A GENERAL EQUILIBRIUM MODEL OF SOVEREIGN DEFAULT AND BUSINESS CYCLES

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AND

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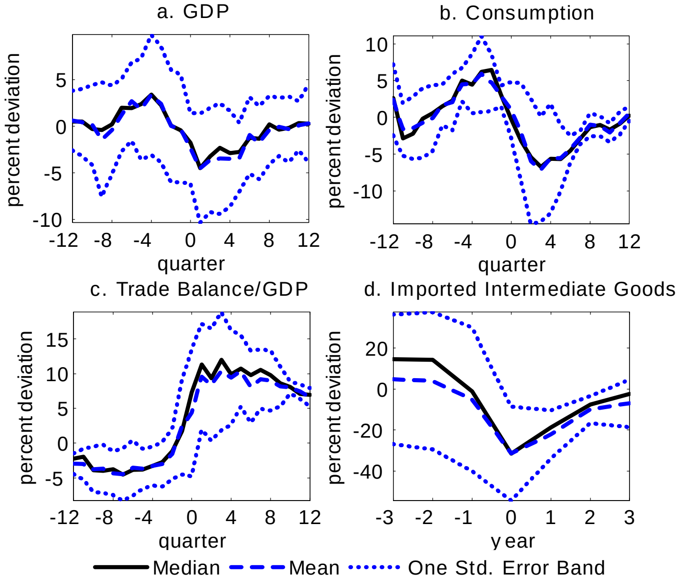
Key Stylized Facts of Sovereign Default

1. V-shaped output dynamics around default episodes
   Deep recessions. Most defaults with output 7% below trend
2. Countercyclical interest rates
   Average correlations between spreads and GDP: -0.5
3. Foreign debt/GDP ratios high on average and before default
   Average: 1/3. After defaults: 2/3

Cyclical Dynamics Around Default Events



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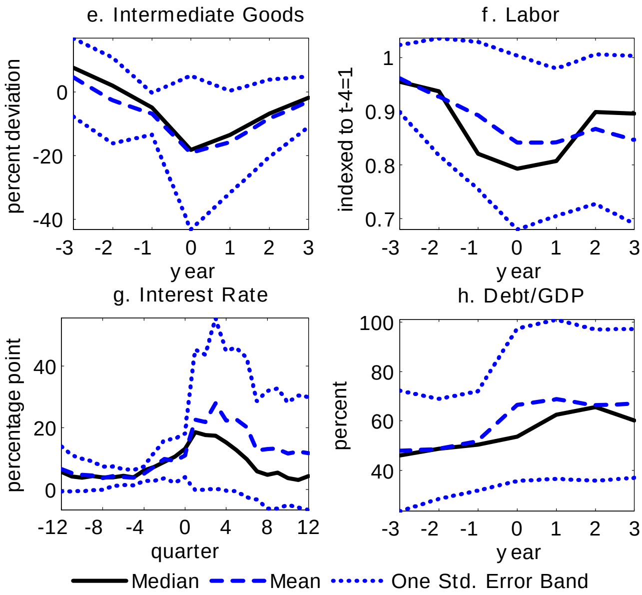


The Disconnect between Default & Business Cycle Theories

- Business cycle models with working capital constraint take as given country interest rates

  Match Fact No. 2 and generate higher output volatility

  ...but country spreads are unexplained

  ...cannot account for Fact No. 1 and No. 3

  ...entire wages bill needs to be paid in advance

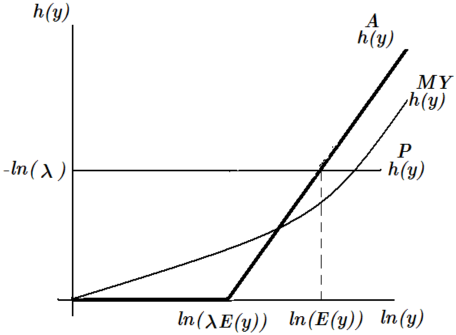
  - Neumeyer & Perri (05), Uribe & Yue (06), Oviedo (05)

The Disconnect between Default & Business Cycle Theories

- Eaton-Gersovitz sovereign default models
  Match Fact No. 2
  ....but output is an endowment with ad-hoc default costs
  ...*cannot explain Fact No. 1*
  ...*cannot account for Fact No. 3 with proportional output cost or Fact No. 1 with asymmetric output cost*
  - Aguiar & Gopinath (06), Arellano (08), Bi (08), D'Erasmo (08), Bai and Zhang (09), Hatchondo, Martinez & Sapriza (09), Arellano & Ramanarayanan (09), Benjamin & Wright (09), Chatterjee & Eyigungor (09), Yue (10), Cuadra, Sanchez & Sapriza (10), Durdu, Nunes & Sapriza (10)...

Percent Output Cost of Default - Comparison

- Proportional cost (Aguiar and Gopinath 2006, Yue 2010): $y_t^{def} = \lambda y_t$.
- Asymmetric cost (Arellano 2008): $y_t^{def} = y_t$ if $y_t \leq \lambda E[y]$; $\quad y_t^{def} = \lambda E[y]$ if $y_t > \lambda E[y]$.



Percentage of output cost of default $h(y_t) = \ln(y_t) - \ln(y_t^{def})$

This Paper

- Default model with endogenous output dynamics
  - Continuum of Imported Input Varieties
  - A fraction of imported inputs requires working capital
  - Domestic inputs are imperfect substitutes and require labor reallocation
  - Default triggers exclusion for government and firms

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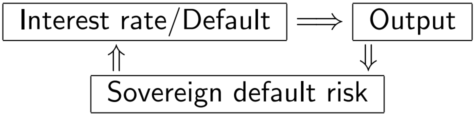

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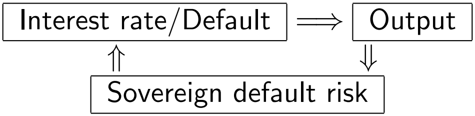

Default causes efficiency loss and an output cost increasing in state of TFP

This Paper

- Quantitative analysis calibrated to Argentina shows that the model produces:
  - Countercyclical spreads and key business cycle statistics
  - Dynamics of GDP and bond spreads around default
  - High debt/GDP ratios on average and at default
  - Strong financial amplification of TFP shocks during default events

Basic Model: Production and Working Capital

- Final goods production technology

$$y = \varepsilon M^{\alpha_m} L_f^{\alpha_l} \overline{k}^{\alpha_k}$$

- Armington aggregator of imported and domestic inputs (imperfect substitutes, $0 < \mu < 1$)

$$M_t = \left[\lambda \left(m_t^d\right)^{\mu} + (1-\lambda)\left(m_t^*\right)^{\mu}\right]^{\frac{1}{\mu}}, \ \ m_t^* \equiv \left[\int_{j\in[0,1]} \left(m_{jt}^*\right)^{\nu} dj\right]^{1/\nu}$$

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- A subset $[0,\theta]$ of imported inputs requires working capital κ borrowed abroad.

$$\frac{\kappa_t}{1+r_t^*} \geq \int_0^{\theta} p_j^* m_j^* \, dj$$

- Domestic intermediate goods do not require working capital but need to be produced hiring domestic labor ($m = AL_m^{\gamma}$).

Producers' Problems

- Competitive producers take all prices and factor costs as given
- Final goods sector

$$\pi_t^f = \varepsilon_t \left(M_t\right)^{\alpha_M} \left(L_t^f\right)^{\alpha_L} k^{\alpha_k} - \int_0^1 p_j^* m_{jt}^* dj - r_t^* \int_0^\theta p_j^* m_{jt}^* dj$$
$$-p_t^m m_t^d - w_t L_t^f .$$

Producers' Problems

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$$
\begin{aligned}
\pi_t^f &= \varepsilon_t \left(M_t\right)^{\alpha_M} \left(L_t^f\right)^{\alpha_L} k^{\alpha_k} - \int_0^1 p_j^* m_{jt}^* dj - r_t^* \int_0^\theta p_j^* m_{jt}^* dj \\
&\quad - p_t^m m_t^d - w_t L_t^f .
\end{aligned}
$$

- The price of the Dixit-Stiglitz aggregator of imported inputs m_t^*

$$
P^*\left(r_t\right) = \left[\int_\theta^1 \left(p_j^*\right)^{\frac{\nu}{\nu-1}} dj + \int_0^\theta \left(p_j^*\left(1+r_t^*\right)\right)^{\frac{\nu}{\nu-1}} dj\right]^{\frac{\nu-1}{\nu}} .
$$

Producers' Problems

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- The price of the Dixit-Stiglitz aggregator of imported inputs m_t^*

$$P^*(r_t) = \left[\int_\theta^1 \left(p_j^*\right)^{\frac{\nu}{\nu-1}} dj + \int_0^\theta \left(p_j^* \left(1 + r_t^*\right)\right)^{\frac{\nu}{\nu-1}} dj\right]^{\frac{\nu-1}{\nu}}.$$

- Intermediate goods sector

$$\pi_m = \max_{Lm} \left[p_m A L_m^\gamma - w L_m\right]$$

Production Optimality Conditions

$$
\begin{aligned}
\varepsilon F_{m^*}\left(m^*, m^d, L_f, \overline{k}\right) &= P_m^*\left(r^*\right) \\
\varepsilon F_{L_f}\left(m^*, m^d, L_f, \overline{k}\right) &= w \\
\varepsilon F_{m^d}\left(m^*, m^d, L_f, \overline{k}\right) &= p_m^d \\
p_m^d \gamma A L_m^{\gamma-1} &= w \\
w &= \omega L^{\omega-1} \\
L &= L_f + L_m \\
m_d &= A L_m^{\gamma}
\end{aligned}
$$

How does Default Cause Efficiency Loss in Production?

- Channels
  - **direct:** demand for m^* falls with default
  - **indirect:** L_f, M fall
  - **general equilibrium**: L falls, L_m, m^d rise or fall depending on gross substitutes or complements

How does Default Cause Efficiency Loss in Production?

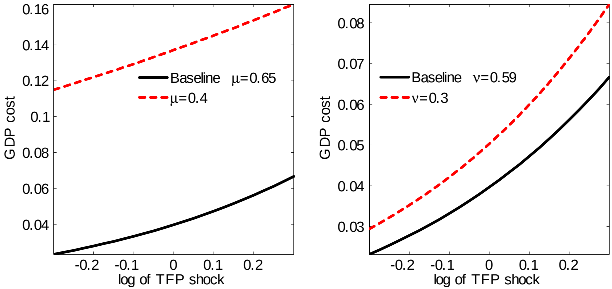
- Channels
  - **direct:** demand for m^* falls with default
  - **indirect:** L_f, M fall
  - **general equilibrium**: L falls, L_m, m^d rise or fall depending on gross substitutes or complements
- At default: firms use only m^d and m_j^*, $j \in [\theta, 1]$, causing efficiency loss because m^d is imperfect substitute.
  - Gopinath and Neiman (2010): evidence of drop in imported inputs within-firm in the Argentine debt crisis

Effect of Default on Equilibrium Factor Allocations

	(I)	(II)	(III)	(IV)	(V)
	Baseline	Threshold η_{m^d,m^*}	Cobb-Douglas Aggregator	High Within-$\eta_{m_j^*}$	Inelastic Labor
η_{m^d,m^*}	2.86,	1.96	1		
$\eta_{m_j^*}$	2.44			10	
M	-11.36%	-21.90%	-40.72%	-3.08%	-9.61%
m^*	-90.64%	-81.59%	-68.21%	-30.38%	-90.46%
m^d	1.73%	0.01%	-13.65%	0.46%	3.73%
L	-2.77%	-7.11%	-19.12%	-0.73%	0.0%
L^f	-6.29%	-11.40%	-19.22%	-1.67%	-3.65%
L^m	2.48%	0.02%	-18.91%	0.65%	5.37%

(percent changes relative to a state with $r^* = 0.01$)

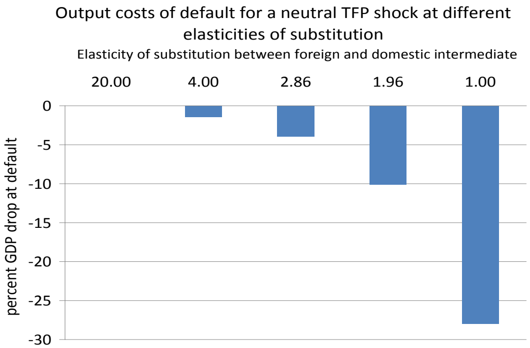
Output Cost of Default



Output Costs of Default as a Function of TFP Shock

- Output cost of default is increasing and strictly convex in TFP
- Output cost of default is higher and a steeper function of ε at lower elasticities
- Debt provides more hedging. Model supports more debt.

Output Cost of Default



Output Costs of Default at a Neutral TFP Shock

Households' Problem

- Preference: GHH utility function

$$\sum_{t=1}^{\infty} \beta^t \frac{\left(c_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}$$

- Static problem: given gov transfers T_t, wages and profits

$$\max_{c_t, L_t} \frac{\left(c_t - \frac{L_t^{\omega}}{\omega}\right)^{1-\sigma} - 1}{1-\sigma}$$

$$s.t. \quad c_t = w_t L_t + \pi_{f,t} + \pi_{m,t} + T_t$$

Sovereign Debt Market

- Risk neutral foreign investors face world interest rate r^*.
- Government issues one-period discount bonds with face values b' and price $q\left(b', \varepsilon\right)$. Asset markets are incomplete.
- Gov. defaults if value of default exceeds value of repayment.

Sovereign Debt Market

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- Government issues one-period discount bonds with face values b' and price $q(b', \varepsilon)$. Asset markets are incomplete.
- Gov. defaults if value of default exceeds value of repayment.
- Default causes temporary exclusion from world credit markets (exogenous re-entry with probability η), affecting both consumption smoothing and access to imported inputs
  - *Implicit or explicit trade sanctions during defaults (Kaletsky (1985), Bulow and Rogoff (1989), Rose (2005), Martinez and Sandleris (2008), Kohlscheen and O'Connell (2008))*

Government's Problem

Given $q(b', \varepsilon)$, the gov. solves a social planner's problem

$$V(b, \varepsilon) = \max\left\{v^{nd}(b, \varepsilon), v^{d}(\varepsilon)\right\}$$

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$$v^{nd}(b, \varepsilon) = \max_{c, m^d, m^*, L^f, L^m, L, b'} \left[u(c, L) + \beta EV\left(b', \varepsilon'\right)\right]$$

$$\text{s.t. } c + q\left(b', \varepsilon\right) b' - b \leq \varepsilon f\left(M, L^f, k\right) - m^* P^*(r^*)$$

$$L^f + L^m = L, \quad A(L^m)^{\gamma} = m^d, \; M = M\left(m^d, m^*\right)$$

Government's Problem

Given $q\left(b',\varepsilon\right)$, the gov. solves a social planner's problem

$$V\left(b,\varepsilon\right)=\max\left\{v^{nd}\left(b,\varepsilon\right),v^{d}\left(\varepsilon\right)\right\}$$

$$v^{d}\left(\varepsilon\right)=\max_{c,m^{d},m^{*},L^{f},L^{m},L}\left[u\left(c,L\right)+\beta\left(1-\eta\right)Ev^{d}\left(\varepsilon'\right)+\beta\eta EV\left(0,\varepsilon'\right)\right]$$

$$\text{s.t. } c+x=\varepsilon f\left(M,L^{f},k\right)-m^{*}P^{*}$$

$$L^{f}+L^{m}=L,\quad A(L^{m})^{\gamma}=m^{d},\ M=M\left(m^{d},m^{*}\right)$$

Default Probability and Bond Pricing

Default set

$$D\left(b\right)=\left\{\varepsilon:v^{nd}\left(b,\varepsilon\right)\leq v^{d}\left(\varepsilon\right)\right\}$$

Default probability (two-dimensional default set)

$$p\left(b',\varepsilon\right)=\int_{D\left(b'\right)}d\mu\left(\varepsilon'|\varepsilon\right)$$

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Lenders' no arbitrage conditions:

$$q\left(b',\varepsilon\right)=\begin{cases}\frac{1}{1+r^{*}} & \text{if } b'\geq 0\\ \frac{1-p\left(b',\varepsilon\right)}{1+r^{*}} & \text{if } b'<0\end{cases}$$

Recursive Equilibrium for the DSGE

A recursive equilibrium is defined by: *(i)* decision rules $b'(b, \varepsilon)$, value function $V(b, \varepsilon)$ and default set $D(b)$; and *(ii)* sovereign bonds price $q(b', \varepsilon)$ such that:

1. Given $q(b', \varepsilon)$, the sovereign government's problem is solved;
2. Given $D(b)$, the lender's no arbitrage condition is satisfied.

Calibration: Parameters set using Data and RBC values

Calibrated Parameters		*Value*	*Target statistics*
CRRA risk aversion	σ	2	Standard RBC value
Risk-free interest rate	r^*	1%	Standard RBC value
Capital share	α_k	0.17	Capital share in GDP (0.3)
Int. goods share	α_m	0.43	Int. goods in gross output
Labor share	α_L	0.40	Labor share in GDP (0.7)
Labor share	γ	0.7	Labor share in GDP (0.7)
Labor elasticity parameter	ω	1.455	Frisch wage elasticity (2.2)
Re-entry probability	η	0.83	Length of exclusion (3 years)
Armington weight in M	λ	0.62	Regression estimate
Armington curvature in M	μ	0.65	Regression estimate
CES elasticity parameter	ν	0.59	Gopinath and Neiman (2010)

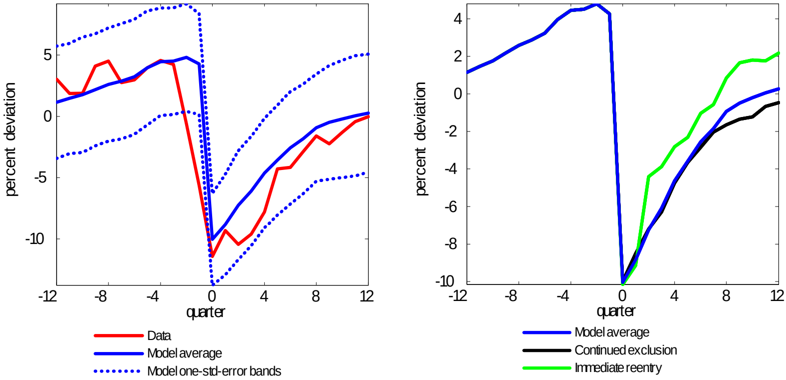
Calibration: Parameters set by SMM

Estimated Parameters		*Value*	*Targets from data*
Productivity persistence	ρ_ε	0.95	GDP autocorrelation (0.95)
Productivity innovations std.	σ_ϵ	1.70%	GDP std. deviation (4.70%)
Intermediate goods TFP	A	0.31	Output drop in default (13%)
Subjective discount factor	β	0.88	Default frequency (0.69%)
Working capital parameter	θ	0.70	Working Capital share (6%)
Sensitivity of payment to IFOs	ξ	-0.67	TB increase in default (10%)

- Adjustment to account for capital outflows during default (repayments to IFOs)

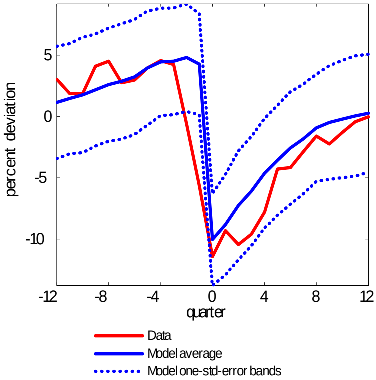
$$x_t = \xi \ln \varepsilon_t$$

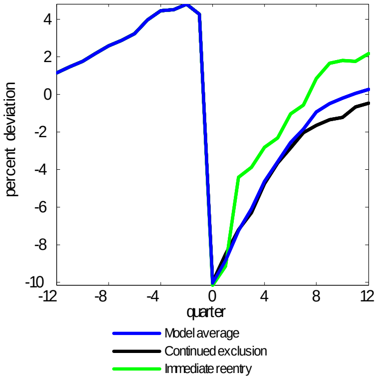
Dynamics of Output **Before** and **After** Default Events



- Deep recession following default
- Gradual recovery after default
- Calvo, Izquierdo and Talvi (2006) "Phoenix Miracles"

Dynamics of Output **Before** and **After** Default Events





- Default triggered by "typical" TFP shock -7.67% (≈1.3 std).
- 81% amplification in output drop due to default
- Gradual recovery driven by TFP recovery and re-entry

Business Cycle Moments

Statistics	Data	Model	Model w/o x_t
Average debt/GDP ratio	35%	22.88%	21.34%
Average bond spreads	1.86%	0.74%	1.68%
Std. dev. of bond spreads	0.78%	1.23%	1.63%
Consumption std./GDP std.	1.44	1.05	1.05
Correlations with GDP			
bond spreads	-0.62	-0.17	-0.21
trade balances	-0.87	-0.54	-0.31
labor	0.39	0.52	0.52
intermediate goods[1]	0.90	0.99	0.99

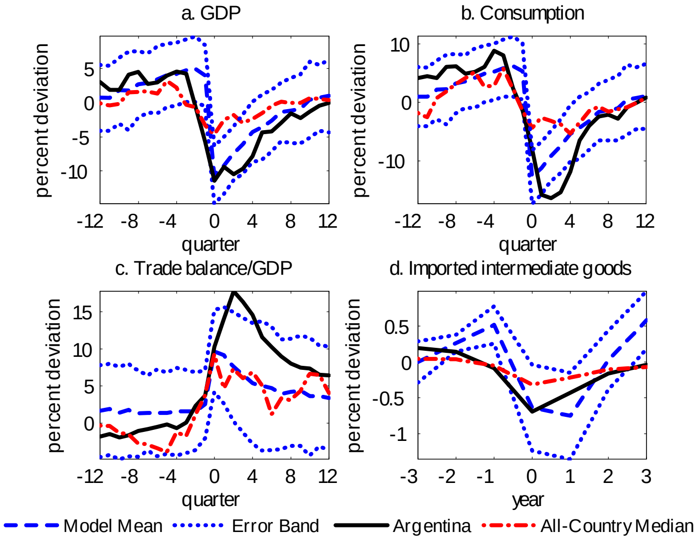
Business Cycle Moments

Statistics	Data	Model	Model w/o x_t
Correlations with bond spreads			
trade balances	0.82	0.15	0.12
labor[1]	-0.42	-0.19	-0.26
intermediate goods[1]	-0.39	-0.16	-0.18
Historical default-output co-movements			
correlation between default and GDP[1]	-0.11[2]	-0.09	-0.12
frac. of defaults with GDP below trend[1]	61.5%[2]	83%	82%
frac. of defaults with large recessions[1]	32.0%[2]	21.1%	20%

Note 1: Statistical moment computed at annual frequency.

Note 2: Cross-country historical estimate for 1820-2004 from Tomz and Wright (2007).

Macro Dynamics Around Default Events



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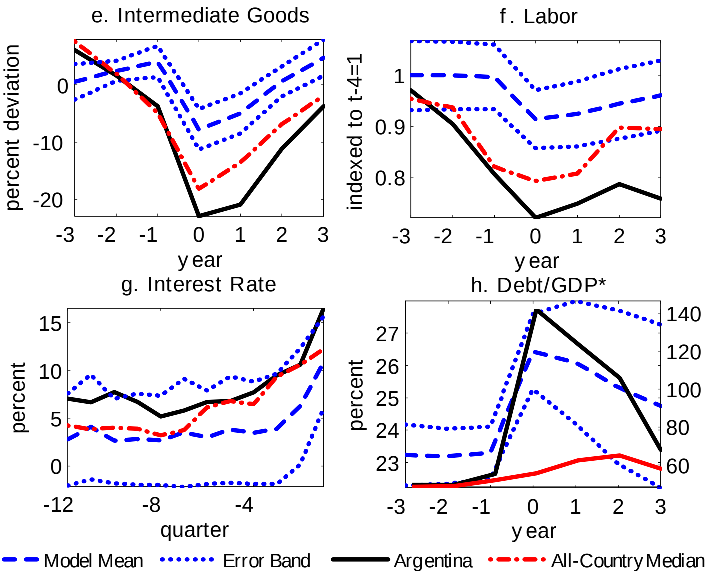

Sensitivity Analysis

	Output drop at default	Mean Debt/GDP ratio	Mean spread	Std. dev of spread
(1) Data	13%	35%	1.86%	0.78%
(2) Baseline	13%	22.88%	0.74%	1.23%
Working capital				
(3) $\theta = 0$	13%	8.99%	0.05%	0.08%
(4) $\theta = 0.6$	13.9%	20.39%	0.59%	1.17%
(5) $\theta = 0.8$	14.3%	26.84%	0.61%	1.19%
Armington elasticity				
Armington elasticity				
(6) 2.63 $(\mu = 0.62)$	14.6%	31.25%	0.55%	0.99%
(7) 3.10 $(\mu = 0.68)$	12.9%	16.15%	1.14%	1.36%
Armington share				
(8) $\lambda = 0.58$	17.20%	39.01%	0.28%	0.79%
(9) $\lambda = 0.66$	12.7%	14.16%	0.99%	1.42%

Sensitivity Analysis

	GDP corr. with		frequency of default w. GDP below trend
	spread	default	
(1) Data	-0.62	-0.11	62%
(2) Baseline	-0.17	-0.09	83%
Working capital			
(3) $\theta = 0$	0.24	-0.02	75%
(4) $\theta = 0.6$	-0.11	-0.11	88%
(5) $\theta = 0.8$	-0.14	-0.10	84%
Armington elasticity			
Armington elasticity			
(6) 2.63 $(\mu = 0.62)$	-0.16	-0.09	90%
(7) 3.10 $(\mu = 0.68)$	-0.11	-0.09	78%
Armington share			
(8) $\lambda = 0.58$	-0.08	-0.04	83%
(9) $\lambda = 0.66$	-0.11	-0.08	77%

Sensitivity Analysis

	Output drop at default	Mean Debt/GDP ratio	Mean spread	Std. dev of spread
(1) Data	13%	35%	1.86%	0.78%
(2) Baseline	13%	22.88%	0.74%	1.23%
Within-variety elasticity				
(10) 2.22 $(\nu = 0.55)$	14.1%	25.83%	0.60%	1.17%
(11) 2.89 $(\nu = 0.65)$	12.8%	19.81%	0.72%	1.22%
Frisch elasticity of labor supply				
(12) 1.67 $(\omega = 1.6)$	12.8%	22.34%	0.91%	1.29%
(13) 2.5 $(\omega = 1.4)$	14.3%	24.46%	0.45%	1.05%
Probability of re-entry				
(14) $\phi = 0.05$	14.3%	37.02%	0.39%	0.88%
(15) $\phi = 0.1$	13.5%	19.78%	0.65%	1.21%

Sensitivity Analysis

	GDP corr. with		frequency of
	spread	default	default w. GDP below trend
(1) Data	-0.62	-0.11	62%
(2) Baseline	-0.17	-0.09	83%
Within-variety elasticity			
(10) 2.22 $(\nu = 0.55)$	-0.11	-0.09	84%
(11) 2.89 $(\nu = 0.65)$	-0.12	-0.07	82%
Frisch elasticity of labor supply			
(12) 1.67 $(\omega = 1.6)$	-0.17	-0.13	85%
(13) 2.5 $(\omega = 1.4)$	-0.02	-0.06	68%
Probability of re-entry			
(14) $\phi = 0.05$	-0.11	-0.05	82%
(15) $\phi = 0.1$	-0.11	-0.12	94%

Concluding Remarks

- Proposed default model with endogenous output dynamics that solves country risk-business cycle disconnect
- Increasing endogenous cost of default driven by efficiency loss due to factor substitution/reallocation
- Strong financial amplification mechanism linking default with deep recessions
- Model explains three key stylized facts of sov. default & key business cycle features
- Hints at important connection between trade structure/openness, default incentives and debt dynamics