Physics 362  
Midterm Exam  

February 14, 2012

This exam contains three problems to be completed in less than 75 minutes. Write your solutions, clearly identifying your answers in your white books. A formula sheet containing important results from lecture is provided.

1. **Vector Potential for a Steady Current Distribution**

A cylinder of radius $a$ contains a current distribution $\vec{J}$ that produces a vector potential $\vec{A} = \alpha s^3 \hat{e}_\phi$ where $s$ is the perpendicular distance from the centerline of the cylinder. There is no current outside this cylinder.

(a) Find the magnetic field $\vec{B}$ in the region $s < a$.

(b) Find the current density $\vec{J}$ in the region $s < a$.

(c) Find the flux of $\vec{B}$ through a circular cross section of the cylinder.

(d) Find the flux of $\vec{J}$ through a circular cross section of the cylinder.

2. **Fields and Currents for a Steady Current Distribution**

A thick walled cylinder with inner radius $a$ and outer radius $b$ has a uniform volume charge density $\rho$ in the region $a < s < b$ and spins at angular velocity $\vec{\omega}$. You can regard the cylinder as long and thin so that the exterior magnetic field ($s > b$) is approximately zero.

(a) Find the volume current density $\vec{J}$ in the region $a < s < b$ for this system.

(b) Find the magnetic field in the region $s < a$.

(c) Find the magnetic field in the region $a < s < b$.

(d) Find the jump discontinuity $B_z(b + \epsilon) - B_z(b - \epsilon)$ at the surface of the cylinder.

![Figure 1: A thick-walled cylinder with inner radius $a$ and outer radius $b$ carries a uniform volume charge density (shaded region) and spins at angular velocity $\omega$.](image)

(a) Find the volume current density $\vec{J}$ in the region $a < s < b$ for this system.

(b) Find the magnetic field in the region $s < a$.

(c) Find the magnetic field in the region $a < s < b$.

(d) Find the jump discontinuity $B_z(b + \epsilon) - B_z(b - \epsilon)$ at the surface of the cylinder.
3. Dipole-dipole interactions

Two dipoles are clamped in the positions shown below: \( \vec{m}_1 \) is at the origin and directed along the positive \( z \) direction and \( \vec{m}_2 \) is at position \( \vec{R} = y \hat{e}_y \) and directed along the positive \( x \) direction.

![Figure 2: Two dipoles with perpendicular polarizations separated by \( \vec{R} = y \hat{e}_y \).](image)

(a) The field produced by dipole \( \vec{m}_1 \) as seen by \( \vec{m}_2 \) at position \( \vec{R} \) can be resolved in spherical coordinates in the manner \( \vec{B} = B_r \hat{e}_r + B_\theta \hat{e}_\theta + B_\phi \hat{e}_\phi \). Find the three amplitudes \( B_r, B_\theta \) and \( B_\phi \) in this expression.

(b) Similarly the dipole moment \( \vec{m}_2 \) can be resolved in spherical coordinates \( \vec{m}_2 = m_2,r \hat{e}_r + m_2,\theta \hat{e}_\theta + m_2,\phi \hat{e}_\phi \). Find the three amplitudes \( m_2,r, m_2,\theta \) and \( m_2,\phi \).

(c) Find the magnetostatic force \( \vec{F}_2 \) acting on \( \vec{m}_2 \).

(d) Find the torque \( \vec{N}_2 \) on dipole 2 about its center.