This exam contains three problems to be completed in less than 75 minutes. Write your solutions, clearly identifying your answers in your white books. A formula sheet containing important results from lecture is provided.

1. **Boundary Conditions for $\vec{B}$ (20 points)**

   The magnetic field of a current distribution has the following form

   $$
   \vec{B}^> = (B^o + \alpha z) \hat{y} \text{ for } z > 0 \\
   \vec{B}^< = (B^o + \alpha z) \hat{x} \text{ for } z < 0
   $$

   where $B^o$ and $\alpha$ are constants.

   (a) Find the volume current densities $\vec{J}^>$ and $\vec{J}^<$ in the regions $z > 0$ and $z < 0$.

   (b) Find the surface current density $\vec{K}$ in the $xy$ plane.

2. **Vector Potential for a Spatially Varying Field (20 points)**

   An azimuthally symmetric magnetic field in a region of space has the form

   $$
   \vec{B}(s, z) = B_s(s, z) \hat{z} + (c_0 + c_2 z^2) \hat{z}
   $$

   with nonzero constant coefficients $c_0$ and $c_2$. Here $c_2 < 0$ describes a splay-field configuration while $c_2 > 0$ describes a pinch-field configuration as shown below. Note that in both cases $B_s(s = 0, z) = 0$.

   ![Figure 1](image)

   (a) Find the $s$-component of the field $B_s(s, z)$.

   (b) The vector potential for this system can be written $\vec{A} = A_\phi \hat{\phi}$. Find an expression for the vector potential $A_\phi(s, z)$ (note that we are seeking a solution in a gauge where $A_\phi$ is not a function of $\phi$).
3. Dipole Fields (20 points)

Two current loops, each with current $I$ and radius $a$ are clamped with their centers at the origin. Current loop “1” has its normal directed along the $\hat{e}_z$ direction while current loop “2” has its normal directed along the $\hat{e}_y$ direction as shown. We work in the far field limit where the distance to the observer $r >> a$.

![Diagram of two circular current loops centered at the origin with the plane of the loop perpendicular to the z and y directions.](image)

Figure 2: Two circular current loops are centered at the origin with the plane of the loop perpendicular to the z and y directions.

(a) Find the magnetic field (magnitude and direction) at a point $P$ a distance $z$ along the $z$ axis expressing your result by resolving the field $\vec{B} = B_x \hat{e}_x + B_y \hat{e}_y + B_z \hat{e}_z$.

(b) Find the direction towards an observer $S$ where the total magnetic field $\vec{B}$ is purely radial, expressing your result by giving the angular coordinates $(\theta, \phi)$ of the observer.