{\ell,0,0}(e,q)} u(c) + \beta\, w_{\ell',0}(q).$$

$$\chi_{\ell,0,1}(e,q) = u(e) + \beta\, w_{0,1}(q).$$

•Depending on Credit History (Good or bad)

$$v_{\ell,0}(e,q) = \max \begin{cases} \{\chi_{\ell,0,0}(e,q), \chi_{\ell,0,1}(e,q)\} & \text{if } \ell < 0, \\ \chi_{\ell,0,0}(e,q), & \text{otherwise} \end{cases}$$

$$v_{\ell,1}(e,q) = \max_{c,a' \in B_{\ell,1,;}(e,a)} u(c) + \beta \left[\lambda\, w_{\ell',1}(q) + (1-\lambda)\, w_{\ell',0}(q)\right].$$

•Given w^0, this procedure yields $w^1(w^0)$ defining implicitly an operator

Proposition 1. *The household problem is a contraction.*

Characterizing Default Sets

$$D_\ell(q) = \{e \in E \quad : v_{\ell,0}(e,q) \leq u(e) + \beta\, w_{0,1}(q)\}.$$

Proposition 2. *The default set is a closed interval, and $D_\ell(q)$ is non–increasing in ℓ and q. Moreover, the default probability $\mu[D_\ell(q)] = F[e_\ell^U(q)] - F[e_\ell^L(q)]$ is a continuous function of q.*

Sometimes too poor to default.

Unsecured Credit Industry

- Competitive firms with zero costs and free entry.
- Firms make loan of size ℓ at price q_ℓ.
- Firms only observe the asset position and the credit history (ℓ, h).
- Firms are one period lived.
- Bad credit history households cannot borrow (neglect renegotiation).
- Profits: $\pi_\ell[a, q_\ell] = \; a \; [\; (1 - p_\ell) \; \ell \, \hat{q} \; - \; \ell \;\; q_\ell]$

  where a is the measure of agents receiving a loan and $p(\ell)$ is the fraction of borrowers on loans of size ℓ who default.

- Zero Profit Condition: $q_\ell(p) = \hat{q} \;\; [1 - p_\ell]$

Equilibrium

Definition 1. *A vector of prices for assets positions $q^* \in Q$ is a competitive equilibrium for the unsecured credit economy if given q^*, household optimization induces default sets $D_\ell(q^*)$, and hence default probabilities $\mu[D_\ell(q^*)]$, so that profit maximizing firms that use prices q^* obtain zero profits.*

•It is obvious from the sequential nature of this definition that the key element in establishing the existence of equilibrium is finding a fixed point.

Proposition 3. *A competitive equilibrium exists (Kakutani).*

•A key element of the proof is the continuity of the $F[D_\ell(q)]$. It requires continuity of F so small changes in q do not induce large changes in default behavior.

Remarks I: Characteristics of Equilibria

•If for some $\widetilde{\ell}$, all households default, then $q_{\widetilde{\ell}} = 0$.

•There is an absolute level of debt that poses a natural lower bound on assets: that implied by having the maximum level of debt that could be paid by the luckiest household with the lowest possible interest rate. This is $\frac{\overline{e}}{1-\hat{q}}$. (polar opposite of the one in Aiyagari-94 and Athreya-99). So we get around the standard problem of the arbitrarity of the debt limits.

•Equilibria is non trivial. $q = 0$ is not an equilibrium (for fine enough grid).

•In any equilibrium there is always default (for fine enough grid). This follows from the fact that for the maximum debt level everybody defaults, and from Assumption 1.

Remarks II: Stationarity and Computation

•Note that we have not referred to a stationary measure of agents: no need to. Storage technology guarantees that prices are unaffected by the distribution of agents, hence any initial distribution of agents will do.

•We find equilibria by successive approximations on prices q.

1. Guess an initial discount price $q = \hat{q}$.

2. Given q, solve the household problem. Find the value function and default intervals for every $\ell \in L$. We approximate v functions with splines, good for default intervals.

3. Compute the new q that yields zero profits. If equal go to 4. If different, update q and go to 2.

4. Compute the stationary distribution by successive approximations. Compute its relevant statistics.

Mapping the Model to Data

We use 1998 data and take all U.S. households except for

- Older than 65.
- In the top wealth quintile.
- With total debts lower than average yearly household earnings.

	Value
Average Earnings	100.0
Total assets	153.0
Assets held by households with negative wealth	2.8
Percentage of households with negative assets	11.4
Earnings Gini	0.44
Mean to Median Earnings	1.19
Wealth Gini	0.63
Mean to Median Wealth	1.86

Only some Default/Debt within our Theory

•[1] of those defaulters, about 2/3 file under Chapter 7.

•[2] of those defaulters there are various reasons adduced:

Reasons adduced for defaulting	
Loss of job	12.2%
Marital Distress	14.3%
Credit Missmanagement	41.3%
Health Care	16.4%
Lawsuits and Harassment	15.9%

•So we target .5% of defaulters as being within the realm of our theory.

•Also, we target two thirds of the debt held by households with negative asset position. So we target 2.% of earnings as the level of debt. (We are possibly underrepresenting debt (Gross and Souleles-01).

The Baseline Model Economy

Statistic	Data	Model
Population Turnover in % per year	2.0	2.0
Earnings Gini	0.44	0.44
Mean to Median Earnings	1.19	1.19
Degree of Risk Aversion	2.0	2.0
Length of the Punishment period in years	10.	10.
Rate of return of the storage technology (in %)	0.5	0.5
Wealth to earnings ratio in %	153.0	153.0
Assets held by households with negative wealth	2.6	2.6
Percentage of households with negative assets	10.0	9.9
Percentage of defaulters	0.50	0.51
Overidentifying Restrictions		
Wealth Gini	0.63	0.48
Mean to Median Wealth	1.86	1.11
Lowest to Mean Earnings (in %)	–	9.01

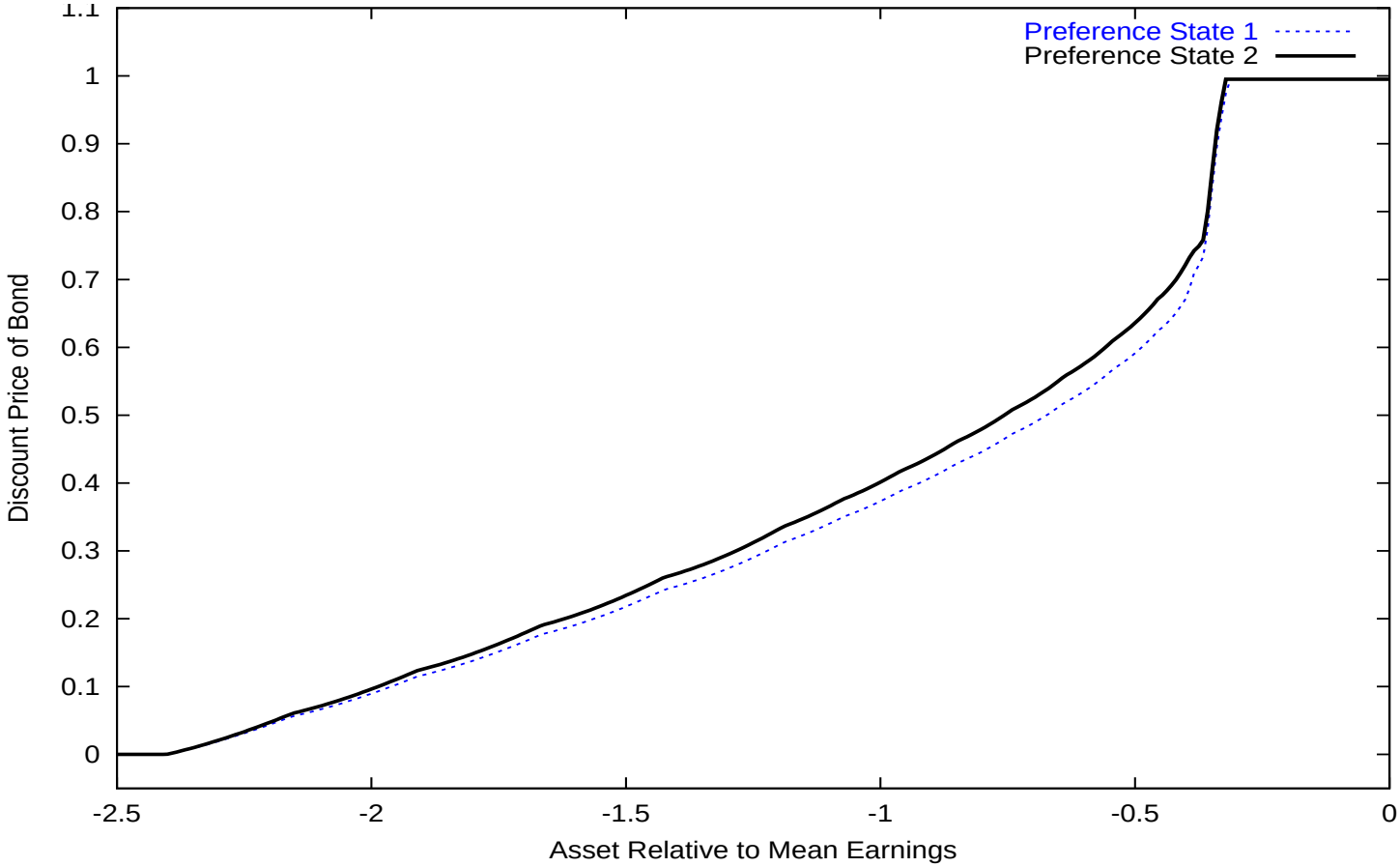

Figure 1: **Price of loans depending on loan size and value of the preference shock.**

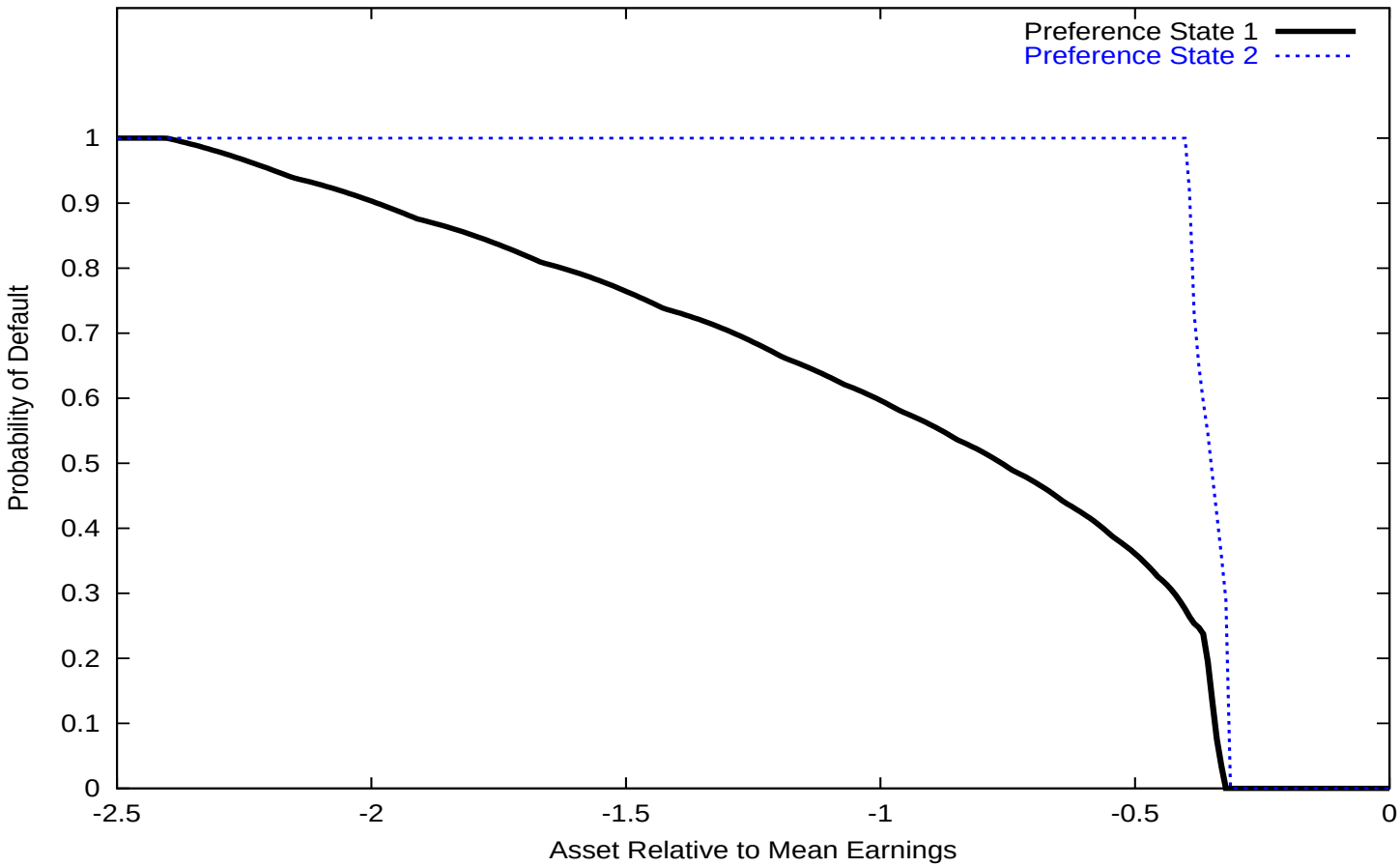

Figure 2: **Probability of default depending on debt and newly realized value of the preference shock.**

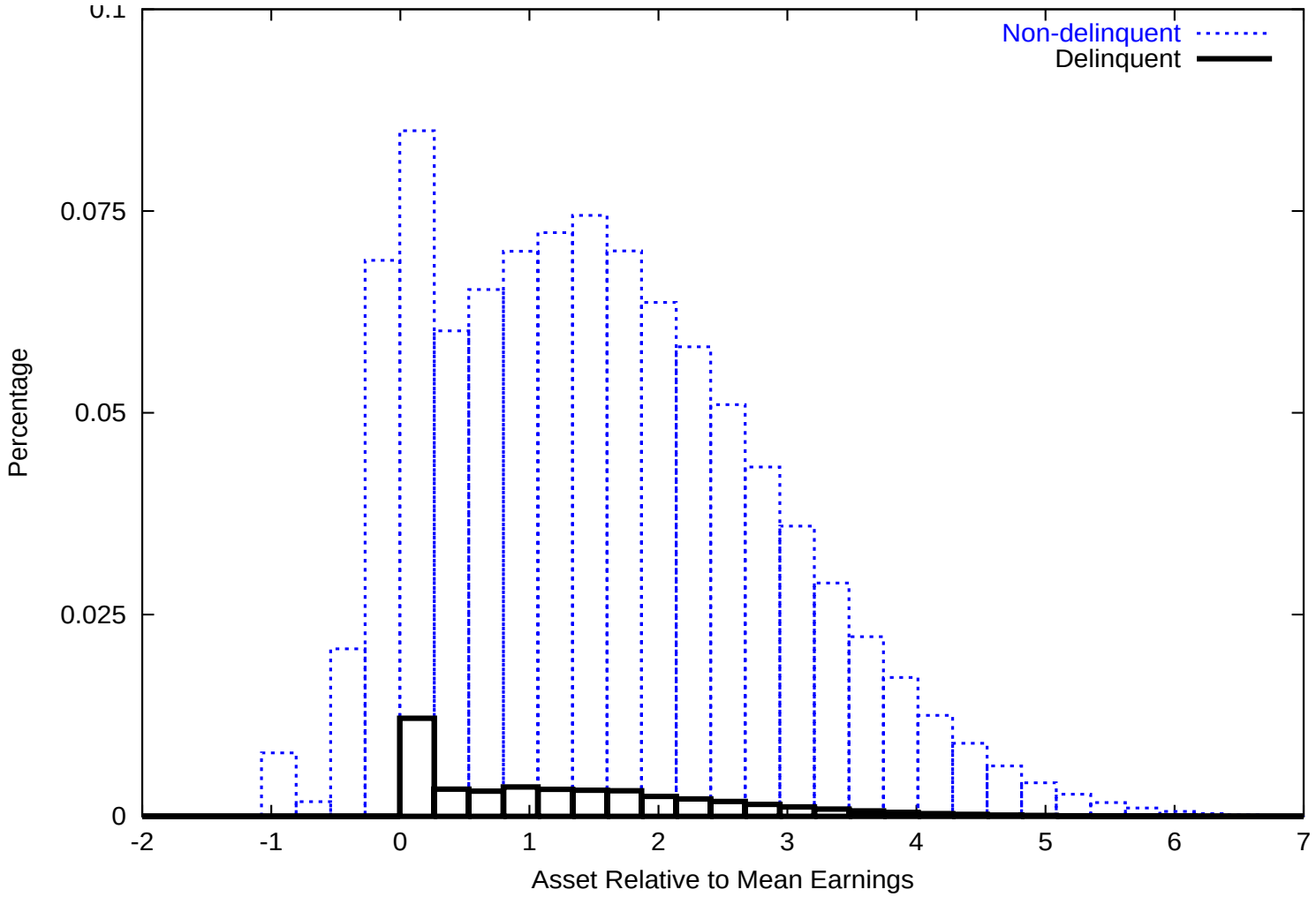

Figure 3: **Distribution of Wealth among Households.**

Changing the law: Shorter Punishment Period

Statistic	Baseline 10 years	Shorter Punishment 5 yrs
Prob$(h' = 0 \| h = 1)$ in %	10.	20.
Earnings	100.00	100.00
Total assets	153.204	154.043
Negative assets	-2.524	-2.449
Total Defaulted amount	0.522	0.614
Percentage of Defaulters	0.514	0.654
Percentage of Delinquent	4.422	2.98

- More Savings
- Less credit
- More Defaulters but Less households with Bad Credit
- More Default

But not by much.

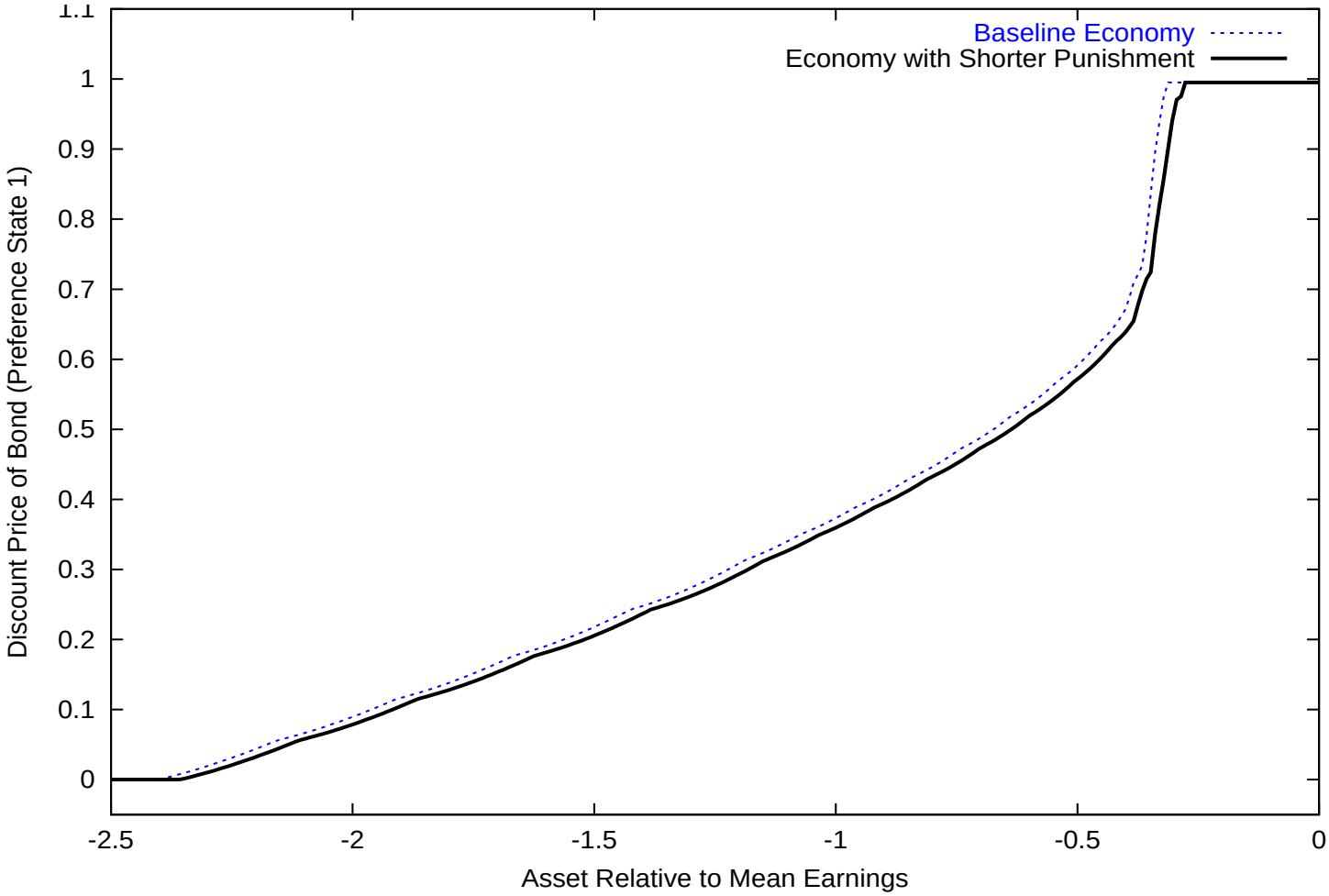

Figure 4: **Price of loans for the economy with Shorter Punishment.**

Changing the law: Earnings Limits for Defaulters

Statistic	Baseline	Tight Limit	Loose Limit
Maximum earnings for filing	no limit	89.	150.
Average Earnings	100.00	100.00	100.00
Total assets	153.204	124.823	142.850
Negative assets	-2.524	-6.879	-4.215
Total Defaulted amount	0.522	0.816	0.972
Percentage of Defaulters	0.541	0.523	0.546
Percentage of Delinquent	4.422	4.273	4.594

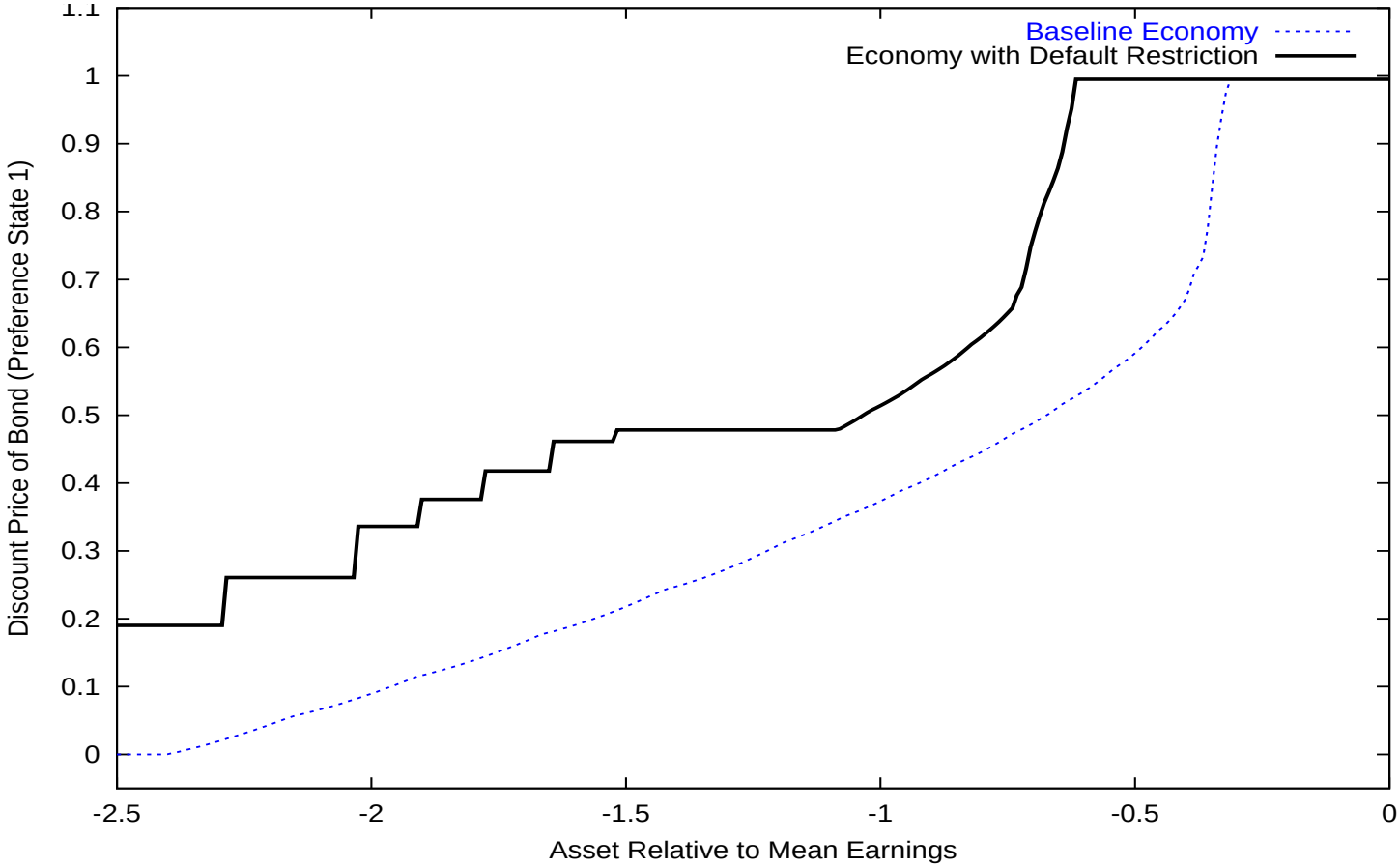

Figure 5: *Average?* **Price of loans for the baseline economy and the economy with a proposed restriction on the ability to default.**

Conclusions

•We have a computable model of default where it is the poor people that default, that resembles the legal system, where the loan industry is in equilibrium, capable of replicating some of the main U.S. bankruptcy facts, and useful for answering questions about policy.

Next?

- Measure Welfare (with transitions of course), almost done.
- Aggregate shocks.
  - Monetary policy $\hat{q}$.
  - Earnings.
  - Employment.

Optimal and Multiperiod Contracts

•This is not yet optimal contracts given an environment. Problems

1. Common problem: Renegotiation. Why can't bankrupt households borrow. They cannot run away from debts.

2. We do not know yet what physical or informational features of an environment would restrict the type of contracts to those that we model. In particular why do not have multiperiod debt coexisiting with one period debt.

•The introduction of multi- period loans, is likely to change the default options of a household and hence the discounted price of these loans cannot be obtained by compounding the price of one period loans. We plan to expand the class of loans that are allowed in future work.

A Word About Other Contracting Possibilities if firms lived longer

- Within a period state contingent contracts.

  There are no mechanisms to introduce incentive compatible contracts where agents report different endowments. This would be possible if people differed in risk aversion and lotteries were used as in Cole 1988.