What is a sustainable public debt?

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Public debt sustainability

- Literal definition: *a sustainable debt is that which can be maintained at a certain rate or level*

- In macro literature:
  1. Under commitment: debt consistent with solvency (IGBC) and/or a stationary equilibrium
  2. Without commitment: debt supported in equilibria with default risk

- Critical question in fiscal policy analysis
  – 2008-11, debt ratios rose by 31 (20) ppts. in U.S. (Europe)
  – Global market of local-currency gov. bonds was 1/2 of world's GDP in 2011 ($30 trillion, 6 times investment-grade external sov. debt)

Layout of the lecture

1. Critical review of "classic" approach
2. *Empirical approach*: Bohn's Fiscal Reaction Function
3. *Structural approach*: Two-country DGE with fiscal sector that matches actual tax base elasticities
4. *Domestic default approach*: Model of optimal default driven by distributional incentives
5. New applications to U.S. and cross-country data, and analysis of their implications

Generic government budget constraints

- Period GBC with Arrow gov. securities:

$$\sum_{s_{t+1}} Q_1(s_{t+1}|s_t)b_t(s_{t+1}|s^t) - b_{t-1}(s_t|s^{t-1}) = g(s_t) - \tau_t(s^t).$$

  – In GDP ratios and under perfect foresight:

$$b_t - (1+r_t)b_{t-1} = -pb_t \qquad r_t \equiv (1+i_t^r)/(1+\gamma_t) - 1$$

- NPG condition + arbitrage yields IGBC:

$$b_{t-1} = pb_t + \sum_{j=1}^{\infty} E_t[MRS(c_{t+j}, c_t)pb_{t+j}] \qquad pb_t \equiv \tau_t - g_t$$

Classic approach

- Proposed by Buiter (1985), Blanchard (1990), and widely used in policy institutions (IMF, 2015)
- At steady-state & under perfect foresight, GBC yields "Blanchard ratio" (debt-stabilizing *pb*):

$$b^{ss} = \frac{pb^{ss}}{r}$$

- First flaw: Disconnected from initial debt and IGBC
  - FRFs with different coefficients satisfy IGBC for same initial debt but converge to different steady states, and can even go to infinity!

Classic approach (contn'd)

- Second flaw: Ignores uncertainty & asset markets
- Mendoza & Oviedo (06, 09): under incomplete markets, adding shocks + smoothing (or tolerable min. outlays) yields "Natural Public Debt Limit:"

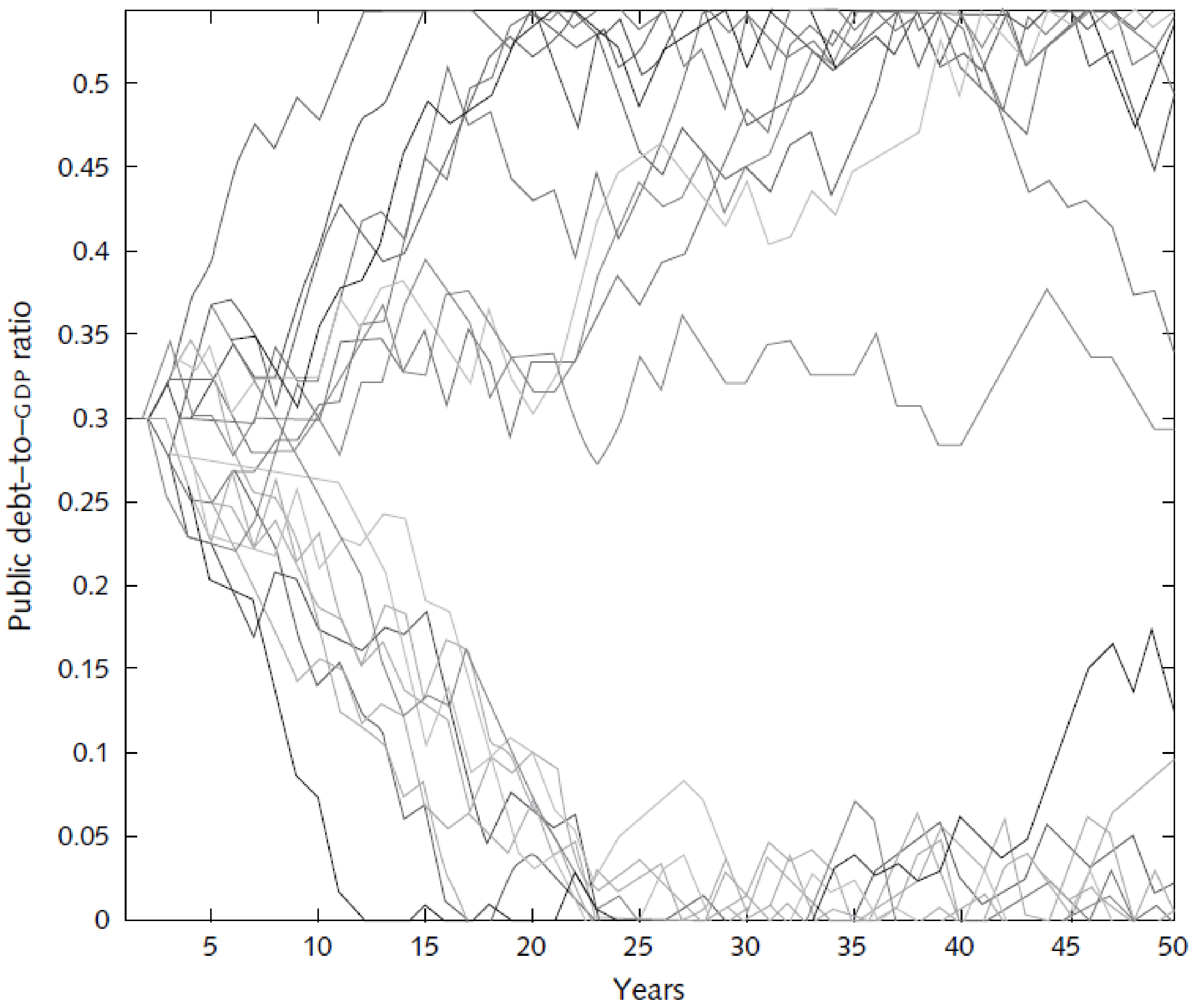
$$b_t \leq NPDL \equiv \frac{\tau^{\min} - g^{\min}}{i^r - \gamma}$$

  - Blanchard ratio uses l.r. means (always violates NPDL)
  - NPDL tighter for economies with more volatile revenues or less able to adjust outlays
  - Debt follows random walk with boundaries:

$$b_t = \max[NPDL, (1 + r_t)b_{t-1} - pb_t] \geq \bar{b}$$

Argentina: Simulated debt dynamics

(starting from 30% debt, calibrated revenue process, g^{min}=12.4, NPDL=55.7)



Empirical Approach: Bohn's Contributions

1. IGBC tests discounting at risk free rate are misspecified:

$$b_{t-1} = pb_t + \sum_{j=1}^{\infty} \left[\frac{E_t[pb_{t+j}]}{R_{t+j}} + cov_t \left(MRS(c_{t+j}, c_t), pb_{t+j} \right) \right]$$

2. IGBC holds if debt or outlays+interest are integrated of <u>any</u> finite order (no particular integration order needed)

3. Linear FRF $pb_t = \mu_t + \rho b_{t-1} + \varepsilon_t,$ with $\rho > 0$ is <u>sufficient</u> for IGBC (debt is stationary if $\rho > r$, or diverges to infinity if $0 < \rho < r$ but is still sustainable!)

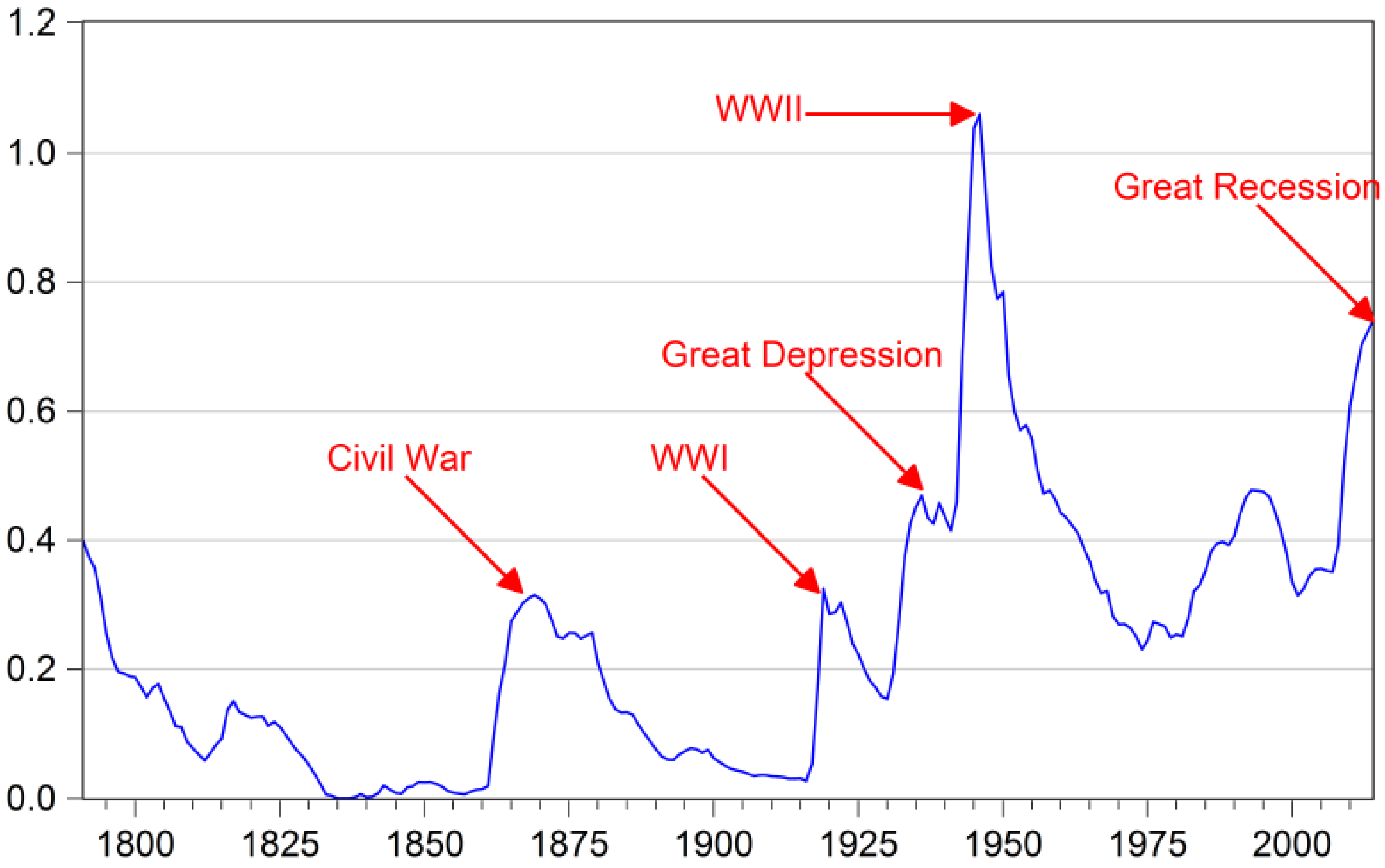
4. Empirical tests based on historical U.S. data 1791-2003 support linear FRF and some nonlinear variations

New FRF Estimates

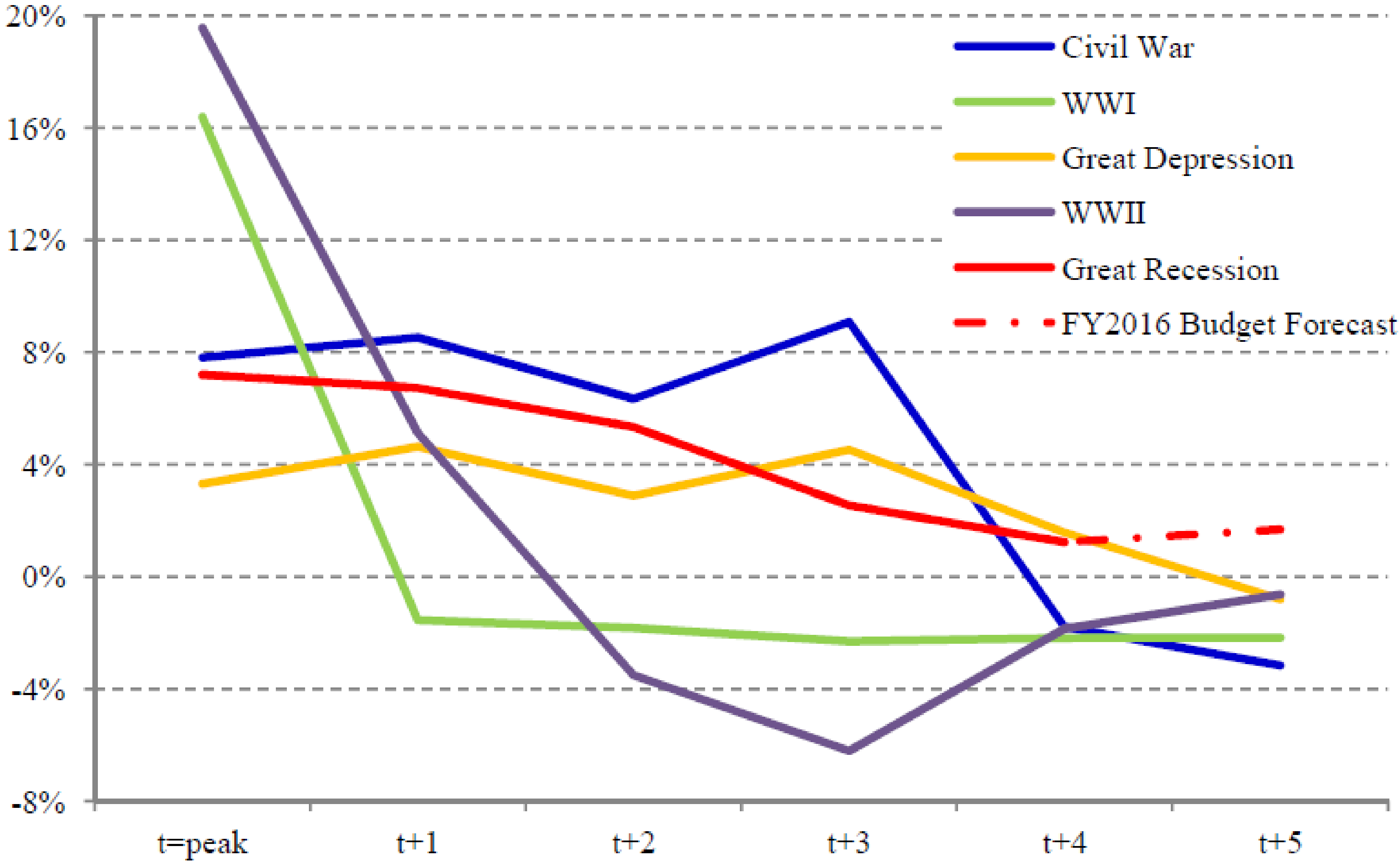
- U.S. estimates (1791-2014) and cross-country panels (1951-2013) again pass sufficiency test
  - EMs have stronger response, less access to debt
- Structural break post-2008 (lower response, large residuals, large primary deficits)
- U.S. deficits larger than in previous “debt crises,” much larger than out-of-sample pre-08 forecast
- FRFs with lower response coefficient satisfy IGBC at same initial debt, but with larger deficits & higher long-run debt

Public Debt Crises in U.S. History

(net federal debt-GDP ratio, 1791-2014)



U.S. Primary Deficits after Debt Crises



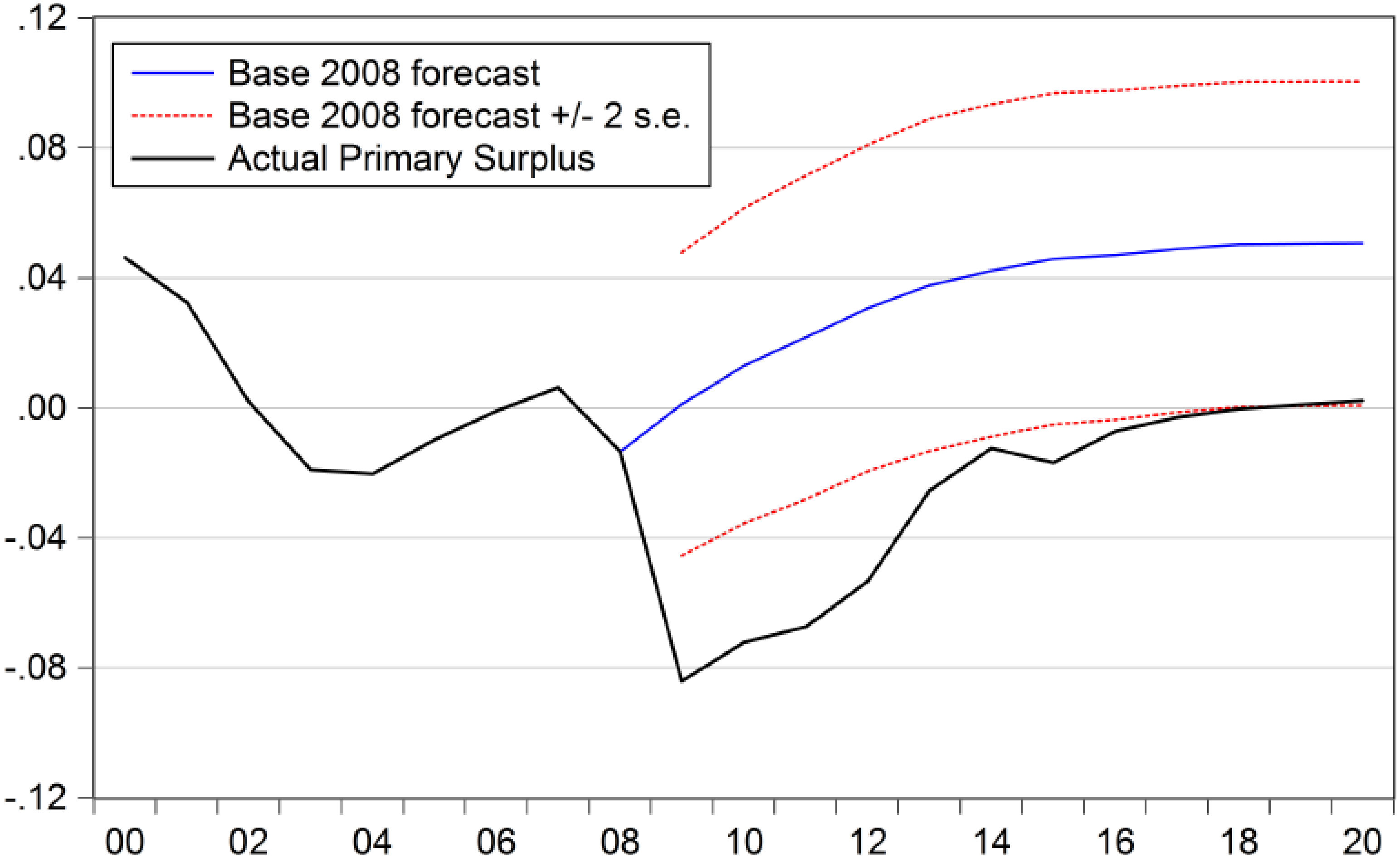
New FRF Estimates: U.S. 1792-2014

Model: / Coefficient	Base model (1)	Asymmetric response (2)	AR(1) term (3)	Debt Squared (4)	Time trend (5)	Bohn's Sample (1793-2003) (6)	Pre-Recession (1793-2008) (7)
Constant	0.00648 (0.004)	0.00540 (0.003)*	0.00974 (0.008)	0.00653 (0.004)	0.00601 (0.006)	0.00485 (0.003)*	0.00470 (0.003)
Initial debt d_t^*	0.07779 (0.040)*	0.08689 (0.030)***	0.10477 (0.032)***	0.07715 (0.038)*	0.07674 (0.035)**	0.10498 (0.023)***	0.10188 (0.022)***
GDP gap	0.07404 (0.078)	0.07300 (0.079)	0.15330 (0.043)***	0.07390 (0.079)	0.07490 (0.077)	0.07987 (0.086)	0.07407 (0.086)
Military Expenditure	-0.72302 (0.133)***	-0.72001 (0.136)***	-0.98955 (0.110)***	-0.72320 (0.133)***	-0.72462 (0.135)***	-0.77835 (0.135)***	-0.76857 (0.135)***
$\max(0, d_t^* - \bar{d})$		-0.14487 (0.061)					
AR(1)			0.89154 (0.029)***				
$(d_t^* - \bar{d})^2$				0.00261 (0.044)			
Time trend					6.89E-06 (5.9E-05)		
s.e	0.0239	0.0240	0.198	0.0120	0.0240	0.0210	0.0209
Adj. R-squared:	0.606	0.605	0.901	0.614	0.605	0.695	0688
Observations:	223	223	222	223	223	213	217

Note: HAC standard errors shown in parenthesis, 2-lag window prewhitening. "*","**", "***" denote that the corresponding coefficient is statistically significant at the 90, 95 and 99 percent confidence levels. Output gap is percent deviation from Hodrick-Prescott trend. Military expenditure includes all Department of Defense and Department of Veterans Affairs outlays.

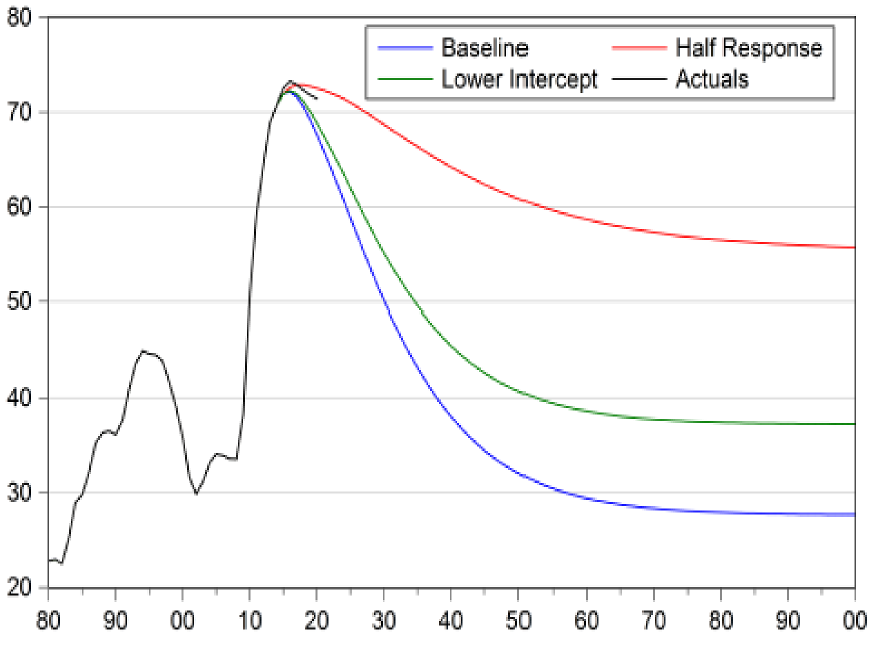
U.S. Primary Balance Post-2008 Forecast

(2009-2020 forecast from 1791-2008 FRF regression)

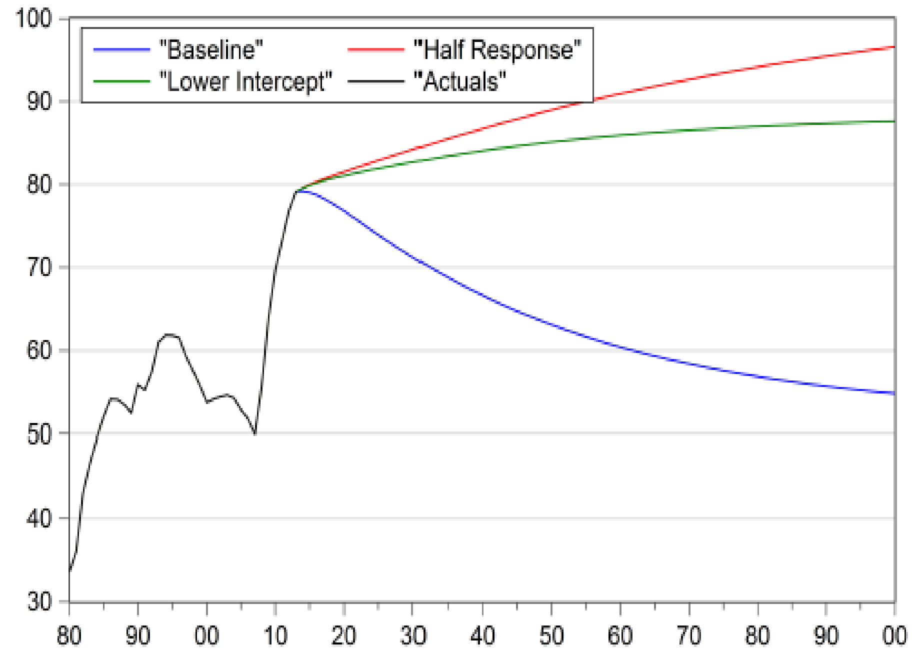


Out-of-sample forecast uses actual values for the independent variables for 2009-2014 and 2016 President's Budget for 2015-2020

Debt Projections: Alternative FRFs



(a) US debt to GDP



(b) Europe debt to GDP

Note: For the US: Model (3) in table 1 is used in conjunction with estimated AR(2) processes for the output gap and military expenditure, plus the government budget constraint. For Europe: Model (5) in table 2 is used in conjunction with estimated AR(1) processes for the output gap and government consumption gap in each country, and a simple average among advanced European countries is taken.

Structural Approach

- FRFs with different parameters satisfy IGBC for same initial debt, but macro dynamics and welfare differ and FRFs can't compare them
- Use calibrated variant of workhorse two-country Neoclassical model to compare fiscal adjustment policies in response to initial debt shocks
- Match observed elasticities of tax bases to tax changes by introducing endogenous utilization and limited depreciation tax allowance

Model highlights

- Deterministic setup with exogenous long-run growth driven by labor-augmenting technological change
- Fiscal sector includes taxes on capital, labor and consumption, gov. purchases, transfers and debt
- Utilization choice & limited tax allowance for depreciation
- Trade in goods and bonds (residence-based taxation)
- Capital immobile across countries, but trade in bonds arbitrages post-tax returns & induces capital reallocation
- Unilateral tax changes have cross-country externalities (relative prices, wealth distribution, tax revenues)

Households

- Maximize

$$\sum_{t=0}^{\infty} \left[\beta(1+\gamma)^{1-\sigma}\right]^t \frac{(c_t(1-l_t)^a)^{1-\sigma}}{1-\sigma}, \quad \sigma > 1, \ a > 0, \ \text{and } 0 < \beta < 1$$

subject to:

$$(1+\tau_c)c_t + x_t + (1+\gamma)(q_t b_{t+1} + q_t^g d_{t+1})$$

$$= (1-\tau_L)w_t l_t + (1-\tau_K)r_t m_t k_t + \theta\tau_K\bar{\delta}k_t + b_t + d_t + e_t,$$

$$x_t = (1+\gamma)k_{t+1} - (1-\delta(m_t))k_t + \phi(k_{t+1}, k_t, m_t)$$

given $\{k_0, b_0, d_0\}$

Firms

- Production technology:

$$y_t = F(\tilde{k}_t, l_t) = \tilde{k}_t^{1-\alpha} l_t^{\alpha}, \text{ with } \tilde{k} = mk.$$

- Firms maximize profits $y_t - w_t l_t - r_t \tilde{k}_t$

- Optimality conditions equate marginal products with pre-tax factor prices

Fiscal sector

- Gov. purchases and transfers are exogenous and kept constant at initial steady-state levels
- GBC:

$$d_t - (1+\gamma) q_t^g d_{t+1} = pb_t$$

$$\equiv \tau_C c_t + \tau_L w_t l_t + \tau_K (r_t m_t - \theta \bar{\delta}) k_t - (g_t + e_t)$$

- IGBC:

$$\frac{d_0}{y_{-1}} = \psi_0 \left[\frac{pb_0}{y_0} + \sum_{t=1}^{\infty} \left(\left[\prod_{i=0}^{t-1} v_i \right] \frac{pb_t}{y_t} \right) \right]$$

$$v_i \equiv (1+\gamma) \psi_i q_i^g, \quad \psi_i \equiv y_{i+1} / y_i$$

Tax distortions and externalities

- Asset markets arbitrage (ignoring capital adj.

$$\frac{(1+\gamma)u_1(c_t,1-l_t)}{\tilde{\beta}u_1(c_{t+1},1-l_{t+1})} = (1-\tau_K)F_1(m_{t+1}k_{t+1},l_{t+1})m_{t+1} + 1 - \delta(m_{t+1}) + \tau_K\theta\bar{\delta}$$

$$= \frac{1}{q_t} = \frac{1}{q_t^g}$$

$$= (1-\tau_K^*)F_1(m_{t+1}^*k_{t+1}^*,l_{t+1}^*)m_{t+1}^* + 1 - \delta(m_{t+1}^*) + \tau_K^*\theta\bar{\delta} = \frac{(1+\gamma)u_1(c_t^*,1-l_t^*)}{\tilde{\beta}u_1(c_{t+1}^*,1-l_{t+1}^*)}$$

- Labor market:

$$\frac{u_2(c_t,1-l_t)}{u_1(c_t,1-l_t)} = \frac{1-\tau_L}{1+\tau_C}F_2(k_t,l_t) \qquad (1-\tau_W) \equiv (1-\tau_L)/(1+\tau_C)$$

- Capacity utilization :

$$F_1(m_tk_t,l_t) = \frac{1+\Phi_t}{1-\tau_K}\delta'(m_t),$$

Calibration: Fiscal Heterogeneity

	GDP-weighted	
	EU15	US
(a) Macro Aggregates		
τ_C	0.17	0.04
τ_L	0.41	0.27
τ_K	0.32	0.37
c/y	0.57	0.68
x/y	0.21	0.21
g/y	0.21	0.16
tb/y	0.00	−0.05
Rev/y	0.45	0.32
$\text{Total Exp}/y$	0.47	0.39
(b) Debt Shocks		
d_{2007}/y_{2007}	0.38	0.43
d_{2011}/y_{2011}	0.58	0.74
$\Delta d/y$	0.20	0.31

Parameters from Data & Literature

α	labor income share	0.61		Trabandt and Uhlig (2009)
γ	growth rate	0.0038		Real GDP p.c. growth (Eurostat 2000–11)
ω	country size	0.46		GDP shares of the U.S. in 2008
σ	risk aversion	2.000		standard DSGE value
a	labor supply elasticity	2.675		$\bar{l} = 0.18$ (Prescott, 2004)
θ	depreciation allowance	0.20		$\frac{REV_K^{corp}}{REV_K} \frac{K^{NR}}{K}$ (Rev. Stat & EUKLEMS)
Fiscal Policy:		US	EU15	
g/y	gov't exp share in GDP	0.16	0.21	OECD National Income Accounts
τ_C	consumption tax	0.04	0.17	MRT modified
τ_L	labor income tax	0.27	0.41	MRT modified
τ_K	capital income tax	0.37	0.32	MRT modified

Parameters from Steady-State Conditions

$\delta(\bar{m})$	depreciation rate	0.0163	steady state law of motion of capital
	$\frac{x/y}{k/y} = \gamma - 1 + \delta(\bar{m})$		$\frac{x}{y} = 0.19$, $\frac{k}{y} = 2.62$ (OECD and AMECO)
β	discount factor	0.998	steady state Euler equation
	$\frac{(1+\gamma)^\sigma}{\beta} = 1 + (1-\tau_K)(1-\alpha)\frac{y}{k} - \delta(\bar{m}) + \theta\tau_K\bar{\delta}$		
(χ_0, χ_1)	utilization	(0.023, 1.44)	match $\delta(\bar{m})$ and $\bar{m}$
η	adjustment cost	2.0	match the midpoint of empirical short-run elasticities of capital tax base to a 1% increase in τ_K: [0.2, 0.5] Gruber & Rauh 2007, Dwaenger & Steiner 2012

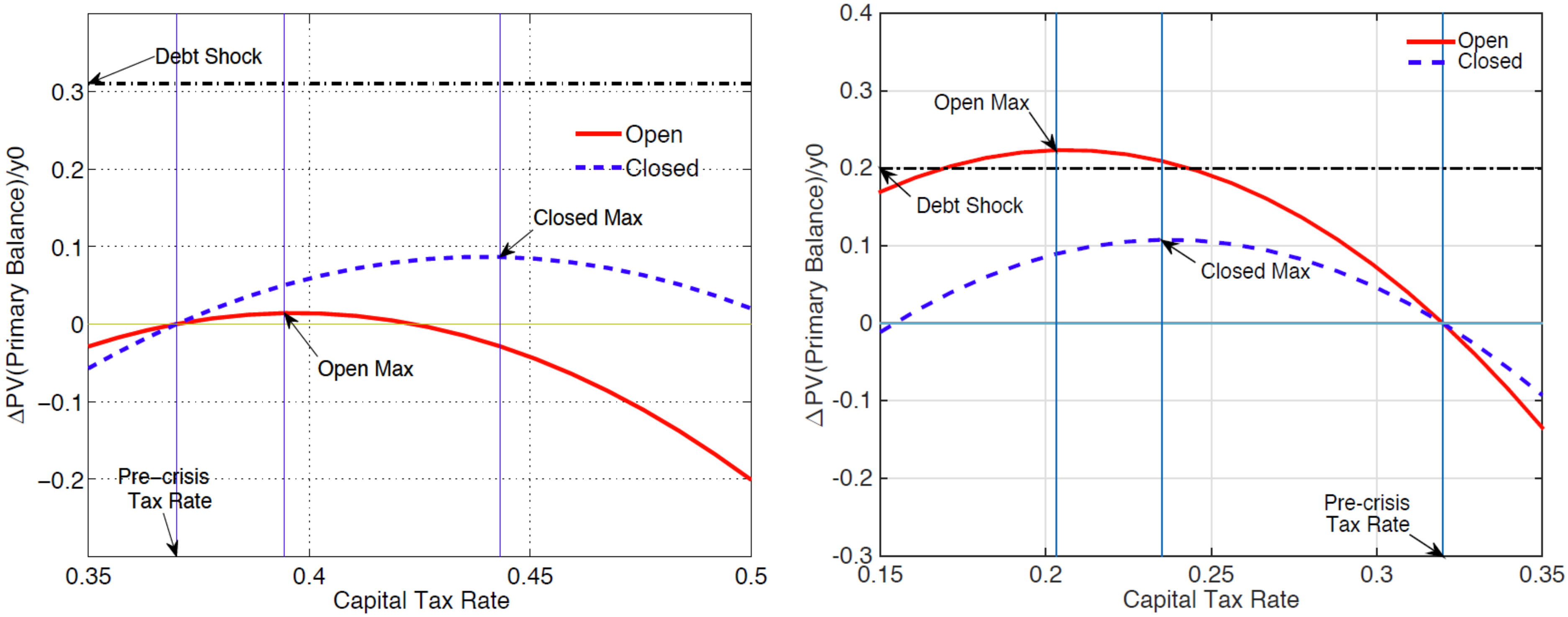
Quantitative exercises

- Unilateral changes in capital or labor taxes
- “Passive” country adjusts to tax externalities in order to maintain revenue neutrality (changes labor tax)
- Construct dynamic Laffer curves (DLCs): *change* in PDV of primary balance (i.e. in sustainable debt)
- Compare against what is needed to make actual increases in debt sustainable (match “new” IGBC)
- Examine macro dynamics and welfare effects
- Perturbation method with shooting routine (accounts for steady-state dependency on initial conditions)

Main findings

- Capital taxes:
  1. Large externalities (strategic incentives)
  2. US: debt *not* sustainable (DLC max below required level)
  3. EU15: inefficient side of DLC (tax cut makes debt sustainable via external effects--closed-economy DLC also peaks below required level)
  4. Without utilization and limited allowance short-run tax elasticity has wrong sign and DLC is linearly increasing
- Labor taxes:
  1. Negligible externalities
  2. US lower initial taxes yield DLCs that sustain high debt
  3. EU15: DLCs (closed or open) peak below required level

Capital Tax Dynamic Laffer Curves



(a) US

(b) EU15

Effects of Setting US Capital Tax at Max.

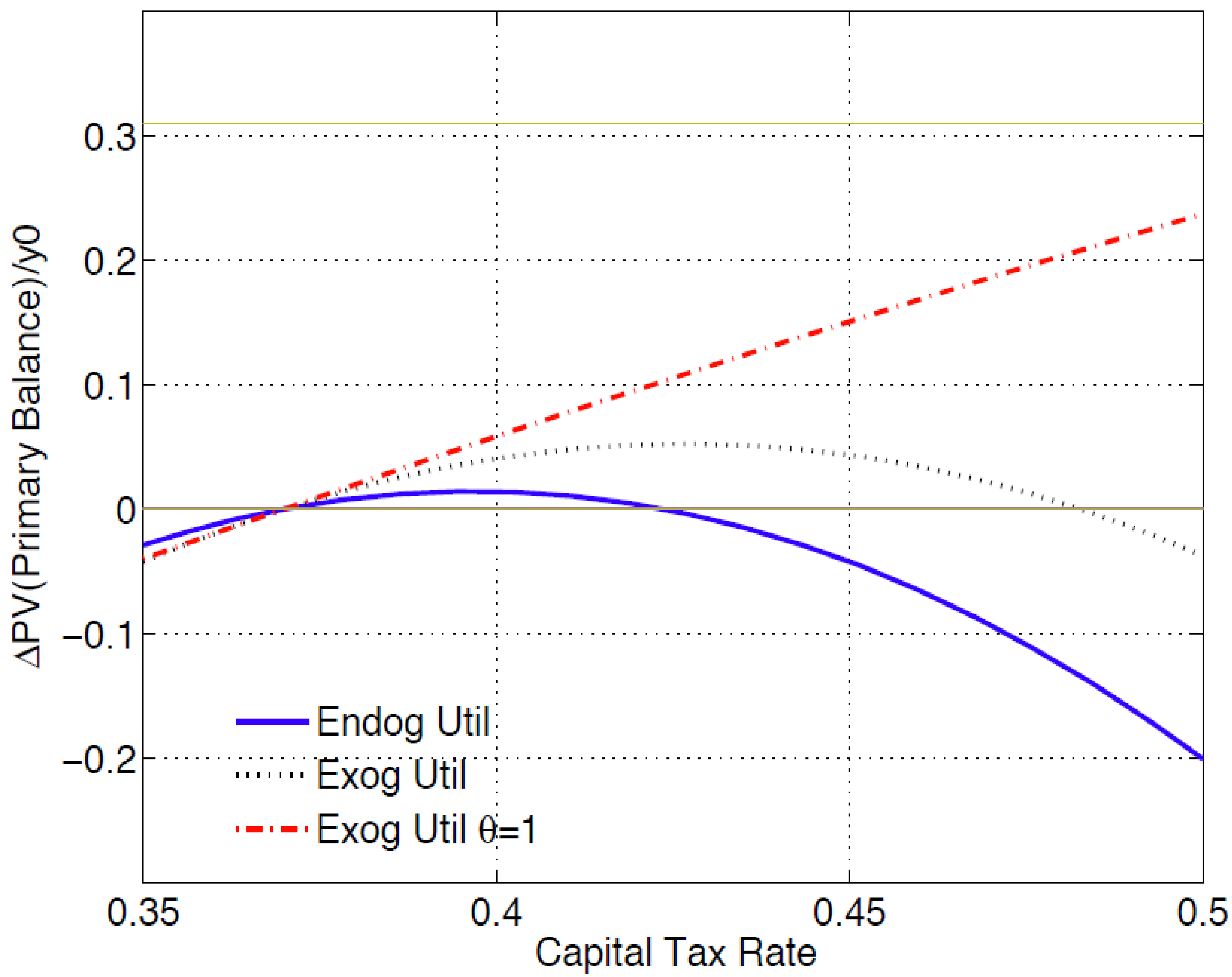
	Open Economy			
	Home		Foreign	
Tax rates	Old	New	Old	New
τ_K	0.37	0.40	0.32	0.32
τ_C	0.04	0.04	0.17	0.17
τ_L	0.27	0.27	0.41	0.40
ΔPV(Primary Bal.)/y_0		0.014		0.00
Welfare Impact		-2.19		0.74
Δy_{ss}		-3.87		1.25

Note: Capital tax increase to maximum point of open-economy Laffer curve. Fore neutrality by lowering labor tax.

Capital Tax Base Elasticities: Models v. Data

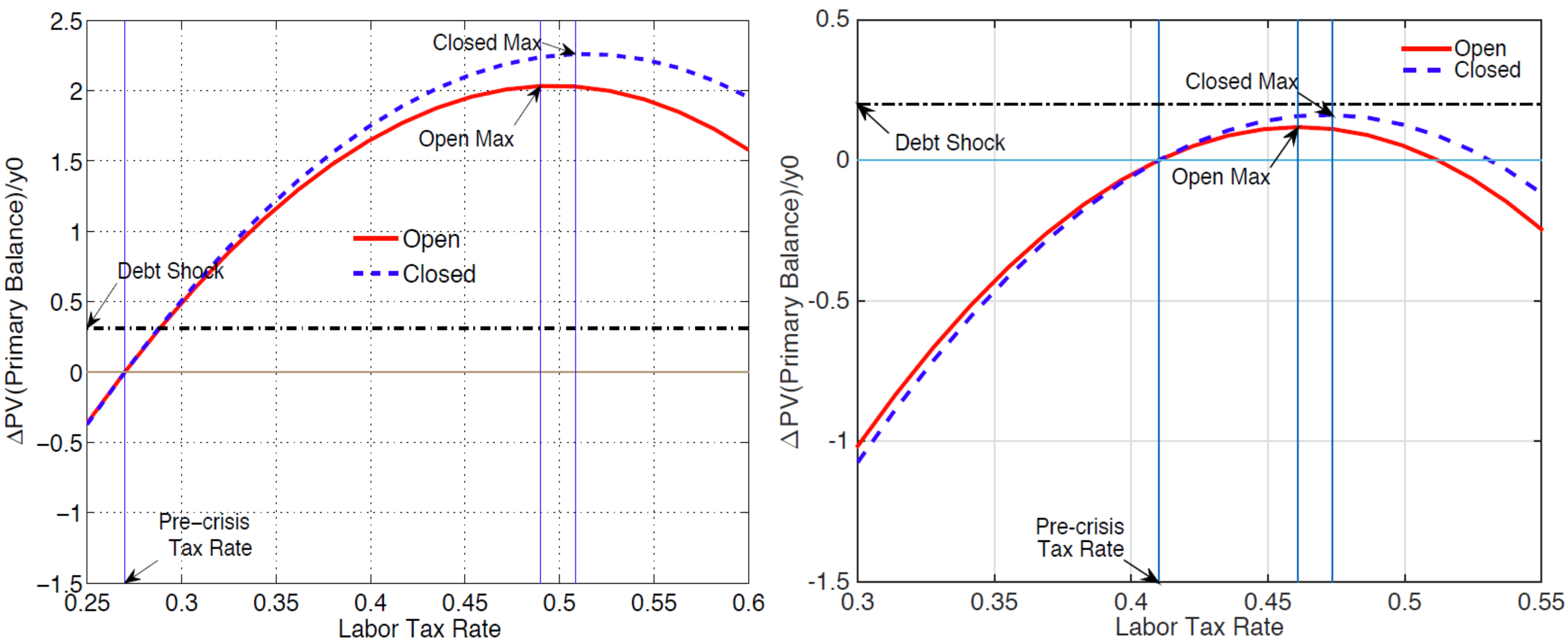
	Elasticity	y_1	l_1	m_1
Empirical estimates	[0.1, 0.5]			
Model Implications for the U.S.				
exog. utilization & $\theta = 1$	-0.09	0.04%	0.011	
exog. utilization & $\theta = 0.2$	-0.09	0.08%	0.028	
endog. utilization & $\theta = 0.2$	0.29	-0.15%	0.010	-0.471

U.S. Capital Tax DLCs: Alternative Models



(a) US

Labor Tax Dynamic Laffer Curves



Domestic Default Approach

- Previous two approaches cast doubt on chances of restoring fiscal solvency via conventional tools
- European crisis + historical evidence (Reinhart & Rogoff (11), Hall & Sargent (14)) raise possibility of domestic defaults
  - A "forgotten history" (R&R) until recently (D'Erasmo & Mendoza (2013,14), Dovis et al. (2014), …)
- Remove commitment: Distributional incentives lead to default unless costs are sufficiently high or gov. favors bond holders
  - Solvency is not enough to make debt sustainable!

Optimal Domestic Default

(D'Erasmo & Mendoza, JEEA 2016)

- Two-period model with two types of risk-averse agents (*L*, *H*), with fraction γ of L-types ($b_0^L < b_0^H$)
- Gov. collects lump-sum taxes τ, faces stochastic *g*, issues bonds B (*g* and default are non-insurable aggregate risks)
- Default is costly as a fraction $\phi(g)$ of income that varies with realization of *g* (a'la Arellano (2008))

$$\phi(g_1) \geq 0, \text{ with } \phi'(g_1) \leq 0 \text{ for } \ g_1 \leq \overline{g}_1, \ \phi'(g_1) = 0 \text{ otherwise}$$

- Gov. attains 2nd-best deviation from equal mg. utilities by redistributing via debt & default

Private Agents

Preferences:

$$u(c_0) + \beta E[u(c_1)], \qquad u(c) = \frac{c^{1-\sigma}}{1-\sigma}$$

Date-0 budget constraints and initial wealth for *i=L,H*:

$$c_0^i + q_0 b_1^i = y + b_0^i - \tau_0$$

Date-1 budget constraints under repayment for *i=L,H*:

$$c_1^i = y + b_1^i - \tau_1$$

Date-1 budget constraints under default for *i=L,H*:

$$c_1^i = (1 - \phi(g_1))y - \tau_1$$

Agents' Optimization Problem

Payoff function for i=*L,H* :

$$v^i(B_1,\gamma) \;=\; \max_{b_1^i}\Big\{u(y+b_0^i-q_0b_1^i-\tau_0)+$$

$$\beta E_{g_1}\Big[(1-d_1(B_1,g_1,\gamma))u(y+b_1^i-\tau_1^{d_1=0})+d_1(B_1,g_1,\gamma)u(y(1-\phi(g_1))-\tau_1^{d_1=1})\Big]\Big\}$$

$$\text{subject to } b_1^i \geq 0$$

with initial bond holdings given by initial wealth distribution and bond market clearing:

$$b_0^H = \frac{B_0-\gamma b_0^L}{1-\gamma}\; b_0^L \geq 0$$

Government

Budget constraints

$$\tau_0 = g_0 + B_0 - q_0 B_1$$

$$\tau_1^{d_1=0} = g_1 + B_1 \qquad \tau_1^{d_1=1} = g_1$$

Default optimization problem in 2nd period (utilitarian SWF):

$$\max_{d \in \{0,1\}} \{W_1^{d=0}(B_1, g_1, \gamma), W_1^{d=1}(g_1, \gamma)\}$$

$$W_1^{d=0}(B_1, g_1, \gamma) = \gamma u(y - g_1 + b_1^L - B_1) + (1-\gamma) u(y - g_1 + b_1^H - B_1)$$

$$W_1^{d=1}(g_1, \gamma) = u(y(1 - \phi(g_1)) - g_1)$$

Debt issuance optimization problem in 1st period:

$$W_0(\gamma) = \max_{B_1} \{\gamma v^L(B_1, \gamma) + (1-\gamma) v^H(B_1, \gamma)\}$$

Default Decision in 2^{nd} Period

- Assume bond demand choices given by:

$$b_1^L = B_1 - \epsilon \text{ and } b_1^H(\gamma) = B_1 + \tfrac{\gamma}{1-\gamma}\epsilon$$

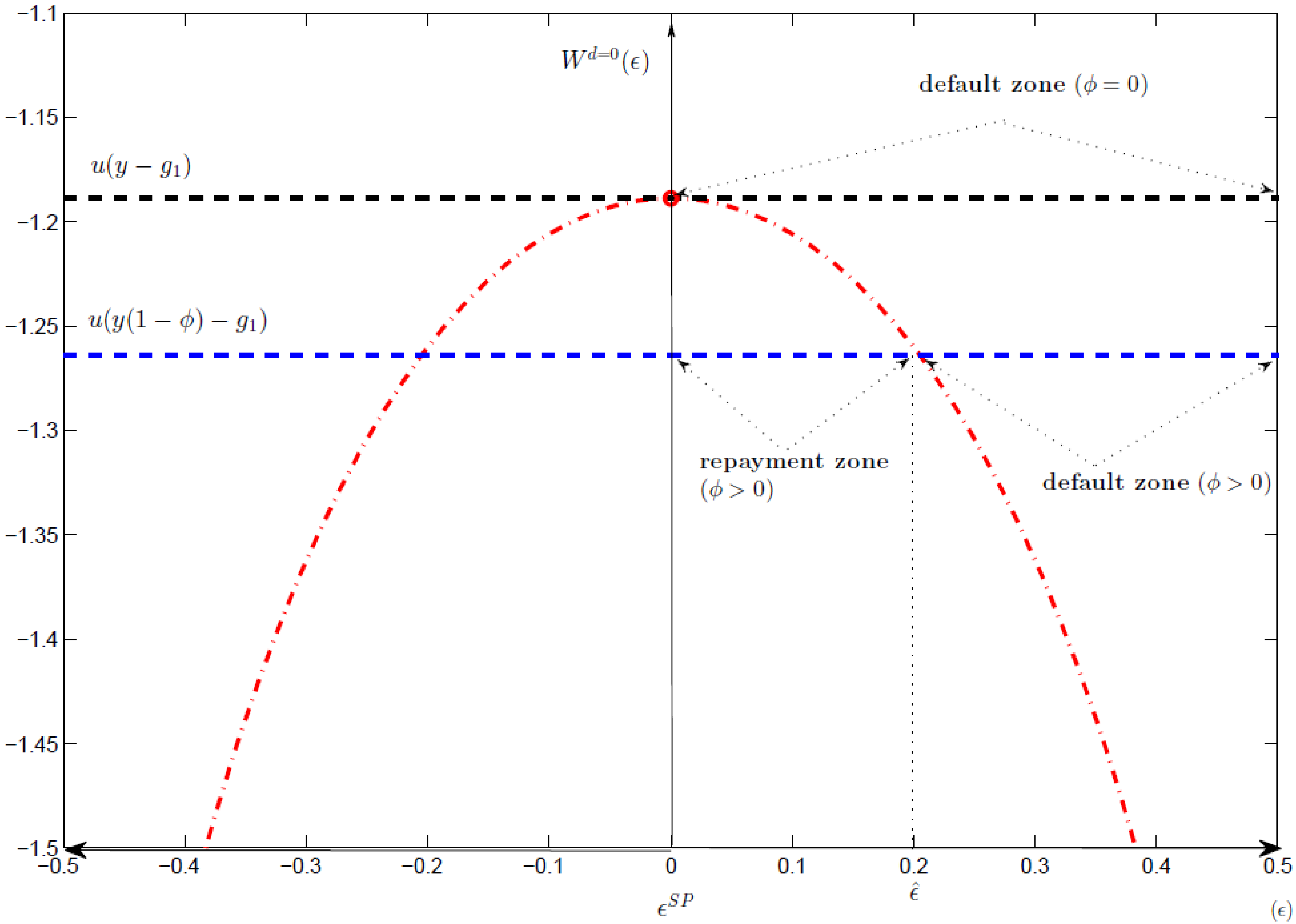
- Socially optimal allocations (under repayment):

$$u'\left(y - g_1 + \frac{\gamma}{1-\gamma}\epsilon^{SP}\right) = u'\left(y - g_1 - \epsilon^{SP}\right) \Longleftrightarrow \epsilon^{SP} = 0$$

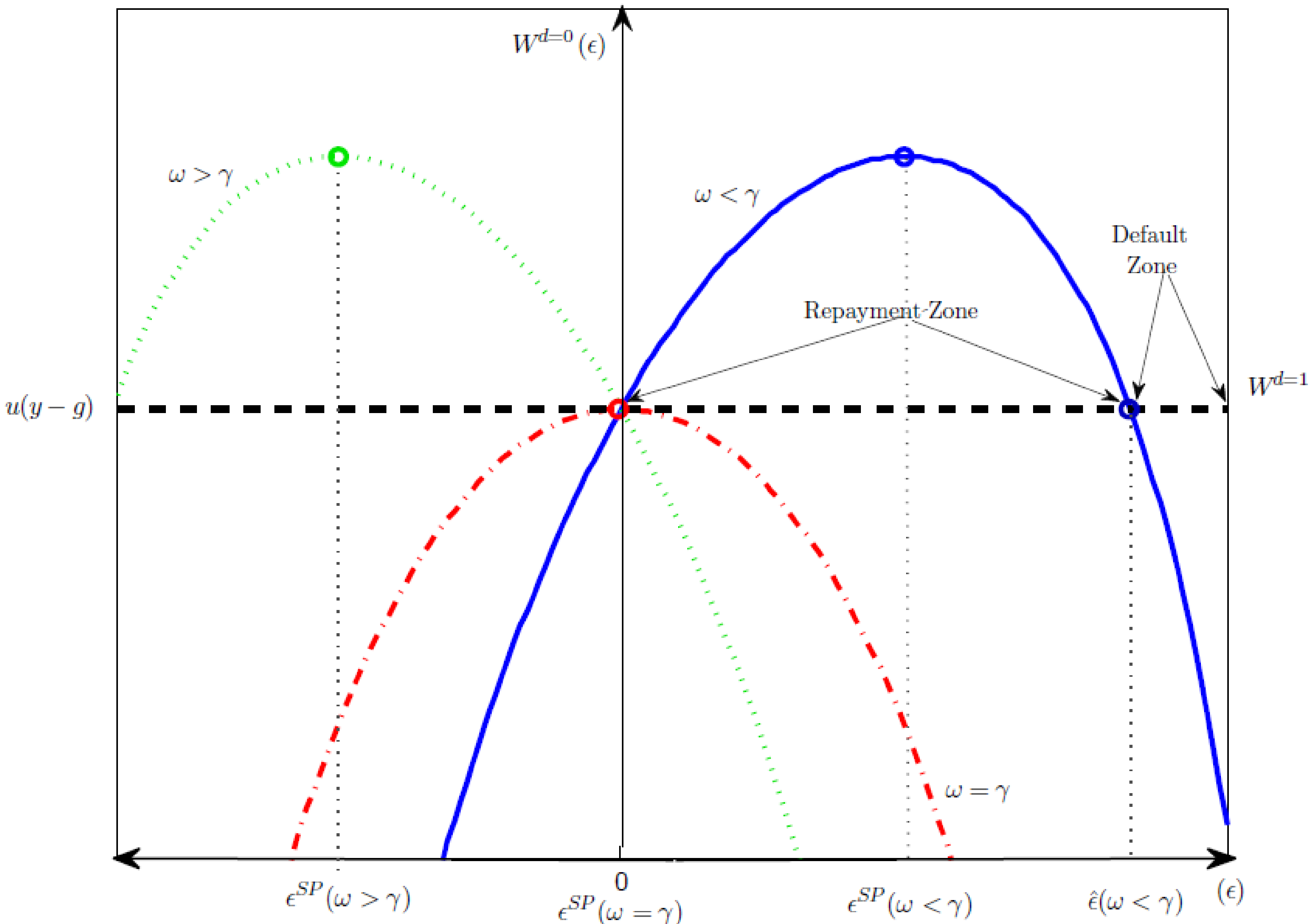
  – Zero consumption dispersion is first best

- If default is costless, it is always optimal (attains 1^{st} best) and debt cannot be sustained.
  – Cost makes default suboptimal (for some bond demand choices dispersion is smaller with repayment)
  – Cost can be endogenized (liquidity, self-insurance) or replaced with gov. bias favoring bond holders

Equilibria with & without default costs



Equilibria with Government Bias



Debt Issuance Decision in 1st Period

- Selling debt reduces dispersion at t=0, but increases it at t=1 under repayment:

$$
\begin{aligned}
c_0^H - c_0^L &= \frac{1}{1-\gamma}\left[B_0 - \boxed{q(B_1,\gamma)B_1}\right] \\
c_1^{H,d=0} - c_1^{L,d=0} &= \frac{1}{1-\gamma}B_1 \\
c_1^{H,d=1} - c_1^{L,d=1} &= 0.
\end{aligned}
$$

- Gov. internalizes how the gain of issuing debt is hampered by default risk, which lowers bond prices (debt Laffer curve).

Debt Issuance Optimality Condition

- Without default, some dispersion is optimal (debt helps relax L-types borrowing constraint)

$$u'(c_0^H) = u'(c_0^L) + \frac{\eta}{q(B_1,\gamma)\gamma}\left\{\gamma\mu^L\right\}$$

$$\mu^L \equiv q(B_1,\gamma)u'(c_0^L) - \beta E_{g_1}\left[(1-d^1)u'(c_1^L)\right] > 0.$$

- With default risk, more dispersion at t=0 is traded off for possibly zero at t=1 in default states

$$u'(c_0^H) = u'(c_0^L) + \frac{\eta}{q(B_1,\gamma)\gamma}\left\{\beta E_{g_1}\left[\Delta d \Delta W_1\right] + \gamma\mu^L\right\}$$

$$\Delta d \equiv d(B_1+\delta, g_1, \gamma) - d(B_1, g_1, \gamma) \geq 0, \quad \text{for } \delta > 0 \text{ small},$$

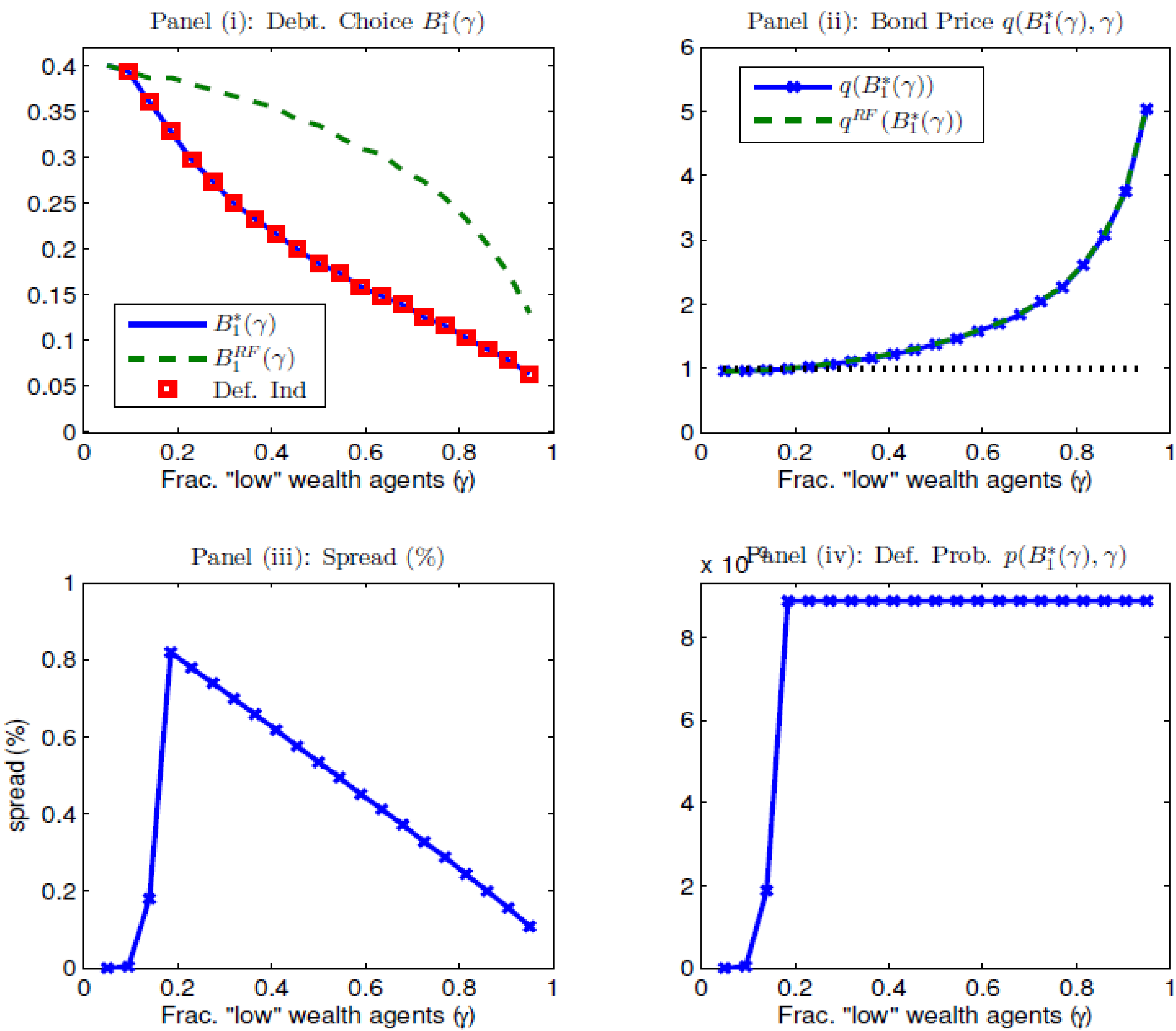
$$\Delta W_1 \equiv W_1^{d=1}(g_1,\gamma) - W_1^{d=0}(B_1, g_1, \gamma) \geq 0,$$

Calibration to European Data

Parameter		Value
Discount Factor	β	0.96
Risk Aversion	σ	1.00
Avg. Income	y	0.79
Low household wealth	b_0^L	0.00
Avg. Gov. Consumption	μ_g	0.18
Autocorrel. G	ρ_g	0.88
Std Dev Error	σ_g	0.017
Initial Gov. Debt	B_0	0.79
Output Cost Default	ϕ_0	0.02

Note: Government expenditures, income and debt values are derived using data from France, Germany, Greece, Ireland, Italy, Spain and Portugal.

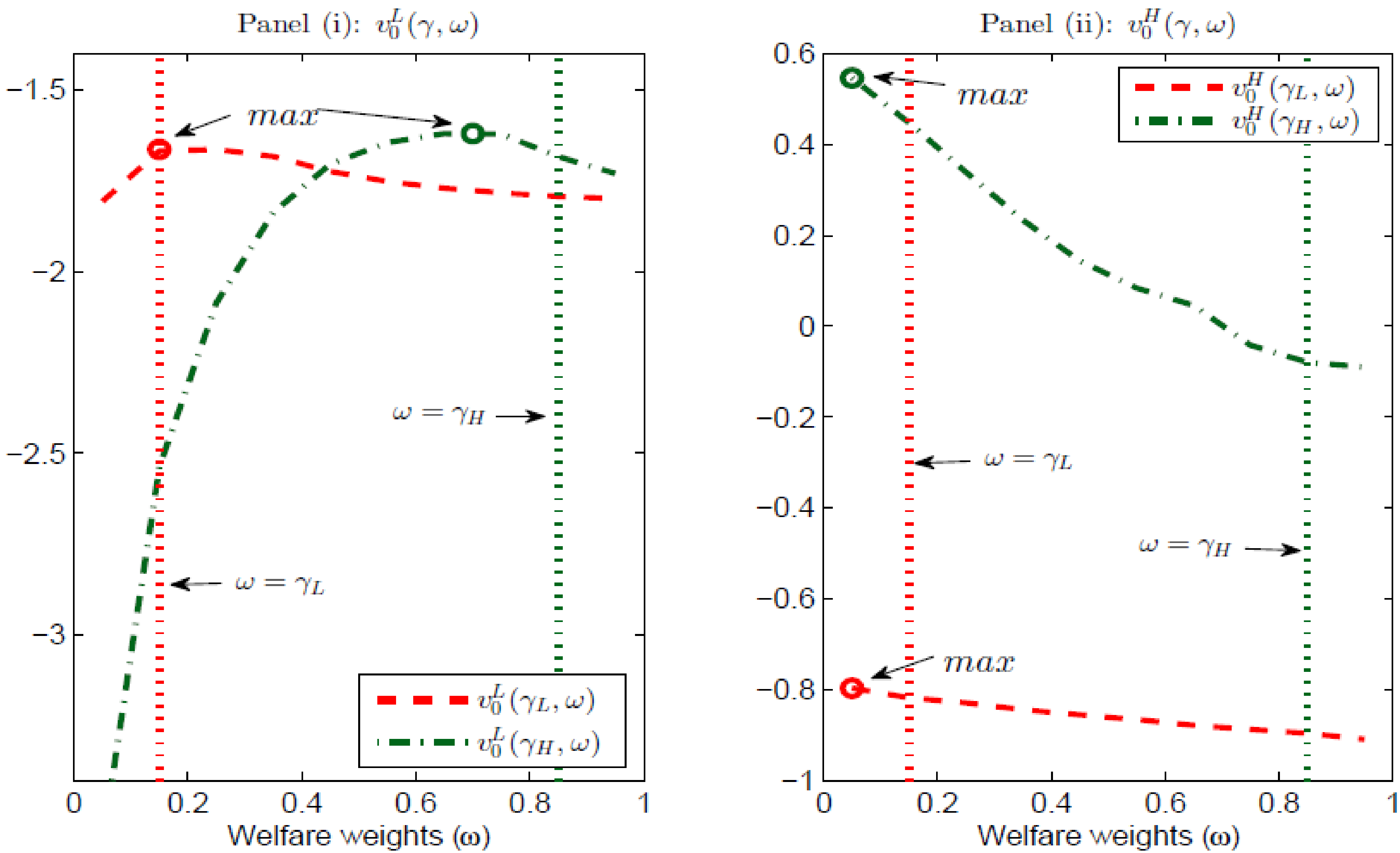
Equilibrium Manifold as Share of Non-debt-holders Rises
(calibration to European data)



Equilibria with Government Bias

Non-bond-holders may prefer bias!

(if ownership is sufficiently concentrated)



Conclusions

- Three approaches to examine sustainable debt paint a bleak picture of fiscal prospects:

1. FRF structural break post-2008, deficits much larger than predicted, and larger than in previous crises
2. Capital tax DLCs peak well below required increase to offset higher debt (except if EU exploits externalities)
3. Default costs or gov. bias make debt exposed to default risk due to distributional incentives sustainable
4. Economies with concentrated debt ownership elect biased governments that sustain high debt at low spreads and default probs.