Workhorse Models of the Small Open Economy

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Objectives and plan of the lecture

- Introduce key issues for analyzing open economy models with incomplete markets: stationarity & debt/wealth dynamics, prec. savings
- Model 1: Deterministic, 1-sector endowment SOE
- Model 2: Stochastic variant of Model 1 but with incomplete markets
- Quantitative example using a variant of Model 2
- Shortcomings of local solution methods, based on Model 2 (*FiPIt* method introduction)

WORKHORSE MODEL 1: DETERMINISTIC SMALL OPEN ECONOMY MODEL

Key Assumptions

1. SOE with perfect access to world credit market
2. One-period bonds, fixed world real interest rate
3. Perfect foresight OR Complete Markets
4. Credible commitment to repay
5. Frictionless economy, no distortions

– CA supports perfect consumption smoothing
– Long-run NFA is simply annuity value of steady-state trade balance

Intertemporal optimization problem

- Sequential social planner's problem:

$$(I)\sum_{t=0}^{\infty}\beta^t u(c_t)$$

$$(II)\; c_t = y_t - b_{t+1} + b_t R, \quad b_0 \;\; given, \;\; \{y_t\}_{t=0}^{\infty}$$

  - Combining constraints + NPG condition yields IBC:

$$\sum_{t=0}^{\infty} R^{-t} c_t = \sum_{t=0}^{\infty} R^{-t} y_t + b_0\, R$$

- Recursive planner's problem:

$$(III)\; V_t(b_t, y_t) = \max_{b_{t+1}} \{u(c_t) + \beta V_{t+1}(b_{t+1}, y_{t+1})\}$$

subject to *(II)*

Equilibrium conditions

- First-order condition of the recursive problem:

$$u'(c_t) = \beta V_{1t+1}(b_{t+1}, y_{t+1})$$

  - From envelope theorem (Benveniste-Sheikman eq.)

$$V_{1t+1}(b_{t+1}, y_{t+1}) = Ru'(c_{t+1})$$

  - So we obtain standard Euler equation:

$$u'(c_t) = \beta R u'(c_{t+1})$$

- Stationarity assumption: $\beta R = 1 \Rightarrow c_t = \bar{c} \quad \forall\, t$

- Closed-form solution (using IBC):

$$\frac{\bar{c}}{(1-\beta)} = \left[\sum_{t=0}^{\infty} \beta^t y_t\right] + b_0 R \qquad \Rightarrow \qquad \bar{c} = (1-\beta)W$$

Current account, trade balance and NFA dynamics

- The equilibrium current account is:

$$b_{t+1} - b_t = y_t - \bar{c} + b_t r$$

- Assume output converges:

$$y_t \to \bar{y} \quad as \quad t \to \infty$$

- Stationary equilibrium of CA is zero, and steady states of NFA and NX are given by:

$$\bar{b} = -\frac{[\bar{y} - \bar{c}]}{r} = -\frac{\overline{nx}}{r}$$

$$= -\frac{[\bar{y} - (1-\beta)W]}{r} = \beta W - \frac{\bar{y}}{r}$$

Stationarity and initial conditions

- Stationary equilibrium is unique, but since wealth depends on initial NFA, $\bar{b}$ and $\bar{c}$ depend on b_0 (i.e. steady state depends on initial conditions)
- Borrow when $y_t < \bar{c}$ and save when $y_t > \bar{c}$
  - CA deficit with low y_t
  - CA surplus with high y_t
  - CA is procyclical!
- Not a good model of actual CA dynamics

General equilibrium extension

- Standard production function $f(k)$ and investment w. capital adj. costs $(\frac{\phi}{2})(k_{t+1} - k_t)^2$ (Tobin's Q)
- Consumption, NFA and CA dynamics are analogous to endowment case, **but** evaluated at eq. sequence of net income (output minus adj. costs) implied by no-arbitrage condition
- Fisherian separation: Decision rule for *k* is independent of *b* but dec. rule for *b* depends on *k*

Recursive social planner's problem

$$V(k,b) = max_{\{k',b',c\}}[u(c) + \beta V(k',b'\}]$$

s.t.

$$c = f(k) - (k'-k)\left[1 + \frac{\phi}{2}(k'-k)\right] - b' + bR$$

- With a solution characterized by decision rules:

$$\widehat{k'}(k,b), \widehat{b'}(k,b)$$

Euler equations

- Bonds

$$u'(t) = \beta R u'(t+1)$$

- Capital

$$\left[1 + \phi\left(k_{t+1} - k_t\right)\right] u'(t)$$

$$= \beta u'(t+1)\left[f'\left(k_{t+1}\right) + 1 + \phi\left(k_{t+2} - k_{t+1}\right)\right]$$

Four key properties

1. k_{ss} is unique and independent of initial conditions, but $\bar{c}$, NFA dynamics, and $\bar{b}$ still depend on b_0

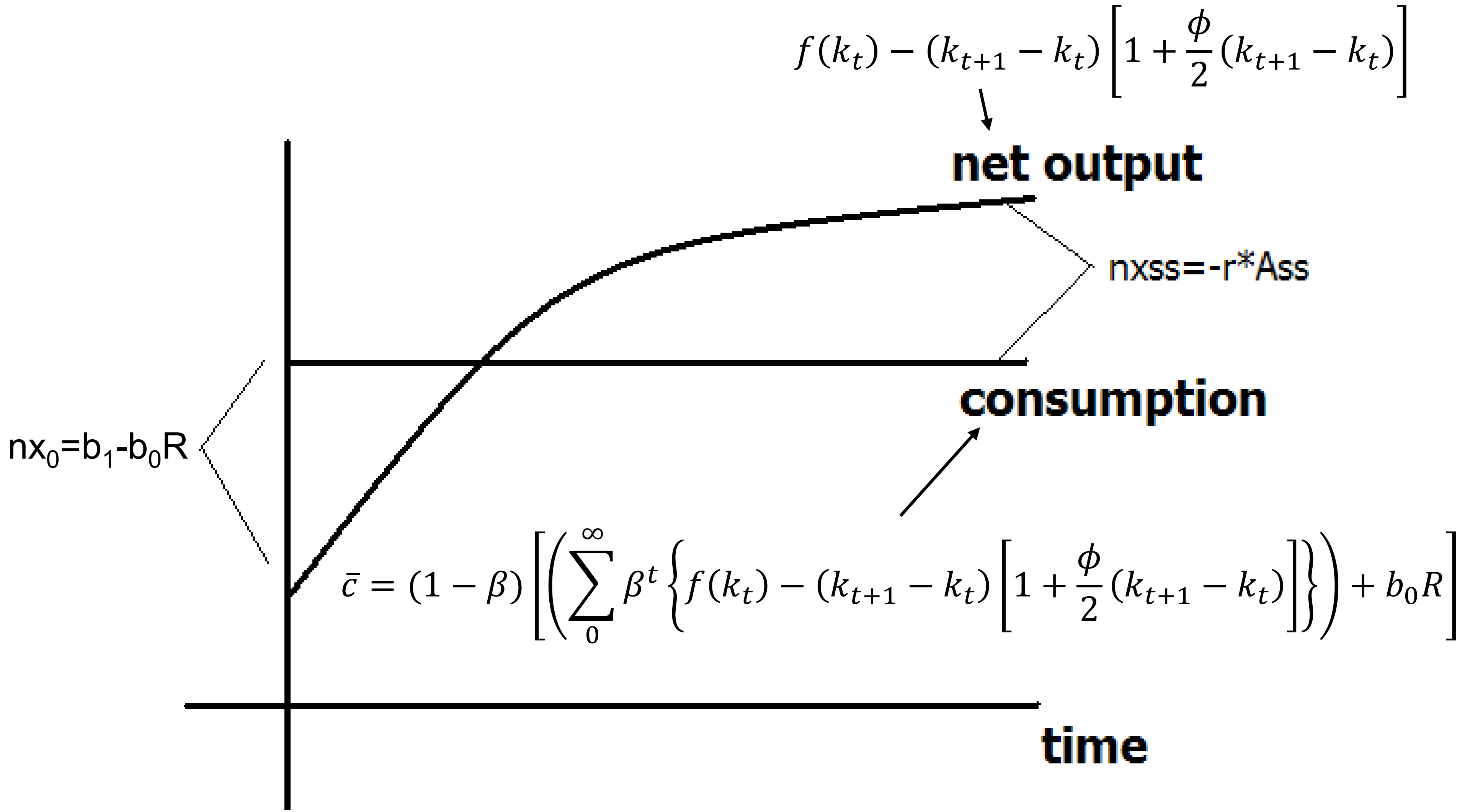
2. Fisherian separation: Investment and production dynamics determined by this no-arbitrage condition:

$$\frac{d' + q'}{q} \equiv \frac{f'(K') + 1 + \phi(K'' - K')}{1 + \phi(K' - K)} = 1 + r^*$$

3. Well-defined dynamics, unique steady-state
   – But steady-state Euler eq. does not yield a solution for $\bar{b}$. Instead, we solve jointly with model's dynamics

4. Standard local methods around det. steady states are not useful for solving these models
   – Even temporary shocks have permanent effects
   – But shooting methods do work

Time-series dynamics
(and a gains from trade argument)



Effects of Shocks

1. Additive (e.g. government expenditures)
   – Permanent: No effect on debt or capital dynamics, equal effects on income profile and consumption.
   – Transitory: No effect on investment dynamics but affects debt dynamics through the effect on permanent income and steady state of *b*.

2. Multiplicative (e.g. productivity, terms of trade)
   – Permanent or transitory: Affect both investment and debt dynamics and steady state of *b*, but only permanent shocks affect k_{ss} .

- CA can turn countercyclical (e.g. persistent TFP shocks induce borrowing for investment)

WORKHORSE MODEL 2: STOCHASTIC MODEL WITH INCOMPLETE MARKETS

Uncertainty and Incomplete Markets

- NFA are non-state-contingent, one-period "real" bonds chosen from a finite state space defined by a discrete grid:

$$B = [b_1 < b_2 < \dots < b_z]$$

- Income and world interest rate are exogenous
- Income follows exogenous Markov process with "*m*" states and known transition prob. matrix:

$$\bar{y} = [y_1 < y_2 < \cdots < y_m] \qquad P(y_i, y_j)$$

- Asset markets are incomplete: *B* cannot provide full insurance against income fluctuations

Sequential planner's Problem

- Choose $\{b_{t+1}\}_{t=0}^{\infty}$ so as to

$$max \quad E_0\left[\sum_{t=0}^{\infty}\beta^t\, u(c_t)\right]$$

s.t.

$$c_t = y_t - b_{t+1} + b_t R$$

$$b_{t+1} \in B \qquad P(y_t, y_{t+1})\; known$$

$$(b_0, y_0) \;\; given,$$

…looks very similar to Model 1, but it is **very** different!

Aiyagari's natural debt limit

- *u(.)* is twice differentiable, concave and satisfies the Inada condition:

$$\lim_{c \downarrow 0} u'(c) = \infty$$

- Implies that consumption must be positive at all times, and hence budget constraint yields NDL:

$$b' \geq -\left[\frac{y_{min}}{R-1}\right]$$

  - Otherwise the agent is exposed to the risk of zero consumption with positive probability
  - Highlights "global" nature of decision-making under incomplete markets (all potential future histories matter)
  - Could also use ad-hoc debt limit $b' \geq -\phi \geq NDL$

Recursive planner's problem

$$V(b_n, y_i) = \max_{b' \in B} \left\{ u(y_i - b' + b_n R) + \beta \sum_{j=1} P(y_i, y_j) V(b', y_j) \right\}$$

s.t. $b' \geq -\phi \geq NDL$

for each of the *mxz* pairs (b_n, y_i), with $b_1 = -\phi$.

- The solution is characterized by:
  1. Decision rule $b' = g(b, y)$
  2. Value function $V(b_n, y_i)$
  3. Unconditional stationary distribution of *(b,y)*

$$\lambda(b, y) = Prob(b_t = b, y_t = y)$$

- Fast and easy to solve w. *FiPIt* method

Law of motion of conditional probabilities

- $P(y_t, y_{t+1})$ and $b' = g(b, y)$ induce a law of motion for conditional transition probabilities from date-t states *(b,y)* to date-t+1 states *(b',y')*:

$$\lambda_{t+1}(b', y') = Prob(b_{t+1} = b', y_{t+1} = y')$$

$$= \sum_{b_t \in B} \sum_{y_t \in \bar{y}} Prob(b_{t+1} = b' | b_t = b, y_t = y) \times$$

$$Prob(y_{t+1} = y' | y_t = y) \times Prob(b_t = b, y_t = y)$$

Equilibrium Transition Probabilities

- But since $b' = g(b,y)$ is a unique recursive function of *(b,y)*, the law of motion becomes:

$$\lambda_{t+1}(b',y') = \sum_b \sum_y \lambda_t(b,y)\, Prob(y_{t+1} = y' | y_t = y) \Upsilon(b',b,y)$$

$$\Upsilon(b',b,y) = \begin{cases} 1 \Leftrightarrow b' = g(b,y) \\ 0 \qquad otherwise \end{cases}$$

  – Which can be rewritten as:

$$\lambda_{t+1}(b',y') = \sum_y \sum_{\{b:b'=g(b,y)\}} \lambda_t(b,y)\, P(y,y')$$

Stochastic Stationary State

- The stochastic steady state is a joint stationary distribution of NFA and income, $\lambda(b,y)$, which is the fixed point of the law of motion

$$\lambda_{t+1}(b',y') = \sum_{y} \sum_{\{b:b'=g(b,y)\}} \lambda_t(b,y)\, P(y,y')$$

- Methods to solve for $\lambda(b,y)$:
  - Iterating to convergence in the law of motion
  - Computing Eigen values of *(mxz)*^{2} trans. prob. matrix
  - Powering to convergence transition prob. Matrix

- Use it to compute moments and IRFs:

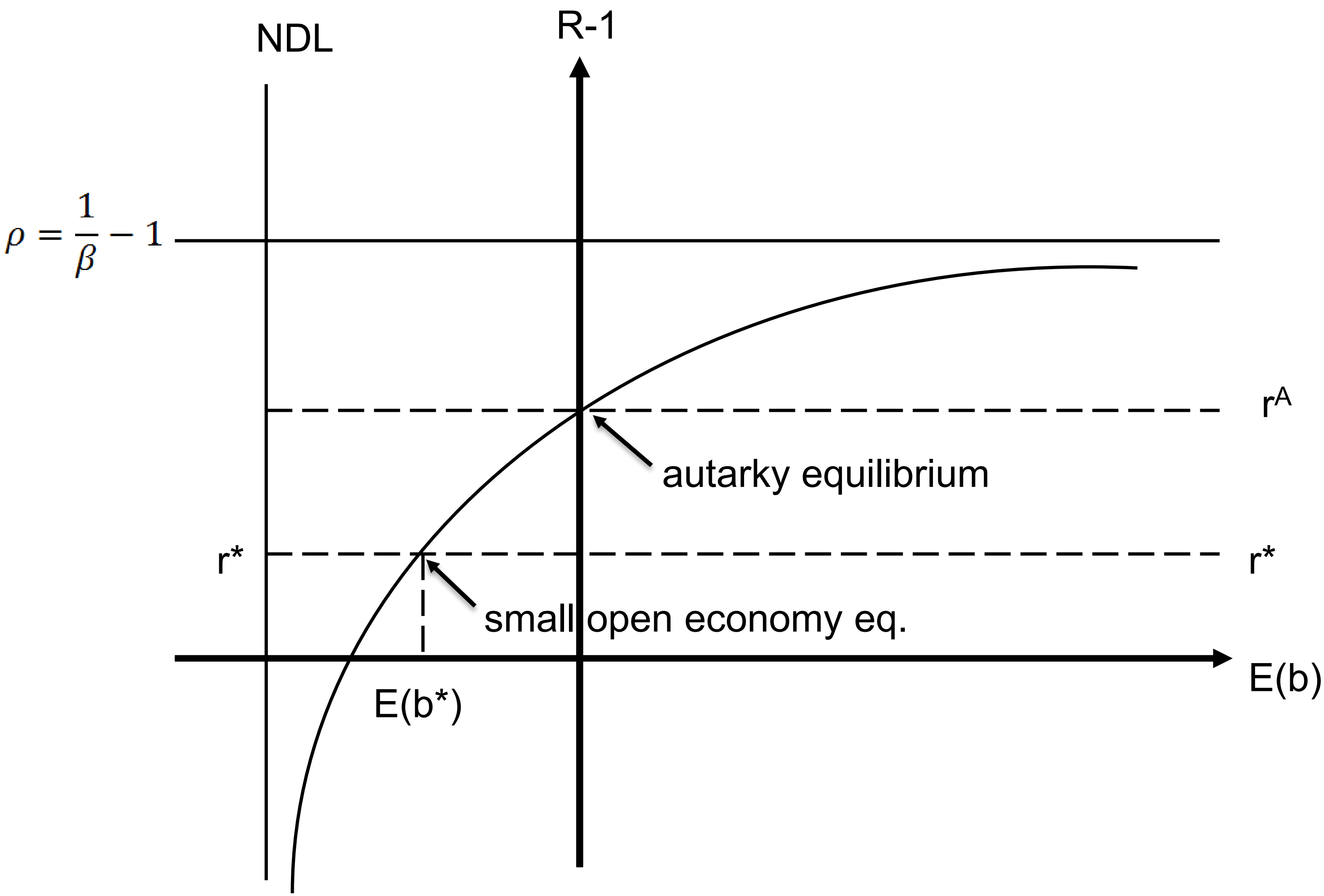
$$E[b] = \sum_{(b,y)\in B\times Y} \lambda(b,y)b \qquad E[c] = \sum_{(b,y)\in B\times Y} \lambda(b,y)(y - b'(b,y) + Rb)$$

$$E_t[b] = \sum_{(b,y)\in B\times Y} \lambda_t(b,y)b \qquad E_t[c] = \sum_{(b,y)\in B\times Y} \lambda_t(b,y)(y - b'(b,y) + Rb)$$

Precautionary savings
(failure of the standard stationarity condition)

- Standard stationarity assumption $\beta R = 1$ fails
  - Euler eq. implies "constant consumption," but income is always stochastic and NFA is non-state-contingent.
  - Formally: marginal benefit of saving $\beta^t R^t u'(t)$ follows a Supermartingale process, and since Supermartingales converge, it follows that $b' \to \infty$
- Agents self insure, build precautionary savings
  - If $\beta R < 1$, force pushing to borrow and force pushing for prec. savings support stationary distribution
  - Natural Debt Limit imposes lower bound on NFA
  - But the deterministic st. state is always the debt limit!

Equilibrium & stationary NFA demand curve



The RBC model of a small open economy

- Originated in Mendoza AER 1991
  - Mendoza IER 1995, Kose JIE, 2002, Sch. Grohe & Uribe IER 2017, Di Pace et al. 2022 added TOT shocks
  - Uribe & Yue JIE 2006 and Neumeyer & Perri JME 2005 added working capital financing
  - Mendoza AER 2010 introduced imported inputs
- Rep. agent maximizes standard time-separable CRRA with GHH labor supply specification

$$E_0\left[\sum_{t=0}^{\infty}\beta^t\frac{(c_t-\frac{L_t^{\omega}}{\omega})^{1-\sigma}}{1-\sigma}\right]$$

s.t. resource constraint

$$c_t(1+\tau)+k_{t+1}-(1-\delta)k_t+\frac{a}{2}\frac{(k_{t+1}-k_t)^2}{k_t}=$$

$$A_tF(k_t,L_t,v_t)-p_tv_t-\phi(R_t-1)(w_tL_t+p_tv_t)-q_t^b b_{t+1}+b_t$$

RBC-SOE model contn'd

- Three shocks:
  - TFP: $A_t F(k_t, L_t, v_t) = \exp(\epsilon_t^A) A k_t^\gamma L_t^\alpha v_t^\eta$
  - Interest rate: $R_t = R \exp(\epsilon_t^R)$
  - Terms of trade (imported inputs price): $p_t = p \exp(\epsilon_t^P)$
- Optimality conditions for labor and inputs:

$$w_t = L^{\omega-1}(1+\tau)$$

$$A_t F_{L_t}(k_t, L_t, v_t) = w_t \left(1 + \phi(R_t - 1)\right)$$

$$A_t F_{v_t}(k_t, L_t, v_t) = p_t \left(1 + \phi(R_t - 1)\right)$$

  - GHH removes wealth effect on labor supply (MRS between *c* and *L* depends on *L* only).
  - *L*, *v*, *w* (and output) depend only on *k* and (*A,p,R*) shocks

RBC-SOE model contn'd

- Euler eq. for *b* (still used for prec. savings):

$$\lambda_t = \frac{1}{q_t^b}\beta E_t[\lambda_{t+1}]$$

- Euler eq. for *k* (risky asset akin to equity):

$$\lambda_t = \frac{1}{q_t}\beta E_t[\lambda_{t+1}(d_{t+1} + q_{t+1})]$$

where:

$$d_t \equiv \exp(\epsilon_t^A)F_{k_t} - \delta + \frac{a}{2}\frac{(k_{t+1} - k_t)^2}{k_t^2}$$

$$q_t = 1 + a\left(\frac{k_{t+1} - k_t}{k_t}\right)$$

- No-arbitrage cond. (small equity premium, quasi Fisherian separation)

$$E[R_{t+1}^q] = R_t - \frac{cov_{t+1}(R_{t+1}^q \lambda_{t+1})}{E[\lambda_{t+1}]}$$

Remarks about solving models with incomplete markets

- Solving these models generally requires global methods that can track dynamics of wealth dist.
- Certainty equivalence fails (e.g., higher variance or persistence of shocks increases average NFA)
- Local methods feature a unit root unless a "stationarity inducing" assumption is added (Schmitt G & Uribe (03))
- …but those local solutions differ significantly from global solution (de Groot, Durdu & Mendoza (19,23))
- Prec. savings also affects portfolio structure (wealthier agents/countries tolerate more risk, hold larger shares of risky assets at lower premia)

Example from Durdu, Mendoza & Terrones (2008)

- SOE with exogenous Markov endowment:

$$V(b,\varepsilon) = \max_{b'} \left\{ \frac{c^{1-\gamma}}{1-\gamma} + \exp(-v(c)) E[V(b',\varepsilon')] \right\}$$

$$s.t. \qquad c = \varepsilon y - b' + bR + A$$

$$b_{t+1} \geq \phi \geq -\min(\varepsilon_t y + A)/r$$

- Allows for 2 formulations of rate of time pref.:
  1. Uzawa-Epstein endogenous rate of time preference
  2. Bewley-Aiyagari-Hugget setup with $\beta R < 1$

$$v(c) = \rho^{UE} \ln(1+c) \;\; or \;\; \ln(1+\rho^{BAH})$$

Calibration

- Discrete state space:

$$(b, b') \in B = \{b_1 < b_2 < ... < b_n\} \qquad n{=}1000$$

$$\varepsilon \in \mathrm{E} = \{\varepsilon_1 < \varepsilon_2 < ... < \varepsilon_j\} \qquad \pi(\varepsilon_{t+1} \mid \varepsilon_t)$$

- Income process (set to Mexico's detrended GDP)

$$y_t = \rho_y y_{t-1} + e_t \quad \sigma_y = 3.301\% \quad \rho_y = 0.597$$

$$\sigma_e = \sqrt{\sigma_y^2(1-\rho_y^2)} = 2.648 \text{ percent}$$

  – Discretized using Tauchen-Hussey quadrature method with j=5 (yields process with 3.28% s.d. and AR=0.55)
  – Can also use canonical Markov chains (e.g. “simple persistence” rule) to discretize time-series processes

- E[y] = 1 for simplicity (variables are GDP ratios)
- E[b] = -0.44 Mexico's average NFA/GDP 1985-2004 in Lane & Milesi Ferretti (06)
- E[c] = 69.2 Mexico's average C/GDP 1965-2005
- R = 1.059 Mexico's country real interest rate from Uribe and Yue (06)
- It follows that $A = y + b(R\text{-}1)\text{-}c = 0.282.$
- Discount factors and rates of time preference:
  - UE: $\rho^{UE} = \ln(R)/\ln(1+c) = 0.109 \quad (1+c)^{-0.109} = 0.944$
  - BAH: $\rho^{BAH} = 0.064$ set by searching for values of ad-hoc debt limit & discount factor that match E[b]=-0.44 and sd(c)=3.28% (ϕ =-0.51 β=0.94)

Calibrated state space

- Vector of income realizations

1	-0.075642
2	-0.035892
3	0.0
4	0.035892
5	0.075642

- Transition prob. matrix of income shocks

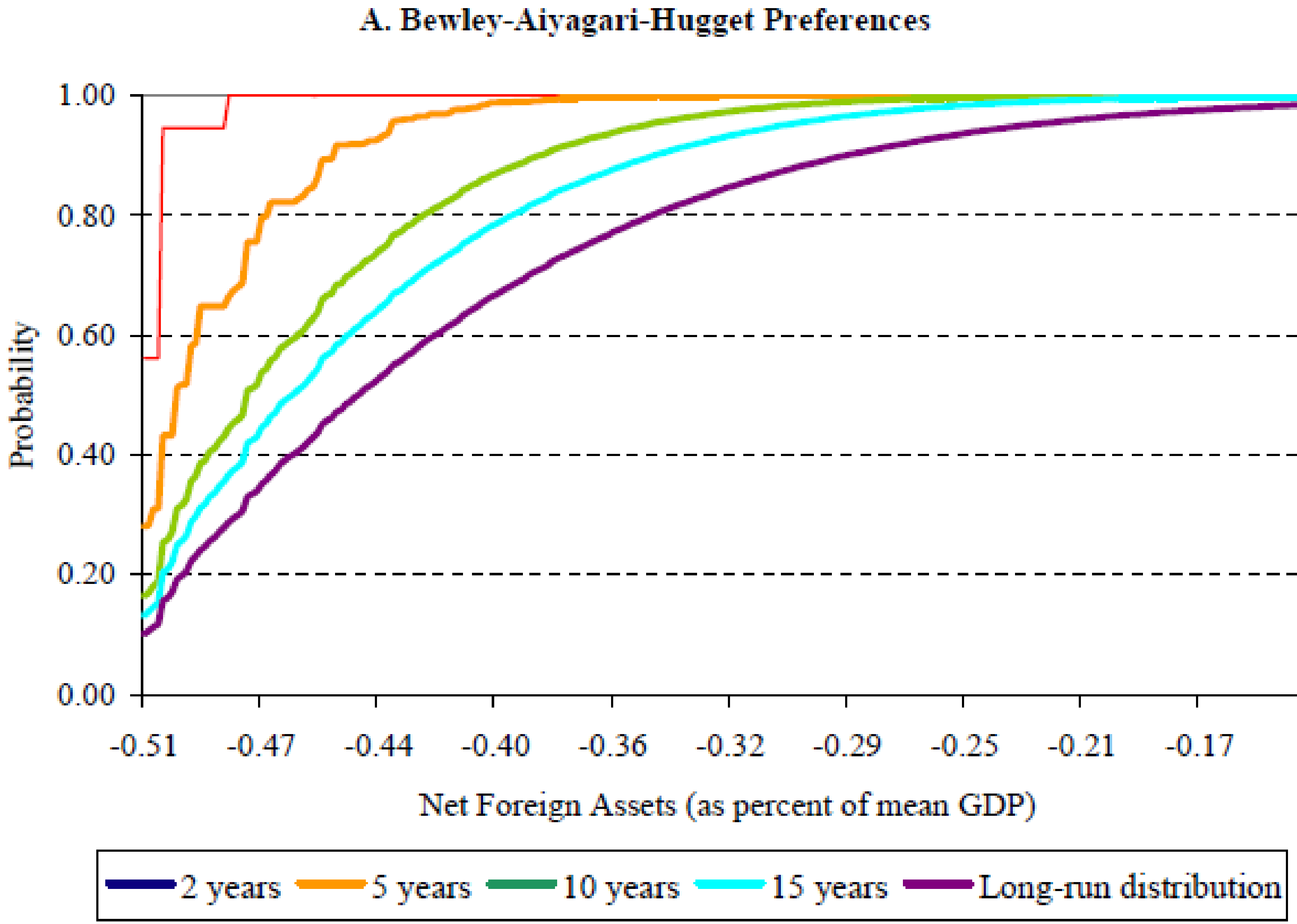
	COL 1	COL 2	COL 3	COL 4	COL 5
ROW 1	0.34500	0.52508	0.12475	0.00513915	2.0099D-05
ROW 2	0.081986	0.47956	0.38426	0.053385	0.00080242
ROW 3	0.011257	0.22208	0.53333	0.22208	0.011257
ROW 4	0.00080242	0.053385	0.38426	0.47956	0.081986
ROW 5	2.0099D-05	0.00513915	0.12475	0.52508	0.34500

- Grid of bonds: spacing=0.001514, nodes=1000, lower bound=-0.5123

Calibrated parameter values

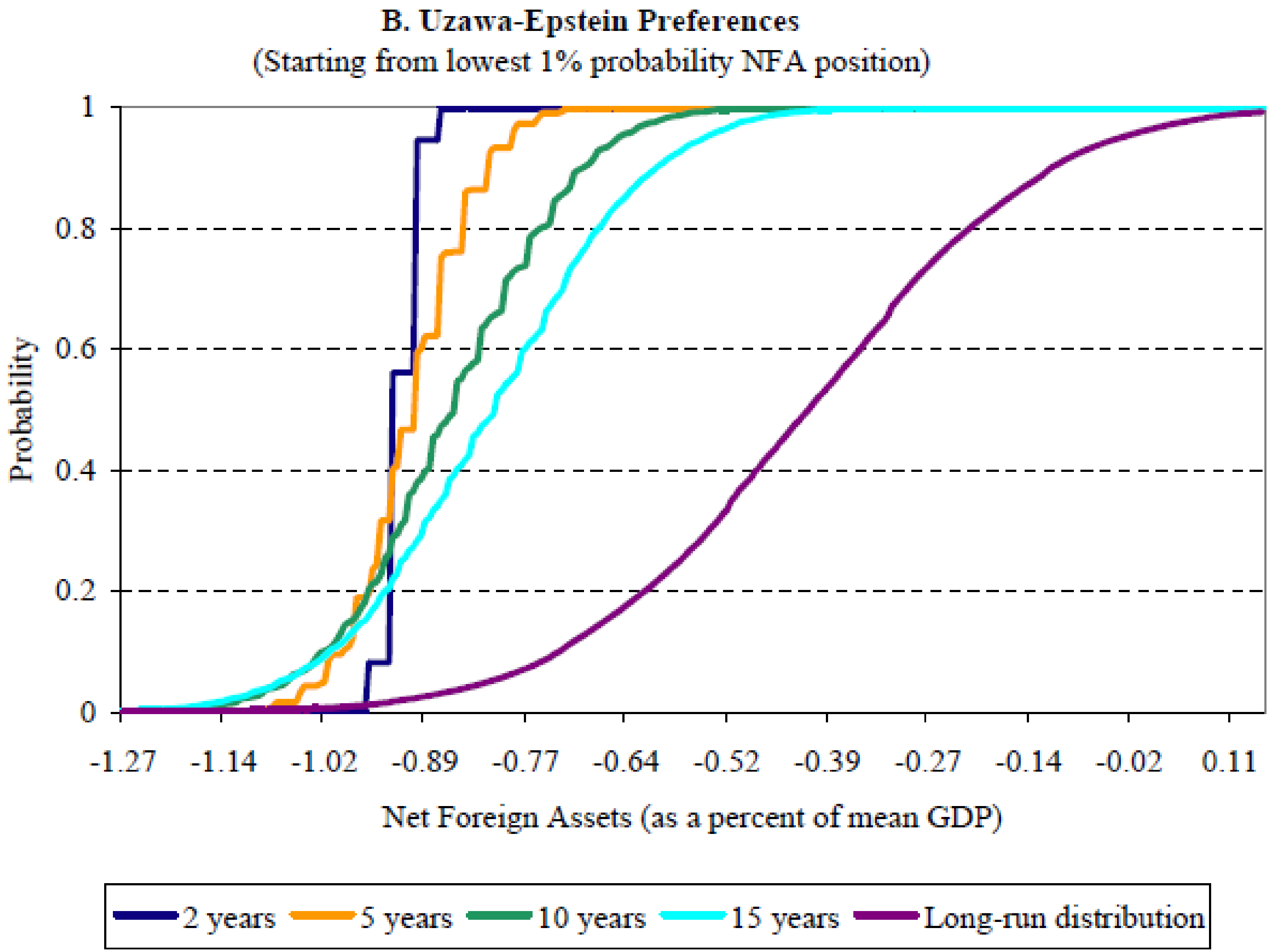
ρ^{BAH}	Rate of time preference in the BAH setup	0.064
ρ^{UE}	Rate of time preference elasticity in the UE setup	0.109
γ	Coefficient of relative risk aversion	2.000
ϕ	Ad-hoc debt limit	-0.510
R	Gross world interest rate	1.059
y	Mean output	1.000
c	Consumption-output ratio	0.692
b	Net foreign assets-output ratio	-0.440
σ_e	Standard deviation of output innovations	0.026
ρ	Autocorrelation of output	0.597
A	Lump-sum absorption	0.282

Transitional and stationary distributions



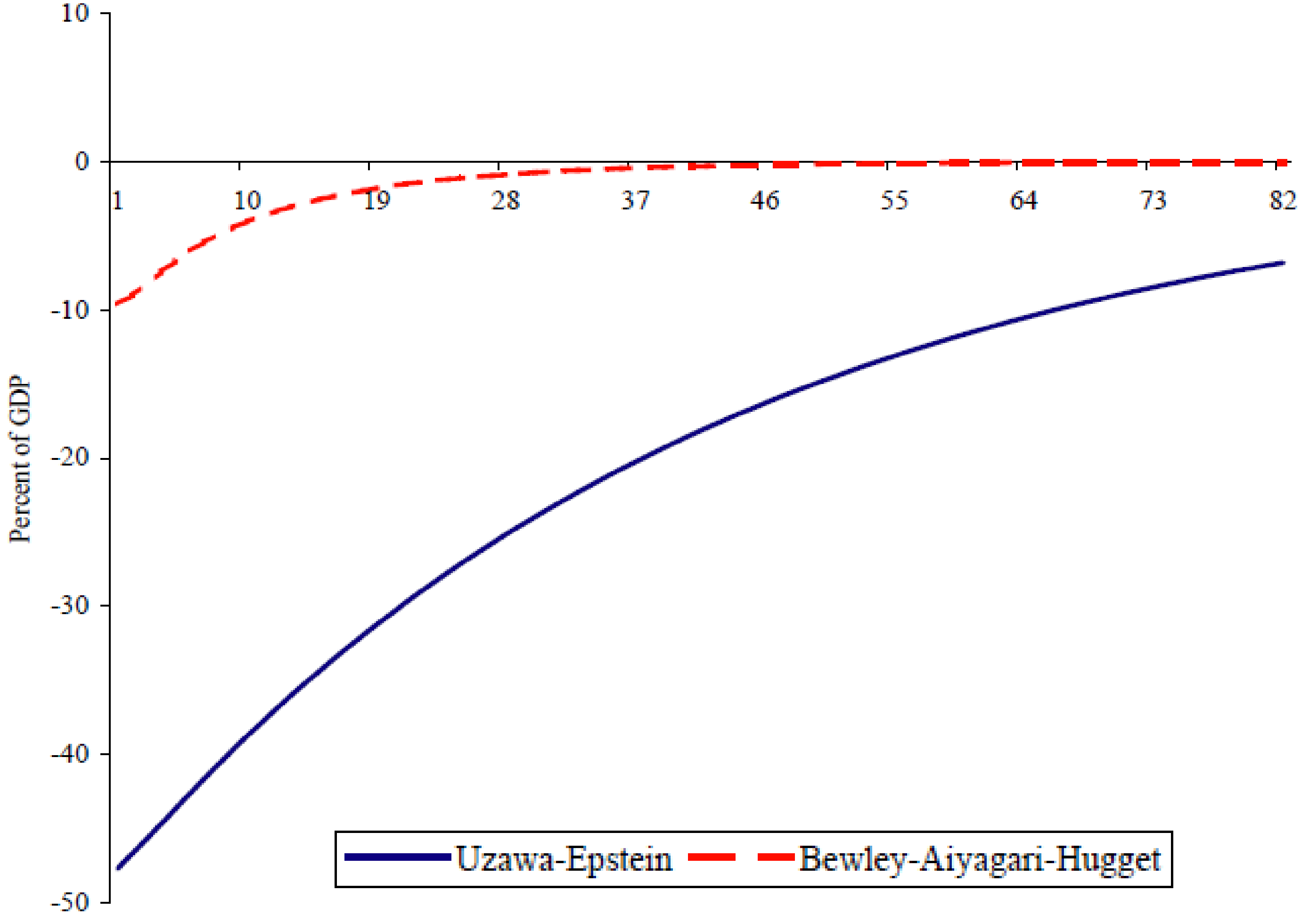
Note: Initial conditions are lowest (b,y) with positive long-run probability

Transitional and stationary distributions



Note: Initial conditions are lowest (b,y) with positive long-run probability

Transitional dynamics of NFA



Note: Dynamics show forecasting function starting from lowest positive prob. B and neutral income shock and plotted as differences relative to long-run averages.

Unconditional moments

	Baseline		Auto Corr 0.7		Std Dev. 5%		Std Dev. 2.5%		Risk Aver. 5.0	
	UE	BAH	UE	BAH	UE	BAH	UE	BAH	UE	BAH
Precautionary savings [1/]	0.02	0.10	0.04	0.12	0.05	0.22	0.01	0.05	0.10	0.24
Means										
Output	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Consumption	0.69	0.69	0.69	0.70	0.70	0.70	0.69	0.69	0.70	0.70
Foreign assets	-0.42	-0.42	-0.41	-0.39	-0.39	-0.30	-0.43	-0.46	-0.34	-0.28
Trade balance [2/]	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.03	0.02	0.02
Discount factor	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94	0.94
Coefficients of variation (in percent)										
Output	3.28	3.28	3.63	3.63	4.97	4.97	2.49	2.49	3.28	3.28
Consumption	3.13	3.26	3.92	3.92	4.72	4.66	2.38	2.59	4.11	3.11
Foreign assets	24.41	10.11	29.73	13.39	36.97	20.28	18.52	6.33	40.92	20.10
Current account [2/]	2.68	2.02	2.77	2.08	4.08	3.42	2.03	1.40	2.81	2.48
Trade balance [2/]	3.04	2.11	3.27	2.23	4.62	3.66	2.30	1.44	3.72	2.78
Discount factor	0.14	0.00	0.18	0.00	0.21	0.00	0.11	0.00	0.18	0.00

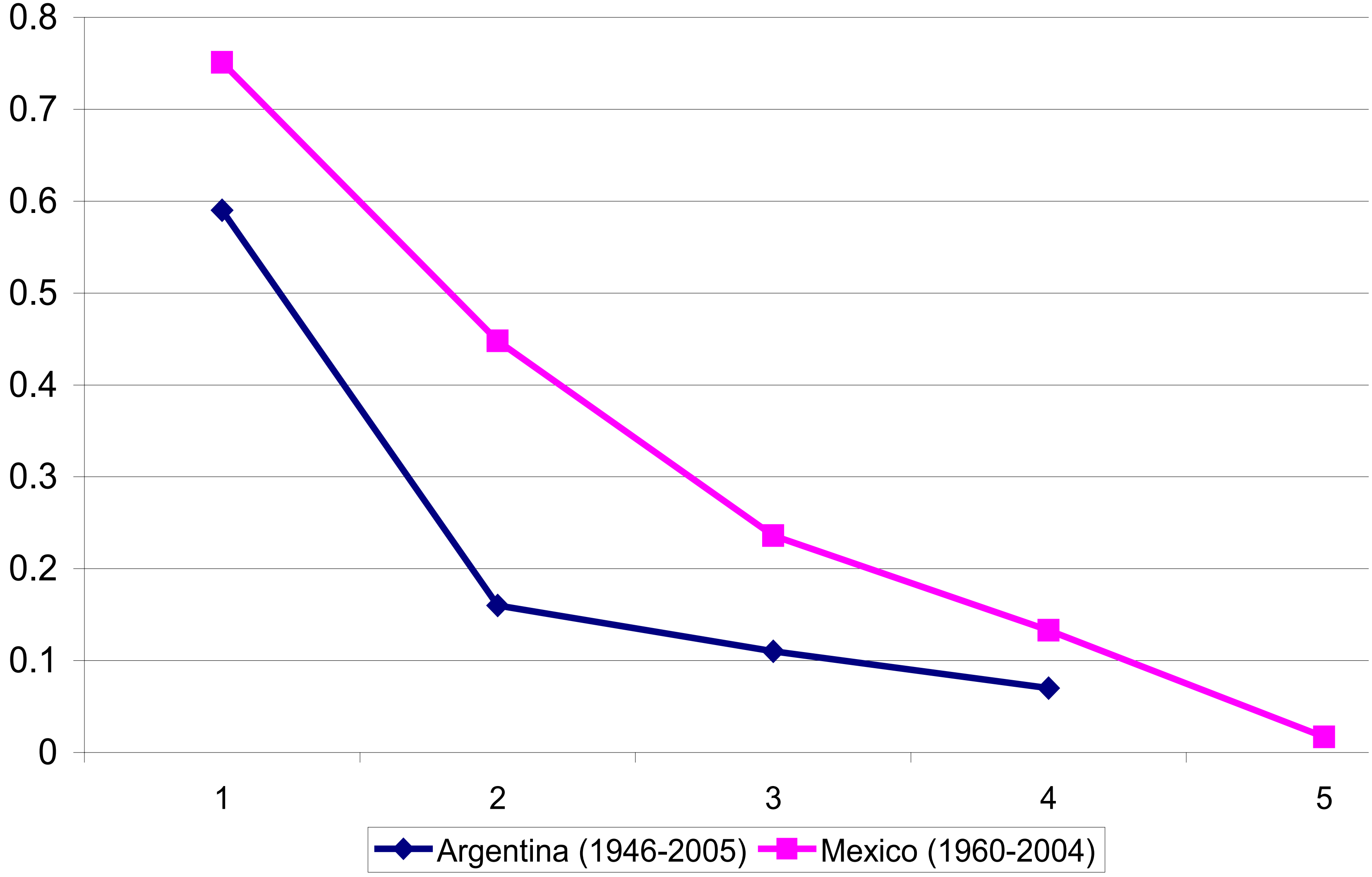
	Baseline		Auto Corr 0.7		Std Dev. 5%		Std Dev. 2.5%		Risk Aver. 5.0	
	UE	BAH	UE	BAH	UE	BAH	UE	BAH	UE	BAH
Normalized coefficients of variation (relative to output)										
Consumption	0.95	0.99	1.08	1.08	0.95	0.94	0.96	1.04	1.25	0.95
Foreign assets	7.43	3.08	8.19	3.69	7.43	4.08	7.44	2.55	12.46	6.12
Current account 2/	0.82	0.62	0.76	0.57	0.82	0.69	0.82	0.56	0.86	0.75
Trade balance 2/	0.92	0.64	0.90	0.61	0.93	0.74	0.93	0.58	1.13	0.85
Discount factor	0.04	0.00	0.05	0.00	0.04	0.00	0.04	0.00	0.06	0.00
Output correlations										
Consumption	0.42	0.75	0.48	0.78	0.42	0.67	0.42	0.81	0.26	0.54
Foreign assets	0.32	0.56	0.34	0.53	0.32	0.44	0.32	0.62	0.19	0.33
Current account 2/	0.97	0.85	0.97	0.83	0.97	0.89	0.97	0.81	0.99	0.93
Trade balance 2/	0.76	0.70	0.68	0.63	0.76	0.73	0.76	0.67	0.66	0.74
Discount factor	-0.42	0.00	-0.48	0.00	-0.42	0.00	-0.42	0.00	-0.26	0.00
Autocorrelations										
Output	0.59	0.59	0.69	0.69	0.59	0.59	0.59	0.59	0.59	0.59
Consumption	0.97	0.84	0.97	0.88	0.97	0.88	0.97	0.81	0.99	0.93
Foreign assets	0.99	0.96	0.99	0.98	0.99	0.98	0.99	0.94	1.00	0.99
Current account 2/	0.57	0.51	0.67	0.62	0.57	0.54	0.57	0.49	0.59	0.56
Trade balance 2/	0.67	0.55	0.76	0.67	0.67	0.59	0.67	0.52	0.76	0.64
Discount factor	0.98	0.00	0.98	0.00	0.98	0.00	0.97	0.00	0.99	0.00

WHY GLOBAL AND LOCAL SOLUTIONS OF INCOMPLETE MARKETS MODELS DIFFER, AND WHY IT MATTERS

Inducing stationarity for local solutions

- Schmitt-Grohe & Uribe (03) proposed three ad-hoc ways to induce stationarity so that local methods can be used:
  1. Debt-elastic interest rate (**DEIR**) function: $r(b - \bar{b})$
  2. Resource cost of holding assets (**AHC**): $h(b - \bar{b})$
  3. Endogenous discounting (**ED**) as function of aggregate consumption: $C(b - \bar{b})$
- They found similar moments for RBC-SOE model
- DEIR widely used and assumed to yield accurate results
- Results differ sharply from global solution because of near-unit root nature of NFA under incomplete markets and local solutions overstating NFA autocorrelation
  - Using DEIR, Garcia-Cicco, Pancrazi & Uribe (10) concluded that RBC-SOE model cannot explain AR behavior of net exports

Autocorrelation functions of TB/Y



Autocorr. of Net Exports: Data v. Models

- Garcia-Cicco et. al. (10): NX is AR(1) in data but RBC-SOE model with DEIR yields near-unit root
- de Groot et al. (19,23): near-unit root of NX ***is not*** a property of the model. It is imposed by introducing DEIR to induce stationarity
- Heuristic argument:
  1. Definition of net exports: $tb_t = b_{t+1} - b_t R^*$
  2. Assume AR(1) process for NFA: $b_{t+1} = \rho b_t + \varepsilon_{t+1}$ and notice DEIR implicitly sets ρ when specifying its elasticity ψ. Garcia-Cicco et al. set it so that $\rho \approx 1$, so that DEIR is "inessential"

Autocorrelations of net exports and NFA

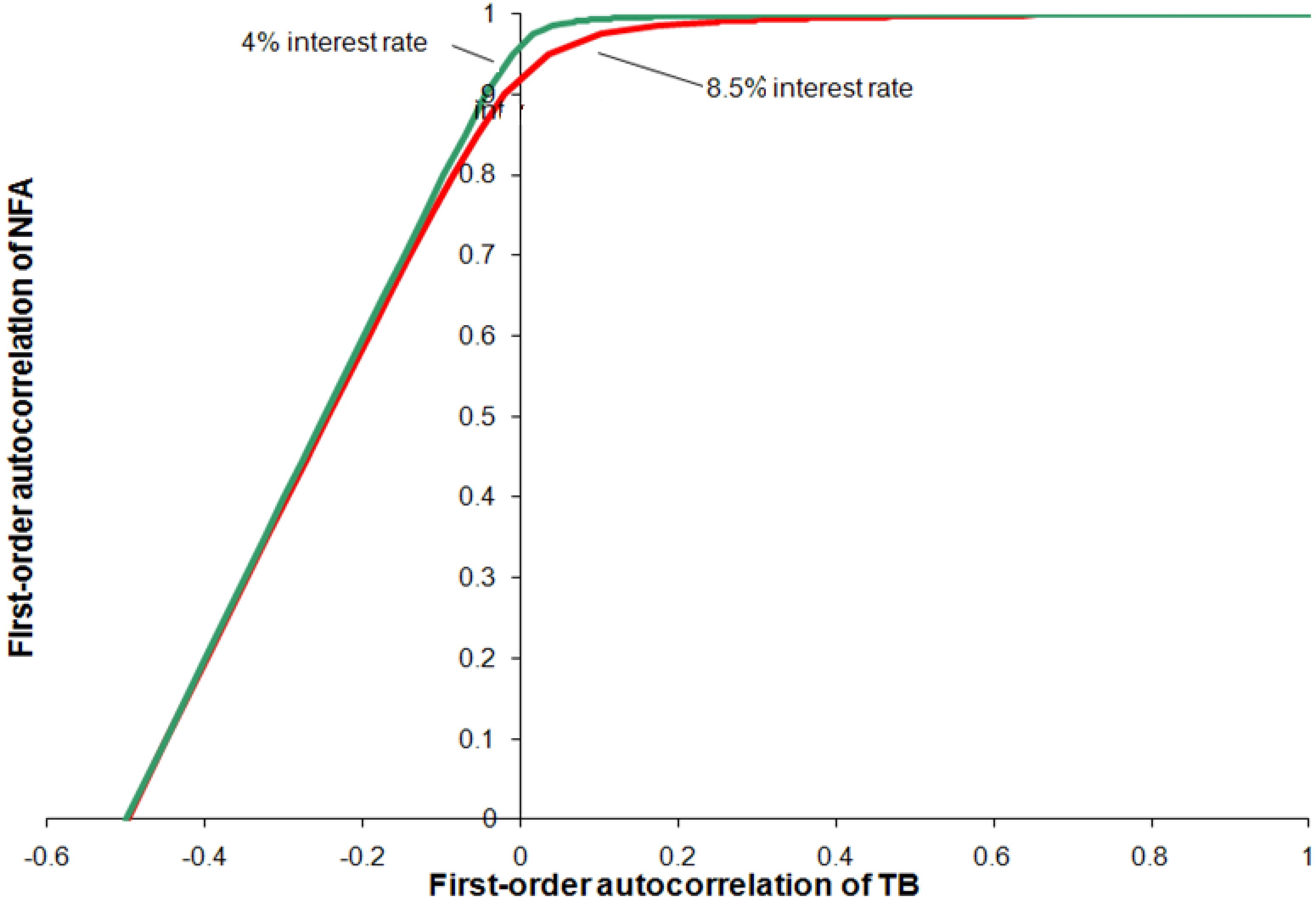
- Combine 1 & 2, solve for AR(1) of net exports:

$$\rho(nx) = \frac{q^2\rho + \rho - q - q\rho^2}{1 + q^2 - 2q\rho}$$

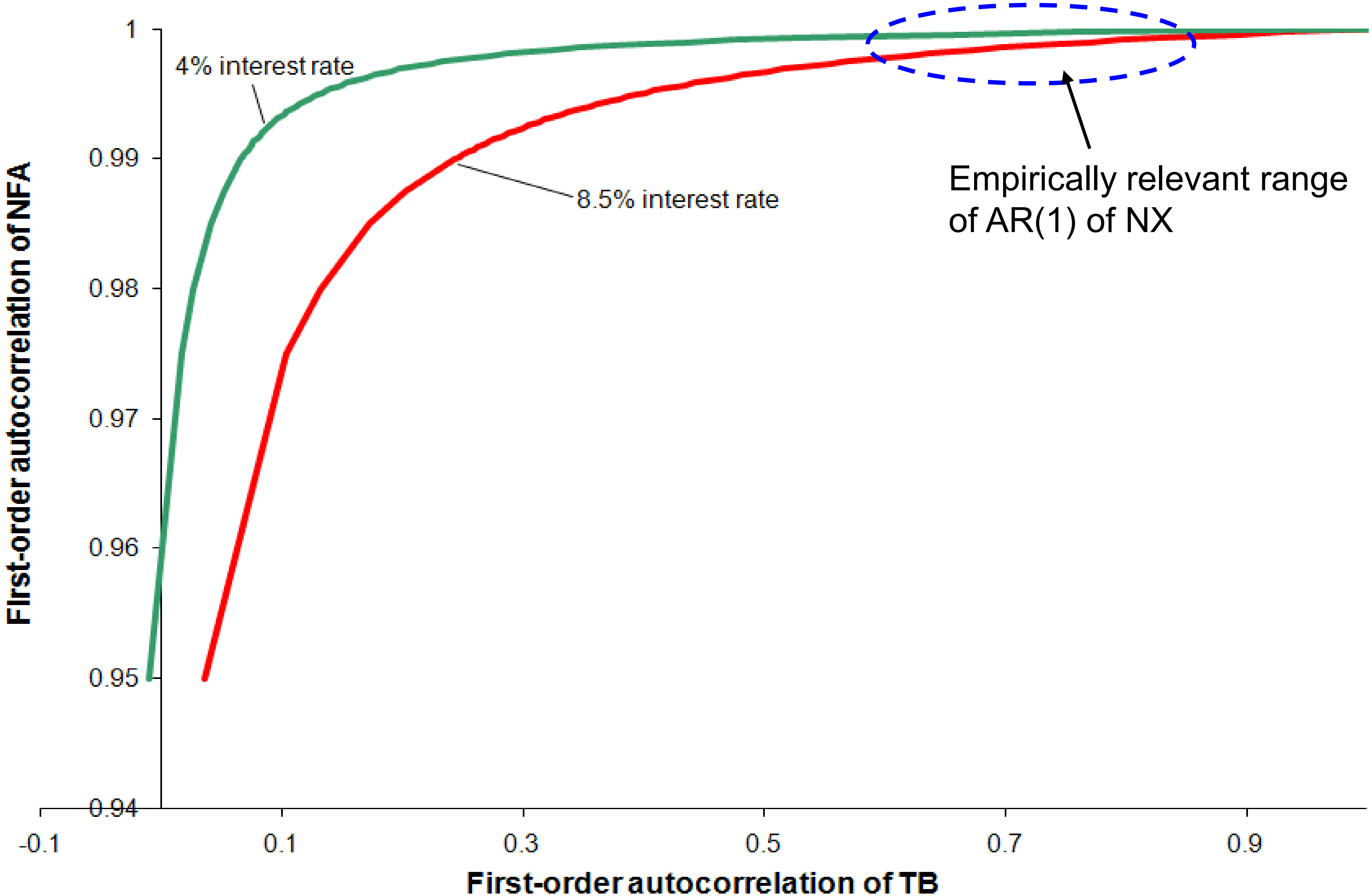
where $q = 1/R^*$

- $\rho(nx)$ is a nonlinear function of ρ, so we need very accurate solution for ρ in order to derive valid conclusions about $\rho(nx)$
  - Changing ρ from 0.95 to 0.999 changes $\rho(nx)$ from near zero to 0.999!!
  - Knowing true solution of NFA dynamics is critical

Autocorrelations of NFA and NX



Autocorrelations of NFA and NX



Why local and global solutions differ?
(de Groot, Durdu,& Mendoza (19, 23))

- Global solution is better at capturing history-dependence of prec. savings reflected in high persistence of NFA dec. rule
  - NFA autocorrelation is a moment of limiting distribution
- Stationarity-inducing assumptions effectively impose long-run average and AR of NFA
- This is critical for issues directly related to NFA:
  1. Global imbalances (accumulation of reserves)
  2. Financial crises & macro-prudential regulation
  3. Sovereign risk
  4. Financial development

…but still ad-hoc approach is widely used

- Allows using local methods that solve quickly and can be applied to large models
- DEIR is by far more common than AHC and ED
- Majority sets DEIR elasticity ψ to “inessential value” of 0.001 following SGU (2003), others calibrate it or estimate it (0.00014-2.8 range)
- Most applications use 1OA, some have used 2OA, 3OA or risky steady state (RSS)
- Quasi-linear methods for occ. binding constraints: OccBin (Iacoviello-Guerrieri), DynareOBC (Holden)

Goals & findings from de Groot et al. (19, 23)

- Compared global solution (*FiPIt*) v. 1OA, 2OA, RSS, OccBin/DynareOBC for endowment economy, RBC, and Sudden Stops (occ. binding collateral constraint)
- Local methods approximate poorly prec. savings
- Business cycle moments, IRFs, SDFs, and crises dynamics & frequency differ (except supply side)
- Best performance requires targeting moments from global sol. (e.g., autocorr. of NFA, s.d. of consumption)
- Various local methods differ mainly on 1st moments, and using targeted calibrations even those are similar

Intro to *FiPIt*: Model 2 again

- Optimization problem:

$$E_0 \left\{ \sum_{t=0}^{\infty} \beta^t u(c_t) \right\} \qquad u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$$

$$c_t = e^{z_t} \bar{y} + b_t - q b_{t+1}$$

$$b_{t+1} \geq -\varphi$$

- Optimality conditions in recursive form:

$$c(b,z) = e^{z} \bar{y} + b - q b'(b,z)$$

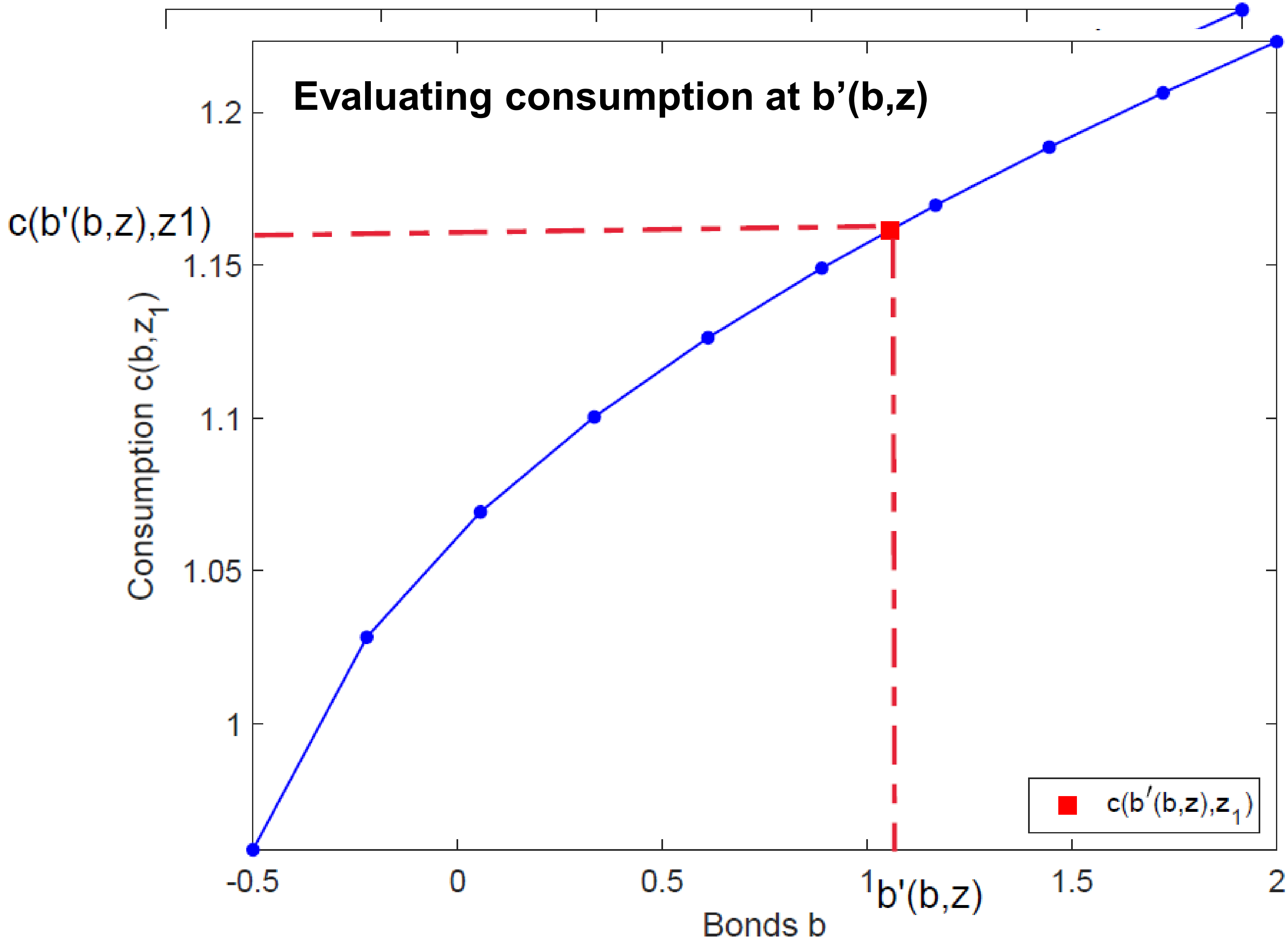
$$c(b,z)^{-\sigma} \geq \beta R \sum_{z'} \pi(z',z) \left[\left(c(b'(b,z),z') \right)^{-\sigma} \right]$$

FiPIT, a simple & fast global method

Mendoza-Villalvazo (2020)

1. Start iteration *j* with a conjectured decision rule $\hat{b}'_j(b,z)$
2. Generate the consumption dec. rule implied by that conjecture using the resource constraint
$$c_j(b,z) \ = \ e^z \bar{y} + b - q\hat{b}'_j(b,z)$$
3. Solve for a new consumption dec. rule "directly" using the Euler eq. (assuming φ is not binding)
$$c_{j+1}(b,z) = \left\{ \beta R \sum_{z'} \pi(z',z) \left[\left(c_j\left(\hat{b}'_j(b,z), z'\right)\right)^{-\sigma} \right] \right\}^{-\frac{1}{\sigma}}$$
   - In RHS, form c_{t+1} by evaluating the j-th iteration cons. dec. rule using the values of the state variables at t+1
   - Use linear interpolation ($c_j(b,z)$ is only known at grid nodes!)
   - No need for a non-linear solver as with time iteration method

Evaluating consumption decision rule



FiPIT Method Contn'd

4. Generate new bond's decision rule $b'_{j+1}(b,z)$ using the resource constraint. If $b'_{j+1}(b,z) \leq -\varphi$, the debt limit binds and we set $b'_{j+1}(b,z) = -\varphi$

5. Update the initial conjecture for iteration j+1:

$$\hat{b}'_{j+1}(b,z) = (1-\rho)\hat{b}'_j(b,z) + \rho b'_{j+1}(b,z).$$

   – $0 < \rho < 1$ if unstable, $\rho > 1$ for slow convergence

6. Iterate until this convergence criterion holds

$$\max |b'_{j+1}(b,z) - \hat{b}'_j(b,z)| \leq \epsilon^b, \quad \forall (b,z) \in B \times Z$$

7. Compute ergodic distribution, moments, IRFs etc

- Analogous to Parameterized Expectations (fixed-point iteration using simulation & regression in Step 3)
- Extends easily to 2 endogenous states w. bilinear interpolation

Local methods

- **1OA, 2OA**: standard approximations of NFA dec. rule applied to approximations of same order to opt. conditions around b^{dss} (DEIR with $\beta(1+r)=1$)

- Use DEIR to support b^{dss}

$$\frac{1}{q_t} \equiv 1 + r_t = 1 + r + \psi \left[e^{b^{dss} - b_{t+1}} - 1 \right]$$

  – ψ can be SGU *baseline* inessential value (0.001) or *targeted* to a particular moment (e.g. autocorr. of nfa)

- **Fulls RSS**: b^{rss} from 2OA of cond. expectation of steady-state Euler eq., solved *jointly* with 1OA of decision rule around b^{rss} assuming $\beta(1+r) < 1$

  – **Partial RSS** combines b^{rss} with DEIR and $\beta(1+r)=1$

Local methods contn'd

- 2OA to NFA decision rule in dev. form:

$$\tilde{b}_{t+1} = h_b \tilde{b}_t + h_y \tilde{y}_t + \frac{1}{2}\left(h_{bb} \tilde{b}_t^2 + h_{yy} \tilde{y}_t^2 + h_{\sigma_z \sigma_z} \sigma_z^2 \right) + h_{by} \tilde{b}_t \tilde{y}_t$$

  a) 1OA and pRSS have only the first two terms in RHS
  b) pRSS uses risky ss. instead of det. ss to define devs.
  c) h_b has same value regardless of approx. order
  d) $h_{\sigma_z \sigma_z} \sigma_z^2$ captures effect of income variability on NFA (prec. savings). In RSS it also matters for risky ss.
  e) Quantitatively, all other 2nd order terms are negligible

- Assuming log utility and i.i.d. shocks:

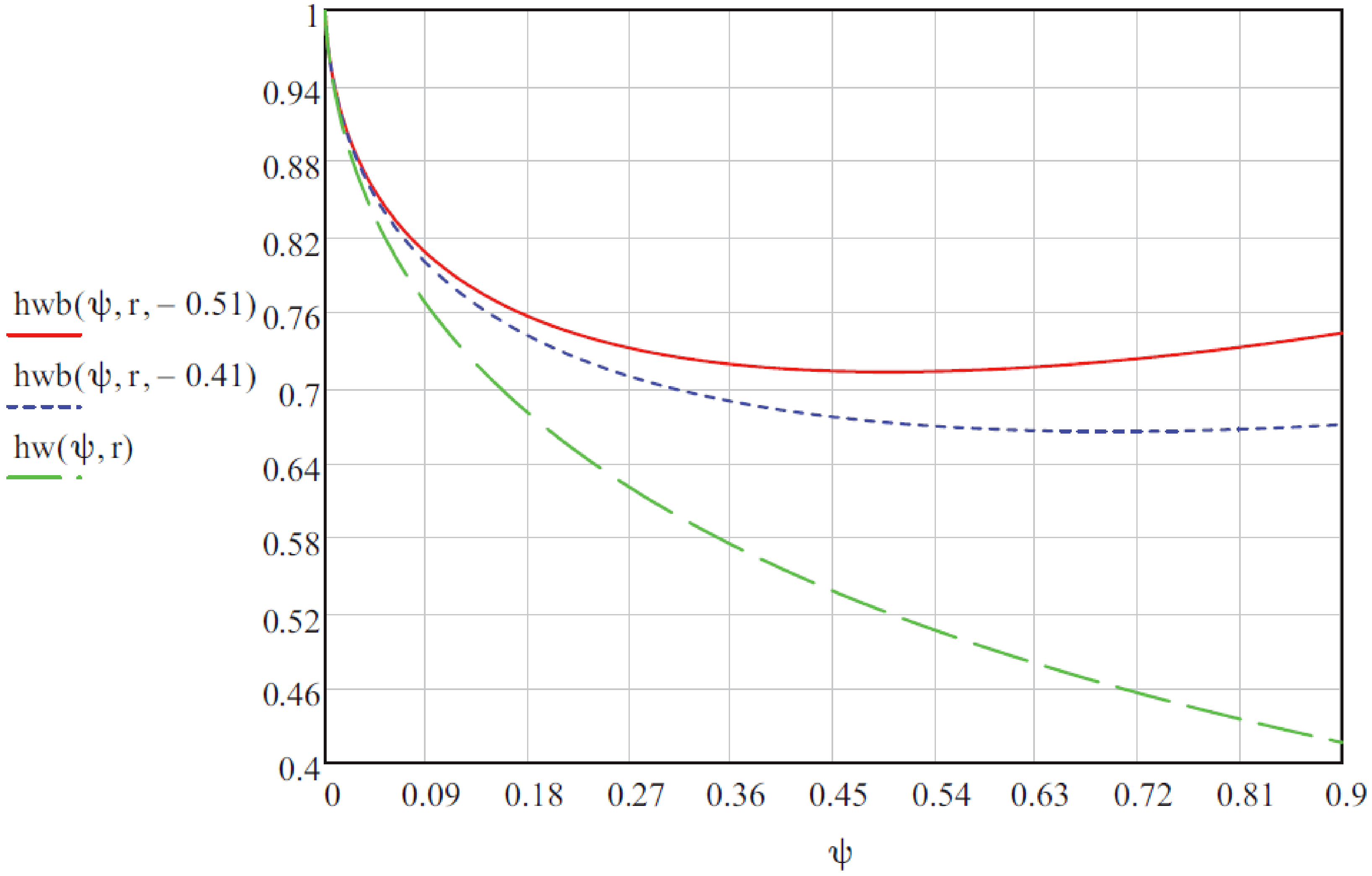
$$h_b(\psi, b^*) = \frac{R + e^{b^*\psi}(1 - b^*\psi + \psi) - \sqrt{R^2 + 2e^{b^*\psi}(b^*\psi + \psi - 1)R + e^{2b^*\psi}\left(1 - b^*\psi + \psi\right)^2}}{2e^{b^*\psi}}$$

  – Hence, $\rho_b(\psi, b^*) \approx h_b(\psi, b^*)$

NFA autocorr. & the three local methods

- $\rho_b(\psi, b^*)$ maps debt elasticity parameter into NFA autocorr. in local solutions
  - If $\psi = 0$, we get 2 roots given by (1+r,1), so NFA is non-stationary.
- Given (R, b*), $\rho_b(\psi, b^*)$ is a U-shaped function of ψ, but in quantitatively relevant range is downward sloping, convex.
- Plot $\rho_b(\psi, b^*)$ as ψ varies for b*=0, -0.51 (det. ss.) and -0.41 (risky ss.)
- For $0 \leq \psi \leq 0.1$, $\rho_b(\psi, b^*)$ is nearly identical for 1OA, 2OA & pRSS!
- Since 2nd order terms (except $h_{\sigma_z \sigma_z} \sigma_z^2$) are negligible, all three methods have very similar 2nd & higher-order moments and IRFs, and pruning is irrelevant!

Elasticity of DEIR function & NFA dec. rule



Calibration

1. Common parameters

σ	Coefficient of relative risk aversion	2.0
y	Mean endowment incom	1.00
A	Total absorption	0.28
R	Gross world interest rate	1.059
σ_z	Standard deviation of income (percent)	3.27
ρ_z	Autocorrelation of income	0.597

2. Global solution parameters

β	Discount factor	0.940
ϕ	Ad-hoc debt limit	-0.51

3. Local solution parameters

Common parameters

β	Discount factor	0.944
$\bar{b}$	Deterministic steady state value of NFA	-0.51

Baseline calibration

ψ	Inessential DEIR coefficient	0.001

Targeted calibration

ψ	DEIR coefficeint for 2OA	0.0469
ψ	DEIR coefficient for RSS	0.0469

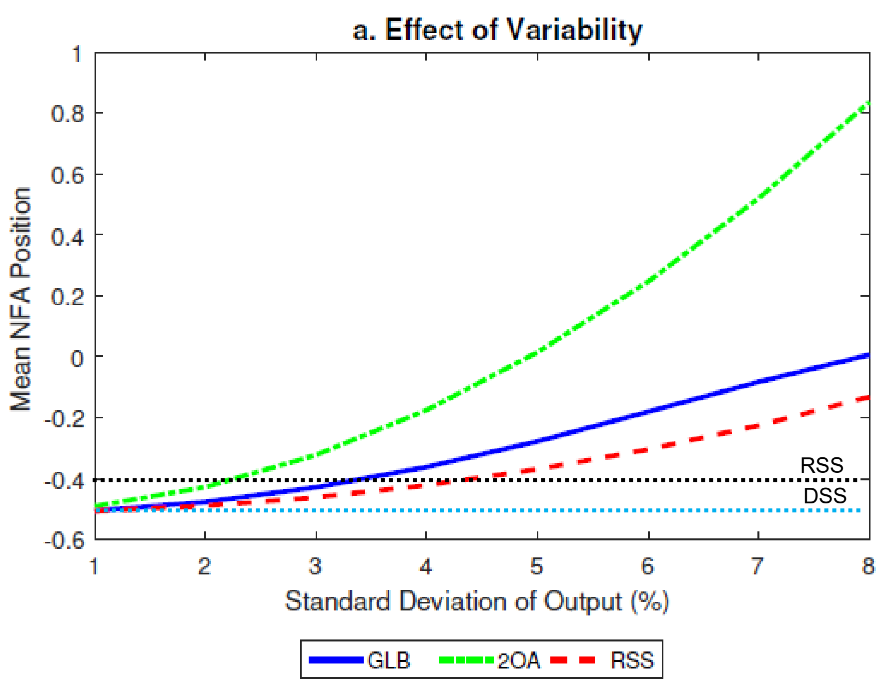
Comparison of long-run moments

	GLB	Baseline Calibration: 2OA DEIR	Baseline Calibration: RSS $\beta R < 1$	Baseline Calibration: RSS DEIR	Targeted Calibration: 2OA DEIR	Targeted Calibration: RSS DEIR
$\psi =$	*na*	0.001	*na*	0.001	0.0469	0.0469
Averages						
$\mu(c)$	0.694	0.701	0.093	0.692	0.689	0.689
$\mu(nx/y)$	0.022	0.015	0.625	0.025	0.028	0.028
$\mu(b/y)$	-0.413	-0.282	-11.210	-0.451	-0.502	-0.506
Standard deviations relative to standard deviation of income						
$\sigma(c)/\sigma(y)$	0.992	1.594	1.161	1.617	1.001	0.997
$\sigma(nx)/\sigma(y)$	0.660	1.327	1.202	1.346	0.730	0.730
$\sigma(nx/y)/\sigma(y)$	0.643	1.311	1.161	1.331	0.709	0.709
$\sigma(b)/\sigma(y)$	7.461	62.327	1.706	40.078	6.647	6.576
$\sigma(b/y)/\sigma(y)$	7.735	61.989	1.892	40.213	7.174	7.118
Income correlations						
$\rho(y,c)$	0.755	0.202	0.188	0.197	0.684	0.684
$\rho(y,nx)$	0.729	0.572	0.312	0.567	0.705	0.708
$\rho(y,nx/y)$	0.704	0.572	0.006	0.567	0.705	0.708
$\rho(y,b)$	0.449	0.128	0.070	0.124	0.489	0.488
$\rho(y,b/y)$	0.549	0.156	0.445	0.149	5.593	0.592
First-order autocorrelations						
ρ_c	0.840	0.995	0.996	0.995	0.929	0.929
ρ_{nx}	0.543	0.819	0.934	0.823	0.583	0.582
$\rho_{nx/y}$	0.551	0.826	0.995	0.830	0.591	0.590
ρ_b	0.977	0.999	0.999	0.999	0.977	0.977
$\rho_{b/y}$	0.961	0.998	0.953	0.998	0.958	0.959

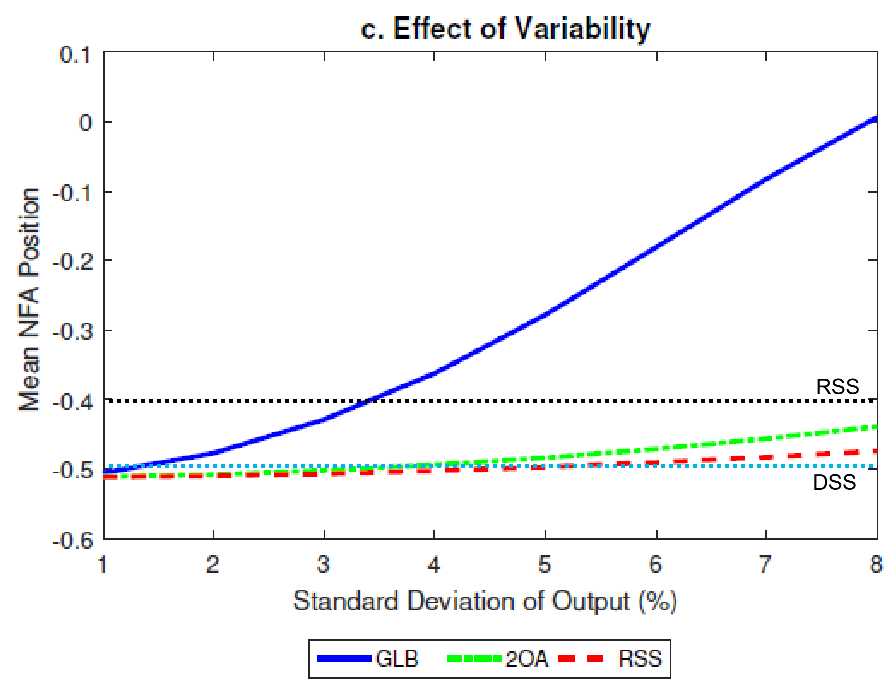
DSS

Effect of higher income variability on mean NFA

Baseline Calibration

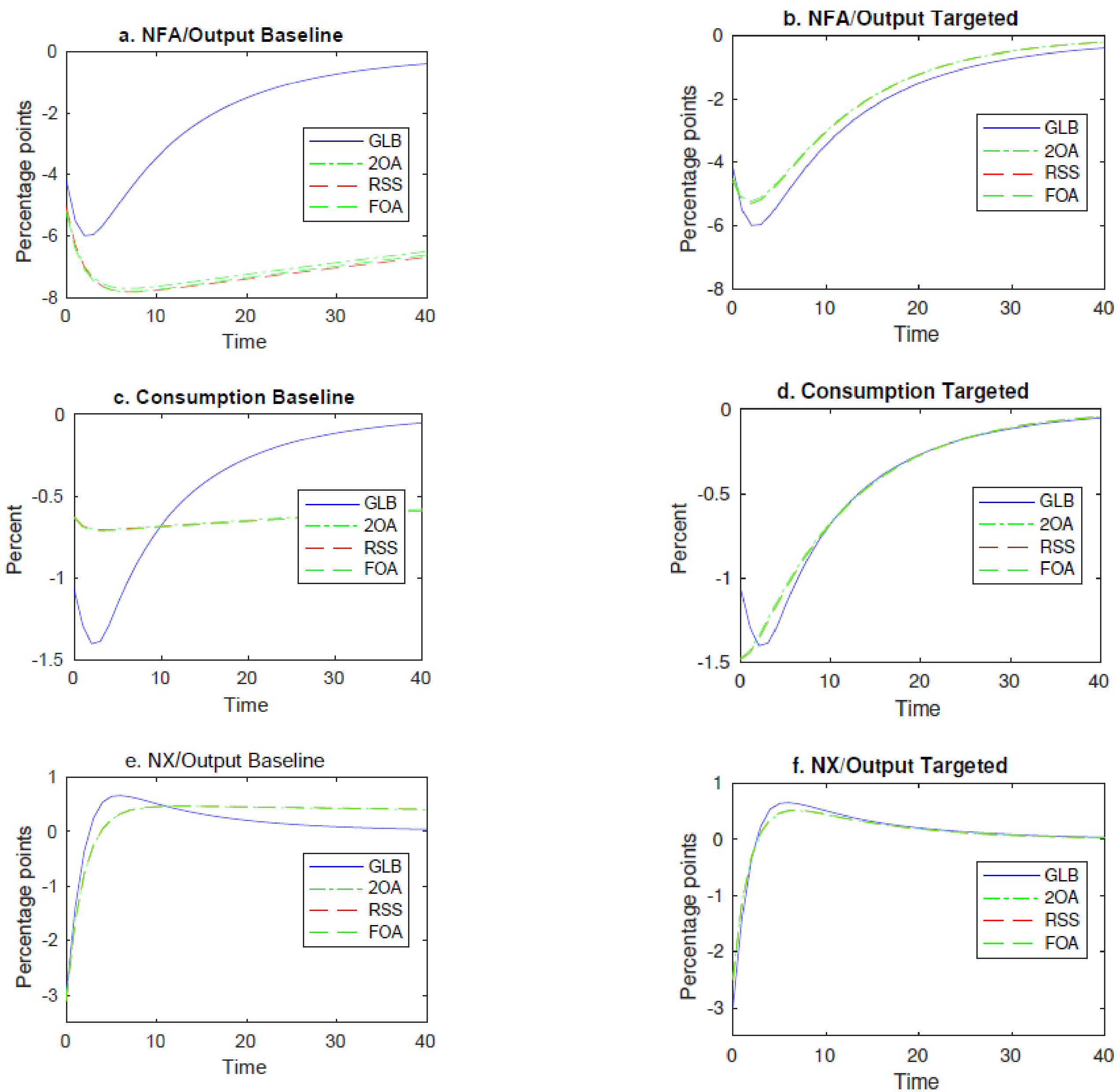


Targeted Calibration



As SGU (03) showed, DEIR and AHC are equivalent up to 1OA. Hence higher ψ is like higher adj. cost, which keeps NFA close to its mean (even 1st moments are similar across local methods!)

Impulse response functions



Comparing RBC-SOE solutions

- *FiPIt* extends easily to models with two endogenous states like RBC-SOE (will discuss in detail in models w. financial frictions)
  - Much faster than time iteration and endogenous grids
- de Groot et al. (19,23) compare global and 1OA, 2OA, 3OA, full and partial RSS solutions
- Similar qualitative findings as for endowment model, except labor, inputs, output and investment are similar because of GHH and near Fisherian separation

Calibration

1. Common parameters

σ	Coefficient of relative risk aversion	2.0
R	Gross world interest rate	1.0857
α	Labor share in gross output	0.592
γ	Capital share in gross output	0.306
η	Imported inputs share in gross output	0.102
δ	Depreciation rate of capital	0.088
ω	Labor exponent in the utility function	1.846
ϕ	Working capital constraint coefficient	0.2579
a	Investment adjustment cost parameter	2.75
τ	Consumption tax	0.168
κ	Collateral constraint coefficient	0.20
y^{dss}	GDP at the deterministic steady state	396

2. RBC global solution parameters

β	Discount factor	0.920
φ	Ad-hoc debt limit as a share of y^{dss}	-0.758

3. RBC local solution parameters

Common Parameters

β	Discount factor	0.9211
b^{dss}/y^{dss}	NFA/GDP at the deterministic steady state	-0.758

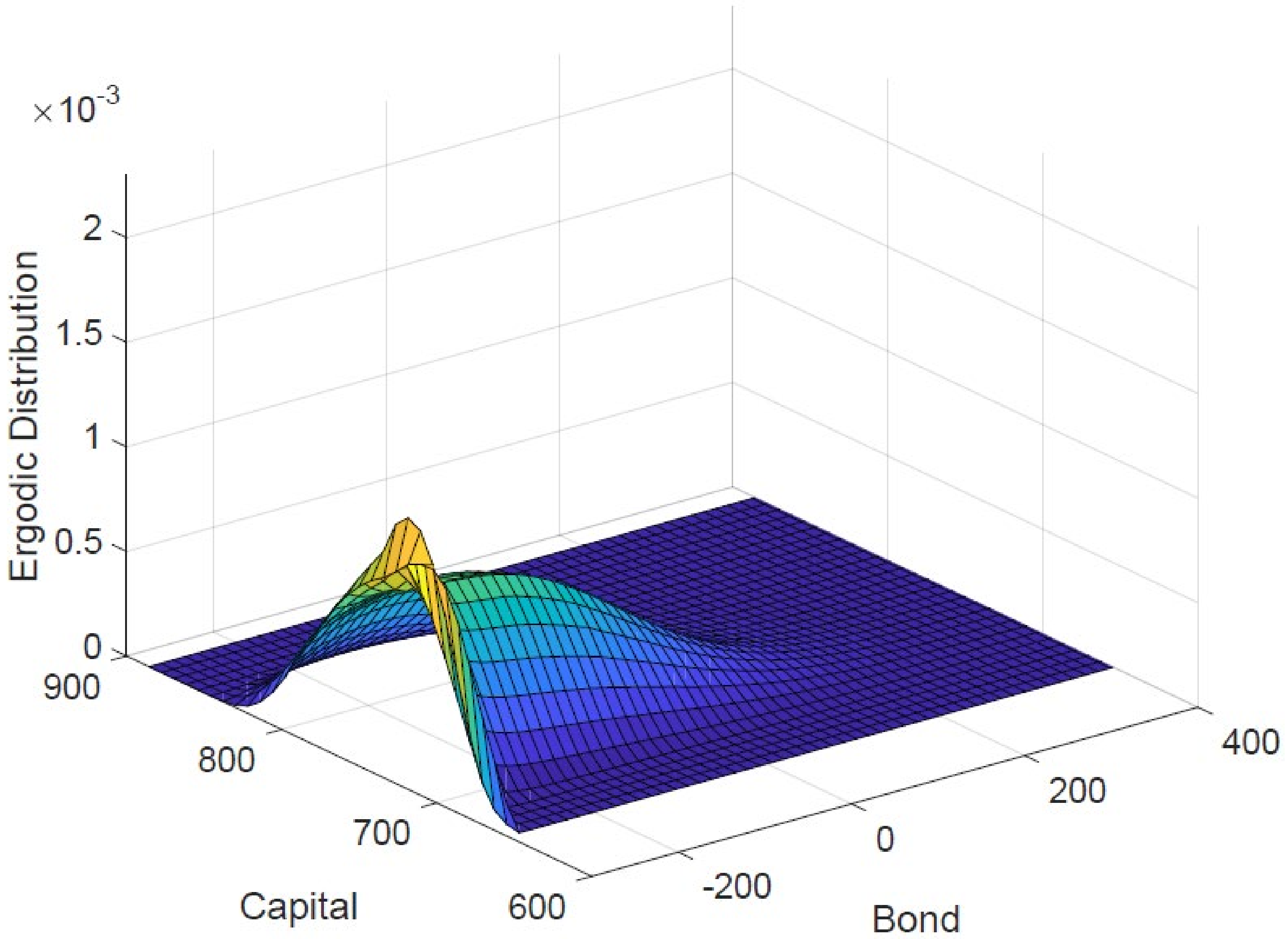
Baseline Calibration

ψ	Inessential DEIR coefficient	0.001

Targeted Calibration

ψ	DEIR coefficient for 2OA	0.0109
ψ	DEIR coefficient for RSS	0.008

Limiting distribution of capital and NFA



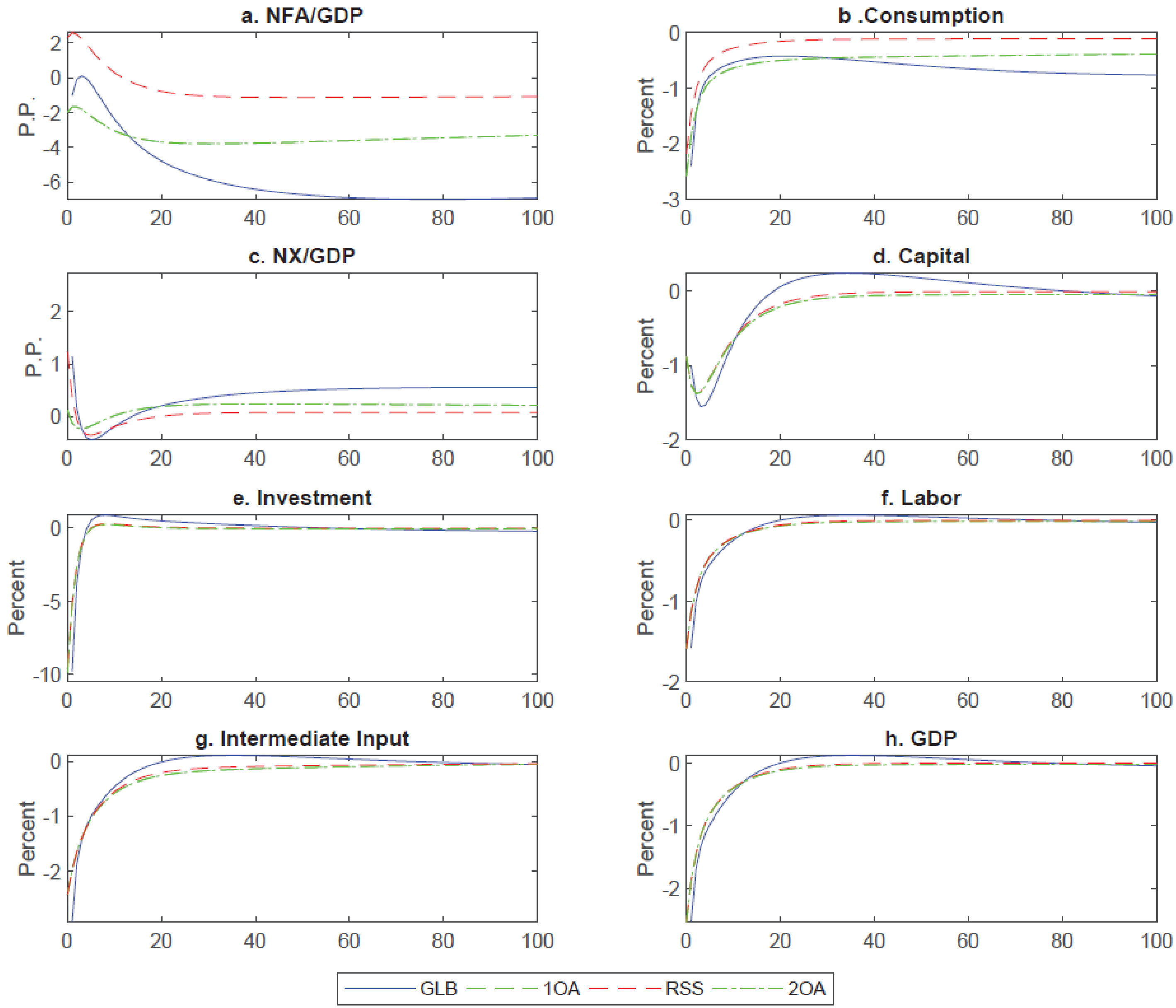
Unconditional moments

	GLB	Baseline Calibration 2OA	Baseline Calibration RSS	Targeted Calibration 2OA	Targeted Calibration RSS
$\psi =$	na	0.001	0.001	0.0109	0.008
Averages					
$E(y)$	393.847	397.269	396.190	397.370	397.210
$E(c)$	264.021	295.599	342.850	259.519	265.420
$E(i)$	67.53	68.631	67.747	68.666	68.063
$E(nx/y)$	0.045	-0.042	-0.185	0.065	0.046
$E(b/y)$	-0.372	0.732	2.559	-0.620	-0.397
$E(lev.rat.)$	-0.286	-0.237	-1.100	0.400	0.295
$E(v)$	42.649	43.009	42.852	43.021	42.975
$E(L)$	18.433	18.523	18.499	18.525	18.528
Variability relative to variability of GDP					
$\sigma(y)$	0.040	0.039	0.039	0.041	0.040
$\sigma(c)/\sigma(y)$	1.291	1.752	1.412	1.252	1.212
$\sigma(i)/\sigma(y)$	3.386	3.448	3.493	3.305	3.388
$\sigma(nx/y)/\sigma(y)$	0.885	1.389	1.212	0.718	0.731
$\sigma(b/y)/\sigma(y)$	7.589	15.064	12.909	3.822	4.269
$\sigma(lev.rat.)/\sigma(y)$	3.614	7.149	6.084	1.884	2.053
$\sigma(v)/\sigma(y)$	1.481	1.493	1.504	1.461	1.482
$\sigma(L)/\sigma(y)$	0.596	0.600	0.600	0.597	0.598

Unconditional moments contn'd

	GLB	Baseline Calibration 2OA	Baseline Calibration RSS	Targeted Calibration 2OA	Targeted Calibration RSS
$\psi =$	na	0.001	0.001	0.0109	0.008
Correlations with GDP					
$\rho(y, c)$	0.773	0.613	0.509	0.928	0.904
$\rho(y, i)$	0.640	0.632	0.628	0.660	0.648
$\rho(y, nx/y)$	-0.227	-0.280	0.026	-0.476	-0.381
$\rho(y, b/y)$	0.090	0.207	-0.160	0.508	0.343
$\rho(y, lev.rat.)$	0.112	0.212	0.150	0.528	-0.366
$\rho(y, v)$	0.834	0.831	0.830	0.839	0.835
$\rho(y, L)$	0.995	0.995	0.995	0.995	0.995
First-order autocorrelations					
$\rho(y)$	0.830	0.825	0.820	0.841	0.853
$\rho(b)$	0.996	0.999	0.998	0.996	0.996
$\rho(c)$	0.885	0.947	0.918	0.874	0.862
$\rho(i)$	0.516	0.511	0.509	0.519	0.513
$\rho(nx/y)$	0.711	0.869	0.843	0.560	0.563
$\rho(lev.rat.)$	0.997	0.999	0.998	0.991	0.995
$\rho(v)$	0.780	0.777	0.774	0.788	0.782
$\rho(L)$	0.808	0.803	0.799	0.819	0.810

Baseline IRFs to negative TFP shock



Targeted IRFs to negative TFP shock