On the Determination of Government Debt

Per Krusell
Rochester, IIES, CAERP, CEPR

Fernando Martín
Penn

José-Víctor Ríos-Rull
Penn, CAERP

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Motivation ...

Most private individuals have positive net asset holdings.

Why do governments end up in debt? And why is it so large everywhere?

If lump-sum taxes are available, debt is not important because of Ricardian equivalence (Barro, 1974).

With distortionary taxes, debt is important and we could examine how a benevolent government would choose it over time.

We explore the simplest nontrivial framework with distortionary taxation: the Lucas-Stokey (1983) model.

We are interested in government decision making when there is lack of commitment (and no reputation mechanism).

Motivation

We take a public finance perspective: g is given, and the task is whether to finance it now or later. The instrument is labor taxes.

We assume that saving is not possible in the aggregate. Interest rates are affected anyway, though, leading the government to possibly "manipulate" interest rates. In particular, a government with debt wants to *lower* the interest rate so as to decrease the debt service which reduces taxes and distortions.

This model can be viewed as a microfundation for Barro (1979), who suggested that taxes should be smoothed.

So, given some initial government debt what is the optimal way of choosing debt over time? The answer depends on whether one assumes that the government has access to commitment.

Findings...

Governments think they are worse taxers than later governments. Why?

- So as to ease the tax burden, they try to lower the interest rate by increasing current consumption.

- This is done by taxing less now, so that people work harder and produce more output: a shift from tax to debt finance.

With commitment, governments issue a little bit more debt at the beginning and then never again.

- Never again because lower taxes at a future date decreases the interest rate at that date but increases it in the previous period: a wash.

- The implied time path: an initial debt increase and then the debt stays constant.

...Findings

Without commitment, one might expect increases in debt every period, since the economy "restarts" every period with a new, manipulative government.

So we wonder whether any debt is feasible and in general what happens.

The economy

Lucas-Stokey model.

Infinitely lived representative agent.

No capital accumulation.

Linear production technology

$$y = \ell \Rightarrow c + g = \ell$$

Benevolent government finances a given exogenous expenditure with debt and labor income taxes

$$g + b = \tau\ell + qb'$$

No default on debt.

The problem of the agent

Agents maximize

$$\sum_{t=0}^{\infty} \beta^t u(c_t, 1-\ell_t)$$

subject to

$$c_t = (1-\tau_t)\,\ell_t + b_t - q_t b_{t+1}$$

Switching to recursive notation, the FOCs are

$$\begin{aligned}(1-\tau)u_c &= u_\ell \\ q &= \frac{\beta u_c'}{u_c}\end{aligned}$$

These FOCs and the government budget constraint imply

$$\eta\left(b,\ell,b',\ell'\right) = \left(1 - \frac{b+g-b'\frac{\beta u_c'}{u_c}}{\ell}\right) u_c - u_\ell = 0$$

The case with commitment

Given b_0, the government maximizes

$$\sum_{t=0}^{\infty} \beta^t u(\ell_t - g, 1 - \ell_t) \qquad \textbf{subject to}$$

$$\begin{aligned} \eta\left(b_t, \ell_t, b_{t+1}, \ell_{t+1}\right) &= 0 \\ \lim_{t\to\infty} \left(\prod_{s=0}^{t} q_s\right) b_{t+1} &\leq 0 \end{aligned}$$

If ℓ_{t+1} had not appeared in the constraint at t, this would have been a standard recursive problem.

Time inconsistency: the FOC has one more term for $t > 0$ than for $t = 0$, since ℓ_t appears in the constraint at $t - 1$ (simply put, there is one less constraint at $t = 0$).

Solution under commitment

Interest rate manipulation: b_{t+1} influences ℓ_t, which in turn influences q_t and q_{t-1}. In particular, an increase in b_{t+1} increases ℓ_t and so it

1. **Increases q_t, so lowers the interest rate between t and $t+1$.**
2. **Lowers q_{t-1}, so raises the interest rate between $t-1$ and t.**

For a government with debt, the first effect is good: it is cheaper to borrow; the second is bad.

It is possible to show that for all u the solution is that both effects cancel out when choosing b_{t+1}, $t > 0$. So, $b_{t+1} = b_t$, $t > 0$.

Time inconsistency: with initial debt $b_0 > 0$, $b_{t+1} > b_0$ for all t : borrowing increases relative to

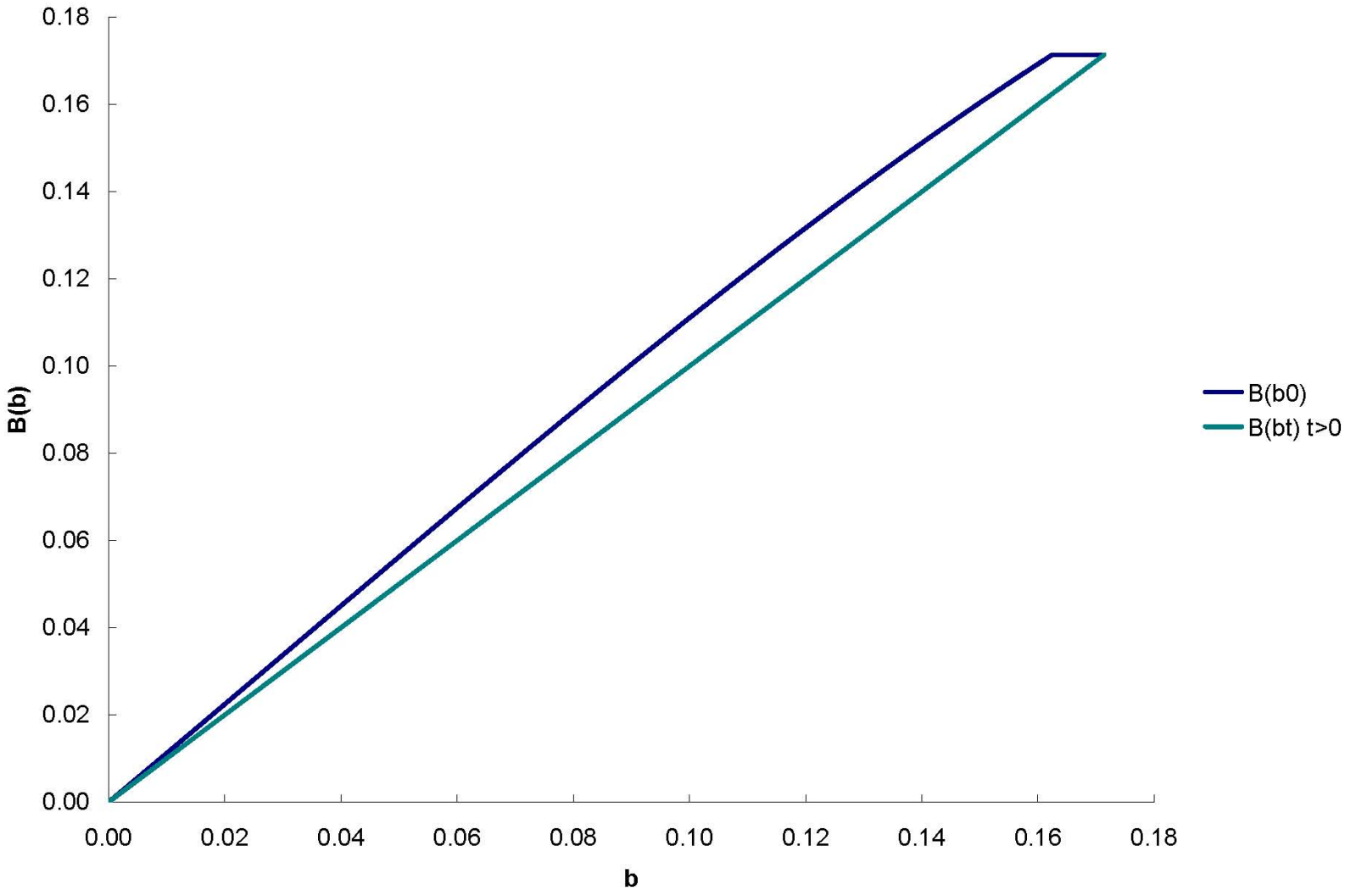

Figure 1: The Solution with Commitment

A special case where the solution is time-consistent

Assume linear utility in consumption

$$u(c_t, 1-\ell_t) = c_t + v(1-\ell_t)$$

The price of bonds is now "exogenous", i.e. $q_t = \beta$, $\forall t$.

The constraint becomes recursive

$$1 - \frac{b_t + g - \beta b_{t+1}}{\ell_t} - v_\ell(1-\ell_t) = 0$$

The solution is easy: "distortion smoothing", i.e.

$$b_t = b_0$$

Hence, we reproduce the Friedman/Barro tax smoothing result.

Time-consistent solutions without commitment

Given a perception that future governments will induce private labor supply $L(b)$ for any b, the government solves

$$v(b) = \max_{\ell,\, b'} u(\ell - g, 1 - \ell) + \beta\, v(b')$$

subject to

$$\eta(b, \ell, b', L(b')) = 0$$

A Markov Perfect Equilibrium is a set of functions $\mathcal{B}(b)$, $L(b)$ and $v(b)$ that solve the above problem.

Notice that we have a fixed-point problem: $L(b)$ is given in the dynamic-programming problem (the decision maker solving the dynamic problem takes the function as exogenous).

Question: what happens with debt, i.e. what does $\mathcal{B}(b)$ look like?

Maximum possible steady state

There is a maximum sustainable level of debt.

Define implicitly $s(b)$

$$\eta\left(b, s(b), b, s(b)\right) = 0$$

which due to the Laffer curve has two solutions, $s^+(b)$ and $s^-(b)$, with $s^+(b) \geq s^-(b)$.

$s^+(b)$ implies lower taxes and hence is on the "good side" of the Laffer curve.

There is a $\overline{b}$ that is the maximum b for which the above equation has a solution.

That is, $\overline{b}$ is the maximum sustainable level of debt when revenues are maximized (maximum of static Laffer curve, where $s^+(\overline{b}) = s^-(\overline{b})$).

Note that if $b^* \leq \overline{b}$ is a steady state, then labor has to be $s^+(b^*)$.

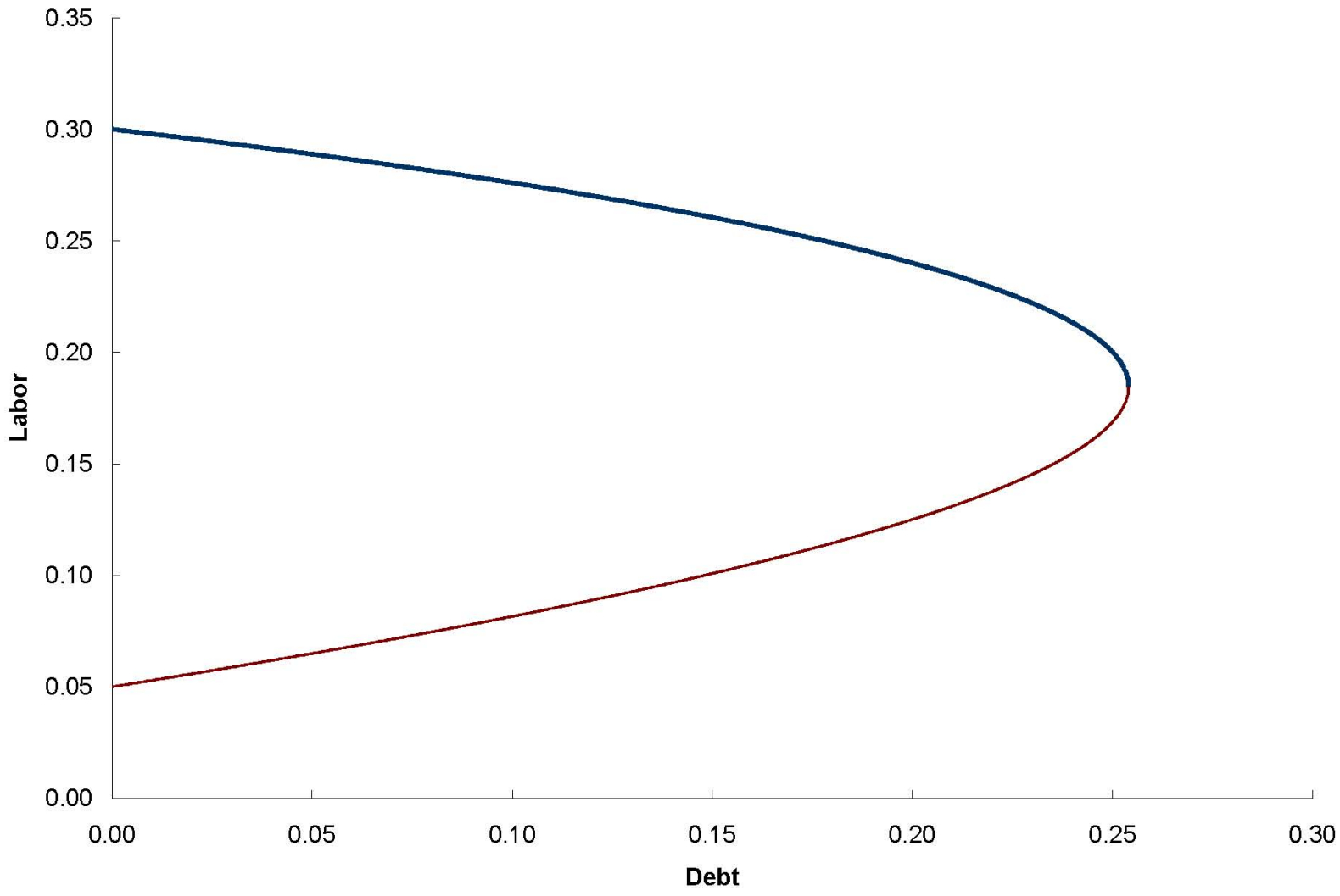

Figure 2: Static Laffer Curve

Maximum possible steady state, 2

The implied maximum debt $\overline{b}$ is too large because of dynamic considerations. Define $\widehat{s}(b)$ as

$$\eta\left(b, \widehat{s}(b), b, s^{+}(b)\right) = 0$$

which has two solutions as well.

One would expect $\widehat{s}^{+}(b) = s^{+}(b)$, but for high enough b we get $\widehat{s}^{+}(b) > s^{+}(b)$. How is this possible?

Say $b' = b$ and $\ell' = s^{+}(b)$. If we lower taxes today, labor increases relative to labor tomorrow. This implies the current interest rate decreases. For high enough b, cheaper borrowing offsets the loss of tax revenue and thus keeping debt constant is feasible.

The problem is $\widehat{s}^{+}(b) > s^{+}(b)$ cannot be a steady state since it's not feasible: $\eta\left(b, \widehat{s}^{+}(b), b, \widehat{s}^{+}(b)\right) \neq 0$.

Maximum possible steady state 3

•Hence, only those b for which $\widehat{s}^{+}(b) = s^{+}(b)$ are feasible steady states. Call the maximum of those $b_{\max}$.

•$b_{\max}$ can also be a steady state since it is an absorbing state. In fact, $b_{\max}$ is the maximum possible level of debt (this is the pay-back constraint).

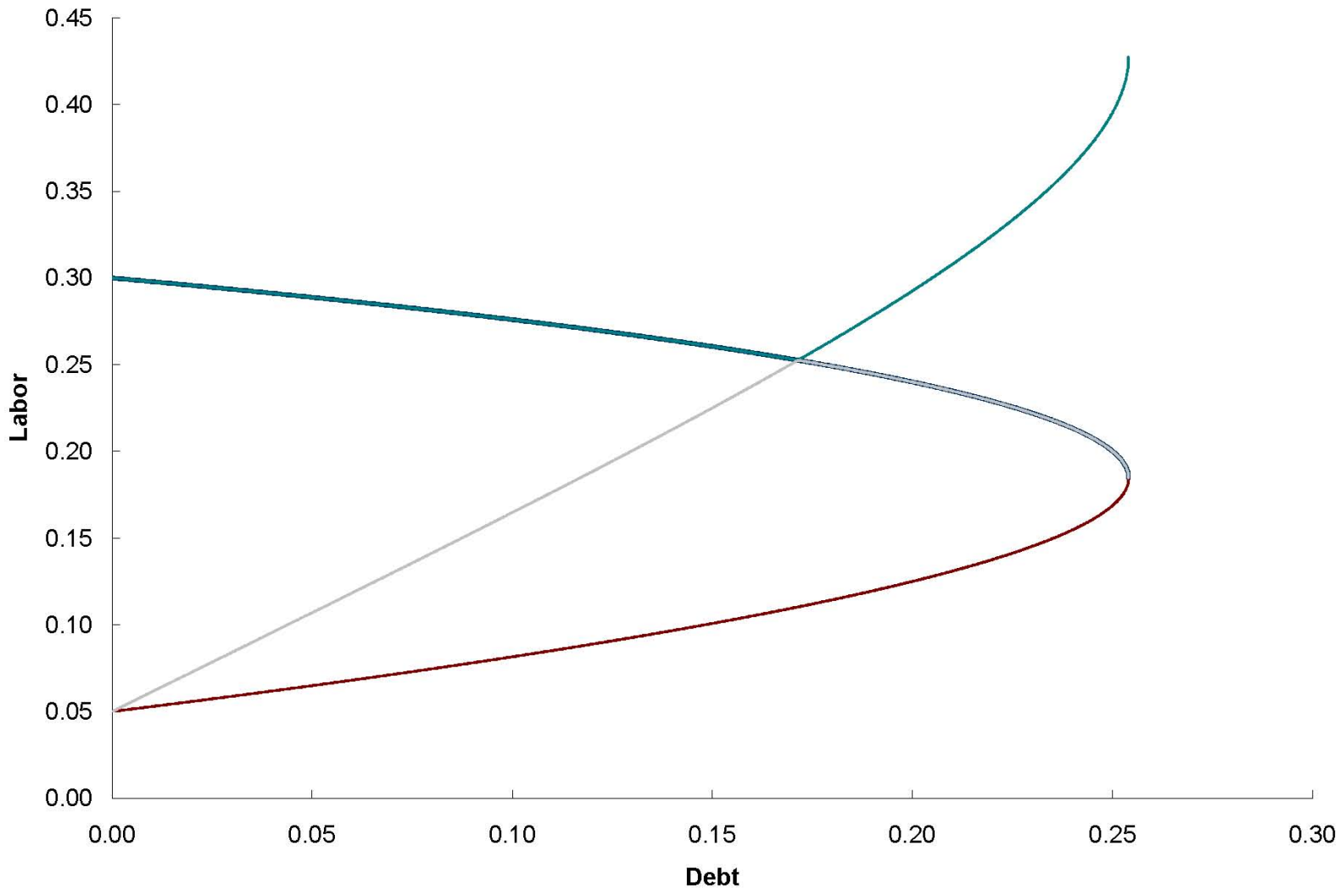

Figure 3: Static and dynamic Laffer curves

The GEE

The FOC of the government (GEE) is

$$(u_c - u_\ell) + \beta\,(u'_c - u'_\ell)\,\frac{\eta_\ell \eta'_b}{\eta'_\ell(\eta_{b'} + \eta_{\ell'} L'_b)} = 0$$

In steady state the GEE reduces to

$$(u_c - u_\ell)\left(1 - \frac{x}{x + \beta \frac{b}{\ell}\frac{du_c}{d\ell} L_b}\right) = 0$$

where $x = \beta \frac{u_c}{\ell}$.

Hence, there are only 2 possible steady states that satisfy the GEE: $u_c - u_\ell = 0$ and $b = 0$ (we can rule out $L_b = 0$ since it implies locally exploding debt).

So there are at least 3 steady states: the Pareto optimum, no debt and $b_{\max}$.

Are there more steady states?

- **YES. Countably many, converging to zero.**

- **We will construct them. Starting from the right, there is a steady state $b_{\max}$. Then we will construct the equilibrium function $\mathcal{B}(b)$ and show its properties.**

- **Let's preview it.**

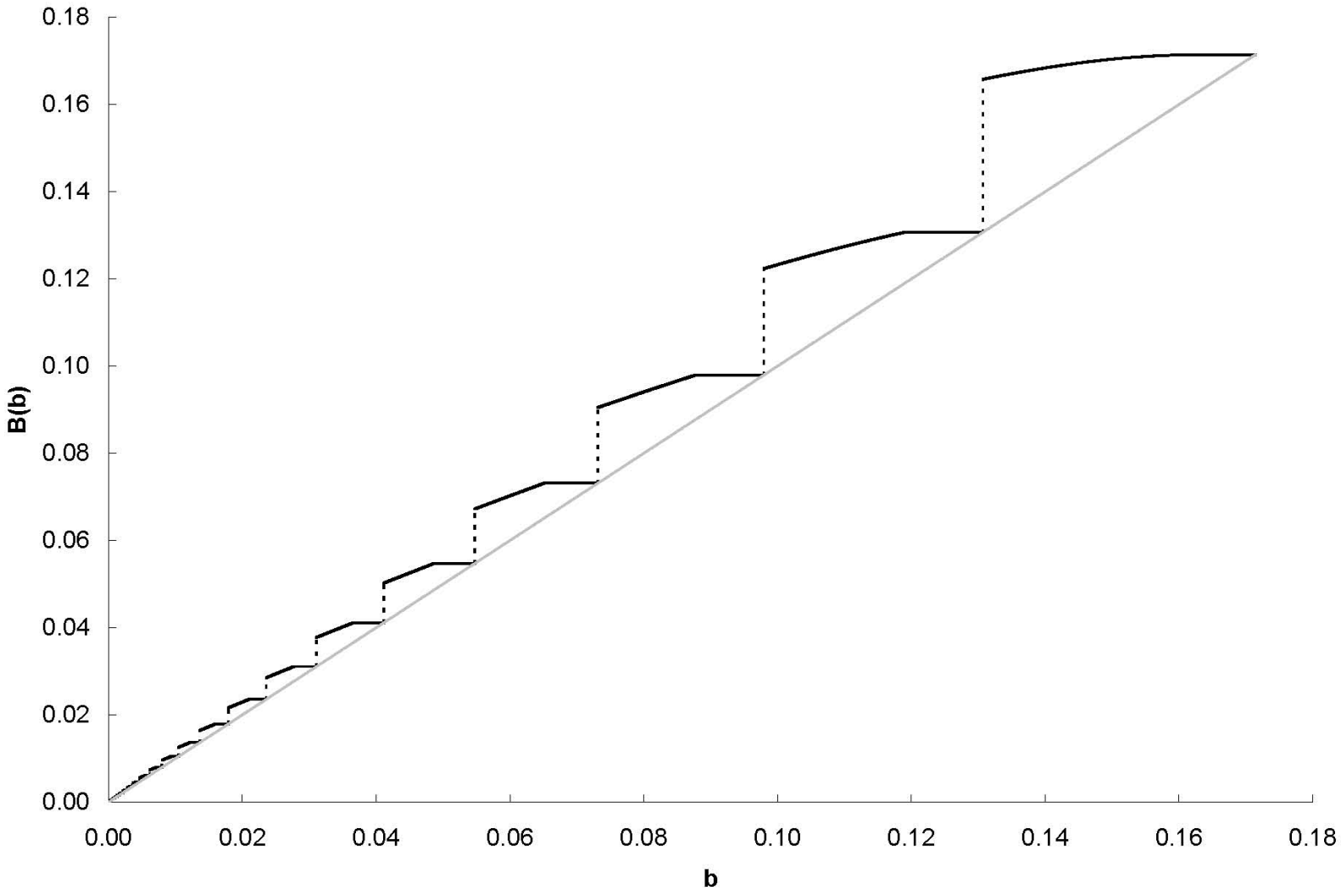

Figure 4: Solution without commitment: debt function

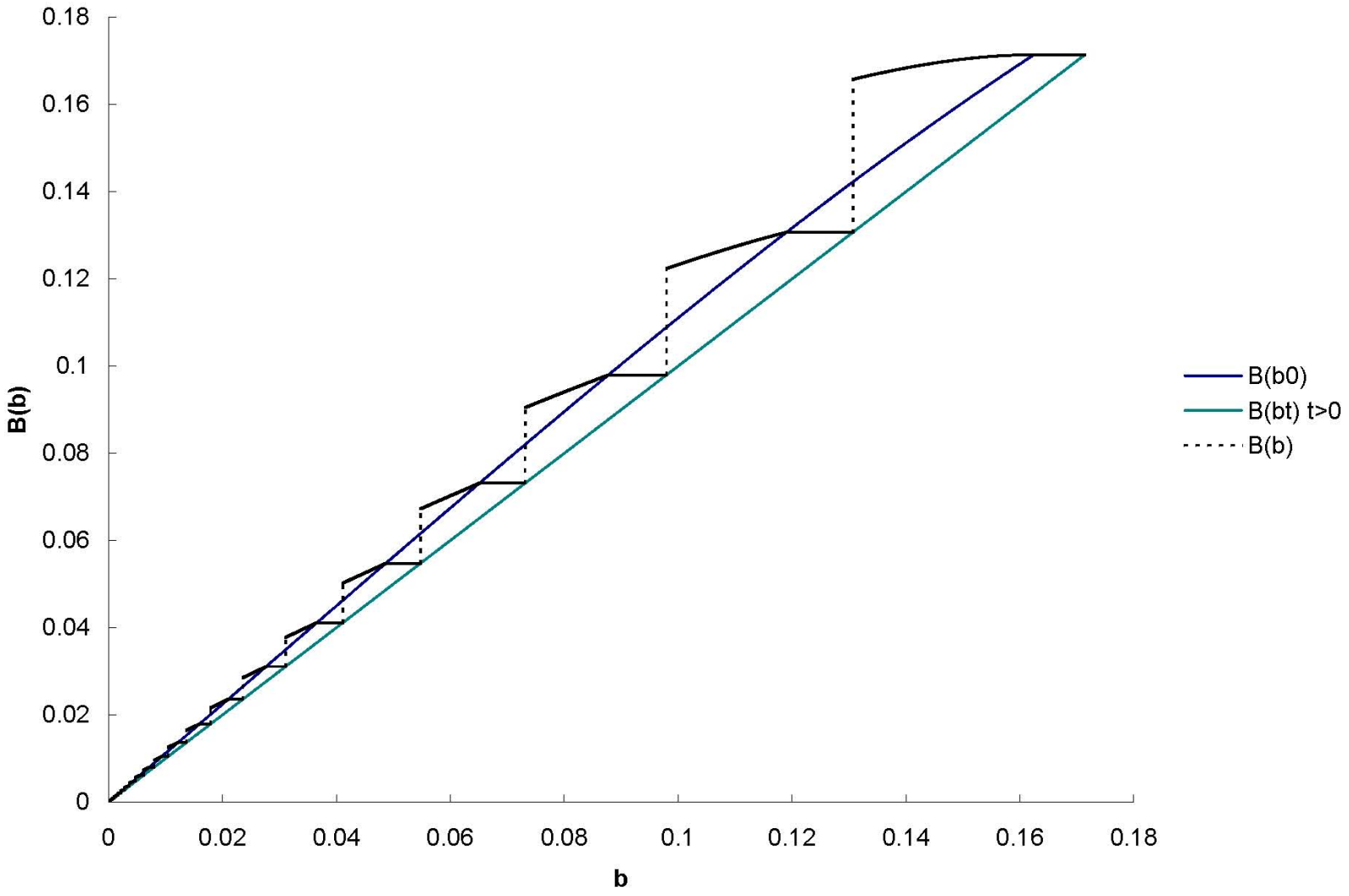

Figure 5: Commitment and no commitment solutions

Constructing $\mathcal{B}(b)$

Theorem: fix $b_n \in (0, b_{\max}]$. Our $\mathcal{B}(b)$ is an equilibrium along some interval $[b_{n+1}, b_n]$, where $b_{n+1} \in (0, b_n)$. Logic of the proof:

1. **$\exists \widehat{b} \in (0, b_n)$ such that $GEE(\widehat{b}_n, b_n) = 0$.**

2. **$\exists \theta(b)$ such that if $b'' = b_n$ then $GEE(b, \theta(b)) = 0$ and $\theta(b) > b$ is continuous and strictly increasing in b.**

3. **$\exists b_n^* \in (0, b_n)$ such that $\theta(b_n^*) = \widehat{b}_n$. Restrict the domain of $\theta(b)$ to $\left[b_n^*, \widehat{b}_n\right]$.**

4. **Find b_{n+1} (steady state to the left of b_n) and show $b_{n+1} \in (b_n^*, b_n)$.**

5. **Construct $\mathcal{B}(b)$.**

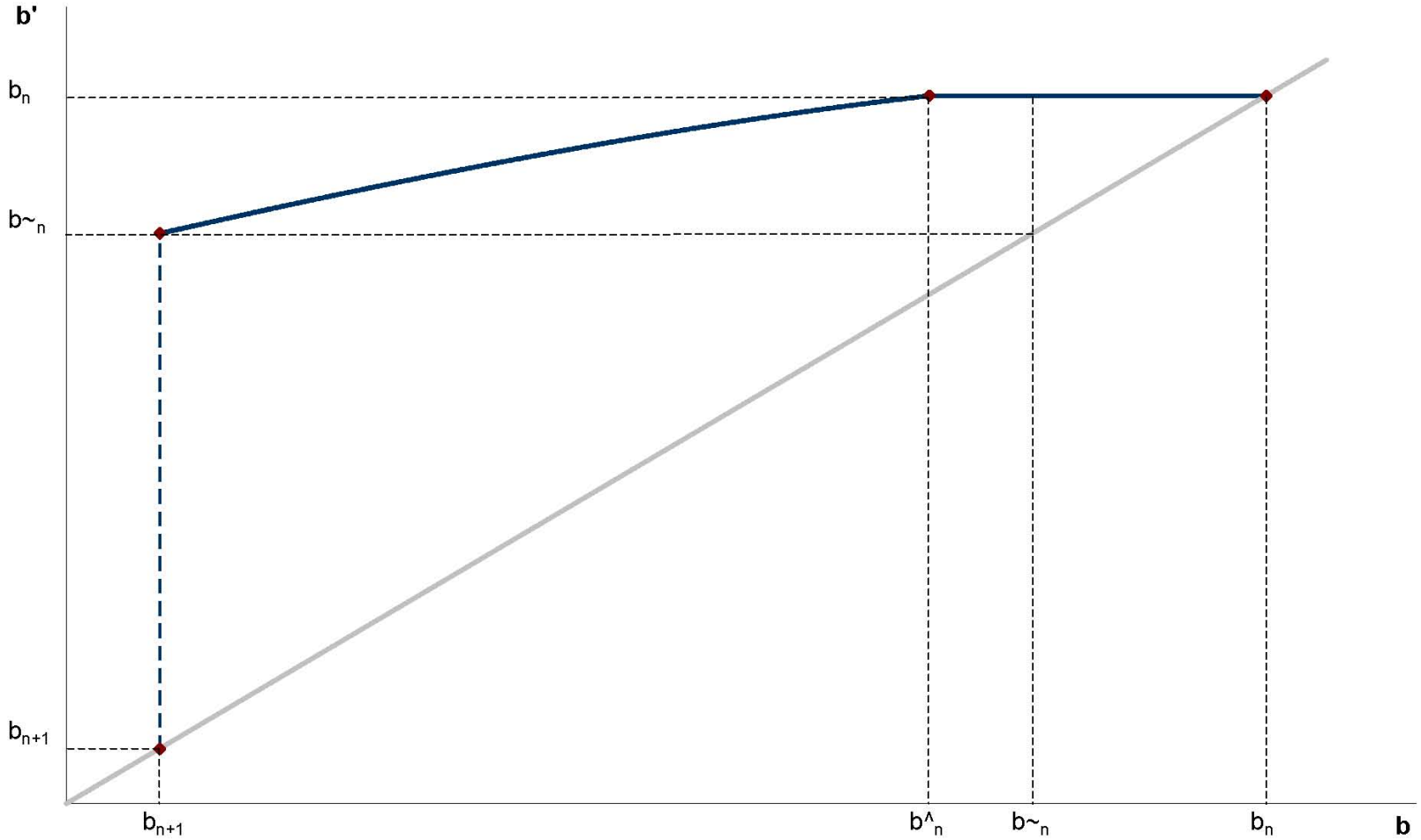

Figure 6: Typical $\mathcal{B}(b)$ for $b \in [b_n, b_{n+1}]$

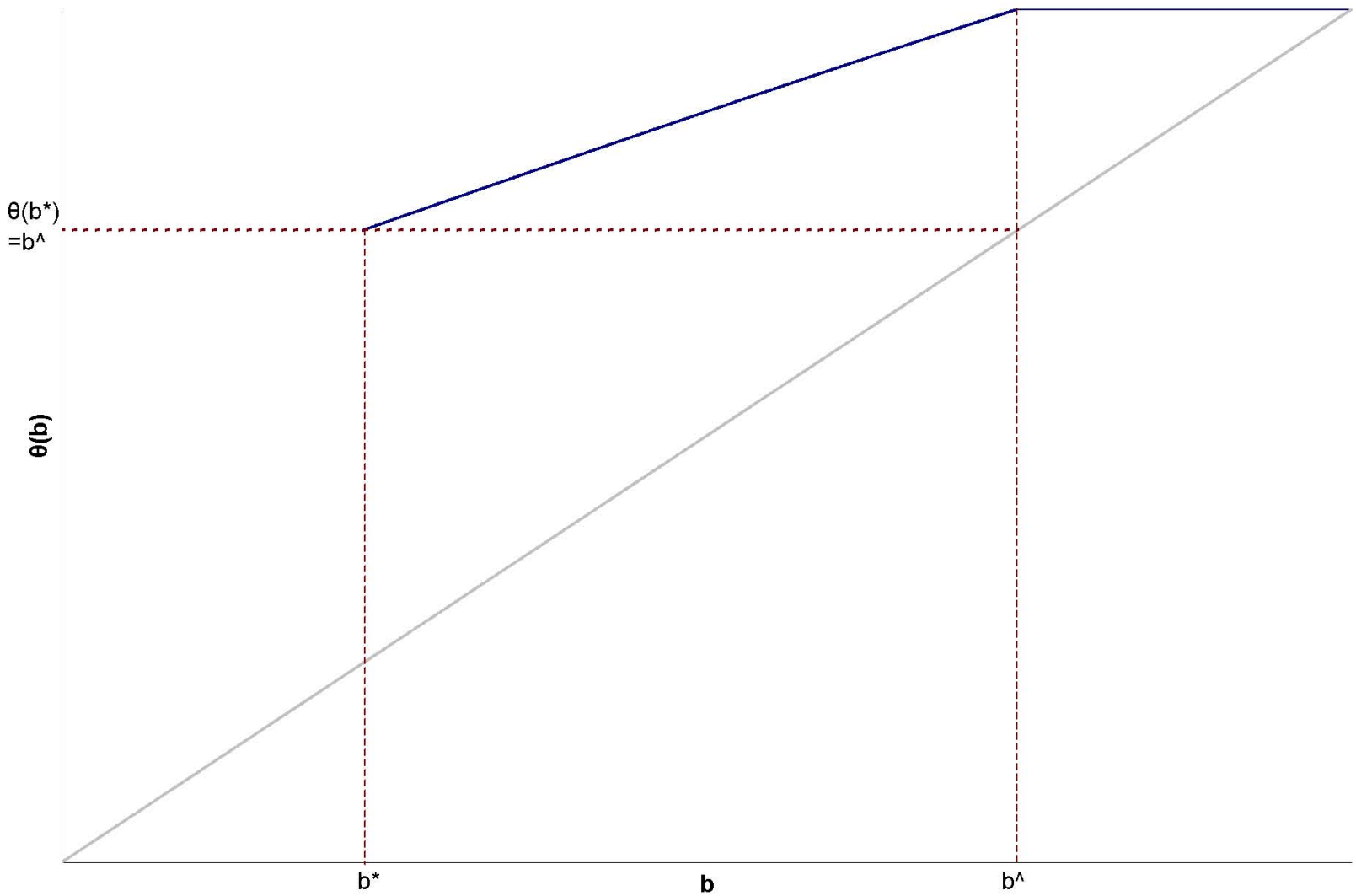

Figure 7: $GEE(b, \theta(b)) = 0$

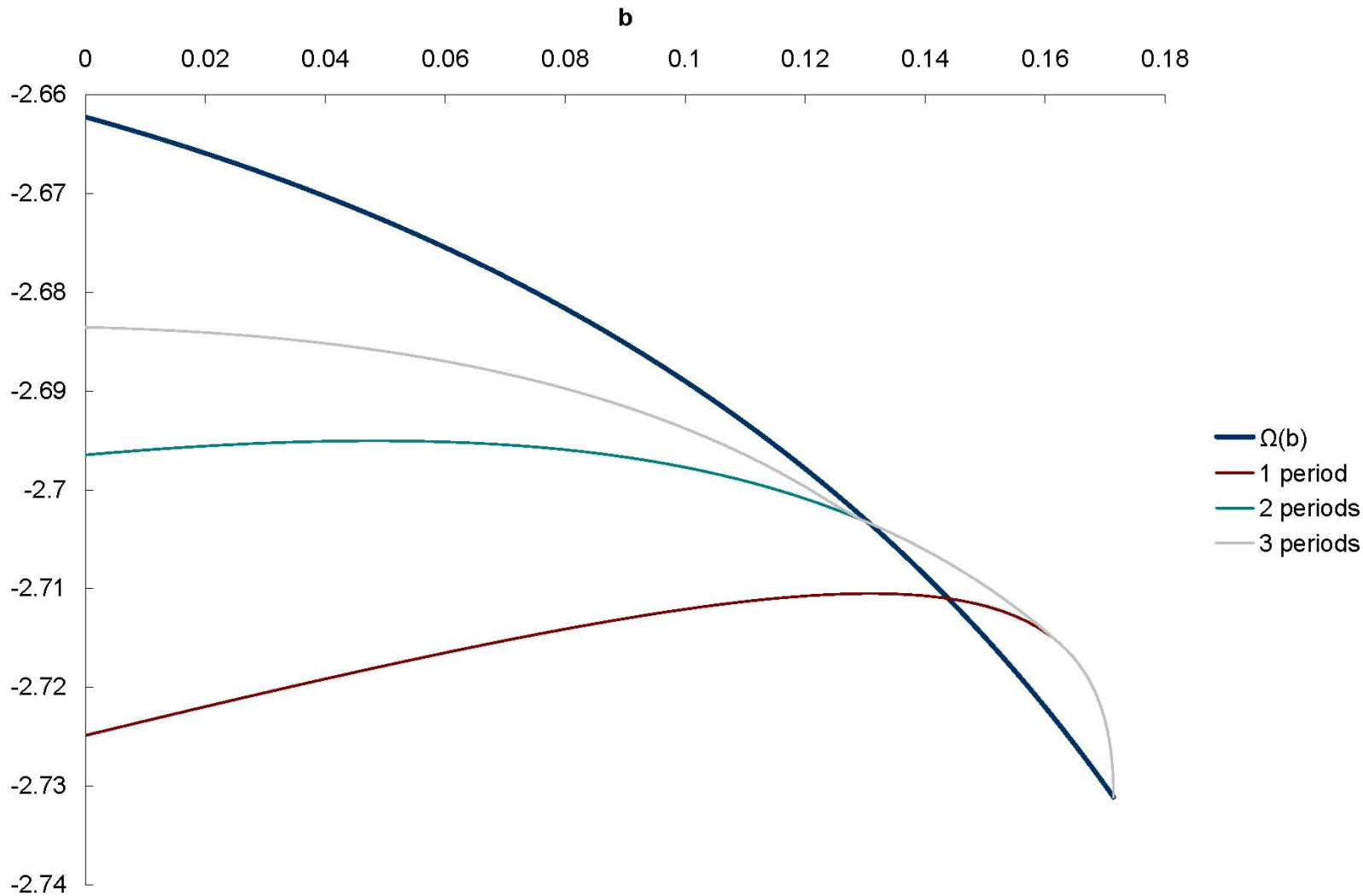

Figure 8: Value of constant debt and of going to b_n in at most 1, 2 or 3 periods

Construct $\mathcal{B}(b)$

Let

$$\mathcal{B}(b) = \begin{cases} b_n & \text{for} \quad \widehat{b} < b \leq b_n \\ \theta(b) & \text{for} \quad b_{n+1} < b \leq \widehat{b} \\ b_{n+1} & \text{for} \quad b = b_{n+1} \end{cases}$$

•To construct $\mathcal{B}(b)$ for all $b \in [0, b_{\max}]$, start with $b_0 = b_{\max}$ and get $\mathcal{B}(b)$ between b_1 and $b_{\max}$; then continue the process.

Properties of $\mathcal{B}(b)$

- **It is an equilibrium.**

- **By construction $\mathcal{B}(b)$ maximizes the government problem within the $[b_{n+1}, b_n]$ interval. At $\widehat{b}$ the solution coincides with the solution under commitment.**

- **There is a countable number of steady states.**

- **There are infinitely many close to zero.**

$\mathcal{B}(b)$ is the limit of finite horizon economies

If we iterate backwards from the end of the world at T, we get

- $b_{\max,t}$ **converges to** $b_{\max}$ **as we move away from** T**.**

- **First discrete jump appears at** $T-2$**.**

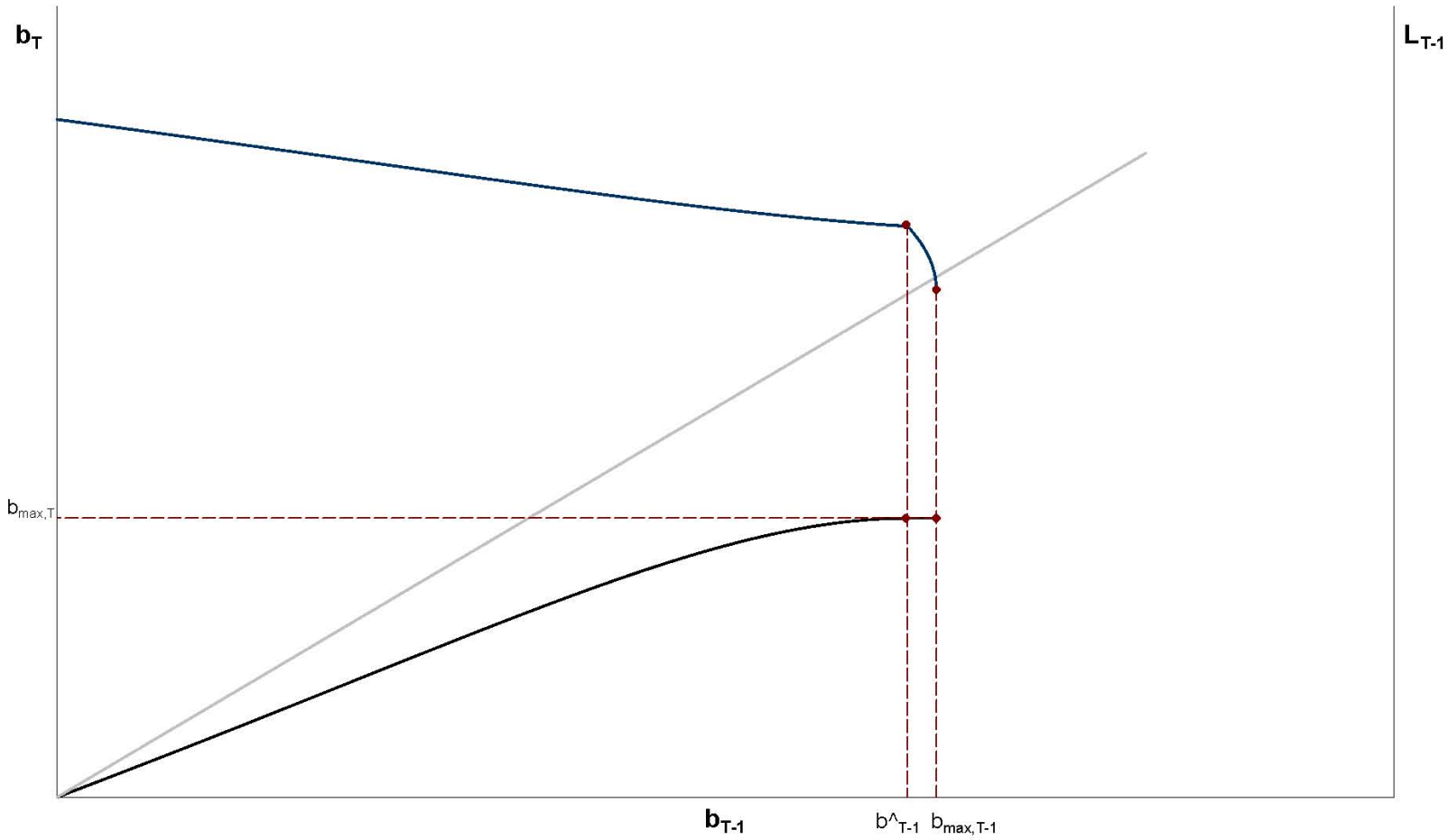

Figure 9: Debt and labor one period from end ($T-1$)

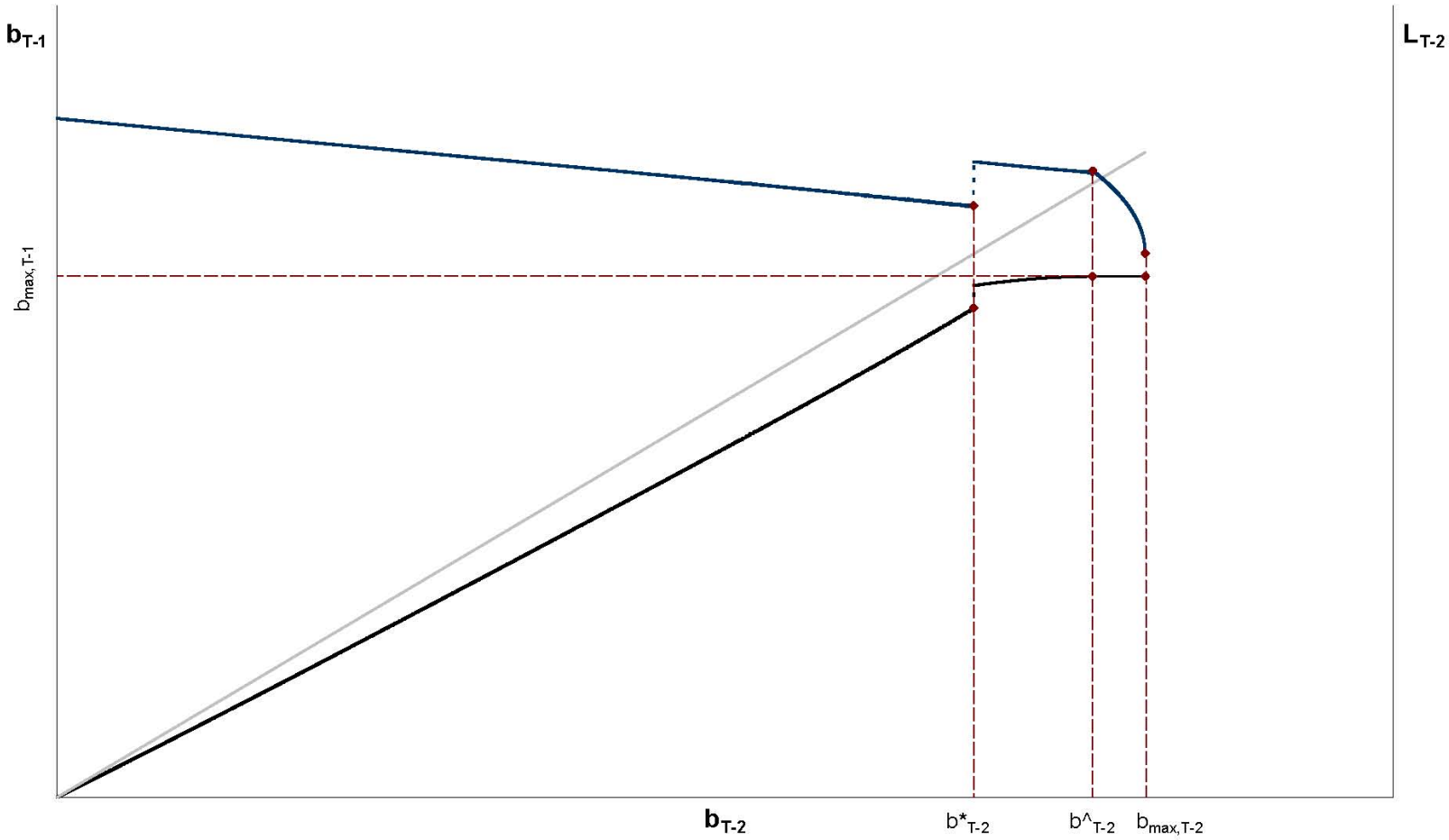

Figure 10: Debt and labor two periods from end ($T-2$)

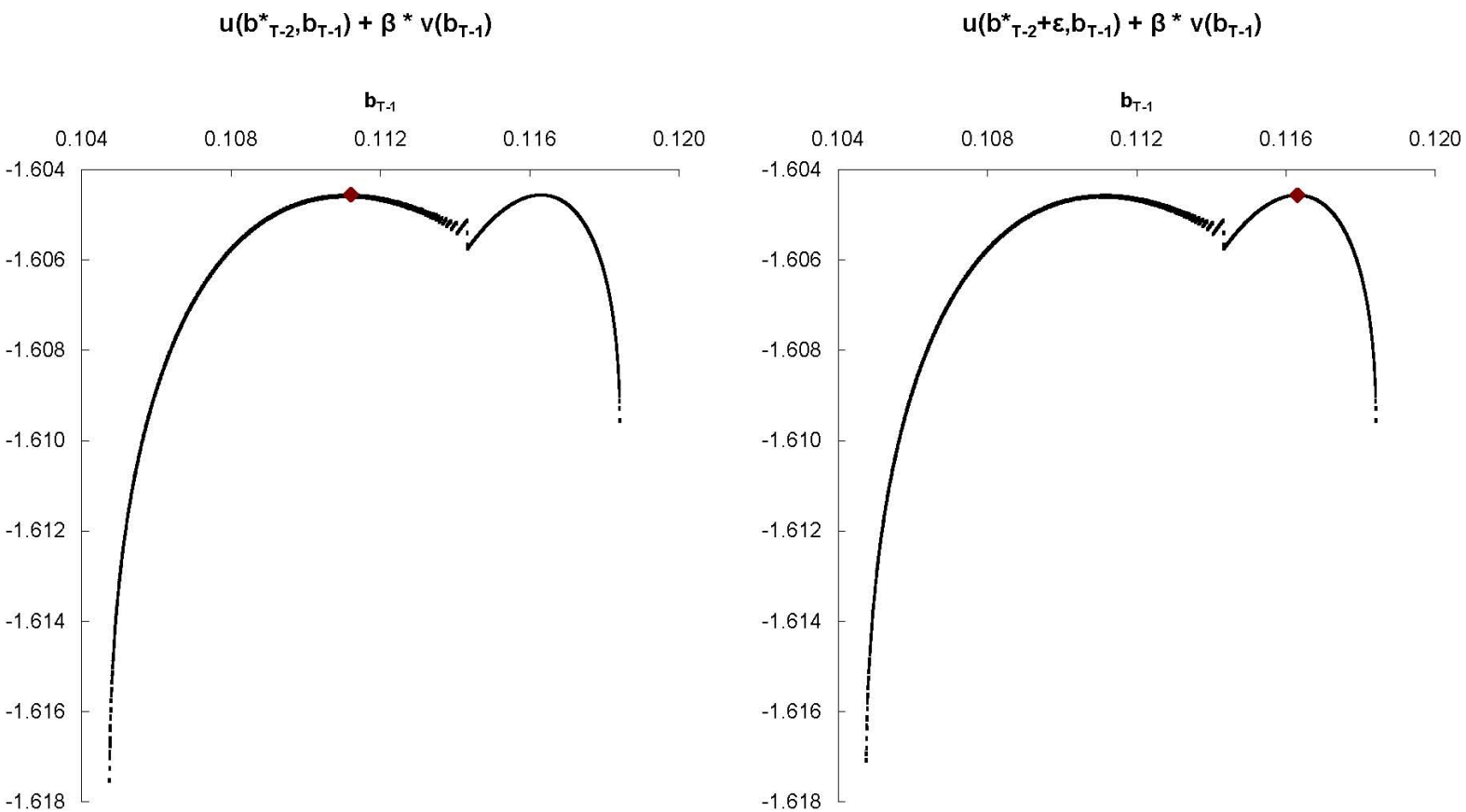

Figure 11: Right hand side of Bellman equation for b^*_{T-2}

What happens for negative debt?

- **There are infinite equilibria.**

- **We can use the same logic as before to construct self-contained intervals $[b_{n+1}, b_n]$.**

- **We can do it by starting on any arbitrary level of debt $b^* \in (b^{PO}, 0)$ and move in either direction.**

- **Each equilibria is indexed by the anchor b^* and contains a countable number of steady states converging to the extremes $\{b^{PO}, 0\}$.**

Conclusions ...

•We analyze an optimal tax problem where interest-rate manipulation is possible: the commitment solution is time-inconsistent.

•We use the popular Lucas-Stokey model. Nobody knows what happens without commitment (and without reputation mechanisms that resuscitate commitment).

•We do and find that, analytically, it has very different features than the commitment solution. Equilibrium debt management—next period's debt as a function of current debt—is a *step function*. A countable, infinite set of stationary debt levels exist.

•Time paths with and without commitment, are surprisingly similar.

•We conjecture that similar properties will hold in economies with capital.

... Conclusions

•**We think that failures in the constitutional design, i.e. the lack of institutions capable of guaranteeing commitment are NOT the reason for the high debt in most countries.**

•**We think that the next line of attack is disagreement about what type of expenditures and the placing of constraints in future governments.**

•**But we have to investigate these issues (capital, richer maturity of debt, existence of uncertainty) which we have not done yet.**

maybe slides

...Findings B

Without commitment, one might expect increases in debt every period, since the economy "restarts" every period with a new, manipulative government. But this guess is wrong!

•Instead, debt increases for one or two periods and is then constant.

•This result, which seems general, is driven by the pay-back (no-Ponzi-game) constraint.

Thus, behavior with and without commitment are surprisingly similar: debt increases once or twice and then stays constant.

However, there are differences, too: without commitment,

•only a countable number of long-run debt levels can occur in equilibrium; and

•for a large part of the space of initial debt levels, long-run debt does not respond to increases in initial debt.