A GENERAL EQUILIBRIUM MODEL OF SOVEREIGN DEFAULT AND BUSINESS CYCLES*

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Why are episodes of sovereign default accompanied by deep recessions? The existing literature cannot answer this question. On one hand, sovereign default models treat income fluctuations as an exogenous endowment process with ad hoc default costs. On the other hand, emerging markets business cycle models abstract from modeling default and treat default risk as part of an exogenous interest rate on working capital. We propose instead a general equilibrium model of both sovereign default and business cycles. In the model, some imported inputs require working capital financing, and default triggers an efficiency loss as these inputs are replaced by imperfect substitutes, because both firms and the government are excluded from credit markets. Default is an optimal decision of a benevolent planner for whom, even after internalizing the adverse effects of default on economic activity, financial autarky has a higher payoff than debt repayment. The model explains the main features of observed cyclical dynamics around defaults, countercyclical spreads, high debt ratios, and key long-run business cycle moments. *JEL* Codes: E32, E44, F32, F34.

I. INTRODUCTION

Episodes of sovereign default are characterized by a striking set of empirical regularities. In particular, the event windows shown in Figure I using data from 23 default events in the 1977–2009 period highlight three key facts:

FACT 1. *Default events are associated with deep recessions.* On average, GDP and consumption fall about 5% below trend, and imported inputs and total intermediate goods fall nearly 20% below trend. Labor falls to a level about 15% lower than in the three years prior to the defaults. Net exports jump about 10 percentage points of GDP in the span of the two quarters before and after default events. These observations are in line with the findings of Levy-Yeyati and Panizza (2011)

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showing that default events coincide with large GDP drops in data from 39 developing countries covering the 1970–2005 period. In addition, Tomz and Wright (2007) studied defaults from 1820 to 2004 and found the maximum default frequency when output is at least 7% below trend.

FACT 2. *Interest rates on sovereign debt peak at about the same time as output hits its through and defaults occur, and they are negatively correlated with GDP.* These two empirical regularities are visually evident by comparing the output and interest rate plots in Figure I. In addition, Neumeyer and Perri (2005) and Uribe and Yue (2006) report cyclical correlations between GDP and country interest rates ranging from 0 to −0.8, with averages of −0.55 in Neumeyer and Perri and −0.42 in Uribe and Yue.[1]

FACT 3. *External debt as a share of GDP is high on average, and higher when countries default.* The mean debt ratio before the default events in Figure I was about 50%, and reached about 72% at the time of the defaults. Looking at all emerging and developing countries, as defined in International Monetary Fund (2006), foreign debt was one-third of GDP on average over 1998–2005. Highly indebted poor countries had the highest average debt ratio, at about 100% of GDP, followed by the Eastern European and Western Hemisphere countries, with averages of about 50% and 40% of GDP, respectively. Looking at defaults historically, Reinhart, Rogoff, and Savastano (2003) report that the external debt ratio averaged 71% of GDP for all developing country defaults in the 1824–1999 period. This is very close to the 72% mean estimate for our default events in Figure I.

Providing a joint explanation of these stylized facts has proven difficult in international macroeconomics, because of a crucial disconnect between two key bodies of theory: On one hand, quantitative models of business cycles in emerging economies explain countercyclical country interest rates by modeling the interest rate as an exogenous variable that represents the financing cost of both the government's sovereign debt and firms'

1. Neumeyer and Perri (2005) used data for Argentina, Brazil, Korea, Mexico, and the Philippines. Uribe and Yue (2006) added Ecuador, Peru, and South Africa, but excluded Korea.

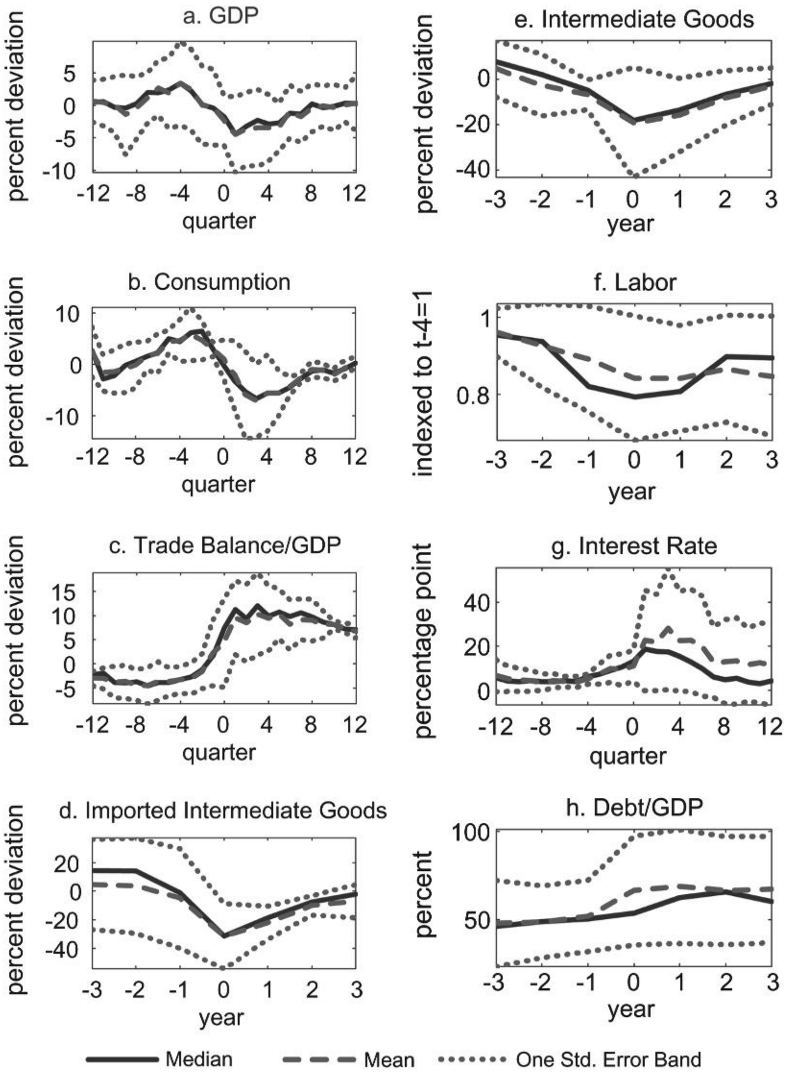

FIGURE I

Macroeconomic Dynamics around Sovereign Default Events

GDP, consumption, and trade balance/GDP are H-P detrended. Imported inputs and intermediate goods are log-linearly detrended. Labor data are indexed so that employment four years before default equals 1. The event window for GDP is based on data for 23 default events over the 1977–2009 period. Due to data limitations, the sample period and/or the number of events varies in some of the other windows. Full details are provided in Appendix 3.

working capital loans.[2] In these models, default is exogenous and hence facts (1) and (3) are left unexplained. On the other hand, quantitative models of sovereign default based on the classic setup of Eaton and Gersovitz (1981) generate countercyclical sovereign spreads by assuming that a sovereign borrower faces shocks to an exogenous output endowment with ad hoc output costs of default.[3] Since output is an exogenous endowment, these models cannot address fact (1) and they do poorly at explaining fact (3). In short, business cycle models of emerging economies cannot explain the default risk premia that drive their findings, and sovereign default models cannot explain the cyclical output dynamics that are critical for their results.

This article proposes a general equilibrium model of sovereign default and economic fluctuations that provides a solution to the disconnect between those two classes of models. This framework is the first to model the joint determination of the dynamics of output and sovereign default in an environment in which they influence each other via the interaction between foreign lenders, the domestic sovereign borrower, domestic firms, and households. The model links default with private economic activity via the financing cost of working capital used to pay for a subset of imported inputs. This subset of imported inputs can be replaced with other imported inputs or with domestic inputs, but these are only imperfect substitutes, and as a result default causes an endogenous efficiency loss in production and bears an endogenous output cost. This efficiency loss builds into the model a financial amplification mechanism by which sovereign default amplifies the effects of adverse productivity shocks on output, and this in turn feeds back into default incentives and sovereign spreads.

Quantitative analysis shows that the model does well at explaining the three key stylized facts of sovereign defaults. Moreover, the model's financial amplification mechanism amplifies the effect of TFP shocks on output by 80% when the economy defaults, and the model matches salient features of emerging markets business cycles, such as the high variability of consumption,

2. See Neumeyer and Perri (2005), Uribe and Yue (2006), and Oviedo (2005).

3. See, for example, Aguiar and Gopinath (2006), Arellano (2008), Yue (2010), Bai and Zhang (2012). Aguiar and Gopinath studied an extension with endogenous output produced by labor in a setup without wealth effects on labor supply. This model is similar to the endowment models because output is endogenous only because of the RBC-like response of labor to TFP shocks, and remains independent of the default decision and unaffected by default events.

the countercyclical dynamics of net exports, and the correlation between output and default.

These results hinge on three important features of the model: First, the assumption that producers of final goods require working capital financing to pay for imports of a subset of intermediate goods. Second, the assumption that both firms and the government are excluded from world credit markets when the country defaults. Third, the efficiency loss in final goods production that occurs when the country defaults, because final goods producers cannot finance the purchases of those inputs, and thus are forced to replace them with other imported and domestic inputs that are imperfect substitutes.

Evidence provided in various empirical studies is in line with the above key features of the model. Amiti and Kronings (2007) and Halpern, Koren, and Szeidl (2008) found firm-level evidence of imperfect substitutability between foreign and domestic inputs, and the associated total factor productivity (TFP) effects of changes in relative factor costs. They used data for Indonesia and Hungary to study the impact of reducing imported input tariffs on firm-level productivity, and found that imperfect substitution of inputs accounts for the majority of the effect of tariff cuts on TFP. In the context of a sovereign default event, Gopinath and Neiman (2010) studied firm-level data around Argentina's debt crisis of 2001–2002. They provided evidence of significant adjustments in imported inputs and estimated increases of up to 30 percent in shares of input spending on domestic goods.

With regard to the connection between sovereign default and private credit conditions, Reinhart and Rogoff (2010) and Reinhart (2010) showed that there is a close association between banking crises, with widespread defaults in the nonfinancial private sector, and sovereign defaults, and that private debts become public debt after sovereign defaults. Kaletsky (1985) argued that exclusion from trade credit is one of the heaviest penalties that countries in default actually experience. He documented the exclusion from trade credit in countries that defaulted in the 1980s, and showed estimates of short-term private credit nearly as large as unpaid interest in medium-term sovereign debt. More recently, Arteta and Hale (2007) and Kohlscheen and O'Connell (2008) provided evidence of significant adverse effects of sovereign default on private access to foreign credit. Kohlscheen and O'Connell found median declines in trade credit from commercial banks of about 35% and 51% two and four years after default events, respectively.

Arteta and Hale showed that there are strong negative effects on private corporate bond issuance during and after default episodes.

There is also related evidence suggesting that sovereign default disrupts external trade, which is relevant because the implications of our model are identical whether default triggers exclusion from credit markets to finance imports or trade sanctions affecting imports directly. Rose (2005) conducted a cross-country analysis of trade flows and default and found that default has a large, persistent negative effect on bilateral trade between creditor and debtor countries. Martinez and Sandleris (2008) provided further empirical evidence on the association between sovereign defaults and the decline in trade.

The model's financial amplification mechanism operates as follows: Final goods producers use labor and an Armington aggregator of imported and domestic inputs as factors of production, with the two inputs as imperfect substitutes. Domestic inputs require labor to be produced. Imported inputs come in different varieties described by a Dixit-Stiglitz aggregator, and a subset of them needs to be paid in advance using foreign working capital loans. Under these assumptions, the optimal input mix depends on the cost of working capital financing and on TFP. When the country has access to world financial markets, final goods producers use a mix of all varieties of imported inputs and domestic inputs and have access to working capital financing at the world real interest rate. In contrast, when the country defaults, final goods producers substitute away from the imported inputs that require working capital because of the loss of access to credit. This reduces production efficiency sharply because of the imperfect substitutability across varieties of imported inputs and across domestic and foreign inputs, and because to increase the supply of domestic inputs labor reallocates away from final goods production.[4]

This mechanism produces endogenous feedback between the output collapse induced by a default event, default probabilities before default, and country risk premia, because, for a given debt position, the probability of default depends on the likelihood of

4. As a result, part of the output drop that occurs when the economy defaults shows as a fall in the Solow residual (i.e., the fraction of *aggregate* GDP not accounted for by capital and labor). This is consistent with the data from emerging markets crises showing that a large fraction of the observed output collapse is attributed to the Solow residual (Meza and Quintin 2006, Mendoza 2010). Moreover, Benjamin and Meza (2007) show that in Korea's 1997 crisis, the productivity drop followed in part from a sectoral reallocation of labor.

TFP realizations for which the sovereign's payoff in the default state exceeds that of repayment. In particular, the expectation of endogenously lower output in the event of default at higher levels of country risk alters repayment incentives for the sovereign, affecting the equilibrium determination of debt positions and default risk premia.

A central implication of the model's financial amplification mechanism is that the output cost of default, which is a key determinant of default dynamics in the Eaton-Gersovitz class of models, is an endogenous increasing, convex function of TFP. This differs sharply from the two approaches followed to model ad hoc costs of default in the literature. One approach models default costs as a fixed percentage of the realization of an exogenous endowment when a country defaults (e.g., Aguiar and Gopinath 2006, Yue 2010). In this case, default is just as costly, in percentage terms, in a low-endowment state as in a high-endowment state (i.e., the percent cost is independent of the endowment realization), and hence average debt ratios are unrealistically low when the models are calibrated to actual default frequencies. The second approach is the asymmetric formulation proposed by Arellano (2008). There is no cost of default below a certain threshold endowment level, and above it the sovereign's income is reduced to the same constant level regardless of the endowment realization at the time of default. Thus, in the latter case the percent cost of default rises linearly with the endowment realization. This formulation makes default more costly in good states, making default more likely in bad states and increasing debt ratios. However, debt ratios in calibrated models are still much lower than in the data, unless features like multiple maturities, dynamic renegotiation, or political uncertainty are added.[5]

The endogenous default cost in our model preserves the advantages of Arellano's exogenous cost in terms of adding "state contingency" to the default option that allows the model to support higher mean debt ratios at observed default frequencies. Our

5. Arellano (2008) obtained a mean debt-output ratio of 6% using her asymmetric cost. Aguiar and Gopinath (2006) obtained a mean debt ratio of 19% using the fixed percent cost, but at a default frequency of only 0.23%. Yue (2010) used the same cost in a model with renegotiation calibrated to observed default frequencies, and obtained a mean debt ratio of 9.7%. Studies that have obtained higher debt ratios with modifications of the Eaton-Gersovitz environment, but still assuming exogenous endowments, include Cuadra and Sapriza (2008), D'Erasmo (2008), Bi (2008a, 2008b), Chatterjee and Eyigungor (2008), Benjamin and Wright (2008), and Lizarazo (2005).

baseline calibration, however, supports a mean debt-output ratio of 23%, nearly four times larger than in Arellano's baseline. In addition, in our model output costs of default are always incurred at equilibrium, whereas with Arellano's formulation defaults occur mostly when the endowment is *lower* than the threshold endowment value, so actual costs of default are zero at equilibrium. Moreover, in our setup, output itself falls sharply when the economy defaults, because of the model's financial amplification of the effects of TFP shocks on output. In contrast, in existing sovereign default models, large output drops can only result from large, exogenous endowment shocks.

The assumptions that both foreign and domestic inputs *and* the varieties of imported inputs are imperfect substitutes are necessary for the model's default cost to be an increasing, convex function of TFP. The cost rises with TFP because the efficiency loss is larger when TFP is higher. The cost is higher and becomes a steeper function of TFP at lower elasticities of substitution across inputs, because the inputs become less similar.[6] The elasticity of labor supply also influences the output cost of default. In particular, the cost is larger the higher this elasticity, because default triggers a reduction in total labor usage. However, the output cost of default, and the efficiency loss behind it, are still present even if labor is inelastic. Final goods producers still have to shift from a subset of imported input varieties to other imported inputs and to domestic inputs, and labor still reallocates from final goods to intermediate goods production.

The treatment of working capital in our model differs from the treatment in Neumeyer and Perri (2005) and Uribe and Yue (2006). They assumed the interest rate on working capital to be identical to the exogenous interest rate on sovereign debt, and as a result its fluctuations affect output regularly in a manner akin to a cyclical labor demand shock. In contrast, in our setup the interest rate on sovereign debt is endogenous and working capital influences output because of the exclusion from credit markets during default events, not because of exogenous cyclical interest rate shifts. In addition, in the Neumeyer-Perri and Uribe-Yue models, working capital loans pay the wages bill in full, which is at odds with empirical estimates suggesting that working

6. If the inputs are perfect substitutes there is no output cost of default, because firms can shift inputs without affecting production. If they are complements, production is either zero (with unitary elasticity of substitution) or not defined (with less-than-unitary elasticity) when the economy defaults.

capital is a small fraction of GDP (Schmitt-Grohe and Uribe 2007 estimate 9.3% annually for the United States, we estimate 6% for Argentina in Section IV). Oviedo (2005) found that the business cycle effects of interest rate fluctuations via working capital are weak with working capital requirements this low. In contrast, in our model the working capital requirement is low (calibrated to match the data), because firms use working capital to pay only for a subset of imported inputs, and yet working capital has large real effects as a consequence of the exclusion from credit markets when default occurs. This is similar to the mechanism in Mendoza (2010), where a working capital constraint with a low working capital requirement has negligible real effects, except when an occasionally binding collateral constraint suddenly limits access to working capital financing.

The rest of the article proceeds as follows. Section II presents the model. Section III examines the mechanism that drives the model's efficiency loss in production in partial equilibrium. Section IV explores the full model's quantitative implications. Section V concludes.

II. A Model of Sovereign Default and Business Cycles

There are four groups of agents in the model, three in the "domestic" small open economy (households, firms, and the sovereign government) and one abroad (foreign lenders). There are also two production sectors in the domestic economy, a sector f of final goods producers and a sector m of intermediate goods producers.

II.A. Households

Households choose consumption and labor supply so as to maximize a standard time-separable utility function $E\left[\sum_{t=0}^{\infty}\beta^t u\left(c_t - g(L_t)\right)\right]$, where $0 < \beta < 1$ is the discount factor, and c_t and L_t denote consumption and labor supplied in period t, respectively. $u(\cdot)$ is the period utility function, which is continuous, strictly increasing, strictly concave, and satisfies the Inada conditions. Following Greenwood, Hercowitz, and Huffman (1988), we remove the wealth effect on labor supply by specifying period utility as a function of consumption net of the disutility of labor $g(L_t)$, where $g(\cdot)$ is increasing, continuously differentiable, and convex. This formulation of preferences plays an important role in allowing international real business cycle models to explain observed business cycle facts, and it also simplifies the "supply side" of the model.[7]

Households take as given the wage rate w_t, profits paid by firms in the f and m sectors $\left(\pi_t^f, \pi_t^m\right)$ and government transfers (T_t). Households do not borrow directly from abroad, but the government borrows, pays transfers, and makes default decisions internalizing their utility.[8] Consequently, the households' optimization problem reduces to:

$$\max_{c_t, L_t} E\left[\sum \beta^t u\left(c_t - g\left(L_t\right)\right)\right], \tag{1}$$

$$s.t. \quad c_t = w_t L_t + \pi_t^f + \pi_t^m + T_t. \tag{2}$$

The optimality condition for labor supply is:

$$g'\left(L_t\right) = w_t. \tag{3}$$

For purposes of the quantitative analysis, we define $g(L) = \frac{L^\omega}{\omega}$ with $\omega > 1$. Hence, the Frisch elasticity of labor supply is given by $\frac{1}{(\omega-1)}$. The period utility function takes the standard constant-relative-risk-aversion form $u\left(c, L\right) = \frac{\left(c - L^\omega/\omega\right)^{1-\sigma} - 1}{1-\sigma}$ with $\sigma > 0$.

II.B. Final Goods Producers

Firms in the f sector produce using labor L_t^f and intermediate goods M_t, and a time-invariant capital stock k.[9] They face Markov TFP shocks ε_t with a transition probability distribution function $z\left(\varepsilon_t|\varepsilon_{t-1}\right)$. The production function is Cobb-Douglas:

$$y_t = \varepsilon_t \left(M\left(m_t^d, m_t^*\right)\right)^{\alpha_M} (L_t^f)^{\alpha_L} k^{\alpha_k} \tag{4}$$

with $0 < \alpha_L, \alpha_M, \alpha_k < 1$ and $\alpha_L + \alpha_M + \alpha_k = 1$.

7. Removing the wealth effect on labor supply is useful because otherwise the wealth effect pushes labor to display a counterfactual rise when TFP falls or when consumption drops sharply, as is the case in default episodes.

8. This assumption is very common in the Eaton-Gersovitz class of models but it is not innocuous, because whether private foreign debt contracts are allowed, and whether they are enforceable vis-à-vis government external debt, affects the efficiency of the credit market equilibrium (see Wright 2006).

9. Sovereign debt models generally abstract from capital accumulation for simplicity. Adding capital makes the recursive contract with default option significantly harder to solve because it adds an additional endogenous state variable. Moreover, changes in the capital stock have been estimated to play a small role in output dynamics around financial crises (see Meza and Quintin 2006 and Mendoza 2010).

The mix of intermediate goods is determined by a standard CES Armington aggregator that combines domestic inputs m_t^d and imported inputs m_t^*, with the latter represented by a Dixit-Stiglitz aggregator that combines a continuum of differentiated imported inputs m_j^* for $j \in [0,1]$:

$$(5)\quad M_t = \left[\lambda\left(m_t^d\right)^{\mu} + (1-\lambda)\left(m_t^*\right)^{\mu}\right]^{\frac{1}{\mu}}, \quad m_t^* \equiv \left[\int_{j\in[0,1]} \left(m_{jt}^*\right)^{\nu} dj\right]^{\frac{1}{\nu}}$$

The firms' purchases of variety j of imported inputs are denoted by m_{jt}^*. The "within" elasticity of substitution across all varieties is given by $\eta_{m_j^*} = |\frac{1}{(\upsilon-1)}|$. The Armington elasticity of substitution between m_t^* and m_t^d is defined as $\eta_{m^d,m^*} = |\frac{1}{(\mu-1)}|$ and λ is the Armington weight of domestic inputs.[10] The following parameter restrictions are assumed to hold: $0 < \nu, \mu < 1$, and $0 \leq \lambda < 1$. $\lambda < 1$ is necessary because without use of imported inputs default would be costless. In addition, foreign and domestic inputs and the varieties of imported inputs need to be imperfect substitutes (i.e., $0 < \nu, \mu < 1$) for the output cost of default to increase with ε, as we show later.

Imported inputs are sold in world markets at exogenous time-invariant prices p_j^* for $j \in [0,1]$ defined in terms of the price of final goods, which is the numeraire. The relative price of domestic inputs p_t^m is an endogenous equilibrium price.

A subset Ω of the imported input varieties defined by the interval $[0,\theta]$, for $0 < \theta < 1$, needs to be paid in advance using working capital financing.[11] The rationale for splitting imported inputs this way is so that in default episodes, when access to the set Ω of imported inputs is hampered by exclusion from credit

10. This structure of aggregation of imported and domestic inputs is similar to the one used in the empirical studies by Gopinath, Itskhoki, and Rigobon (2010) and Halpern, Koren, and Szeidl (2009).

11. We assume that the full cost of purchasing the varieties in Ω is paid in advance. Hence, θ determines the "intensitity" of the working capital friction in a similar way as standard working capital models use θ to define the fraction of the cost of a single input that is paid in advance. We could also introduce an extra parameter so that the varieties in Ω require that only a fraction of their cost be paid in advance, but lowering this fraction would have *qualitatively* similar effects as keeping the fraction at 100% and lowering Ω instead. Quantitatively, however, trading one formulation for the other is akin to emphasizing the extensive versus the intensive margin of trade adjustment, so reducing Ω by some amount is not equivalent to increasing by the same amount the fraction of all inputs inside Ω that would require working capital.

markets, imported inputs do not vanish entirely, even though they adjust sharply, as observed in the data.

We model working capital following the classic pay-in-advance setup of Fuerst (1992). Working capital loans κ_t are within-period loans provided by foreign creditors, and we assume a within-period timing of transactions according to which these loans are contracted and repaid after the uncertainty about the government's repayment of its current debt service is resolved. Under this assumption, working capital loans are contracted at the risk-free world real interest rate r_t^*. If the government repays, firms borrow at r_t^*, and if it does not they are excluded from world credit markets. Alternatively, one could assume timing structures or institutional features that would cause working capital loans to be exposed to default risk and lead to the default risks of firms and the government to be positively correlated.[12] In a previous version of this article (Mendoza and Yue 2011), the two are perfectly correlated because the government confiscates the repayment of firms when it defaults.[13] This formulation implicitly imposes a distortionary tax on firms' financing costs that is at play even when the country repays (as long as the probability of default next period is positive). However, the quantitative predictions of the model under this formulation are very similar to the ones reported in this article.

The standard pay-in-advance condition driving the demand for working capital is:

$$\frac{\kappa_t}{1+r_t^*} \geq \int_0^\theta p_j^* m_j^* dj. \tag{6}$$

Profit-maximizing producers of final goods choose κ_t so that this condition holds with equality. Domestic inputs and the varieties

12. Notice that existing models of emerging markets business cycles with working capital (e.g., Neumeyer and Perri 2005 and Uribe and Yue 2006) impose by assumption that the interest rates on sovereign debt and working capital are equal. Since sovereign default risk is exogenous in these models, they do not need to spell out the timing or institutional features of credit markets that support that assumption, but the question of what could cause private and sovereign risk to be correlated is still relevant.

13. There we also provided evidence showing that corporate and sovereign interest rates move together in emerging markets data for the 1994–2005 period. The median correlation of the two interest rates across countries is about 0.7. Arellano and Kocherlakota (2007) and Agca and Celasun (2009) provide further evidence of positive co-movement between private and sovereign interest rates, and Corsetti et al. (2010) show that this feature is also present in OECD data.

of imported inputs in the $[\theta, 1]$ interval do not require working capital. A plausible rationale for this asymmetry could be that trade in domestic inputs and some imported inputs is largely intrafirm trade and is at least partially collateralized by the goods themselves, whereas this mechanism may not work as well for other imported inputs because of government interference with payments via confiscation or capital controls, which are common during default episodes—as was clearly evident in Argentina's 2001 default.

Final goods producers choose factor demands in order to maximize date-t profits taking w_t, r_t^*, p_j^*, and p_t^m as given. Date-t profits are:

$$(7)\qquad \pi_t^f = \varepsilon_t \left(M_t\right)^{\alpha_M} \left(L_t^f\right)^{\alpha_L} k^{\alpha_k} - r_t^* \int_0^\theta p_j^* m_{jt}^* dj - \int_0^1 p_j^* m_{jt}^* dj - p_t^m m_t^d - w_t L_t^f.$$

Following Uribe and Yue (2006), we show in Appendix 1 that the above static profit maximization problem follows from a standard problem maximizing the present value of dividends subject to the working capital constraint. Moreover, Appendix 1 also establishes that since the firms' payoff function is linear and factor demands are characterized by standard conditions equating marginal products to marginal costs (see later discussion), firms do not have an incentive to build precautionary savings to self-insure against changes in factor costs or the loss of credit market access. Furthermore, even if this incentive were at play, building up a stock of *foreign* deposits to provide self-finance of working capital to pay foreign suppliers is ruled out by the standard assumption of the Eaton-Gersovitz setup that countries cannot build deposits abroad, otherwise debt exposed to default risk cannot exist at equilibrium (as shown by Bulow and Rogoff 1989).

The price of m_t^* is the standard CES price index. Because some imported inputs carry the financing cost of working capital, we can express this price index as follows:

$$P^*\left(r_t^*\right) = \left[\int_0^\theta \left(p_j^*\left(1 + r_t^*\right)\right)^{\frac{\nu}{\nu-1}} dj + \int_\theta^1 \left(p_j^*\right)^{\frac{\nu}{\nu-1}} dj\right]^{\frac{\nu-1}{\nu}}.$$

As we show in the next section, the set Ω of imported inputs is not used when a country defaults because both firms and government are excluded from world credit markets, and thus firms cannot obtain working capital financing to import inputs in the Ω set (alternatively, we could assume that part of the punishment for default is a trade penalty that excludes the country from the Ω set of world input markets). Hence, when a country is in financial autarky the price index of imported inputs is:

$$P^*_{aut} = \left[\int_\theta^1 \left(p^*_j\right)^{\frac{\nu}{\nu-1}} dj\right]^{\frac{\nu-1}{\nu}}$$

We use a standard two-stage budgeting approach to characterize the solution of the final goods producers' optimization problem. In the first stage, firms choose L^f_t, m^d_t, and m^*_t, given the factor prices w_t, p^m_t, and $P^*\left(r^*_t\right)$, to maximize date-t profits:

$$\text{(8)}\quad \pi^f_t = \varepsilon_t \left(M\left(m^d_t, m^*_t\right)\right)^{\alpha_M} \left(L^f_t\right)^{\alpha_L} k^{\alpha_k} - P^*\left(r^*_t\right) m^*_t - p^m_t m^d_t - w_t L^f_t,$$

where $M\left(m^d_t, m^*_t\right) = \left(\lambda \left(m^d_t\right)^\mu + (1-\lambda)\left(m^*_t\right)^\mu\right)^{\frac{1}{\mu}}$. Then, in the second stage they choose their demand for each variety of imported inputs.

The first-order conditions of the first stage are:

$$\text{(9)}\quad \alpha_M \varepsilon_t k^{\alpha_k} \left(M\left(m^d_t, m^*_t\right)\right)^{\alpha_M-\mu} \left(L^f_t\right)^{\alpha_L} (1-\lambda)\left(m^*_t\right)^{\mu-1} = P^*\left(r^*_t\right)$$

$$\text{(10)}\quad \alpha_M \varepsilon_t k^{\alpha_k} \left(M\left(m^d_t, m^*_t\right)\right)^{\alpha_M-\mu} \left(L^f_t\right)^{\alpha_L} \lambda \left(m^d_t\right)^{\mu-1} = p^m_t$$

$$\text{(11)}\quad \alpha_L \varepsilon_t k^{\alpha_k} M_t^{\alpha_M} \left(L^f_t\right)^{\alpha_L-1} = w_t.$$

Given m^*_t, the second stage yields a standard CES system of demand functions for imported inputs that can be split into a subset for varieties that do not require working capital and the subset in Ω:

$$m^*_{jt} = \left(\frac{p^*_j}{P^*\left(r_t\right)}\right)^{-\frac{1}{1-\nu}} M^*, \quad \text{for } j \in [\theta, 1],$$

$$m^*_{jt} = \left(\frac{p^*_j\left(1+r_t\right)}{P^*\left(r_t\right)}\right)^{-\frac{1}{1-\nu}} M^*, \quad \text{for } j \in [0, \theta].$$

When the country is in default, and thus final goods producers cannot access working capital financing, the demand function system becomes the limit of the above system as $r^*_t \to \infty$:

$$m^*_{jt} = \left(\frac{P^*_{aut}}{p^*_j}\right)^{-\frac{1}{1-\nu}} M^*, \quad \text{for } j \in [\theta, 1],$$
$$m^*_{jt} = 0, \ \text{for } j \in [0, \theta].$$

II.C. Intermediate Goods Producers

Producers in the m^d sector use labor L^m_t and operate with a production function given by $A(L^m_t)^\gamma$, with $0 \leq \gamma \leq 1$ and $A > 0$. A represents both the role of a fixed factor and an invariant state of TFP in the m^d sector. Given p^m_t and w_t, the profit maximization problem of intermediate goods firms is:

$$\max_{L^m_t} \ \pi^m_t = p^m_t A(L^m_t)^\gamma - w_t L^m_t. \tag{12}$$

Their labor demand satisfies this standard optimality condition:

$$\gamma p^m_t A(L^m_t)^{\gamma-1} = w_t. \tag{13}$$

II.D. Equilibrium in Factor Markets and Production

Take as given the interest rate r^*_t and a TFP realization ε_t. The corresponding (partial) equilibrium factor allocations and prices are given by the values of $[m^*_t,\ m^d_t,\ L^f_t,\ L^m_t,\ L_t]$ and $[p^m_t, w_t]$ that solve the following nonlinear system:

$$\alpha_M \varepsilon_t k^{\alpha_k} \left(M\left(m^d_t, m^*_t\right)\right)^{\alpha_M-\mu} \left(L^f_t\right)^{\alpha_L} (1-\lambda)\left(m^*_t\right)^{\mu-1} = P^*\left(r^*_t\right) \tag{14}$$

$$\alpha_M \varepsilon_t k^{\alpha_k} \left(M\left(m^d_t, m^*_t\right)\right)^{\alpha_M-\mu} \left(L^f_t\right)^{\alpha_L} \lambda \left(m^d_t\right)^{\mu-1} = p^m_t \tag{15}$$

$$\alpha_L \varepsilon_t k^{\alpha_k} \left(M\left(m^d_t, m^*_t\right)\right)^{\alpha_M} \left(L^f_t\right)^{\alpha_L-1} = w_t \tag{16}$$

$$\gamma p^m_t A(L^m_t)^{\gamma-1} = w_t \tag{17}$$

$$g'\left(L_t\right) = w_t \tag{18}$$

$$L^f_t + L^m_t = L_t \tag{19}$$

$$A(L^m_t)^\gamma = m^d_t. \tag{20}$$

Conditions (14)–(20) drive the effects of TFP shocks and the exclusion from credit markets during default events on production and factor allocations. We study these effects in detail in Section III. Note that when the country is in financial autarky, factor allocations and prices are determined as the limiting case of the above nonlinear system as $r_t^* \to \infty$. The sector f does not have access to foreign working capital financing and hence to the set Ω of imported inputs.

Using the optimality conditions, it follows that total value added valued at equilibrium relative prices is given by $(1 - \alpha_M)\ \varepsilon_t(M_t)^{\alpha_M}(L_t^f)^{\alpha_L}k^{\alpha_k} + p_t^m A(L_t^m)^{\gamma}$. Moreover, given the CES formulation of M_t, the value of imported inputs satisfies $P^*(r_t^*)m_t^* = \alpha_M \varepsilon_t(M_t)^{\alpha_M}(L_t^f)^{\alpha_L}k^{\alpha_k} - p_t^m m_t^d$. Given these results, we can calculate GDP as gross production of final goods minus the cost of imported inputs, adjusting for the fact that in most emerging economies GDP at constant prices is computed fixing prices as of a base year using Laspeyres indexes (which is consistent with the model if we abstract from fluctuations in r_t^*). Hence we define GDP as $gdp_t \equiv y_t - P^* m_t^*$, using a time-invariant price index of imported inputs.[14]

II.E. The Sovereign Government

The sovereign government issues one-period, non–state-contingent discount bonds, so markets of contingent claims are incomplete. The face value of these bonds specifies the amount to be repaid next period, b_{t+1}. When the country purchases bonds $b_{t+1} > 0$, and when it borrows $b_{t+1} < 0$. The set of bond face values is $B = [b_{\min}, b_{\max}] \subset R$, where $b_{\min} \leq 0 \leq b_{\max}$. We set the lower bound $b_{\min} > -\frac{\bar{y}}{r}$, which is the largest debt that the country could repay with full commitment. The upper bound $b_{\max}$ is the highest level of assets that the country may accumulate.[15]

The sovereign cannot commit to repay its debt. As in the Eaton-Gersovitz model, when the country defaults it does not repay at date t and the punishment is exclusion from world credit markets in the same period. The country reenters credit markets

14. We use $P^* = 1$, which follows from the fact that $p_j^* = 1$ for all $j \in [0.1]$ and assuming a zero real interest rate in the base year. Note, however, that changes in our quantitative results are negligible if we use the equilibirum price index $P^*(r_t^*)$ instead, because default is a low-frequency event, and outside default episodes the interest rate on working capital is constant and equal to the world real interest rate.

15. $b_{\max}$ exists if the condition $(1 + r^*)\beta < 1$ holds.

with an exogenous probability ϕ, and when it does it starts with a fresh record and zero debt.[16]

We add to the Eaton-Gersovitz setup an endogenous link between sovereign default and private economic activity. As we explain in the next section, this link follows from the assumption that both firms and the government are excluded from world credit markets when default occurs. This causes an efficiency loss in final goods production by forcing firms to substitute away from imported inputs that require working capital. As noted in the introduction, this is in line with the empirical evidence of severe adverse effects from sovereign default on private credit markets and foreign trade documented by Reinhart and Rogoff (2010), Reinhart (2010), Kaletsky (1985), Kohlscheen and O'Connell (2008), and Rose (2005), and with evidence on inefficient reallocation across foreign and domestic inputs in the aftermath of sovereign default found by Gopinath and Neiman (2010).

The sovereign government chooses a debt policy (amounts and default or repayment) along with private consumption and factor allocations so as to solve a recursive social planner's problem. The state variables are the bond position and TFP, denoted by the pair (b_t, ε_t), and the planner takes as given the bond pricing function $q_t\left(b_{t+1}, \varepsilon_t\right)$. The planner's payoff is given by:

$$V\left(b_t, \varepsilon_t\right) = \max\left\{v^{nd}\left(b_t, \varepsilon_t\right), v^d\left(\varepsilon_t\right)\right\}, \tag{21}$$

where $v^{nd}\left(b_t, \varepsilon_t\right)$ is the value of continuing in the credit relationship with foreign lenders (i.e., "no default"), and $v^d\left(\varepsilon_t\right)$ is the value of default. If $b_t \geq 0$, the value function is simply $v^{nd}\left(b_t, \varepsilon_t\right)$ because in this case the economy uses the credit market to save, receiving a return equal to the world's risk-free rate r_t^*. For simplicity we assume for the remainder of the article that this interest rate is time- and state-invariant, $r_t^* = r^*$.

The continuation value is given by the choice of $[c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t, b_{t+1}]$ that solves this constrained maximization problem:

$$v^{nd}\left(b_t, \varepsilon_t\right) = \max_{c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t, b_{t+1}} \left\{\begin{array}{l} u\left(c_t - g(L_t)\right) \\ \qquad +\beta E\left[V\left(b_{t+1}, \varepsilon_{t+1}\right)\right] \end{array}\right\}, \tag{22}$$

subject to:

16. We asbtract from debt renegotiation. See Yue (2010), Bi (2008b), and Benjamin and Wright (2008) for quantitative studies of sovereign default with renegotiation.

$$(23)\quad c_t + q_t\left(b_{t+1}, \varepsilon_t\right) b_{t+1} - b_t \leq \varepsilon_t f\left(M\left(m_t^d, m_t^*\right), L_t^f, k\right) - m_t^* P^*\left(r^*\right),$$
$$L_t^f + L_t^m = L_t$$
$$A(L_t^m)^\gamma = m_t^d$$

where $f(\cdot) = M^{\alpha_M}(L_t^f)^{\alpha_L} k^{\alpha_k}$. The first constraint is the resource constraint of the economy. The last two constraints are the resource constraints in the markets for labor and domestic inputs, respectively.

Notice that the planner faces the same allocations of output and factors as the private sector. In particular, for given values of r^* and ε_t, a bond pricing function $q_t\left(b_{t+1}, \varepsilon_t\right)$ and any pair $(b_{t+1}, b_t) \in B$, including the optimal choice of b_{t+1}, the optimal factor allocations chosen by the planner $[m_t^*,\ m_t^d,\ L_t^f,\ L_t^m,\ L_t]$ satisfy the conditions (14)–(20) that characterize a competitive equilibrium in factor markets, with w_t and p_t^m matching the shadow prices given by the Lagrange multipliers of the resource constraints for labor and domestic inputs. In addition, the planner internalizes the households' desire to smooth consumption, and hence transfers to them an amount equal to the negative of the balance of trade (i.e., the flow of resources private agents need to finance the gap between GDP and consumption).

The value of default is:

(24)
$$v^d\left(\varepsilon_t\right) = \max_{c_t, m_t^d, m_t^*, L_t^f, L_t^m, L_t} \left\{ \begin{array}{l} u\left(c_t - g(L)\right) \\ \qquad + \beta\left(1-\phi\right) E v^d\left(\varepsilon_{t+1}\right) + \beta\phi E V(0, \varepsilon_{t+1}) \end{array} \right\},$$

subject to:

$$(25)\qquad c_t = \varepsilon_t f\left(M\left(m_t^d, m_t^*\right), L_t^f, k\right) - m_t^* P_{aut}^*$$
$$L_t^f + L_t^m = L_t$$
$$A(L_t^m)^\gamma = m_t^d.$$

Note that $v^d\left(\varepsilon_t\right)$ takes into account the fact that in case of default at date t, the country has no access to financial markets that period, and hence the country consumes the total income given by the resource constraint in the default scenario. The value of default at t also takes into account that at $t+1$ the economy may reenter world capital markets with probability ϕ and associated value $V\left(0, \varepsilon_{t+1}\right)$, or remain in financial autarky with probability $1-\phi$ and associated value $v^d\left(\varepsilon_{t+1}\right)$.

The definitions of the default set and the probability of default are standard from Eaton-Gersovitz models (see Arellano 2008). For a debt position $b_t < 0$, default is optimal for the set of realizations of ε_t for which $v^d(\varepsilon_t)$ is at least as high as $v^{nd}(b_t, \varepsilon_t)$:

$$D(b_t) = \left\{ \varepsilon_t : v^{nd}(b_t, \varepsilon_t) \leq v^d(\varepsilon_t) \right\}. \tag{26}$$

The probability of default at $t+1$ perceived as of date t, $p_t(b_{t+1}, \varepsilon_t)$, can be induced from the default set and the transition probability function of productivity shocks $z(\varepsilon_{t+1}|\varepsilon_t)$ as follows:

$$p_t(b_{t+1}, \varepsilon_t) = \int_{D(b_{t+1})} dz(\varepsilon_{t+1}|\varepsilon_t). \tag{27}$$

The economy is considered to be in financial autarky when it has been in default for at least one period and remains without access to world credit markets. The optimization problem of the sovereign is the same as the problem in the default period. This is the case because, since the Bulow-Rogoff result referred to earlier requires the economy not to be able to access funds saved abroad during periods of financial autarky, before defaulting the economy could not have built up a stock of savings abroad to provide working capital financing to firms to purchase imported inputs. Alternatively, we can assume that the default punishment includes exclusion from the subset Ω of world markets of intermediate goods.

The model preserves these standard features of the Eaton-Gersovitz model: Given ε_t, the value of defaulting is independent of the level of debt, and the value of not defaulting increases with b_{t+1}, and consequently the default set and the equilibrium default probability grow with the country's debt. The following theorem formalizes these results.

THEOREM 1. Given a productivity shock ε and a pair of bond positions $b^0 < b^1 \leq 0$, if default is optimal for b^1, then default is also optimal for b^0 and the probability of default at equilibrium satisfies $p^*(b^0, \varepsilon) \geq p^*(b^1, \varepsilon)$.

Proof. See Appendix 2. ■

II.F. Foreign Lenders

International creditors are risk-neutral and have complete information. They invest in one-period sovereign bonds and in within-period private working capital loans. Foreign lenders behave competitively and face an opportunity cost of funds equal to r^*. Competition implies that they expect zero profits at equilibrium and that the returns on sovereign debt and the world's risk-free asset are fully arbitraged:

$$q_t\left(b_{t+1}, \varepsilon_t\right) = \begin{cases} \frac{1}{1+r^*} & \text{if } b_{t+1} \geq 0 \\ \frac{\left[1-p_t\left(b_{t+1}, \varepsilon_t\right)\right]}{1+r^*} & \text{if } b_{t+1} < 0 \end{cases}. \tag{28}$$

This condition implies that at equilibrium bond prices depend on the risk of default. For a high level of debt, the default probability is higher. Therefore, equilibrium bond prices decrease with indebtedness. This result, formalized in Theorem 2, is again in line with the Eaton-Gersovitz model and is also consistent with the empirical evidence documented by Edwards (1984).

THEOREM 2. Given a productivity shock ε and bond positions $b^0 < b^1 \leq 0$, the equilibrium bond prices satisfy $q^*\left(b^0, \varepsilon\right) \leq q^*\left(b^1, \varepsilon\right)$.

Proof. See Appendix 2. ■

II.G. Recursive Equilibrium

DEFINITION 1 The model's recursive equilibrium is given by (i) a decision rule $b_{t+1}\left(b_t, \varepsilon_t\right)$ for the sovereign government with associated value function $V\left(b_t, \varepsilon_t\right)$, consumption and transfers rules $c\left(b_t, \varepsilon_t\right)$ and $T\left(b_t, \varepsilon_t\right)$, default set $D\left(b_t\right)$, and default probabilities $p^*\left(b_{t+1}, \varepsilon_t\right)$; and (ii) an equilibrium pricing function for sovereign bonds $q^*\left(b_{t+1}, \varepsilon_t\right)$ such that:

1. Given $q^*\left(b_{t+1}, \varepsilon_t\right)$, the decision rule $b_{t+1}\left(b_t, \varepsilon_t\right)$ solves the social planner's recursive maximization problem (21).
2. The consumption plan $c\left(b_t, \varepsilon_t\right)$ satisfies the resource constraint of the economy.
3. The transfers policy $T\left(b_t, \varepsilon_t\right)$ satisfies the government budget constraint $T\left(b_t, \varepsilon_t\right) = q_t\left(b_{t+1}, \varepsilon_t\right) b_{t+1} - b_t$.
4. Given $D\left(b_t\right)$ and $p^*\left(b_{t+1}, \varepsilon_t\right)$, the bond pricing function $q^*\left(b_{t+1}, \varepsilon_t\right)$ satisfies the arbitrage condition of foreign lenders (28).

Condition 1 requires that the government's default and borrowing decisions be optimal given the interest rates on sovereign debt. Condition 2 requires that the private consumption and factor allocations implied by these optimal borrowing and default choices be feasible. In addition, since factor allocations satisfy conditions (14)–(20), these allocations are consistent with a competitive equilibrium in factor markets. Condition 3 requires that the decision rule for government transfers shifts the appropriate amount of resources between the government and the private sector (i.e., an amount equivalent to net exports when the country has access to world credit markets, or zero when the economy is in financial autarky). Notice also that given Conditions 2 and 3, the consumption plan satisfies the households' budget constraint. Finally, Condition 4 requires the equilibrium bond prices that determine country risk premia to be consistent with optimal lender behavior.

A solution to the recursive equilibrium includes solutions for sectoral factor allocations and production with and without credit market access. Solutions for equilibrium wages, profits, and the price of domestic inputs follow then from the firms' optimality conditions and the definitions of profits described earlier.

III. Default, Factor Allocations, and Output in Partial Equilibrium

The efficiency loss in final goods production caused by sovereign default plays a central role in our analysis, because it is the main driver of the endogenous output cost of default, and hence of the model's financial amplification mechanism. We illustrate how this mechanism works using a partial-equilibrium numerical example. In this example, we solve for factor allocations, factor prices and output using conditions (14)–(20) and the parameter values set in the calibration described in Section IV, and perform sensitivity analysis changing the values of parameters that determine the magnitude of the efficiency loss (the elasticities of the Armington aggregator, the aggregator of imported inputs, and labor supply). In each scenario, we compare outcomes with and without access to credit markets. For the former we assume $r^* = 0.01$ and for the latter we solve the equilibrium conditions as $r^* \to \infty$.

III.A. *Effects of Default on Factor Allocations*

Table I lists the percent changes in M, m^*, m^d, L, L^f, and L^m that occur when sovereign default causes firms to be excluded from credit markets to finance purchases of the Ω set of imported inputs, keeping TFP at its mean level $\varepsilon = 1$.

TABLE I
EFFECTS OF DEFAULT ON FACTOR ALLOCATIONS

	(I)	(II)	(III)	(IV)	(V)
	Baseline η_{m^d,m^*} = 2.86, $\eta_{m_j^*}$ = 2.44	Threshold Armington elasticity η_{m^d,m^*}=1.96	Cobb-Douglas Armington aggregator η_{m^d,m^*}=1	High within-variety elasticity $\eta_{m_j^*}$ = 10	Inelastic labor $\omega \to \infty$
M	−11.36	−21.90	−40.72	−3.08	−9.61
m^*	−90.64	−81.59	−68.21	−30.38	−90.46
m^d	1.73	0.01	−13.65	0.46	3.73
L	−2.77	−7.11	−19.12	−0.73	0.0
L^f	−6.29	−11.40	−19.22	−1.67	−3.65
L^m	2.48	0.02	−18.91	0.65	5.37

Note: percent changes relative to a state with r^* = 0.01

Column (I) shows results for our baseline calibration, where we set μ=0.65 and ν=0.59, which imply elasticities of η_{m^d,m^*} = 2.86 and $\eta_{m_j^*}$ = 2.44, respectively. Columns (II)–(IV) show results for three scenarios in which we vary these elasticities one at a time. Column (II) sets η_{m^d,m^*}= 1.96, which is the threshold elasticity below which m^d and m^* switch from gross substitutes to gross complements.[17] Column (III) shows results for the Cobb-Douglas case of unitary elasticity in the Armington aggregator (in which m^d and m^* are gross complements). Column (IV) assumes a high within elasticity of substitution across imported input varieties ($\eta_{m_j^*}$=10). In addition, column (V) shows results for a scenario with inelastic labor supply.

The changes in factor allocations shown in Table I illustrate three main effects that result from the loss of access to credit markets. First, the aggregate demand for m^* always falls, because of the direct effect by which credit market exclusion prevents firms from using the set Ω of imported inputs. This is the standard effect by which an increase in the interest rate rises the effective financing cost of these inputs via the working capital channel, but evaluated as $r^* \to \infty$. Default causes this decline in m^* for any $0 < \mu < 1$. Second, there are also indirect effects that lower the demands for total intermediate goods (M) and labor in the final goods sector (L^f), because of the Cobb-Douglas structure of the

17. Note that the threshold would be at the unitary elasticity of substitution if labor supply were inelastic.

production function of final goods. The direction of these effects is also the same for any $0 < \mu < 1$. Third, the loss of credit market access also has effects on the output and labor allocations of the intermediate goods sector, but the direction of these effects changes with the value of μ (within the $(0, 1)$ range). In particular, there is an increase or a decline in m^d and L^m depending on whether the value of μ makes m^d and m^* gross substitutes or complements. If μ is high (low) enough for the two inputs to be gross substitutes (complements), both m^d and L^m increase (fall) as m^* falls. As a result, the decline in M and L caused by a sovereign default is larger when domestic and foreign inputs are gross complements (in column (III)) than when they are gross substitutes (columns (I)–(II) and (IV)–(V)).

Compare now the baseline case in column (I) with the scenario with high within elasticity in column (IV). The effects of default are weaker when imported inputs across the varieties inside and outside the set Ω become better substitutes, because default does not impair access to imported inputs that do not require payment in advance. As a result, aggregate m^* falls less, causing a smaller decline (rise) in M (m^d), and less reallocation of labor from sector f to sector m.

Notice that qualitatively the effects of increasing η_{m^d,m^*} or $\eta_{m_j^*}$ are similar. The higher either of these two elasticities, the weaker the working capital channel, because inputs that do not require credit are better substitutes of those that do, and hence the weaker the effects of default on production and factor allocations. The two elasticities are not quantitatively equivalent, however, because when imported input varieties become better substitutes, final goods producers can substitute into varieties outside the Ω set facing exogenous prices p_j^*, whereas when domestic inputs become better substitutes, substituting away into domestic inputs is done facing an endogenous price p^m. As a result, changing η_{m^d,m^*} alters the size of the default effects on factor allocations more than changing $\eta_{m_j^*}$.

Finally, compare the baseline case in column (I) with the inelastic labor supply scenario in column (V). The effect of default on m^* is nearly unchanged. M falls less with inelastic labor, however, because m^d rises more, and this is possible because with inelastic labor supply L cannot fall, and this results in a larger increase in L^m and a smaller decline in L^f. Thus, even with inelastic labor supply, default and the loss of access to working capital financing affect the efficiency of production by inducing an

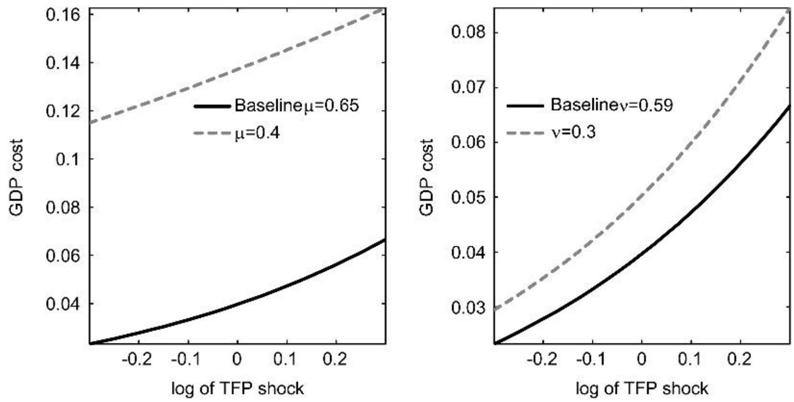

FIGURE II

Output Costs of Default as a Function of TFP Shock

inefficient reallocation of inputs, and by reallocating a given total labor endowment from production of final goods to production of domestic intermediate goods.

III.B. Output Costs of Default

We examine now the output costs of default that result from the effects of default on factor allocations just examined. In particular, we study how output costs of default vary with ε, because this relationship is a central factor affecting default incentives. We are also interested in studying how the relationship between output costs of default and TFP varies with the model's key elasticity parameters.

Figure II shows two plots of the output cost of default as a function of ε. The plot on the left compares the case with baseline parameters with a scenario in which we lower μ from the 0.65 baseline value to 0.4 (η_{m^d,m^*} falls from 2.86 to 1.66, which is below the gross substitutes-complements threshold). The plot on the right compares the baseline scenario with a scenario in which we lower ν from 0.59 to 0.3 ($\eta_{m_j^*}$ falls from 2.43 to 1.43). For each value of ε in the horizontal axis, the output cost of default is measured as the percent fall in output that occurs when the government defaults.

Figure II illustrates three key properties of the model's output cost of default: First, the output cost of default is increasing and convex in ε. This is the case because, with Cobb-Douglas technologies and competitive markets, the negative effect of increases in marginal costs on factor demands is larger at higher TFP levels.[18] Second, the cost of default is higher the lower is η_{m^d,m^*}. This is an implication of the previous results showing that the default effects on factor allocations are larger when domestic inputs are poorer substitutes of imported inputs. Third, the cost of default is also higher the lower is $\eta_{m_j^*}$, although the effect of varying ν on the output cost of default is considerably smaller than the effect of varying μ, in line with what we discussed earlier.

The fact that the output cost of default increases with ε implies that default is more painful at higher TFP levels. This result is critical for the model's ability to support high debt levels at the observed default frequencies and producing defaults in "bad" times, because it makes default more attractive at lower states of productivity. In this way, default works as a desirable implicit hedging mechanism given the incompleteness of asset markets.

Figure III illustrates further how the cost of default declines as η_{m^d,m^*} rises. This figure plots the output cost of default for the mean value of TFP ($\varepsilon = 1$) at different values of η_{m^d,m^*}. Again, the cost of default becomes smaller at higher Armington elasticities because the inputs are closer substitutes, and hence the efficiency loss when firms substitute away from the set Ω of imported inputs is smaller. Quantitatively, the figure shows that already for $\eta_{m^d,m^*} > 4$, the mechanism driving efficiency losses in the model becomes very weak and is effectively the same as if the inputs were perfect substitutes.

A similar analysis of the output costs of default as the one illustrated in Figures II and III but for different values of ω (instead of μ and ν) shows that a higher labor supply elasticity (i.e., lower ω) increases the cost of default, converging to about 11.5% for infinitely elastic labor supply. The output cost of default is increasing in TFP for any value of ω, but in contrast with what we found for μ, the slope of the relationship does not change as ω changes. In addition, we found that adjusting A has qualitatively similar effects as changing ω.

The labor market equilibrium illustrated in Figure IV provides the intuition behind the result that higher labor supply

18. This is the case in turn because of the "strong" convexity of Cobb-Douglas marginal products. Consider for simplicity the case in which production $\varepsilon F(m)$ requires a single input m. In this case, "strong convexity" means that $F(m)$ satisfies $F'''(m) > \frac{(F''(m))^2}{F'(m)}$, which holds in the Cobb-Douglas case.

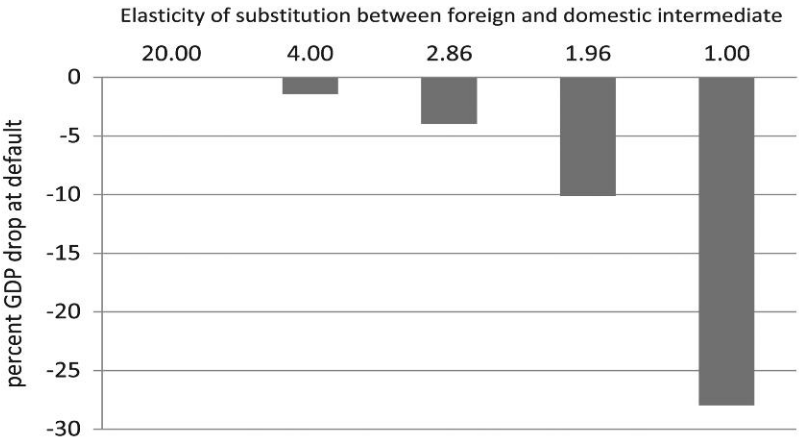

FIGURE III
Output Costs of Default for a Neutral TFP Shock at Different Elasticities of Substitution

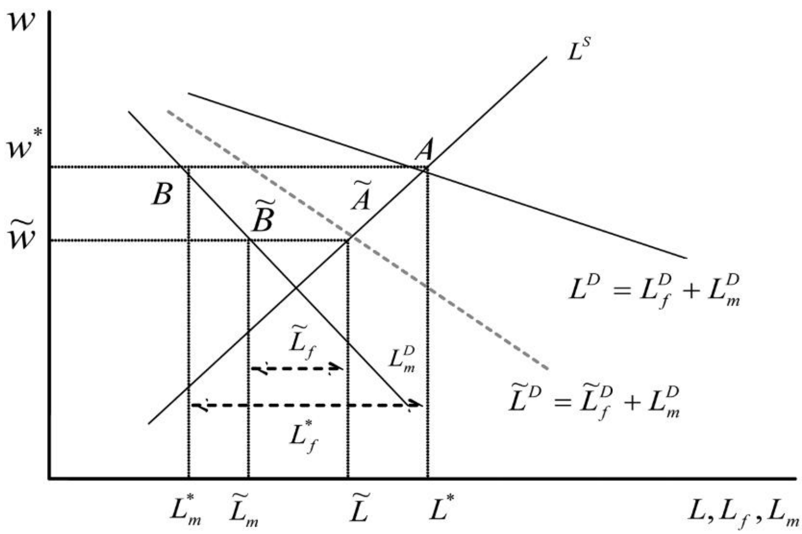

FIGURE IV
Default and the Labor Market Equilibrium

elasticity produces larger output costs of default. For simplicity, we plot labor demands and supply as linear functions. The labor demand functions are given by the marginal products in the left-hand side of (11) and (13), and the labor supply is given by the marginal disutility of labor in the left-hand side of (3). Since labor is homogeneous across sectors, total labor demand is just the sum of sectoral demands. The initial labor market equilibrium is at point A with wage w^*, total labor L^* and sectoral allocations L_m^* and L_f^*.

Assume now that the government defaults. Following from our previous results on the effects of default on factor allocations, default causes a reduction in labor demand in final goods from L_f^D to $\tilde{L}_f^D$. This occurs because, as explained earlier, the loss of access to working capital financing for a subset of imported inputs causes a reduction in M and the marginal product of L_f is a negative function of M (since the production function is Cobb-Douglas). As a result, total labor demand shifts from L^D to $\widetilde{L}^D$.[19] The new labor market equilibrium is at point $\widetilde{A}$. The wage rate, the total labor allocation, and the labor allocated to final goods are lower than before, and labor allocated to production of domestic inputs rises (if foreign and domestic inputs are gross substitutes). In contrast, assuming that labor is infinitely elastic would make L^s an horizontal line at the level of w^* and the interest rate hike would leave w unchanged. As a result, L falls more, L_m is unchanged instead of rising, and L_f falls less.[20] Hence, the negative effect of default on output is stronger. At the other extreme, with inelastic labor L^s is a vertical line at the level of L^*. Now L cannot change, but w falls more than in Figure IV, L_m rises more, and L_f falls more. Hence, the decline in output is smaller.

IV. Quantitative Analysis

IV.A. Baseline Calibration

We study the quantitative implications of the model by conducting numerical simulations setting the model to a quarterly

19. In Figure IV, we hold constant p_m for simplicity. At equilibrium, the relative price of domestic inputs changes, and this alters the value of the marginal product of L_m, and hence labor demand by the m sector. The results of our numerical analysis do take this into account and still are roughly in line with the intuition derived here.

20. The last effect hinges on the fact that the gap between L_m^D and L^D widens as the wage falls. This is a property of factor demands with Cobb-Douglas production.

TABLE II
BASELINE CALIBRATION

Calibrated parameters		*Value*	*Target statistics*	
Int. goods share in gross output of final goods	α_m	0.43	Argentina's national accounts	
Capital share in gross output of final goods	α_k	0.17	Standard capital share in GDP (0.3)	
Labor share in gross output of final goods	α_L	0.40	Standard labor share in GDP (0.7)	
Labor share in GDP of int. goods	γ	0.7	Standard labor share in GDP (0.7)	
Coefficient of relative risk aversion	σ	2	Standard RBC value	
Risk-free interest rate	r^*	1%	Standard RBC value	
Curvature parameter of labor supply	ω	1.455	Frisch wage elasticity (2.2)	
Reentry probability	ϕ	0.083	Dias and Richmond (2007)	
Armington weight of domestic inputs	λ	0.62	Regression estimate	
Armington curvature parameter	μ	0.65	Regression estimate	
Dixit-Stiglitz curvature parameter	ν	0.59	Gopinath and Neiman (2010)	
Parameters set with SMM		*Value*	*Targets from data*	
Autocorrelation of TFP shocks	ρ_ε	0.95	GDP autocorrelation	0.82
Standard deviation of TFP innovations	σ_ϵ	1.7%	GDP std. deviation	4.7%
Intermediate goods TFP coefficient	A	0.31	Output drop in default	13%
Subjective discount factor	β	0.88	Default frequency	0.69%
Upper bound of imported inputs with working capital	θ	0.7	Working capital to GDP ratio estimate	6%
TFP semi-elasticity of exogenous capital flows	ξ	-0.67	Net exports/GDP at default	10%

frequency and using a baseline calibration based largely on data for Argentina, as is standard practice in quantitative studies of sovereign default. Table II shows the calibrated parameter values.

The share of intermediate goods in gross output α_M is set to 0.43, which corresponds to the average ratio of intermediate goods to gross production calculated using annual data for Argentina for the period 1993–2005 from the United Nation's *UNData*.[21] Given α_M, we set α_k=0.17 so that the capital income share *in value added* of the f sector ($\frac{\alpha_k}{(1-\alpha_M)}$) matches the standard 30% ($\frac{0.17}{(1-0.43)} = 0.3$). These factor shares imply a labor share in gross output of final goods of $\alpha_L = 1 - \alpha_M - \alpha_k = 0.40$, which yields a labor share *in value added* of $\frac{\alpha_L}{(1-\alpha_M)} = 0.7$ in line with the standard 70% labor share. The labor share in intermediate goods production γ is also set to 0.7.

The risk aversion parameter σ is set to 2 and the quarterly world risk-free interest rate r^* is set to 1%, which are standard values in quantitative business cycle and sovereign default studies. The curvature of labor disutility in the utility function is set to $\omega = 1.455$, which implies a Frisch wage elasticity of labor supply of $\frac{1}{(\omega-1)} = 2.2$. This is the value typically used in RBC models of the small open economy (Mendoza 1991 and Neumeyer and Perri 2005), and is based on estimates for the United States quoted by Greenwood, Hercowitz, and Huffman (1988).

The probability of reentry after default is 0.083, which implies that the country stays in exclusion for three years after default on average. This is the estimate obtained by Dias and Richmond (2007) for the median duration of exclusion using a partial access definition of re-entry. A three-year exclusion period is also in the range of the estimates reported by Gelos, Sahay, and Sandleris (2003).[22]

The values of μ and λ are set using data on the ratio of imported to domestic intermediate goods at constant prices and the associated relative prices, together with the condition equating the marginal rate of technical substitution between m^* and m^d with the corresponding price ratio (which follows from conditions (14) and (15)). National Accounts data for Argentina, however, do not provide a breakdown of intermediate goods into domestic and imported, so we obtained them instead from Mexican data for the

21. Mendoza (2010) reports a very similar share for Mexico, and Gopinath, Itskhoki, and Rigobon (2010) show shares in the 40–45% range for several countries.

22. The two studies use different definitions of reentry. Gelos, Sahay, and Sandleris use actual external bond issuance of public debt. Dias and Richmond define reentry when either the private or public sectors can borrow again, and they also distinguish partial reaccess from full reaccess (with the latter defined as positive net debt flows larger than 1.5% of GDP). Gelos et al. estimate an avearge exclusion of 5.4 years in the 1980s and nearly 1 year in the 1990s.

period 1988–2004 and assumed that the ratios are similar.[23] A nonlinear regression of the optimality condition implied by (14) and (15) produced estimates of $\mu = 0.65$ and $\lambda = 0.62$, both statistically significant (with standard errors of 0.11 and 0.12, respectively). These two estimates also allow the model to match the average ratios of imported to domestic inputs at current and constant prices in the Mexican data, which are 18% and 15.7%, respectively.

The values μ=0.65 and λ=0.62 imply that η_{m^d,m^*}=2.9 and that there is a small bias in favor of domestic inputs. This Armington elasticity is in the range of existing empirical estimates for several countries, but the estimates vary widely. McDaniel and Balistreri (2002) review the literature and quote estimates ranging from 0.14 to 6.9. They explain that elasticities tend to be higher when estimated with disaggregated data, in cross-sectional instead of time-series samples, or when using long-run instead of short-run tests. In the next section we conduct sensitivity analysis to study the effects of changing this and other key parameters on our main quantitative findings.

The value of v is difficult to set because it requires analysis of disaggregated data on imported intermediate goods. Gopinath and Neiman (2010) examined a large firm-level data set for Argentina that included the disaggregation of import varieties. They focused in particular on the dynamics of trade adjustment at the firm level around the 2002 default event. We set $\eta_{m_j^*} = 2.44$ ($v = 0.59$) in line with the elasticity across varieties that they reported. They also concluded, in line with our argument, that trade adjustment via the extensive margin at the firm level, with firms shifting from imported to domestic inputs, was very significant in the aftermath of Argentina's default. Interestingly, they also set $\eta_{m^d,m^*} = 2.08$ ($\mu = 0.519$), which is close to the value in our calibration.

Calibrating the model to the data also requires accounting for the fact that, contrary to what the model predicts, international capital flows, and the trade imbalances they finance, do not vanish completely when sovereigns default on private lenders–trade balances actually rise into surpluses, as shown in

23. Several countries have input expenditure ratios similar to Mexico's, but the ratios can vary widely. Goldberg and Campa (2008) report ratios of imported inputs to total intermediate goods for 17 countries that vary from 14% to 49%, with a median of 23%. This implies ratios of imported to domestic inputs in the 16–94% range, with a median of 30%.

Figure I.[24] An important component of these continuing capital flows are those vis-à-vis international organizations, on which countries very rarely default.[25] In the case of Argentina, the country made repayments to international organizations for 2.7% of GDP in 2002 and as large as 5% of GDP by 2006. To adjust our quantitative analysis accordingly, we introduce an amount x_t of exogenous capital flows that are independent of the borrowing and default decisions. For simplicity, and to prevent these capital flows from altering default incentives significantly, we assume that x_t is perfectly correlated with TFP and given by $x_t = \xi \ln \varepsilon_t$, and calibrate the semi-elasticity parameter ξ to data for Argentina as described later. In addition, we show later that the key features of our quantitative results, except for the surge in net exports during the exclusion period, are invariant to removing x_t.

We calibrate the remaining six parameters (σ_ϵ^2 , ρ_ε, A, β, θ, and ξ) using the simulated method of moments (SMM) to target a set of moment conditions from the data. Productivity shocks in final goods production follow an AR(1) process:

$$\log \varepsilon_t = \rho_\varepsilon \log \varepsilon_{t-1} + \epsilon_t, \tag{29}$$

with $\epsilon_t \overset{iid}{\sim} N(0, \sigma_\epsilon^2)$. We use Tauchen's (1986) quadrature method to construct a Markov approximation to this process with 25 realizations. Data limitations prevent us from estimating (29) directly using actual TFP data, so we set σ_ϵ^2 and ρ_ε in the SMM procedure to target the standard deviation and first-order autocorrelation of quarterly H-P detrended GDP. We use seasonally adjusted quarterly real GDP from Argentina's Ministry of Economy and Finance (MECON) for the period 1980Q1 to 2005Q4. The standard deviation and autocorrelation of the cyclical component of GDP are 4.7% and 0.79, respectively. The TFP process obtained using SMM features $\rho_\varepsilon = 0.95$ and $\sigma_\epsilon = 1.7\%$.

The targets for setting A, β, θ, and ξ are, respectively, the decline in output at default, the frequency of default, the share

24. The prediction that net exports go to zero when the country is excluded from credit markets is not particular to our model. All existing quantitative models of sovereign default in the Eaton-Gersovitz class have the same feature, because the only way to finance a trade imbalance in this class of models is with foreign credit.

25. Only 23 countries have defaulted on the IMF since it was created in 1945 (see Aylward and Thorned 1998), and these are low-income countries or countries in armed conflicts without access to private lenders (e.g., Liberia, Somalia, Congo, Sudan, Afghanistan, Iraq). In all the sovereign defaults included in Figure I, payments to the IMF continued even after countries defaulted on private lenders, except in the case of Peru in the 1980s.

of working capital financing in GDP, and the rise in the trade balance–GDP ratio at default.[26] Output in the first quarter of 2002 was 13% below trend.[27] The default frequency is 0.69%, since Argentina has defaulted five times since 1824 (the average default frequency is 2.78% annually or 0.69% quarterly). The trade balance–output ratio rose by 10 percentage points in the quarter when Argentina defaulted. Lacking working capital data, we follow Schmitt-Grohe and Uribe's (2007) strategy to proxy working capital as the fraction of M1 held by firms, using an estimate for the United States showing that firms own about two-thirds of M1. Using Argentina's M1 data and the same two-thirds of firm ownership, we estimate Argentina's working capital at about 6% of GDP. Given all these targets, the SMM procedure yields $A = 0.31$, $\beta = 0.88$, $\theta = 0.7$, and $\xi = -0.67$.[28]

IV.B. Cyclical Co-movements in the Baseline Simulation

We start the assessment of the quantitative performance of the model by comparing moments from the data with moments from the model's stochastic stationary state. To compute the latter, we feed the TFP process to the model and conduct 2,000 simulations, each with 500 periods and truncating the first 100 observations.

Table III compares the moments from Argentine data (column (I)) with those produced by the model (column (II)). The data for National Accounts aggregates is from the sources noted in the calibration. The debt data are from the World Bank's *GDF* data set for the 1980–2005 period. The bond spreads data are quarterly EMBI+ spreads on Argentine foreign currency denominated bonds from 1994Q2 to 2001Q4, taken from J.P. Morgan's

26. A is useful for targeting the output drop at default because, as mentioned in Section III, changes in A have similar effects as changes in ω. In particular, lower values of A yield larger output drops at default without altering the slope of the relationship between TFP and these output drops.

27. Argentina declared default in the last week of December 2001, but it is reasonable to assume that in quarterly data, the brunt of the real effects of the debt crisis were felt in the first quarter of 2002. Arellano (2008) also follows this convention to date the default as of the first quarter of 2002. She estimated the output cost at 14%, measured as a deviation from a linear trend.

28. Note that β is relatively low compared to typical RBC calibrations, but is in the range of values used in sovereign default models (e.g., β in Aguiar and Gopinath 2006, Arellano 2008, and Yue 2010 ranges from 0.8 to 0.953). These lower discount factors are often justified by arguing that political economy incentives lead government decision makers to display higher rates of time preference.

TABLE III

STATISTICAL MOMENTS IN THE BASELINE MODEL AND IN THE DATA

Statistics	(I) Data	(II) Baseline model	(III) Model without x_t
Average debt/GDP ratio	35%	22.88%	21.34%
Average bond spreads	1.86%	0.74%	1.68%
Std. dev. of bond spreads	0.78%	1.23%	1.63%
Consumption std. dev./GDP std. dev.	1.44	1.05	1.05
Correlations with GDP			
bond spreads	−0.62	−0.17	−0.21
trade balance	−0.87	−0.54	−0.31
labor[a]	0.39	0.52	0.52
intermediate goods[a]	0.90	0.99	0.99
Correlations with bond spreads			
trade balance	0.82	0.15	0.12
labor[a]	−0.42	−0.19	−0.26
intermediate goods[a]	−0.39	−0.16	−0.18
Historical default-output co-movements			
correlation between default and GDP[a]	−0.11[b]	−0.09	−0.12
fraction of defaults with GDP below trend[a]	61.5%[b]	83%	82%
fraction of defaults with large recessions[a]	32.0%[b]	21.1%	20%

a. Statistical moment computed at annual frequency.

b. Cross-country historical estimate for 1820–2004 from Tomz and Wright (2007). Large recessions in Tomz and Wright are deviations from trend in the range −11% to −7%, and in the model are recessions with GDP at least two standard deviations below trend (i.e., deviations from trend of at least −9.2%).

EMBI+ data set. Labor is measured using the employment rate from Argentina's *INDEC* Permanent Survey of Households. These data yield a similar correlation between labor and the country real interest rate as the one in Neumeyer and Perri (2005) (−0.49 in our data versus −0.45 in their paper). Note also that in Table III we follow the sovereign default literature in listing correlations relative to bond spreads, rather than relative to country interest rates, as in Neumeyer and Perri, but the correlations are similar using either variable (e.g., the correlation between GDP and the interest rate is −0.57 and with spreads is −0.62).

Table III shows that the model produces a debt-to-GDP ratio of almost 23% on average while *at the same time* matching the 0.7% default frequency observed in the data. This is mainly the result of the effects on default incentives of the large output drop that occurs when the country defaults and the increasing output cost of default as a function of TFP. Although a 23% average debt ratio is still below Argentina's 35%, it is much larger than the mean debt ratios typically obtained in quantitative models of sovereign default with exogenous output costs already targeted to improve the models' quantitative performance and calibrated to match the default frequency. For instance, Arellano (2008) obtained a mean debt ratio of 6% of GDP calibrating her asymmetric cost of default so that income when the economy defaults is the maximum of the actual endowment realization or 97% of the average endowment. Yue's (2010) model with renegotiation and an exogenous 2% proportional output cost of default produced an average debt ratio of 9.7%. Using the same default cost, Aguiar and Gopinath (2006) obtained a higher mean debt ratio (27%), but with a default frequency of only 0.02%.

The variability of the spreads is higher in the model than in the data, while the mean spread in the model is smaller than in the data. The mean spread of 0.74% in the model is close to the default frequency, because we assume a zero recovery rate on defaulted debt and risk-neutral creditors, which imply that bond spreads are linked one-to-one with default probabilities (see equation (28)). Thus, the model can only generate an average bond spread of a similar magnitude as the default frequency.

The model yields a negative correlation between spreads and GDP, albeit less negative than in the data, because sovereign bonds have higher default risk in bad states. As we noted in the introduction, both quantitative models of sovereign default and of business cycles in emerging economies also produce countercyclical interest rates, but in the former output is an exogenous endowment and in the latter country risk is exogenous. In contrast, our model produces countercyclical country risk in a setting in which both output and country risk are endogenous, and influence each other.

The model is also consistent with two key stylized facts of emerging markets business cycles: countercyclical net exports and consumption variability that exceeds output variability (although the model underestimates both moments relative to the data). The first result follows from the fact that when the country is in a bad TFP state, it faces higher interest rates and tends to borrow less from abroad. The country's trade balance thus increases, leading to a negative correlation between net exports and output. The second result occurs because the ability to use external debt to smooth consumption is negatively affected by the higher interest rates induced by increased default probabilities. The sovereign borrows less when the economy faces an adverse TFP shock, and thus households adjust consumption by more than in the absence of default risk. On the other hand, because agents are impatient, the benevolent government borrows more to increase private consumption when the TFP shock is good. Hence, the variability of consumption rises. As with the countercyclical spreads, existing models of sovereign default and emerging markets business cycles can also account for countercyclical net exports and consumption variability in excess of GDP variability, but working under the assumption that either country risk or output fluctuations are exogenous.

The model predicts correlations of intermediate goods and labor with bond spreads and with GDP in line with those observed in the data. The correlation of intermediate goods with GDP is 0.90 in the data, compared with 0.99 in the model, and with spreads (before the 2002 default) is -0.39, compared with -0.16 in the model.[29] The correlation of labor with GDP is 0.39 in the data versus 0.52 in the model, and with spreads is -0.42 versus -0.19 in the model.

It is important to note that the negative correlations of GDP, labor, and intermediate goods with spreads are not directly influenced by the financial amplification mechanism triggered by the loss of working capital financing when the country defaults. This is because these correlations exclude default states by construction (since spreads are infinite in those states). They are influenced indirectly, however, because the financial amplification mechanism drives default incentives, and thus at equilibrium it influences default probabilities and spreads in nondefault states. In nondefault states, adverse TFP shocks reduce factor demands and GDP via standard real-business-cycle effects, but they may or may not increase default probabilities (and hence spreads) depending on how the government's optimal decision rule alters its debt position. With default risk, higher debt choices carry higher borrowing costs, and if outstanding debt is already high enough, the increasing cost of contracting more debt can lead the government to choose lower debt in some low TFP states. As a result, the correlation of factor demands and GDP with spreads is weakly negative.

29. We exclude financial autarky periods in computing correlations with spreads because in the model spreads go to infinity when the economy defaults, and hence correlations with spreads are undefined.

Interestingly, fluctuations in country risk and working capital financing have important effects in our model even though sovereign spreads are only about 70 basis points on average, the interest rate on working capital is equal to a constant 1% risk-free rate in all nondefault states, and working capital is only 6% of GDP. In contrast, Neumeyer and Perri (2005) and Uribe and Yue (2006) found working capital to be important for emerging markets business cycles, but using average interest rates that are 10 times larger (around 7%), with the interest rate on working capital set to be equal to the interest rate on sovereign debt, and assuming working capital pays 100% of the wages bill (i.e., working capital is about two-thirds of GDP). These differences are important because Oviedo (2005) showed that without high average interest rates on working capital and a large working capital requirement the effects of interest rate fluctuations in emerging markets business cycle models are much weaker.

We also report in Table III three moments that pertain to the relationship between default and output in historical cross-country data for the period 1820–2004 based on the work of Tomz and Wright (2007): The correlation between defaults and GDP and the fractions of defaults that occur when GDP is below trend and when recessions are "unusually large." Because these are moments based on annual data, we show comparable annual-equivalent moments from the model. The correlation between defaults and GDP in the model is a close match to the actual correlation. The model can produce defaults that occur in good times (defined as GDP above trend) at the annual frequency, but this happens only with 17% of defaults. Hence, 83% of defaults occur with GDP below trend, which is roughly 20 percentage points more than in the data. In terms of the fraction of defaults that occur with deep recessions, the model predicts that 21% of defaults occur with GDP two standard deviations or more below trend (which amounts to −9.2% below trend), while Tomz and Wright found that 32% of defaults occur with GDP in the lowest quintile of the distribution of GDP deviations from trend (which in their data corresponds to the range of −11% to −7% below trend).

Column (III) of Table III shows the effects of removing the exogenous component of capital flows x_t, while maintaining all the other parameters at their baseline values. This has small effects on most of the moments, except the correlation between net exports and GDP and the mean spreads. Because x_t can be viewed as an exogenous trade deficit that is perfectly correlated with

TFP, removing it makes the trade balance less countercyclical. The mean spreads change because, without x_t and with the rest of the parameters unchanged, the probability of default increases to about 1.7%, and the mean spreads rise accordingly because of the risk-neutral lenders' arbitrage condition. Thus, the exogenous component of capital flows plays a small role in our results, except for enabling the model to produce the trade surplus regularly observed after default events.

IV.C. Macroeconomic Dynamics around Default Events

We study next the model's ability to match output dynamics around default events. We do this by applying event study techniques to simulated time series data. The left panel in Figure V plots the model's average path of output around default events together with the data for Argentina's HP detrended GDP around the 2002 default (1999Q1 to 2005Q3). The event window covers 12 quarters before and after debt defaults, with the default events normalized to date 0. We plot the average for output in the model at each date $t = -12, \ldots, 12$ around default events in the simulations. Hence, the simulated GDP line represents the average behavior of output around defaults in the model's stochastic stationary state. Because Argentina's data is for a single default event, instead of a long-run average, we add dashed lines with one-standard-error bands around the model simulation averages.

The model's mean output dynamics are a good match to those observed in the data. The size of the output drop in the date of the default is matched by calibrating A, but the model also tracks closely Argentina's output dynamics before and after the default. Except for the quarter just before the default, for which the model predicts higher output than in the data, output in the data remains inside the model's one-standard-error bands throughout the 25-quarter event window.

Defaults in the model are triggered by adverse TFP shocks, but these shocks are not large in a statistical sense. On average, TFP declines by 7.61% in the model's default events, which is roughly 1.3 times the 5.8% standard deviation of the calibrated TFP process (σ_ε). Thus, defaults occur with a TFP shock of just about 1.3 standard deviations in size, which suggests that the model's financial amplification mechanism amplifies significantly the real effects of TFP shocks when these shocks trigger default.

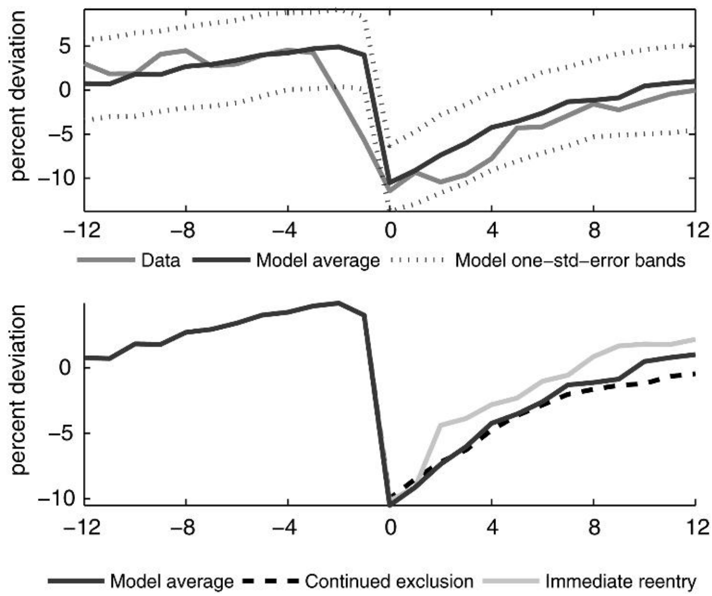

FIGURE V

Output around Default Events

The amplification effect can be quantified by computing the average output drop that the model produces in response to a 7.61% TFP shock when there is no default, and comparing it with the 13% mean output drop that the same shock produces when default occurs. Without default, a 7.61% TFP shock produces a mean output drop of about 7.17%. Thus, the amplification coefficient due to default is $\frac{13}{7.17} = 1.81$, or 81%.

The recovery of output after defaults is driven by two effects. First, since ε is mean-reverting and defaults occur with TFP below trend, TFP improves on average after defaults. Therefore, even though the country remains in financial autarky on average for three years after a default, the economy recovers because TFP improves. The second effect is the surge in output that occurs if the country reenters credit markets (as final goods producers switch back to a more efficient mix of imported and domestic inputs). This can happen with 8.3% probability every quarter.

These two effects are illustrated in the right panel of Figure V, which shows the simulated paths of GDP with continued exclusion for 12 quarters after default and with immediate reentry one period after default. In the first scenario, the recovery reflects only the effect of the mean reversion of ε. Since the probability of reentry is low (at 8.3%), GDP in this continued exclusion scenario is about the same as in the model average for about seven quarters after default, but then it moves below the model average because eight quarters and beyond after a default the probability of reentry, with the associated efficiency gain, starts to weigh more on the model average. In contrast, the scenario with immediate reentry shows a big rebound in GDP at $t = 1$, because of the efficiency gain that occurs in that period. The model average lies uniformly below the immediate reentry scenario because it always assigns some weight to the continued exclusion scenario, in which output is lower, and it generally weighs more the effect of TFP recovery than the effect of credit market reentry because of the low reentry probability.

The model's V-shaped output dynamics are qualitatively consistent with the data of emerging markets that experienced sudden stops. Calvo, Izquierdo, and Talvi (2006) conducted a cross-country empirical analysis of the recovery of emerging economies from sudden stops and found that most recoveries are not associated with improvements in credit market access. In our model as well, recovery occurs (on average) even though the economy continues to be excluded from world credit markets. Notice, however, that the recovery without credit is always modest relative to what occurs if credit market access is regained.

Figure VI shows event windows that compare the actual default event dynamics from our cross-country data set (as shown in Figure I) with the average of the model simulations, the corresponding one-standard-error bands and also the data from Argentina's 2002 default.[30] The plots show event windows for GDP, consumption, the trade balance–GDP ratio, imported inputs, total intermediate goods, labor, the country interest rate, and the debt-GDP ratio.

The model does well at replicating the observed behavior of GDP, consumption, and net exports, in the sense that the actual paths of these variables are mostly within the model's standard

30. We provide both the cross-country medians and the observations for Argentina's default because we aim to illustrate how well the model can match both the behavior across the default events in our cross-country data set and the data for Argentina. The former is harder because the model's calibration is based on Argentine data, and thus misses cross-country variation in the model's key parameters.

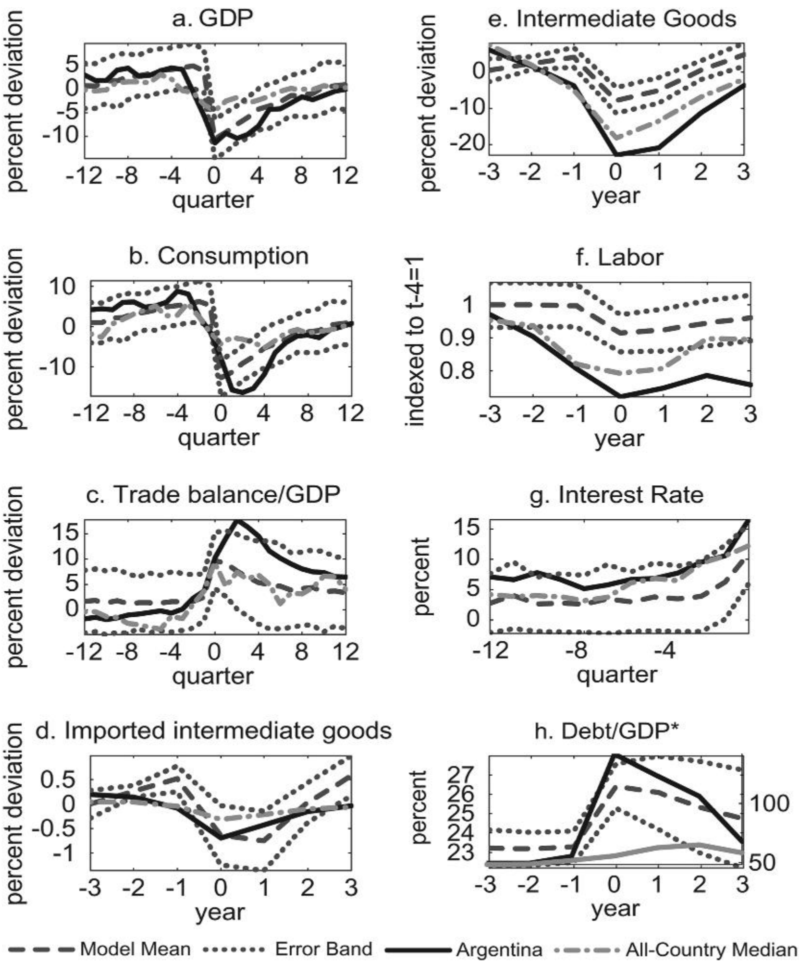

FIGURE VI

Macro Dynamics around Default Events

Consumption is H-P detrended. Trade balance and intermediate goods are computed as the ratio of GDP and H-P detrended. For labor, imported inputs, total intermediate goods, and debt, we aggregate the simulated data into annual frequency. Imported inputs and total intermediate goods are log-linearly detrended. Labor is rescaled by the level of the corresponding measure at three years before default. We take the annual data from Argentina and apply the same procedure. *The scale for Argentina and cross-country median debt/GDP is drawn on the right axis.

error bands and close to the model averages. This is true for Argentina 2002 and for the cross-country medians. The GDP and consumption drops that occur at $t = 0$ are larger in the model than in the cross-country medians, but this is because Argentina's recession was larger than the median and we calibrated the model to match Argentina's output collapse.

The model is consistent with the data in predicting a relatively stable path for net exports before the default, a sudden increase around the default date, and a sustained surplus after that. The adjustment to include the exogenous component of capital flows is important for the sudden increase, but not for the stable trade balance before default. The rise in net exports when default occurs is triggered by a low TFP realization ε_0, which is associated with a surge in x_0. Without this, net exports would go to zero at $t = 0$ and a small deficit on average during the remainder of the exclusion period, because during exclusion there would be no capital flows to finance a trade imbalance (the average deficit during exclusion would follow from averaging a zero trade balance under financial autarky with a trade deficit in case of reentry to credit markets).

The qualitative features of the dynamics of imported inputs, total intermediate goods, and labor are also in line with the data. Quantitatively, however, the model predicts smaller declines in labor and intermediate goods when defaults occur than in the data–with the caveat that nontrivial data limitations in terms of country coverage and sample periods for these variables makes them a weaker benchmark against which to compare the model's dynamics. The drop in imported inputs in the model is similar to the one observed in Argentina, but larger than the cross-country median, reflecting again the fact that the model is calibrated to match the recession in Argentina, which was larger than the cross-country median.

The model also does well at capturing the qualitative features of the dynamics of the interest rate (inclusive of country risk) and the debt ratio, but quantitatively it tends to underestimate both. The model produces a steady increase in the interest rate for the six quarters before default.[31] The debt ratio is relatively stable in the years before default, then surges with the default,

31. As explained earlier, we do not show interest rates for $t \geq 0$ because in the default state the default risk is infinite.

and drops in the years that follow.[32] As explained earlier, interest rates in the model are relatively low because they are pinned down by the default frequency, given the arbitrage condition of risk-neutral lenders, and the observed default frequency is low. The low debt ratio is also in line with the previous result showing that although this model can support significantly higher debt ratios than existing models of sovereign default, it still underestimates actual debt ratios.

The sharp declines in GDP, consumption, labor, and intermediate goods that occur when access to credit markets is lost indicate that the model yields predictions consistent with the sudden stops observed in emerging economies. In most of the sudden stops literature, however, the loss of credit market access is modeled as the result of an exogenous shock, whereas in this model the exclusion from credit markets and the economic collapse are endogenous and influence each other.[33]

IV.D. Sensitivity Analysis

In this section we conduct a sensitivity analysis to evaluate the robustness of the model's key quantitative predictions to changes in the range of imported inputs that require working capital θ, the parameters of the Armington aggregator μ and λ, the within-variety elasticity parameter ν, the labor elasticity parameter ω, and the reentry probability ϕ. The results are summarized in Table IV.[34] This table shows the main statistical moments we use to evaluate the performance of the model for each alternative

32. To make debt ratios comparable across data and model during periods of exclusion, we adjusted the model's measure to match the practice followed in the World Bank data set, which includes defaulted debt and the corresponding interest in arrears in the debt estimates. Thus, the mean debt ratio for the model after default in the event plot is the average of the predefault debt ratio and the debt ratio chosen in the case of reentry.

33. Mendoza (2010) proposed an alternative model of endogenous sudden stops based on collateral constraints and Irving Fisher's debt-deflation mechanism instead of sovereign default risk.

34. We also generated plots with the default event dynamics of output comparable to Figure VI. The quantitative differences are small and qualitatively they all have the same pattern, so we decided not to put them in the article. We also conducted the full sensitivity analysis without the exogenous component of capital flows x_t. Most of the results are similar to those reported in Table IV, which again verifies that x_t plays a minor role, except for enabling the model to produce the trade surplus after default.

TABLE IV

SENSITIVITY ANALYSIS

	Output drop at default	Mean debt/GDP ratio	Mean spread	Std. dev. of spread	GDP corr. with		Frequency of default w. GDP below trend
					spread	default	
(1) Data	13%	35%	1.86%	0.78%	−0.62	−0.11	62%
(2) Baseline	13%	22.88%	0.74%	1.23%	−0.17	−0.09	83%
Working capital							
(3) $\theta = 0$	13%	8.99%	0.05%	0.08%	0.24	−0.02	75%
(4) $\theta = 0.6$	13.9%	20.39%	0.59%	1.17%	−0.11	−0.11	88%
(5) $\theta = 0.8$	14.3%	26.84%	0.61%	1.19%	−0.14	−0.10	84%
Armington elasticity							
(6) 2.63 ($\mu = 0.62$)	14.6%	31.25%	0.55%	0.99%	−0.16	−0.09	90%
(7) 3.10 ($\mu = 0.68$)	12.9%	16.15%	1.14%	1.36%	−0.11	−0.09	78%
Armington share							
(8) $\lambda = 0.58$	17.20%	39.01%	0.28%	0.79%	−0.08	−0.04	83%
(9) $\lambda = 0.66$	12.7%	14.16%	0.99%	1.42%	−0.11	−0.08	77%
Within-variety elasticity							
(10) 2.22 ($\nu = 0.55$)	14.1%	25.83%	0.60%	1.17%	−0.11	−0.09	84%
(11) 2.89 ($\nu = 0.65$)	12.8%	19.81%	0.72%	1.22%	−0.12	−0.07	82%
Frisch elasticity of labor supply							
(12) 1.67 ($\omega = 1.6$)	12.8%	22.34%	0.91%	1.29%	−0.17	−0.13	85%
(13) 2.5 ($\omega = 1.4$)	14.3%	24.46%	0.45%	1.05%	−0.02	−0.06	68%
Probability of re-entry							
(14) $\phi = 0.05$	14.3%	37.02%	0.39%	0.88%	−0.11	−0.05	82%
(15) $\phi = 0.1$	13.5%	19.78%	0.65%	1.21%	−0.11	−0.12	94%

scenario, and also reproduces the statistics from the Argentine data and the baseline model (see rows (1) and (2)).

Working Capital. Rows (3)–(5) report results varying the value of θ. Row (3) removes working capital altogether by setting $\theta = 0$, whereas rows (4) and (5) set θ smaller ($\theta = 0.6$) and larger ($\theta = 0.8$) than the 0.7 baseline.

Without working capital, there is no output cost of default and no financial amplification mechanism. Hence, to keep the results comparable with the other results in Tables III–IV, in which default is costly, and with those reported in quantitative studies of sovereign debt that use exogenous proportional costs of default (e.g., Aguiar and Gopinath 2006, Yue 2010), we introduce an exogenous proportional cost of default when θ=0. This cost is set so that TFP falls by as much as needed to produce an output drop of 13% when default occurs, which is the same size as the drop in our baseline calibration. The other parameters are kept unchanged.

The model without working capital performs much worse than the baseline. In particular, the frequency of defaults (proxied by the mean spread because of the lender's risk neutrality) falls from 0.7% to 0.05%. The variability of spreads also nearly vanishes, spreads become procyclical, and the mean debt ratio drops to 9%. This marked worsening in the performance of the model follows from the fact that without working capital, default no longer affects factor demands and production, and thus the cost of default becomes independent of TFP. In short, this scenario shows that if we reduce our model to a variant of the standard Eaton-Gersovitz model with an exogenous proportional cost of default, we obtain results very similar to what other authors have found.[35]

Row (5) shows that widening the range of imported input varieties that require working capital to $\theta = 0.8$ (i.e., tightening the working capital constraint), instead of 0.7 as in the baseline, increases the mean debt ratio by about 4 percentage points and generates a larger output cost of default. The probability of default and the variability of spreads fall slightly, and the correlation between GDP and spreads increases to -0.14. If instead we assume $\theta = 0.6$, row (4) shows that we obtain a lower mean debt ratio

35. Note that the calibrated proportional drop in TFP is less than 1% in this experiment. If we set a 2% drop as in the studies with propotional output costs, the output drop in default becomes 15%, the mean debt ratio is 24%, and the average spread falls to 0.01%, which are again results in line with the literature (e.g., Aguiar and Gopinath 2006).

than in the baseline, but the cost of default and the correlation between GDP and spreads still rise, and the frequency of default and the variability of spreads still fall, as in the case with $\theta = 0.8$. Hence, while rising θ has a monotonic effect increasing the mean debt ratio, the effects on the other moments are nonmonotonic. This is because higher θ has potentially opposing effects on default incentives and production plans. On one hand, since a larger fraction of imported inputs requires foreign financing, the loss of credit market access has a larger impact on production. On the other hand, default can still lead to a higher output cost even with lower θ if the TFP shock that triggers default is larger with $\theta = 0.6$ than with $\theta = 0.7$. At the same time, the higher output cost of default makes it optimal for the sovereign to exercise the default option less often, lowering the default probability and the volatility of bond spreads, and increasing the mean debt/GDP ratio.

The distribution of defaults across "bad times" and "good times" changes slightly with the value of θ. In particular, lower θ shifts the distribution toward states with GDP below trend. With $\theta = 0.8$, about 84% of defaults occur with output below trend, compared with 83% in the baseline and 88% with $\theta = 0.6$. The correlation between GDP and default remains near -0.1 with either $\theta = 0.8$ or 0.6.

Parameters of the Armington Aggregator of Domestic and Imported Inputs. Rows (6) and (7) report results for Armington elasticities lower ($\eta_{m^d,m^*} = 2.63$) and higher ($\eta_{m^d,m^*} = 3.1$) than the baseline ($\eta_{m^d,m^*} = 2.9$). Comparing the three scenarios, we find predictable results from making domestic and imported inputs better substitutes, and in this case the effects are monotonic. As inputs become better substitutes, the output cost of default and the mean debt ratio fall, and the frequency of default and the variability of the spreads increase. The effects of lowering the Armington share of domestic inputs to $\lambda = 0.58$ (row (8)) or increasing it to $\lambda = 0.66$ (row (9)), relative to the 0.62 baseline are similar: Increasing λ makes the working capital channel operating via imported inputs less relevant, and as a result the output cost at default and the mean debt ratio fall, and the frequency of default and the variability of the spreads increase. Changes in η_{m^d,m^*} and λ have small effects on business cycle comovements. The correlations of GDP with spreads are slightly higher than in the baseline, but remain weakly negative.

It is worth noting that these variations in η_{m^d,m^*} and λ result in expenditure ratios of imported to domestic inputs that differ from those in the baseline and in the Mexican data used to calibrate them. However, these expenditure ratios are still in the range of those for the countries included in the study by Goldberg and Campa (2006), so scenarios like those in rows (6)–(9) should be regarded as plausible. Moreover, the range of expenditure ratios they documented would also support a wider range of values of η_{m^d,m^*} and λ than those shown in Table IV, including scenarios that can support very high debt ratios, albeit at lower default probabilities. For example, using $\lambda = 0.5$ we found a mean debt ratio of about 93% at a 0.2% default frequency. Similarly, if we lower η_{m^d,m^*} to the threshold value at which imported and domestic inputs become gross complements (1.96), the mean debt ratio rises to 80% at a negligible default frequency, and the output cost of default climbs to 22.6%.

Changing η_{m^d,m^*} and λ also affects the distribution of default events across output realizations, and the effects go in the same direction. The fraction of defaults that occurs with output below trend falls from 90% to 78% as we increase η_{m^d,m^*} from 2.63 to 3.1, and from 83% to 77% as we increase λ from 0.58 to 0.66. The correlations between GDP and default events change slightly and remain in the -0.04 to -0.09 range.

Within Elasticity of Imported Input Varieties. Rows (10) and (11) show results for lower (2.22) and higher (2.89) values of $\eta_{m_j^*}$ than the 2.44 baseline. Making imported input varieties better substitutes also weakens the working capital channel, because it implies that the efficiency loss of shifting away from the set Ω of imported inputs to its complement is smaller. Hence, as $\eta_{m_j^*}$ increases, the output drop at default and the mean debt ratio fall, while the mean and standard deviation of spreads increase. The fraction of defaults that occur with output below trend falls slightly from 84% to 82% as $\eta_{m_j^*}$ rises from 2.22 to 2.89.

These effects are again similar to those we obtained by increasing η_{m^d,m^*}, but quantitatively they are weaker. In particular, we increased $\eta_{m_j^*}$ by 62 basis points in rows (10) and (11) versus 50 basis points for η_{m^d,m^*} in rows (6) and (7), and yet the changes in the cost of default, mean debt ratio, mean and variability of spreads, and fraction of defaults with GDP below trend were *smaller* in absolute value. This is in line with the argument presented in the partial equilibrium analysis of the default effects on production, suggesting that changes in η_{m^d,m^*} have larger effects on the impact of default on GDP and factor allocations than changes in $\eta_{m_j^*}$. Thus, for parameterizations around the baseline calibration, the Armington elasticity of substitution across domestic and imported inputs affects more the model's quantitative performance than the within elasticity across imported input varieties.

Frisch Elasticity of Labor Supply. Rows (12) and (13) show results for lower (1.67) and higher (2.5) labor supply elasticities than in the baseline (2.2). As we showed in Section III, the efficiency loss triggered by default is larger the higher the elasticity of labor supply, because it implies that the labor market adjusts to interest rate hikes with a smaller wage decline and a larger decline in total labor, which in turn produce a smaller increase in L^m and a larger fall L^f . Hence, the output drop at default and the mean debt ratio are bigger as we increase the elasticity of labor supply. At the same time, the default probability (or the mean spread) and the variability of spreads fall with a bigger labor elasticity. The change in labor elasticity also affects the distribution of default events across output realizations, shifting it toward states with GDP below trend as the elasticity falls. The correlation between GDP and default remains near -0.1.

Reentry Probability (Mean Length of the Exclusion Period). In rows (14) and (15) we change the probability of reentry to 5% and 10%, respectively, which imply exclusion periods of 5 and 2.5 years, respectively (versus three years in the baseline). Here we encounter again nonmonotonic effects. The output drop at default is higher, and the frequency of default and the variability of spreads lower, when the mean exclusion period is either shorter or longer than in the baseline. In contrast, the mean debt ratio does decline monotonically as we lower the length of the exclusion period, from 37% with 5 years of exclusion to about 20% with 2.5 years. These results follow from the fact that increasing ϕ has ambiguous effects on the value of default at date t (equation (24)), by increasing the weigh of the reentry option at $t + 1$ but lowering the value of continuing in autarky. Intuitively, taking the default option today becomes more attractive because credit market reentry in the future is more likely (i.e., "easier") but the expected value of staying in autarky is itself lower as the probability of continuing in this state is lower.

The distribution of defaults also changes with the reentry probability, increasing the fraction of defaults with GDP below trend as the probability rises (or the length of the exclusion period falls). With $\phi = 0$, 94% of defaults occur with output below trend (compared with 83% in the baseline case), and for $\phi = 0.05$ the corresponding fraction is 82%. As with the other scenarios, the correlations of GDP with defaults and with spreads change slightly, but remain weakly negative.

Summing up, this sensitivity analysis shows that although the model's statistical moments vary somewhat as we change key parameters, the main quantitative findings are robust to these changes. The model produces large endogenous output drops at default as a result of the model's financial amplification mechanism, produces higher mean debt ratios at higher default frequencies than is generally the case in the quantitative sovereign debt literature, and generates weakly countercyclical spreads. The correlation between GDP and defaults remains slightly negative, and the frequency of defaults with GDP below trend is in the 75–95% range. The exception is the case in which the working capital channel is removed ($\theta = 0$ in row (3)), which removes the endogenous output cost of default. Replacing it with an exogenous proportional cost calibrated to match the observed output drop, the model falls apart and can only generate a small debt ratio at negligible levels of the default frequency and the variability of spreads, and with procyclical spreads.

V. Conclusions

This article proposed an equilibrium model of sovereign default and business cycles and showed that its quantitative predictions are broadly in line with observed empirical regularities of actual sovereign defaults. This model provides a solution to the disconnect between sovereign debt models (which rely on exogenous endowment dynamics with ad hoc costs) and models of emerging markets business cycles (which assume an exogenous financing cost of working capital calibrated to match the interest rate on sovereign debt).

The model features a financial amplification mechanism that links default with production plans and factor allocations via an endogenous efficiency loss, because default impairs the firms' access to working capital financing required to purchase a subset of imported inputs. This mechanism produces a novel feedback loop between default risk, debt dynamics, and the output cost of default.

In the model, producers of final goods choose an optimal mix of imported and domestic inputs that are imperfect substitutes in an Armington aggregator and varieties of imported inputs that are also imperfect substitutes in a Dixit-Stiglitz aggregator. Some imported input varieties require foreign working capital financing, and production of domestic inputs requires domestic labor. In this setup, strategic default causes an efficiency loss because final goods producers cannot operate with the imported input varieties that require credit, substituting them for other imported inputs and domestic inputs, and because labor reallocates from the final goods sector to the sector producing domestic inputs. These characteristics of the model are in line with empirical evidence on the substitution of inputs around default episodes (e.g., Gopinath and Neiman 2010) and the adverse effects of sovereign default on private credit markets and trade (e.g., Rose 2005, Kohlscheen and O'Connell 2008, Reinhart 2010).

The model is consistent with three key stylized facts of sovereign debt: (1) the dynamics of macroeconomic aggregates around default events, (2) the negative correlation between interest rates on sovereign debt and output, and (3) high debt-output ratios on average and when defaults take place. The model also explains key emerging markets business cycle moments such as countercyclical net exports, high variability of private consumption, weakly negative correlations between defaults and GDP, and the correlations of intermediate goods and labor with spreads and GDP. Moreover, the results show that default occurs with adverse TFP shocks that are not unusually large on average (1.3 times the standard deviation of TFP), and that the model embodies a powerful financial amplification mechanism that amplifies the effect of TFP shocks on GDP by a factor of 1.8 in default events.

The model produces an endogenous output cost of default that is increasing in the state of productivity. This is a key result that follows from the efficiency loss that drives the model's financial amplification mechanism. In this way, our model provides a foundation for the ad hoc default cost (increasing in endowment income above a threshold) that Arellano (2008) identified as critical to induce default incentives that trigger default in bad states, at nonnegligible debt ratios, and at realistic default frequencies.

In addition, the endogenous feedback between production and default in our model produces a mean debt ratio four times larger than in Arellano's model.

The model also offers an interesting new perspective on how structural features of the private sector and industrial and trade policies may interact with sovereign debt ratios, default incentives, and default risk. The cost of default is lower in economies where intermediate goods are better substitutes, where imported inputs are a smaller share of total inputs (either because of the nature of technology or because of trade protection), or where labor supply has a lower Frisch elasticity. As a result, economies with these characteristics can support lower mean debt ratios at higher default frequencies. This may help explain why the 1980s cluster of debt crises in Latin America affected countries that engaged in import substitution policies in the previous two decades. The same argument could explain why some European countries with a high degree of trade integration were able to attain high debt ratios at levels of risk premia that remained low and stable for some time.

We acknowledge, however, that the linkages between sovereign default and private sector credit, and the mechanisms by which default induces efficiency losses, should be the subject of further research. For instance, introducing elements of political uncertainty, debt maturity, secondary debt markets, dynamic renegotiation, and risk-averse lenders, all of which have been shown to add significant elements to the analysis of sovereign default in models with exogenous output dynamics, can be very promising lines of research (see, for example, Amador 2003, Benjamin and Wright 2008, Bi 2008a, 2008b, Broner, Martin, and Ventura 2008, Chatterjee and Eyigungor 2008, Cuadra and Sapriza 2008, D'Erasmo 2008, and Lizarazo 2005).

Appendix 1: The Firms' Dynamic Optimization Problem

We review here the intertemporal optimization problem faced by a representative firm that maximizes the present value of profits and needs working capital to pay for a fraction of the cost of imported inputs. The setup is based on a similar derivation in Uribe and Yue (2006). For simplicity, we characterize the problem in partial equilibrium, assuming that all intermediate goods are imported, and that there is a single homogeneous foreign input. We are interested in particular in showing that: (a) the problem can be reduced to a static profit-maximization problem, and (b) firms do not accumulate precautionary asset holdings (if they start with zero liabilities, they maintain zero liabilities at the end of each period in perpetuity).

Consider a representative firm in a small open economy that produces output by means of a production function that uses imported intermediate goods and labor as inputs,

$$y_t = F(m_t, L_t), \tag{30}$$

where the function F is homogeneous of degree 1, increasing in both arguments, and concave. Firms buy their inputs from perfectly competitive markets.

Production is subject to a working capital constraint that requires firms to pay in advance for a fraction θ of the cost of imported inputs, which have a world-determined relative price p. The working capital constraint is:

$$\frac{\kappa_t}{R_t} \geq \theta p m_t; \quad \theta \geq 0,$$

where κ_t denotes the amount of working capital held by the representative firm in period t.

The above formulation of the working capital constraint corresponds to a timing of transactions akin to a "cash-in-advance constraint," by which firms *must* hold non–interest-bearing *foreign* assets by an amount equal to the fraction θ of the cost of imported inputs. There is also an alternative formulation known as the "shopping time" formulation, according to which firms need to have the working capital $\theta p m_t$ at the end of the period (see Uribe and Yue 2006 and Oviedo 2005). The two differ only in that the former increases the cost of inputs by $\theta\,(R_t - 1)$, as we show below, and the latter by $\frac{\theta(R_t-1)}{R_t}$, but in both cases the relevant interest rate is determined by the same interest rate as for one-period debt R_t.

The debt position of the firm, denoted by d_t, evolves according to the following period-by-period budget constraint:

$$d_t = R_{t-1}d_{t-1} - F(m_t, L_t) + w_t L_t + p m_t + \pi_t - \kappa_{t-1} + \kappa_t,$$

where π_t denotes profits in period t, and R_t is the risk-free world interest rate in one-period bonds. Thus, there is no assumption requiring that the interest rate on one-period bonds be the same as that on working capital loans.

Define the firm's total net liabilities at the end of period t as $a_t \equiv R_t d_t - \kappa_t$. Then, we can rewrite the budget constraint as:

$$\frac{a_t}{R_t} = a_{t-1} - F(m_t, L_t) + w_t L_t + pm_t + \pi_t + \left(\frac{R_t - 1}{R_t}\right)\kappa_t.$$

Assume the interest rate is positive at all times. This implies that the working capital constraint always binds, or otherwise the firm would incur in unnecessary financial costs, which would be suboptimal. Since the working capital constraint holds with equality, we can eliminate κ_t from the above expression to get:

$$\frac{a_t}{R_t} = a_{t-1} - F(m_t, L_t) + pm_t\left[1 + \theta\left(R_t - 1\right)\right] + w_t L_t + \pi_t. \tag{31}$$

It follows from this result that the working capital constraint increases the unit cost of imported inputs by the amount $\theta(R_t - 1)$, which is increasing in the interest rate R_t, where R_t is the world real interest rate.

The firm's problem is to maximize the present discounted value of the stream of profits. In the article, the owners are domestic residents, so firms discount at the households' stochastic discount factor $\beta^t \frac{\lambda_t}{\lambda_0}$, where $\lambda_t = \beta R_t E_t \lambda_{t+1}$ is the Euler equation for bond holdings. Alternatively, firms can be assumed to discount profits at the world interest rates R_t. For the results we show here this does not matter. Under the first alternative, the firm's problem is,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \pi_t.$$

Using constraint (31) to eliminate π_t from the firm's objective function, the firm's problem can be stated as choosing a_t, m_t, and L_t so as to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t \frac{\lambda_t}{\lambda_0} \left\{\frac{a_t}{R_t} - a_{t-1} + F(m_t, L_t) - pm_t\left[1 + \theta\left(R_t - 1\right)\right] - w_t L_t\right\},$$

subject to the no-Ponzi-game constraint $\lim_{j\to\infty} E_t \frac{a_{t+j}}{\prod_{s=0}^{j} R_{t+s}} \leq 0$.

The first-order conditions associated with this problem are the Euler equation for net liabilities, $\lambda_t = \beta R_t E_t \lambda_{t+1}$, the no-Ponzi-game constraint holding with equality, and

$$F_m(m_t, L_t) = p\left[1 + \theta\left(R_t - 1\right)\right] \tag{32}$$

$$F_L(m_t, L_t) = w_t. \tag{33}$$

It is clear from condition (32) that the working capital requirement drives a wedge between the marginal product of imported inputs and their world relative price p. This wedge is larger the larger the financing cost of working capital, $(R_t - 1)$, or the higher the fraction of the cost of imported inputs that needs to be paid with credit, θ.

It is critical to note that since total net liabilities are irrelevant for the optimal choices of labor and imported inputs and the payoff function of the firm is linear with respect to net liabilities (and all the terms in $\lambda_t = \beta R_t E_t \lambda_{t+1}$ are exogenous to the firm's choices), any process a_t satisfying equation (31) and the firm's no-Ponzi-game constraint is optimal. Hence, if firms start out with zero net liabilities, then an optimal plan consists in holding no liabilities at all times ($a_t = 0$ for all $t \geq 0$), with distributed profits given by

$$\pi_t = F(m_t, L_t) - pm_t\left[1 + \theta\left(R_t - 1\right)\right] - w_t L_t.$$

This implies that firms do not accumulate precautionary holdings of assets, regardless of the input prices they face. Their choices of labor and imported inputs follow from the optimality conditions (32)–(33), which depend only on current values of factor prices, the rate of interest, and TFP. These are not assumptions attached to the working capital requirement, but a result that follows from the linear nature of the firm's payoff: If firms maximize the present value of profits, with discount rates independent of the firm's choices and net liabilities entering linearly in profits, there is no incentive for precautionary asset holdings by firms. One can of course propose alternative formulations that deviate from these conditions and would produce precautionary asset demand by firms, but these are not conditions assumed in the setup of the model.

Appendix 2: Theorem Proofs

Proof of Theorem 1.

Given a productivity shock ε, the utility from defaulting $v^d(\varepsilon')$ is independent of b. We can also show that the utility from not defaulting $v^{nd}(b', \varepsilon')$ is increasing in b_{t+1}. Therefore, if $V(b^1, \varepsilon') = v^d(\varepsilon')$, then it must be the case that $V(b^0, \varepsilon') = v^d(\varepsilon')$. Hence, any ε' that belongs in $D(b^1, \varepsilon)$ must also belong in $D(b^0, \varepsilon)$.

Let $d^*(b', \varepsilon')$ be the equilibrium default decision rule. The equilibrium default probability is then given by

$$p(b', \varepsilon) = \int d^*(b', \varepsilon')\, d\mu(\varepsilon' | \varepsilon).$$

From $D(b^1, \varepsilon') \subseteq D(b^0, \varepsilon')$, if $d^*(b^1, \varepsilon') = 1$, then $d^*(b^0, \varepsilon') = 1$. Therefore,

$$p(b^0, \varepsilon) \geq p(b^1, \varepsilon).$$

Proof of Theorem 2.

From Theorem 1, given a productivity shock ε and $b^0 < b^1 \leq 0$, $p^*(b^0, \varepsilon) \geq p^*(b^1, \varepsilon)$. The equilibrium bond price is given by

$$q(b', \varepsilon) = \frac{1 - p(b', \varepsilon)}{1 + r}.$$

Hence, using Theorem 1, we obtain that:

$$q(b^0, \varepsilon) \leq q(b^1, \varepsilon).$$

Appendix 3: Data Definition and Sources

Table A.1 describes the variables and data sources of our data set for cross-country event studies. Table A.2 summarizes the list of countries, default episodes, and the available variables in the analysis.

TABLE A.1
Variables and Sources

Variable	Definition	Frequency	Sources
GDP	Real GDP	quarterly	International Financial Statistics and Levy-Yeyati and Panizza (2011)
CON	Real consumption	quarterly	International Financial Statistics
MS	Imported intermediates (in dollars, deflated by US PPI)	annual	United Nations comtrade (BEC code)
TB	Trade balance/GDP	quarterly	International Financial Statistics
IM	Intermediate goods (deflated by PPI)	annual	United Nations—National Accounts Statistics: Main Aggregates
L	Labor (paid employment)	annual	International Labor Organization
D	External debt/GNI	annual	Global Development Finance
S	Sovereign bond spreads	quarterly	EMBI+ and EMBI global (J.P. Morgan)

The default episodes are based on Levy-Yeyati and Panizza (2011), Benjamin and Wright (2008), and Standard and Poors report. Because we need the measure of economic variables which typically reflect the impact of default with some lag, we use the quarter after the default announcement date in the event analysis. This treatment is the same as Levy-Yeyati and Panizza (2011) who study the drop of GDP in the postdefault quarters.

GDP, consumption, and trade balance/GDP are from International Financial Statistics and Levy-Yeyati and Panizza (2011). Levy-Yeyati and Panizza (2011) compiled the real GDP for countries from national sources for periods when IFS does not record their GDP. GDP, consumption, and trade balance/GDP are H-P detrended.

The imported intermediate inputs are sum of categories for intermediate goods based on the classification of Broad Economic Categories (BEC). The categories for intermediate goods are: (111*) Food and beverages, primary, mainly for indus-

TABLE A.2

LIST OF COUNTRIES AND VARIABLES INCLUDED IN THE EVENT ANALYSIS

Sovereign default	Available series
Argentina 1982Q2	GDP, L, D
Argentina 2002Q1	GDP, CON, MS, TB, IM, L, D, S
Chile 1983Q1	GDP, D
Croatia 1992Q2	GDP, L
Domin. Rep. 1993Q1	GDP, D
Ecuador 1999Q3	GDP, CON, MS, TB, IM, D, S
Indonesia 1998Q3	GDP, CON, TB, D
Mexico 1982Q4	GDP, L, D
Moldova 2002Q1	GDP, CON, MS, L, TB, D
Nigeria 1983Q1	GDP, D
Nigeria 1986Q4	GDP, D
Pakistan 1998Q3	GDP, D
Peru 1983Q2	GDP, D
Philippines 1983Q4	GDP, CON, TB, D
Russia 1998Q4	GDP, CON, TB, IM, L, D, S
South Africa 1985Q4	GDP, CON, TB, L, D
South Africa 1993Q1	GDP, CON, TB, IM, L, D
Thailand 1998Q1	GDP, CON, MS, TB, D, S
Ukraine 1998Q4	GDP, MS, IM, L, D, S
Uruguay 1990Q2	GDP, D
Uruguay 2003Q2	GDP, MS, IM, L, D, S
Venezuela 1995Q3	GDP, IM, D, S
Venezuela 1998Q3	GDP, IM, D, S

try; (121*) Food and beverages, processed, mainly for industry; (21*) Industrial supplies not elsewhere specified, primary; (22*) Industrial supplies not elsewhere specified, processed; (31*) Fuels and lubricants, primary; (322*) Fuels and lubricants, processed (other than motor spirit); (42*) Parts and accessories of capital goods (except transport equipment); (53*) Parts and accessories of transport equipment.

Intermediate goods are from United Nations, National Accounts Official Country Data. The data are taken from Table 4.1 Total Economy (S.1), I. Production account—Uses Intermediate consumption, at purchaser's prices.

Labor data are the total paid employment data from LABORSTA data set collected by International Labor Organization.

Because of the serious data limitation for imported inputs, total intermediate goods, and labor, we cannot use HP filter. Imported inputs and total intermediate goods are log-linearly detrended. Labor data are indexed so that the employment four years before default is 1.

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