Saving for a Sunny Day: An Alternative Theory of Precautionary Savings

Preliminary

Luigi Briglia (CEMFI)
Satyajit Chatterjee (FRB Philadelphia)
Dean Corbae (Wisconsin)
Kyle Dempsey (The Ohio State University)
José Víctor Ríos Rull (Penn UCL CAERP)

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Vanderbilt University

Introduction

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    - Extension (in progress) to explain top wealth inequality: Listening to this temporary spending opportunities is optional (only rich end up doing it).

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  - Extend EV shocks into realm of fundamentals; change ex ante behavior rather than provide tractable error structure

Data: the Nature of Errors in Consumption Functions

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	ind. var. = cash on hand	
	decile mean	decile rank
intercept	0.1091	0.0980
	(0.0057)	(0.0096)
slope	0.0048	0.0845
	(0.0007)	(0.0167)

Notes: Actual regressors for decile rank regressions are 0.05 for decile 1, 0.15 for decile 2, etc.

▸ details of measurement approach ▸ visual

Measurement error: for $c = g(a)$ and random ζ, suppose

- consumption is mismeasured, $\tilde{c} = \zeta c$: then $CV(\tilde{c}|a) = CV(\zeta)$.

- wealth is mismeasured, $\tilde{a} = a/\zeta$: then $CV(c|\tilde{a}) = CV(g^{-1}(\zeta))$

Several widely used classes of shocks cannot replicate this pattern:

- income: sensitivity of c to income **declines** as agents move away from constraint $\implies$ so do errors

- marginal utility: $c(a;\theta) = \lambda(\theta)a$, so $\overline{c}(a) = \overline{\lambda}a \implies$ errors are **independent** of a

Simplest Dynamic Model: A two period savings model

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- Today we follow the first approach

Construction of the Finite Choice Economies

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- economics: utility **bonus** of a unit interval budget set is 0

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    - **key:** size of budget set (a) determines the number of alternatives and therefore the number of shocks received, $J(N, a)$

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$$\max_{c^i \in \{c^1, \cdots, c^N\}} \quad u(c^i) + \eta^i + u(a - c^i),$$
$$\text{s.t.} \quad c^i \le a.$$

- Or $\max_{i \in \{1, \cdots, J(N)\}} \; u(c^i) + \eta^i + u(a - c^i)$, when $J(N) = \arg\max_{i=1,\dots,N}\{c_i \le a\}$.
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    - **key:** size of budget set (a) determines the number of alternatives and therefore the number of shocks received, $J(N, a)$
    - More options increases expected value
    - Options have cardinal interpretation and shocks are factored in **ex-ante**

- The ex-ante value

$$v^N(a) = \int \max_{c^i \in \{c^1, \cdots, c^{J(N,a)}\}} \{u(c^i) + \eta^i + u(a - c^i)\} \, dF(\eta^1, \cdots, \eta^N),$$

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Standard results in Extreme Value Theory yield formulas

- The value satisfies

$$v^N(a) = \alpha \ \ln \left(\frac{1}{J(N,a)} \sum_{i=1}^{J(N,a)} \exp\left\{ \frac{u(c^i) + u(a - c^i)}{\alpha} \right\} \right) + \alpha \ \ln c^{J(N,a)}.$$

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$$H^N(a,a') = \frac{\sum_{i=1}^{n(a')} \exp\left\{\frac{u(c^i)+u(a-c^i)}{\alpha}\right\}}{\sum_{i=1}^{J(N,a)} \exp\left\{\frac{u(c^i)+u(a-c^i)}{\alpha}\right\}}.$$

- The Value converges to (because it is essentially a Riemann integral)

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(multiply and Divide by $J(N, a)$).

- Note that these are differentiable functions.
- Main insights go through whether discrete or continuous case; in remainder, we'll go with continuous.

- Using standard results from discrete choice and our normalization of the EV shocks, we obtain

$$V_N(a) = \alpha \ln \left[\frac{c_{J(N,a)}}{J(N,a)} \sum_{i=1}^{J(N,a)} \exp\left(\frac{v(c_i; a)}{\alpha} \right) \right] \quad \to \quad V(a) = \alpha \ln \left[\int_0^a \exp\left(\frac{v(c; a)}{\alpha} \right) dc \right]$$

$$h_N(c_i; a) = \frac{\exp\left(\frac{v(c_i;a)}{\alpha}\right)}{\sum_{j=1}^{J(N,a)} \exp\left(\frac{v(c_j;a)}{\alpha}\right)} \quad \to \quad h(c; a) = \frac{\exp\left(\frac{v(c;a)}{\alpha}\right)}{\int_0^a \exp\left(\frac{v(c;a)}{\alpha}\right) dc}$$

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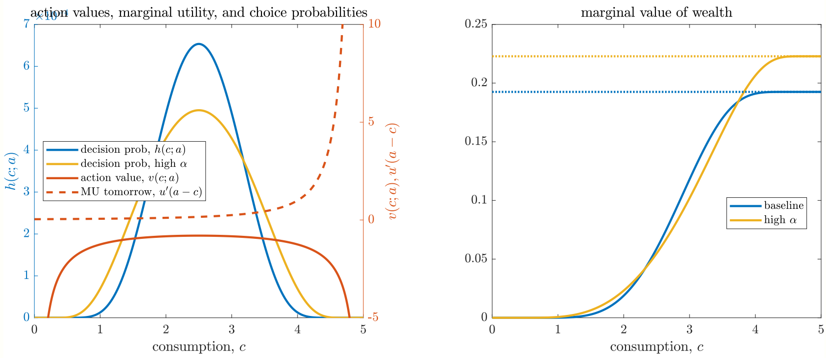
- Main insights orthogonal to discrete v. continuous; use continuous for remainder of talk.

$$\begin{aligned} V'(a) &= \int_0^a u'(a-c)h(c;a)dc \\ &= u'(a-c^*(a)) + \int_0^a \left[u'(a-c) - u'(a-c^*(a))\right] h(c;a)dc \end{aligned}$$

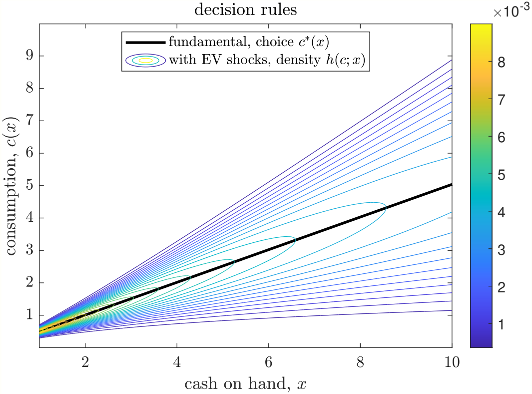
MVW is **positive** and **increasing** in α.

- **1st term:** standard effect: $\uparrow a \implies \uparrow c$ tomorrow given c today
- **2nd term:** novel to our framework from "noise" in decisions
  - positive by Jensen's inequality given prudence ($u'(a-c)$ convex in c)
  - comes from **not** being constrained upon choosing c that lead to low a'
  - key mechanism: sunny day v. rainy day



Higher α fans out $h(c; a)$ $\implies$ more weight on high future MU states $\implies$ MVW increases due to convexity of $u'(a - c)$.

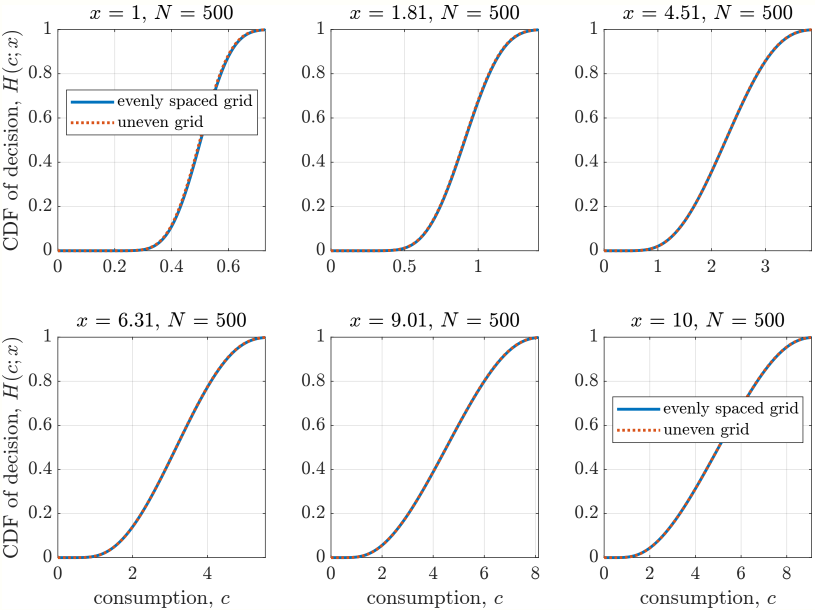
Consumption choices "fan out" with wealth



Violations of Euler equation / deviations from predicted consumption grow on average

- potential driver of right tail of wealth?

Wealth Disregards Euler Equation: Fanning wide of Consmpt



The infinitely-lived savings problem

Extending the environment: $T = 2 \to T = \infty$

Almost everything is the same as the 2-period case, except:

- now assume flow utility and shocks occur each period: $u(c_t) + \epsilon_{c,t}$

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- First consider a finite number of periods, then take limit as $T \to \infty$

- Proceeding as before, we have

$$
\begin{aligned}
V_t(a) &= \alpha \ln \int_0^a \exp\left(\frac{v_t(c_i; a)}{\alpha}\right) dc + \alpha \ln a \\
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  - $\implies V_t(a)$ is strictly concave and differentiable
  - infinite horizon limits exist $V(a)$, h and takes analogous forms

	EV shocks	No EV shocks	EV change
$V(a)$	$\frac{1+\alpha}{1-\beta} \ln a + B$	$\frac{1}{1-\beta} \ln a + \tilde{B}$	steeper slope
$\overline{c}(a)$	$\frac{(1+\alpha)(1-\beta)}{1+\alpha+\alpha(1-\beta)} a$	$(1-\beta)a$	lower avg consumption
$h(c; a)$	$\sim \mathcal{B}\left(\frac{1+\alpha}{\alpha}, \frac{\beta(1+\alpha)}{\alpha(1-\beta)} + 1; 0, a\right)$	-	Beta distribution

To first order, EV shocks act as a specific form of increased patience, but variation around average skews towards savings.

Quantifying the Novel Precautionary Motive

Excercise I: use slope from PSID to estimate α

ind. var.	cash on hand: decile mean			cash on hand: decile rank		
moment	intercept	slope	required α	intercept	slope	required α
PSID data	0.1091	0.0048	-	0.0980	0.0845	-
	(0.0057)	(0.0007)		(0.0096)	(0.0167)	
model with EVS shocks only						
EVS only	0.0742	0.0048	0.1824	0.1265	0.0845	0.3562
add in earnings risk:						
iid	0.0637	0.0048	0.1635	0.1118	0.0845	0.3237
STY (2004)	0.0483	0.0048	0.1143	0.0444	0.0845	0.1441

Notes: Slopes match data to numerical precision by design. Actual regressors for decile rank regressions are 0.05 for decile 1, 0.15 for decile 2, etc. STY (2004) refers to the labor income process of Storesletten, Telmer, and Yaron (2004 JPE).

What does $\alpha = 0.1143$ **mean?** Consider the following exercise:

- solve no EV, Aiyagari economy with earnings process from last row
- increase variance of income until economy has r^* from EV case
- required increase in the unconditional variance of earnings measures the contribution of EV shocks to savings

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- solve no EV, Aiyagari economy with earnings process from last row
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- required increase in the unconditional variance of earnings measures the contribution of EV shocks to savings

Result: the variance of earnings risk must increase by **26-33%.**

- related exercise: with mean 1 iid normally distributed marginal utility shocks, need a standard deviation of θ of 0.465.

- **EV model:** estimate (β, γ, α) to match wealth moment and PSID regression coefficients

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	Data	K-Y (1)		top 20%(2)	
		EV	No EV	EV	No EV
bottom 20%	-0.41	1.18	0.74	0.93	0.64
2nd quintile	0.87	4.54	3.68	3.91	3.55
3rd quintile	3.74	10.4	9.70	8.77	8.74
4th quintile	10.3	20.8	21.5	19.1	19.8
top 20%	85.5	63.1	64.4	67.3	67.2
top 10%	73.3	43.4	44.7	48.8	47.8
top 5%	61.2	28.6	29.5	35.0	32.9
top 1%	34.9	9.60	9.68	13.9	12.6
top 0.1%	12.7	1.25	1.32	3.50	2.90
top 0.01%	4.24	0.150	0.146	0.813	0.593

Exercise II: implications for wealth inequality (shares)

- **EV model:** estimate (β, γ, α) to match wealth moment and PSID regression coefficients
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- **Punchline 1:** EV effect brings UP bottom of distribution (counterfactual)
- **Punchline 2:** also fans out right tail of distribution, conditional on share

Taking Stock: What have we learned?

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- Help with the shape of Euler Equation Errors
- Does not Help with Wealth Dispersion
- The Poor are concerned with this Option to consume
- We now put together these ideas with some form of notion of Superior Goods so that it only affects the "Rich".

- Think of two Subperiods:

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- Think of two Subperiods:
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  - In the afternoon we get extreme value shocks over levels of consumption that we can **Choose to** Ignore
- The fundamental problem we look to solve is

$$\begin{aligned} V(a) &= \max_{y \in [0,a]} u(a-y) + W(y) \\ \text{where } W(y) &= \mathbb{E}_{\epsilon}\left[\max\left\{\beta V(y), \underbrace{\max_{c \in [0,y]} \epsilon(c) + \beta V(y-c)}_{\equiv \tilde{W}(y;\epsilon)}\right\}\right] \end{aligned}$$

- we obtain after many a step (which can be interpreted)

$$
\begin{aligned}
W(y) \quad = \quad & \overline{v}(y) \exp\left\{-\frac{\sum_j w_j(y)}{\overline{w}(y)}\right\} + \alpha \ln\left(\sum_j w_j(y)\right)\left(1 - \exp\left(-\frac{\sum_j w_j(y)}{\overline{w}(y)}\right)\right) \\
& -\alpha \int_0^{\frac{\sum_j w_j(y)}{\overline{w}(y)}} \ln s \exp\{-s\}\, \mathrm{d}s
\end{aligned}
$$

- we obtain after many a step (which can be interpreted)

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$$-\alpha \int_0^{\frac{\sum_j w_j(y)}{\overline{w}(y)}} \ln s \exp\{-s\}\, \mathrm{d}s$$

1st Term "Floor" value $\overline{v}$ times the probability that theoption is chosen.

Problem with Option to Ignore EV Shocks

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$$
\begin{aligned}
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2nd Term Typical expected value of the extreme value branch of the max operator in (??), again times the probability that this branch is chosen

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- Computationally, this formula is still very useful: as long as we have a precise numerical integral of the function in the third term, then we effectively still have a closed form; given the action-specific values embodied in all the w terms, we can compute W directly.

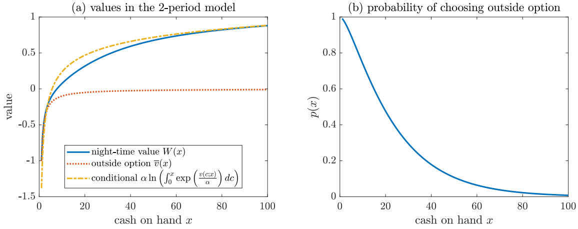


Figure 1: **An example from the 2-period model**

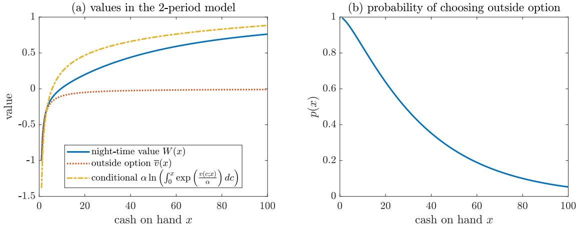


Figure 2: **Low** θ

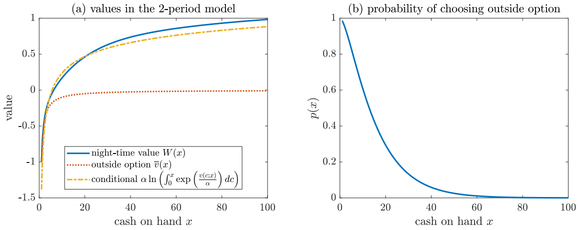


Figure 3: **High θ**

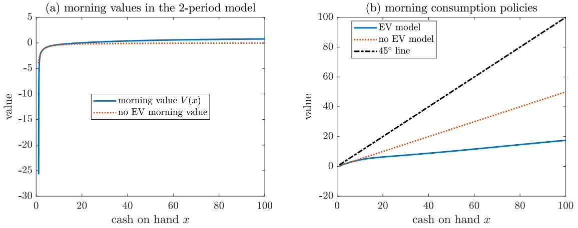
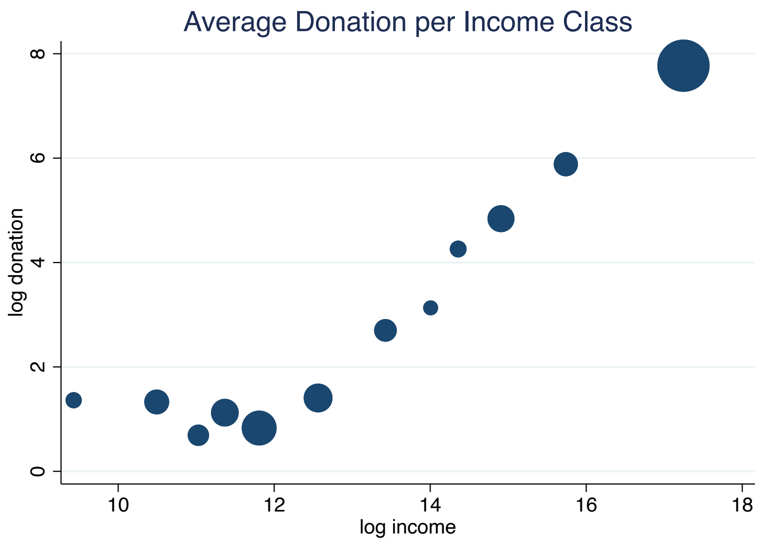
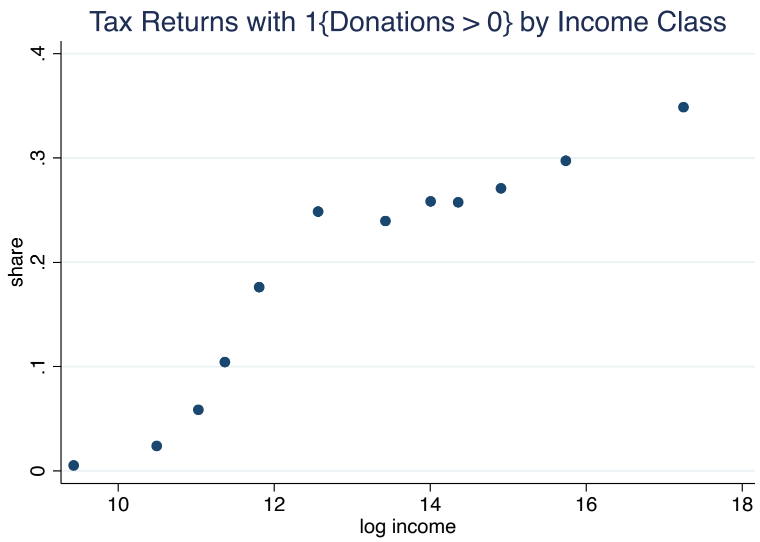


Figure 4: **Beginning of period values and policies**


Average Donation per Income Class
log donation
0
2
4
6
8
10
12
14
16
18
log income


Tax Returns with 1{Donations > 0} by Income Class
share
0
.1
.2
.3
.4
10
12
14
16
18
log income

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  - Improve the fit of the errors
- Still Work to do Here

Conclusions

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- Promising Direction of hte Notions of Option to Choose

Thank you Very Much

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Appendix

Log case: derivation

Guess and verify $V(a) = A \ln a + B$, which implies

$$V(a) = \alpha \ln \int_0^a c^{\frac{1}{\alpha}} (a-c)^{\frac{\beta A}{\alpha}} dc + \beta A \ln(1+r) + \beta B + \alpha \ln a$$

Then the change of variables $y = c/a$ implies

$$V(a) = (1 + \beta A + 2\alpha) \ln a + \alpha \underbrace{\ln \int_0^1 y^{\frac{1}{\alpha}} (1-y)^{\frac{\beta A}{\alpha}} dy}_{=\mathcal{B}(1/\alpha+1, \beta A/\alpha+1)} + \beta A \ln(1+r) + \beta B$$

where $\mathcal{B}$ is the beta function. Proceeding, we obtain

$$\begin{aligned} A &= \frac{1+2\alpha}{1-\beta} \\ B &= \frac{\alpha}{1-\beta} \ln \mathcal{B}\left(\frac{1}{\alpha}+1, \frac{\beta(1+2\alpha)}{\alpha(1-\beta)}+1\right) + \frac{\beta}{1-\beta}\frac{1+2\alpha}{1-\beta} \ln(1+r) \end{aligned}$$

▸ Back to log case main ▸ Decision rule

Log case: decision rule

By plugging in the form of the value function from the log case, we obtain

$$h(c;a) = \frac{1}{a}\frac{\left(\frac{c}{a}\right)^{p-1}\left(\left(1-\frac{c}{a}\right)^{q-1}\right.}{B} \sim \mathcal{B}(p,q;[0,a])$$

- $p = \frac{1}{\alpha} + 1$ and $q = \frac{\beta(1+2\alpha)}{\alpha(1-\beta)} + 1$ are the shape parameters
- B is the constant from the previous slide
- $\mathcal{B}(p,q;[0,a])$ is the (generalized) beta distribution with shape parameters p and q defined over the extended interval $[0,a]$

Back to log case main | Back to log case derivation

MU failure details (I): form of the value function

If we guess that $V(x,\theta) = A(\theta)\frac{x^{1-\gamma}}{1-\gamma}$ for a set of constants $A(\theta)$ with mean $\overline{A} = \sum_\theta \pi(\theta)A(\theta)$. Then, solving the Euler equation yields

$$\frac{c}{(1+r)(x-c)} = \underbrace{\left[\frac{\beta(1+r)\overline{A}}{\theta}\right]^{-\frac{1}{\gamma}}}_{\equiv\Gamma(\theta;\overline{A})} \implies c^*(x,\theta) = \underbrace{\frac{(1+r)\Gamma(\overline{A},\theta)}{1+(1+r)\Gamma(\overline{A},\theta)}}_{\equiv\Lambda(\theta;\overline{A})} x$$

Tomorrow's cash on hand will be

$$x'^*(x,\theta) = (1+r)(x - c^*(x,\theta)) = \underbrace{(1+r)(1-\Lambda(\theta;\overline{A}))}_{\equiv\Delta(\theta;\overline{A})} x$$

and so under the guess of $V(x,\theta)$ (which implies $\overline{V}(x) = \sum_\theta \pi(\theta)V(x,\theta) = \overline{A}\frac{x^{1-\gamma}}{1-\gamma}$),

$$\begin{aligned}
\max_c \theta u(c) + \beta\overline{V}((1+r)(x-c)) &= \theta\frac{(c^*)^{1-\gamma}}{1-\gamma} + \beta\overline{A}\frac{(x'^*)^{1-\gamma}}{1-\gamma} \\
\implies A(\theta)\frac{x^{1-\gamma}}{1-\gamma} &= \left[\theta\Lambda(\theta;\overline{A})^{1-\gamma} + \beta\Delta(\theta;\overline{A})^{1-\gamma}\right]\frac{x^{1-\gamma}}{1-\gamma}
\end{aligned}$$

Given N levels of θ and existing expressions for $\overline{A}$, Λ, and Δ, this is a system of N equations in N unknowns (the $A(\theta)$), and so it must have a unique solution.

MU failure details (II): figure

- MU shocks affect consumption share of wealth along wealth distribution in a homogenous fashion

- make the log consumption figure streamlined, include analog for EV case.

Parameters

▸ Back: alternative economies

parameter	model		value	notes
CRRA		γ	2.0	standard
subjective discount factor		β	0.96	standard for annual model
capital share		λ	0.30	"
depreciation rate		δ	0.072	"
STY (2004) earnings process				
standard deviation, perm comp.	STY	$\sigma(\epsilon_1)$		log-normal, 5-point discret
persistence, persi comp.	STY	$\rho(\epsilon_2)$		AR(1), 10-point discret
st dev, pers comp.	STY	$\sigma(\epsilon_2)$		normally distributed innovation
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coef. of variation, labor productivity	ER	$\sigma(\zeta)$	0.2	2/3 or 1% precautionary savings
coef. of variation, marginal utility	MUR	$\sigma(\theta)$	0.328	match r from ER economy
scale parameter, simple model	EVS	α	0.048	"
scale parameter, full model	EVS+STY	$\tilde{\alpha}$	0.114	calibration to PSID data
augmented transt earnings risk	STY aug	$\sigma(\tilde{\epsilon}_3)$	0.456	match r from EVS+STY Ec
augmented marg ut risk	MUR+STY	$\sigma(\tilde{\theta})$	0.465	match r from EVS+STY Ec

(a) By decile mean of cash on hand

(b) By decile mean of cash on hand

Back

More on simple 2-good case (I)

Assume the following functional forms:

- **EVS good:** $u_1(c_1) = \frac{c_1^{1-\gamma_1}}{1-\gamma_1}$, γ_1 low
- **non-EVS good:** $u_2(c_2) = \frac{(c_2 - \underline{c}_2)^{1-\gamma_2}}{1-\gamma_2}$, γ_2 high
  - $\underline{c}_2 \geq 0$: floor to capture the "necessity" nature of this good
  - $\implies c_1 \leq a - \underline{c}_2$, since an Inada condition holds at $\underline{c}_2$ rather than 0
- **tomorrow:** $u_3(c') = \frac{(c')^{1-\gamma'}}{1-\gamma'}$, $\gamma' \in [\gamma_1, \gamma_2]$ (or just non-EVS)

Fundamental solution: equalize marginal utilities and use up budget

$$c_1^{-\gamma_1} = (c_2 - \underline{c}_2)^{-\gamma_2} = (a - c_1 - c_2)^{-\gamma'}$$
$$\implies c_2 = \underline{c}_2 + c_1^{\frac{\gamma_1}{\gamma_2}} \implies c_1 + c_1^{\frac{\gamma_1}{\gamma_2}} + c_1^{\frac{\gamma_1}{\gamma'}} = a - \underline{c}_2$$

Can solve for c_1 via bisection, then plug into c_2 expression.

More on simple 2-good case (II)

EVS solution: equalize marginal utilities only for non-EVS good and future consumption, use up budget

$$(c_2 - \underline{c}_2)^{-\gamma_2} = (a - c_{1i} - c_2)^{-\gamma'}$$

Can solve for $c_2^*(c_1)$ via bisection, then plug back into budget to get $a'^*(c_1)$

Formulas: 2 goods, 2 periods

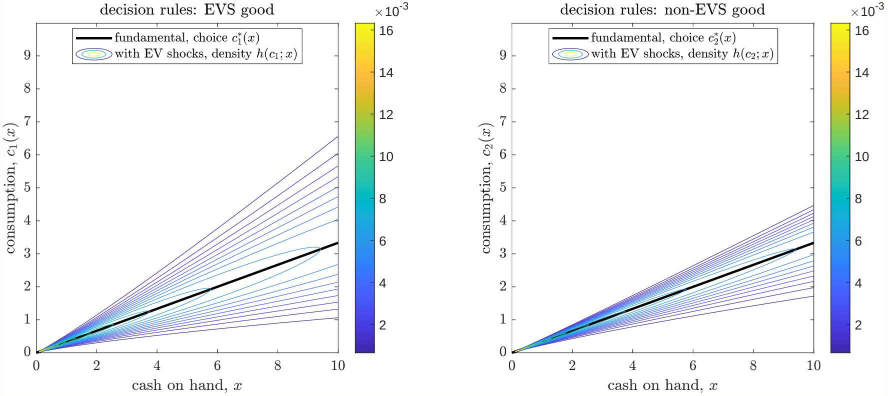
The ex-ante value function and decision rules can then be defined as in the baseline:

$$
\begin{aligned}
V(a) &= \alpha \ln \int_0^a \exp\left(\frac{v_{c_1}(a)}{\alpha}\right) dc_1 + \alpha \ln a \\
h(c_1; a) &= \frac{\exp\left(\frac{v_{c_1}(a)}{\alpha}\right)}{\int_0^a \exp\left(\frac{v_{c_1}(a)}{\alpha}\right) dc_1}
\end{aligned}
$$

Note that the density over c_1 induces a density over c_2 via $c_2^*(c_1)$.

▸ Back

Decision contours: 2 goods, 2 periods, same $u(\cdot)$ function



Back

Empirical approach: predicting consumption

Goal: flexible prediction model of consumption expenditures from PSID

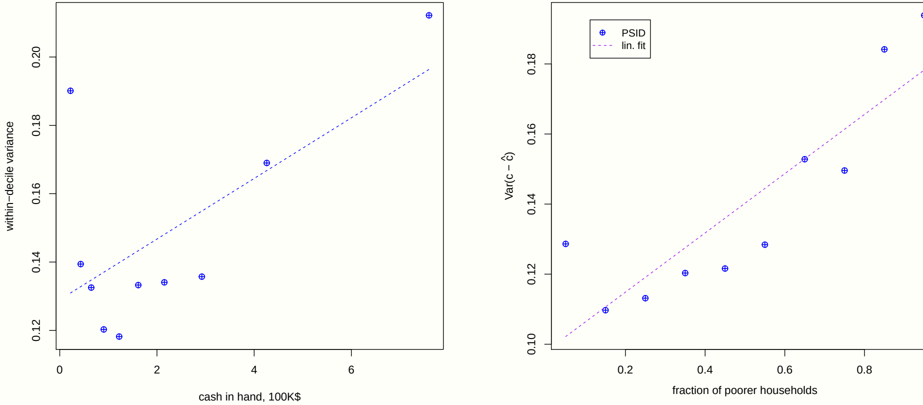
Methodology: proceed in 2 steps

1. adapt Kaplan and Violante (2010) to measure log income
   - 3 components: (i) permanent; (ii) AR(1); and (iii) transitory
2. estimate consumption function $\ln c = g(x_{it}, \eta_{it}, Z_{it})$ where x_{it} is cash on hand, η_{it} is a transitory shock, and Z_{it} is a control vector

Key measurement: define residual $\xi_{it} = \ln c_{it} - \hat{g}(x_{it}, \eta_{it}, \mathrm{Z_{it}})$, then compute variance within deciles

- implementing analogous measure in-model is trivial (no regressions!)

▸ figure: empirical results ▸ back to main



(c) decile mean **(d) decile rank**

back to main back to measurement

Fundamental shocks and finite choice economies

Preferences: consuming $c \in [0, \overline{c}]$ (with $\overline{c}$ non-binding) yields $u(c) + \epsilon_c$

- $u(\cdot)$: standard: strictly concave, differentiable
- ϵ_c: random variables attached to each level of consumption
- no borrowing, $r = 0 \implies$ future utility $u(a - c)$ for wealth / c.o.h. a

Proceed by considering this economy as the **limit of discrete economies**

- indexed by the cardinality N of consumption grid $\{c_i\}_{i=1}^{N}$
- assume $c = 1$ is on the grid at location $M(N)$: $c_{M(N)} = 1$
- grid is "close" to the upper bound, $c_N \geq \overline{c} \geq c_{N-1}$
- equally-spaced grid, take limit as $N \to \infty$ to get continuous objects

back to main

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