Banking Dynamics and Capital Regulation

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WORK IN PROGRESS

Capital Buffers as a form of Regulation

- A threshold of a ratio between own capital and risk weighted assets.
- Below this threshold, bank activities are limited to not issue dividends, nor to make new loans, while the capital recovers.
- If own capital gets very low (another thereshold, say 2%) banks may get intervened or liquidated.
- Rationale is to Protect the Public Purse safe when there is Deposit Insurance in the presence of moral hazard on the part of the bank.

New Regulations, Basel III: Counter-cyclical capital buffer

- To ease the regulation in recessions.
- Why?
  1. Automatically the Recession makes the capital requirement tighter by reducing the value of assets (and hence of capital), and/or by relabeling those assets as riskier.
  2. Banking Activity (lending) is more socially valuable.
- A tight requirement would induce some banks to reduce drastically their lending to comply if adversely affected.
- We want to Measure the trade-offs involved when taking into account many (quantitatvely) relevant features.
- Change in capital requirements on the onset of a recession
  - How much extra credit?
  - How much extra banking loses?

Not so new a Question

- Davydiuk (2017).
  - There is overinvestment due the moral hazard of investors (banks) that do not pay depositors
  - The overinvestment is larger in expansions because of decreasing returns and bailout wedge increasing in lending.
  - Nicely built on top of an infinitely lived RA business cycle model.
- Corbae et al. (2016) is quite similar except, single bank problem with market power, and constant interest borrowing and lending. Done to have structural models of stress testing. They miss the crucial ingredient of market discipline.

What is a bank? Related to Corbae and D'Erasmo (2016)

- A costly to start technology that has an advantage at

  1. Attracting deposits at zero interest rates (provides services). We think that this margin is not very elastic over the cycle.

  2. Matching with borrowers and can grant long term "risky loans" at interest rate r with low, but increasing, emission costs. This is the main margin of banks behavior.

  3. It can borrow (issue bonds) in addition to deposits and default. Crucial feature as it adds market discipline to the environment.

- Its deposits are insured but its loans and its borrowing are not: There is a moral hazard problem.

- Assets are long term, liabilities are short term

Features that are not there

- Banks cannot issue equity. Just accumulated earnings.
- Banks cannot resell loans.
- Endogenous determination of the rest of the economy, especially interest rates

Banks may be worth saving even if bankrupt

- New loans are partially independent of old loans.
- Capacity to attract deposits is valuable.
- May get better over time on average.
- Large bankruptcy costs.
- Banks may take time to develop. They grow slowly in size due to exogenous loan productivity process and need for internal accummulation of funds.
- Useful also for Shadow Banking

Model: There are also aggregate shocks z that shape things

- A bank is ξ, exogenous, idyosincratic, Markovian $\Gamma^{z,\xi}$.
  - Access to deposits;
  - Costs of making new loans and managing bonds issuances.
  - Characteristics of loans: duration and failing rates.
  - Characteristics of management (patience)
  - Zealousness of regulators they confront.

- A bank has liquid assets a that can (and are likely to) be negative and long term loans ℓ (decay at rate λ).

- Banks make new loans n, distribute dividends c and issue risky bonds b' at price $q(z, \xi, \ell, n, b')$.

- The bank is subject to shrinkage shocks to its portfolio of loans δ, $\pi_{\delta/z}$, that may bankrupt it. Costly liquidation ensues.

- New banks enter small ξ at cost $\overline{c}^e$

Model: What are Aggregate Shocks

- Determines the distribution of δ
- Determines the countercyclical capital requirement $\theta(z)$;.
- Could also determine the details of measuring risk ($\omega^r(z)$ risk weight of assets)
- Note that in this version there is no interaction between banks. The distribution is not a state variable of the banks' problem.
- The state of the economy is a measure x of banks that evolves over time itself via banks decisions and shocks (an extension of Hopenhayn's classic)

Model: Bank's Problem

$$V(z,\xi,a,\ell) = \max\{0, W(z,a,\ell,\xi)\}$$

$$W(z,\xi,a,\ell) = \max_{n\geq 0, c\geq 0, b'} \left\{ u(c) + \beta \sum_{z',\xi',\delta'} \Gamma_{z\xi,z'\xi'} \pi_{\delta'|z'} \, V[z',\xi',a'(\delta'),\ell'(\delta')] \right\} \text{ s.t.}$$

$$(TL) \qquad \ell' = (1-\lambda)\,(1-\delta')\,\ell + n$$

$$(TA) \qquad a' = (\lambda + r)(1-\delta')\ell + r\; n - \xi_d - b'$$

$$(BC) \qquad c + \overline{c}^f + n + \xi_n(n) \leq a + q(z,\xi,n,\ell,b')b' + \xi_d$$

$$(KR) \qquad \frac{\text{Equity}}{\omega^r(z)\,(n+\ell) + \omega^s\; 1_{b'<0} b' q(z,\xi,\ell,n,b')} \geq \theta(\xi,z) \qquad \text{or}$$

$$(KR) \qquad c = n = 0 \qquad \text{and capital ratio} > .02$$

Note that the bank can lend $b' < 0$, it has operating costs $\overline{c}^f$ (nonlinear u and functions ξ^n are convex).

Model: Solution of Banks Problem given $q(\xi', \ell, n, b')$

- The solution to this problem is a set of functions
  - $b'(z, \xi, a, \ell)$ bonds borrowing (or safe lending)
  - $n(z, \xi, a, \ell)$ new loans
  - $c(z, \xi, a, \ell)$ dividends
- The solution yields a probability of a bank failing
  - $\delta^*(z, \xi, \ell, n, b')$

Model: Equilibrium

The only relevant equilibrium condition is

1. Zero profit in the bonds markets:

$$q(z, \xi, \ell, n, b') = \frac{1 - \delta^*(z, \xi, \ell, n, b')}{1 + \overline{r}}$$

Model: Aggregate State, $\{z, x\}$

- The choices of the bank $\{n(z, \xi, a, \ell), b'(z, \xi, a, \ell), c(z, \xi, a, \ell)\}$ and the exogenous shocks $\{z', \xi', \delta'\}$ generate a transition for the state of each bank and in turn of the distribution of banks..

Definition
A, equilibrium is a function $x' = G(z, x)$, a price of bonds q, and decisions for $\{n, b', c\}$ such that banks maximize profits, lenders get the market return, and the measure is updated consistently with decisions and shocks.

Putting the Model to use

- We pose an economy that (after many periods in good times) resembles a current distribution of banks.
- Then explore what happens upon the economy entering a recession, under various scenarios:
  1. No Countercyclical Capital Requirement and adjusted ω^r to reflect that the loans are riskier.
     - More loans are destroyed
     - Outlook of loans is worse
  2. No Countercyclical Capital Requirement and no adjustment in ω^r.

Plan

- Describe Targets
- Describe properties of an stationary allocation in good times.
- Describe the transition when the economy switches to a recession.
- This is more like an example. We are now estimating the model to Replicate the Canadian Banking Industry with (6) Large and (40+) Small Banks.

Long Good Times Targets Capital Requirement: $\theta = .105$

- We have the following industry properties

	(Canadian) Data	Model
Bank failure rate	0.22%	0.26%
Capital ratio	14.4%	14.4%
Wholesale Funding	27.0%	21.8%

Normalized T-Account of Banking Industry

Canadian Data

New Loans	1.07	Deposits	3.31
Existing Loans	4.87	Wholesale Funding	1.63
		Own Capital	1.00

Model

New Loans	1.26	Deposits	4.40
Existing Loans	5.69	Wholesale Funding	1.51
		Own Capital	1.00

The issue of Calibrating Risk Weights: Forward looking

How do regulators assess risks for the purposes of computing the capital requirement?

- By Revealed Preference (we implement what they seem to do not what they seem to say)
- For each group of banks, we calibrate the risk weight on risky loans to the implied average risk weight in the data:

$$\widehat{\omega}_r(z = g, \xi) = \frac{\text{total risk weighted assets in 2017Q1}}{\text{total risky assets in 2017Q1}}$$

  Both terms in RHS are published by regulators.

- We want to think of featuring two groups of banks:
  1. Canadian Big 6 banks
  2. Non-Big 6 banks
- The risk weight on safe assets, ω_s, is set to zero.

Model: Capital Requirement, $\theta(z, \xi)$

- $\theta(z, \xi)$ is the capital requirement where banks need to maintain their capital ratio above it to avoid supervisory penalty.
- CCyB changes this requirement based on the aggregate state of the economy, i.e., z.
- The requirement also differs for Global Systemically Important (GSIB) or Domestic Systemically Important (DSIB) Banks.
- When regulators identify banks as GSIB or DSIB, their capital requirement increases by 1 to 3.5% above non-GSIB/DSIB banks.
- The size of bank is a determining factor among others, i.e., ξ.
- Currently, six largest banks are DSIBs in Canada, charged with the additional capital requirement of 1%.

The issue of Calibrating loan failure rates

- Given $\widehat{\omega}_r(\xi)$, we compute the implied probability of loan default, $\widehat{\delta}$, for each bank group, using the regulatory formula defining risk weights.

  Internal rating-based approach formula defines the risk weight on corporte loans as follows:

$$\widehat{\omega}_r(\xi) = 12.5 \text{ LGD } \left[\Phi\left(\frac{\Phi^{-1}(\widehat{\delta}) + \sqrt{R}\Phi^{-1}(0.999)}{\sqrt{1-R}}\right) - \widehat{\delta}\right] \frac{1+(M-2.5)b}{1-1.5b}$$

  where Φ is the standard normal distribution,

$$R = 0.12\frac{1-\exp(-50\widehat{\delta})}{1-\exp(-50)} + 0.24\left[1 - \frac{1-\exp(-50\widehat{\delta})}{1-\exp(-50)}\right],$$

$$b = \left[0.11852 - 0.05478\log(\widehat{\delta})\right]^2,$$

  LGD is the loss given default and M is the maturity of loans

- Then, we match the ratio of average loan failure rates across bank groups to the ratio of $\widehat{\delta}$ between Big 6 and Non-Big 6 in the data:

$$\frac{\mathbb{E}\ \delta'_{\text{big banks}}}{\mathbb{E}\ \delta'_{\text{small banks}}} = \frac{\widehat{\delta}_{\text{Big 6}}}{\widehat{\delta}_{\text{Non-Big 6}}}$$

Another what are Recessions, $z = b$

- First what is the tail distribution of bank failures. Perhaps we have to explore different scenarios

- How do regulators perceive those risks and get their

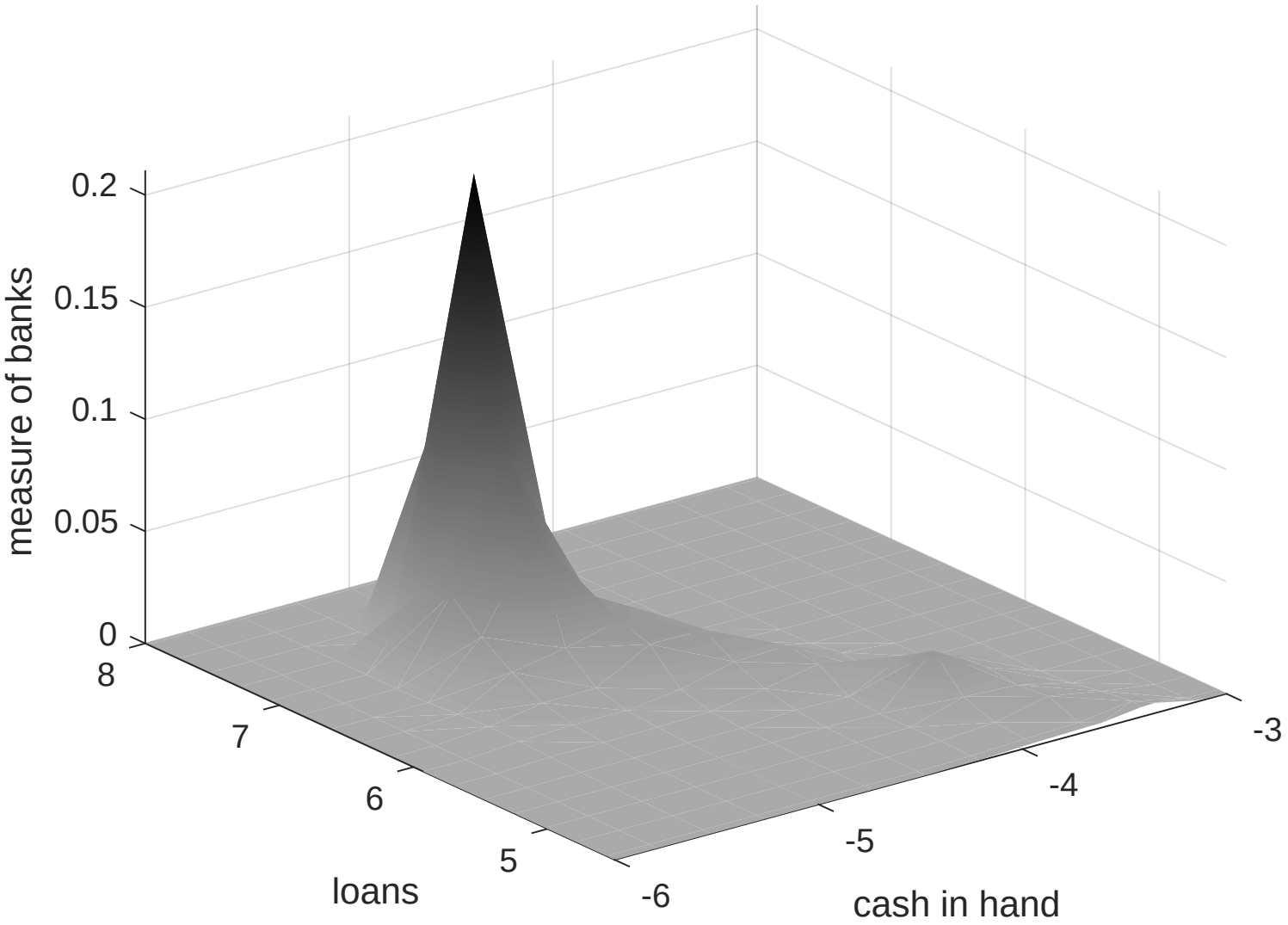
$$\widehat{\omega}(z = b, \xi)$$

We will have to explore various ones. So far this has not mattered much.

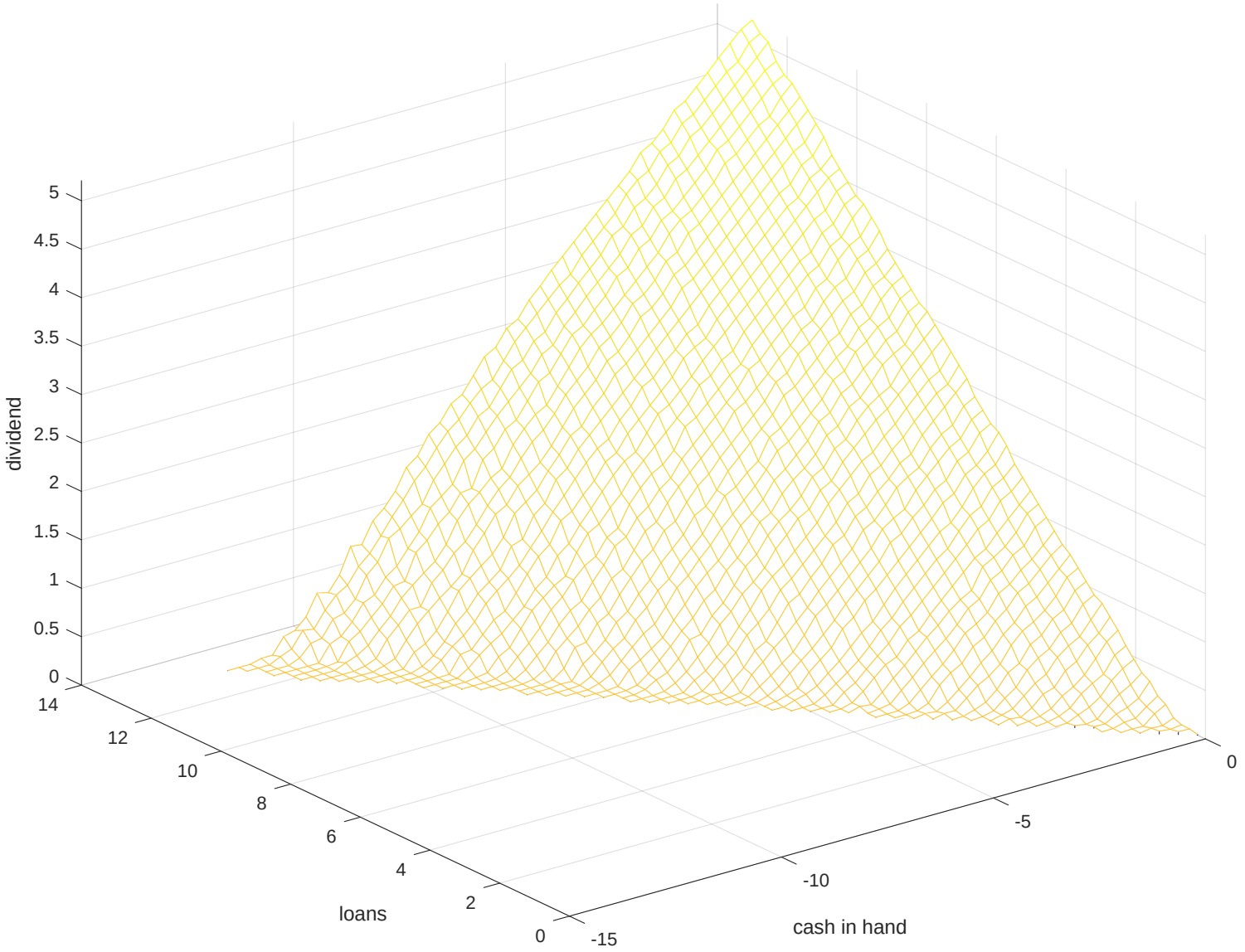
Model Parameters

Parameter	Value	Description
$\xi_n^{\mathbf{0}}$	0.075	Loan issuance cost: $\chi(n, \xi_n) = \xi_n^{\mathbf{0}}\, n + 0.5\, \xi_n^{\mathbf{1}}\, n^{\mathbf{2}}$
$\xi_n^{\mathbf{1}}$	0.15	Loan issuance cost: $\chi(n, \xi_n) = \xi_n^{\mathbf{0}}\, n + 0.5\, \xi_n^{\mathbf{1}}\, n^{\mathbf{2}}$
ξ_d	5	Deposits
β	0.95	Subjective discount factor
λ	0.2	Maturity rate of long-term loans
r	0.1	Bank lending rate
r_f	0.005	Risk-free rate
σ	0.9	$u(c) = c^{\sigma}$
ω_r	1	Risk weight on risky loans
ω_s	0	Risk weight on safe assets
$\Gamma_{z=G,z'=G}$	0.99	$\Pr(z' = G \mid z = G)$
$\Gamma_{z=B,z'=B}$	0.80	$\Pr(z' = B \mid z = B)$
$E(\delta \mid z = G)$	0.025	$\Sigma_\delta\ \delta \cdot \pi(\delta \mid z = G)$
$V(\delta, Z = G)$	0.0015	$\alpha(Z = G) = 0.3847$, $\beta(Z = G) = 15.0011$
$E(\delta \mid z = B)$	0.040	$\Sigma_\delta\ \delta \cdot \pi(\delta \mid z = B)$
$V(\delta, Z = B)$	0.0040	$\alpha(Z = B) = 0.3417$, $\beta(Z = B) = 8.2009$

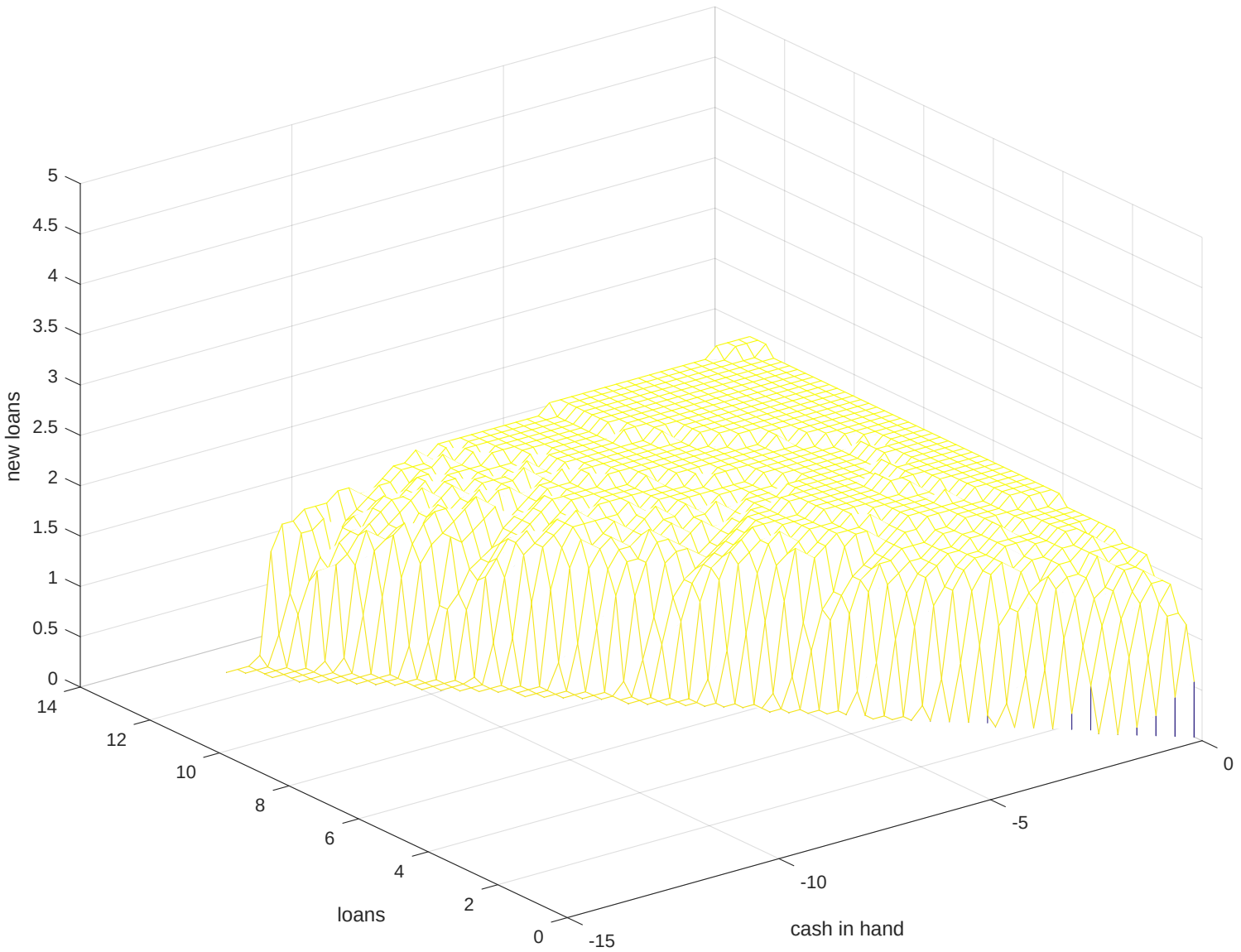
Distribution of Banks



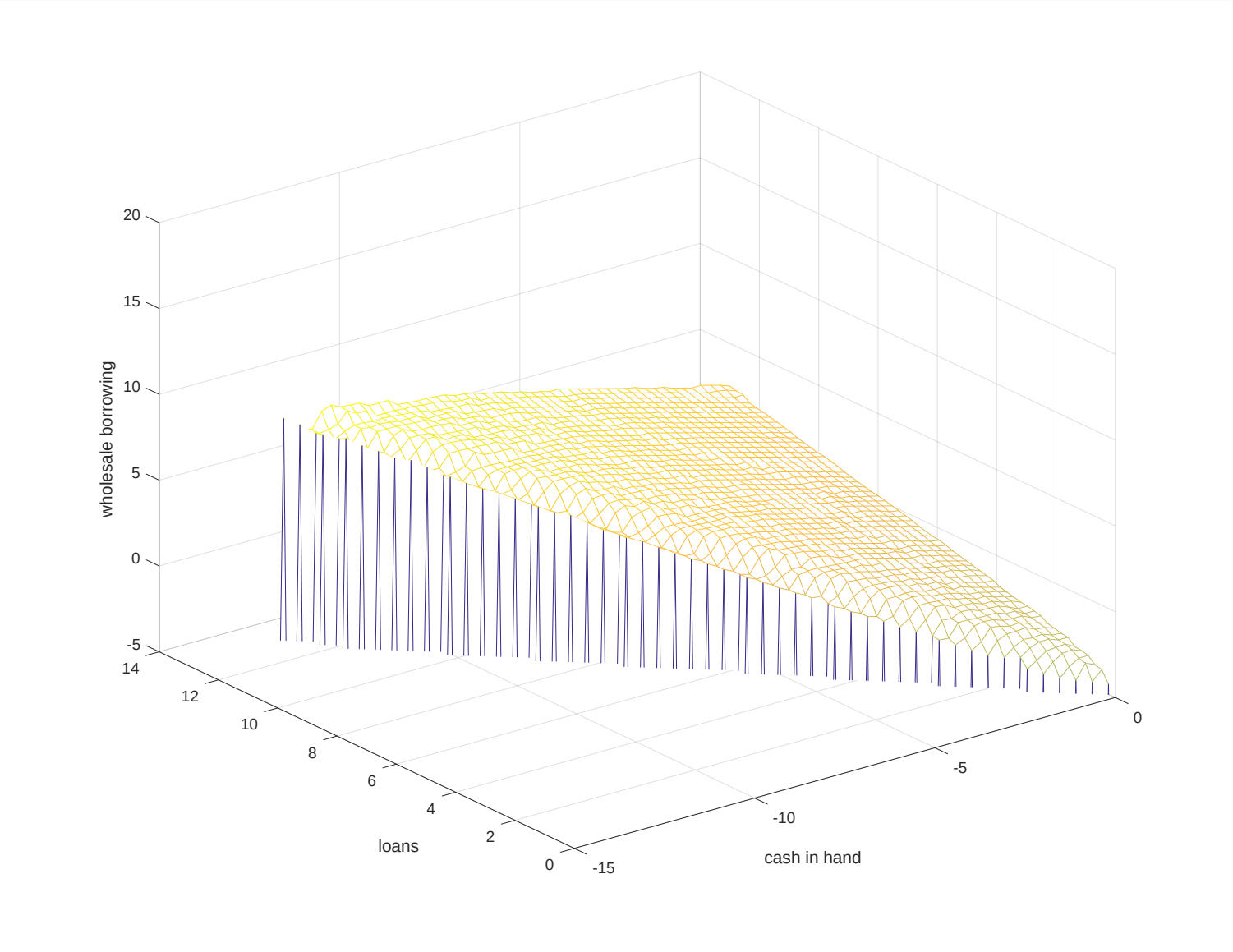
Banks Dividends



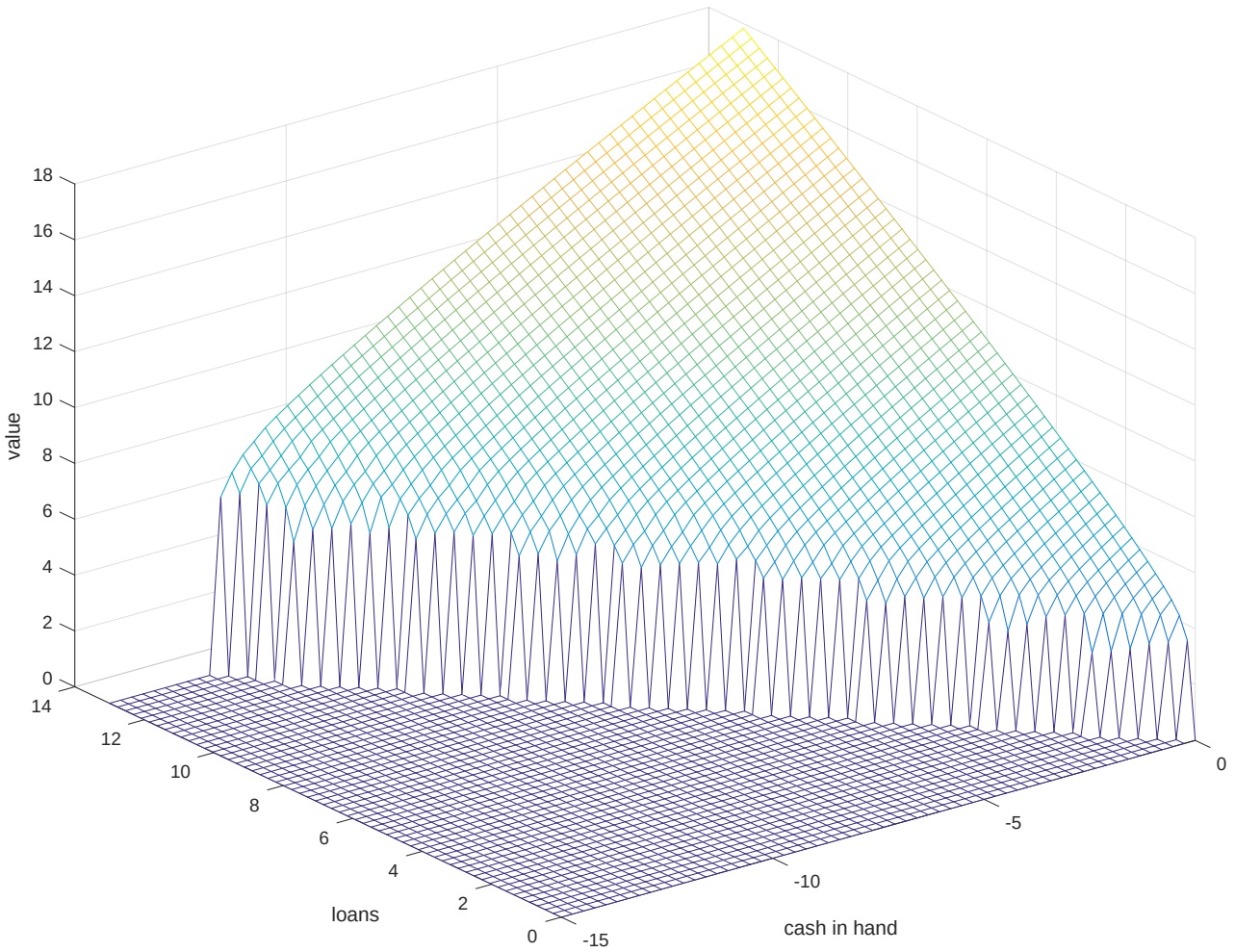
Banks New Loans Issue



Banks Wholesale Funding (Deposits plus Bonds)



Banks Value Function



Public Loses when Banks touch Intervention Threshold (2%)

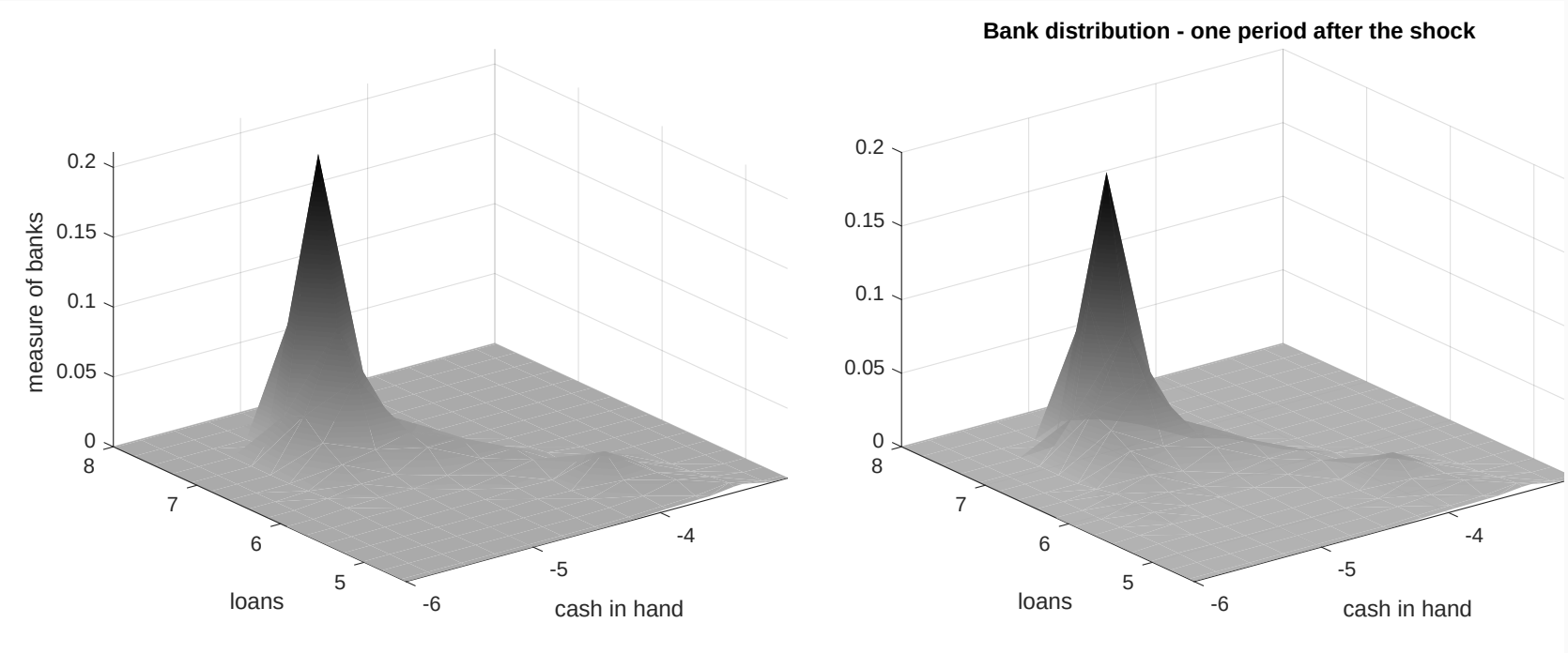
Recovery Rate of Bank Assets at Default	Discount Rate of Regulator		
	0.5% (Risk-Free Rate)	2.0%	5.0% (Bank's Discount Rate)
0.3	23.01	7.92	3.43
0.6	9.84	3.40	1.49
1.0	-1.11	-0.94	-0.71

- The Public does well in closing the bank

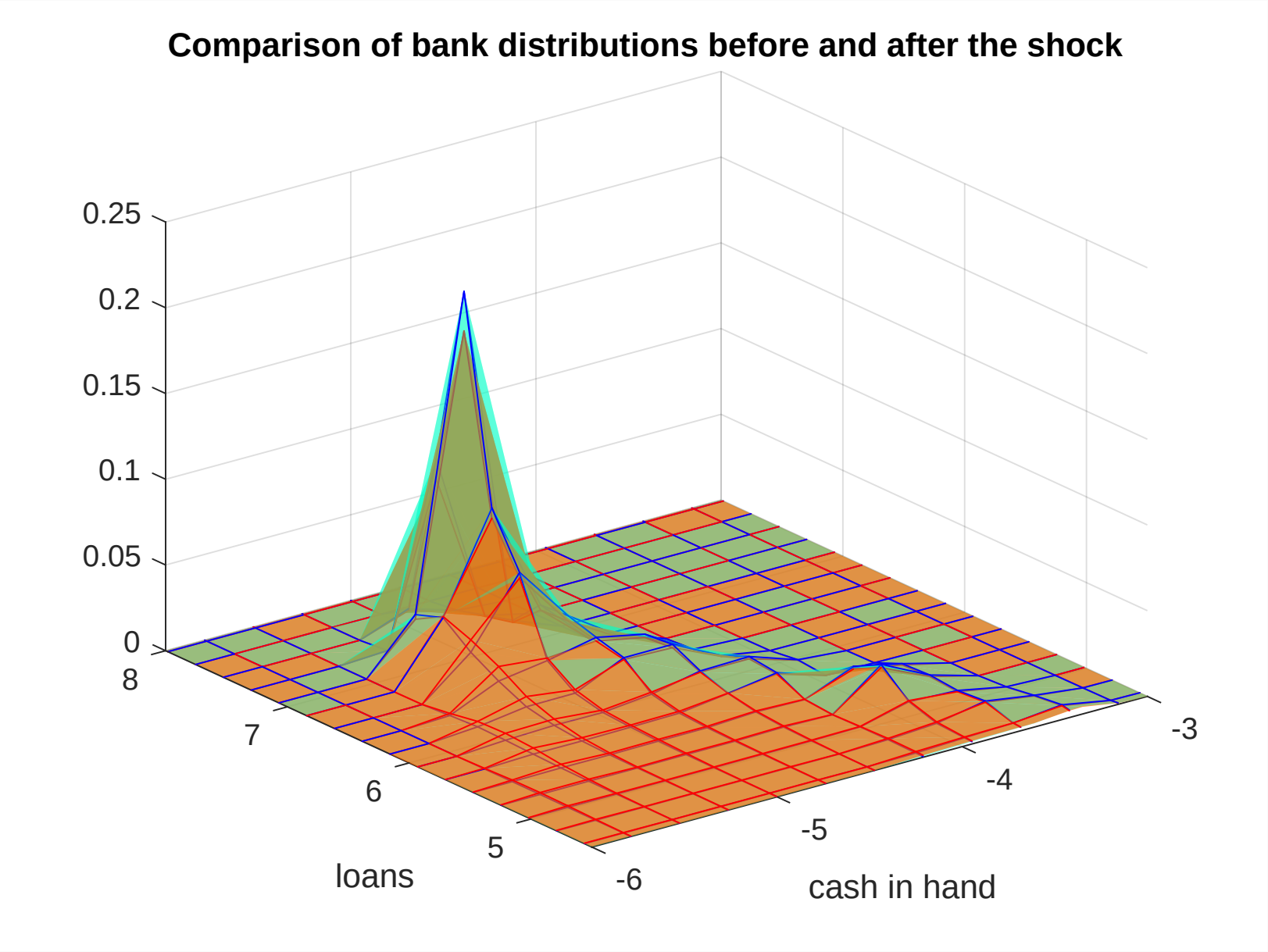
A Nasty Crisis with and without CCyB

- Imagine the shock $\triangle E(\delta) = 0.015$ (from .025 to .04) hits all banks, which happens with a very small probability, 0.01. The crisis continues for two periods and ends to go back to the good aggregate state thereafter.
- Some banks are in better financial shape than others.
- We explore the recovery of the Banking sector under the four scenarios.
- What happens upon

A Nasty Crisis with and without CCyB



A Nasty Crisis with and without CCyB

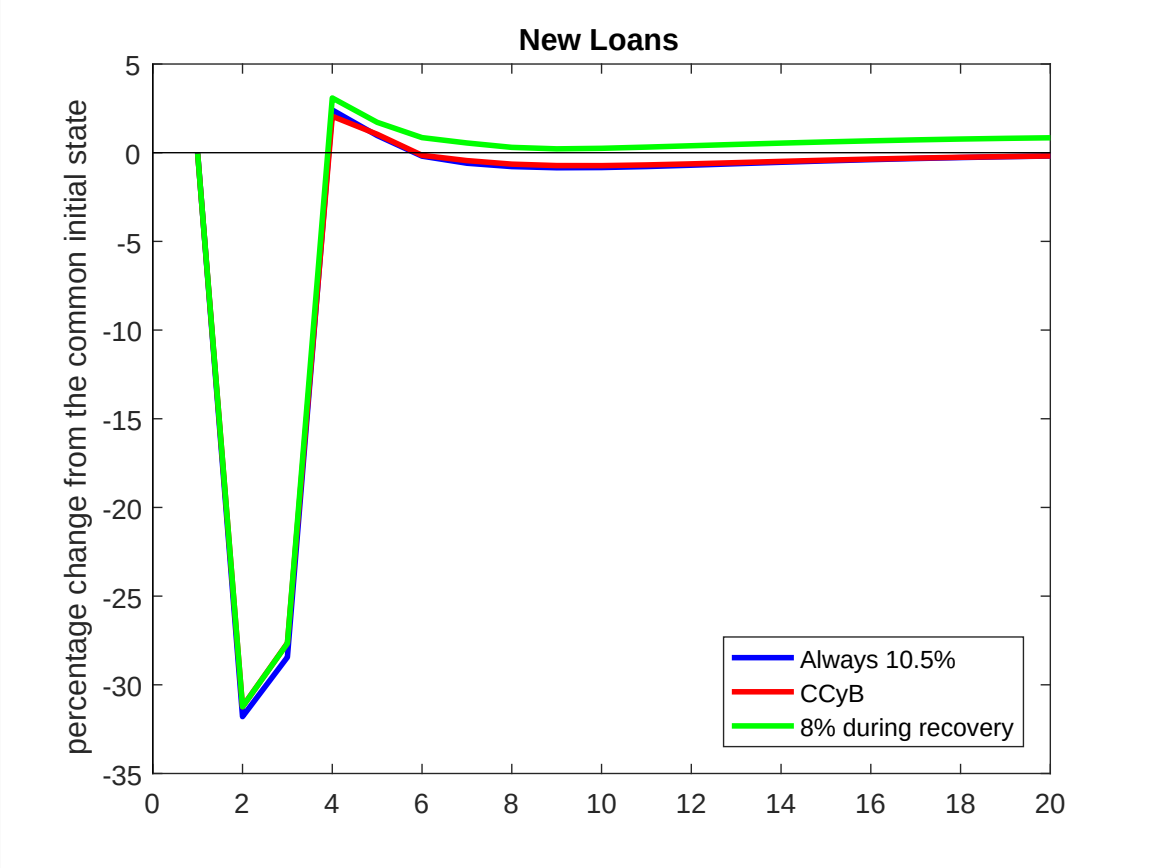


Ulterior Path of the Economies after the shock

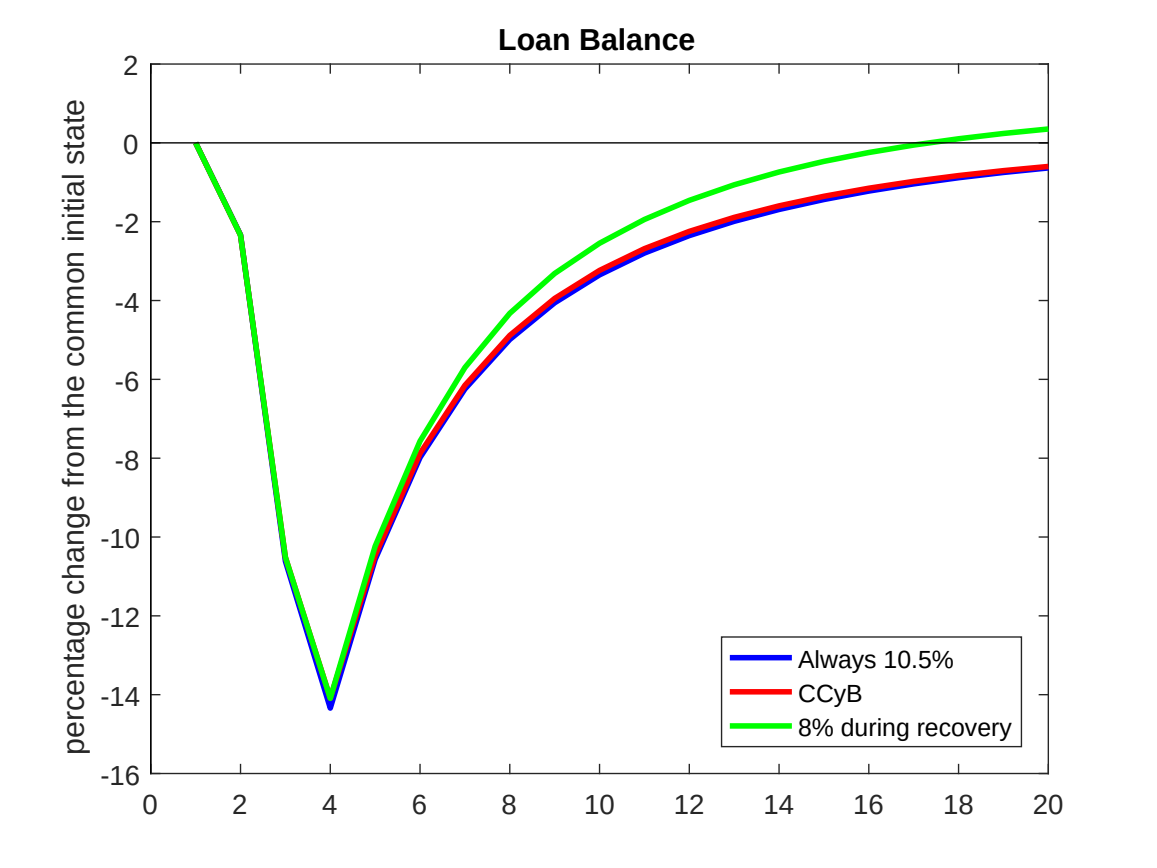
- Recall that it is a recession for two periods and then we have a recovery.
- We compare Countercyclical Capital Requirement with a constant weight to risk assests (left)and with a variable weight (right)
- We look at impulse responses

New Lending

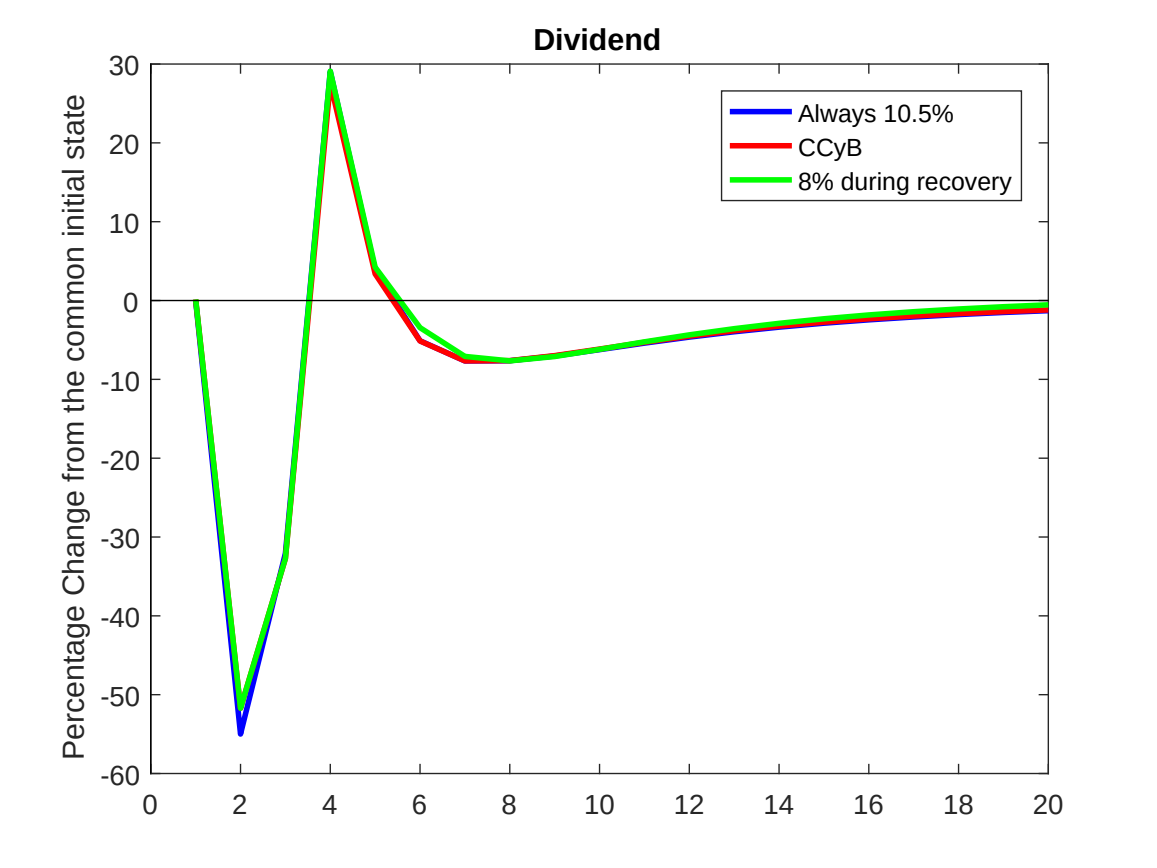
Small difference between non-contingent policy and CCyB during the downturn. CCyB (if low capital requirement extends for a longer period) provides some help during the recovery.



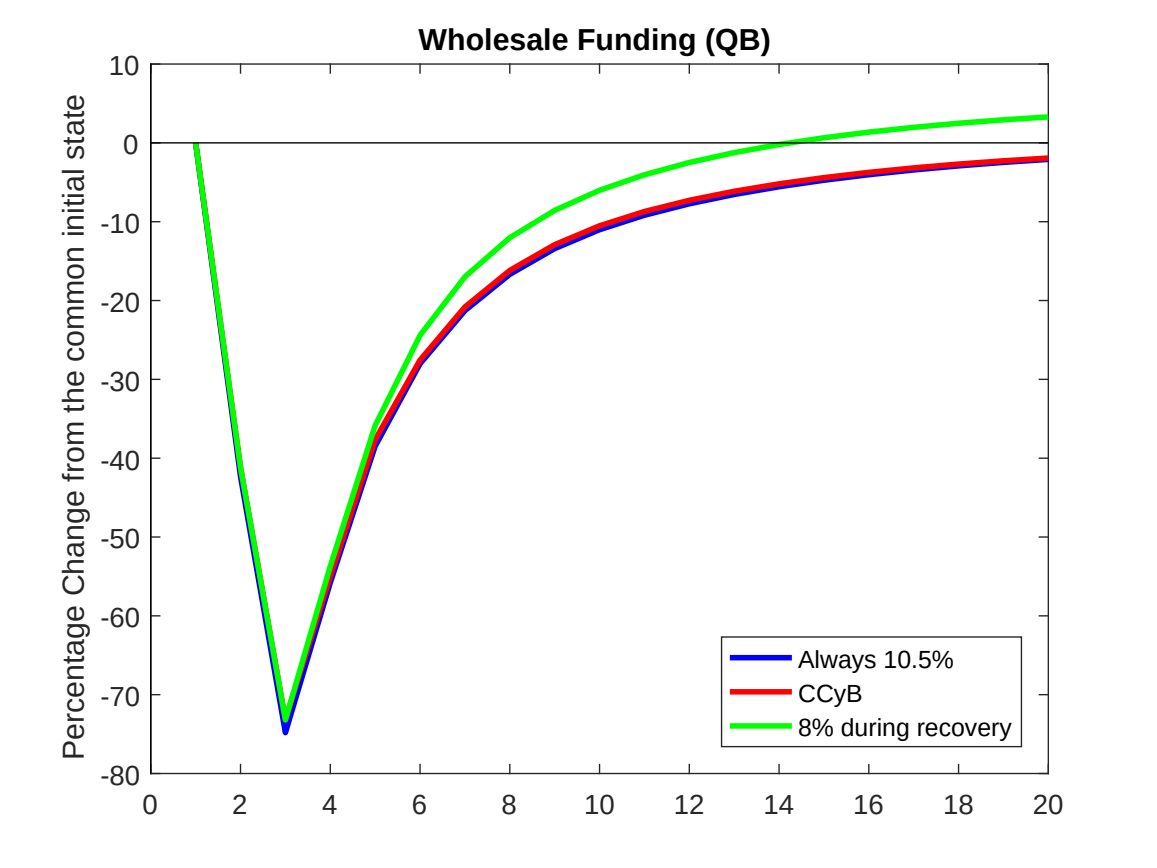
Stock of Loans



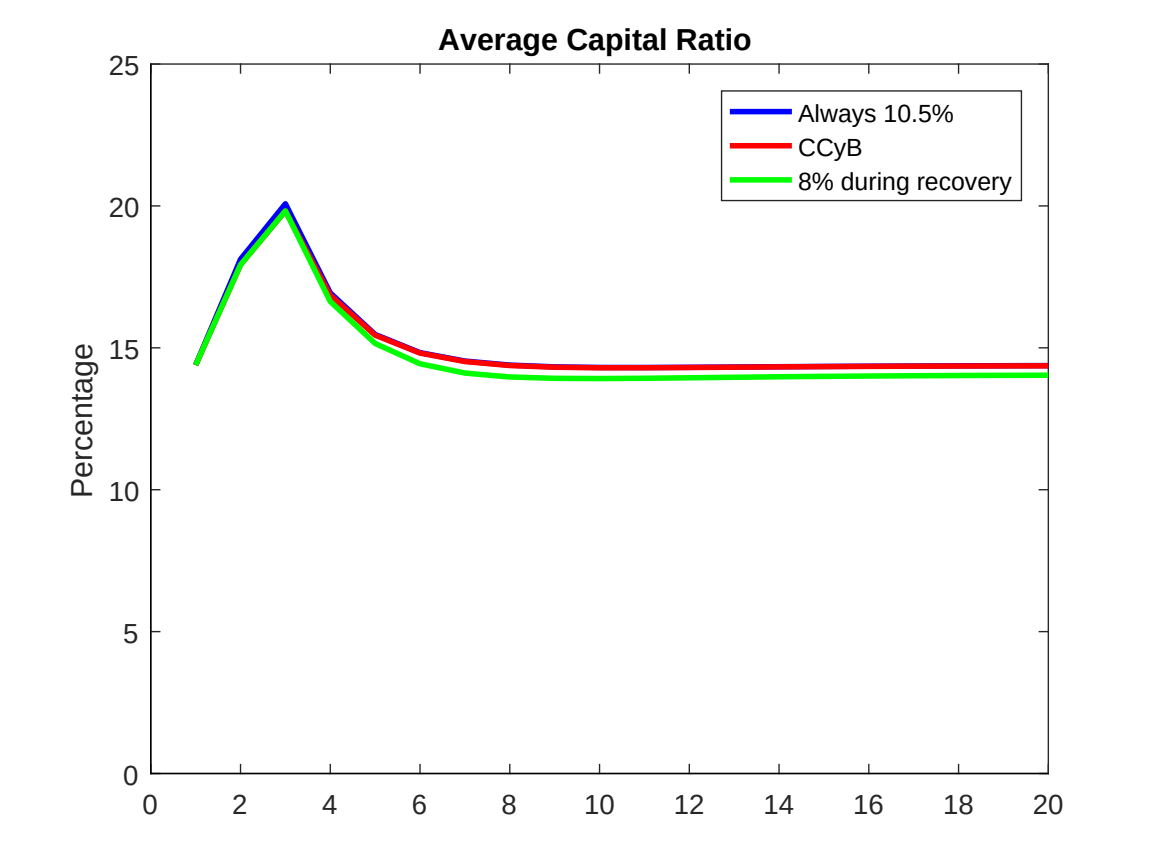
Dividends


Dividend
Percentage Change from the common initial state
30
20
10
0
-10
-20
-30
-40
-50
-60
0
2
4
6
8
10
12
14
16
18
20
Always 10.5%
CCyB
8% during recovery

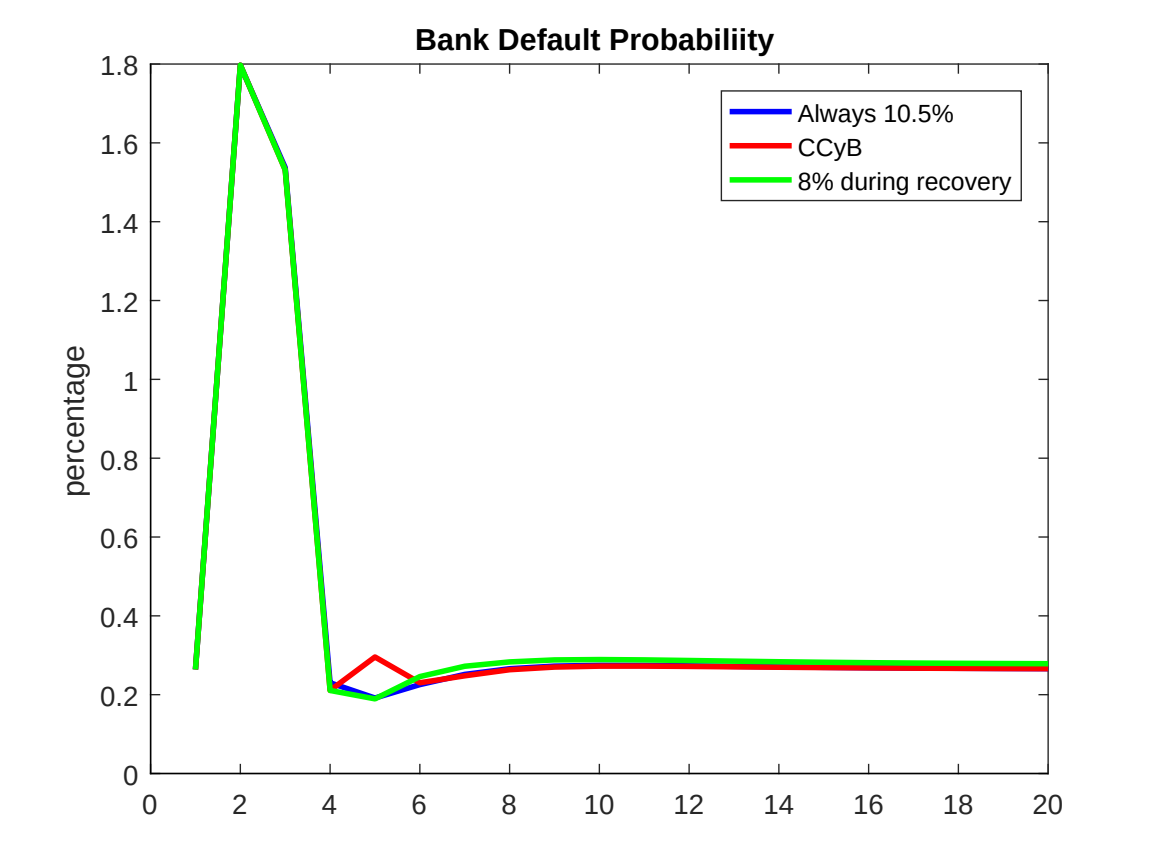
Wholesale Funding



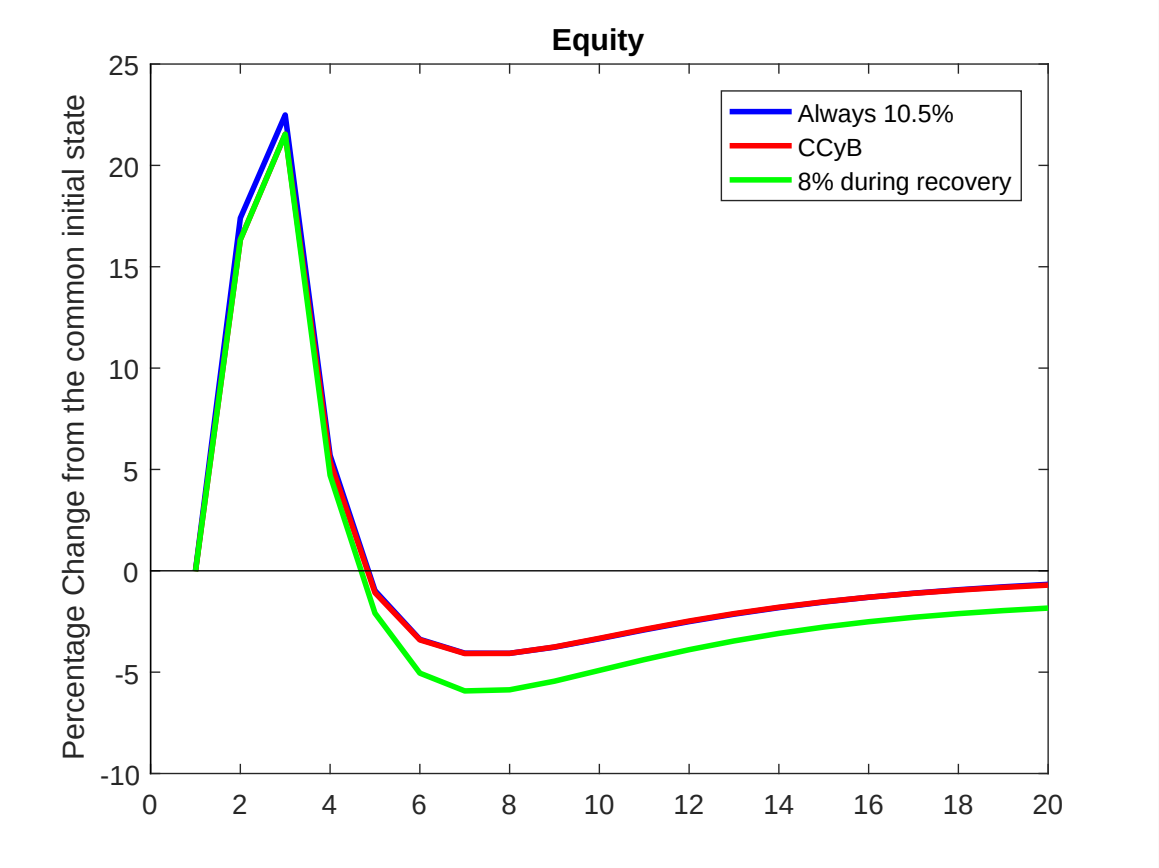
Capital Ratio



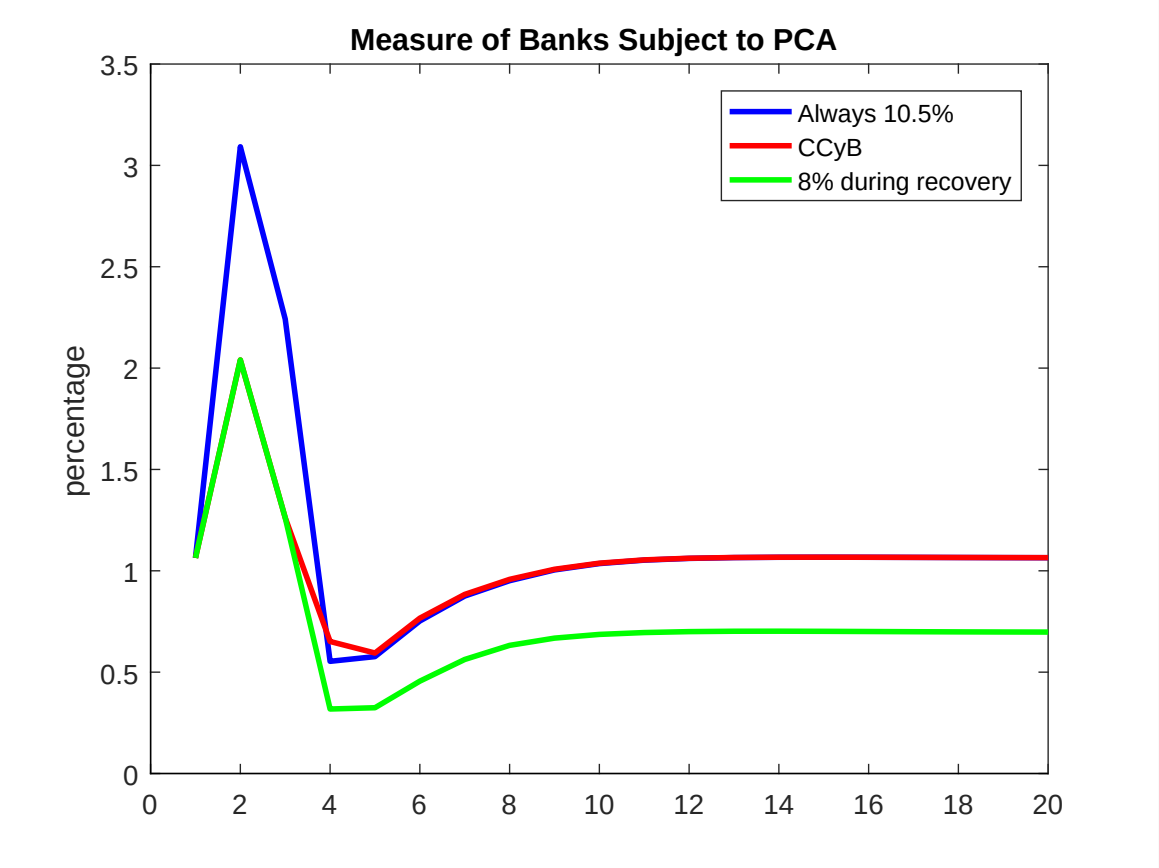
Bank Failure Rates



Bank Equity


Equity
Percentage Change from the common initial state
25
20
15
10
5
0
-5
-10
0
2
4
6
8
10
12
14
16
18
20
Always 10.5%
CCyB
8% during recovery

Fraction of Capital Requirement Violation



Directions of Current Work

- To replicate the Industry structure properly
  - Size of Banks in terms of Numbers and Dollars (large and small banks)
  - Cross-Sectional (and temporal) Dispersion of
    - New Loan issues
    - Dividends
    - Outside financing (bonds)

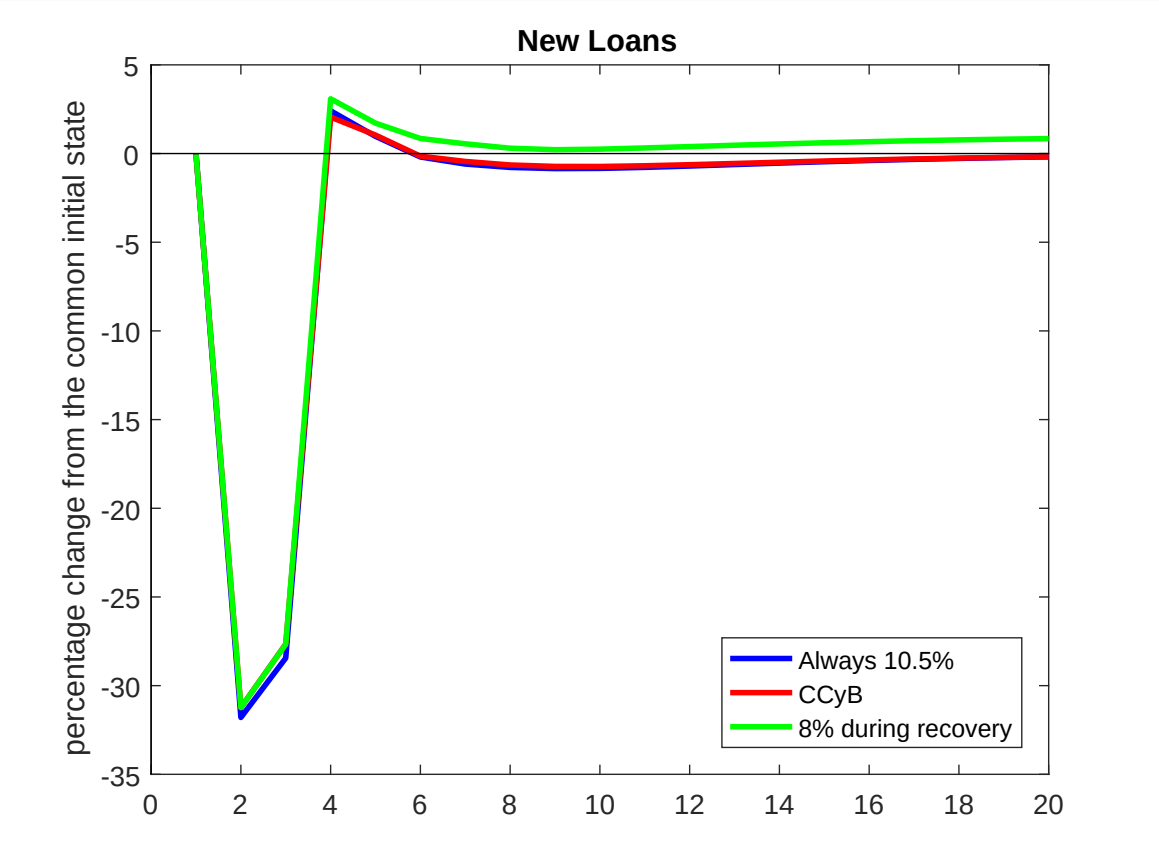
Shortcomings and Extensions

- Competitive Theory of Lending (Corbae and D'Erasmo (2016))
- Firms have zero measure. We could wipe out a positive measure of financial institutions and call it one bank.
- Need to pose this industry into a GE framework so ALL interest rates can be determined endogenously.
- Bank Runs:
  - Can be interpreted as a low probability state with $\xi_d = 0$
  - For shadow banking we need some multiple equilibrium notions á la Cole and Kehoe (2000)
- Notion of "systemic" banks. It needs a good theory of drops in price of collateral.
- Contagion, financial crisis. This needs serious thinking.

Temporary Conclusions

- We (want to) measure the effects of countercyclical capital requirements.
- We insist in capturing the margins that we deem important:
  1. Moral Hazard
  2. Bank's risk taking that can lead to its failure
  3. Banks choose dividends/loans/outside financing
  4. Endogenous bank funding risk premium: market discipline
  5. Maturity mismatch between long-term loans & short-term funding
  6. Accurate representation of both banks actual choices and regulator behavior
- Lowering capital requirements has little effect because banks are already concerned.
- Perhaps our findings will change when we fine tune the calibration so that banks' capital shrinks.

New Lending by Banks: with 8% Capital Requirement during Recovery



General Equilibrium

- Consider a household with per period utililty function $u(c, d)$, where d stands for deposits' services.
- Deposits are created via matches with banks. Total (and per capita) deposits are the aggregate of bank services. We can think of a matching function with banks.

$$D = \int \xi_d \, dx$$

- Households own shares of a mutual fund

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Representative Bank-Representative Household version of Dynamics and Capital Regulation

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1 Linear Costs for Banks

General Equilibrium Model

- There is a household sector with indivisible labor (many workers in a household).
- There is a banking sector that produces deposits' services and make loans with CRS.
- There is a productive sector with a putty clay technology.
- Otherwise it is a growth model.
- There may be shocks to TFP, to the destruction of new and old firms, and to the banking management losses.
- But we start lookng at a steady state

Households

- Period utililty $u(c, n, d)$, where n is the fraction employed and d stands for deposits' services. Discount rate β.

- Deposits are created via matches with banks. We can think of a matching function with banks.

- A household has a measure one of workers that may or may not have a job. Employment in loan firms is n^ℓ while employment in equity firms is n^e, $n^\ell + N^e \leq 1$. A household member that does not work gets $\overline{c}$ units of utility consumption.

$$u(c, n, d) = \log c + (1 - n)b + v(d)$$

Investment and firms: Putty-Clay

- Firms create plants with one worker using loans in a putty-clay fashion $y = A\ k^{\alpha}$.
- There is free entry of these firms. Upon entry, firms (which are worth zero) join a mutual fund with their liabilities.
- With prob λ loans are paid off.
- All firms get destroyed with probability $\delta \sim \gamma_{\delta}$.
- Extensive margin: There are N^n new firms each period.
- Intensive margin: Each period firms invest k units.
- Total amount of new loans is $L^n = k * N^n$.
- The whole distribution of firms can be summarized by two aggregates (as in Choi and Ríos-Rull (2010) and others)
- Employment or the number of plants is

$$N' = (1 - \delta)N + N^n.$$

- Output is

$$Y' = (1 - \delta')Y + N^n\ A\ k^{\alpha}.$$

Investment and firms

- Firms borrow at rate r^ℓ.
- The value a newly opened firm with capital k using the effective household interest rate r^b is

$$\frac{\Pi^f(k)}{1+r^b} = \frac{\left[Ak^\alpha - w(k) + \frac{1-\delta'}{1+r^b}\Pi^f(k)\right]}{1+r^b}$$

where $w(k)$ are wages and r^b is the market discount rate. So

$$\Pi^f(k) = \frac{1+r^b}{r^b+\delta}\left[Ak^\alpha - w(k)\right].$$

- The cost of a loan of size k is

$$\sum_{t=1}^{\infty} k\left[r^\ell + \frac{\lambda}{1-\lambda}\right]\left(\frac{1-\lambda}{1+r^b}\right)^t = k\left[r^\ell + \frac{\lambda}{1-\lambda}\right]\frac{1-\lambda}{r^b+\lambda}.$$

Investment decision

- So the optimal size satisfies

$$\max_k \quad \frac{Ak^\alpha - w(k)}{r^b + \delta} - k\left[r^\ell + \frac{\lambda}{1-\lambda}\right]\frac{1-\lambda}{r^b+\lambda}.$$

- With FOC

$$A\,\alpha\,k^{\alpha-1} - w_k(k) = \left[(1-\lambda)r^\ell + \lambda\right]\frac{r^b+\delta}{r^b+\lambda}.$$

- Firms enter until there are zero profits from doing so

$$\frac{Ak^\alpha - w(k)}{r^b + \delta} = k\left[r^\ell + \frac{\lambda}{1-\lambda}\right]\frac{1-\lambda}{r^b+\lambda}.$$

Firms Profits and loses

- Because upon creation firms are worth zero there is no need to worry about their value.
- Once created, firm's profits or loses go to the households who do not buy and sell firms and take those profits as given.
- Profits of all firms are

$$\pi^f = Y - W\ N - L[(1-\lambda)r^\ell + \lambda]$$

Wage Determination

- A bargaining process between the firm and the worker. V: (We may change this to get more wage rigidity and avoid the Shymer puzzle)

- The bargaining process is repeated every period and if unsuccesfull neither firm nor worker can partner with anybody else within a period. Let μ be the bargaining weight of the worker. Then, because of log utility, we have

$$w(k) = \mu\ A\ k^{\alpha} + (1-\mu)\frac{b}{c}$$

- Total (per capita) Labor Income paid in the Economy are

$$W\ N = N\left[\mu\ A\ k^{\alpha} + (1-\mu)\frac{b}{C}\right] = \mu Y + (1-\mu)\frac{Nb}{C}$$

Banking Industry I

- A CRS banking industry uses output to produce deposits and to make loans
- Loans are long term and decay at rate λ. Deposits are short term.
- It borrows and lends short term bonds B' at interest rate r^b.
- A fraction δ^ℓ of the loans are destroyed V: (Still have to discuss the relation between δ and δ^ℓ

$$
\begin{aligned}
D' &= \kappa_d Y^d \\
L^n &= \kappa_\ell Y^\ell \\
L' &= (1-\delta'^\ell)(1-\lambda)L + L^n
\end{aligned}
$$

- Banks cash position

$$A' = (\lambda + r^\ell(1-\lambda))(1-\delta'^\ell)L + r^\ell L^n - D'(1+r^d) - B'(1+r)$$

Banking Industry II

- Bank's Budget Constraint (π^B are dividends)

$$\pi^B + L^n \left(1 + \frac{1}{\kappa_\ell}\right) = A + B' + D' \left(1 - \frac{1}{\kappa_d}\right)$$

- Due to linearity of technology banks have zero steady state profits. $\pi^B = 0$.

- This is not the case outside steady state.

Banking Industry III

- Let $r^\ell(r^b)$ and $r^b(r^b)$ be the interest rates of bonds and deposits when the Capital Requirement constraint is not binding.

$$
\begin{aligned}
1 + \frac{1}{\kappa_\ell} &= \sum_{t=1}^{\infty} \left[\frac{(1-\lambda)(1-\delta)}{1+r^b} \right]^{t-1} \left[(1-\lambda) r^\ell + \lambda \right] \\
r^d &= r^b - \kappa_d \\
r^\ell &= \left[\left(1 + \frac{1}{\kappa_\ell} \right) \frac{r^b + \lambda + \delta - \lambda\delta}{(1+r^b)} - \lambda \right] \frac{1}{1-\lambda}
\end{aligned}
$$

Bonds Markes

- Households lend funds to banks at rate r^b. We call them bonds, B.

General Equilibrium: Markets

- Budget constraint of households

$$c + d' + b' = b(1 + r^b) + d(1 + r^d) + W\,n + \pi^f + \pi^b$$

Definition of a Steady-State Equilibrium

- Stocks: Y, N, Π, A, B, L, D,
- Choices: K, C, A', B', D', L^N, N^n, s.t.
- Prices $r^\ell, r^b, r^d, W, w(k)$
- Profits π^f, π^B

  1. Plant sizes are optimal
  2. Entry yields zero profits
  3. Households solve their problem $r^b = \beta^{-1}$, $u_c = u_d \frac{1}{\kappa_d}$
  4. Wages are determined by Nash bargaining
  5. The choices imply that the stocks repeat themselves

Non-Steady-State Equilibrium: Shocks for $\eta = \{z, \delta, \delta^{\ell}\}$

- As is standard in putty-clay models, there is no need to keep track of the whole distribution of firms. Only of output and number of plants/workers. The aggregate state vector S consists of
  - The shocks η
  - Y Output
  - N Employment or number of plants
  - A Banks Cash
  - B Bonds
  - D Deposits
  - L Loans
- Households also have an idiosyncratic state vector $s = \{b, d, n\}$.

Household Problem

$$v(S,s) = \max_{c,b',d'} \quad u(c,d,n) + \beta E\left\{v(S',s')|S,s\right\} \qquad \text{s.t.}$$

$$\begin{aligned}
c + d' + b' &= b[1 + r^b(S)] + d[1 + r^d(S)] + W(S)\,n + \pi^f(S) + \pi^b(S) \\
N'(S) &= (1-\delta')N + N^n(S) \\
n'(S,s) &= (1-\delta')n(S,s) + N^n(S) \\
Y'(S) &= (1-\delta')Y + N^n(S)\; z\; A\; k(S)^\alpha \\
L'(S) &= (1-\delta'^\ell)(1-\lambda)L + L^n(S) \\
A'(S) &= A'(S) \\
B'(S) &= B'(S) \\
D'(S) &= D'(S)
\end{aligned}$$

- With solution $d'(S,s)$ and $b'(S,s)$, as well as $v(S,s)$

Firms' Problem

- The value of firms with loans Π^ℓ and of firms without loans Π^e is

$$\begin{aligned}
\Pi^\ell(S,k) &= zAk^\alpha - w(S,k) - kr^\ell(S) + \\
&\qquad \mathrm{E}\left\{(1-\delta')\frac{(1-\lambda)\Pi^\ell(S',k) + \lambda\left[\Pi^e(S',k) - k\right]}{1+r^b(S')} \;\middle|\; S\right\} \\
\Pi^e(S,k) &= zAk^\alpha - w(S,k) + \mathrm{E}\left\{(1-\delta')\frac{\Pi^e(S',k)}{1+r^b(S')} \;\middle|\; S\right\}
\end{aligned}$$

- The cost of a loan of size k is $\mathrm{E}\left\{\frac{kr^\ell(S') + \frac{\Phi(S'',k)}{\mathbf{1}+r^b(S'')}}{1+r^b(S')} \;\middle|\; S\right\}$

$$\Phi(S,k) = k[(1-\lambda)r^\ell + \lambda] + (1-\lambda)\,\mathrm{E}\left\{\frac{(1-\delta'^\ell)\Phi(S',k)}{1+r^b(S')} \;\middle|\; S\right\}$$

Firms' Problem II

- So the optimal size satisfies

$$\max_{k} \quad \mathrm{E}\left\{ \frac{\Pi^{\ell}(S', k)}{1 + r^{b}(S')} - \frac{k r^{\ell}(S') + \frac{\Phi(S'', k)}{1 + r^{b}(S'')}}{1 + r^{b}(S')} \,\middle|\, S \right\}$$

- V: COMPUTE THE FOC

- Firms enter until there are zero profits from doing so

$$\mathrm{E}\left\{ \frac{\Pi^{\ell}(S', k)}{1 + r^{b}(S')} \,\middle|\, S \right\} = \mathrm{E}\left\{ \frac{k r^{\ell}(S') + \frac{\Phi(S'', k)}{1 + r^{b}(S'')}}{1 + r^{b}(S')} \,\middle|\, S \right\}$$

Recursive Competitive Equilibrium

- Laws of motion $N'(S), Y'(S), L'(S), B'(S), D'(S)$,
- Decision rules and value functions for households $d'(S,s)$, $b'(S,s)$, and $v(S,s)$, and firms $k(S), N^n(S), \Pi^\ell(S), \Pi^e(S)$.
- Prices $r^b(S), r^\ell(S), r^d(S), w(S,k), W(S)$, and Profits $\pi^f(S)$, $\pi^B(S)$

1. Households and Firms solve their problems
   1.1 Euler equation of Households $u_c(S) = \mathrm{E}\{\beta(1+r^b(S'))u_c(S') \mid S\}$.
   1.2 Marginal utility of deposits equals $\mathrm{E}\left\{\frac{r^b(S')-r^d(S')}{1+r^b(S')} \,\middle|\, S\right\}$
   1.3 Optimal choice of k
2. Rep Agent: $B'(S) = b'(S, s(S))$, $D'(S) = D'(S, s(S))$, $n'(S, s(S)) = N'(S)$.
3. Interest rates yield zero expected profits to banks
4. Realized profits are

$$\begin{aligned}\pi^f(S) &= zY - N\,W - L[(1-\lambda)r^b + \lambda L] \\ \pi^B(S) &= A - (1-\lambda)(1-\delta)L\end{aligned}$$

5. Wages are set by Nash bargaining.

2 Non-linear Costs for Banks

Banking Industry I

- Banks use output to produce deposits and to make loans, $d' = \kappa_d y^d$ and $\ell^n = \kappa_\ell y^\ell$.
- Loans are long term and decay at rate λ. Deposits are short term.
- It borrows and lends short term bonds B' at interest rate r^b.
- A random fraction δ^ℓ of the loans are destroyed. There are increasing costs with that destruction: $\ell' = (1 - \delta'^\ell)(1 - \lambda)\ell + \ell^n$
- Banks cash position

$$a' = (\lambda + r^\ell(1-\lambda))(1-\delta^\ell)\ell + r^\ell \ell^n - d'(1+r^d) - b'(1+r^b) - \xi(\delta^\ell)\ell$$

- There is a capital requirement

$$\frac{\ell + \ell^n - d' - b'}{\ell + \ell^n} \geq \theta$$

- There is curvature in the bank's dividends $\Phi(m)$

Banking Industry: Banks Problem

$$\Omega(S,a,\ell) = \max_{d',b',\ell^n} \Phi\left[a - \ell^n\left(1+\frac{1}{\chi_\ell}\right) + d'\left(1-\frac{1}{\chi_\ell}\right) + b'\right] + \\ + \mathrm{E}\left\{\frac{\Omega[S',a'(S'),\ell'(S')]}{1+r^b(S')}\,\middle|\, S\right\} \qquad \text{s.t.}$$

$$\begin{aligned} a'(S') &= (\lambda + r^\ell(S')(1-\lambda))(1-\delta^\ell)\ell + r^\ell(S')\ell^n - \\ &\qquad d'[1+r^d(S')] - b'[1+r^b(S')] - \xi(\delta^\ell)\ell \\ \ell'(S') &= \left(1-\delta'^\ell\right)(1-\lambda)\ell + \ell^n \\ \theta &\leq \frac{\ell+\ell^n-d'-b'}{\ell+\ell^n} \end{aligned}$$

First order conditions

- Dividends and bonds interest rates are linked mechanically as they are perfect substitutes for banks. Wrt new loans ℓ^n we have

$$-\Phi_m\left(1+\frac{1}{\chi_\ell}\right)+\mathrm{E}\left\{\frac{r^\ell\Omega_2'+\Omega_3'}{1+r^b(S')}\right\}+\mu(KREQ)=0$$

- WRT bonds we have

$$\Phi_m-\mathrm{E}\{\Omega_2'\}-\mu(KREQ)=0$$

- The envelope conditions tell us that

$$\begin{aligned}\Omega_2 &= \phi_m+\frac{\partial\ell^n}{\partial a}\left[\phi_m\left(1+\frac{1}{\chi_\ell}\right)+\mathrm{E}\left\{\frac{r^\ell\Omega_2'+\Omega_3'}{1+r^b(S')}\right\}+\mu(KREQ)\right]\\ \Omega_3 &= \mathrm{E}\{(\lambda+r^\ell(S')(1-\lambda))(1-\delta^\ell)-\xi(\delta^\ell)\}+\mathrm{E}\{(1-\delta'^\ell)(1-\lambda)\Omega_3'\}\end{aligned}$$

Banking Industry III

- Let $r^\ell(r^b)$ and $r^b(r^b)$ be the interest rates of bonds and deposits when the Capital Requirement constraint is not binding.

$$
\begin{aligned}
1 + \frac{1}{\kappa_\ell} &= \sum_{t=1}^{\infty} \left[\frac{(1-\lambda)(1-\delta)}{1+r^b} \right]^{t-1} \left[(1-\lambda) r^\ell + \lambda \right] \\
r^d &= r^b - \kappa_d \\
r^\ell &= \left[\left(1 + \frac{1}{\kappa_\ell} \right) \frac{r^b + \lambda + \delta - \lambda\delta}{(1+r^b)} - \lambda \right] \frac{1}{1-\lambda}
\end{aligned}
$$

Model: An Extension Shadow Banking

- Brought to center stage by the troubles of Home Capital in Canada
- No deposits ($\xi_d = 0$), just bonds, but particularly good at issuing high risk loans.
- The only thing to add is a distinction between low and high risk loans.
  - Because financial institutions specialize, this does not add state variables.
  - Still need a theory of why are they trouble.

Return

$$\omega = 25\eta$$