Wealth, Wages, and Employment

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Preliminary

Introduction

- We pose an environment where the joint distribution of employment, wages, and wealth, is determined and where
  - Workers are risk averse, so only use self-insurance.
  - Workers sometimes lose their jobs or quit or switch generating gross flows that are a form of employment and wage risk.
  - The economy aggregates into a modern economy (total wealth, labor shares, consumption/investment ratios)
  - Business cycles can be studied. In particular, we want to study gross employment flows jointly with the other standard objects.
- We use the volatility of gross flows to estimate the extent of wage rigidity.

Literature

- The steady state of this economy has as its core Aiyagari (1994) meets Merz (1995), Andolfatto (1996) meets Moen (1997).

- Related Lise (2013), Hornstein, Krusell, and Violante (2011), Krusell, Mukoyama, and Şahin (2010), Ravn and Sterk (2016, 2017), Den Haan, Rendahl, and Riegler (2015).

- Especially Eeckhout and Sepahsalari (2018), Chaumont and Shi (2022), Griffy (2021).

What are the uses?

- The study of Business cycles including gross flows in and out of employment, unemployment and outside the labor force
- Policy analysis where now risk, employment, wealth (including its distribution) and wages are all responsive to policy.
- Get some insights into the extent of wage rigidity
- Life-Cycle versions of these ideas (under construction) will allow us to assess how age dependent policies fare.

Today: Build the Theory Sequentially and discuss & Fluctuations from two types of shocks

- In Steady State
  1. Exogenous Job Destruction and Worker Quits. Built on top of Growth Model. (GE version of Eeckhout and Sepahsalari (2018)): Not a lot of wage dispersion. Not a lot of job creation in expansions.
  2. Add Endogenous Quits: Higher wage dispersion may arise to keep workers longer (quits via extreme value shocks). trumps wages and wage
  3. On the Job Search workers may get outside offers and take them. (Similar but not the same as in Chaumont and Shi (2022)).
  4. Outside of the Labor Force
  5. All of the Above
- Outside Steady State Employers commit to a wage schedule $w(z)$ that depends on the aggregate state.

Key Findings

- If wages are fully fixed and committed (Drastic Wage rigidity)
  - Both endogenous quits and on-the-job yield counter factual procyclical unemployment and massive on the job search.
  - Allowing the wage of an already formed job match to respond some to aggregate shocks corrects this.
  - Getting the right relative volatility of old and new wages and the amount of job-to-job moves and quits provides a way to measure wage rigidity.
- With partial wage rigidity the model fares reasonably well with the data. A few things still to improve. (Excessive Job-to-JOB transitions)
- Similar behavior to that in the Shimer/Hagedorn-Manowski debate. Here we can try to move towards an accommodation of both points of view.

A Brief Look At Data: Relevant Volatility Properties in U.S.

	Mean Perc	St Dev Relt to Output	Correl w Output	Source
Average Wage	-	0.44-0.84	0.24-0.37	Haefke et al. (2013)
New Wage	-	0.68-1.09	0.79-0.83	Haefke et al. (2013)
Unemployment	4-6	4.84	-0.85	Campolmi and Gnocchi (2016)
Annual Quits	10-40	4.20	0.85	Brown et al. (2021)
Annual Switches	25-35	4.62	0.70	Fujita and Nakajima (2016)
Consumption	75	0.78	0.86	NIPA
Investment	25	4.88	0.90	NIPA

Developing the Model in Incremental Steps

1. Exogenous Quits (GE version of Eeckhout and Sepahsalari (2018))
2. Endogenous Quits using Extreme Value Shocks
    - A detour with Aiming Shocks
3. On the Job Search (related to but different from Chaumont and Shi (2022)).
4. Outside the Labor Force

1- Exog Quits: Precautionary Savings, Competitive Search

- Jobs are created by firms (plants). A plant with capital plus a worker produce one unit of the good
  - Firms pay flow cost $\bar{c}$ to post a vacancy in market $\{w, \theta\}$.
  - Firms cannot change the wage afterwards (like a machine programmed to pay w)
  - Plants (and their capital) are destroyed at rate δ^f.
  - Workers quit exogenously at rate δ^h leaving firms idle.
- Households differ only in wealth and wages (if working).
- No state contingent claims, nor borrowing.
  - If employed, workers get w and save.
  - If unemployed, workers produce b and search in some market $\{w, \theta\}$.
- General equilibrium: Workers own firms.

Order of Events of Exog Quits Model

1. Households enter the period with or without a job: $\{e, u\}$.
2. Production & Consumption: Employed produce z on the job. Unemployed produce b at home. They choose savings.
3. Firm Destruction and Exogenous Quits:
   Some Firms are destroyed (rate δ^f) They cannot search this period.
   Some workers quit their jobs for exogenous reasons δ^h.
   Total job destruction is δ.
4. Search: Firms and the unemployed choose wage w and tightness θ.
5. Job Matching : $M(V, U)$: Some vacancies meet some unemployed job searchers. A match becomes operational the following period.
   Job finding and job filling rates $\psi^h(\theta) = \frac{M(V,U)}{U}$, $\psi^f(\theta) = \frac{M(V,U)}{V}$.

Exog Quits Model: Household Problem

- Individual state: wealth and wage
  - If employed: (a, w)
  - If unemployed: (a)
- Problem of the employed: (Standard)

$$V^e(a,w) = \max_{c,a'} u(c) + \beta \ [(1-\delta)V^e(a',w) + \delta V^u(a')]$$

$$\text{s.t.} \quad c + a' = a(1+r) + w, \quad a \geq 0$$

- Problem of the unemployed: Choose which wage to look for

$$V^u(a) = \max_{c,a',w} u(c) + \beta \left\{\psi^h[\theta(w)] \ V^e(a',w) + [1-\psi^h[\theta(w)]] \ V^u(a')\right\}$$

$$\text{s.t.} \quad c + a' = a(1+r) + b, \quad a \geq 0$$

$\theta(w)$ is an equilibrium object

Characterization of a worker's decisions

- Standard Euler equation for savings

$$u_c = \beta\,(1+r)\;E\,\{u_c'\}$$

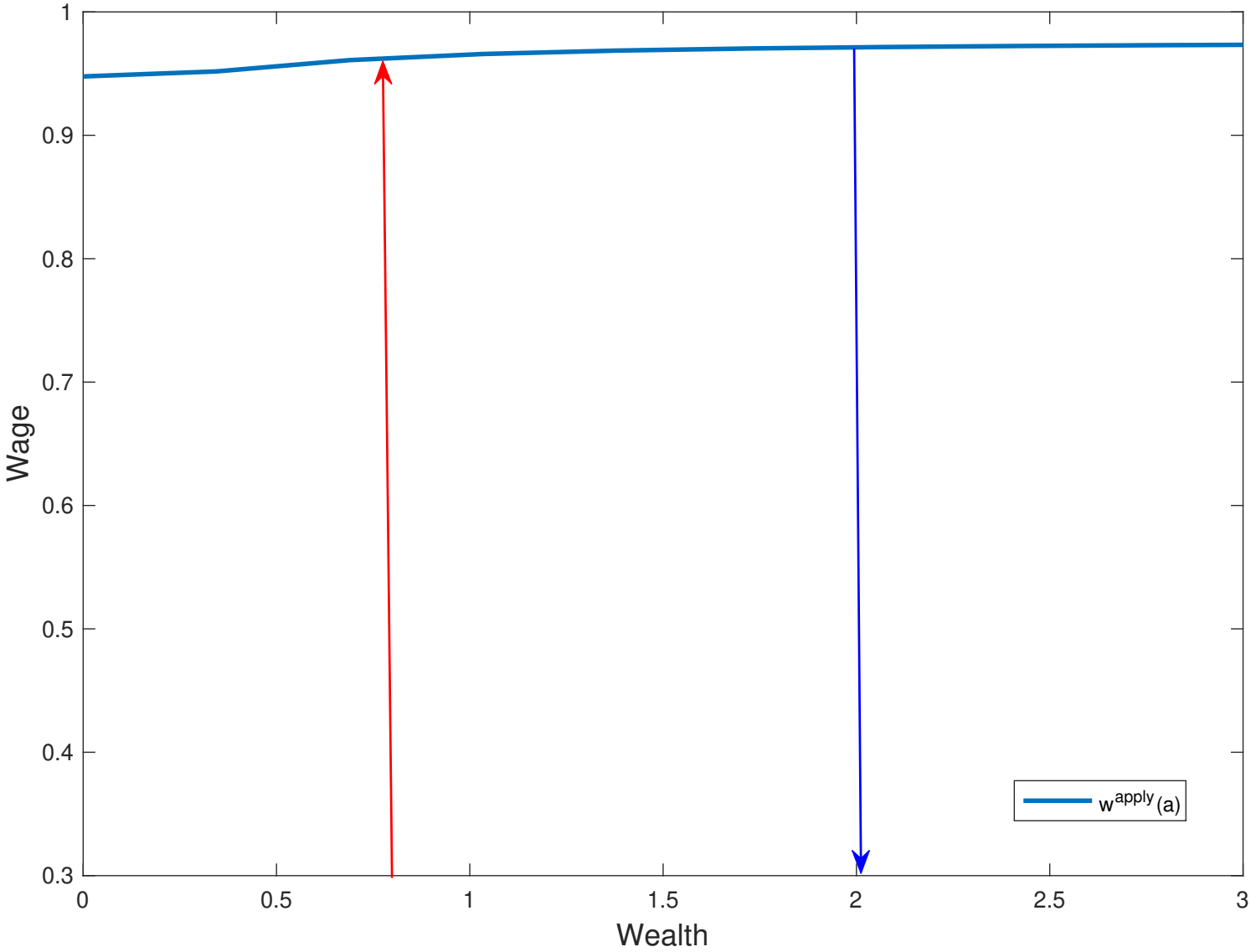
  - Households with more wealth are able to insure better against unemployment risk.

- From wage applicants $\max_w \psi^h[\theta(w)]\;[V^e(a',w) - V^u(a')]$ so

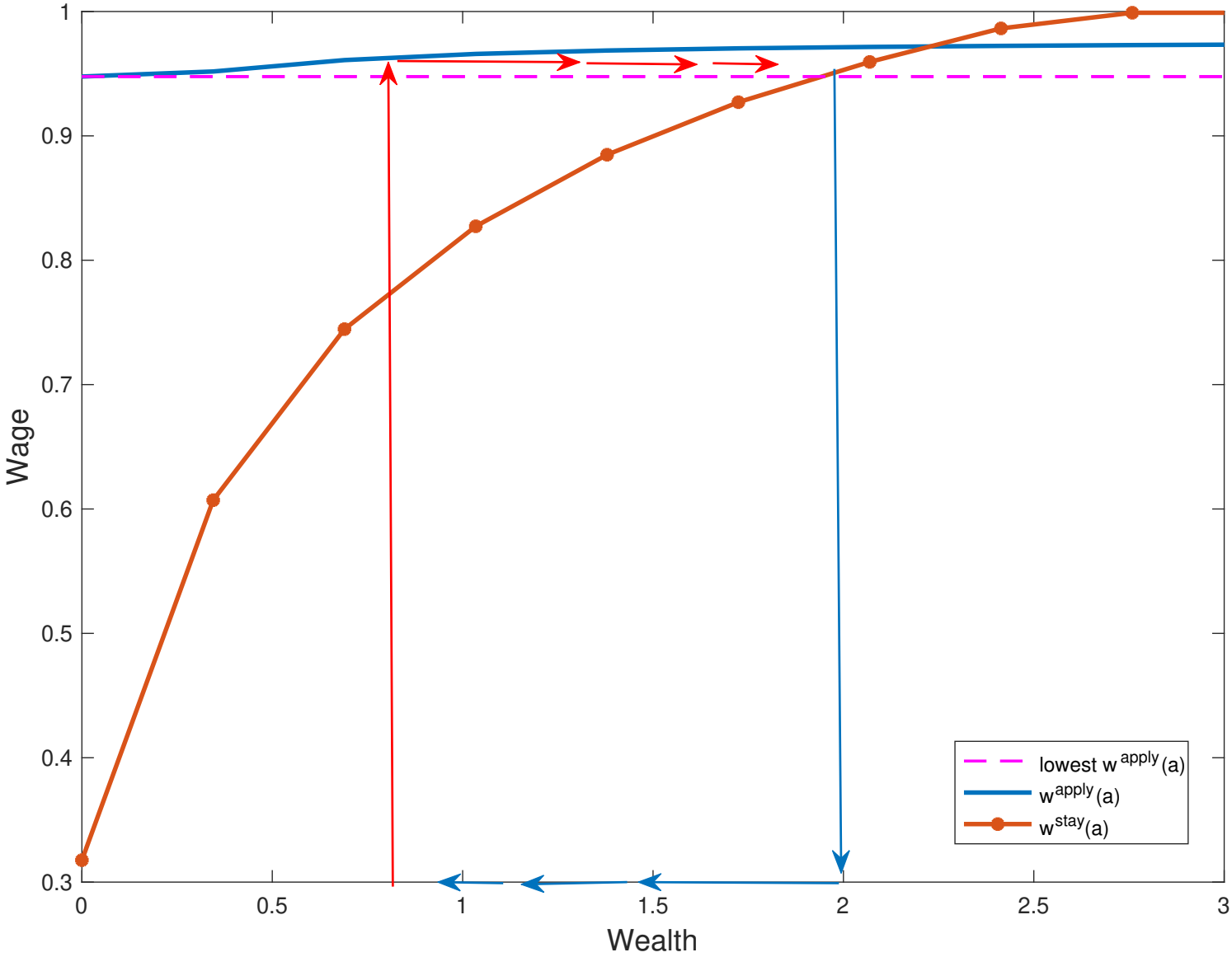
$$\psi^h[\theta(w)]\;V_w^e(a',w) = \psi_\theta^h[\theta(w)]\;\theta_w(w)\;[V^u(a') - V^e(a',w)]$$

- Up to a certain level of wealth, richer households apply to higher wages. After that, it seems not. Consistent with theory

Worker's wage application decision



Worker's saving decision



Firms Post vacancies: Choose wages & filling probabilities

- Value of wage-w job: uses constant $\overline{k}$ capital that depreciates at rate δ^k ($\Omega = \overline{k}$)

$$\Omega(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r}\left[(1-\delta^h)\,\Omega(w) + \delta^h\,\Omega\right]$$

- Affine in w: $\Omega(w) = \left[z + \overline{k}\left(\frac{1-\delta^f}{1+r}\delta^h - \delta^k\right) - w\right]\frac{1+r}{r+\delta^f+\delta^h-\delta^f\delta^h}$

Block Recursivity Applies (firms can be ignorant of Eq)

- Value of creating a firm: $\psi^f[\theta(w)]\;\;\Omega(w) + [1-\psi^f[\theta(w)]]\;\Omega$
- Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \psi^f[\theta(w)]\;\;\frac{\Omega(w)}{1+r} + [1-\psi^f[\theta(w)]]\;\;\frac{\Omega}{1+r},$$

Standard Stationary Equilibrium

- Functions $\{V^e, V^u, \Omega, g'^e, g'^u, w^u, \theta\}$, an interest rate r, and a stationary distribution x over (a, w), s.t.

  1. $\{V^e, V^u, g'^e, g'^u, w^u\}$ solve households' problems, $\{\Omega\}$ solves the firm's problem.

  2. Zero profit condition holds for active markets

$$\bar{c} + \overline{k} = \psi^f[\theta(w)] \ \frac{\Omega(w)}{1+r} + [1 - \psi^f[\theta(w)]] \ \frac{\overline{k}(1-\delta-\delta_k)}{1+r}, \quad \forall w \text{ offered}$$

  3. An interest rate r clears the asset market

$$\int_{A\times(W\cup 0)} a \ dx = \int_{A\times(W\cup 0)} \Omega(w) \ dx + \mu^0 \ \overline{k}.$$

Summary: Properties of Exogenous Quits Model

- Risk-averse, only partially insured workers, endogenous unemployment
- Wage dispersion small—wealth doesn't matter too much
- When solved with aggregate shocks ...
- It is almost like a two-agent model (employed, unemployed) of Pissarides despite curved utility and savings

Model 2: Endogenous Quits: Extreme Value Taste Shocks

- Temporary Shocks to the utility of working or not working: Some workers quit. (in addition to any intrinsic taste for leisure)
- Wealth is not observable and contracts cannot be contingent on it.(Unlike Chaumont and Shi (2022)).
- As long as very few agents on the decreasing part of wealth applying function, wealth can be inferred from the wage agents applied to.
- Hence it is still Block Recursive
- Adds a (smoothed) quitting motive so that conditional on wealth, high wage workers quit less often.
- Firms may want to pay high wages to retain workers.

Endogenous Quits Model: Time-line

1. Workers enter period with or without a job: $\{e, u\}$.
2. Production occurs and consumption/saving choice ensues:
3. Exogenous job/firm destruction happens.
4. Quitting:
   - The employed, e, draw shocks $\{\epsilon^e, \epsilon^u\}$ and make quitting decision. Job losers cannot search this period.
5. Search: New or Idle firms post vacancies. Choose $\{w, \theta\}$.
6. Matches occur

Quitting Model: Workers

- Workers receive i.i.d shocks $\{\epsilon^e, \epsilon^u\}$ to the utility of working or not
- Value of the employed right before receiving those shocks:

$$\widehat{V}^e(a', w) = \int \max\{V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u\}\, dF^\epsilon$$

  V^e and V^u are values after quitting decision as described before.
- If $\{\epsilon^e, \epsilon^u\} \sim G(\mu, \alpha)$ (Gumbel) then the ex-ante quitting probability $q(a, w)$ is

$$q(a, w) = \frac{e^{V^u(a)/\alpha}}{e^{V^e(a,w)/\alpha} + e^{V^u(a)/\alpha}}$$

  - Lower $\alpha \to$ lower chance of quitting (less capricious).
  - The higher the wage the higher the difference bw V^e and V^u, so longer job durations.
  - Firms could pay more to keep workers longer.

Quitting Model: Workers Problem

- Problem of the employed: just change $\widehat{V}^e$ for V^e

$$V^e(a,w) = \max_{c,a'} u(c) + \beta \left[(1-\delta)\widehat{V}^e(a',w) + \delta V^u(a)\right]$$
$$\text{s.t.} \quad c + a' = a(1+r) + w, \quad a \geq 0$$

- We let $\mu = -\alpha\gamma - \ln(2)$ so that $E\{\max[\epsilon_1^u, \epsilon_2^u]\} = 0$. To avoid the option value of working we have also add $E\{\max[\epsilon_{\mathbf{1}}^u, \epsilon_{\mathbf{2}}^u]\}$ to the utility of the unemployed

- Alternatively we could accept the fact that a job is an option to get utility.

Quitting Model: Value of the firm

- Free entry condition requires that for all offered wages

$$\bar{c} + \overline{k} = \frac{1}{1+r} \left\{ \psi^f [\theta(w)] \, \Omega^0(w) + [1 - \psi^f [\theta(w)]] \, \Omega \right\},$$

$\Omega^j(w)$: Value with with j-tenured worker.

- Probability of retaining a worker with tenure j at wage w is $\ell^j(w)$.

(One to one mapping between wealth and tenure)

$$\ell^j(w) = 1 - q[g^{e,j}(a, w), w]$$

$g^{e,j}(a, w)$ savings rule of a $j -$ *tenured* worker that was hired with wealth a

- Firm's value

$$\Omega^j(w) = z - \overline{k}\delta^k - w + \frac{1-\delta^f}{1+r} \left\{ \ell^j(w) \Omega^{j+1}(w) + [1 - \ell^j(w)] \, \Omega \right\}$$

Quitting Model: Solving forward for the Value of the firm

$$\Omega^0(w) \quad = \quad (z - w - \delta^k k)\, Q^1(w) + (1 - \delta^f - \delta_k) k\, Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta^f}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta^f}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

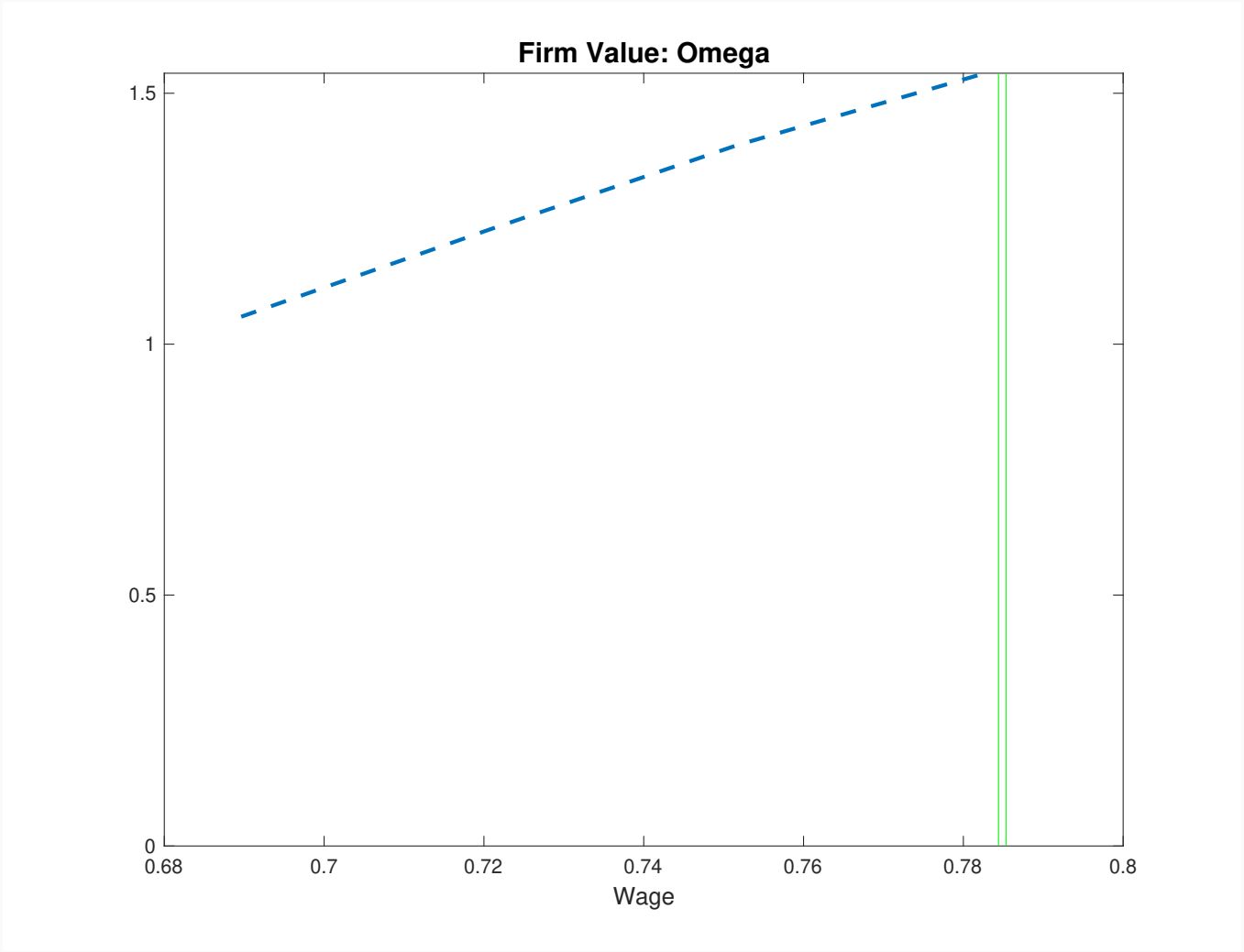
- New equilibrium objects $\{Q^0(w), Q^1(w)\}$. Rest is unchanged.
- Except for when there are agents in a decreasing part of the wage applying function, it is Block Recursive because wealth can be inferred from w and j. (No need to index contracts by wealth (as in Chaumont and Shi (2022))).

Do we get More Wage Dispersion?

- This Model has the potential to get more wage dispersion
- Conditional on wealth higher wages lead to less quitting.
- So firms are willing to pay more to keep workers longer
- **BUT** we will see a problem

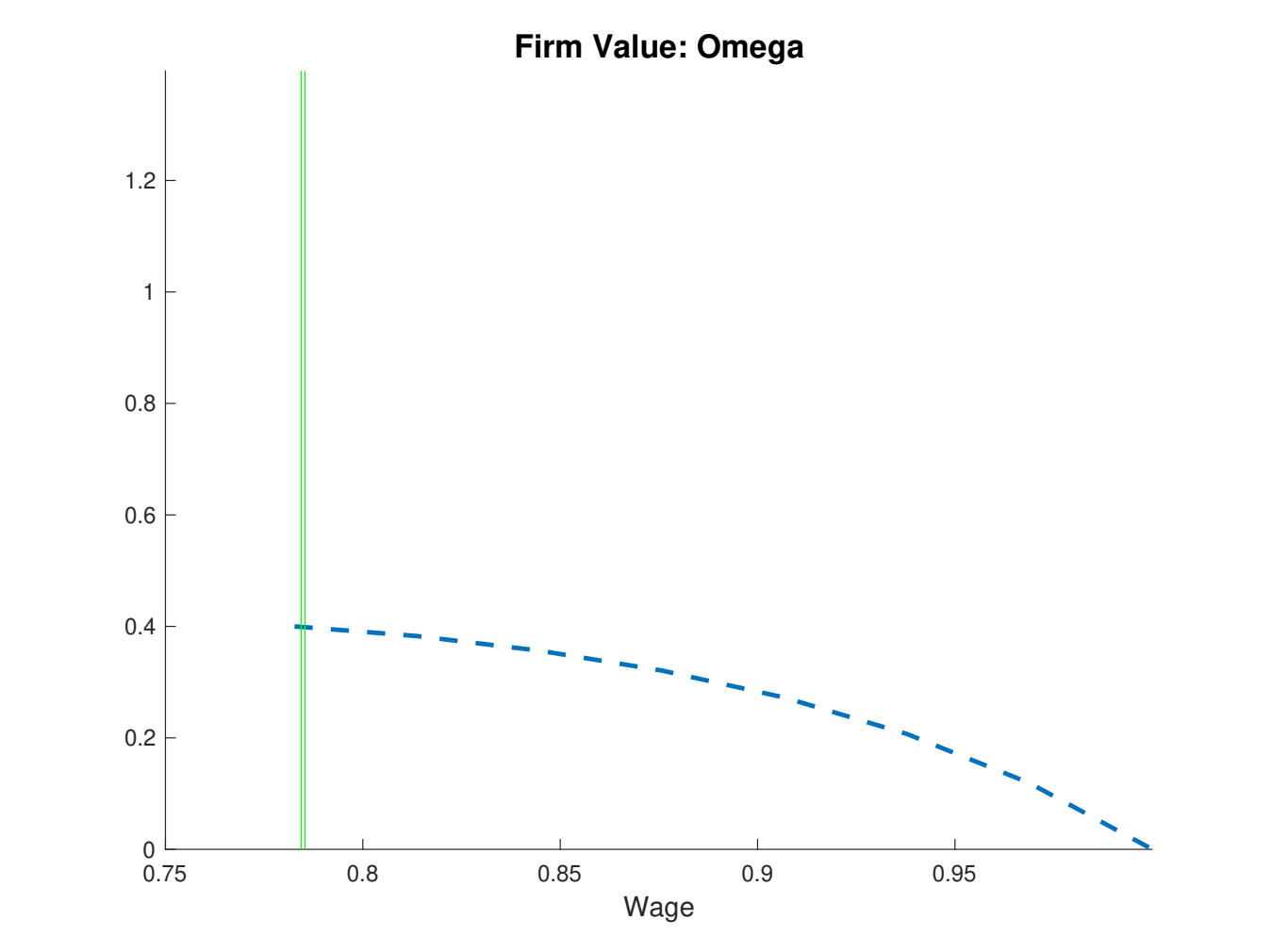
Value of the firm as wage varies: The Poor

- For the poorest, employment duration increases when wage goes up.
- Firms value is increasing in the wage



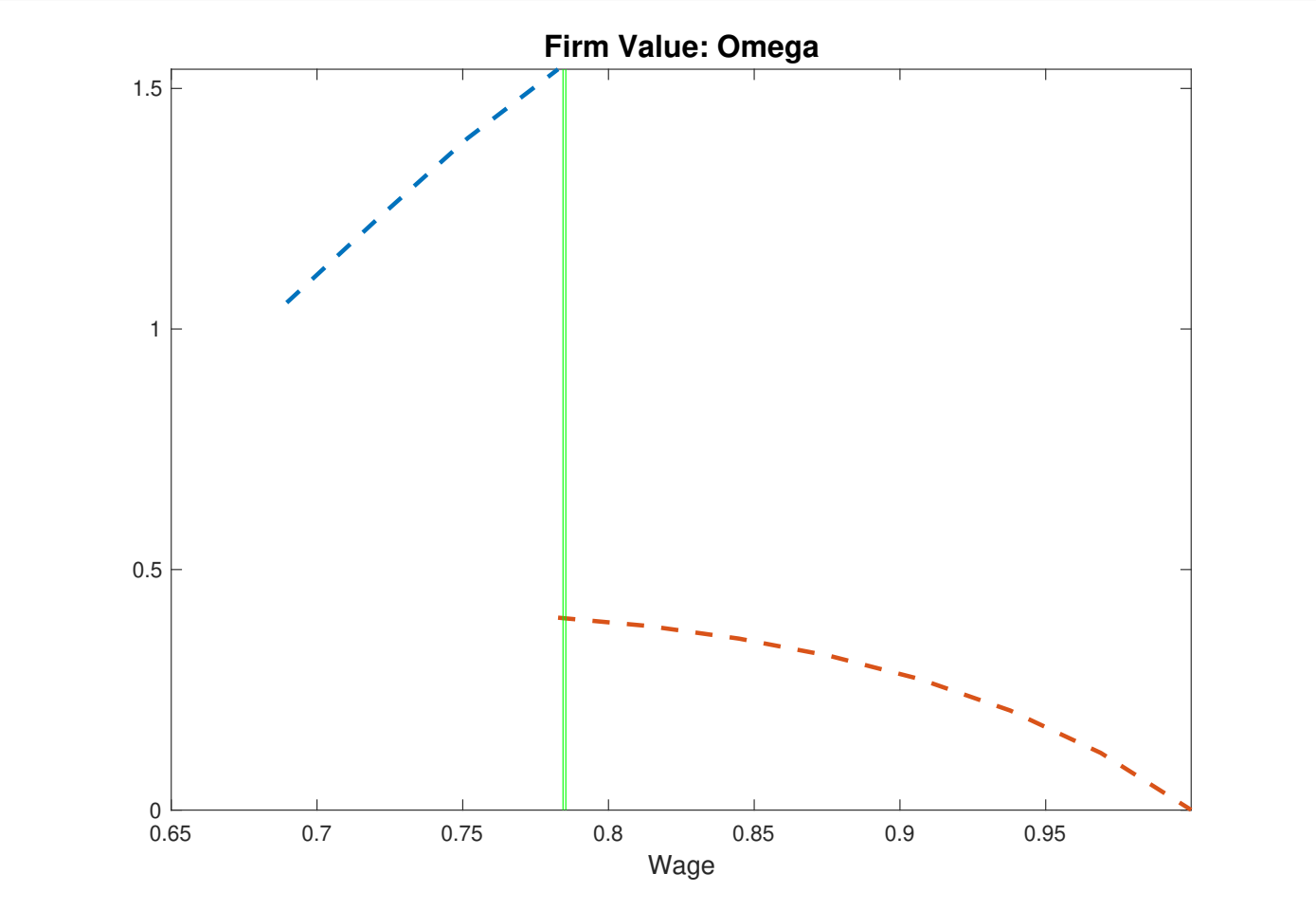
Value of the firm as wage varies: The Rich

- For the richest, employment duration increases but not fast enough.
- Firm value is slowly decreasing in wages (less than static profits).



Value of the firm: Accounting for Worker Selection

- Large drop from below to above equilibrium wages.
- In Equilibrium wage dispersion COLLAPSES due to selection.



- Related to the Diamond dispersion paradox but for very different reasons.

Effect of Quitting: The Mechanism

- Two forces shape the dispersion of wages
  - Agents quit less at higher paid jobs, which enlarge the spectrum of wages that firms are willing to pay (for a given range of vacancy filling probability).
  - However, by paying higher wages, firms attract workers with more wealth.
- Wealthy people quit more often, shrink employment duration.
- In equilibrium, the wage gap is narrow (disappears?) and the effect of wealth dominates. graph

Main Shortcoming

- Comes from the perfect correlation between age and wealth (at time of starting the job).
- Need to overcome it. Two ways that may be complementarity
  1. On the Job Search
  2. Aiming Shocks: (EV) Shocks that distort the wage applying decision.
     - Direct search with noise.

Model 3: On the Job Search: Time-line

1. Workers enter period with or without a job: V^e, V^u.
2. Production & Consumption:
3. Exogenous Separation
4. Quitting? Searching? Neither?: Employed draw shocks $(\epsilon^e, \epsilon^u, \epsilon^s)$ and make decision to quit, search, or neither. Those who quit become u', those who search join the u, in case of finding a job become $\{e', w'\}$ but in case of no job finding remain e' with the same wage w and those who neither become e' with w. $\widehat{V}^E(a', w)$, is determined with respect to this stage.
5. Search : Potential firms decide whether to enter and if so, the market (w) at which to post a vacancy; u and s assess the value of all wage applying options, receive match specific shocks $\{\epsilon^{w'}\}$ and choose the wage level w' to apply. Those who successfully find jobs become e', otherwise become u'.
6. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
7. Match

- After saving, the unemployed problem is

$$\widehat{V}^u(a') = \int \max_{w'} \left[\psi^h(w') V^e(a', w') + (1 - \psi^h(w')) V^u(a') + \epsilon^{w'} \right] \, dF^\epsilon$$

- After saving, the employed choose whether to quit, search or neither

$$\widehat{V}^e(a', w) = \int \max\{ V^e(a', w) + \epsilon^e, V^u(a') + \epsilon^u, V^s(a', w) + \epsilon^s \} dF^\epsilon$$

- The value of searching is

$$V^s(a', w) = \int \max_{w'} \left[\psi^h(w') V^e(a', w') + [1 - \psi^h(w')] V^e(a', w) + \epsilon^{w'} \right] \, dF^\epsilon$$

On the Job Search: Household choices

- The probabilities of quitting and of searching

$$q(a', w) = \frac{1}{1 + \exp(\alpha[V^e(a', w) - V^u(a')]) + \exp(\alpha[V^s(a', w) - V^u(a') + \mu^s])},$$

$$s(a', w) = \frac{1}{1 + \exp(\alpha[V^u(a') - V^s(a', w)]) + \exp(\alpha[V^e(a', w) - V^s(a', w) - \mu^s])}.$$

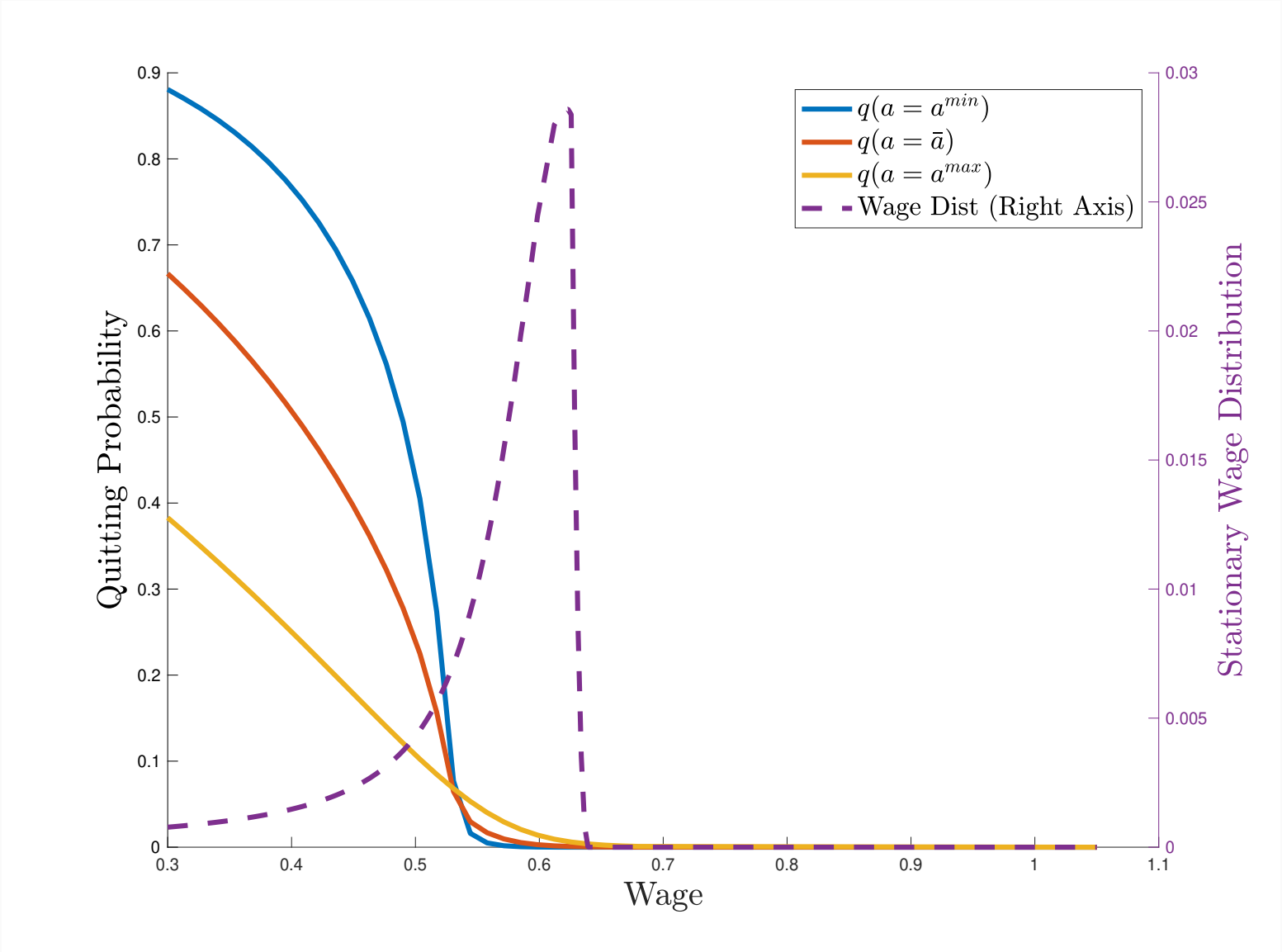
$\mu^s < 0$ is the mode of the shock ϵ^s which reflects the search cost.

- Households solve

$$V^e(a, w) = \max_{a' \geq 0} u[a(1 + r) + w - a'] + \beta \left[\delta V^u(a') + (1 - \delta)\widehat{V}^e(a', w)\right]$$

$$V^u(a) = \max_{c, a' \geq 0} u[a(1 + r) + b - a'] + \beta \widehat{V}^u(a')$$

OJS Quitting Probabilities, Various wealths & Wage Density



- The rich pursue often other activities (leisure?)

- The value of the firm is again given like in the Quitting Model

$$\Omega^0(w) = (z - w - \delta^k k)\, Q^1(w) + (1 - \delta - \delta_k)k\, Q^0(w),$$

$$Q^1(w) = 1 + \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} \prod_{i=0}^{\tau} \ell^i(w) \right],$$

$$Q^0(w) = \sum_{\tau=0}^{\infty} \left[\left(\frac{1-\delta}{1+r} \right)^{1+\tau} [1 - \ell^\tau(w)] \left(\prod_{i=0}^{\tau-1} \ell^i(w) \right) \right].$$

- Except that now the probability of keeping a worker after j periods is

$$\ell^j(w) = 1 - \int h(w;a)\, q[g^{e,j}(a,w), w]\, dx^u(a) -$$
$$\int h(w;a)\, s[w; g^{e,j}(a,w)] \left[\int \hat{h}[\widetilde{w}; g^{e,j}(a,w), w] \xi \phi^h(\widetilde{w})\, d(\widetilde{w}) \right] dx^u(a)$$

- Not block recursive but Q^1 and Q^2 are sufficient.

Mapping the Model to Data: Adding Some Bells and Whistles

- Life cycle (Yaari (1965), Blanchard (1985)) with 50 years of expected duration
    - Provides a mechanism for having poor agents
- Searching while on the job is slightly more inefficient than while unemployed.
- Workers hired from the ranks of unemployment require some training

$$\Omega^0(w) = z - \overline{k}\delta^k - w - CT + \frac{1-\delta^f}{1+r}\left\{\ell^0(w)\Omega^1(w) + [1-\ell^0(w)]\,\Omega\right\}$$

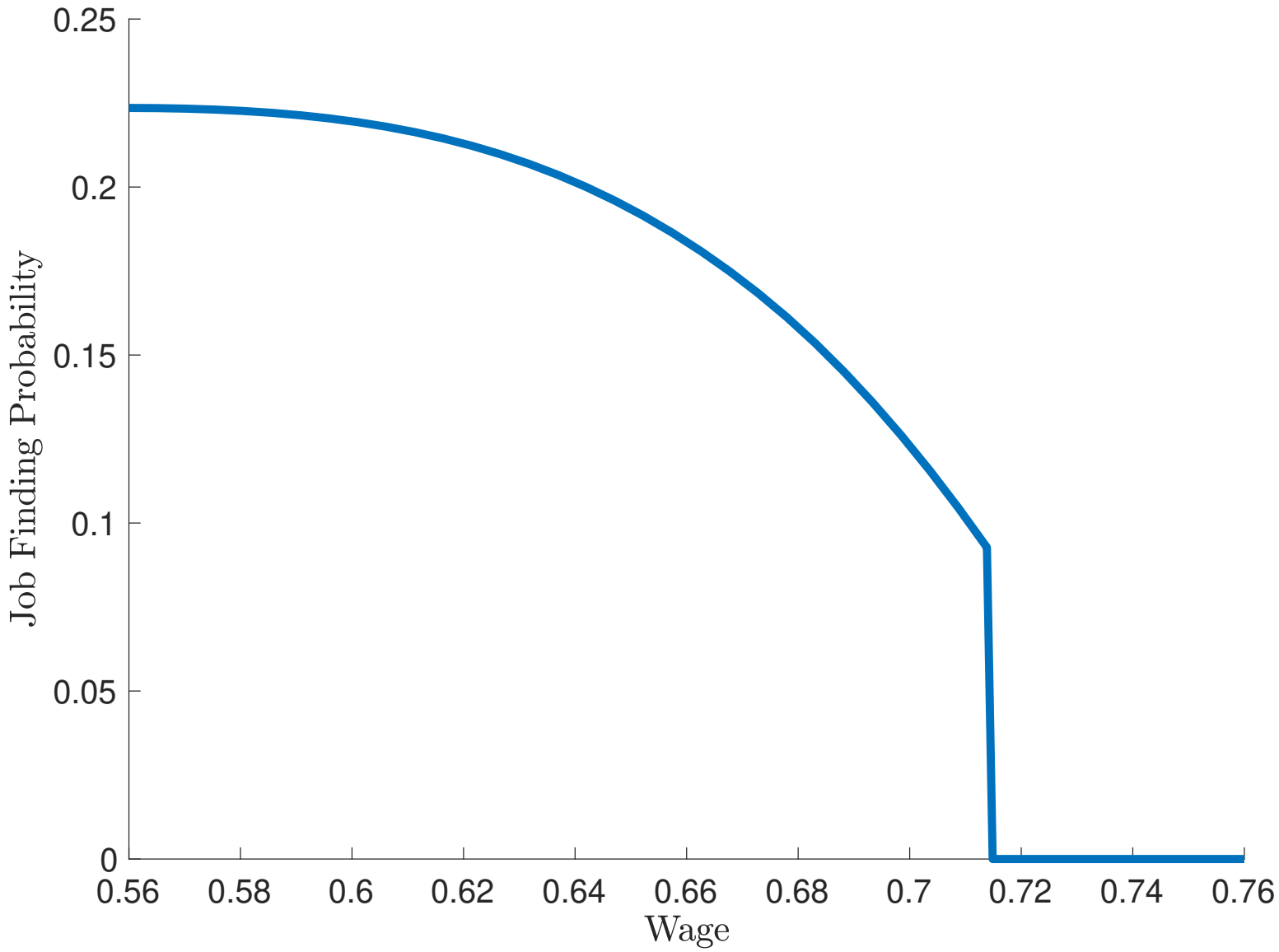
Parameter Values: Period is half a quarter

	Definition	Value in Yearly Units
r	interest rate	3%
K	fixed capital required	3
δ^f	firm destruction rate	2.88%
δ^k	capital maintenance rate	6.38%
c^v	job posting cost	0.03
y	productivity on the job	1
b/w	productivity at home	0.4
σ	risk aversion	2
Matching function	$m = \chi u^\eta v^{1-\eta}$, OJS	$\chi = 0.3$
		$\eta = 0.5$

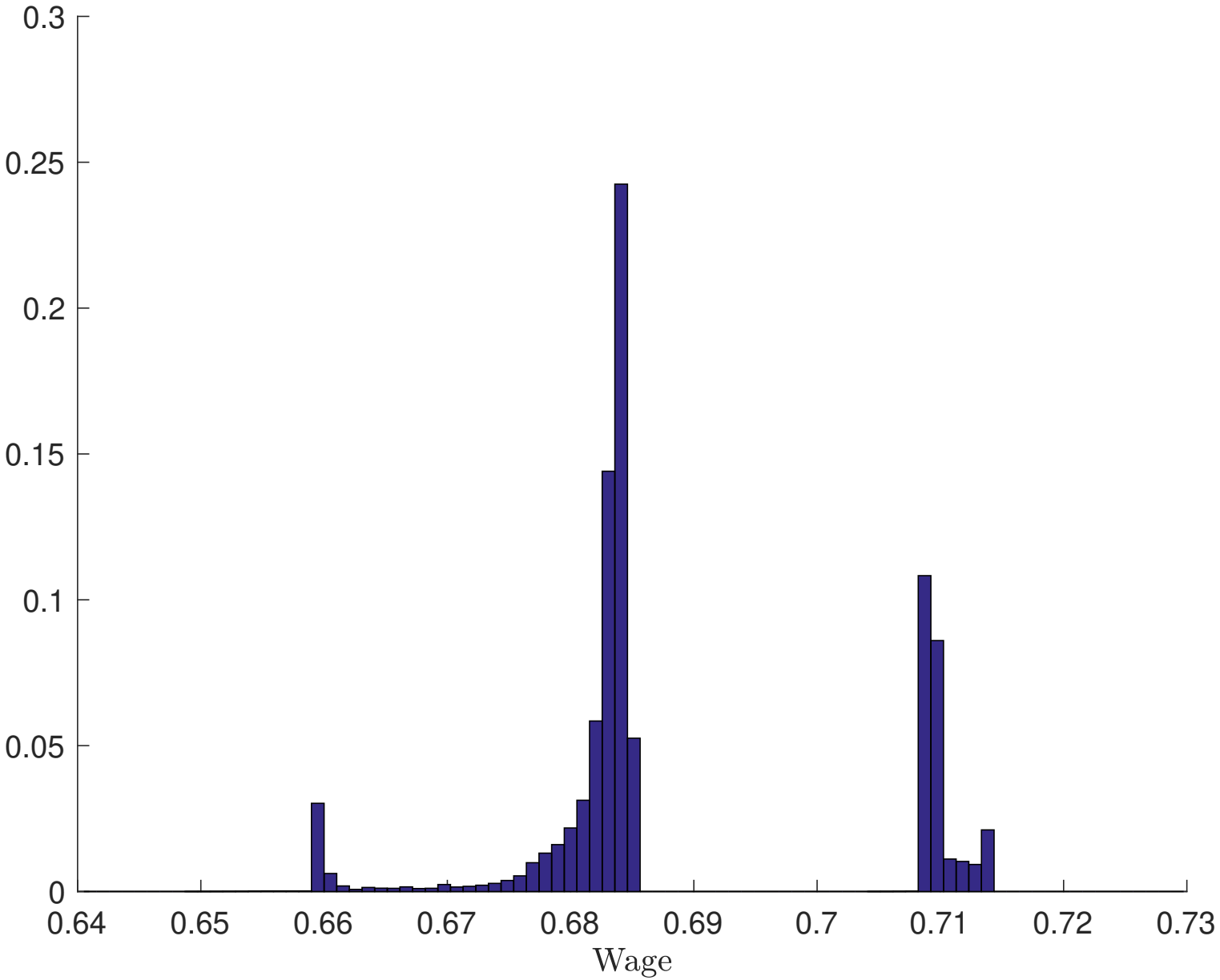
Steady State Allocations in Yearly Units: Endog Quits & OJS

interest rate	0.030
avg consumption	0.652
avg wage	0.683
avg wealth	2.938
stock market value	3.015
avg labor income	0.653
consumption to wealth ratio	0.222
labor income to wealth ratio	0.222
quit ratio	0.061
unemployment rate	0.087
job losers	0.089
wage of newly hired unemp	0.619
std consumption	0.013
std wage	0.004
std wealth	3.875
mean-min consumption	1.956
mean-min wage	1.153
UE transition	1.152
total vacancy	0.826
avg unemp duration	0.531
avg emp duration	9.108
avg job duration	0.317
OJS move rate	2.368

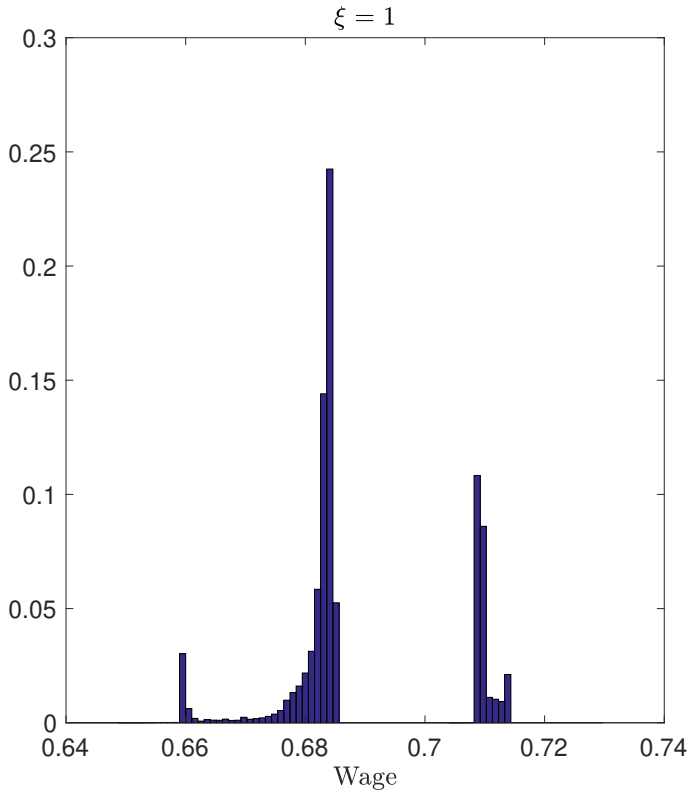
Job Finding Probability Curves

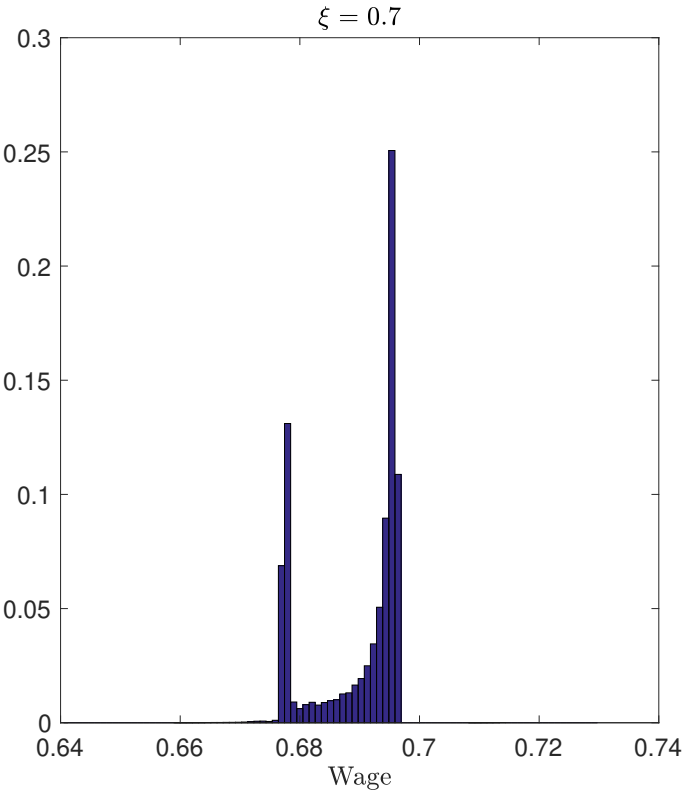


Wage Distributions: Baseline

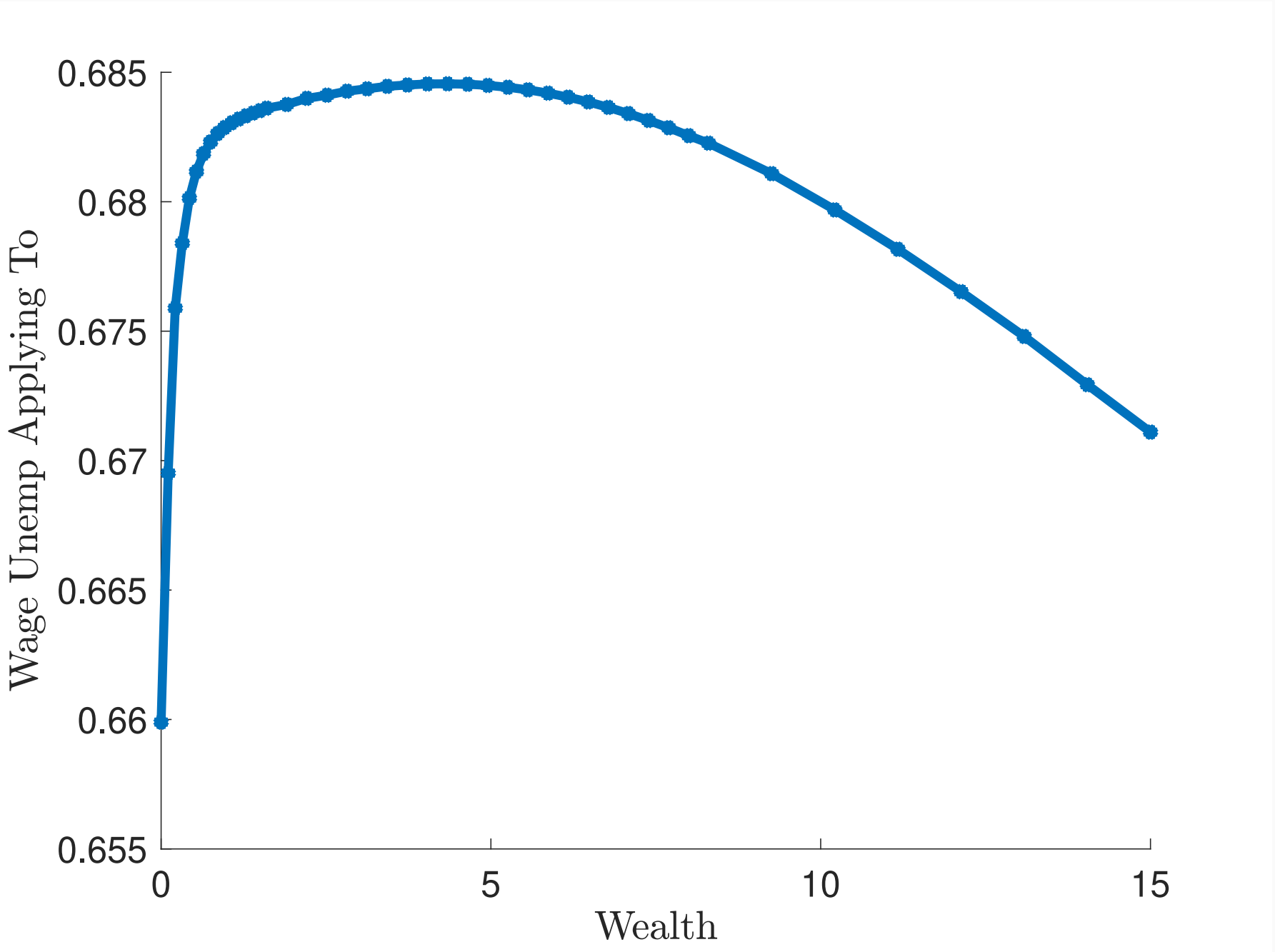


Wage Distributions: Comparing with lower OJS

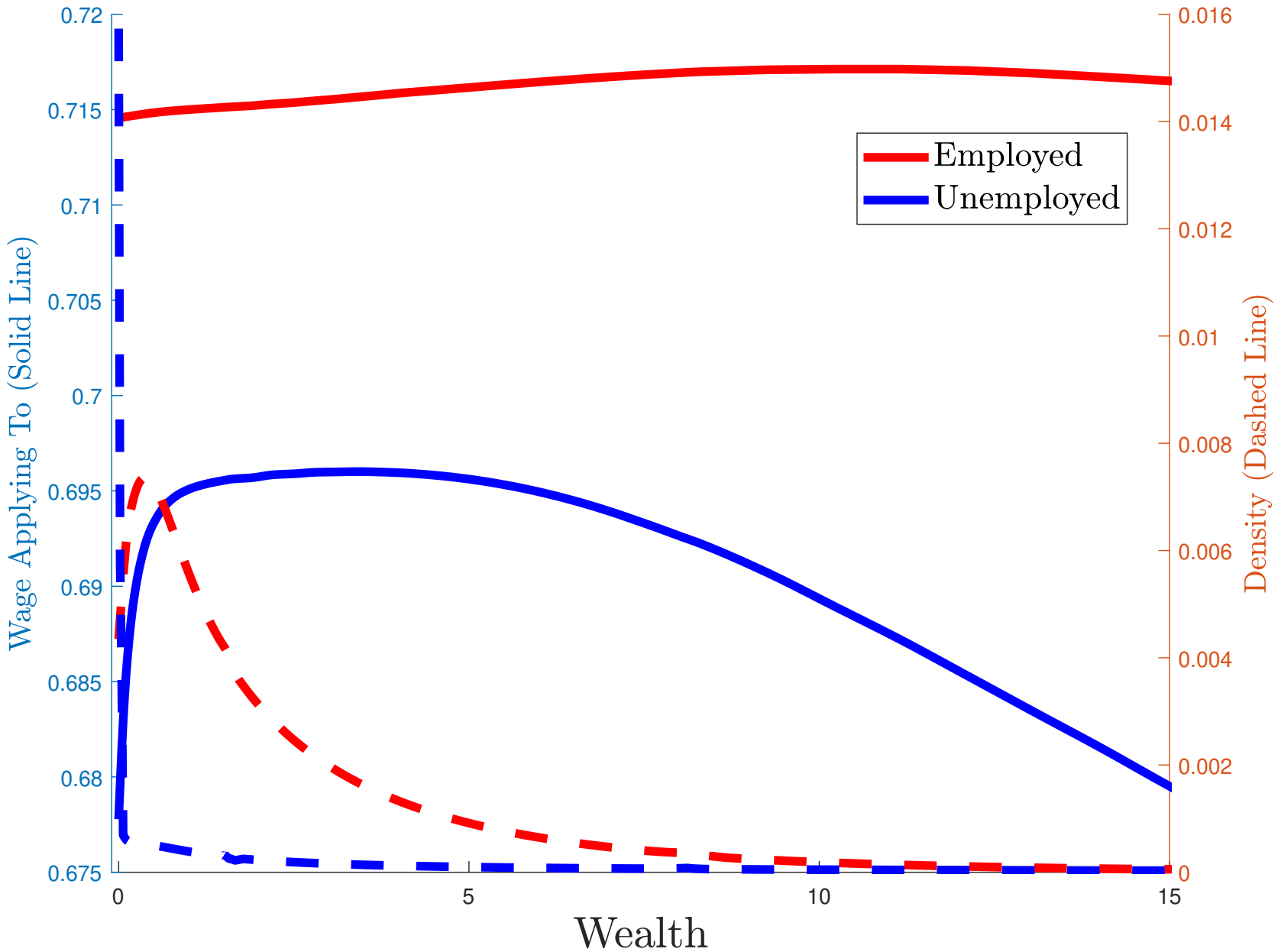




Wage Applications of the Unemployed by Wealth



Wage Applications of U and $\overline{w}$ and densities of all



Summary of Steady States

- Wage dispersion: 1.153, which vs ≈ 1.2 in the data.
- Obviously, not a good theory of wealth inequality. Should complement it with other mechanisms.
- But it can deliver gross flows (3% per month OJS and a bit less for quits).

Aggregate Fluctuations

Introduce Aggregate Shocks (in a small open economy)

- We now pose a standard aggregate shock

  1. Productivity shocks z_t: Output = EmpRate $\times$ $(1 + z_t)$

- We introduce a wage peg assumption: $w(z) = \varphi^z \; z \; w$

  - If wages were completely rigid there would be massive quits: counterfactual.
  - By aiming at the Job to Job Volatility we can estimate the degree of wage rigidity φ^z
  - We use the Boppart et al. (2018) way of solving aggregates

Baseline: IRF to z shock: Typical Response when wages sufficiently flexible

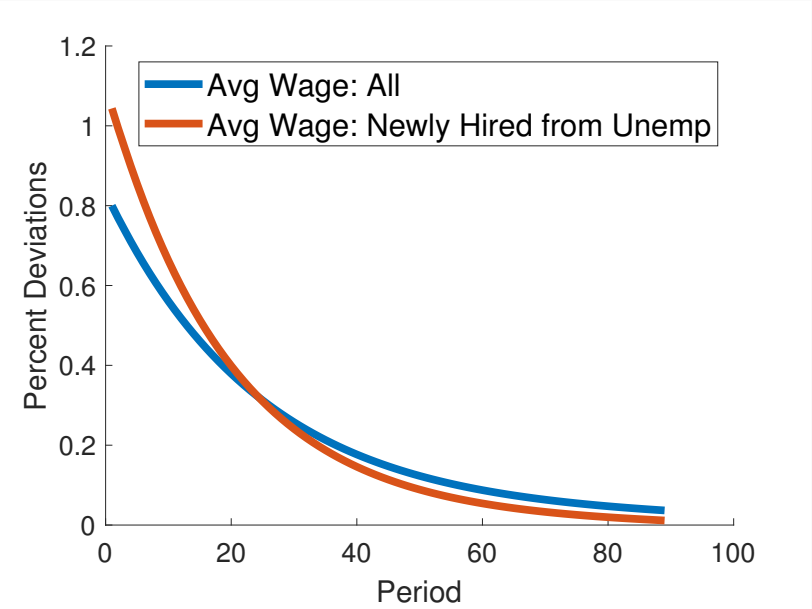


Figure 1: Wages

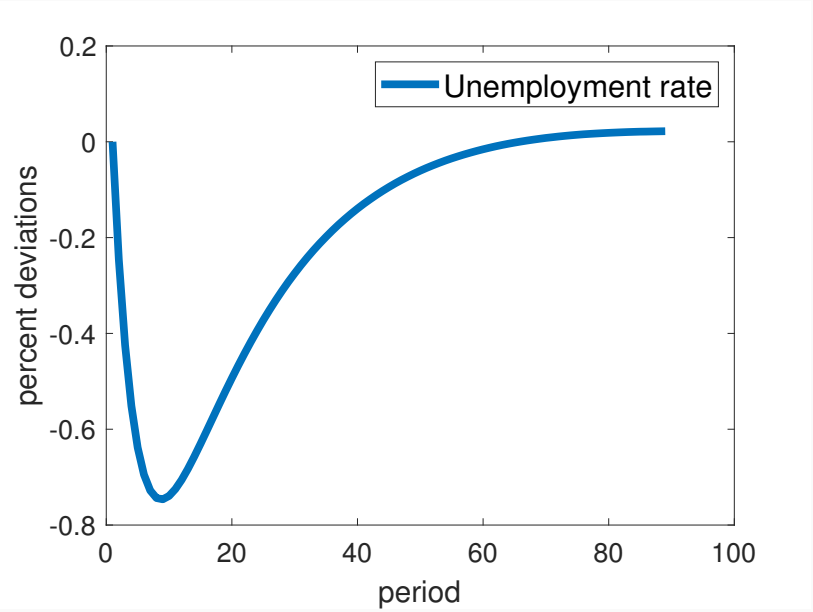


Figure 2: Unemployment Rate

- Obviously New wages move more than average wages
- Some response of unemployment

Baseline: IRF to z shock

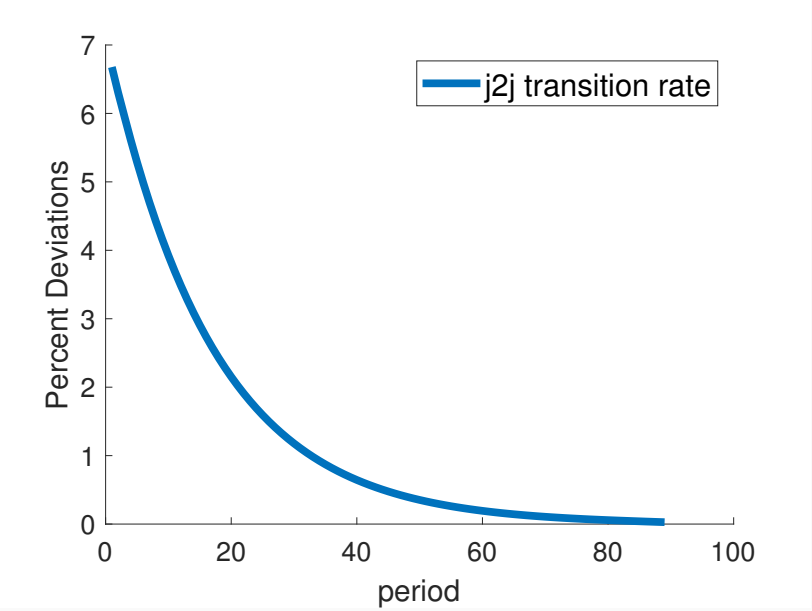


Figure 3: J2J transitions

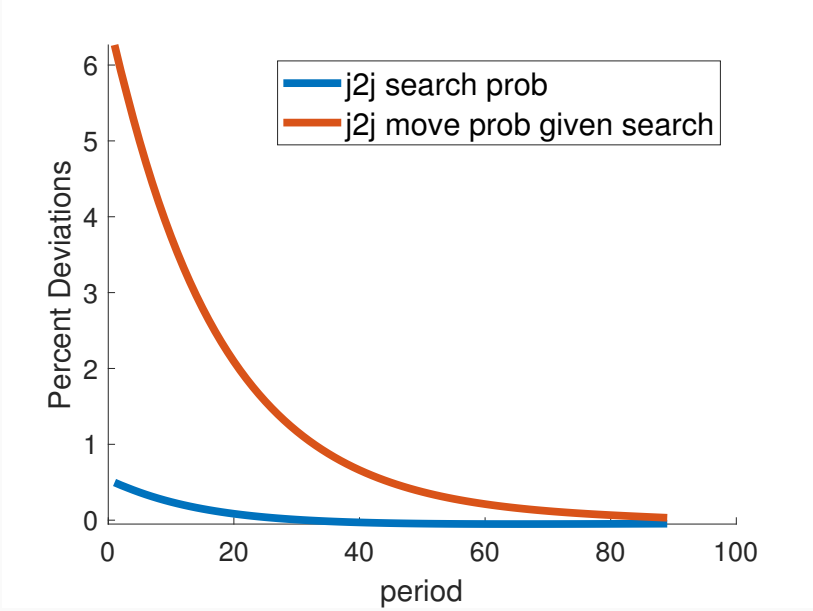


Figure 4: J2J search & JFP

- Too much responsive j2j transitions
- Due to improved job finding probabilities, not more searchers

Assessing Performance in terms of standard HP-filtered 2nd moments

- 1st order data moments are from standard database: CPS, JOLTS, LEHD and NIPA.

- 2nd order data moments are from Haefke et al. (2013), Campolmi and Gnocchi (2016), Brown et al. (2017) and Fujita and Nakajima (2016).

Productivity Shock $\rho = 0.95$: Relative Volatility

- Wage adjustment estimate $\varphi^w = .8$:

	Model	Data
Output	1	1
Average Wage	0.77	0.44-0.84
New Wage	1.07	0.68-1.09
Unemployment	0.35	4.84
Quits + OJS moves	4.05	4.20
OJS moves	4.87	4.62

Table 1: Standard Deviation Relative to Output: Only Productivity Shock

- Unemployment moves way way too little

Productivity Shock ($\rho = 0.95$): Correlation

- Wage adjustment estimate $\varphi^w = .8$:

	Model	Data
Output	1	1
Average Wage	1	0.24-0.37
New Wage	1.	0.79-0.83
Unemployment	-0.58	-0.85
Quits + OJS moves	0.99	0.85
OJS moves	1.	0.70

Table 2: Correlation with Contemprary Output: Only Productivity Shock

- Correlations are too large but appropriate

Summary of Fluctuations

- Same properties of standard real business cycle models on aggregates.
- Unemployment volatility is terrible.
  - Need to expand the model to a more detached workforce by adding outside the labor force.
    - Either multi person households
    - Markovian process on value of non working with many agents close to indifferent (easier)
- Job to job transitions volatility can be replicated
  - The amount of wage rigidity implied is small

Conclusions I

- Develop tools to get a joint theory of wages, employment and wealth that marry the two main branches of modern macro:
  1. Aiyagari models (output, consumption, investment, interest rates)
  2. Labor search models with job creation, turnover, wage determination, flows between employment, unemployment and outside the labor force.
  3. Add tools from Empirical Micro to generate quits
- Useful for business cycle analysis: We are getting procyclical
  - Quits
  - Employment
  - Investment and Consumption
  - Wages

Conclusions II

- Exciting set of continuation projects:

  1. Incorporate movements in and out of the labor force.
  2. Endogenous Search intensity on the part of firms and in general abandon the constant zero profit entry condition (Qiu (2022))
  3. Aiming Shocks to soften correlation between wages and wealth
  4. Efficiency Wages: Endogenous Productivity (firms use different technologies with different costs of idleness)
  5. Move towards more sophisticated household structures (more life cycle movements, multiperson households).

Extensions

Outside the Labor Force

Outside the Labor Force Model: Time-line

1. Workers enter period with or without a job: V^e, V^u.
2. Production & Consumption:
3. Exogenous Separation
4. In the beginning of the period non Workers get a shock to the utility of either searching or not searching. They then choose whether to sit out and not search or to search. It is an extreme value shock.

   Workers get a utility injection equal to the expected utility of the maximum of those two shocks to get no bias in the value of working versus not.
5. Quitting? Searching? Neither?:
6. Search
7. $\widehat{V}^u(a'), \{\Omega^j(w)\}$ are determined with respect to this stage.
8. Match

Firms choose Search Intensity

- The number of vacancies posted is chosen by firms
- Easy to implement
- Slightly Different steady state

Free entry with variable recruiting intensity

- Let $\upsilon(\overline{c})$ be a technology to post vacancies where $\overline{c}$ is the cost paid.

- Then the free entry condition requires that for all offered wages

$$0 = \max_{\overline{c}} \left\{ \upsilon(\overline{c})\ \psi^f[\theta(w)] \ \ \frac{\Omega(w)}{1+r} + \left[1 - \upsilon(\overline{c})\ \psi^f[\theta(w)]\right] \ \ \frac{\overline{k}(1-\delta_k)}{1+r} - \overline{c} - \overline{k} \right\},$$

- With FOC given by

$$v_{\overline{c}}(\overline{c}) \ \left\{ \psi^f[\theta(w)] \ \ \left[\frac{\Omega(w)}{1+r} - \ \ \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

How to make it consistent with the current steady state

- If $v(\overline{c}) = \frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \overline{c}$, we have

$$(\upsilon_1 \overline{c} + \upsilon_2) \left\{ \psi^f[\theta(w)] \left[\frac{\Omega(w)}{1+r} - \frac{\overline{k}(1-\delta_k)}{1+r} \right] \right\} = 1,$$

- By Choosing υ so that for the numbers that have now

$$\left\{ \left[\frac{\upsilon_1 \overline{c}^2}{2} + \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\Omega(w)}{1+r} + \left[1 - \frac{\upsilon_1 \overline{c}^2}{2} - \upsilon_2 \overline{c} \right] \psi^f[\theta(w)] \frac{\overline{k}(1-\delta_k)}{1+r} \right\} = \overline{c} + \overline{k}$$

- Solving for $\{\upsilon_1, \upsilon_2\}$ that satisfy both equations given our choice of $\overline{c}$ we are done

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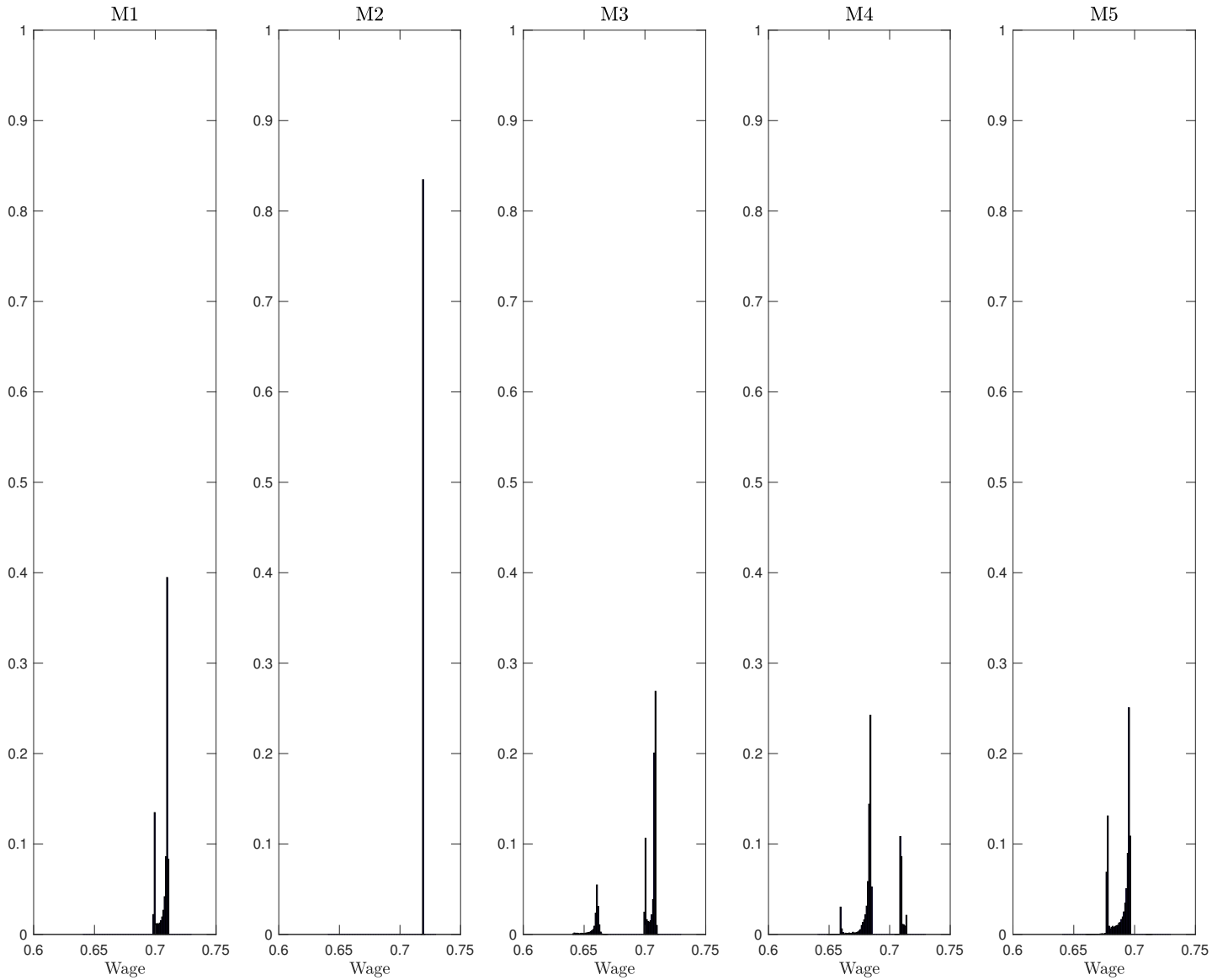
Steady-States

	m1	m2	m3	m4	m4 (low xi)
β	0.975	0.972	0.975	0.976	0.976
interest rate	0.030	0.030	0.030	0.030	0.030
avg consumption	0.686	0.682	0.691	0.684	0.680
avg wage	0.707	0.719	0.696	0.689	0.690
avg wealth	2.789	2.763	2.361	3.041	2.919
stock market value	2.971	2.692	3.065	2.953	2.931
avg labor income	0.659	0.655	0.668	0.654	0.652
consumption to wealth ratio	0.246	0.247	0.293	0.225	0.233
labor income to wealth ratio	0.236	0.237	0.283	0.215	0.223
quit ratio	0.090	0.088	0.090	0.090	0.092
unemployment rate	0.129	0.165	0.076	0.097	0.106
job losers	0.117	0.115	0.117	0.117	0.119
wage of newly hired unemployed	0.707	0.719	0.656	0.677	0.689
std consumption	0.013	0.010	0.011	0.011	0.011
std wage	0.000	0.000	0.003	0.002	0.001
std wealth	2.989	2.715	2.624	3.606	3.677
mean-min consumption	2.057	2.045	2.072	2.051	2.039
mean-min wage	1.012	1.001	1.094	1.058	1.042
UE transition	0.121	0.114	0.128	0.125	0.126
total vacancy	0.544	0.308	0.704	0.578	0.707
avg unemp duration	1.062	1.449	0.589	0.773	0.745
avg emp duration	7.228	7.335	7.228	7.228	7.131
OJS move rate	0.000	0.000	0.420	0.395	0.292
avg job duration	7.228	7.335	1.814	1.898	2.342

Wage Distributions



Derive the Idle Value

- Value of an idle firm is

$$\Omega^0 = -\delta^k k + \frac{1-\delta^f}{1+r}\left[-c^v + \psi^f \Omega + (1-\psi^f)\Omega^0\right]$$

- Free entry

$$k = \frac{1}{1+r}\left[-c^v + \psi^f \Omega + (1-\psi^f)\Omega^0\right]$$

  - Newly entered firms do not receive the destruction shock immediately
  - Vacancy posting cost is paid immediately before searching
- Combine the above

$$\Omega^0 = (1-\delta^f-\delta^k)k$$

M4 Low Ave J-2-J 1% Productivity Shock ($\rho = .9$) [IRF]

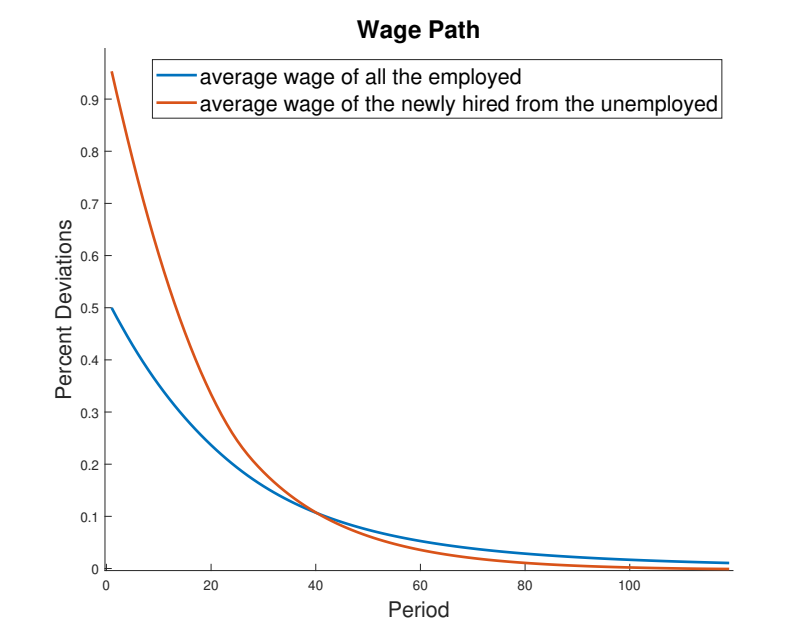


Figure 5: Wages

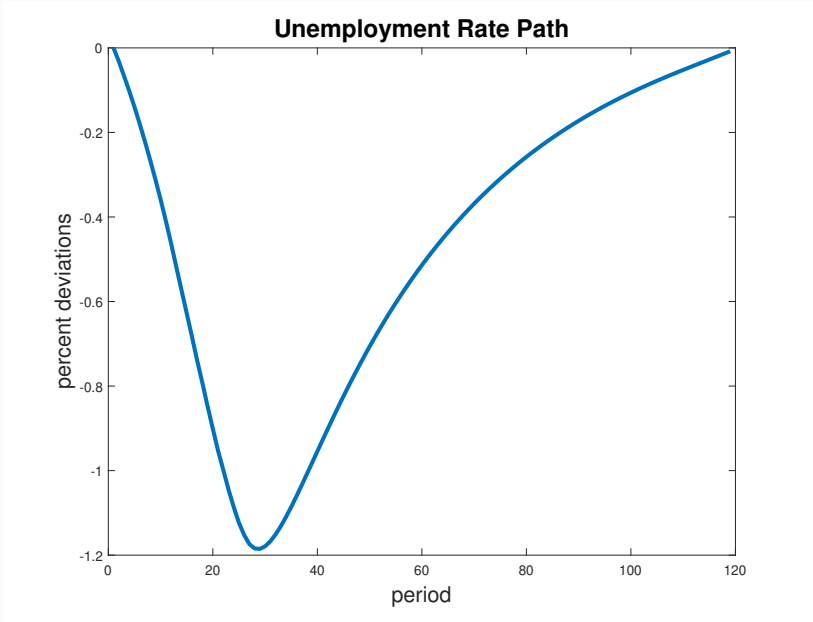


Figure 6: Unemployment Rate

- Similar Wage Responses
- 70% more unemployment volatility: J: mainly comes from more responsive quits

M4 Low Ave J-2-J 1% Productivity Shock ($\rho = .9$) IRF

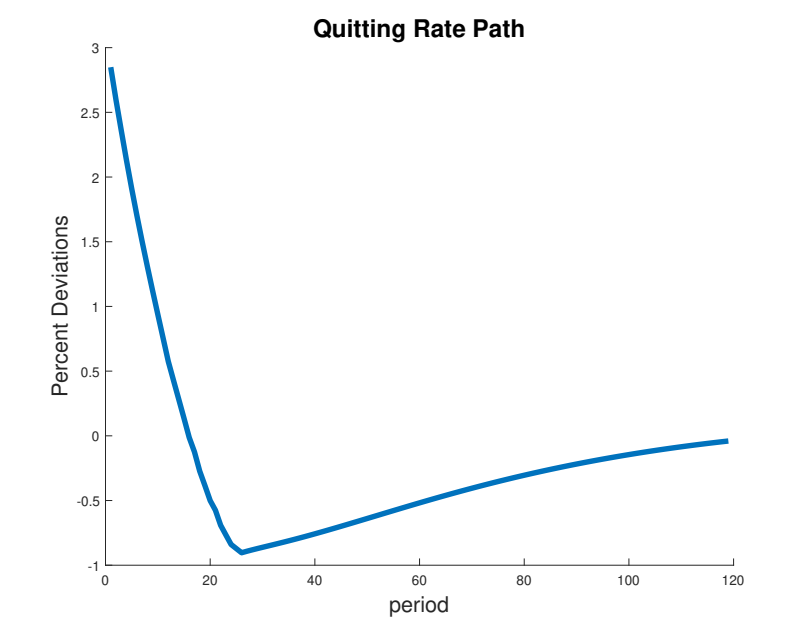


Figure 7: Quits

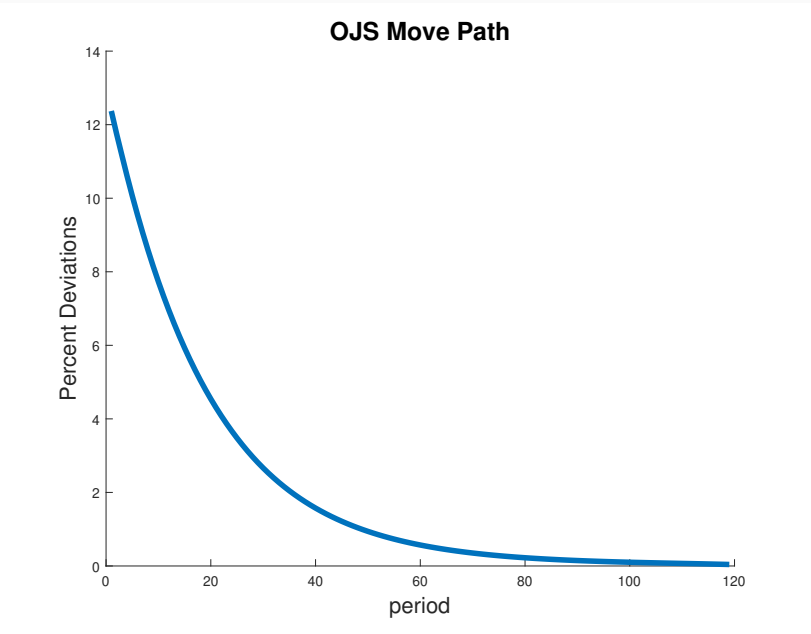


Figure 8: Job-to-job Moves

- More quitting
- Similar (excessive) J-2-J transitions

M4 Low Ave J-2-J 1% Delta Shock ($\rho = .95$)

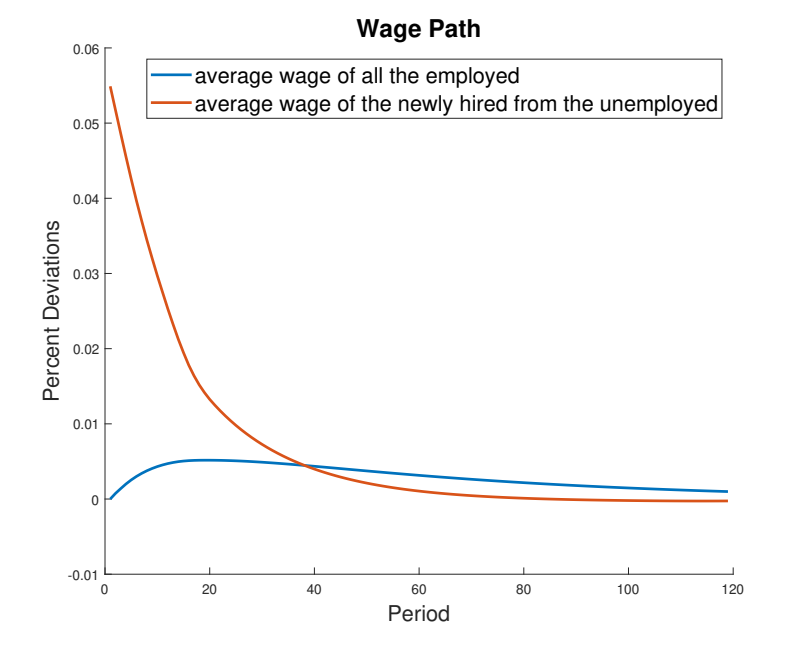


Figure 9: Wages

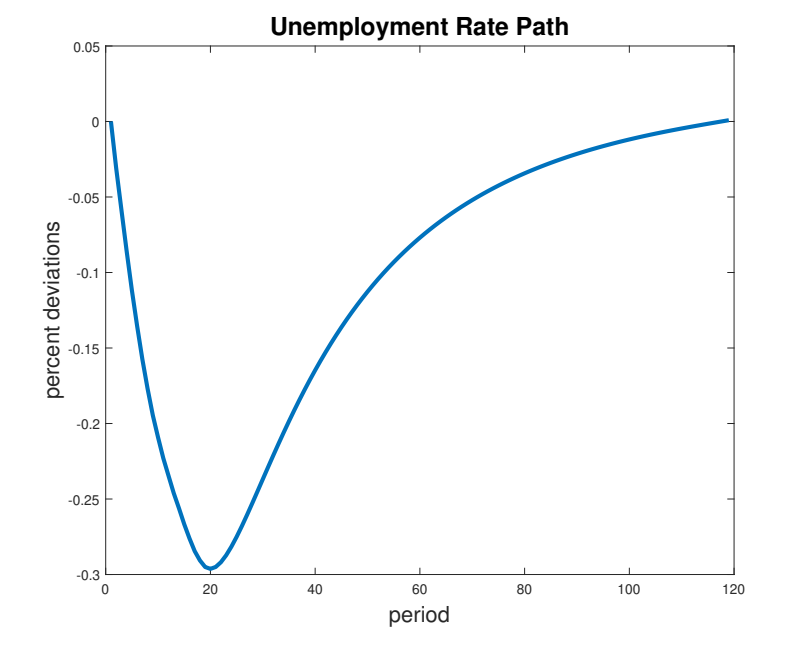


Figure 10: Unemployment Rate

- Similar Wage Response
- 16% more unemployment response
- Note wage is not pegged to the delta shock

M4 Low Ave J-2-J 1% Delta Shock ($\rho = .95$)

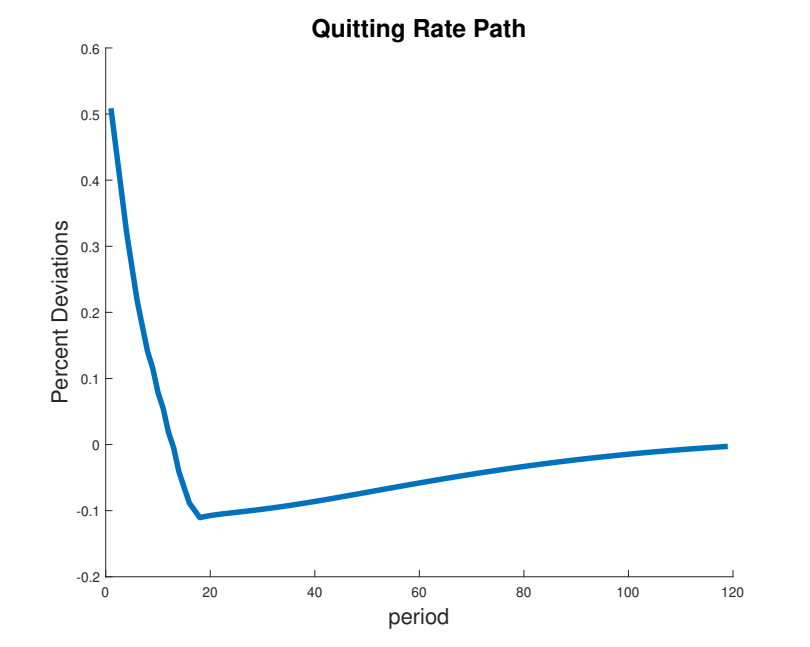


Figure 11: Quits

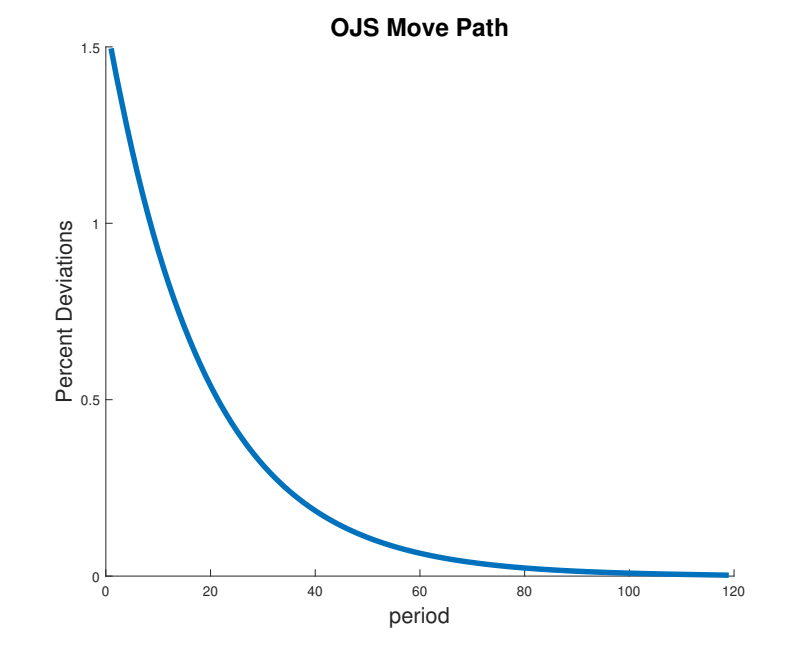


Figure 12: Job-to-job Moves

- More Quit similar (excessive) volatility for job-to-job transitions

M4 Low Ave J-2-J: Business Cycle Statistics

- Two ways to aggregate shocks

	shock corr = 0.95		shock corr = 0	
	Std	corr	Std	corr
output	1.00	1.00	1.00	1.00
avg wage	0.41	0.93	0.41	0.90
new wage	1.69	0.76	1.38	0.52
unemployment	2.59	-0.73	2.80	-0.63
quits + j2j movers	29.85	0.77	26.72	0.38
J2J movers	36.30	0.79	32.51	0.41

- Not too successful in reducing volatility of quits and J2J movers.
- Need to look for alternatives.

M4 Higher Wage Peg: 1% Productivity Shock ($\rho = .95$)

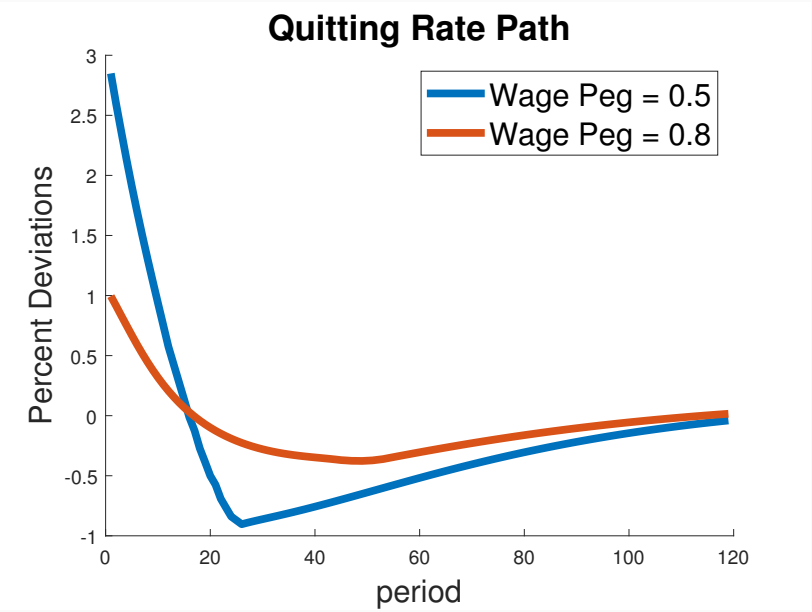


Figure 13: Quits

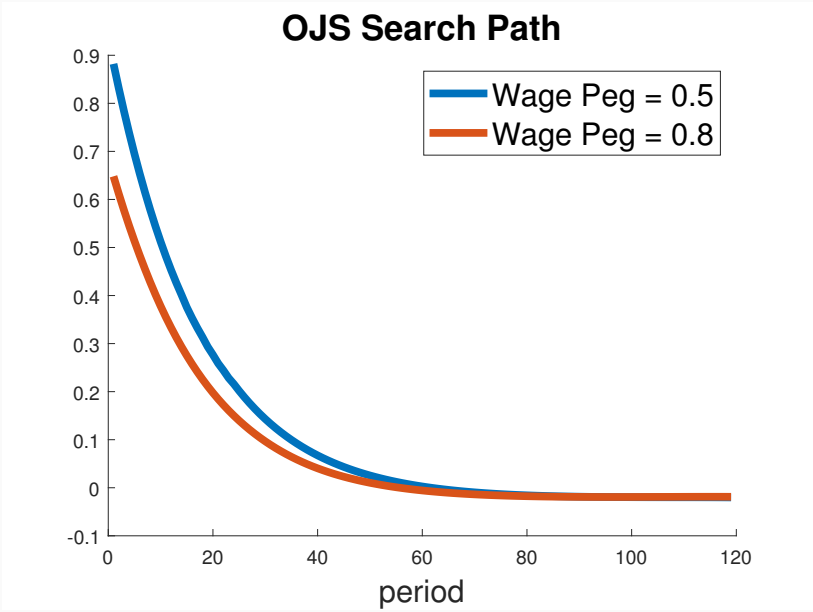


Figure 14: OJS Searchers

- Higher wage peg lowers the reponse of on-the-job search and quit.
  - Workers find it less so attractive to move/quit as existing wages now comove more with the productivity shock

M4 Higher Wage Peg: 1% Productivity Shock ($\rho = .95$)

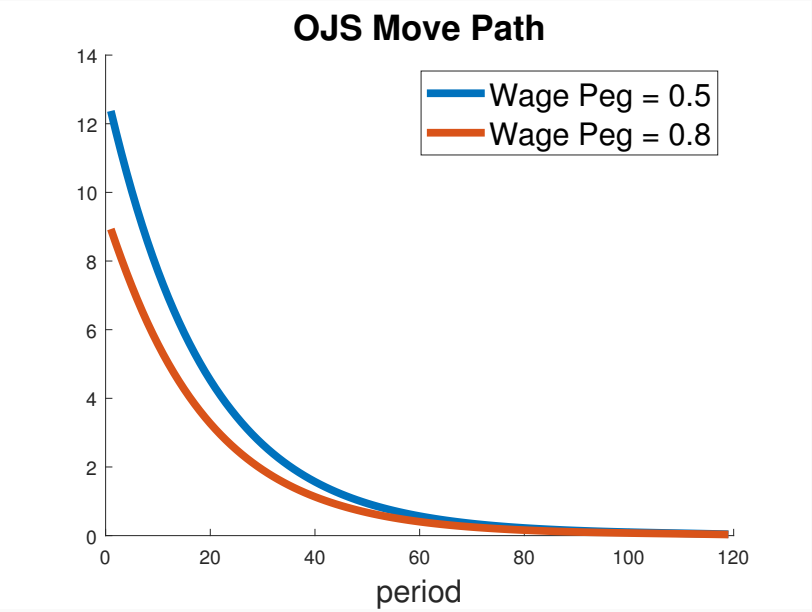


Figure 15: Job-to-job transitions

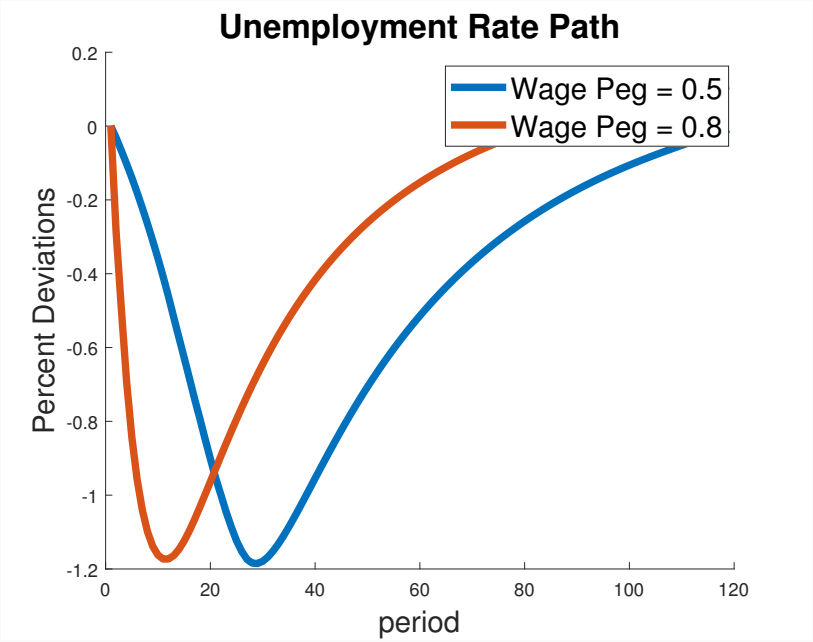


Figure 16: Unemployment

- Job-to-job transition rate also lowers: from 12% to 9%. This is from
  - less search on the job (see Fig 14)
  - less improvement of job finding rate due to smaller s-s firm profits
- Also less persistence of the unemployment response (less turnover).
- However the j2j transition rate is still far more responsive than the unemployment

M4 Higher Wage Peg: Business Cycle Statistics

	Wage Peg = 0.5			Wage Peg = 0.8		
	Mean	Std	Corr	Mean	Std	Corr
Output	1	1	1	1	1	1
Avg Wage	0.690	0.51	1.00	0.690	0.76	0.99
New Wage	0.689	0.95	1.00	0.689	1.04	0.99
Unemp Rate	10.6%	0.35	-0.48	10.6%	0.42	-0.64
Quits+J2J moves	38.4%	8.94	0.99	38.4%	6.65	-0.99
J2J moves	29.2%	10.66	0.99	29.2%	8.50	-0.99

Table 3: M4 Compare Wage Pegs: Productivity Shock ($\rho = 0.95$)

- Higher wage pegs lower the j2j transition volatility while raise the unemployment volatility
- However even we make the existing wages comove with productivity closely, the j2j transition volatility is still much higher than the unemployment volatility
- In the next several pages we take a closer look at this problem

Quitting Makes a Big Difference

- Job finding Rates back