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The Effects of Macroeconomic Shocks in a Basic Equilibrium Framework

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The macroeconomic effects of random shocks to output, the terms of trade, and the real interest rate are examined using an intertemporal equilibrium model of a small open, endowment economy. Equilibrium stochastic processes are computed numerically and compared with actual stylized facts. The model rationalizes the Harberger-Laursen-Metzler effect, but cannot produce countercyclical net exports and large real exchange rate fluctuations. The quantitative implications of the model are compared with the implications of the elasticities approach, and their sensitivity to changes in preference parameters and in the persistence of shocks is examined. [JEL E32, N1]

> Though we are interested in models because we believe they may help us understand matters about which we are currently ignorant, we need to test them as useful imitations of reality by subjecting them to shocks for which we are fairly certain how actual economies, or parts of economies, would react. The more dimensions on which the model mimics the answers actual economies give to simple questions, the more we trust its answers to harder questions.
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> —Robert E. Lucas, Jr. (1980, p. 696)

ONE KEY objective of open economy macroeconomics is to provide policymakers with a framework relating indicators of the performance of an economy—such as consumption, net exports, and the real

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exchange rate—to exogenous changes in elements of the economic environment—such as productivity, the terms of trade, or the world's interest rate, as well as policy instruments. The Keynesian model of the open economy that evolved from the seminal work of Harberger (1950), Laursen and Metzler (1950), Mundell (1962), and Fleming (1962) was the framework that dominated policy discussions during the 1960s and 1970s. During this period, econometric models based on the open economy *IS-LM* approach become the standard instrument for analyzing the effects of economic policies.

In the latter part of the 1970s and the early 1980s, critics cast serious doubts on the Keynesian framework and the way it was being used as a tool for policy analysis. Lucas (1976) criticized policy analysis based on conventional macroeconometric models for ignoring the principles of rationality and intertemporal optimization that govern economic behavior. The extensive theoretical literature that originated in the innovative work of Calvo (1981), Helpman (1981), Obstfeld (1981), and Svensson and Razin (1983) showed that key predictions of the Keynesian approach to the open economy, as well as their policy implications, are altered when explicit consideration is given to agents' optimizing behavior.[1] For instance, the Harberger-Laursen-Metzler hypothesis that an improvement in the terms of trade reduces the trade deficit was shown to be valid only in the case in which individuals perceive the improvement to be transitory.[2]

Despite extensive theoretical work, technical limitations prevented empirical research and, hence, practical policy discussions using intertemporal equilibrium models from developing quickly. In general, most dynamic optimizing models that can be regarded as a minimum framework for evaluating policies quantitatively—modeling equilibrium under uncertainty—cannot be solved analytically and cannot be reduced to closed forms suitable for policy simulations. Empirical work such as that by Hercowitz (1986), Barrionuevo (1992), and Ostry and Reinhart (1992) showed how advanced econometric methods could be used to test these models and to estimate their structural parameters. Their results suggest that the data generally cannot reject the models. However, this literature has not provided an assessment of the models' ability to duplicate observed empirical regularities, nor has it produced behavioral equations useful for policy analysis.

Recently, recursive numerical solution methods have received increasing attention as a vehicle for helping to overcome these technical limita-

[1] For a review of this literature, see Frenkel and Razin (1987).

[2] Three classic articles on this issue are Obstfeld (1982), Svensson and Razin (1983), and Persson and Svensson (1985).

tions (see Stokey, Lucas, and Prescott (1989)). These methods allow researchers to examine the dynamics of models that analytical tools cannot handle, thereby providing a means for developing theory. More important, they also provide a way to quantify the implications of existing theoretical models and to examine the effects of policies (see Mendoza (1991b) for an application to capital controls). Before one can study practical cases of policy analysis, however, one needs to explore the dynamics of intertemporal equilibrium models and to contrast them with stylized facts, so as to determine their potential as a reliable policymaking tool. This is the primary aim of this paper.

The equilibrium co-movements of macroeconomic aggregates are examined in a stochastic intertemporal equilibrium model of a small open, endowment economy similar to those developed in the theoretical work of the 1980s. The analysis focuses on the co-movement of consumption, the trade balance, and the real exchange rate in response to stochastic disturbances affecting output, the terms of trade, and the real interest rate.

The co-movement among the macroeconomic variables mentioned above displays several important empirical regularities in the data. Three of the most widely discussed—because of their implications for the design of policies—are the positive correlation between net exports and the terms of trade—a phenomenon known in the literature as the Harberger-Laursen-Metzler (HLM) effect; the countercyclical fluctuations of the trade balance; and the large variability of real exchange rates.[3] According to the intertemporal equilibrium approach, which views foreign borrowing and lending as a way to smooth consumption, the interpretation of these stylized facts relies on the duration of exogenous shocks. Countercyclical net exports are explained by the assumption that positive (negative) income shocks exhibit enough persistence, so that the proborrowing (prosaving) effect induced by expectations of higher (lower) future income dominates the prosaving (proborrowing) effect induced by an increase (decline) in current income. The HLM effect is explained if, when the terms of trade worsen (improve), agents are assumed to borrow (save) abroad rather than adjust consumption fully because they perceive the terms of trade shock as temporary.[4] Moreover, wealth and substitution effects unchained by exogenous shocks can cause sizable fluctuations in the real exchange rate, depending on how they affect the relative price of nontradable goods.

Thus, intertemporal equilibrium models of small open, endowment

[3] These stylized facts are documented in detail in the next section.

[4] The assumption of an endowment economy simplifies this analysis significantly by making savings identical to the balance of trade.

economies can *potentially* rationalize the stylized facts—in the sense that relevant comparative statics derivatives do not have unambiguous, counterfactual signs—but whether they can mimic observed empirical regularities and, hence, be treated as a useful instrument for policy analysis is still an open question. This paper attempts to answer this question by applying recursive numerical solution methods in the following manner. First, parameter estimates obtained in existing econometric studies, in combination with the steady-state restrictions of the model, are used to assign values to preference parameters. Second, stochastic fluctuations in output, the terms of trade, and the real interest rate that display the same properties as those observed in the data are incorporated. And third, equilibrium co-movements of the model are computed numerically and compared with stylized facts. The objective is to examine whether "realistic" parameters produce "realistic" co-movements.

The results show that the model recreates the HLM effect and mimics consumption behavior, but that other co-movements differ from the stylized facts, in particular the correlation between the trade balance and output and the variability of the real exchange rate. The analysis suggests that the stylized facts cannot be explained solely on the basis of the consumption-smoothing effect emphasized by the endowment economy, and that modeling investment may be critical. Sensitivity analysis shows that these results are robust to changes in the persistence of income shocks and in the values of preference parameters.

Because of their key implications for commercial and exchange rate policies, the income and price elasticities of trade flows are also among the most widely studied empirical regularities in open economy macroeconomics. As Goldstein and Khan's (1985) exhaustive review of the literature shows, most studies in this "elasticities" approach have applied a variety of econometric techniques to estimate import and export demand using data on trade flows and relative prices.[5] The main policy implication of this approach is an extension of the Marshall-Lerner condition, which states that if price elasticities of import and export demand add up to more than unity, then a decline in the terms of trade—exogenous or policy induced—should reduce the trade deficit. In contrast, recent work on intertemporal equilibrium models with uncertainty by Backus, Kehoe, and Kydland (1992b) has argued that, when actual time series are viewed as equilibrium processes reflecting the balance of time-varying supply and demand curves, the Marshall-Lerner condi-

[5] Most studies in this area abstract from cross-price and cross-expenditure effects, as well as intertemporal considerations. More recently, Marquez (1991) incorporated cross-price and cross-expenditure effects, and Khan and Reinhart (1992) introduced intertemporal effects.

tion—a static market equilibrium stability condition—may not by itself determine the co-movement between net exports and relative prices.

This study examines to what extent estimates of income and price elasticities of imports provide information on the co-movement between the trade balance and the terms of trade in an artificial, intertemporal equilibrium economy. The model's co-movements are used to construct ordinary-least-squares (OLS) estimates of the elasticities that would be obtained using the data generated by the artificial economy. These estimates are interpreted in the context of a static version of the model using the elasticities approach to illustrate the differences between this approach and the intertemporal equilibrium framework. The analysis shows that the elasticities are related to the behavior of net exports but they are not properties of import demand, nor is knowledge of the preference parameters that determine import demand sufficient to predict the response of net exports to relative price changes.

The emphasis given here to numerical solutions of open economy dynamic stochastic equilibrium models is common to other recent work. Backus, Kehoe, and Kydland (1992b) and Stockman and Tesar (1990) examined two-country models with complete contingent-claims markets and multiple goods; despite some positive results, they found that actual terms of trade fluctuations are underestimated by a factor of 2 to 6. Mendoza (1991a) simulated a one-good model of a small open economy where world financial markets are fully integrated and competitive but incomplete in the sense that only noncontingent debt contracts exist. This model mimics many features of Canadian business cycles, but produces nearly perfect consumption-output correlation, because the intertemporal relative price of consumption is independent of saving decisions. The model examined here contributes to this recent work by examining the effects of terms of trade shocks of the magnitude observed in the data and by introducing a three-good structure in which domestic saving plans affect the economy's real interest rate.

Finally, this study also contributes to the debate on the implications of the completeness of financial markets. Backus (1992) showed that under complete markets equilibrium co-movements are determined by preference parameters and are independent of country-specific shocks. In contrast, in the model examined here markets are incomplete, and, hence, wealth effects from country-specific disturbances affect equilibrium co-movements. Sensitivity analysis shows that the direction of these effects depends on the persistence of shocks, as suggested by theoretical work on deterministic models, and that the relative size of preference parameters affects the co-movements in the opposite direction than under complete markets.

Table 1. *Statistical Properties of Output, the Trade Balance, and the Terms of Trade in the Seven Largest Industrialized Countries*

	Output		Terms of Trade			Trade Balance			
Country	σ	ρ	σ	ρ	$\rho_{tot,y}$	σ	ρ	$\rho_{tb,y}$	$\rho_{tb,tot}$
United States	2.17	0.446	4.00	0.263	0.197	7.99	0.377	−0.277	−0.363
United Kingdom	1.98	0.524	4.41	0.551	−0.230	6.32	0.509	−0.538	0.731
France	1.49	0.654	3.46	0.341	0.287	4.43	0.132	−0.019	0.566
Germany	1.92	0.439	4.37	0.490	0.239	4.78	0.424	−0.299	0.346
Italy	2.17	0.537	5.03	0.504	0.112	8.73	0.305	−0.210	0.404
Canada	2.01	0.540	3.09	0.469	−0.034	4.75	0.394	−0.709	−0.102
Japan	3.58	0.812	10.98	0.583	0.559	10.99	0.275	0.054	0.527

Sources: International Monetary Fund, *International Financial Statistics* (various issues) and database for *World Economic Outlook* (various years).

Note: Data for the terms of trade and the trade balance are for the period 1960–89, and for gross domestic product (GDP) for the period 1965–89, expressed in per capita terms and detrended using the Hodrick-Prescott filter with the smoothing parameter set at 100, as in Backus, Kehoe, and Kydland (1992a). Output is GDP at constant prices from national income accounts; the terms of trade are the ratio of U.S. dollar unit value of exports to U.S. dollar unit value of imports; the trade balance is exports minus imports of merchandise from the balance of payments deflated using import unit values (the detrended trade balance corresponds to detrended exports minus detrended imports); σ is the percentage standard deviation; ρ is the first-order serial autocorrelation; $\rho_{tot,y}$ is the correlation of the terms of trade with GDP; $\rho_{tb,y}$ is the correlation of the trade balance with GDP; and $\rho_{tb,tot}$ is the correlation of the trade balance with the terms of trade.

The paper is organized as follows. Section I documents the stylized facts of relevant macroeconomic variables in the seven largest industrialized countries (the so-called Group of Seven countries). Section II presents the model and its optimality conditions. Section III discusses functional forms and specification of parameter values. Section IV presents the results of numerical simulations, compares them with actual data, and performs sensitivity analysis. Section V presents concluding remarks.

I. The Stylized Facts

Tables 1–3 list the stylized facts that characterize fluctuations in output (Y), the terms of trade, the trade balance, consumption, and the real exchange rate in the Group of Seven. In the standard intertemporal equilibrium model of a small open, endowment economy, fluctuations in the first two variables are determined exogenously by a probabilistic process, whereas the behavior of the latter three reflects the outcome of optimal intertemporal plans. Thus, the statistical moments of output and the terms of trade determine the stochastic processes that shocks to endowments and the relative price of exports in terms of imports follow, whereas the moments of the trade balance, consumption, and the real exchange rate are the empirical regularities that the model should mimic.

Table 1 shows that, excluding Japan, there is certain uniformity in the variability, co-movement, and persistence of output and the terms of trade.[6] Excluding Japan, output fluctuates between 1.5 and 2.2 percent, and the terms of trade, between 3.1 and 5 percent. The first-order serial autocorrelation of output is between 0.44 and 0.65, and that of the terms of trade is between 0.26 and 0.55. The correlation between the two variables is very weak, in a range between −0.23 and 0.20. In contrast, output and terms of trade shocks are larger, more persistent, and more correlated in Japan, perhaps reflecting higher dependency on fuel imports.[7]

Table 1 also shows that the trade balance is countercyclical, or acyclical as in Japan, and that there is an HLM effect (that is, the trade balance and the terms of trade are positively correlated), except in Canada and the United States; excluding these two countries, the correlation between the trade balance and the terms of trade is between 0.35 and 0.73. The

[6] The model describes a small open economy, and, hence, stylized facts for the United States, Germany, and Japan must be interpreted with caution.

[7] Japan's fuel imports in 1988 were 27 percent of all imports, while in the rest of the Group of Seven this share was between 5 percent and 13 percent.

standard deviation of the trade balance ranges from 4.4 percent to 11 percent, with Japan again the outlier. However, despite this wide range, the trade balance is always between one and two times more variable than the terms of trade. There is also some uniformity in the first-order serial autocorrelation of the trade balance, which ranges from 0.13 to 0.51.

Despite the uniformity in some of the stylized facts across countries, the basic prediction of intertemporal equilibrium models with incomplete markets—that more persistent income shocks should weaken the comovement between the trade balance and output and between the trade balance and the terms of trade—is not observed in the data. For instance, for the United States, the country with the lowest persistence in the terms of trade and the second-lowest persistence in output, the data show the lowest correlation between the trade balance and the terms of trade at −0.36. For Japan, the country with more persistence in output and the terms of trade, the data produce the largest correlations between the trade balance and output and between the trade balance and the terms of trade.

This puzzle may reflect weaknesses in the design of endowment economy models. For instance, these models ignore the fact that the actual behavior of the trade balance incorporates optimal investment plans. The impact on the trade balance resulting from a persistent positive income shock is smaller in an investment economy than in an endowment economy—in the former there is an extra proborrowing effect caused by the desire to expand the capital stock. Moreover, following Backus (1992), the structure of financial markets in these models may be "too incomplete" and, hence, they may overemphasize the persistence of country-specific shocks relative to differences in preference parameters. To substantiate these arguments, however, it is necessary to establish how well the endowment economy model mimics the stylized facts, and how sensitive the model is to changes in the persistence of exogenous shocks relative to changes in preference parameters.

Table 2 lists some of the stylized facts that characterize aggregate consumption and consumption of tradables and nontradables as reported by Stockman and Tesar (1990) for four of the Group of Seven countries. Given data limitations, they used Organization for Economic Cooperation and Development (OECD) information to approximate consumption of nontradables with data on personal expenditures in some services and fuel. This evidence suggests that the consumption of tradables and nontradables is positively correlated, in a range from 0.62 to 0.91. Consumption of nontradables is about half as variable as output, except for Japan where nontradables consumption is almost as variable as gross domestic product (GDP), while consumption of tradables in three of the four countries is more variable than output. The latter result may reflect

Table 2. *Statistical Properties of Private Consumption in Selected Industrialized Countries*

Country	Aggregate Consumption		Nontradables		Tradables
	σ/σ_y	$\rho_{c,y}$	σ/σ_y	$\rho_{t,nt}$	σ/σ_y
United States	0.74	0.935	0.43	0.724	1.12
Italy	0.62	0.371	0.35	0.864	0.76
Canada	1.07	0.930	0.57	0.620	1.56
Japan	1.22	—	0.97	0.909	1.50

Note: σ/σ_y is the standard deviation relative to the standard deviation of output; $\rho_{c,y}$ is the correlation of consumption with output; and $\rho_{t,nt}$ is the correlation of consumption of tradables with consumption of nontradables. These moments are from Stockman and Tesar (1990), and correspond to data from the OECD *Quarterly Accounts (Citibase* for U.S. data), converted to annual data taking averages, and detrended with the Hodrick-Prescott (HP) filter using the smoothing parameter set at 400. Aggregate consumption is private final consumption, including durables and semidurables. Consumption of nontradables is measured as the sum of expenditures on "rent, fuel, and power" and "transportation and communication." The data are for the period 1970–88, except in the case of Italy, which are for the period 1981–87.

a measurement problem common to data on aggregate private consumption, which is often found to be more variable than output as shown in Table 2, apparently in violation of the principle of consumption smoothing. It is well established, however, that once consumption of durables is subtracted, consumption becomes less variable than output and exhibits distinctly procyclical behavior. Prescott (1986) and Backus, Kehoe, and Kydland (1992a) reported that consumption of nondurables was between 0.3 and 0.6 times as variable as output in U.S. quarterly detrended data, and its correlation with output ranged from 0.66 to 0.81. Mendoza (1991a) found that in Canada nondurables consumption was about 0.87 times as variable as GDP using annual data, and that its output correlation was 0.59.

Table 3 reports indicators of variability and persistence of U.S. dollar real exchange rates obtained with the moments reported by Schlagenhauf and Wrase (1992). Considering four of the Group of Seven countries, as well as a trade-weighted index, real exchange rates fluctuate between 2.9 percent and 9.7 percent, about 1.5 and 4.9 times as much as U.S. output. These variability ratios are significantly larger than the consumption-output variability ratios and slightly larger than the trade balance-output variability ratios implicit in Table 1.[8] The persistence of the real exchange

[8] For the four countries in Table 3, the data in Table 1 imply that the standard deviation of the trade balance relative to the standard deviation of output is between 2.4 and 3.2

Table 3. *Persistence and Variability of U.S. Dollar Real Exchange Rates*

Country	Real Exchange Rate		
	σ	$\rho(1)$	$\sigma/\sigma y$
Canada	2.94	0.779	1.49
Germany	9.67	0.799	4.90
Japan	9.67	0.795	4.90
United Kingdom	8.71	0.817	4.41
Trade weighted	6.95	0.807	3.52

Note: σ is the percentage standard deviation; $\rho(1)$ is the first-order serial autocorrelation; and $\sigma/\sigma y$ is the standard deviation relative to the standard deviation of U.S. output. These moments are as reported in Schlagenhauf and Wrase (1992) and correspond to quarterly, HP-filtered data covering the period 1972:1–1989:4. Real exchange rates are computed using each country's consumer price index and end-of-quarter U.S. dollar nominal exchange rates. The trade-weighted real exchange rate uses 1980 U.S. trade shares.

rate shares the property of uniformity observed in other stylized facts—quarterly first-order autocorrelations are between 0.78 and 0.82 (0.37 and 0.45 annually).

II. An Intertemporal Equilibrium Model of a Small Open, Endowment Economy

The economy is inhabited by identical, infinitely lived individuals seeking to maximize expected lifetime utility. Individuals consume three goods; two are tradable, an exportable or home good, x, and an imported or foreign good, f, and one is a nontradable good, n. There are fixed endowments of exportables, X, and nontradables, N, with the relative price of exportables in terms of importables (that is, the terms of trade) and the relative price of nontradables in terms of importables denoted as p^x and p^n, respectively. There is also a perfectly competitive world capital market to which individuals have free access. In this market, agents borrow or save as much as they want by buying or selling foreign financial assets, a, denominated in units of the foreign good. These assets are noncontingent, one-period bonds that yield the world's real interest rate, r^*. The economy is a small participant in world markets, and hence r^* and p^x are exogenous. Stochastic disturbances affect the endowments, the terms of trade, and the world's interest rate, and these are denoted e^y, e^p, and e^r, respectively.

The optimization problem that describes this economy is to maximize[9]

[9] As in Obstfeld (1981) and Mendoza (1991a), expected lifetime utility embodies an endogenous rate of time preference so as to produce a well-defined stationary equilibrium.

$$\mathbf{U}(x,f,n) = \mathbf{E}\left[\sum_{t=0}^{\infty}\left\{u(x_t,f_t,n_t)\exp\left(-\sum_{\tau=0}^{t-1} v(x_\tau,f_\tau,n_\tau)\right)\right\}\right], \tag{1}$$

with respect to the stochastic processes, $\{a_{t+1}\}_{t=0}^{\infty}$, $\{x_t\}_{t=0}^{\infty}$, $\{f_t\}_{t=0}^{\infty}$, and $\{n_t\}_{t=0}^{\infty}$, and subject to the constraint below for $t = 0, \ldots, \infty$:

$$f_t + e_t^p p_t^x x_t + p_t^n n_t = e_t^y(e_t^p p_t^x X + p_t^n N) - a_{t+1} + a_t(1 + r^* e_t^r). \tag{2}$$

Optimal consumption and savings are characterized as follows:

$$\mathbf{U}_f(t) = \exp(-v(t))E_t[\mathbf{U}_f(t+1)(1 + r^* e_{t+1}^r)] \tag{3}$$

$$\frac{\mathbf{U}_x(t)}{\mathbf{U}_f(t)} = e_t^p p_t^x \tag{4}$$

$$\frac{\mathbf{U}_n(t)}{\mathbf{U}_f(t)} = p_t^n \tag{5}$$

These conditions have standard interpretation, except that the lifetime marginal utilities of foreign goods, $\mathbf{U}_f(t)$, exportable goods, $\mathbf{U}_x(t)$, and nontradables, $\mathbf{U}_n(t)$, include a term that accounts for the impact of changes in current consumption on the rate of time preference. The condition defined in equation (3) equates the marginal costs and benefits of sacrificing a unit of consumption of imports, and equations (4) and (5) set the atemporal marginal rates of substitution in consumption of exportable and foreign goods and nontradable and foreign goods equal to the corresponding relative prices.

The macroeconomic effects of temporary and permanent income shocks in two-period, deterministic models similar in structure to the one presented here have been analyzed extensively in theoretical work. Researchers used comparative statics and duality to show how preference parameters interact to affect the dynamics of macroeconomic aggregates. However, many of these comparative statics exercises yield ambiguous signs for most co-movements of interest, and cannot be used to interpret actual time series because of the assumption of perfect foresight.[10] Nevertheless, these studies provide some insights helpful for an understanding of how the infinite-horizon, stochastic model works. For instance, in a three-good model with nontradables, Ostry (1988) determined that the responses of consumption, the trade balance, and the real exchange rate to changes in the terms of trade depend on the intertemporal elasticity

[10] As Shiller (1981) showed, to infer properties of actual time series from the optimality conditions of deterministic models is equivalent to ignoring the covariance between asset returns and the marginal utility of consumption, which measures agents' risk aversion and influences their saving plans.

of substitution in aggregate consumption, the atemporal elasticities of substitution between importables, exportables, and nontradables, the shares of total expenditure allocated to each of the three goods, the fraction of the endowment of importables that is consumed, and the average propensity to save. The direction of the co-movements can be positive or negative depending on the relative size of these parameters.

III. Functional Forms and Parameter Values

The functions $u(\cdot)$ and $v(\cdot)$ in equation (1) are given the following form:

$$u(x,f,n) = \frac{\left([(x^{\alpha}f^{1-\alpha})^{-\mu} + n^{-\mu}]^{-\frac{1}{\mu}}\right)^{1-\gamma}}{1-\gamma}, \tag{6}$$

$$v(x,f,n) = \beta \ln\left(1 + [(x^{\alpha}f^{1-\alpha})^{-\mu} + n^{-\mu}]^{-\frac{1}{\mu}}\right), \tag{7}$$

where

$$0 \leq \alpha \leq 1,\ \mu > -1,\ \gamma > ,\ \beta > 0.$$

There is a constant elasticity of substitution (CES) between tradables and nontradables, $1/(1+\mu)$, and the shares of expenditures on exportables and importables in total expenditure on tradables, using import prices as numeraire, are fixed at α and $1-\alpha$, respectively. The intertemporal elasticity of substitution is $1/\gamma$, and β approximates the elasticity of the rate of time preference with respect to the CES utility.

Given equations (6) and (7), the problem stated in equation (1) can be represented in a dynamic programming form suitable for numerical solution through a two-step procedure. First, since the supply on nontradables in each period is $e_t^y N$, market clearing dictates that $n_t = e_t^y N$; thus, using (5), the equilibrium relative price of nontradables, p_t^n, that clears the market is

$$p_t^n = \frac{(e_t^y N)^{-\mu-1}}{(x_t^{\alpha} f_t^{1-\alpha})^{-\mu-1}(1-\alpha)x_t^{\alpha} f_t^{-\alpha}}. \tag{8}$$

This expression determines the equilibrium relative price of nontradables once e_t^y is observed and optimal plans for x_t and f_t are known. The relative price of nontradables is the key determinant of the real exchange rate. In the theoretical literature (see Frenkel and Razin (1987) or Greenwood (1984)), the real exchange rate is defined as either the relative price of nontraded goods, this price weighted by the share of nontradables in consumption expenditures, or a duality-based consumer price index that incorporates terms of trade effects.

The second step in defining the dynamic programming problem combines equations (4), (6), and (7) to express x_t as a function of f_t in equilibrium $\chi_t(f_t)$:

$$\chi_t(f_t) = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{f_t}{e_t^p p_t^x}\right). \tag{9}$$

With equations (8) and (9) in mind, and assuming that e^y, e^p, and e^r are first-order Markov processes, the dynamic programming problem is

$$v(a_t, \epsilon_t) = \max_{a_{t+1}}\left\{\frac{[(\chi(f_t)^{\alpha} f_t^{1-\alpha})^{-\mu} + (e_t^y N)^{-\mu}]^{-\frac{(1-\gamma)}{\mu}}}{(1-\gamma)} + \left(1 + [(\chi(f_t)^{\alpha} f_t^{1-\alpha})^{-\mu} + (e_t^y N)^{-\mu}]^{-\frac{1}{\mu}}\right)^{-\beta} E[V(a_{t+1}, \epsilon_{t+1})]\right\}, \tag{10}$$

subject to

$$f_t = (1-\alpha)[e_t^p p_t^x e_t^y X + (1 + r^* e_t^r)a_t - a_{t+1}],$$
$$a_t, a_{t+1} \geq \Delta, \text{ and } f_t \geq 0.$$

At the beginning of each period t, agents start with a certain stock of net foreign assets, or debt, and observe a particular set of realizations of shocks affecting endowments, the terms of trade, and the world's real interest rate—a triple (e_t^y, e_t^p, e_t^r) that is defined as ϵ_t. Thus, the state of the economy at date t is fully described by (a_t, ϵ_t). Individuals then formulate optimal, state-contingent plans regarding the consumption of imports, f_t, and the acquisition of foreign assets, a_{t+1}, given their rational expectations about future realizations of the disturbances. These optimal plans, given equations (8) and (9), determine equilibrium paths for x_t, p_t^n, and other macroaggregates of interest.

A variation of the algorithm described in Mendoza (1991a) is employed to solve the dynamic programming problem stated in equation (10). This algorithm computes the unique invariant joint-limiting distribution of the state variables (a, ϵ) in the stochastic steady state, and uses this distribution to calculate the statistical moments that characterize equilibrium co-movements in the model. These moments represent population moments of an artificial economy.

The solution procedure starts by setting realistic values for parameters describing preferences and stochastic shocks. For simplicity, the disturbances are assumed to follow two-point, symmetric Markov processes as detailed in an annex available from the author on request. Under this assumption, the variability, co-movement, and persistence of the shocks are given by

$$\sigma_{e^y} = e^y, \quad \sigma_{e^p} = e^p, \quad \sigma_{e^r} = e^r,$$

$$\rho_{e^y}(1) = \rho_{e^p}(1) = \rho_{e^r}(1) = \theta \tag{11a}$$

$$\rho_{e^y,e^p} = 1 - 4[\Pi(\epsilon^3) + \Pi(\epsilon^4)], \rho_{e^y,e^r} = 1 - 4[\Pi(\epsilon^2) + \Pi(\epsilon^4)],$$

$$\rho_{e^p,e^r} = 1 - 4[\Pi(\epsilon^2) + \Pi(\epsilon^3)]. \tag{11b}$$

The standard deviation of each shock, σ, is given by its size; the first-order serial autocorrelation, $\rho(1)$, is forced to be the same for the three shocks by the symmetry assumption and is given by θ; the contemporaneous correlations are determined by the structure of limiting probabilities, Π, assigned to each state of nature or combination of shocks, ϵ^u, for $u = 1, 8$.

The stylized facts documented in Table 1 determine the values of the moments that describe shocks to endowments and the terms of trade. For interest rate shocks, the moments have been selected following Barro and Sala-i-Martin (1990).[11] Taking averages excluding Japan, where macroeconomic aggregates seem more volatile than in the rest of the Group of Seven countries, the data indicate that the stochastic components of the model are as follows:

$$\sigma_{e^y} = 1.96, \sigma_{e^p} = 4.06, \sigma_{e^r} = 1.50,$$

$$\rho_{e^y}(1) = \rho_{e^p}(1) = \rho_{e^r}(1) = 0.5, \tag{12a}$$

$$\rho_{e^y,e^p} = 0.095, \rho_{e^y,e^r} = 0.191, \rho_{e^p,e^r} = -0.404, \tag{12b}$$

where

$$X = 1.0, N = 0.824, r^* = 0.04.$$

The mean of the endowment of exportables is set at unity, and the mean of the endowment of nontradables, N, is determined so as to ensure that in the deterministic steady state the ratio of nontradable to tradable expenditures is equal to the average observed in data for industrial countries—0.87 (see Table 4). The average r^* is the mean real rate of return on a risk-free asset typical of real business cycle models.

[11] These authors estimated the standard deviation of the world's expected real interest rate, r_w^e, at 1.5 percent. The correlations between e^y and e^r, and e^p and e^r were approximated with estimates of the correlation between r_w^e and the investment-output ratio, and the negative of the correlation between r_w^e and the price of oil, obtained with the regression coefficients of the corresponding variables in column (1) of Table 2 and the standard deviations in Table 1 of their paper.

Values for parameters in equations (6) and (7) are selected using cross-sectional data for 1975 from a sample of 13 industrialized countries reported in Kravis, Heston, and Summers (1982) and UNCTAD (1987), as summarized in Table 4, jointly with the steady-state conditions of the model and estimates obtained in some econometric literature. The parameters are

$$\gamma = 1.5,\ \mu = 0.35,\ \alpha = 0.72,\ \beta = 0.432. \tag{13}$$

The value of γ is in the range of existing estimates and is in line with values that have been found effective to mimic the stylized facts (see Backus, Kehoe, and Kydland (1992a) and Mendoza (1991a)).[12] The values of μ and α are set using Table 4. Columns (1) and (2) in this table list indices of relative consumer expenditures and relative prices for traded and nontraded goods produced with data from Tables 6-10 and 6-12 in Kravis, Heston, and Summers (1982). Following Stockman and Tesar (1990), μ is determined by running a logarithmic regression of relative expenditures on relative prices and GDP per capita adjusted for purchasing power (also from Table 6-10 in Kravis, Heston, and Summers (1982)). This produces an estimate of $1/(1 + \mu)$ of about 0.74 with a standard error of 0.438.[13] The share of home tradable goods in total consumer expenditure on tradables, α, is set by obtaining an estimate of its complement, the share of imports in tradable consumer expenditures, $1 - \alpha$. Column (3) of Table 4 lists the fraction of total imports pertaining to imports of consumer goods obtained from UNCTAD (1987), and this is combined with data from Kravis, Heston, and Summers to estimate $1 - \alpha$ for the 13 countries in the table, resulting in an average $1 - \alpha$ equal to 0.28.

Given X, r^*, γ, μ, and α, a system of four simultaneous equations determines β, N, and the deterministic steady-state equilibria of p^n and a. The four equations are: (1) the stationary equilibrium condition that equates the rate of time preference with r^*; (2) the marginal rate of substitution between nontradables and imports in the steady state; (3) the ratio of net factor payments to output measured in units of importables, $\phi = r^*a/(p^xX + p^nN)$; and (4) the ratio of expenditure on nontradables

[12] Estimating an equilibrium model with nontraded goods, Barrionuevo (1992) found evidence that γ may be around 1. Sensitivity analysis to examine the implications of this nearly risk-neutral preferences is undertaken in the next section.

[13] Stockman and Tesar (1990) included 17 developing countries in their sample. Their estimate of the elasticity of substitution is 0.44 with a standard error of 0.225. However, this is inconsistent with estimates obtained by Ostry and Reinhart (1992).

Table 4. *Industrial Countries: Selected Data on the Composition of Consumption Expenditures and Imports, 1975*

			Imports of Consumer Goods	
Country	(1) Relative Expenditure	(2) Relative Prices (Index, U.S. = 100)	(3) (as a percent of total imports)	(4) (as percentage of expenditure on tradables)
Japan	0.90	89.3	25.1	11.0
Austria	0.74	81.6	35.4	24.9
Belgium[a]	0.74	88.0	36.1	53.9
Denmark	1.15	74.5	32.9	31.7
France	0.83	77.1	31.7	15.7
Germany	0.79	81.7	39.6	22.4
Ireland	0.97	68.5	38.6	46.4
Italy	0.88	62.7	29.3	15.7
Luxembourg	0.94	81.2	—	—
Netherlands	0.64	92.2	38.4	48.7
Spain	0.81	62.1	26.0	9.5
United Kingdom	1.03	70.7	39.9	28.3
United States	0.74	100.0	28.3	5.5
Mean[b]	0.87	77.5	35.4	28.0

Note: Columns (1) and (2) correspond to the ratios of column (8) to column (9) in Tables 6-10 and 6-12 of Kravis, Heston, and Summers (1982). Column (3) is the sum of the shares of imports of food and manufactures (excluding chemical products and machinery and equipment) in total imports from UNCTAD (1987, pp. 158–79). Column (4) is generated by applying the shares in column (3) to data on total imports from UNCTAD (1987), and then using the U.S. dollar amount of imports of consumer goods to produce the shares of imports in consumption of tradables with data on population, private consumption, exchange rates, and share of tradables in total private consumption, from Tables 1-2, 1-7, and 6-10 in Kravis, Heston, and Summers (1982).

[a] For Columns (3) and (4) Belgium includes Luxembourg.

[b] Excluding the United States, which is the base for the purchasing power correction in Kravis, Heston, and Summers (1982).

to expenditure on tradables in the steady state also measured in units of imports, $\Omega = p^nN/(p^xX + r^*a)$. To solve these equations, p^x is assumed to be equal to 1 in the steady state, and ϕ and Ω are set using cross-country and time-series averages from Tables 4 and 5.[14] Column (1) in Table 4 shows that the average of Ω for 13 industrialized countries is 0.87, and from the data in the fifth column of Table 5 one can compute the cross-section mean of ϕ for time-series average of the Group of Seven, which comes to -0.18.[15]

IV. Numerical Simulations

This section is divided into three parts. The first part compares the equilibrium co-movements of the model with the stylized facts. The second part examines the model's implications for income and price elasticities and compares them with the predictions of the elasticities approach. The last part analyzes the sensitivity of the results to changes in the parameters that theory highlights as key for explaining equilibrium co-movements.

The Model and the Data

Table 6 lists the equilibrium co-movements of macroeconomic aggregates implied by the limiting probability distribution of the model's state variables (Figure 1). Consider first the balance of trade. The percentage standard deviation and first-order autocorrelation of the trade balance are in line with those observed in the Group of Seven countries. This contrasts with the low variability of the trade balance obtained in two-country, complete-markets models (Backus, Kehoe, and Kydland (1992b) and Stockman and Tesar (1990)), and suggests that when terms of trade shocks of the magnitude observed in the data are taken into account, equilibrium models produce substantial variability in net exports. The model's correlation between the trade balance and the terms of trade is also in line with actual observations. Income shocks with a coefficient of persistence of $\theta = 0.5$ are sufficiently transitory to re-create,

[14] Table 5 reports moments for deviations from trend in real net factor payments and real GDP measured at import prices—proxies for $e^r r^* a$ and $e^y(p^xX + p^nN)$ in the model. With output measured at import prices, the trade balance becomes weakly procyclical instead of countercyclical as in Table 1.

[15] Given Canada's relatively large and negative ϕ, ϕ is set at zero in the numerical solutions to reflect more closely the typical ratio of net factor payments to GDP in industrial countries.

Table 5. *Statistical Properties of Net Factor Payments and GDP at Import Prices in the Group of Seven Industrialized Countries*

	Net Factor Payments (r^*a)				GDP at Import Prices			
Country	σ	ρ	$\rho_{r^*a,tot}$	r^*a/y	σ	ρ	$\rho_{ym,tb}$	$\rho_{ym,tot}$
United States	5.08	0.105	0.375	0.69	9.07	0.545	−0.569	0.847
United Kingdom	6.41	0.118	−0.011	0.92	6.95	0.438	0.346	0.771
France	3.45	0.024	0.249	0.04	7.14	0.378	0.417	0.873
Germany	8.86	−0.105	0.272	0.15	8.72	0.624	0.217	0.876
Italy	4.45	−0.676	−0.969	−0.93	10.58	0.578	0.302	0.958
Canada	14.54	0.599	−0.199	−2.27	3.54	0.403	0.161	0.065
Japan	14.74	0.582	0.735	0.15	18.15	0.572	0.522	0.972

Sources: International Monetary Fund, *International Financial Statistics* (various issues) and database for the *World Economic Outlook*.

Note: Data are for the period 1965–89 (except net factor payments for the United States, France, and Italy, where the series start in 1970, 1978, and 1984, respectively), converted into U.S. dollars and deflated by the dollar unit value of imports. The data are expressed in per capita terms and detrended using the Hodrick-Prescott filter with the smoothing parameter set at 100. σ is the percentage standard deviation; ρ is the first-order serial autocorrelation; $\rho_{r^*a,tot}$ is the correlation of net factor payments with the terms of trade; r^*a/y is the average ratio of net factor payments to GDP; $\rho_{ym,tb}$ is the correlation of GDP with the trade balance (both at import prices); and $\rho_{ym,tot}$ is the correlation of GDP at import prices with the terms of trade.

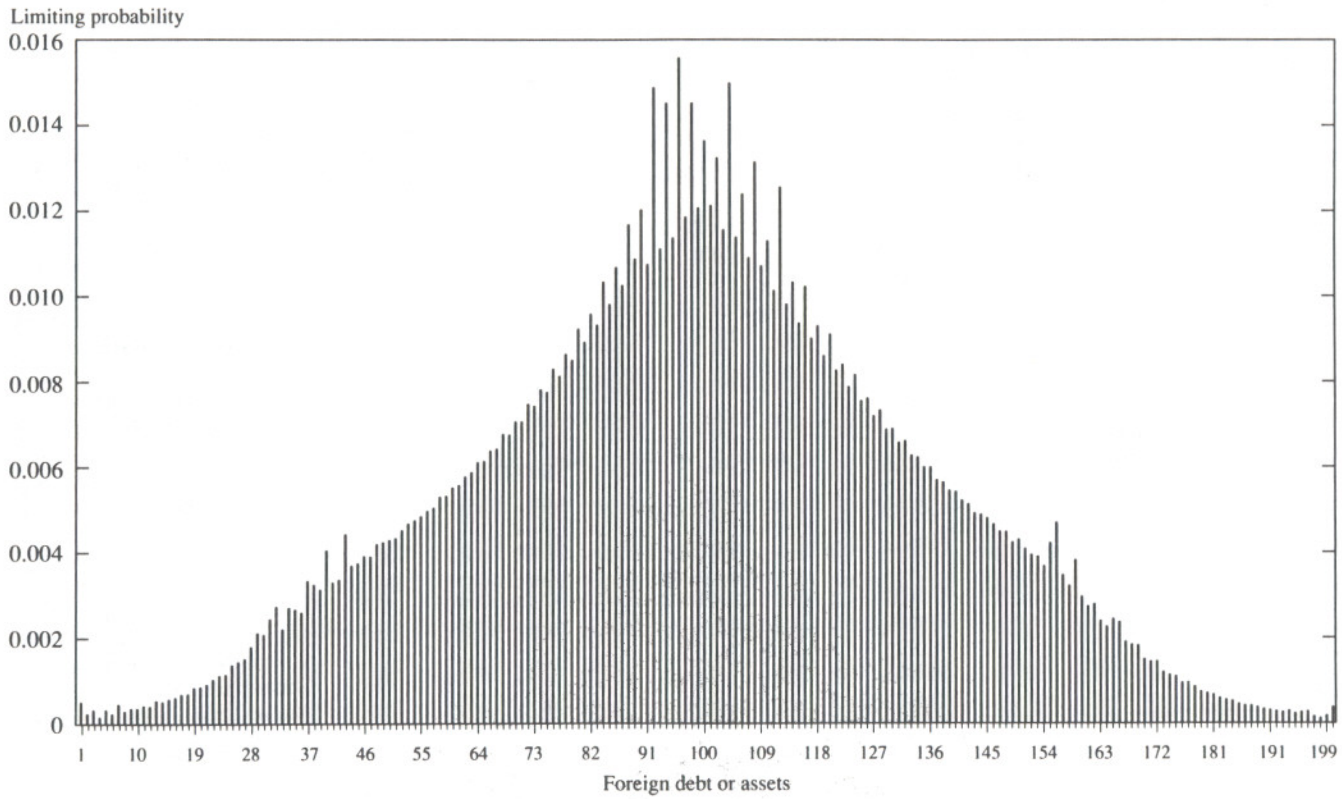
Table 6. *Equilibrium Co-Movements in the Benchmark Model*

Variables	σ	σ/σ_{tot}	$\rho(1)$	ρ_{tot}	ρ_{ym}
Output at domestic prices	1.96	0.44	0.500	0.095	0.531
Output at consumer prices	2.11	0.52	0.500	0.377	0.708
Output at import prices	2.85	0.70	0.674	0.742	1.000
Consumption	2.68	0.66	0.948	0.055	0.638
Tradables	2.46	0.61	0.921	0.231	0.782
Nontradables	3.05	0.75	0.938	−0.110	0.480
Trade balance	4.41	1.09	0.549	0.836	0.484
Current account	4.04	0.99	0.486	0.949	0.745
Net factor payments	1.38	0.34	0.995	0.098	0.562
Foreign assets	39.00	9.61	0.995	0.098	0.562
Terms of trade	4.06	1.00	0.500	1.000	0.742
Price of nontradables	3.31	0.81	0.843	−0.157	0.128
Real exchange rate	1.55	0.38	0.841	−0.164	0.123
Consumer price index	2.02	0.50	0.781	0.656	0.674
Consumption at consumer prices	2.18	0.54	0.607	−0.538	0.161
Consumption basket[a]					
Home goods	4.23	1.04	0.566	−0.825	−0.257
Imports	2.46	0.61	0.921	0.231	0.782
Nontradables	1.96	0.48	0.500	0.095	0.720
Other correlations:					
Consumption at consumer prices and output at domestic prices			0.593		
Consumption of tradables and nontradables			0.920		
Trade balance and output at domestic prices			0.307		
Trade balance and first lag of terms of trade			0.418		

Note: All variables are measured at import prices, unless otherwise noted. For each variable, σ is the standard deviation in a percent; σ/σ_{tot} is the standard deviation relative to the standard deviation of the terms of trade; $\rho(1)$ is the first-order serial autocorrelation; ρ_{tot} is the correlation with the terms of trade; and ρ_{ym} is the correlation with output at import prices.

[a] Each component of the consumption basket is in units of the corresponding good.

Figure 1. *Limiting Distribution of Foreign Assets*



and even exaggerate slightly, the HLM effect. However, with the "realistic" structure of parameters and exogenous shocks derived from the data, the proborrowing effects resulting from expectations of future shocks cannot offset the prosaving effects resulting from contemporaneous shocks, and, hence, net exports are procyclical instead of countercyclical—the correlation of the trade balance with output at import prices is 0.48, and with the standard measure of real GDP, 0.31.[16] The model also fails to mimic the behavior of net factor payments and produces excessive variability in foreign asset holdings.

The weak performance in explaining some aspects of trade balance behavior suggests that savings in dynamic models of endowment economies are not properly characterized. In particular, actual savings plans embody optimal investment decisions that are ignored in the model. When physical capital interacts with foreign assets as a means of saving, any persistent improvement in productivity or the terms of trade produces a proborrowing effect reflecting the desire to increase investment. Hence, savings cannot be well duplicated by the model because it ignores the equilibrium covariance between savings, investment, and the trade balance. The fact that about two thirds of imports are capital goods, as can be inferred from column (3) in Table 4, suggests that this issue may be quantitatively important—for example, in the one-good model with investment studied by Mendoza (1991a), the trade balance is countercyclical.

A one-good investment model, however, also produces the counterfactual result that savings and consumption are almost perfectly correlated with GDP.[17] In contrast, the model examined here does not display this anomaly: consumption fluctuates less than the terms of trade, exhibits positive persistence, and is positively, but imperfectly, correlated with output at import prices. Consumption is not perfectly procyclical because terms of trade and output shocks generate not only procyclical-oriented consumption-smoothing effects, resulting from projected changes in wealth, but also consumption-substitution effects, due to changes in relative prices between the three goods included in the model.

The *J*-curve dynamics of the co-movement between net exports and the terms of trade, as identified, for instance, in the work of Backus, Kehoe, and Kydland (1992a), is partially explained by the model. The first-order autoregressive structure of the shocks implies that the correlation be-

[16] The standard measure of real GDP is referred to as output at domestic prices henceforth.

[17] Recall from Section I that, excluding durables, the GDP correlation of consumption is between 0.6 and 0.8.

tween the trade balance at t, and the terms of trade at lag k is simply $\theta^k \rho_{tot,tb}$, and, hence, that the cross-correlation function of these variables is "tent" shaped. The evidence shows that this is a good approximation for some Group of Seven countries, but is not for Canada and the United States.[18]

In the behavior of the real exchange rate, the model produces mixed results. The three measures of the real exchange rate (that is, the relative price of nontradables, this price weighted by its share in consumption expenditures, and a duality-based consumer price index) exaggerate the persistence of actual real exchange rate movements and fluctuate significantly less than observed real exchange rates. The variability of the three measures is between 1.6 and 2.2 percent, whereas according to Table 3 this coefficient exceeds 8.5 percent for real exchange rate fluctuations in Germany, Japan, and the United Kingdom and is almost 3 percent in Canada. With an elasticity of substitution between tradables and nontradables of 0.74, the relative price of nontradables in the model does not vary enough to account for large deviations from purchasing power parity. Mussa's (1990) argument that the observed fluctuations of real exchange rates in the era of floating nominal rates cannot be accounted for solely by real disturbances is consistent with this result.

Income and Price Elasticities of Imports

The factors that determine the response of net exports to a change in the terms of trade in the model are explored next by an examination of the income and price elasticities of imports. This analysis also links the elasticities approach with the intertemporal equilibrium framework.

Consider researchers that are given the data produced by the model and asked to study under what conditions an improvement of the terms of trade would induce a narrowing of the trade deficit. They follow the literature on macroeconomic models of trade, as reviewed by Goldstein and Khan (1985), and assume that the utility function is as in equation (6). Given a static budget constraint (that is, equation (2) ignoring asset accumulation and the shocks), they solve for the demand for imports:

$$f = \frac{p^x \mathbf{X} + p^n \mathbf{N}}{\dfrac{1}{(1-\alpha)} + \left(\dfrac{(1-\alpha)^{1+\alpha\mu}}{\alpha^{\alpha\mu}}\right)^{-\frac{1}{1+\mu}} \left(\dfrac{p^n}{p^{x^\alpha}}\right)^{\frac{\mu}{1+\mu}}}. \tag{14}$$

[18] Cross-correlation functions of net exports and the terms of trade for the Group of Seven countries computed with the data used in Table 1 also support this argument.

Given the Cobb-Douglas interaction of exportables and importables, the response of the trade balance to a contemporaneous change in the terms of trade in this static model is

$$\frac{\partial TB}{\partial p^x} = e^y \mathbf{X}(1 - \eta_{f,p^x}), \tag{15}$$

where $\eta_{f,px}$ is the elasticity of imports with respect to the terms of trade: $\eta_{f,px} = (p^x/f)(\partial f/\partial p^x)$. From equation (15) it follows that a sufficient condition for net exports and the terms of trade to be positively correlated is that $\eta_{f,px}$ be less than unity.[19] This is the model's modified Marshall-Lerner condition. It differs from the usual condition, because in the small open economy with endowments both the supply of imports to and the demand for exports from the domestic economy are infinitely elastic at the level of the world's terms of trade. For an increase in p^x to improve the trade balance, it is necessary that exports increase more than imports, which, given the unitary elasticity of substitution between importables and exportables, can be guaranteed as long as $\eta_{f,px} < 1$.

The researchers then estimate income and price elasticities for the atemporal import demand function (equation (14)), unaware of the true intertemporal structure of the economy. Leaving aside the nonlinear form in which relative prices appear, they estimate linear regressions.[20] Their estimates of the elasticities are reported in Table 7, along with those they would obtain in an alternative experiment for which the artificial economy's data are produced assuming that there are only highly persistent terms of trade shocks ($\theta = 0.9$). In the two cases they conclude that $\eta_{f,px} < 1$, and, hence, that improvements in the terms of trade improve the trade balance. The numerical solutions of the true model support this conclusion, but also show that the intuition behind it is not always correct.

In the second experiment, where only the terms of trade experience shocks and intertemporal effects are minimized by their high persistence—as prosaving and proborrowing effects offset each other—the intuition of the static model is correct. With the true model driven only by terms of trade shocks, the regressions identify import demand, and with almost permanent shocks this import demand is not affected by large intertemporal effects. In fact, by differentiating equation (14), it is possible to show that the smaller is μ (that is, the higher is the elasticity of

[19] In the intertemporal model, condition (15) has the additional term, $(p^x)^{-1}(A_{t+1} - A_t(1 + r^*))$.

[20] Linearity is not a restrictive assumption around the stochastic steady state—a nonlinear estimate of $\eta_{f,\,px}$ computed as the expected value of the ratio of percentage deviations from means of f and p^x is identical to the linear regression estimate.

Table 7. *Linear Price and Income Elasticities of Imports in the Artificial Economy*

	Elasticities of Imports		
Experiment	Domestic income	Terms of trade	Relative price of nontradables
Benchmark model	0.674	0.140	0.003
Terms of trade shocks only	1.005	0.627	0.000

Note: The elasticity of variable x with respect to variable y is computed as $\beta x, y, \frac{E(y)}{E(x)}$, where $\beta x, y$ is a linear regression coefficient. These coefficients are derived from the model by using the cofactors of the correlation matrix computed with the limiting distribution of the model's state variables. For example, the income elasticity of imports ($\eta_{f,y}$) is

$$\eta_{f,y} = \beta_{f,y}\frac{E(y)}{E(f)},$$

and in this expression $\beta_{f,y}$ is

$$\beta_{f,y} = \left(\frac{-\sigma f}{\sigma y}\right)\left(\frac{\Re_{f,y}}{\Re_{f,f}}\right),$$

where $\Re_{f,y}$ and $\Re_{f,f}$ are the cofactors of the matrix of correlation coefficients between imports, income, the terms of trade, and the relative price of nontradables.

substitution between tradables and nontradables), the more likely it is that $\eta_{f,px} < 1$, because $\partial f/\partial p^x$ is an increasing function of μ. Solving analytically for $\eta_{f,px}$, given $\partial f/\partial p^x$ and the model's parameters, yields 0.61, which is very close to what the regressions show—the income elasticity is unity, which is also close to what the regressions produce.

Comparing the first experiment with the second, where the data come from the model described in the previous section, the regression shows that both income and terms of trade elasticities of imports decline. The implication of the static theory is that $\partial TB/\partial p^x$ should continue to be positive, and should be larger than before, since $\eta_{f,px}$ declined. The numerical solution of the true model confirms this (in the pure terms of trade shock case the correlation between the trade balance and the terms of trade is 0.42, about half of what is reported in Table 6). However, in this case the elasticities are not properties of import demand because there are other shocks, in addition to terms of trade disturbances shifting import supply, that affect import demand directly (that is, the regressions cannot identify the demand for imports). The data on imports represent the equilibrium of the market of importables as it fluctuates through time under the influence of supply and demand shifts.[21] Moreover, once in-

[21] For a discussion of the problems of identification and simultaneity in the estimation of import demand functions, see Goldstein and Khan (1985).

tertemporal effects are incorporated, the persistence of shocks affects the relationship between net exports and the terms of trade even if the preference parameters—μ and α—that determine $\eta_{f,px}$ in a static model remain unchanged.

The researchers would reject the prediction of unitary income elasticity of imports implied by the CES utility function in the experiment where they use the benchmark model's data—the income elasticity of imports is 0.67. However, the CES utility *is* part of the true model. The estimate of the income elasticity of imports declines because of the prosaving effects resulting from the consumption-smoothing nature of the intertemporal model. Like price elasticities, the income elasticity represents movements in the equilibrium of the imports market and not changes in import demand. The income elasticity at 0.67 is lower than the income elasticities reported in Goldstein and Khan's (1985) review, which ranged from 0.76 to 2.19, and is also lower than the average income elasticities estimated by Marquez (1991).

In summary, the above discussion suggests that the positive comovement between net exports and the terms of trade in the dynamic optimizing model is influenced by atemporal and intertemporal effects. In the atemporal context, the not too low elasticity of substitution between tradables and nontradables and the small share imports have in total consumption expenditures suggest a positive correlation between the trade balance and the terms of trade. In addition, the intertemporal effects suggest that given the persistence of the shocks and their mutual correlation, the prosaving effect dominates the proborrowing effect. The condition that $\eta_{f,px} < 1$ provides useful information for predicting how net exports respond to a change in the terms of trade, but $\eta_{f,px}$ itself is not a property of the demand for imports—it is simply a characteristic of the equilibrium path of the market of importables—nor is knowledge of preference parameters alone sufficient to predict this response.

Sensitivity Analysis and Incomplete Markets

Tables 8–10 list eight indicators that describe how net exports are affected by changes in the persistence of the shocks, the intertemporal elasticity of substitution in consumption, and the atemporal elasticity of substitution between tradables and nontradables. According to Table 8, the effects of changes in the persistence of the shocks are consistent with the predictions of theoretical work. As θ increases, both the correlation between the trade balance and the terms of trade and the correlation between the trade balance and output (in terms of imports or domestic prices) decline. This result reflects the combined effect of proborrowing

Table 8. *Trade Balance Behavior and the Persistence of Income Shocks*

θ	σ_{tb}	$\rho(1)$	$\rho_{tb,tot}$	$\rho_{tb,ym}$	$\rho_{tb,ydp}$	$\eta_{f,ym}$	$\eta_{f,tot}$
0.1	4.56	0.135	0.910	0.683	0.328	0.463	0.024
0.2	4.53	0.239	0.898	0.643	0.324	0.511	0.044
0.3	4.50	0.343	0.882	0.597	0.320	0.561	0.068
0.5[a]	4.41	0.549	0.836	0.484	0.307	0.674	0.140
0.7	4.24	0.746	0.745	0.315	0.274	0.808	0.270
0.8	3.99	0.841	0.645	0.189	0.236	0.887	0.411
0.9	3.64	0.936	0.401	−0.069	0.146	0.987	0.685

Note: θ is the first-order serial autocorrelation of the disturbances; σ_{tb} is the standard deviation of the trade balance in percent; $\rho(1)$ is the first-order serial autocorrelation of the trade balance; $\rho_{tb,tot}$ is the correlation of the trade balance with the terms of trade; $\rho_{tb,ym}$ is the correlation of the trade balance with GDP at import prices; $\rho_{tb,ydp}$ is the correlation of the trade balance with GDP at domestic prices; $\eta_{f,ym}$ is the elasticity of imports with respect to income at import prices; and $\eta_{f,tot}$ is the terms of trade elasticity of imports.

[a] Value in the benchmark simulation.

and prosaving movements induced by the agent's desire to smooth consumption. A purely transitory shock generates only a prosaving effect, and, hence, transitory disturbances induce stronger correlations between the trade balance, the terms of trade, and output. As the shocks exhibit more persistence, agents weight the prosaving effect of a contemporaneous increase in the endowments or in the purchasing power of exports against the expectations of increased future income from which they would like to borrow. Thus, shocks with more persistence induce less co-movement between the trade balance and output and a weaker HLM effect.

Table 8 also shows that the trade balance remains procyclical unless the shocks are highly persistent—net exports become weakly countercyclical when $\theta = 0.9$. This high persistence is not consistent with what characterizes actual fluctuations in the terms of trade, output, and interest rates. Thus, even though theory shows that persistent shocks could produce a countercyclical trade balance, the simulations show that this is not the case when the persistence observed in actual disturbances is considered.

The fact that the persistence of shocks affects net exports is a key implication of the limited risk-sharing possibilities imposed by the incompleteness of insurance markets. If these markets were complete, the persistence of domestic disturbances would not affect the co-movements of the trade balance. Hence, the moments listed in Table 8 illustrate the extent to which market incompleteness emphasizes the role of domestic shocks. At the extremes, θ is a key determinant of trade balance behavior, but within the range $0.3 \leq \theta \leq 0.7$, changing θ does not affect significantly the variability of the trade balance nor its correlations with the terms of trade and output at domestic prices. Still, within that range, which contains most of the first-order autocorrelation coefficients for the terms of trade and output at domestic prices observed in data of the Group of Seven countries, both incomplete-markets and complete-markets models are able to mimic some stylized facts of net exports and consumption.[22] Hence, the stylized facts of macroeconomic aggregates may not be a good indicator of market completeness. This is consistent with the findings of Cole and Obstfeld (1991), suggesting that competitive allocations are not significantly affected by market completeness, and Baxter and Crucini (1992), who found that the persistence of exogenous shocks needs to be quite high for incomplete markets to affect equilibrium co-movement significantly.

[22] In this context, the inability of the endowment economy to mimic the countercyclical trade balance is irrelevant, since Mendoza (1991a) showed that in a one-good model with investment net exports were countercyclical.

Table 9. *Trade Balance Behavior and the Intertemporal Elasticity of Substitution*

$1/\gamma$	σ_{tb}	$\rho(1)$	$\rho_{tb,tot}$	$\rho_{tb,ym}$	$\rho_{tb,ydp}$	$\eta_{f,ym}$	$\eta_{f,tot}$
0.2	3.91	0.603	0.534	0.329	0.673	0.813	0.532
0.3	3.93	0.593	0.633	0.428	0.596	0.757	0.433
0.5	4.07	0.562	0.778	0.538	0.434	0.674	0.266
0.6	4.28	0.555	0.818	0.511	0.353	0.668	0.184
0.67[a]	4.41	0.549	0.836	0.484	0.307	0.674	0.140
0.8	4.72	0.542	0.860	0.396	0.223	0.711	0.046
1.0	5.21	0.534	0.876	0.240	0.125	0.803	−0.077

Note: $1/\gamma$ is the intertemporal elasticity of substitution in consumption; the other variables are as in Table 8.
[a] Value in the benchmark simulation.

These results also question the view that dynamic models of endowment economies with incomplete markets emphasize the persistence of shocks because they deal with experiments comparing deterministic equilibria, as Backus (1989) suggested. By adapting the deterministic Obstfeld-Svensson-Razin models of the early 1980s to a setting with uncertainty, the simulations show that the predictions of these models extend to the equilibrium co-movement between net exports and the terms of trade. The persistence of shocks affects the trade balance because of the incompleteness of markets, not because of the presence or absence of uncertainty.

Consider now the effects on net exports resulting from changes in the intertemporal elasticity of substitution ($1/\gamma$). Table 9 shows that as this parameter increases, the correlation between the trade balance and the terms of trade rises. According to equation (15), this is due to the decline in the *equilibrium* elasticity of imports with respect to the terms of trade reported in the last column. The weaker response of imports to changes in the terms of trade reflects the higher degree of substitutability between current and future imports. A rise in $1/\gamma$ reduces the degree of relative risk aversion and induces individuals to distribute the impact of an improvement in the terms of trade at date t more evenly across consumption in all periods. The increase in σ_{tb} that follows the increase in $1/\gamma$ is also consistent with this view, because lower risk aversion leads agents to adjust savings more to accommodate a smoother consumption stream. Table 9 also shows that as $1/\gamma$ increases, the persistence of the trade balance is marginally affected while the co-movement between the trade balance and output at domestic prices falls, but the co-movement between the trade balance and output at import prices first increases and then falls. The small changes in the autocorrelation coefficient are not surprising, since the trade balance tends to reflect the persistence common to the three random disturbances driving the model, but the pattern of changes in the output correlations is more difficult to interpret.

Turning now to Table 10, an increase in the atemporal elasticity of substitution between tradables and nontradables ($1/(1 + \mu)$) tends to increase the correlation between the trade balance and the terms of trade. As before, this reflects a fall in the equilibrium elasticity of imports relative to the terms of trade. This elasticity declines because the response of imports to a change in the terms of trade declines as tradables and nontradables become better substitutes; σ_{tb} also increases with $1/(1 + \mu)$, because, given an unlimited import supply, consumption smoothing is more effective when tradables and nontradables are good substitutes. Table 10 also shows that the persistence of the trade balance declines slightly as $1/(1 + \mu)$ increases, but remains close to θ—as in Table 9. The

Table 10. *Trade Balance Behavior and the Atemporal Elasticity of Substitution Between Tradable and Nontradable Goods*

$1/(1 + \mu)$	σ_{tb}	$\rho(1)$	$\rho_{tb,tot}$	$\rho_{tb,ym}$	$\rho_{tb,ydp}$	$\eta_{f,ym}$	$\eta_{f,tot}$
0.2	2.35	0.636	0.757	−0.068	0.278	0.674	0.607
0.3	3.16	0.597	0.795	0.073	0.300	0.843	0.428
0.5	3.67	0.571	0.815	0.325	0.318	0.847	0.309
0.6	3.99	0.563	0.823	0.401	0.313	0.787	0.237
0.74[a]	4.41	0.549	0.836	0.484	0.307	0.674	0.140
0.8	4.64	0.544	0.842	0.502	0.298	0.622	0.085
1.5	5.07	0.559	0.758	0.703	0.473	0.294	0.098

Note: $1/(1 + \mu)$ is the atemporal elasticity of substitution in consumption of tradable and nontradable goods; the other variables are as in Table 8.

[a] Value in the benchmark simulation.

behavior of the output correlations, however, is now reversed. The correlation of the trade balance with output at import prices falls, while the correlation of the trade balance with output at domestic prices first increases and then falls. As before, it is not clear why one correlation declines monotonically while the other does not.

The effects of changes in μ and γ reported in Tables 9 and 10 differ significantly from those obtained in complete-markets models. Backus (1992) proved analytically that in an endowment economy with complete markets, the terms of trade are more strongly correlated with the trade balance the lower are the intertemporal and atemporal elasticities of substitution. This is also the case in the real business cycle models of Stockman and Tesar (1990) and Backus, Kehoe, and Kydland (1992a).[23] Under incomplete markets the strength of wealth effects is such that an increase in $1/(1 + \mu)$ leads to an increase in the correlation between the trade balance and the terms of trade, whereas in models of complete markets substitution effects seem to dominate, and the result is the opposite.

The results of the sensitivity analysis also indicate that measurement error in the parameters θ, γ, and μ is not likely to account for the inability of the model to mimic some stylized facts. The trade balance-output correlation measures exaggerate actual GDP correlations for most of the values of those parameters that were considered, and the variability of the real exchange rate is always underestimated—the highest standard deviations of the three measures of real exchange rates, ranging from 3.9 percent for the consumer price index to 7.7 percent for the relative price of nontradables, are obtained when $1/(1 + \mu) = 0.2$. Finally, the sensitivity analysis confirms the view that income and price elasticities are related to the co-movement between the trade balance and the terms of trade, but they reflect properties of the equilibrium of the imports market, not import demand, and are not invariant to changes in the persistence of exogenous shocks.

V. Concluding Remarks

This paper examined macroeconomic effects of random shocks to output, the terms of trade, and the real interest rate in an intertemporal equilibrium model of a small open, endowment economy with incomplete insurance markets. Agents consume three goods, two tradables, a

[23] These authors' measure of the terms of trade is the inverse of the measure used in this paper. So to make their results comparable, one must change the signs of the correlations between the trade balance and the terms of trade.

home good, and an imported good, and one nontradable. The model was parameterized using as guides the stylized facts for output, terms of trade, and interest rate fluctuations, and existing empirical evidence on preference parameters. Recursive numerical solution methods were used to compute moments of variability, co-movement, and persistence of the endogenous variables. The model mimics some stylized facts, particularly the Harberger-Laursen-Metzler effect and the cyclical properties of consumption, but it cannot explain large deviations from purchasing power parity and the countercyclical movements of net exports. The absence of investment seems to play a key role in producing these anomalies.

The model's quantitative implications were compared with the predictions of the static elasticities approach. Estimates of income and price elasticities of imports provide information useful to determine the response of net exports to terms of trade shocks, but they are not properties of the demand function and they are not invariant to assumptions regarding the duration of the terms of trade and other income shocks.

The sensitivity of the model's equilibrium co-movements to changes in the persistence of income disturbances and changes in preference parameters was also examined. This analysis showed that measurement error in preference parameters is not likely to account for the model's inability to mimic some stylized facts, and that the persistence of income shocks affects equilibrium co-movements as predicted by previous theoretical work. Preference parameters seem to affect the equilibrium co-movements in the opposite direction from the one models with complete markets predict, reflecting the importance of income effects under incomplete markets.

The results of this investigation suggest that an analysis of a model that incorporates investment is necessary. The integration of investment is likely to produce a countercyclical trade balance, as analyses of one-good investment models have shown, and it would also produce realistic consumption-output correlations that are not present in the one-good model. Such a model could also be used to explore the role of terms of trade shocks as a driving force of business cycles, and would also serve to assess the macroeconomic effects of trade liberalization and other commercial policies.

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