Lab Session: Using FiPIt to Solve a Model of Macroprudential Policy

(application to Bianchi-Liu-Mendoza (2016))

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Introduction

- This presentation covers the algorithm Bianchi, Liu & Mendoza (JIE, 2016) used to solve a macroprudential policy model with:
  - Regime Switching in global liquidity.
  - Noisy news about future income.
  - DTI occasionally binding constraint.
- Matlab codes are provided together with a handout (step-by-step guide) and a "readme" file listig the codes
- This presentation focuses mainly `MPPsolve2new.m,` which is the main code for solving the model.
- `MPPsolve2new.m` is divided into seven sections (cells).

Section 1

MPPsolve2new.m

Cell 1: Model & algorithm parameters

- Sets parameter values, creates state space, and sets values for algorithm parameters controlling execution.
- 100 nodes in the grid for bonds.
- 3 nodes for tradable output y^T shocks
- 3 nodes for news shocks: good, average and bad.
- 2 nodes for interest rates shocks: high, low.
- $iter\_tol$: maximum number of decision rule iterations
- tol: tolerance for convergence criterion of decision rules
- tol_EEbind: tolerance for solution of nonlinear equation

Baseline Parameterization

Parameter	Value	Parameter	Value
y^N	1	N_{y^T}	3
$E[y^T]$	1	ρ_{y^T}	0.54
σ_{y^T}	0.059	β	0.91
γ	2	η	0.205
κ_L	0.32	ω	0.32
θ	$\frac{2}{3}$	R^h	1.0145
R^l	0.9672	F_{hh}	0.9333
F_{ll}	0.6		

I) Discretize y^T shocks using Tauchen and Hussey's method.

- Implemented in function `tauchenhussey.m`
- Time-series properties of the y^T process are estimates obtained by Bianchi (2011) for Argentina, corresponding moments are reported in Table 5.

II) Incorporate news shocks according to the formulas in the Section 2.3 of the paper. Recall by Bayes rule:

$$p(y^T_{t+1} = l | s_t = i, y^T_t = j) = \frac{p(s_t = i | y^T_{t+1} = l) p(y^T_{t+1} = l | y^T_t = j)}{\sum_n p(s_t = i | y^T_{t+1} = n) p(y^T_{t+1} = n | y^T_t = j)} \tag{1}$$

Cell 2: Construction of Markov Chain

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Cell 2: Construction of Markov Chain

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II) (cont'd) Hence we can write:

$$
\begin{aligned}
\Pi(y^T_{t+1}, s_{t+1}, y^T_t, s_t) &\equiv p(s_{t+1} = k, y^T_{t+1} = l | s_t = i, y^T_t = j) \\
&= p(y^T_{t+1} = l | s_t = i, y^T_t = j) p(s_{t+1} = k | y^T_{t+1} = l) \\
&= p(y^T_{t+1} = l | s_t = i, y^T_t = j) \times \ldots \\
&\quad \ldots \sum_m \left[p(y^T_{t+2} = m | y^T_{t+1} = l) p(s_{t+1} = k | y^T_{t+2} = m) \right]
\end{aligned} \tag{2}
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III) Finally, add global liquidity shocks to construct entire transition matrix, assuming y^T shocks and liquidity shocks are independent.

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&= p(y^T_{t+1} = l | s_t = i, y^T_t = j) \times \dots \\
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iii) Finally, add global liquidity shocks to construct entire transition matrix, assuming y^T shocks and liquidity shocks are independent.

Cell 3: Decentralized Equilibrium (DE)

- Solves DE using FiPIt algorithm
- Iterate on $c^T(b,z)$ and $B(b,z)$ that satisfy:

$$P^N(b,z) = \frac{1-\omega}{\omega}\left(\frac{c^T(b,z)}{y^N}\right)^{1+\eta} \tag{3}$$

$$u_T(c^T(b,z),y^N) \geq \beta R(z) E_z[u_T(c^T(B(b,z),z'),y^N)] \tag{4}$$

$$B(b,z) \geq -\kappa R(z)(P^N(b,z)y^N + y^T(z)) \tag{5}$$

$$c^T(b,z) + q(z)B(b,z) = b + y^T(z) \tag{6}$$

- z: triple (y^T, s, q) includes realizations of the three shocks (endowment y^T, the news signal s, and bond price q).

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- z: triple (y^T, s, q) includes realizations of the three shocks (endowment y^T, the news signal s, and bond price q).

- Subscript "j" is index for iterations. Then construct the fixed-point iteration loop as follows:

Step 1: Define initial decision rule conjectures (for $j = 1$)

- **If the constraint does not bind**:

$$B_1(b, z) = b$$

$$c_1^T(b, z) = b + y^T(z) - (1/R(z))B_1(b, z) \quad \text{from eq. (6)},$$

$$p_1^N(b, z) = \frac{1-\omega}{\omega}\left(\frac{c_1^T(b, z)}{y^N}\right)^{1+\eta} \quad \text{from eq. (3)}$$

- **If the constraint binds** at price set above:

$$B_1^{max}(b, z) = -\kappa R(z)(p_1^N(b, z)y^N + y^T(z)) \quad \text{from eq. (5)},$$

$$c_1^{bind,T}(b, z) = b + y^T(z) - (1/R(z))B_1^{max}(b, z) \quad \text{from eq. (6)},$$

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Cell 3: Decentralized Equilibrium

Step 2: Start FiPIt loop, construct $u'_j(b, z)$ and $E[u'_j(B_j(b, z), z')]$

Step 3: Compute Euler Equation gap assuming constraint binds at t:

$$EE(b, z) \equiv u_T(c_{j+1}^{bind,T}(b, z), y^N) - \beta R(z) E_z \left[u_T(c_j^T(B_j(b, z), z'), y^N) \right]$$

- This is the multiplier μ on the credit constraint if it does bind.
- Linear interpolation of $u_T(c_j^T(B_j(b, z), z'), y^N)$ over $B_j(b, z)$

Step 4: If $EE(b, z) > tol\_EEbind$, the constraint binds and new decision rules are set to $B_{j+1} = B_j^{max}(b, z)$ and $c_{j+1}^T = c_j^{bind,T}(b, z)$

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Step 5: If $EE(b,z) \leq tol\_EEbind$, the constraint does not bind.

- Find $c^T_{j+1}(b,z)$ by solving Euler equation (4) with equality.
  - `fzero` nonlinear solver
  - Linear interpolation of $u_T(B_j(b,z), z', y^N)$ over $B_j(b,z)$ in RHS of Euler eq.
  - Same as *FiPIt* but needs nonlinear solver because of $CES(c^T, c^N)$
  - Could be avoided using duality $u_T(c^T, c^N) = u'(CES)/P^c(\cdot)$
- Compute $B_{j+1}(b,z)$ using resource constraint (6) and making sure is inside bonds grid.
- Compute $p^N_{j+1}(b,z)$ using equation (3).

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Step 6: Execute Steps 1-5 for all (b, z), which yields new functions

$$c^T_{j+1}(b, z) \qquad B_{j+1}(b, z)$$

that will generally differ from conjectures $c^T_j(b, z)$, $B_j(b, z)$.

- Check convergence criterion:

$$d2 = \sup |x_{j+1} - x_j| \leq \text{tol, for } x \in \{B, c\}.$$

- If it fails, update conjectures as convex combinations of new solutions $c^T_{j+1}(\cdot)$, $B_{j+1}(\cdot)$ and initial conjectures $c^T_j(\cdot)$,$B_j(\cdot)$, and return to Step 1.
- If it holds, a solution to the DE in recursive form has been found.

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Cell 4: Social Planner's Problem (SP)

- Code similar to DE but with the difference that marginal benefit of c_t^T is not just $u_T(t)$ but adds to it the term $\mu_t^{SP}\psi_t$
  - $\mu^{SP} \geq 0$, with > 0 if collateral constraint (5) binds.
  - ψ is the externality term given by

$$\psi(b,z) = \kappa(\eta+1)\left(\frac{1-\omega}{\omega}\right)\left(\frac{c_j^T(b,z), y^N)}{y^N}\right)^{\eta} \tag{7}$$

  - Start conjectures at DE solution
- SP's Euler equation gap:

$$EESP \equiv \frac{u_T(c_{j+1}^{bind,T}(b,z), y^N) - \beta R(z) E_z\left[u_T(c_j^T(B_j(b,z),z'),y^N)\right]}{1-\psi(b,z)}$$

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Cell 5: Welfare gain of optimal policy

- Calculate value functions for DE and SP as recursive discounted sums of expected lifetime utility flows of each case:
    1. Start with a conjecture $V^K(b,z)$.
    2. Take the optimal policy functions c^T, B derived from cells 3 for DE and 4 for SP.
    3. For each state find the new value function:

$$V^{K+1}(b,z) = u\big(c^T(b,z)\big) + \beta\,\mathrm{E}\left[V^K\big(B(b,z),z'\big)|z\right]$$

    4. Continue until $|V^{K+1}(b,z) - V^K(b,z)| \leq tol$

- Calculate welfare gain as compensating variation in CES consumption that equates lifetime utility in DE and SP:

$$\big(V^{SP}(b,z))/V^{DE}(b,z)\big)^{\frac{1}{1-\sigma}} - 1$$

Cell 6: Optimal Macroprudential Tax

- Assume financial regulator imposes a debt tax τ_t.
- The Euler equation in the Decentralized Equilibrium would be:

$$u_T(t) - \mu_t^{DE} = (1+\tau_t)\frac{\beta}{q_t}\mathbb{E}_t\left[u_T(t+1)\right] \tag{8}$$

- The optimal taxes should yield the SP's allocations as the DE eq. with taxes (i.e., make SP's allocations solve equation (8)).
- The planner's allocations solve the SP's Euler Equation:

$$u_T(t) + \mu_t^{SP}(\psi_t + 1) = \frac{\beta}{q_t}\mathbb{E}_t\left[u_T(t+1) + \mu_{t+1}^{SP}\psi_{t+1}\right]. \tag{9}$$

- The tax policy should make those two equations the same.

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Cell 6: Optimal Macroprudential Tax

- Recall we are interested in the optimal *macroprudential* tax (in good times, when $\mu_t^{DE} = 0$)
- Multipliers μ_t^{DE} and μ_t^{SP} do not need to be equal.
  - But if for some allocation the constraint is not binding in the DE, it won't bind in SP's (since there is overborrowing in DE)
- Optimal macroprudential tax is:

$$1 + \tau_t = \frac{q_t}{\beta} \frac{u_T(t)}{\mathbb{E}_t \left[u_T(t+1) \right]} = 1 + \frac{\mathbb{E}_t \left[\mu_{t+1} \psi_{t+1} \right]}{\mathbb{E}_t \left[u_T(t+1) \right]} \tag{10}$$

  - First equality is obtained from (8).
  - Second one replaces $u_T(t)$ by equation (9) (eq. (23) in the paper).
- When $\mu_t^{DE} > 0$, SP and DE allocations are identical. Set $\tau_t = 0$ if this is consistent with $\mu_t^{DE} > 0$

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  - Second one replaces $u_T(t)$ by equation (9) (eq. (23) in the paper).
- When $\mu_t^{DE} > 0$, SP and DE allocations are identical. Set $\tau_t = 0$ if this is consistent with $\mu_t^{DE} > 0$

Cell 6: Optimal Macroprudential Tax

- Recall we are interested in the optimal *macroprudential* tax (in good times, when $\mu_t^{DE} = 0$)
- Multipliers μ_t^{DE} and μ_t^{SP} do not need to be equal.
  - But if for some allocation the constraint is not binding in the DE, it won't bind in SP's (since there is overborrowing in DE)
- Optimal macroprudential tax is:

$$1 + \tau_t = \frac{q_t}{\beta} \frac{u_T(t)}{\mathbb{E}_t \left[u_T(t+1) \right]} = 1 + \frac{\mathbb{E}_t \left[\mu_{t+1} \psi_{t+1} \right]}{\mathbb{E}_t \left[u_T(t+1) \right]} \tag{10}$$

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Section 2

MPPsimulation.m

University of Pennsylvania

The file `MPPsimulation.m`:

- Loads the results from `MPPsolve2new.m`.
- Simulates the economy recursively
  - Exogenous states (y^T, s, q) are simulated as a Markov chain.
  - Endogenous state b is simulated using the policy function found with `MPPsolve2new.m`.
- Produces unconditional moments of the economy.
- Also moments conditional on Sudden Stops.
- Performs event analysis of Sudden Stops.
- Produces figures.

Cell 1: Initialization

- Preallocates vectors and initial period values for simulations.
- 201,000 periods are simulated.
  - The first 1,000 will be discarded.
- Vectors of size 201,000 for all exogenous and endogenous variables are created.
- The initial state of the economy $t = 1$ also is set.
  - Debt, tradables output, news and bond price: (b_1, y_1^T, s_1, q_1),
- Calls the function `markov.m` which simulates a Markov chain given a transition matrix and a initial state.
  - the transition matrix was built in cell 2 of `MPPsolve2new.m`.
- A sample path for the evolution of the exogenous states is obtained.

Cell 2: Simulation Loop

Makes the recursive loop. Each period:

- Enter with current debt.
- Reads exogenous state from path created by previous cell.
- Use policy function to obtain next period debt.
- Solves all other endogenous variables for the period.

After the simulation is done, the first 1000 periods are discarded to remove potential dependence on the arbitrary first state.

Cell 3: Aggregate Moments

Calculates aggregate moments of this economy and prints results.

- Basically means, variances and covariances of vectors created by the simulations.
- Output used by Table 2 in the paper.

Also defines Sudden Stops:

- Defined as periods where two things occur:
  i. The borrowing constraint binds.
  ii. The current account is more than two standard deviations above its mean.
- Sudden Stops events are identified in the vector `SS`.
- $SS(t) = 1$ if and only if there was a Sudden Stop in period t.

Cell 4: Sudden Stop Moments

- Calculates moments conditional on a Sudden Stop.
  - Means, variances and covariances taking into account only those periods with $SS(t) = 1$.
- Printed output used by Table 3 of the paper.
  - Baseline model (3) with initial configuration $\theta = 0.66$.
  - We can change the news parameter θ to non informative $\theta = 0.33$ or almost perfectly informative $\theta = 0.95$ news to obtain other columns in Table 3.

Cell 5: Sudden Stop Event Analysis

- Identifies Sudden Stop events for analysis.
- Variable `nbd=3` indicates the window:
  - From 3 periods before to 3 periods after the sudden stop.
  - Two periods with a Sudden Stop indicator ($SS(t) = 1$) are considered the same event if they are less than 3 periods apart.
- Variable `nE` has the number of Sudden Stop events.
- Makes matrices of size 7×`nE` for each variable to store its evolution and calculate average evolution paths around Sudden Stop events.

Cell 6: Makes figures for event analysis. Compare to Figures 2, 3 and 4 in the paper.

News Signal Comparison

θ	(1) 0.35		(2) 0.55		(3) 0.66		(4) 0.75		(5) 0.95	
LR Moments	DE	SP	DE	SP	DE	SP	DE	SP	DE	SP
$E[B/Y]$	-28.98	-28.87	-29.13	-28.96	-29.37	-29.15	-29.65	-29.40	-30.81	-30.80
$\sigma(CA/Y)$	0.028	0.011	0.030	0.014	0.032	0.018	0.034	0.021	0.032	0.028
Prob of Crisis	7.96	0.16	4.7	0.72	3.68	2.22	2.77	2.24	0.96	0.72
Welfare Gain	n/a	0.138	n/a	0.132	n/a	0.126	n/a	0.119	n/a	0.06
Financial Crisis Moments										
ΔC	-10.38	-5.92	-12.34	-7.44	-14.28	-9.12	-16.77	-11.42	-19.68	-16.94
ΔRER	-31.99	-17.06	-39.61	-22.14	-47.70	-28.12	-58.76	-36.70	-79.21	-66.73
$\Delta CA/Y$	8.34	2.99	10.99	4.87	13.58	6.91	16.98	9.73	22.74	19.03
$E[\tau]$ pre-crisis	n/a	5.39	n/a	5.47	n/a	5.43	n/a	4.82	n/a	4.75
Switch from $\mathbf{R_l}$ to $\mathbf{R_h}$										
Prob of Crisis	0.126	0.007	0.084	0.033	0.043	0.040	0.035	0.029	0.014	0.009
ΔC	-12.30	-6.76	-13.13	-7.65	-17.68	-11.45	-18.41	-12.65	-19.97	-17.26
ΔRER	-39.33	-19.81	-42.98	-23.00	-62.49	-36.52	-66.72	-41.60	-83.25	-66.05
$\Delta CA/Y$	10.56	3.72	11.70	4.79	17.90	9.40	19.12	11.09	22.97	18.82
$E[\tau]$ pre-crisis	n/a	7.48	n/a	7.46	n/a	7.41	n/a	6.87	n/a	6.58

variables except $\sigma(CA/Y)$ are in percentages. $\theta = 1$ means perfect signal, and $\theta = \frac{1}{3}$ means useless signal