Equilibrium Search and Tax Credit Reform

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Abstract

An empirical equilibrium job search model with wage posting is developed to analyse the labour market impact of UK tax reforms. The model allows for a rich characterization of the labour market, with hours responses, accurate representations of the tax and transfer system, and both worker and firm heterogeneity. The model is estimated with pre-reform longitudinal survey data using a semi-nonparametric estimation technique, and the impact of actual tax reform policies is simulated. The model predicts that the British Working Families’ Tax Credit and contemporaneous reforms increased employment, with equilibrium effects found to play a relatively minor role.

Keywords: Labour market equilibrium, empirical job search, wage dispersion, monopsony, incidence, tax credits

1 Introduction

Over the past two decades earned income tax credit programs have grown substantially in the UK, US and many other countries.1 These programs are typically motivated

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by a desire from policy makers to increase labour market participation among target
groups, and to alleviate in-work poverty. While the effect of these policies on labour
supply has been studied extensively, much less is known regarding the incidence of
these tax credit programs and their broader equilibrium impact. The objective of this
paper is to develop an empirical equilibrium job search model that provides us with an
appropriate framework to address these issues, and to apply it in our analysis of a series
of UK tax reforms that included the Working Families’ Tax Credit (WFTC) reform, which
considerably increased the generosity of in-work support for families with children (see
Brewer, 2001).

This paper contributes to the literature on the impact of tax credit policies, but starts
from the premise that labour markets may be characterized by considerable search fric-
tions (see for example, van den Berg and Ridder, 2003). Even absent equilibrium con-
siderations, very little is known about the quantitative impact of these policies in a
search environment. Furthermore, the presence of frictions may have important equilib-
rium implications for our understanding of programs like WFTC. In particular, if firms
set wages then these frictions bestow them with some degree of monopsony power. If
labour supply were to increase following such reforms, firms may respond by lowering
wage offers, in which case the effective transfer to eligible families is reduced, whilst
non-eligible families may become worse off if they are competing within the same labour
market. In terms of both evaluation, and program design, an understanding of the quan-
titative importance of these equilibrium effects is essential.

The equilibrium job search literature allows us to capture these and other effects in a
dynamic and imperfectly competitive economy that is characterized by search frictions.
Competition between firms is the fundamental determinant of wages, with the extent
of this competition limited by the presence of search frictions. In the spirit of Burdett
and Mortensen (1998), we consider a model with ex-ante wage posting: firms set wages
before meeting potential workers, which workers then either accept or reject. We ad-

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2Exceptions include the recent studies by Azmat (2006), Leigh (2010), and Rothstein (2008, 2009).
3Blundell, Francesconi and van der Klaauw (2011) study the same tax reforms that we focus upon in
this paper. They present evidence of important announcement effects on the employment decisions of sin-
gle parents (employment increasing between the announcement and implementation of the programme),
which is consistent with the presence of significant labour market frictions existing for this group. We
describe other evaluations of WFTC in Section 5.3.
4While a competitive model of the labour market produces some of the same qualitative theoretical
predictions as the model we develop here, the analysis here is primarily quantitative, with the potential
impact of the reforms studied and size of equilibrium effects depending crucially upon the extent of search
frictions that we uncover here.
5Manning (2003) argues that while wage posting is not always appropriate, it provides a good charac-
vance this literature in several dimensions, with the model developed here designed to reflect some key features of the UK labour market and to allow for the possibility of rich equilibrium effects following reforms such as WFTC.

At a methodological level, this paper contributes to the empirical equilibrium job search literature by developing a wage-posting model with wage-hours packages, accurate representations of the tax and transfer system, and both observed and unobserved worker and firm heterogeneity. The paper most closely related is the on-the-job search model presented in Bontemps, Robin and van den Berg (1999), which this paper builds upon. As in their model, we allow for continuous distributions of firm productivity and worker leisure flows, but do not impose the over-identifying restriction that the arrival rate of job offers is independent of employment status.6 Furthermore, we endogenize these job offer arrival rates at the macroeconomic level by complementing the model with aggregate matching functions. As will become clear, allowing for heterogeneity in worker leisure flows is important as it provides the main mechanism through which tax reforms induce non-degenerate labour supply responses.

As we describe in the following section, both WFTC and its predecessor were only available to families with children. To investigate possible differential impacts to the tax reforms, and to also explain differences in labour market outcomes, we make a further methodological contribution by incorporating further dimensions of worker heterogeneity. Both the tax and transfer system, and the key worker parameters, may all potentially vary with observable demographic characteristics. In contrast to the segmented markets approach adopted by van den Berg and Ridder (1998), we will allow workers of all types to operate within the same labour market. It is this feature that allows workers who are not eligible for tax credits to be indirectly affected by them through changes in the optimal strategies of firms.

6As we shall see in Section 3, the over-identifying restriction in Bontemps, Robin and van den Berg (1999) simplifies the analysis as it implies that the optimal strategy of unemployed workers is independent of the equilibrium wage offer distribution. This restriction led to a poor fit of the duration data in their application, as empirically job offer arrival rates for unemployed workers are often estimated to exceed that of the employed.
The UK labour market has a high prevalence of part-time work, particularly among women with children. As noted above, the presence of children is a central eligibility requirement for receipt of tax credits. These features motivate us to incorporate hours of work into the model. While the use of the canonical labour supply model may be pervasive, there is a body of empirical work that challenges the view that individuals are able to freely choose their hours of work at a fixed hourly wage.\(^7\) We allow for both part-time and full-time jobs, and throughout this paper maintain the assumption that jobs sequentially arrive as wage-hours packages. We do not attempt to provide micro-foundations for this, but rather assume it is a purely technological characteristic of firms.

The level of generality here means that the model is analytically intractable. Nonetheless, we show that the model remains empirically tractable by developing a three step semi-nonparametric estimation technique similar to that proposed by Bontemps, Robin and van den Berg (1999, 2000), and then estimating the model using UK Labour Force Survey data shortly before WFTC was introduced. We show that the estimated model is successful in explaining pre-reform differences in employment states, distributions of wages, as well as labour market transitions. Using the estimated parameters we then simulate the impact of actual tax reforms and later compare our results to post-reform data. We find that the introduction of WFTC, together with the contemporaneous changes to the tax and transfer system, increased employment for most groups, with single parents experiencing the largest employment increase. Our main simulations suggest that while equilibrium considerations do play a role in these reforms, the changes in labour market outcomes are dominated by the direct effect of changing job acceptance behaviour. We also show that the same reforms may have larger within market equilibrium effects if we take a more segmented view of the labour market, and how these effects depend upon the size of labour market frictions.

The remainder of the paper proceeds as follows. In the next section we describe the WFTC reform, as well as the contemporaneous changes to the UK tax and transfer system. In Section 3 we present the theoretical model that we use to study tax reforms and describe the optimal strategies of firms and workers. Section 4 discusses identification, describes the estimation procedure, and presents the main estimation results. In Section 5 we then use the estimated model to simulate the impact of actual tax reforms, and

\(^7\)See, for example, Altonji and Paxson (1988) and Dickens and Lundberg (1993). Blundell, Brewer and Francesconi (2008) studied the impact of a series of in-work benefit reforms in the UK during the 1990s, and found that the positive effect on hours worked was largely driven by women who changed their job.
compare our results to observed post-reform changes. Finally, Section 6 concludes.

2 UK Tax Credit Reform

As in the US, the UK has a long history of in-work benefits, starting with the introduction of Family Income Supplement in 1971. Over the years, these programs became more generous, and in October 1999, Working Families’ Tax Credit was introduced, replacing a similar but less generous tax credit program called Family Credit (FC). Both WFTC and FC were only available to families with children and shared a similar eligibility structure, requiring recipients work for at least 16 hours per week, and with the credit tapered away with household earnings above a threshold. Both also offered a further credit when recipients worked at least 30 hours a week. WFTC increased the level of in-work support by offering higher credits, increasing the threshold so that families can earn more before it was phased out, and by reducing the withdrawal rate.\textsuperscript{8} In Figure 1 we illustrate how this reform changed tax credit awards for a single parent family.

When analysing low income support it is important to take an integrated view of the tax system. This is because tax credit awards in the UK are counted as income when calculating entitlements to other benefits. Families in receipt of these benefits would gain

\textsuperscript{8}WFTC also provided more support for formal childcare costs and allowed all child maintenance payments to be disregarded from income when calculating tax credit entitlement.
less from WFTC than otherwise equivalent families not receiving such benefits. There were also other important changes to the tax system affecting families with children that coincided with the expansion of in-work tax credits, and which make the potential labour market impact considerably more complex. In particular, there were increases in the generosity of Child Benefit (a cash benefit available to all families with children regardless of income), as well as notable increases in the child additions in Income Support (a welfare benefit for low income families working less than 16 hours a week). There were also other changes to the tax and transfer system that affected families both with and without dependent children: a new 10% starting rate of income tax was introduced, the basic rate of income tax was reduced from 23% to 22%, and there was a real rise in the point at which National Insurance (payroll tax) becomes payable.9

3 The Model

In the model that we develop to study tax reforms, workers receive job offers when both employed and unemployed. These job offers are characterised by both an hours of work requirement and an hourly wage, and workers may choose to either accept or reject any given job offer that they encounter. Workers differ in both observed and unobserved dimensions, and this generates differences in job acceptance behaviour across workers. Reforms to the tax system, as we investigate here, will change the value to workers from holding different jobs and therefore induce behavioural responses in the form of changes in job acceptance behaviour. For example, a wage subsidy may enlarge the set of wages that workers are now willing to accept. Firms are not passive in this model and may respond should workers change their behaviour in this way. More specifically, following any tax-induced change in worker behaviour, firms (which differ in their productivity levels) may respond by adjusting the wage that they offer (which changes the profit per-worker and workforce size), as well as their recruiting intensity decision (which changes the rate at which they meet with prospective employees). These changes will determine the aggregate rate of job offer arrivals, and feedback into the behaviour of workers until a new equilibrium emerges.

9Our analysis does not consider the non-tax related changes that occurred during this period. Various “New Deal” active labour market policies were introduced which aimed to improve both the incentives and the ability of the long-term unemployed to obtain employment (see Blundell et al., 2004). Furthermore, a national minimum wage was introduced (see Metcalf, 2008).
3.1 Environment

The economy consists of a continuum of infinitely lived individuals with a population size normalized to unity. Individuals firstly differ by their observable demographic characteristics that are finite in number and indexed by \( i \leq I \). The fraction of such workers is denoted \( n_i \) with \( \sum_i n_i = 1 \). Individuals also differ in their unobserved leisure flow \( b \) (as in Albrecht and Axell, 1984), which includes any search costs but not unemployment benefit and other transfers. As will become clear, heterogeneity in leisure flows translates into heterogeneity in job acceptance behaviour, so that tax reforms may introduce non-trivial labour supply effects on this margin. The cumulative distribution function of leisure flows in the population of type \( i \) workers is denoted \( H_i \), which is assumed continuous on its support \([b_i, \overline{b}_i]\). To simplify some of the subsequent exposition we assume that \( b_i \) is sufficiently low such that in equilibrium all firms are active in the labour market. Time is continuous and all individuals have the constant discount rate \( \rho > 0 \). There is no saving or borrowing technology.

Jobs are characterized by a wage rate \( w \) and required hours of work \( h \). We allow for part-time jobs (hours \( h_0 \)) and full-time jobs (hours \( h_1 > h_0 \)), with workers subject to a monetary hours disutility \( C_i^h \). Mirroring the actual conditioning performed by the UK tax authorities, the tax schedule is a function of demographics, hours, and earnings, with \( T_i^h(wh) \) denoting the potentially negative net taxes paid (and transfers received) by an employed worker. We assume that this tax schedule is continuously differentiable in labour earnings. The net transfer paid to an unemployed worker is given by \( -T_i^u \). Utility flows are assumed linear in income, so that in the presence of a tax and transfer system and hours responses these are given by:

\[
wh - T_i^h(wh) - C_i^h \quad \text{if employed}
\]
\[
b - T_i^u \quad \text{if unemployed.}
\]

From the outset we impose the location normalization \( C_i^0 = 0 \) for all \( i \leq I \).

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10 The inclusion of work hours is rare in the empirical search literature (for an exception see, for example, Blau, 1991). The analytical framework we develop generalizes to more than two hours choices, and can also be applied in the context of other non-wage amenities. See also Hwang, Mortensen and Reed (1998) for an analysis of non-wage amenities in an equilibrium search framework.

3.2 Worker Search

Individuals (or workers) are either employed or unemployed. Both search for jobs. Job offer arrivals are exogenous to the worker: a type $i$ worker accrues hours $h$ at the constant rate $\lambda_{ji}^h$ with $j \in \{u,e\}$ indexing the current worker state of unemployment or employment. To maintain focus on the decision of workers, we postpone any discussion concerning how these arrival rates may depend upon the overall state (or tightness) of the labour market until Section 3.6. Employment spells end at rate $\delta_i$ regardless of whether individuals are employed in part-time or full-time jobs, and we define $\kappa_{ji}^h = \lambda_{ji}^h / \delta_i$ as the ratio of the arrival rates to this destruction rate. We place no restrictions on the relative magnitude of these parameters.

Regardless of their observed or unobserved type, workers sample sector $h$ wages from the wage offer distribution $F_h$ which has support $[w_h, \bar{w}_h]$. Wages are assumed constant throughout an individual’s employment spell within a given firm and we additionally define the survival functions $\overline{F}_h = 1 - F_h$. For now, we treat these distributions as given and will later describe how they emerge as an equilibrium outcome from the wage posting game. The assumption that all workers sample offers from common distributions implies that, while the government may be able to condition taxes and transfers on demographic characteristics, firms are unable to do so. This assumption can be justified by the presence of anti-discrimination laws, such as the Equal Pay Act 1970, Sex Discrimination Act 1975, and various Employment Equality Regulations, which outlaw such practices. It implies that workers who are not targeted by a tax reform may be indirectly affected through changes in the optimal strategy of firms. We explore alternative assumptions regarding market segmentation in Section 5.6.

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12 We make no distinction between unemployment and non-participation. Individuals with high values of the leisure flow $b$ are effectively non-participants in the labour market.

13 We do not allow the search effort of workers to vary with their current wage or to respond to any changes in the tax system. A richer specification would endogenize the job offer arrival rates at the micro-level by relating them directly to an endogenously determined worker search effort, as in Christensen et al. (2005). Incorporating such responses is non-trivial and is left as an extension for future research. See Gentry and Hubbard (2004) for US evidence on the impact of tax rates on job mobility.

14 As emphasized by van den Berg and Ridder (2003), the parameters $\kappa_{ei}^h$ and $\kappa_{ui}^h$ can be thought of as labour market friction parameters. In particular, $\kappa_{ei}^h$ is the number of sector $h$ job offers a type $i$ individual can expect to receive when employed, before exiting to unemployment. The assumption here that both the job destruction rate and job offer arrival rates when employed are independent of whether individuals are currently engaged in part-time or full-time work simplifies the subsequent analysis.
3.3 Worker Strategies

We now describe the optimal strategies of unemployed and employed workers. To proceed we define \( q_i(w) \) such that the value to a type \( i \) individual holding a full-time job paying wage \( w \) is the same as the value of a part-time job paying wage \( q_i(w) \). We refer to this function as the *indifference condition*.

**Proposition 1** The indifference condition \( q_i(w) \) solves:

\[
wh_1 - T_i^1(wh_1) - C_i^1 = q_i(w)h_0 - T_i^0(q_i(w)h_0).
\]

This function describes the wedge between the wages that an individual will accept across hours sectors. For example, a full-time wage \( w \) worker will accept any full-time offer that is (by convention, strictly) greater than \( w \); they will also accept any part-time offer (strictly) greater than \( q_i(w) \). The proof of the proposition follows immediately from the assumption that the job destruction rate and job offer arrival rates are independent of current hours; employed workers can do no better than maximize their instantaneous utility flow.\(^{15}\) Note that \( q_i(w) \) has a unique solution provided that marginal tax rates are strictly less than one for all \( w \), conditional on hours of work. We maintain this assumption throughout.

Unemployed workers follow a reservation wage strategy. Let \( \phi_i(b) \) denote the lowest acceptable wage for full-time work conditional on observed type \( i \) and unobserved leisure flow \( b \). This takes a similar form to the standard reservation wage equation with on-the-job search (see Mortensen and Neumann, 1988), but is here modified both by the presence of taxes (which discount future earnings by the net-of-tax rate) and because workers are sampling job offers from two distributions.

**Proposition 2** The full-time reservation wage for unemployed workers \( \phi_i(b) \) is the solution to the following equation:

\[
\phi_i(b)h_1 - T_i^1(\phi_i(b)h_1) - C_i^1 = b - T_i^u + h_1 \int_{\phi_i(b)}^\infty B_i(w) \, dw
\]

where:

\[
B_i(w) \equiv \frac{(1 - T_i^1(wh_1)) \left[ (\kappa_{ui}^0 - \kappa_{ei}^0)F_0(q_i(w)) + (\kappa_{ui}^1 - \kappa_{ei}^1)F_1(w) \right]}{1 + \rho / \delta + \kappa_{ui}^0F_0(q_i(w)) + \kappa_{ei}^1F_1(w)}.
\]

\(^{15}\)In the more general case where arrival rates and/or job destruction rates vary with current work hours, the indifference condition will be a function of the distribution of wage offers in both sectors.
The proof of this proposition is provided in the Supplementary Material. We immediately establish the following corollary.

**Corollary 1** The part-time reservation wage for unemployed workers is given by $q_i(\phi_i(b))$.

Henceforth, we will refer to the full-time reservation wage for unemployed workers simply as the reservation wage. Before proceeding we note that when job offer arrival rates are independent of employment status, that is $\kappa_{ui} = \kappa_{ei}$, we have $B_i(w) = 0$ for all $w$ so that the optimal strategy of workers is independent of the equilibrium wage offer distributions. This is the case analysed in Bontemps, Robin and van den Berg (1999).

### 3.4 Steady-state Worker Flows

Having described the strategy of workers, we can derive a number of steady state conditions that characterize the labour market. For now, we continue to treat the distributions of wage offers and their arrival rates as being given.

#### 3.4.1 Distribution of Reservation Wages

Reservation wages summarize the optimal strategy of individuals. The cumulative distribution function of reservation wages amongst all type $i$ workers (both employed and unemployed) is denoted $A_i$ and is related to the distribution of unobserved leisure flows according to $A_i(w) = H_i(\phi_i^{-1}(w))$. The respective distributions amongst the stock of unemployed and employed workers are denoted $A_{ui}$ and $A_{ei}$. These are related to $A_i$ according to:

$$A_i(w) = u_i A_{ui}(w) + (1 - u_i) A_{ei}(w). \quad (3)$$

The distribution of reservation wages amongst the unemployed $A_{ui}$ allows us to describe the flows from the unemployment pool into employment at a given wage. As we demonstrate shortly, it also allows us to determine the steady state unemployment rate. In steady state the flow of individuals with a reservation wage no greater than $\phi$ who exit the employment pool following a job destruction shock must exactly equal the flow of such workers who enter employment. Hence,

$$\delta_i (1 - u_i) A_{ei}(\phi) = \lambda_{ui}^0 u_i \int_{-\infty}^{\phi} F_0(q_i(w)) \, dA_{ui}(w) + \lambda_{ui}^1 u_i \int_{-\infty}^{\phi} F_1(w) \, dA_{ui}(w). \quad (4)$$
By differentiating equation 4 using Leibniz’s rule we obtain a relationship between the densities of employed and unemployed worker reservation wages, which when combined with equation 3 allows us to establish the following proposition:

**Proposition 3** The unemployment weighted distribution of reservation wages amongst type i unemployed workers is given by:

\[
u_i A_{ui}(\phi) = \int_{-\infty}^{\phi} \frac{dA_i(w)}{1 + D_{ui}(w)/\delta_i(w)}
\]

(5)

where \(D_{ui}(\phi) \equiv \lambda_{ui}^0 F_0(q_i(\phi)) + \lambda_{ui}^1 F_1(\phi)\) is the rate at which a type i worker with reservation wage \(\phi\) will exit the unemployment pool into employment.

### 3.4.2 Between Jobs and the Distribution of Earnings

While individuals may sample wage offers from common distributions, the cross sectional distribution of wages among the employed (earnings) will differ. For example, some worker types may be more or less selective in the wages they will accept, or may gravitate to higher paying jobs at different rates. In what follows we denote the cumulative distribution function of sector \(h\) earnings for type \(i\) individuals as \(G_{hi}\) with the corresponding density functions \(g_{hi} \equiv G_{hi}'\). The fraction of such workers currently employed in an hours \(h\) job is denoted \(m_{hi}\) and by construction we have \(m_{0i} + m_{1i} = 1 - u_i\).

Rather than presenting flow equations for each sector separately, it is convenient to define \(q_{hi}(w)\) such that \(q_{0i}(w) = q_i(w)\) and \(q_{1i}(w) = w\). In steady-state, the number of type \(i\) individuals who leave a sector \(h\) job paying wage \(q_{hi}(w)\) (either by their job being destroyed at rate \(\delta_i\) or by gravitating to a higher value job) must exactly equal the number of individuals who accept such a job (either from the unemployment pool or from a lower value job). Hence,

\[m_{hi} g_{hi}(q_{hi}(w)) D_{ei}(w) = f_h(q_{hi}(w)) \left[ \lambda^h_{ui} u_i A_{ui}(w) + \lambda^h_{ei} m_{0i} G_{0i}(q_i(w)) + \lambda^h_{ei} m_{1i} G_{1i}(w) \right]
\]

(6)

where \(D_{ei}(w) \equiv [\delta_i + \lambda^0_{ei} F_0(q_i(w)) + \lambda^1_{ei} F_1(\phi)]\) is the rate at which such a worker will exit their current job. Equation 6 feature a weighted distribution of full-time and part-time earnings amongst the employed, \(m_{0i} G_{0i}(q_i(w)) + m_{1i} G_{1i}(w)\), with the hours specific distribution functions in this expression evaluated at wage rates that yield equal value to the worker. And while expressions for \(G_{0i}\) and \(G_{1i}\) are both individually complicated, this weighted distribution admits a considerably simpler form.
Proposition 4  The weighted distribution of earnings $m_0 G_0(q_i(w)) + m_1 G_1(w)$ may be written as:

$$m_0 G_0(q_i(w)) + m_1 G_1(w) = A_i(w) - u_i A_{ui}(w) [1 + D_{ui}(w)/\delta_i].$$  \hfill (7)

A proof is provided in the Supplementary Material. Thus, we are able to use this proposition to obtain expressions for the earnings densities (equation 6) in terms of the transitional parameters, wage offer distributions, and distribution of reservation wages. These may then be integrated to obtain the respective individual cumulative distribution functions and employment shares.

3.4.3 Unemployment rate

The steady state unemployment rate balances the flows from the unemployment pool to employment, to the job destruction induced flows from employment to unemployment.

Proposition 5 The steady state unemployment rate of type $i$ workers is given by:

$$u_i = \frac{1}{1 + \kappa_{ui}^0 + \kappa_{ui}^1} A_i(\omega_i) + \int_{\omega_i}^{\overline{\omega}_i} \frac{dA_i(w)}{1 + D_{ui}(w)/\delta_i(w)} + 1 - A_i(\overline{\omega}_i).$$  \hfill (8)

where $\omega_i \equiv \min \{w_1, q_i^{-1}(w_0)\}$ and $\overline{\omega}_i \equiv \max \{\overline{w}_1, q_i^{-1}(\overline{w}_0)\}$.

This proposition follows immediately by letting $\phi \to \infty$ in equation 5. It decomposes the unemployment rate into the contribution by three (endogenously determined) groups of workers: those who accept all offers; those who accept some and reject others; and those who reject all. Note that $u_i$ is bounded below by $(1 + \kappa_{ui}^0 + \kappa_{ui}^1)^{-1}$, which is the rate that would prevail in the absence of any reservation wage heterogeneity. In contrast to the homogeneous worker model $\kappa_{ei}^h$ affects $u_i$ through two channels: the direct effect through changes in worker reservation wages and the indirect effect through its potential impact on the equilibrium wage offer distributions (described below).

3.5 Firms

In order to make this an equilibrium model we specify the behaviour of firms. It is the profit maximizing behaviour of firms, taking as given the optimal strategies of workers and other firms, that determines the equilibrium distributions of wage offers and job offer arrival rates. The type of job offer made by firms (full-time or part-time) is an
exogenous technological characteristic of the firm and we therefore refer to firms as belonging to a particular hours sector. As in Burdett and Mortensen (1998) we assume that there is wage posting: within each hours sector employers post a single wage \( w \) prior to forming matches with potential employees, who can then either accept or reject the offer. Firms also choose a level of recruiting intensity \( v \) which raises their visibility in the labour market; the probability that workers draw job offers from a particular firm is proportional to this recruiting intensity. This feature allows firms to change the rate at which they meet potential employees independent of the wage, and will also allow us to endogenize the set of job offer arrival rates at the aggregate level.

Within each sector, firms differ in their exogenously determined productivity. The cumulative distribution of firm productivity in sector \( h \) is denoted \( \Gamma_h \) which is continuous on its support \( [p_{h,1}, p_{h,2}] \). This productivity corresponds to the hourly flow marginal product of workers, and is independent of both the number and identity of workers. That is, all workers are assumed equally productive at a given firm regardless of their observed or unobserved type. The flow cost of recruiting effort also potentially differs across firms. We denote this as \( c_h(v; p) \) with this function strictly convex in \( v \) and with \( c_h(0; p) = 0 \) for all \( p \).

### 3.5.1 Firm Size

The number of workers of a given observable type \( i \) that a sector \( h \) firm employs at wage \( w \) and recruiting intensity \( v \) is denoted \( l_{hi}(w, v) \). Since firms may potentially contact workers of all types, total employment at such a firm is given by \( L_h(w, v) = \sum_i n_i l_{hi}(w, v) \). In steady state, sector \( h \) employment \( l_{hi}(w, v) \) solves the flow equation:

\[
l_{hi}(q_{hi}(w), v)D_{ei}(w) = v \frac{\Gamma_h}{V_h} \left[ \lambda_{ui}u_iA_{ui}(w) + \lambda_{ei}m_0G_{0i}(q_i(w)) + \lambda_{ei}^h m_{1i}G_{1i}(w) \right],
\]

which balance the number of workers who enter and exit employment at a given firm. Note that \( l_{hi}(w, v) \) is non-decreasing in \( w \). This is because firms which pay higher wages attract more workers from both the unemployment pool (the mechanism in Albrecht and Axell, 1984) and lower value firms (the mechanism in Burdett and Mortensen, 1998). In

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16We do however, allow the contact rate of firms to change differentially following tax reforms by inclusion of a recruiting intensity decision. Alternative approaches such as allowing firms to substitute between part-time and full-time workers (Roger and Roux, 2009), or choosing which sector to operate in (Meghir, Narita and Robin, 2012) would complicate the analysis considerably.
these flow equations $V_h$ denotes the aggregate recruiting intensity in hours sector $h$:

$$V_h = \int_p^p v_h(p) \, d\Gamma_h(p)$$

(10)

and with $v_h(p)$ denoting the recruiting policy of a sector $h$ productivity $p$ firm. Given that the recruiting intensity $v$ enters the RHS of equation 9 multiplicatively, it is convenient in what follows to write $l_{hi}(w, v) = \bar{l}_{hi}(w) v / V_h$, and similarly define $\bar{T}_h(w) = \sum_i n_i \bar{l}_{hi}(w)$. Substituting equation 7 into equation 9 we may eliminate the weighted cross-sectional earnings distributions and establish the following result.

**Proposition 6** Steady state employment levels in sector $h$ for given wage $w$ and recruiting intensity $v$ are given by $l_{hi}(w, v) = \bar{l}_{hi}(w) v / V_h$ with:

$$\bar{l}_{hi}(q_{hi}(w)) = \frac{\kappa_{ei}^h A_i(w) + [\kappa_{ui}^h D_{ei}(w)/\delta_i - \kappa_{ei}^h (1 + D_{ui}(w)/\delta_i)] u_i A_{ui}(w)}{\left(D_{ei}(w)/\delta_i\right)^2}.$$  

(11)

In contrast to models without reservation wage heterogeneity, the absence of on-the-job search ($\kappa_{ei}^h = 0$) does not imply that employment is uniformly distributed across firms when matching is random. This is intuitive because low wage firms are only able to attract low reservation wage workers (it is straightforward to show that $\bar{l}_{hi}(w)$ is proportional to $A_{ui}(w)$ in this case). More generally, firm size depends upon a weighted distribution of reservation wages amongst employed and unemployed workers, with the weights a function of job offer arrival rates and the distributions of wage offers.

### 3.5.2 Firm Profits

Each firm chooses its wage policy $w_h(p)$ and recruiting policy $v_h(p)$ to maximize its steady state profit flow, taking the arrival rate of job offers, together with the behaviour of other firms (both part-time and full-time) and workers as given.\(^{17}\) Hence:

$$\{w_h(p), v_h(p)\} = \arg\max_{(w,v)} \pi_h(w; p) \frac{v}{V_h} - c_h(v; p)$$

(12)

where $\pi_h(w; p) = (p - w)h \cdot \bar{T}_h(w)$ is the expected profit flow per unit intensity. The optimal recruiting policy $v_h(p)$ equates the marginal cost of increasing recruiting effort

\(^{17}\)The assumption of steady state profit maximisation is standard in the wage-posting literature. See Mortensen (2000) for a discussion.
to the marginal expected profit flow. That is:

$$\left. \frac{\partial c_h(v; p)}{\partial v} \right|_{v=v_h(p)} = \frac{\pi_h(w_h(p); p)}{V_h}.$$  \hspace{1cm} (13)

Maximized expected profit flow per unit intensity is given by

$$\pi_h(p) = \pi_h(w_h(p); p) = (p - w_h(p))h \cdot \mathcal{L}_h(w_h(p))$$

so that $\pi_h'(p) = \mathcal{L}_h(w_h(p))$ by the Envelope Theorem. Since $\mathcal{L}_h(w)$ is increasing in $w$, and $w_h(p)$ is increasing in $p$, it follows that the maximized expected profit flow per unit intensity is a convex function of $p$. Rather than working directly with the first order conditions for the optimal wage policy of firms, we write firms’ expected profit flow per unit intensity as:

$$\pi_h(p) = \pi^*_h(p_h) + h \int_{p_h}^{p} \mathcal{L}_h(w_h(y)) \, dy$$ \hspace{1cm} (14)

where $\pi^*_h(p_h) = \max_w (p_h - w) \mathcal{L}_h(w)$ are the maximized profits for the least productive sector $h$ firm. Setting equation 14 equal to $(p - w_h(p))h \mathcal{L}_h(p)$ we obtain the following expression for the wage policy function:

**Proposition 7** The optimal wage policy of firms $w_h(p)$ satisfies the following equation:

$$w_h(p) = p - \left[ \pi^*_h(p_h) + h \int_{p_h}^{p} \mathcal{L}_h(w_h(y)) \, dy \right] \times \frac{1}{h \mathcal{L}_h(w_h(p))}$$ \hspace{1cm} (15)

which is a form that we exploit when we numerically solve for the equilibrium of our model (see the Supplementary Material for details).

### 3.6 Matching Technology and Equilibrium

In order to close the model we endogenize the arrival rate of job offers by complementing it with aggregate matching functions (see Mortensen, 2000). The total flow of matches in each hours sector $h$ is denoted $M_h(V_h, S_h)$; it depends on the total recruiting intensity $V_h$ and the total intensity adjusted search effort of workers $S_h$:

$$S_h = \sum_i n_i \left[ s^h_{ui} u_i + s^h_{ei} (1 - u_i) \right], \hspace{1cm} (16)$$
where \( s^h_{ji} \) denotes the exogenous search effort of type \( i \) workers that is directed to sector \( h \) when in state \( j \in \{u,e\} \). By assumption, the matching function \( M_h \) is increasing in both its arguments, concave, and linearly homogeneous. The job offer arrival rates for each worker type are then related to the flows of matches according to:

\[
\lambda^h_{ji} = s^h_{ji} \cdot M_h(V_h, S_h) / S_h. \tag{17}
\]

The market equilibrium of the economy is now defined in Definition 1.

**Definition 1**  
A market equilibrium is defined by \( \{F_0, F_1, v_0, v_1\} \) such that simultaneously:

1. The arrival rates of job offers \( \{\lambda^0_{ui}, \lambda^1_{ui}, \lambda^0_{ei}, \lambda^1_{ei}\}_{i \leq 1} \) are given by equation 17.

2. The distribution of part-time and full-time wage offers in the economy is:

\[
F_0(w_0(p)) = \int_{p_0}^{p} \frac{v_0(p) \, d\Gamma_0(p)}{V_0} \quad \text{and} \quad F_1(w_1(p)) = \int_{p_1}^{p} \frac{v_1(p) \, d\Gamma_1(p)}{V_1} \tag{18}
\]

with \( V_h \) as defined in equation 10.

3. The strategy of each type \( i \) worker with leisure flow \( b \) is to accept any full-time (part-time) wage greater than \( \phi_i(b) \) (respectively, \( q(\phi_i(b)) \)) when unemployed; when employed in the full-time (part-time) sector at wage \( w \) (respectively, \( q_i(w) \)), the strategy is to accept any full-time wage strictly greater than \( w \) and any part-time wage strictly greater than \( q_i(w) \).

4. The strategy of each sector \( h \) productivity \( p \) firm is to choose a recruiting intensity \( v_h(p) \) and wage \( w_h(p) \) that maximizes profits given the job offer arrival rates, strategies of other firms’ and workers’, as in equation 12.

### 4 Estimation

This section discusses the structural estimation of our model using longitudinal survey data. We first derive the likelihood function, and proceed to discuss identification and the estimation procedure. We then discuss our application of the UK tax and transfer system and the data used in estimation. Results are presented in Section 4.6.
4.1 Likelihood Function

We derive the likelihood contribution for individuals in different initial labour market positions, and with different observed transitions. Note that we do not use any information beyond the first observed transition, and that the steady state distributions of earnings and employment/unemployment shares have been used to determine the initial conditions. We discuss these issues further later in this section. The presentation closely follows that of Bontemps, Robin and van den Berg (1999), and in what follows the durations beyond the selection date (the residual duration) and the duration at the selection date (the elapsed duration) are given by:

\[ t_{ub} = \text{elapsed unemployment duration} \]
\[ t_{uf} = \text{residual unemployment duration} \]
\[ d_{ub} = 1 \text{ if unemployment duration left-censored, otherwise } 0 \]
\[ d_{uf} = 1 \text{ if unemployment duration right-censored, otherwise } 0 \]
\[ t_{eb} = \text{elapsed employment duration} \]
\[ t_{ef} = \text{residual employment duration} \]
\[ d_{eb} = 1 \text{ if employment duration left-censored, otherwise } 0 \]
\[ d_{ef} = 1 \text{ if employment duration right-censored, otherwise } 0, \]

while earned and accepted wages are denoted as follows:

\[ w_u = \text{full-time wage accepted by unemployed individuals} \]
\[ q_i(w_u) = \text{part-time wage accepted by unemployed individuals} \]
\[ d_u = 1 \text{ if wage accepted by unemployed is unobserved, otherwise } 0 \]
\[ w_e = \text{full-time wage of employees at date of first interview} \]
\[ q_i(w_e) = \text{part-time wage of employees at date of first interview} \]
\[ d_e = 1 \text{ if wage of employees is unobserved, otherwise } 0. \]

Current employment is indexed by:

\[ h_e^0 = 1 \text{ if employed work in the part-time sector, otherwise } 0 \]
\[ h_e^1 = 1 \text{ if employed work in the full-time sector, otherwise } 0, \]
and initial transitions are indexed by:

\[
\begin{align*}
    v^0_u &= 1 \text{ if unemployed accept a part-time job, otherwise 0} \\
    v^1_u &= 1 \text{ if unemployed accept a full-time job, otherwise 0} \\
    v^0_e &= 1 \text{ if employed accept a part-time job, otherwise 0} \\
    v^1_e &= 1 \text{ if employed accept a full-time job, otherwise 0}.
\end{align*}
\]

### 4.1.1 Unemployed Workers

The exact form that the likelihood contribution for unemployed workers of type \(i\) will take will depend upon whether unemployment durations are subject to any censoring and the type of wage offer accepted, if observed. If unemployed workers are observed to exit unemployment to a full-time job paying \(w_u\) or a part-time job paying \(q_i(w_u)\), then we must have both \(d_u = 0\) and \(d_{uf} = 0\). The likelihood contribution is given by:

\[
\int_{-\infty}^{w_u} D_{ui}(\phi)^{2-d_{ub}} \exp \left[-D_{ui}(\phi)(t_{ub} + t_{uf})\right] \times \frac{(\lambda^0_{ui}f_0(q_i(w_u)))^{v^0_u} (\lambda^1_{ui}f_1(w_u))^{v^1_u}}{D_{ui}(\phi)} \frac{dA_i(\phi)}{1 + D_{ui}(\phi)/\delta_i},
\]

where we have integrated over the distribution of possible reservations wages given the observed accepted wage rate using equation 5.

If we do not observe a wage accepted by the unemployed \((d_u = 1)\), but we nonetheless have \(d_{ub} + d_{uf} < 2\), then it still must be the case that the full-time reservation wage of such an individual is no greater than \(\bar{w}_i\). The likelihood contribution is therefore:

\[
\int_{-\infty}^{\bar{w}_i} D_{ui}(\phi)^{2-d_{ub}-d_{uf}} \exp \left[-D_{ui}(\phi)(t_{ub} + t_{uf})\right] \times \left[\frac{(\lambda^0_{ui}F_0(q_i(\phi)))^{v^0_u} (\lambda^1_{ui}F_1(w_u))^{v^1_u}}{D_{ui}(\phi)}\right]^{1-d_{uf}} \frac{dA_i(\phi)}{1 + D_{ui}(\phi)/\delta_i}.
\]

Finally, if we have both \(d_u = 1\) and \(d_{ub} + d_{uf} = 2\), then the individual is never observed in the employment state so we must also consider the probability that such an individual
has a reservation wage that is greater than $\bar{w}_i$. The likelihood contribution then becomes:

$$\int_{-\infty}^{\bar{w}_i} \exp \left[ -D_{ui}(\phi)(t_{ub} + t_{uf}) \right] \frac{dA_i(\phi)}{1 + D_{ui}(\phi)/\delta_i} + [1 - A_i(\bar{w}_i)].$$

### 4.1.2 Employed Workers

The likelihood contribution of a type $i$ individual working full-time (part-time) at wage $w_e(q_i(w_e))$ is given by:

$$\{m_0; \delta_0 q_i(w_e)\}^{h_0} \{m_1; \delta_1 q_i(w_e)\}^{h_1} D_{ei}(w_e)^{2-d_{eb}-d_{ef}} \exp \left[ -D_{ei}(w_e)(t_{eb} + t_{ef}) \right]$$

$$\times \left[ \frac{\delta_i^{1-v_0^e-v_1^e} (\lambda_0^{e} \mathcal{T}_0(q_i(w_e)))^{v_0^e} (\lambda_1^{e} \mathcal{T}_1(w_e)))^{v_1^e}}{D_{ei}(w_e)} \right]^{1-d_{ef}}.$$

The likelihood function takes the same form for an employed worker whose wage is unobserved ($d_e = 1$), except that we now integrate the above likelihood contribution over the support of wages.

### 4.1.3 Discussion

While the derivation of the likelihood function is very standard, we note some points of significance. First, we use the steady state distributions of earnings and employment/unemployment shares to determine the initial conditions. Second, we do not use any information on the wages received following a job-to-job transition. We comment on the first issue further in Section 4.6 in light of our estimation results. The second issue represents a common difficulty with the structural estimation of wage posting models. Namely, that they do not permit job-to-job transitions associated with wage cuts. In the context of the model developed here, wage cuts may be permitted if an individual changes hours sectors, but reductions in the value of jobs are not.\(^{18}\) This partial information approach has been adopted in a number of other empirical applications of wage posting models (see for example, Bontemps, Robin and van den Berg, 1999, 2000; Kiefer and Neumann, 1993). An additional complication here is that the transition between

\(^{18}\)The model could potentially be extended to allow for job-to-job transitions associated with lower values by introducing a reallocation shock as in Jolivet, Postel-Vinay and Robin (2006). These shocks are draws from the wage offer distributions for which the only alternative to acceptance is to become unemployed. The presence of reservation wage heterogeneity complicates the analysis as some individuals may wish to exercise the unemployment option upon receiving such a shock.
hours sectors from certain wages may also be a zero probability event in some regions of the parameter space for some individuals. While there are a number of ways of potentially addressing this issue, we adopt a simple approach by assuming that there is some probability that the hours sector following a job-to-job transition is observed with error.  

4.2 Identification

Before detailing the three-step estimation procedure that we develop, we informally discuss the identification of the wage offer and reservation wage distributions. These ideas are presented more formally in the Supplementary Material. To begin, it is useful to first consider a special case of our model in the absence of hours sectors, a tax system, and demographic heterogeneity, and where the distribution of leisure flows collapses to a degenerate distribution (i.e. workers are homogeneous). This is the model analysed in Bontemps, Robin and van den Berg (2000). Conditional on transitional parameters, identification of the wage offer distribution follows directly from a steady state relationship between the wage offer and earnings distributions. Moreover, in such a setting all job offers will be accepted by all unemployed workers so that the accepted wage distribution will coincide with the wage offer distribution. This special case of our more general model is therefore over-identified.

Regardless of its source, once we allow for heterogeneity in the reservation wage of unemployed workers the distribution of accepted wages will no longer equal the wage offer distribution. This is because workers are selective in the wages that they are willing to accept, so that the distribution of accepted wages (which will stochastically dominate the wage offer distribution) will depend upon two distributions so that we face a deconvolution problem. Under the maintained assumption of steady state behaviour, we are still able to establish non-parametric identification in this case because we observe as many distributions (starting wages and cross-sectional earnings) as distributions that we wish to recover. If we observe further distributions, such as the distribution of wages that

---

19We estimate this misclassification error probability. Alternatives approaches such as modelling the hours disutility as unobserved heterogeneity, or incorporating the measurement error through wages (as in van den Berg and Ridder, 1998) would complicate the analysis considerably. See Bound et al. (1994) for US evidence on measurement error in wages and hours.

20While these are not primitives of the model, given knowledge of these distributions and the transitional parameters, it is then straightforward to identify the underlying distribution of firm productivities and worker leisure flows by using the equilibrium relationships we derived in Section 3.
the employed receive in their next job, then we once again will have over-identification.\textsuperscript{21}

### 4.3 Three Step Estimation Procedure

The likelihood function (as presented in the Section 4.1) depends directly upon the part-time and full-time wage offer distributions, which themselves depend upon the exogenous distributions of firm productivity and the other parameters of the model. Moreover, the conditional earnings distributions, distribution of reservation wages, and the unemployment and employment shares, are all complicated functions of these distributions and parameters. Rather than attempting to solve the full model at each evaluation of the likelihood function, we instead estimate the model using a three step procedure that is an extension of that proposed in Bontemps, Robin and van den Berg (1999, 2000). Specifically:

1. We estimate \( \{w_0, \bar{w}_0\} \) as the sample minimum and maximum values of \( w_e \) amongst part-time jobs and \( \{w_1, \bar{w}_1\} \) as the sample minimum and maximum values of \( w_e \) amongst full-time jobs. None of these estimates condition upon worker type. We then calculate estimates of the unconditional earnings densities in each sector using a non-parametric estimator. We denote these estimated densities as \( \hat{g}_0 \) and \( \hat{g}_1 \).\textsuperscript{22}

2. We assume a parametric form for the distribution of unobserved leisure flows \( H_i \) with a finite parameter vector \( \{\theta_{Hi}\}_{i \leq I} \). Since workers are assumed to sample wages from the same wage offer distributions \( F_0 \) and \( F_1 \) regardless of their demographic type \( i \), we weight equation 6 by the fraction of each type \( (n_i) \) and then sum to obtain appropriately averaged equations of the form:

\[
 f_h(w) = \frac{\sum_i n_i m_{hi} \hat{g}_{hi}(w)}{\sum_i n_i \hat{l}_{hi}(w)},
\]

for \( h \in \{0, 1\} \). We replace the numerator of equation 19 by \( m_h \hat{g}_h(w) \), where \( m_h = \sum_i n_i m_{hi} \). To recover the part-time and full-time offer distributions that induce our estimates of the unconditional empirical earnings distributions, we provide an initial guess of \( f_0 \) and \( f_1 \) and then repeatedly (and simultaneously) iterate on this

\textsuperscript{21}This is related to the approach taken by Barlevy (2008) and Barlevy and Nagaraja (2006) who using record-value theory demonstrate identification of the wage offer distribution by tracking the wage growth of workers as a function of past mobility.

\textsuperscript{22}In our empirical application we use Gaussian kernel estimators with a bandwidth of 0.5.
equation for both $h_0$ and $h_1$, exploiting the conditional linearity seen above. At each iteration step we scale the densities by a normalization factor to ensure that we have proper distribution functions, and then verify that these normalization factors converge to 1. Conditional on the transitional parameters and distributions of leisure flows, we obtain consistent estimates of the offer distributions and their densities, which we respectively denote $\hat{F}_h$ and $\hat{f}_h$. These estimates are then substituted into the likelihood function. They are also used to calculate the conditional employment shares and earnings densities: $u_i(\hat{F}_0, \hat{F}_1), m_{hi}(\hat{F}_0, \hat{F}_1),$ and $g_{hi}(\cdot; \hat{F}_0, \hat{F}_1)$.

3. Given a parametric form for the matching functions $M_h(V_h, S_h)$ and the recruiting cost functions $c_h(v; p)$ (we discuss our calibration of these in Section 5), we obtain the implied distribution of firm productivity and recruiting efforts by rewriting the first order conditions from the firms’ maximization problem in each sector $h$ as:

$$p = w_h^{-1}(w) = w + \mathcal{L}_h(w)/\mathcal{L}_h'(w), \quad (20)$$

and then using equations 13 and 17, together with the relationship $F_h(w_h(p)) = \int_{p}^{P} v_h(p)/V_h \, d\Gamma_h(p)$ from Definition 1. If the discount rate $\rho$ is known, then the distribution of leisure flows $H_i$ can then be recovered using equation 2.

We construct confidence intervals by bootstrapping the three stage estimation procedure. The advantages of this multi step procedure versus a completely parametric approach (whereby we specify the underlying distribution of firm productivity and then solve for the equilibrium of the model) are essentially threefold. Firstly, it is considerably easier to perform this numerical inversion than it is to solve the full model at every evaluation of the likelihood function. Second, it permits greater flexibility than simple parametric forms for the productivity distribution. Thirdly, since this semi-nonparametric estimation procedure does not require assumptions regarding the determination of $F_h$, these estimates and those of the transitional parameters are valid under a range of possible models. Conversely, in addition to the usual efficiency considerations, the main disadvantage of this approach compared to a full model parametric specification, is that it does not guarantee a monotonically increasing relationship between wages and productivity (in which case the empirical distribution of wages can not be an equilibrium outcome from our model), and in general it may not be possible to constrain the parameters to achieve such monotonicity.\textsuperscript{23}

\textsuperscript{23}In principle we could estimate the model using data from both before and after the reforms, treating each as a different steady state from the model. This would require solving the full model, as the model
4.4 Applying the UK Tax and Transfer System

Our empirical application seeks to accurately represent the main features of the UK tax and transfer system so that we may consider the impact of a series of tax reforms. We do not attempt to describe the full UK system here, but the interested reader may consult Adam and Browne (2009) and Jin, Levell and Phillips (2010) for recent surveys. The underlying tax and transfer schedules are calculated prior to estimation using FORTAX (Shephard, 2009), and reflect the complex interactions between the tax and transfer system, varying accurately with earnings, hours of work and demographic characteristics.24

To economize on the number of groups that we need to consider (and parameters to estimate), we make a number of further assumptions regarding the set of demographic types. Specifically, we do not allow taxes and transfers to vary by the age of the claimant or by the age of any children. Taxes and transfers are calculated as if the claimant were at least 25 years old, and as if any children are aged 10. Families with more than two children are treated as if there were only two children. Since some benefits have asset tests, we also assume that no families in our sample are affected by them. All families are assigned average band C council tax (a local property based tax).25

The model developed in Section 3 assumed the presence of a single economic decision maker. This presents difficulties for our empirical application because transfers and in-work tax credits are assessed on family income in the UK. A complete treatment of couples is beyond the scope of this paper (see Guler, Guvenen and Violante, 2009, for a theoretical characterization of the reservation wage strategy of couples with income pooling). Rather than providing a detailed characterization of the household decision making process, we take an admittedly limited approach by conditioning upon the current employment status and (discretized) earnings of the individuals’ partner so that there are essentially two observations for each two-person household. We subsume partner earnings in the tax schedule, but allow this tax schedule to accurately vary with the earnings of both individuals. In our empirical application we discretize the empirical distribution of partner earnings (conditional on gender and the presence and number

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24 A potentially important benefit that we do not model is housing benefit, a transfer given to low income families to assist with housing rent. The Labour Force Survey data used in our empirical application does not contain data on rents. Since tax credit income results in housing benefit entitlement being withdrawn, families in receipt of housing benefit would gain less from WFTC than otherwise equivalent families. This omission means that we are likely to overstate the initial labour supply response for some types.

25 Band C is the most common band; the Labour Force Survey data does not report banding information.
of children) into ten groups, including non-employment (zero earnings); actual partner earnings are then replaced with those observed at either the $10^{th}$, $20^{th}$, $\ldots$, or $90^{th}$ percentile point of the relevant empirical distribution. When the earnings of an individual’s partner is unobserved, we integrate the respective likelihood contribution of individuals over the corresponding distribution of partner earnings.

The above categorization requires that we consider $I = 64$ different worker types. Conditional on hours of work, the resultant tax schedules for each of these groups as a function of the wage rate will be a piecewise linear function, with possible discontinuities. We first remove these discontinuities by appropriately modifying parameters of the tax and transfer system. The modified marginal tax rate schedule for fixed hours is then replaced by a differentiable function using the method proposed by MaCurdy, Green and Paarsch (1990), which smooths the tax schedule in the neighbourhood of any marginal rate changes. Details are provided in the Supplementary Material.

4.5 Data

We estimate our model using a sub-sample of the UK Labour Force Survey (LFS). The LFS is a quarterly survey of around 60,000 households in Great Britain, with these households followed for five successive quarters or “waves”. When individuals first enter the survey they are in wave one, so that in any given quarter, there are roughly equal proportions of individuals in each interview wave. This rolling panel structure means that there is approximately an 80% overlap in the samples for successive quarters.

The LFS provides us with very rich information concerning the respondents labour market status. Crucially, we observe employment status and spell durations, together with hours and earnings information (in the first and fifth waves since 1997) for workers.\textsuperscript{26} Our pre-reform estimation is performed using data before WFTC was introduced or announced. We follow individuals who are observed in the first quarter of 1997 until (at the latest) the first quarter of 1998. We calculate incomes and construct the likelihood function as if individuals always faced the April 1997 system during this period so that the environment is stationary. While we may observe long elapsed spell durations, we nonetheless impose left censoring for durations greater than 24 months as it is difficult to justify the assumption that they were generated from the same steady state.

We classify individuals as being employed if they have a job, and non-employed

\textsuperscript{26}While individuals are interviewed at a quarterly frequency, the start and end date of spells is reported so that we effectively observe continuous time data.
if they do not. Since we do not distinguish between the states of unemployment and non-participation, this definition of non-employment is broader than the standard ILO unemployment definition. Amongst the employed, women who report working less than 30 hours per week are classified as part-time workers, while those working at least 30 hours per week are classified as full-time workers. We set $h_0 = 20$ and $h_1 = 40$, which correspond well to the respective conditional averages. Empirically, very few men work part-time, so we treat all male workers as working full-time and set $C^1_i = 0$. In both cases, we calculate gross wages using reported hours of work, but then proceed to calculate incomes as if they were working at the relevant discrete hours point.

Individuals who are aged below 21 or above 50 are excluded from our sample, as are individuals in full-time education. We also exclude individuals when any adult family member is either self-employed or long-term sick/disabled, or there is a change in household structure. Given the assumption that workers are equally productive at a given firm, we additionally restrict our sample to those individuals with no more than the compulsory level of schooling (that is, GCSE or equivalent), and assume that any higher educated individuals operate in a separate labour market. Together, these assumptions imply that all individuals have a minimum of five years of potential labour market experience. After sample selection we have roughly 24,000 observations. Summary statistics are presented in the Supplementary Material.

While the tax and transfer schedules may vary with each observable type, we only allow the parameters of the model to depend on a subset of demographic types. For couples we do not allow the parameters to vary with the earnings and labour market status of their partner; for parents we do not allow them to vary with the number of their children. The distribution of work opportunity costs $H_i$ is assumed to be Normally distributed, with mean $\mu_i$ and standard deviation $\sigma_i$.\(^{27}\)

### 4.6 Estimation Results and Model Fit

Given our parameter estimates (Table 1), the implied wage policy functions $w_0(p)$ and $w_1(p)$ that are obtained from the first order conditions to the firms’ profit maximization problem (equation 20) are found to be monotonically increasing in $p$ so that the estimated empirical distribution of wages can be an equilibrium outcome from our model. That is, the theoretical model is not rejected by the data. These wage policy functions

\(^{27}\)The leisure flow distribution was poorly identified for single women. In the results presented we have restricted the distribution of leisure flows to be the same for married and single women without children.
Table 1: Maximum Likelihood Parameter Estimates

<table>
<thead>
<tr>
<th></th>
<th>$1/\delta_i$</th>
<th>$1/\lambda^0_{ui}$</th>
<th>$1/\lambda^1_{ui}$</th>
<th>$1/\lambda^0_{ei}$</th>
<th>$1/\lambda^1_{ei}$</th>
<th>$\mu_i$</th>
<th>$\sigma_i$</th>
<th>$C_i^1$</th>
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<td>68.5</td>
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<td>13.5</td>
<td>9.8</td>
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<td>23.6</td>
<td>33.8</td>
<td>35.6</td>
<td>49.3</td>
<td>42.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>[97.5,107.4]</td>
<td>[90.2,208.7]</td>
<td>[74.4,110.5]</td>
<td>[44.1,56.0]</td>
<td>[34.3,51.8]</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: All durations are monthly. Incomes are measured in pounds per week in April 1997 prices. The distribution of work opportunity costs $H_i$ is assumed to be Normal, with mean $\mu_i$ and variance $\sigma_i^2$. The 5th and 95th percentiles of the bootstrap distribution of parameter estimates are presented in brackets, and are calculated using 500 bootstrap replications.
are presented in Figure 2a. The first notable feature evident in this figure is that the wage policy functions become flat as productivity increases so that high productivity firms have a high degree of monopsony power. Second, the extent of this monopsony power is typically lower for part-time firms. The underlying distributions of firm productivity are shown in Figure 2b, with both part-time and full-time distributions highly skewed to the right. The associated estimated wage offer distributions are presented in Figure 2c, which shows that there is a relatively higher concentration of low wage offers among part-time firms. Finally, the unconditional distribution of wage earnings that these distributions induce is shown in Figure 2d.

The estimates suggest that there is considerable heterogeneity across groups. The job destruction rate (which is relatively precisely estimated across all groups) is highest for single parents ($\hat{\delta}_i = 0.014$) with this estimate implying that jobs are exogenously destroyed on average every 70 months ($= 1/0.014$). The destruction rates are lowest for married men and married women without dependent children, where they are estimated to be around two and a half times as small. The arrival rates of job offers also varies considerably across groups. Job offers arrive most frequently for men: for unemployed married men without children we obtain $\hat{\lambda}_{ui}^1 = 0.251$, while for single men and married men without children we obtain lower estimates (0.073 and 0.085 respectively). The estimated total job offer arrival rates $\hat{\lambda}_{ui}^0 + \hat{\lambda}_{ui}^1$ for unemployed childless women is broadly similar to the values of $\hat{\lambda}_{ui}^1$ for men. However, for unemployed single parents and married women with children we obtain much lower job offer arrival rates (particularly for full-time jobs).

For the majority of groups, the estimated job offer arrival rate when employed is less than that when unemployed, although in some cases the estimated parameters are very similar in value. For single men we obtain the relative arrival rate ratio $\hat{\lambda}_{ui}^1 / \hat{\lambda}_{ei}^1 = 1.5$, while this ratio is somewhat higher for married men without children ($=3.9$) and slightly lower for married men with children ($=1.1$). In this latter case we can not reject the null hypothesis that the arrival rates are equal. These general findings are broadly consistent with the estimates reported in van den Berg and Ridder (1998); they contrast with Bontemps, Robin and van den Berg (2000) which found (using French Labour Force Survey data) that job offers arrive around ten times as frequently for the unemployed compared to the employed. For all groups of women we obtain the relative part-time arrival rate ratio $\hat{\lambda}_{ui}^0 / \hat{\lambda}_{ei}^0 > 1$ (ranging from around 1.3 for single parents and married women with children to 4 for women without children). Mirroring the pattern observed
Figure 2: Pre-reform equilibrium distributions and functions. All figures calculated using the maximum likelihood estimates from Table 1 and calculated under the April 1997 (pre-reform) tax and transfer system. All incomes measured in April 1997 prices. Figure 2a shows how the optimal wage policy of firms varies with hours and firm productivity, and truncated at productivities greater than \( w^{-1}_1(\hat{G}_1(0.99)) \); Figure 2b shows the underlying distribution of firm productivity as obtained from the first order conditions to the firms’ profit maximization problem (with the normalization \( v_h(p) = 1 \), and truncated at productivities greater than \( w^{-1}_1(\hat{G}_1(0.95)) \); Figure 2c shows the associated distribution of part-time and full-time wage offers; Figure 2d shows the unconditional distribution of part-time and full-time earnings that these wage offer distributions induce.
for men, the relative full-time job offer arrival rate ratio for single women $\frac{\lambda_{ui}}{\lambda_{ei}} = 1.7$, while this ratio is again somewhat higher for married women without children ($=3.4$) and lower for married women with children ($=0.7$). For single parents we estimate large differences by employment status with $\frac{\lambda_{ui}}{\lambda_{ei}} = 0.1$.28

The estimated monetary disutility of full-time work $\hat{C}_{i}^{1}$ is lowest for single women (around £20 per week in April 1997 prices), while it is at least twice as high for single parents and married women. We obtain considerable dispersion in the unobserved leisure flows for all groups, and this translates into dispersion in reservation wages. In the Supplementary Material we present results which show the proportion of workers of each type whose reservation wage is below given percentiles of the (full-time) wage offer distribution. In all cases we obtain $\hat{A}_{i}(\hat{\omega}_{1}) \ll 1$, so that workers are indeed selective in the wage offers that they are willing to accept. This feature also implies a negative duration dependence in the exit rate out of unemployment. Furthermore, we find that essentially all individuals would be willing to accept the highest full-time wage offer.

Since the wage offer distributions are common to all workers, any difference in employment states and earnings distributions must be explained by variation in the transitional parameters, leisure flow distributions, and the tax and transfer system. Overall, we obtain a good fit to the data. The difference in the empirical and predicted states for the main demographic groups is small and only rarely does it exceed 1 percentage point (see Table 2). Similarly, we do well in replicating the observed distribution of wage earnings (see Figure 3); for most groups the fit is excellent, although the fit of the full-time earnings distribution for married women with children (Figure 3h) is slightly less satisfactory. Finally, we note that the estimated model also provides a generally good fit to the transitions between the different labour market states. In the Supplementary Material we compare the simulated and empirical one-year transition rates. We replicate the general pattern of transitions across the demographic groups, with a particularly good fit to the one-year job-to-job transition rates. For some groups we slightly over-predict the job to unemployment transitions, while the largest differences in the central estimates occur for the annual transitions from unemployment to employment in cases where a low frequency of individuals is observed (again, see Table 2).29

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28 As described in Section 4.1, we also estimate a misclassification error probability for job-to-job hours transitions. We obtain an estimate of 10.5%.

29 Simultaneously providing a good fit to both states and transitions is consistent with the steady state assumption providing a reasonable approximation of the labour market. To better understand this issue we also use our parameter estimates to simulate labour market histories for an initially unemployed cohort of individuals. For individuals who have been in the labour market for five years (as in our sample
Table 2: Empirical and Predicted Employment States

<table>
<thead>
<tr>
<th></th>
<th>Empirical</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u_i$</td>
<td>$m_{0i}$</td>
</tr>
<tr>
<td>single men</td>
<td>0.251</td>
<td>–</td>
</tr>
<tr>
<td>married men, no kids</td>
<td>[0.238,0.262]</td>
<td>[0.738,0.762]</td>
</tr>
<tr>
<td></td>
<td>0.070</td>
<td>–</td>
</tr>
<tr>
<td>married men, kids</td>
<td>[0.061,0.079]</td>
<td>[0.921,0.939]</td>
</tr>
<tr>
<td></td>
<td>0.123</td>
<td>–</td>
</tr>
<tr>
<td>single women</td>
<td>[0.114,0.131]</td>
<td>[0.869,0.886]</td>
</tr>
<tr>
<td>single parents</td>
<td>0.157</td>
<td>0.103</td>
</tr>
<tr>
<td></td>
<td>[0.144,0.170]</td>
<td>[0.092,0.113]</td>
</tr>
<tr>
<td>married women, no kids</td>
<td>0.611</td>
<td>0.229</td>
</tr>
<tr>
<td></td>
<td>[0.586,0.615]</td>
<td>[0.216,0.242]</td>
</tr>
<tr>
<td>married women, kids</td>
<td>0.130</td>
<td>0.224</td>
</tr>
<tr>
<td></td>
<td>[0.121,0.140]</td>
<td>[0.213,0.237]</td>
</tr>
<tr>
<td></td>
<td>0.342</td>
<td>0.409</td>
</tr>
<tr>
<td></td>
<td>[0.332,0.351]</td>
<td>[0.400,0.419]</td>
</tr>
</tbody>
</table>

Notes: Predicted states are calculated using the maximum likelihood estimates from Table 1. Employment states may not sum to one due to rounding. The 5th and 95th percentiles of the bootstrap distribution of employment states are presented in brackets, and are calculated using 500 bootstrap replications.
Figure 3: Simulated and empirical wage earnings by group. Horizontal axis refers to hourly wage rate in April 1997 prices; Vertical axis refers to wage density. Empirical distributions are calculated using a Gaussian kernel with a bandwidth of 0.5.
5 Simulating Tax Reforms

In this section we simulate the impact of real changes to the UK tax and transfer system between April 1997 (the pre-reform sample period) and April 2002. We also consider the impact of introducing WFTC in isolation. All the simulations in this section assume a quadratic recruiting cost technology \( c_h(v; p) = c_h(p)v^2/2 \) which allows us to establish the following proposition:

**Proposition 8** If the recruiting cost technology is given by \( c_h(v; p) = c_h(p)v^2/2 \) then the equilibrium of the estimated model following any tax reform is invariant to the parametrization of \( c_h(p) \) provided that \( c_h(p) > 0 \) for all \( p \).

A proof of this proposition is provided in the Supplementary Material. An implication of this proposition is that without loss of generality we may set \( v_h(p) = 1 \) in the pre-reform period and recover the values of \( c_h(p) \) that are consistent with this being an equilibrium. This also implies that \( \Gamma_h(p) = F_h(w_h(p)) \) under the base system. The simulations that follow also assume the presence of Cobb-Douglas matching functions \( M_h(V_h, S_h) = V_h^{\theta_0}S_h^{1-\theta_0} \) and we set \( \theta_0 = \theta_1 = 1/2 \) (see, Petrongolo and Pissarides, 2001). Our results are not sensitive to these specific values.

To highlight the relative importance that these reforms have on job acceptance behaviour and the strategy of firms, we present our results in two stages. Firstly, we consider the impact of the reforms holding the distribution of job offers and their arrival rates constant; secondly, we additionally allow firms to optimally respond by changing their wage policy and recruiting effort. We refer to the first channel as the **direct impact** of the reforms, and the second channel as the **equilibrium impact** of the reforms. In our discussion we also highlight respective roles of the different equilibrium mechanisms.

5.1 Direct Impact

The direct and equilibrium impact of the reforms on employment states is presented in Table 3, and we first discuss the direct effect. The table shows that the (non-WFTC) reforms between April 1997 and April 2002 had a small positive effect on the employment selection rule, see Section 4.5), we find that the steady state assumption provides a good approximation to employment states and wage distributions. For the demographic groups where frictions are found to be least important we only have very small deviations from the steady state values; in other cases the differences are not more than a few percentage points. We match part-time wages very well, but average full-time wages are still slightly below their steady state values. For groups with a longer labour market history, the fit is naturally closer to their steady state values.
of both singles and couples without children (ranging from 0.2 percentage points for married men without children, to 0.8 percentage points for single men). These increases are primarily due to small reductions in the real value of out-of-work income, together with small reductions in income-tax which act to raise the value of holding low wage jobs and so lower reservation wages which increases the exit rate from unemployment.

Perhaps unsurprisingly, the largest predicted impact of these reforms is on the employment rate of single parents, where we predict an increase in employment of 5.1 percentage points. Despite both full-time and part-time reservation wages falling for many of these workers, this steady state employment increase is exclusively due to a movement into full-time work. This is largely due to on-the-job search, with the estimated arrival rate of full-time wage offers among the employed far exceeding that of part-time wage offers ($\lambda^1_{et} \gg \lambda^0_{et}$). These differences in arrival rates for employed workers mean that individuals who enter employment as a result of the reform tend to gravitate to the full-time sector in the new steady state. Second, the lower withdrawal rate of WFTC compared to FC results in full-time incomes increasing by more than part-time incomes over a large range of wages for this group (Figure 4a shows how the indifference condition $q_i(w)$ changes), therefore making full time work relatively more desirable. This latter feature increases the overall job-to-job transition rate for this group, as does the mechanical compositional effect of workers now accepting lower wage jobs, and then climbing up the job ladder more quickly.

For couples with children the impact of these reforms is more complicated as tax credit entitlement depends upon family income: individuals with a high earning partner are essentially unaffected by the reform; those with a non-working or very low earning partner respond positively, much like single parents; in intermediate cases, movement into work can result in tax credit awards being withdrawn which may induce negative labour supply responses (particularly among the newly eligible families). On balance,

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30In-work incomes for a single parent with one child increase by around 18% (13%) as a result of this reform at full-time (part-time) work hours for a worker earning the average wage (amongst single parents). The percentage increases are even large for multi-child families.

31The reservation wages of individuals with very high $b$ will actually increase. These individuals experience an increase in their out-of-work income, but at the high wages that these workers would actually accept they will be ineligible for tax credits so there is little change in their net taxes when employed.

32To understand the changes to the indifference condition shown here we note that over a large range of wages the reduction in the taper rate is most important so that full-time work relatively more becomes more desirable. At high wages (where individuals only become eligible for tax credits at full-time hours following the reform), part-time incomes increase by more than full-time incomes. At very high wages individuals are not-affected by the tax credit reform, so there are only small changes in the indifference condition due to the other smaller changes to the tax and transfer system.
Table 3: Employment Impact of Reforms

<table>
<thead>
<tr>
<th></th>
<th>Direct Impact</th>
<th>Equilibrium Impact</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta u_i$</td>
<td>$\Delta m_{0i}$</td>
</tr>
<tr>
<td>single men</td>
<td>-0.008</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[-0.019,0.002]</td>
<td>-</td>
</tr>
<tr>
<td>married men, no kids</td>
<td>-0.002</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[-0.009,0.006]</td>
<td>-</td>
</tr>
<tr>
<td>married men, kids</td>
<td>-0.011</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>[-0.019,-0.004]</td>
<td>-</td>
</tr>
<tr>
<td>single women</td>
<td>-0.005</td>
<td>-0.000</td>
</tr>
<tr>
<td></td>
<td>[-0.014,0.006]</td>
<td>[-0.010,0.011]</td>
</tr>
<tr>
<td>single parents</td>
<td>-0.051</td>
<td>-0.014</td>
</tr>
<tr>
<td></td>
<td>[-0.069,-0.016]</td>
<td>[-0.036,-0.001]</td>
</tr>
<tr>
<td>married women, no kids</td>
<td>-0.004</td>
<td>-0.007</td>
</tr>
<tr>
<td></td>
<td>[-0.012,0.006]</td>
<td>[-0.020,0.006]</td>
</tr>
<tr>
<td>married women, kids</td>
<td>0.008</td>
<td>-0.008</td>
</tr>
<tr>
<td></td>
<td>[-0.001,0.017]</td>
<td>[-0.019,0.003]</td>
</tr>
</tbody>
</table>

Notes: All employment responses are expressed in percentage points. Changes may not sum to zero due to rounding. The direct impact considers all changes to the tax and transfer system between April 1997 and April 2002, holding the wage offer distributions and arrival rates at their pre-reform levels. The equilibrium impact allows the wage offer distribution and arrival rates to change.
these factors lead to a small decrease in the employment of married women with children (a 0.8 percentage point decrease), and increase the employment rate of married men with children by a similar absolute magnitude. Among married women, this decrease in labour supply comes primarily through a reduction in those working part-time. The potential disincentive effects of tax credits programs on family labour supply have long been recognized, with the direction of these responses consistent with those reported in other studies (see for example, Eissa and Hoynes, 2004).

Before we discuss the equilibrium impact of the reforms, we briefly discuss the effect on wages. Note that selection alone implies that wage earnings will change even though the distribution of wage offers is held fixed. This highlights the fact that attempting to estimate the incidence of tax credit programs by comparing changes in observed wages amongst eligible and non-eligible groups is potentially misleading without carefully controlling for these dynamic selection effects. Indeed, selection alone implies some large reductions in full-time average wages. Our simulations imply that single parents experience a 6% reduction in average full-time wage earnings and a 1% reduction in part-time wages (see Figure 4b). The changes for other groups are smaller.
5.2 Equilibrium Impact

In Table 3 we also present the equilibrium impact of the tax reforms, which reflect both the adjustments in wage offers and recruiting intensity decisions. The first immediate thing to note is that the impacts are generally very similar to those obtained from the direct impact. That is, equilibrium considerations do not appear to be very important for this particular set of reforms. For the group of single parents, the employment increase is reduced from 5.1 to 4.8 percentage points (a 5\% decrease). Looking more closely we can see that equilibrium considerations tend to increase employment in full-time jobs, and decrease employment in part-time jobs, although the magnitudes of change are admittedly small.

Given the small magnitude of the changes here, we do not attempt to describe the responses in detail. We note, however, that the direction of the changes to firms’ optimal policies can largely be understood by examining the initial direct impact of the reforms on labour supply. As described above, the reduction in reservation wages experienced by many workers raises both part-time and full-time employment at low wages. However, on-the-job search and the changes to the indifference conditions as noted above also induce transitions between the hours sectors. The net effect of these transitions means that part-time employment falls at medium wage levels. Part-time firms which experienced initial increases in their employment generally respond by lowering their wage offers in the new equilibrium and increasing their recruiting intensity; at slightly higher wage rates where employment initially falls we find the opposite effect. Full-time firms have smaller adjustments in their wages (although it is still the case that the firms which experienced the largest initial increase in employment tend to have the greatest reduction in wages). Again, the change in recruiting intensity of these firms closely mirrors the initial labour supply response. In Figure 5 we illustrate these relative changes in firms’ strategies, and relate them to the initial labour supply responses.\textsuperscript{33,34}

The significance of the finding of slight equilibrium effects should not be understated.

\textsuperscript{33}The difference between average profit flows in part-time and full-time sectors changes very little following the reform so that possible entry and exit decisions are unlikely to have important implications for the wage offer distributions.

\textsuperscript{34}The changes to the wage policy function are quantitatively more important than the adjustments in recruiting intensity in generating the described employment and wage responses. The aggregate effect of changes to firm level recruiting intensity is to raise the arrival rate of full-time job offers by 1.6\%, and with a negligible impact on the arrival rate of part-time job offers. While the changes to recruiting intensity produce little further impact on employment rates, they do amplify the compositional shift from part-time to full-time work.
Indeed, one important argument made for the introduction of a national minimum wage in the UK at the time, was that it would prevent employers from capturing some of the benefits of the more generous tax credit programme by reducing wages. The analysis here suggests that those fears may have been unfounded.

5.3 Other Evaluations

The set of tax reforms considered here have been the subject of a number of other studies, with difference-in-differences being the most common empirical approach. These evaluations have largely focused upon the employment impact of single parents and essentially involve comparing the changing employment outcomes of single mothers to single women without children. While there are sometimes differences in the definition of the sample, and the exact period under investigation, the quantitative impact reported in these studies is remarkably similar to that reported in Table 3. Netting off the impact of these reforms on single women, the relevant comparable employment impact for these difference-in-differences studies is 4.6 (=5.1-0.5) percentage points. This compares to the 3.6 percentage point increase reported in Blundell, Brewer and Shephard (2004), and the 5 percentage point increase in Gregg and Harkness (2003). See Brewer and Browne (2006) for a review of these and other studies. The fact that these different approaches deliver similar quantitative predictions is consistent with the absence of substantial equilibrium effects (if they were important then the usual stable unit treatment value assumption would be violated).35 We also note that the employment impact for single parents is also similar to that reported in studies using discrete choice techniques (Blundell and Shephard, 2012; Brewer et al., 2006).

While much less studied, the predicted employment impact for individuals in couples (a small increase for men, and corresponding decrease for women) is very close to that reported in Brewer et al. (2006), but contrast to those in Leigh (2007) where more sizeable (albeit, only marginally significant) employment increases for both groups were estimated. Again, see Brewer and Browne (2006) for a comparison of these and a small number of other studies.

35There is no underlying reasons for this similarity to hold. Had search frictions been estimated to be more or less important, for example, then the simulated employment impact could have diverged substantially from that reported in these ex-post evaluations.
Figure 5: Equilibrium policy responses. Figures 5a and 5c show how the equilibrium recruiting intensity of part-time and full-time firms (as identified by their pre-reform wage policy) changes, and how this compares to the relative direct employment responses. Similarly, Figures 5b and 5d show how the equilibrium wage policy of part-time and full-time firms changes, and again compares this to the relative direct employment responses. All incomes measured in April 1997 prices.
5.4 Post-reform Comparison

We now compare our results to the actual changes to employment states between 1997 and 2002. Tabulations are presented in the Supplementary Material. Over this period, all the broad demographic groups experienced an increase in their employment rate. And for all groups except single men and married women with children, we systematically under-predict the growth in employment by between around one and one-and-a-half percentage points. For single men and particularly married women with children, the extent of under-prediction is larger. Overall, this suggests that non-tax changes over this period, including robust productivity growth, changes in the distribution of partner earnings, the introduction of a national minimum wage, and various “New Deal” active labour market policies (particularly relevant for low skilled men) were also important for understanding the changing labour market outcomes. Finally, we note that the model is somewhat less successful in explaining the changing distribution of work hours. For single parents, the empirical increase in employment is more evenly split between movements into both part-time and full-time employment; this contrasts with our simulations which suggested that it was exclusively due to a movement into full-time work.36

5.5 Isolating the impact of WFTC

The results presented above considered the impact of all reforms to the tax system between April 1997 and April 2002. However, as described in Section 2, the introduction and expansion of WFTC coincided with a number of other tax reforms. In particular, at the same time that the government increased income in-work through the expansion of tax credits, it also increased out-of-work income through increases in out-of-work income through the UK system of Income Support. To understand the importance of these other changes, we simulate the impact of replacing the tax credit system in April 1997 with that of April 2002, holding all other components of the tax and transfer system fixed. By construction, the direct impact of this hypothetical reform on individuals without children is identically zero. Among families with children, we unsurprisingly find larger effects. For both singles and couples with children, the direct employment

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36 An important caveat to this informal comparison is that we are comparing empirical changes to a theoretical steady state, so that the role of any transitional dynamics is being ignored. Given the differences in the estimated arrival rate parameters, we note that the model implies a short-term increase in both part-time and full-time employment amongst single parents, followed by a shift towards full-time work through the process of on-the-job search (recall that \( \lambda^1_{ei} \gg \lambda^0_{ei} \)).
impacts now approximately double in size (an employment increase of 11 percentage points for single parents, an increase of 1.8 percentage points for married men with children, and a 1.5 percentage point decrease for married women with children). Again, equilibrium effects are found to be slight.

5.6 Why Aren’t Equilibrium Effects More Important?

Our analysis suggests that equilibrium effects may be small. We now explore the extent to which this may be due to the integrated nature of the labour market and the targeted nature of the reforms. While allowing all workers to compete within the same market was a natural characterization of the labour market, and one which permitted spillover effects, it limits the potential for sizeable equilibrium effects following a targeted reform like WFTC if firms are constrained to have a single wage policy. To understand the importance of these assumptions we now consider, for illustrative purposes, a labour market comprised of single individuals (both with and without children). Single parents comprise around one-third of this more segmented labour market. Under this alternative assumption we obtain larger within-market equilibrium effects. There are now larger changes in the optimal policies of firms (essentially scaled up versions of the responses shown in Figure 5) which result in a reduced employment response for single parents. The adjustment in wages alone reduce the employment increase to 3.8 percentage points, while allowing for both equilibrium mechanisms results in a 4 percentage point employment increase. The initial direct impact on employment is therefore reduced by over 20% in this case. Underlying these changes is a large reduction in part-time and full-time wage offers (on average, 14% and 11% respectively), while the associated job offer arrival rates increase by 2% and 3%.

We also note that the impact of any policy, as well as the size of any equilibrium effects, will depend upon the characteristics of the labour market. For example, suppose

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37In a model with worker and firm bargaining, wages essentially become individualistic so that the potential for equilibrium effects is much larger. Lise, Seitz and Smith (2005) used such a model in their analysis of the Canadian Self-Sufficiency Project, and found substantial equilibrium effects.

38This is equivalent to firms being able to discriminate against these demographically identifiable groups, so essentially allowing $F_i$ to become indexed by a set of $i$. To ensure that the direct impacts remain as in Table 3 we do not re-estimate the model for these illustrations. Instead, we use the same parameter estimates from Table 1 (with the associated non-parametric estimates of $F_i$), and recover the distribution of firm productivity under alternative market segmentation assumptions. We also attempted this exercise using a market comprised solely of single parents, but encountered large monotonicity violations when using firms’ first order conditions to recover the distributions of productivity.
that in the same segmented labour market that we considered above, that the exogenous search intensity parameters $s_{ij}$ were to fall everywhere by 50%. The changing worker strategies and optimal policies of firms’ increase the pre-reform equilibrium unemployment rates by 10 percentage points for single parents, 4 percentage points for single women, and 5 percentage points for single men. The direct impact of the reform upon single parents is now larger (relative to the size of the initially employed population we now observe a 16% compared to 12% employment increase), although equilibrium effects are now proportionately smaller. Part-time wage offers now fall on average by 8%, while average full-time offers are essentially unchanged. The associated job offer arrival rates increase by 3% and 2%. This discussion underscores the fact that the impact of similar policies may have very different effects depending upon the nature and extent of frictions in the labour market.

6 Conclusion

This paper has developed an empirical equilibrium job search model with wage posting, and has used it to analyze the impact of a series of UK tax reforms that included the Working Families’ Tax Credit reform. It presents a synthesis of existing equilibrium job search models, and extends them in a number of dimensions to reflect key features of the UK labour market and to allow for the possibility of rich equilibrium effects.

We structurally estimate the model using a semi-nonparametric estimation procedure, and predict that the series of tax reforms had a positive effect on the employment of most groups, with single parents experiencing the largest employment increase. Our simulations reveal an important finding that while equilibrium effects do play a role, the changes in labour market outcomes are dominated by the direct effect of changing job acceptance behaviour.

Even though these equilibrium effects may not appear very large for this particular set of reforms, it does not imply that they should always be ignored. Recalling that WFTC is only available to families with children, these equilibrium effects have the potential to be more important for reforms that are less targeted. We also demonstrate that the within market equilibrium effects of the same reforms may be much larger if we consider a somewhat more segmented labour market. We also show how the size of these effects may depend upon the extent of search frictions in the labour market.

We believe that this paper represents an important step in using empirical equilib-
rium job search models to evaluate the impact of tax policies. Despite performing our empirical analysis on individuals with low education levels, it is likely that differences in worker ability persist. A natural extension could therefore involve incorporating heterogeneity in worker productivity which necessitates a more detailed modelling of firm production technologies. Furthermore, given that the tax and transfer systems of many countries depend upon family income to some extent, a more detailed characterization of the behaviour of couples would allow us to more rigorously explore the impact of policies on within household labour supply allocations. Finally, given the importance of labour supply in our simulations, incorporating a micro-level search intensity choice would create a further dimension along which individuals can respond. While each of these represent non-trivial extensions, it does suggest a very exciting agenda of future research.

References


Online Supplement to “Equilibrium Search and Tax Credit Reform”

These online appendices (i) present some additional descriptive and results tables, (ii) provide proposition proofs, (iii) describe the tax rate schedule smoothing, and (iv) outline the numerical algorithm used to solve the model.

A Additional Parameter and Results Table

In Table $S_1$ we present some data summary statistics; in Table $S_2$ we present results which show the proportion of workers of each type whose reservation wage is below given percentiles of the (full-time) wage offer distribution, as described in Section 4.6 from the main text; in Table $S_3$ we compare empirical and simulated one-year transitions, as described in Section 4.6 from the main text; in Table $S_4$ we compare the predicted changes from the model to the actual changes based on our sample selection criteria, and as discussed in Section 5.4 from the main text.

B Proof of Proposition 2

In this appendix we derive the optimal reservation wage strategies of unemployed workers that were presented in Section 3.3 from the main text. For notational simplicity, we do not explicitly write the value functions or the resultant reservation wages as a function of the unobserved leisure flow $b$. The value of unemployment $V_{ui}$ must satisfy:

$$
\rho V_{ui} = b - T_i^u + \lambda^0_{ui}E_{w \sim F_0} \max \left\{ V_{e1}^0(w) - V_{ui}, 0 \right\} \\
\quad + \lambda^1_{ui}E_{w \sim F_1} \max \left\{ V_{e1}^1(w) - V_{ui}, 0 \right\} 
$$

(S1)

where $V_{e1}^0(w)$ and $V_{e1}^1(w)$ are the values of part-time and full-time employment when receiving wage $w$, and where $E_{w \sim F_h}$ indicates that the expectation is taken over the random variable $w$ with cumulative distribution function $F_h$. For workers who are employed in
Table S1: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>Unemployed</th>
<th>Employed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(N_u)</td>
<td>(u \rightarrow h_0)</td>
</tr>
<tr>
<td>Single men</td>
<td>796</td>
<td>-</td>
</tr>
<tr>
<td>Married men, no kids</td>
<td>132</td>
<td>-</td>
</tr>
<tr>
<td>Married men, kids</td>
<td>504</td>
<td>-</td>
</tr>
<tr>
<td>Single women</td>
<td>357</td>
<td>26</td>
</tr>
<tr>
<td>Single parents</td>
<td>1661</td>
<td>100</td>
</tr>
<tr>
<td>Married women, no kids</td>
<td>417</td>
<td>25</td>
</tr>
<tr>
<td>Married women, kids</td>
<td>2176</td>
<td>204</td>
</tr>
</tbody>
</table>

Notes: \(N_u\) refers to the number of unemployed observations in a given category; \(N_e^0\) and \(N_e^1\) respectively refer to the number of part-time and full-time employment observations. \(w_u\) refers to the number of observed accepted wages from unemployment; \(w_e^0\) and \(w_e^1\) refer to the number of cross-sectional wage observations in part-time and full-time employment. \(i \rightarrow j\) refers to the numbers of observed transitions from state \(i\) to state \(j\), with states \(u\), \(e\), \(h_0\) and \(h_1\), denoting unemployment, overall employment, part-time employment, and full-time employment respectively.
Table S1: (continued)

<table>
<thead>
<tr>
<th></th>
<th>Part-time wages</th>
<th></th>
<th>Full-time wages</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( P_{10} )</td>
<td>( P_{25} )</td>
<td>( P_{50} )</td>
<td>( P_{75} )</td>
</tr>
<tr>
<td>single men</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>married men, no kids</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>married men, kids</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>single women</td>
<td>2.82</td>
<td>3.37</td>
<td>3.97</td>
<td>5.16</td>
</tr>
<tr>
<td>single parents</td>
<td>2.75</td>
<td>3.29</td>
<td>3.80</td>
<td>5.01</td>
</tr>
<tr>
<td>married women, no kids</td>
<td>3.07</td>
<td>3.66</td>
<td>4.27</td>
<td>5.38</td>
</tr>
<tr>
<td>married women, kids</td>
<td>3.00</td>
<td>3.61</td>
<td>4.40</td>
<td>5.72</td>
</tr>
</tbody>
</table>

Notes: All wages are hourly and are expressed in April 1997 prices. \( P_{10} \), \( P_{25} \), \( P_{50} \), \( P_{75} \), and \( P_{90} \) respectively refer to the 10\textsuperscript{th}, 25\textsuperscript{th}, 50\textsuperscript{th}, 75\textsuperscript{th}, and 90\textsuperscript{th} percentiles of the cross-sectional hourly wage distribution; SD refers to the standard deviation.
Table S2: Reservation Wage Distribution

<table>
<thead>
<tr>
<th>Percent of full-time wage offer distribution</th>
<th>5</th>
<th>20</th>
<th>40</th>
<th>60</th>
<th>80</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>single men</td>
<td>0.300</td>
<td>0.446</td>
<td>0.589</td>
<td>0.726</td>
<td>0.878</td>
<td>0.989</td>
</tr>
<tr>
<td>married men, no kids</td>
<td>0.138</td>
<td>0.311</td>
<td>0.509</td>
<td>0.688</td>
<td>0.868</td>
<td>0.983</td>
</tr>
<tr>
<td>married men, kids</td>
<td>0.610</td>
<td>0.702</td>
<td>0.767</td>
<td>0.819</td>
<td>0.883</td>
<td>0.978</td>
</tr>
<tr>
<td>single women</td>
<td>0.546</td>
<td>0.670</td>
<td>0.760</td>
<td>0.831</td>
<td>0.906</td>
<td>0.978</td>
</tr>
<tr>
<td>single parents</td>
<td>0.437</td>
<td>0.565</td>
<td>0.636</td>
<td>0.691</td>
<td>0.774</td>
<td>0.986</td>
</tr>
<tr>
<td>married women, no kids</td>
<td>0.288</td>
<td>0.499</td>
<td>0.679</td>
<td>0.805</td>
<td>0.907</td>
<td>0.981</td>
</tr>
<tr>
<td>married women, kids</td>
<td>0.283</td>
<td>0.471</td>
<td>0.636</td>
<td>0.760</td>
<td>0.875</td>
<td>0.973</td>
</tr>
</tbody>
</table>

Notes: Table shows the fraction of individuals whose full-time reservation wage is below various percentiles \( p \) of the full-time wage offer distribution, \( A_i(\hat{F}_i^{-1}(p)) \), and is calculated using the maximum likelihood estimates from Table 1. The 5th and 95th percentiles of the bootstrap distribution are presented in brackets, and are calculated using 500 replications.
Table S3: Empirical and Predicted One-Year Transitions

<table>
<thead>
<tr>
<th></th>
<th>Empirical</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$u \rightarrow e$</td>
<td>$e \rightarrow e$</td>
</tr>
<tr>
<td>single men</td>
<td>0.320</td>
<td>0.093</td>
</tr>
<tr>
<td></td>
<td>[0.219,0.411]</td>
<td>[0.066,0.124]</td>
</tr>
<tr>
<td>married men, no kids</td>
<td>0.800</td>
<td>0.098</td>
</tr>
<tr>
<td></td>
<td>[0.519,1.000]</td>
<td>[0.071,0.124]</td>
</tr>
<tr>
<td>married men, kids</td>
<td>0.490</td>
<td>0.079</td>
</tr>
<tr>
<td>single women</td>
<td>0.367</td>
<td>0.078</td>
</tr>
<tr>
<td></td>
<td>[0.239,0.521]</td>
<td>[0.052,0.109]</td>
</tr>
<tr>
<td>single parents</td>
<td>0.170</td>
<td>0.110</td>
</tr>
<tr>
<td></td>
<td>[0.125,0.218]</td>
<td>[0.065,0.160]</td>
</tr>
<tr>
<td>married women, no kids</td>
<td>0.149</td>
<td>0.092</td>
</tr>
<tr>
<td></td>
<td>[0.068,0.240]</td>
<td>[0.069,0.114]</td>
</tr>
<tr>
<td>married women, kids</td>
<td>0.229</td>
<td>0.069</td>
</tr>
<tr>
<td></td>
<td>[0.191,0.270]</td>
<td>[0.053,0.085]</td>
</tr>
</tbody>
</table>

Notes: Predicted transitions are simulated using the maximum likelihood estimates from Table 1. All transitions measure differences in employment states over a period of one-year (irrespective of any within year transitions). Empirical transitions are calculated using individuals observed in both Q1 1997 and Q1 1998. The 5th and 95th percentiles of the bootstrap distribution of annual transition rates are presented in brackets, and are calculated using 500 replications.
Table S4: Post-reform Comparison

<table>
<thead>
<tr>
<th></th>
<th>Empirical</th>
<th>Predicted</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta u_i$</td>
<td>$\Delta m_{0i}$</td>
</tr>
<tr>
<td>single men</td>
<td>-0.050</td>
<td>-</td>
</tr>
<tr>
<td>married men, no kids</td>
<td>-0.019</td>
<td>-</td>
</tr>
<tr>
<td>married men, kids</td>
<td>-0.031</td>
<td>-</td>
</tr>
<tr>
<td>single women</td>
<td>-0.018</td>
<td>0.005</td>
</tr>
<tr>
<td>single parents</td>
<td>-0.065</td>
<td>0.031</td>
</tr>
<tr>
<td>married women, no kids</td>
<td>-0.023</td>
<td>-0.012</td>
</tr>
<tr>
<td>married women, kids</td>
<td>-0.034</td>
<td>0.007</td>
</tr>
</tbody>
</table>

Notes: All employment responses are expressed in percentage points. Changes may not sum to zero due to rounding. The direct impact considers all changes to the tax and transfer system between April 1997 and April 2002, holding the wage offer distributions and arrival rates at their pre-reform levels. The equilibrium impact allows the wage offer distribution and arrival rates to change.

For a part-time job ($h = h_0$) we have:

\[
\rho V^0_{el}(w) = wh_0 - T^0_{el}(wh_0) + \lambda^0_{el} \mathbb{E}_{x \sim F_0} \max \left\{ V^0_{el}(x) - V^0_{el}(w), 0 \right\} \\
+ \lambda^1_{el} \mathbb{E}_{x \sim F_1} \max \left\{ V^1_{el}(x) - V^0_{el}(w), 0 \right\} + \delta_i (V_{ui} - V^0_{el}(w))
\]

and for workers employed in a full-time job ($h = h_1$):

\[
\rho V^1_{el}(w) = wh_1 - T^1_{el}(wh_1) - C^1_i + \lambda^0_{el} \mathbb{E}_{x \sim F_0} \max \left\{ V^0_{el}(x) - V^1_{el}(w), 0 \right\} \\
+ \lambda^1_{el} \mathbb{E}_{x \sim F_1} \max \left\{ V^1_{el}(x) - V^0_{el}(w), 0 \right\} + \delta_i (V_{ui} - V^1_{el}(w)).
\]

Recalling that $q_i(w)$ is defined such that $V^1_{el}(w) = V^0_{el}(q_i(w))$, it follows that the value of a full-time job at wage $w$ may be written as:

\[
\rho V^1_{el}(w) = wh_1 - T^1_{el}(wh_1) - C^1_i + \lambda^0_{el} \int_{q_i(w)}^{\bar{w}} (V^0_{el}(x) - V^1_{el}(w)) dF_0(x) \\
+ \lambda^1_{el} \int_{w}^{\bar{w}} (V^1_{el}(x) - V^1_{el}(w)) dF_1(x) + \delta_i (V_{ui} - V^1_{el}(w)).
\]

(S2)
Equating equation $S_2$ (evaluated at wage $w$) with the analogous expression for part-time employment (evaluated at wage $q_i(w)$) implies that $q_i(w)$ is the solution to:

$$wh_1 - T_i^1(wh_1) - C_i^1 = q_i(w)h_0 - T_i^0(q_i(w)h_0)$$  \hspace{1cm} (S3)

which is equation 1 from the main text. We obtain this simple expression because, conditional on being in employment, both the destruction rate and arrival rates for full-time and part-time jobs are assumed independent of work hours so that it is only necessary to compare the instantaneous utility flows. Note also that:

$$q'_i(w) = \frac{h_1}{h_0} \cdot \frac{1 - T_i^1(wh_1)}{1 - T_i^0(q_i(w)h_0)}.$$  \hspace{1cm} (S4)

To calculate the reservation wage we proceed by first performing integration by parts on equation $S_2$ to obtain:

$$\rho V_{ei}^1(w) = wh_1 - T_i^1(wh_1) - C_i^1 + \lambda_{ei}^0 \int_{q_i(w)}^{q_i(\phi_i)} F_0(x) dV_{ei}^0(x) + \lambda_{ei}^1 \int_{q_i(\phi_i)}^{\phi_i} F_1(x) dV_{ei}^1(x) + \delta_i(V_{ui} - V_{ei}^1(w))$$  \hspace{1cm} (S4)

which when differentiated with respect to $w$ yields:

$$h_1(1 - T_i^1(wh_1)) = (\delta_i + \rho + \lambda_{ei}^0 F_0(q_i(w)) + \lambda_{ei}^1 F_1(w))V_{ei}^1(w).$$  \hspace{1cm} (S5)

We denote $\phi_i$ as the lowest acceptable wage offer for full-time work. Since $V_{ui} = V_{ei}^1(\phi_i) = V_{ei}^0(q_i(\phi_i))$, the lowest acceptable wage offer for part-time work is then $q_i(\phi_i)$. Using this, we can write equation $S_1$ as:

$$\rho V_{ui} = b - T_i^u + \lambda_{ui}^0 \int_{q_i(\phi_i)}^{\phi_i} T_0(w) dV_{ei}^0(w) + \lambda_{ui}^1 \int_{q_i(\phi_i)}^{\phi_i} T_1(w) dV_{ei}^1(w).$$
Substituting the envelope conditions \( V_{ei}^1(w) \) and \( V_{ei}^0(w) = V_{ei}^1(q^{-1}(w))/q'_i(w) \) in to the above:

\[
\rho V_{ui} = b - T_i^u + \lambda_{ui}^0 \int_{q_i(\phi_i)}^{\bar{w}_0} \frac{h_0(1 - T_i^{0r}(wh_0)) \bar{F}_0(w)}{\delta_i + \rho + \lambda_{ei}^0 \bar{F}_0(w) + \lambda_{ei}^1 \bar{F}_1(q^{-1}_i(w))} \, dw \\
+ \lambda_{ui}^1 \int_{\phi_i}^{\bar{w}_1} \frac{h_1(1 - T_i^{1r}(wh_1)) \bar{F}_1(w)}{\delta_i + \rho + \lambda_{ei}^0 \bar{F}_0(q_i(w)) + \lambda_{ei}^1 \bar{F}_1(w)} \, dw.
\]

By definition of the reservation wage we can set the above equal to \( \rho V_{ei}^1(\phi_i) \) (from equation S2) to obtain the following implicit equation defining \( \phi_i \):

\[
\phi_i h_1 - T_i^1(\phi_i h_1) - C_i^1 = b - T_i^u + (\lambda_{ui}^0 - \lambda_{ei}^0) \int_{q_i(\phi_i)}^{\bar{w}_0} \frac{h_0(1 - T_i^{0r}(wh_0)) \bar{F}_0(w)}{\delta_i + \rho + \lambda_{ei}^0 \bar{F}_0(w) + \lambda_{ei}^1 \bar{F}_1(q^{-1}_i(w))} \, dw \\
+ (\lambda_{ui}^1 - \lambda_{ei}^1) \int_{\phi_i}^{\bar{w}_1} \frac{h_1(1 - T_i^{1r}(wh_1)) \bar{F}_1(w)}{\delta_i + \rho + \lambda_{ei}^0 \bar{F}_0(q_i(w)) + \lambda_{ei}^1 \bar{F}_1(w)} \, dw.
\]

Dividing both the numerator and denominator of the integral terms by \( \delta_i \), and performing a simple change of variable, we then obtain the simplified expression presented in equation 2 in the main text.

C Proof of Proposition 4

The number of individuals who exit either a full-time job paying a wage no-greater than \( w \) or a part-time job paying wage no-greater than \( q_i(w) \) must exactly equal the number of individuals who exit the unemployment pool to receive such wages. That is:

\[
[m_{1i} G_{1i}(w) + m_{0i} G_{0i}(q_i(w))] D_{ei}(w) \\
= \lambda_{ui}^0 u_i \int_{-\infty}^{w} (F_0(q_i(w)) - F_0(q_i(x))) \, dA_{ui}(x) + \lambda_{ui}^1 u_i \int_{-\infty}^{w} (F_1(w) - F_1(x)) \, dA_{ui}(x). \quad (S6)
\]

The RHS of equation S6 may be written as:

\[
\lambda_{ui}^0 u_i \int_{-\infty}^{w} \bar{F}_0(q_i(x)) \, dA_{ui}(x) + \lambda_{ui}^1 u_i \int_{-\infty}^{w} \bar{F}_1(x) \, dA_{ui}(x) \\
- \left[ \lambda_{ui}^0 \bar{F}_0(q_i(w)) + \lambda_{ui}^1 \bar{F}_1(w) \right] u_i A_{ui}(w),
\]
or equivalently by using equations 3 and 4 from the main text as,

$$
\delta_i A_i(w) - u_i A_{ui}(w) [\delta_i + D_{ui}(w)].
$$

Setting equation S7 equal to the LHS of equation S6 and rearranging terms, we then obtain the form presented in equation 7 from the main text.

## D Proof of Proposition 8

Under the assumption that $c_h(p, v) = c_h(p)v^2$, the equilibrium of the model is invariant to the parametrization of $c_h(p)$ provided that $c_h(p) > 0$ for all $p$. To understand this invariance result let us consider the pair of recruiting cost functions $c_h(p)$ and $\tilde{c}_h(p)$ for $h \in \{0, 1\}$. In each case, we recover the distribution of firm productivity and underlying search intensities such that under the base system we induce the estimated set of arrival rates and wage offer distributions. To demonstrate this result, we show that given a set of job offer arrival rates the optimal choice of wage policy functions and distribution of wage offers are the same. We then show that given the optimal choice of recruiting intensity, both cost functions generate the same equilibrium job offer arrival rates.

First note that in order to induce the same distributions of wage offers under the base system, the distributions of firm productivity under the alternative cost function must satisfy:

$$
\tilde{\gamma}_h(p) = \frac{\gamma_h(p)\tilde{c}_h(p)V_{h,b}^2}{c_h(p)V_{h,b}^2}.
$$

In equation S8 and in what follows, we denote objects under the alternative cost function $\tilde{c}_h(p)$ with a tilde. Similarly, $V_{h,b}$ and $\tilde{V}_{h,b}$ are used to denote the respective aggregate recruiting intensities under the base systems.

We now demonstrate that we obtain the same distribution of wage offers following the tax reform under $\tilde{c}_h(p)$ with the alternative recruiting policy function $\tilde{\vartheta}_h(p)$ as given by:

$$
\tilde{\vartheta}_h(p) = \frac{c_h(p)v_h(p)V_h}{\tilde{c}_h(p)\tilde{v}_h(p)}.
$$

This recruiting policy function is consistent with the same expected profit flow per unit intensity under the alternative cost functions (see equation 13 from the main text). This choice of recruiting policy also induces the same wage offer distributions. Using equa-
with the final equality following from the observation that:

\[
\tilde{V}_h = \int_{p_h}^{\tilde{p}_h} \tilde{c}_h(p) \, d\tilde{\Gamma}_h(p) = \int_{p_h}^{\tilde{p}_h} c_h(p) v_h(p) V_h \, d\tilde{\Gamma}_h(p) = \left( \frac{V_h}{\tilde{V}_h} \cdot \frac{V_{h,b}^2}{V_{h,b}^2} \right) \int_{p_h}^{\tilde{p}_h} \tilde{v}_h(p) \, d\Gamma_h(p),
\]

so that the aggregate recruiting intensity ratio in the reform and base systems is the same irrespective of the cost function parametrization:

\[
\frac{V_h}{V_{h,b}} = \frac{\tilde{V}_h}{\tilde{V}_{h,b}}. \tag{S11}
\]

Finally, we verify that the set of job offer arrival rates are the same by showing that the set of search intensities that would induce these arrival rates do not change between the base and reform systems. Note that the arrival rates must satisfy:

\[
\lambda_{ji}^h = s_{ji}^h \left[ \frac{V_h}{M_h} \right] \frac{\tilde{s}_{ji}^h}{s_{ji}^h}
\]

and that if the set of arrival rates are the same then so is the flow of matches. Thus, we obtain the same set of arrival rates with the alternative cost functions if:

\[
\frac{s_{ji}^h}{\tilde{s}_{ji}^h} = \left[ \frac{V_h}{\tilde{V}_h} \right] \frac{\tilde{s}_{ji}^h}{s_{ji}^h}.
\]

From equation S11 we know that \( V_h/\tilde{V}_h = V_{h,b}/\tilde{V}_{h,b} \) so that there is no change in this ratio between the base and reform systems.
E Identification

In Section 4.2 we discussed the identification of our model, and we now illustrate these ideas more formally. Here we set out to show that conditional on the set of transitional parameters, the observed distributions of part-time and full-time wages, together with the distributions of wages accepted by the unemployed are sufficient to separately identify the wage offer and reservation wage distributions if the labour market is in steady state.\(^{39}\) Once these are known, the structure of the model then permits identification of the opportunity cost and productivity distributions. In what follows, we let \(G_{\text{U}1}^i\) and \(G_{\text{U}0}^i\) denote the respective cumulative distribution functions of wages first accepted by type \(i\) unemployed workers in full-time and part-time jobs. Since individuals will accept any wage offer that is at least as high as their reservation wage, \(G_{\text{U}1}^i\) will be given by:

\[
G_{\text{U}1}^i(w) = \int_{-\infty}^{w} \Pr(W_1 < w | W_1 > x) \, dA_{ui}(x) = \int_{-\infty}^{w} \frac{F_1(w) - F_1(x)}{F_1(x)} \, dA_{ui}(x) = A_{ui}(w) - \mathbb{P}_1(w) \left[ \int_{w}^{\infty} \frac{dA_{ui}(x)}{F_1(x)} + A_{ui}(w) \right].
\]

Similarly the fraction of part-time jobs accepted that pay no more than \(q_i(w)\) can be shown to be given by:

\[
G_{\text{U}0}^i(q_i(w)) = A_{ui}(w) - \mathbb{P}_0(q_i(w)) \left[ \int_{w}^{\infty} \frac{dA_{ui}(x)}{F_0(q_i(x))} + A_{ui}(w) \right].
\]

If we combine the above two expressions with the respective density functions of accepted wages \(g_{hii}^U \equiv G_{hii}^U\) we can write:

\[
A_{ui}(w; F_0) = G_{\text{U}0}^i(q_i(w)) + \frac{\mathbb{P}_0(q_i(w))g_{0i}^U(q_i(w))}{f_0(q_i(w))}
\] (S12)

and:

\[
A_{ui}(w; F_1) = G_{\text{U}1}^i(w) + \frac{\mathbb{P}_1(w)g_{1i}^U(w)}{f_1(w)},
\] (S13)

\(^{39}\)The argument that follows is subject to the usual caveat that the distribution of reservation wages is only non-parametrically identified on the support of observed wages. In our empirical application (see Section 4 in the main text) we assume a parametric distribution of leisure flows, and therefore require that this distribution is recoverable in the sense of Flinn and Heckman (1982).
which therefore demonstrates that the distribution of reservation wages amongst the unemployed on support \( \omega_i \) is identified given knowledge of the wage offer functions \( F_0 \) and \( F_1 \). The monetary disutility of full-time work \( C^1_{i} \) is identified by observing how the job-to-job transitions across hours sectors varies with the current wage rate.

Substituting equations S12 and S13 into equation 6 from the main text, we can eliminate the unobserved reservation wage distribution to obtain the following differential equations governing the evolution of the two wage offer distributions:

\[
F_1'(w) = m_{1i}g_{1i}(w)R^1_1(w; F_0, F_1) \tag{S14}
\]

\[
F_0'(q_i(w)) = m_{0i}g_{0i}(q_i(w))R^0_1(w; F_0, F_1) \tag{S15}
\]

where:

\[
R^1_i(w; F_0, F_1) \equiv \frac{(1 + \kappa^0_{ci}T_0(q_i(w)) + \kappa^1_{ci}T_1(w)) - u_iG^U_{1i}(w)\kappa^1_{ui}T_1(w)}{\kappa^1_{ui}u_iG^U_{1i}(w) + \kappa^1_{ci}(m_{0i}G_{0i}(q_i(w)) + m_{1i}G_{1i}(w))}
\]

and:

\[
R^0_i(w; F_0, F_1) \equiv \frac{(1 + \kappa^0_{ci}T_0(q_i(w)) + \kappa^1_{ci}T_1(w)) - u_iG^U_{0i}(q_i(w))\kappa^0_{ui}T_0(q_i(w))}{\kappa^0_{ui}u_iG^U_{0i}(q_i(w)) + \kappa^0_{ci}(m_{0i}G_{0i}(q_i(w)) + m_{1i}G_{1i}(w))}.
\]

Equations S14 and S15 define a system of differential equations, which together with the initial conditions \( F_1(\omega_i) = 0 \) and \( F_0(q_i(\omega_i)) = 0 \), establishes non-parametric identification of both wage offer functions conditional on the set of transitional parameters. Identification of the underlying opportunity cost distribution and the productivity distributions then follows as described in Section 4.3.

### F Tax Schedule Smoothing

Conditional on hours of work \( h \), we remove small discontinuities in the budget constraint by setting the minimum transfer amount to zero for all benefits. We also remove the National Insurance (payroll tax) entry fee that existed prior to April 1999. With \( K \) tax brackets, the marginal tax rate approximation at hours \( h \) and earnings \( wh \) for an individual of demographic type \( i \) is given by:

\[
MTR^h_i(wh) = \sum_{k=1}^{K} \left[ \Phi^h_{i,k}(wh) - \Phi^h_{i,k+1}(wh) \right] x^h_{i,k}(wh),
\]
where $\tau_{i,k}^h$ is the marginal tax rate at the $k$th bracket and $\Phi_{i,k}^h$ is the normal cumulative distribution function with a mean equal to the value of the $k$th tax bracket and with standard deviation $\sigma_{i,k}^h$. The value of $\sigma_{i,k}^h$ determines how quickly the smoothed marginal tax rates $MTR_{i,k}^h$ change in the neighbourhood of the break points, with a small value fitting the underlying step function more closely. We set $\sigma_{i,k}^h = 20$ (measured in pounds per week in April 1997 prices), although our results are not sensitive to this particular choice.

G Numerical Algorithm

We now sketch the algorithm that we use to solve for the steady state equilibrium:

1. We first provide an initial guess of the recruiting policy functions $v_h(p)$ and unemployment shares $u_i$ to obtain the total recruiting intensity $V_h$ and total search intensity $S_h$. Given a parametrization of the matching functions, we can use equation 17 from the main text to obtain a set of arrival rates $\lambda_{i,j}^h$.

2. Conditional upon the guesses from step 1, we solve for the optimal wage policy of firms $w_h(p)$. To do this we first provide an initial guess of the wage policy functions (with $w_h(p) < p$ strictly increasing in $p$), and using the relationship between the (weighted) distribution of firm productivity and wage offers in equation 18, solve for the steady state of the labour market using the flow equations presented in Section 3 from the main text. To obtain an update of $\pi_h^*(p_h)$ we calculate $\bar{w}_h = \arg \max_w (p_h - w)L_h(w)$ and then set $\pi_h^*(p_h) = (p_h - \bar{w}_h)L_h(\bar{w})$. We then obtain an update of the wage policy functions by using equation 15 from the main text. We continue to iterate until (conditional on our guess of the recruiting policy functions and arrival rates) the wage policy functions of both part-time and full-time firms converge.

3. Given the wage policy functions from step 2, we update our guess of the recruiting policy functions by using equation 13 from the main text. We then return to step 1 above, with these updated recruiting policy functions (together with the implied unemployment rates $u_i$ from step 2) replacing our initial guesses. We keep repeating this iterative procedure until the recruiting policy functions of both part-time and full-time firms has converged.