# Appendix for
# Natural Resources and Sovereign Risk in Emerging Economies: A Curse *and* a Blessing

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# Contents

# A Data

We collected data for GDP, consumption, trade balance, oil rents as a percentage of GDP, oil production (extraction), oil reserves, oil consumption, oil net exports, oil prices, total public debt, total external public debt, net foreign assets, default episodes, and country risk for the thirty largest oil-producing emerging economies in 2010. Those thirty countries are Saudi Arabia, Iran, Iraq, Kuwait, Venezuela, United Arab Emirates, Russian Federation, Libya, Nigeria, Kazakhstan, Qatar, China, Brazil, Algeria, Mexico, Angola, Azerbaijan, Ecuador, India, Oman, Sudan, Malaysia, Indonesia, Egypt, Yemen, Argentina, Syrian Arab Republic, Gabon, Colombia and Vietnam.

As an indicator of country risk we use the Institutional Investor Index (III from now on). The III country credit rating, is a measure of sovereign debt risk that is published biannually in the March and September issues of the Institutional Investor magazine. It is also commonly known as the Country Credit Survey. More specifically, the III is an indicator used to identify and measure country risk, where country risk refers to a collection of risks related to investing in a foreign country, including political risk, exchange rate risk, economic risk, sovereign risk and transfer risk. We have biannual data for the 1979-2014 period. The index is based on information provided by senior economists and sovereign-risk analysts at leading global banks and money management and securities firms. The respondents have graded each country on a scale of zero to 100, with 100 representing the least likelihood of default. Respondents responses are weighted according to their institutions' global exposure.

The data on oil reserves, oil production, oil net exports (thousands of barrels per day), and oil prices (Brent crude oil, USD per barrel) is from the US Energy Information Administration (EIA) from 1980 to 2014. For reserves, we used proved reserves. For oil prices we use the real price by deflating the Brent spot price FOB with the US CPI index for all urban consumers all items in US City average, seasonally adjusted (1982-1984=100).

GDP, Oil rents as a percentage of GDP, consumption, and the trade balance, are taken from the World Bank's World Development Indicators Database. Using oil rents we construct oil GDP by multiplying GDP (constant LCU) times oil rents as a percentage of GDP. Non-Oil GDP is obtained by subtracting oil GDP from total GDP. We construct gross oil output by multiplying the nominal price of oil (Brent crude oil, USD per barrel) times oil production (average number of barrels per year). When computing gross oil output as percentage of

GDP, we use GDP in current USD.

Total public debt data comes from the International Monetary Fund's Historical Public Debt Database (HPDD). We have information, covering 1971-2015 period, for Gross Government Debt. Total public external debt data is taken from the World Bank Global Development Finance database (GDF), which has annual data for over 130 countries on total external debt by maturity and type of debtor (private non-guaranteed debt and publicly guaranteed debt). The data goes back as far as 1970 and is collected on the basis of public and publicly-guaranteed debt reported in the World Bank's Debtor Reporting System by each of the countries. This information is not available for Saudi Arabia, Iraq, Kuwait, United Arab Emirates, Libya, Qatar, Oman, Malaysia and Syria.

We use the updated and extended version of the "External Wealth of Nations" dataset, constructed by Lane & Milesi-Ferretti (2007) to obtain information on net foreign asset positions. It contains data for the 1970-2015 period and for 188 countries (including those in our sample), plus the euro area as a whole. Specifically, net foreign assets series are based on three alternative measures: i) the accumulated current account, adjusted to reflect the impact of capital transfers, valuation changes, capital gains and losses on equity and Foreign Direct Investment (FDI), and debt reduction and forgiveness; ii) the net external position, reported in the International Investment Positions section of the International Monetary Fund's Balance of Payments Statistics (BOPS), and net of gold holdings; iii) the sum of net equity and FDI positions (both adjusted for valuation effects), foreign exchange reserves and the difference between accumulated flows of "debt assets", and the stock of debt measured by the World Bank (or the OECD).

Default data is from Borensztein & Panizza (2009) for the 1979-2004 period. We include sovereign defaults on foreign currency bond debt and foreign currency bank debt. A sovereign default is defined as the failure to meet a principal or interest payment on the due date (or within the specified grace period) contained in the original terms of the debt issue, or an exchange offer of new debt that contains terms less favorable than the original issue. Such rescheduling agreements covering short and long term debt are considered defaults even where, for legal or regulatory reasons, creditors deem forced rollover of principal to be voluntary. We use the updated and extended version default data from Reinhart & Rogoff (2010) dataset for the 2005-2014 period. A default is defined as an external sovereign default crisis or a restructuring of external debt.

# B Institutional Investor Index & Sovereign Risk Measures

In this section, we show that the Institutional Investor Index (III) is a robust measure of sovereign risk by showing that it is highly correlated with other measures of sovereign risk. We also explain how we use the III to chain the Emerging Markets Bond Index (EMBI) backwards to be able to use it to calculate the average and standard deviation of the spread used in Section 4.

## B.1 Moody's and Fitch Credit Ratings

Credit ratings by agencies such as Moody's and Fitch are commonly used measures of sovereign risk. These agencies assign risk based on rating symbols. Tables B1 and B2 provide brief descriptions of what each symbol signifies about credit risk. Table B3 provides the date each agency first issued a credit risk rating to a given sovereign.

Table B1: Moody's Global Long-Term Rating Scale

| Rating | Description |
|---|---|
| Aaa | Obligations rated Aaa are judged to be the highest quality, subject to the lowest level of credit risk. |
| Aa | Obligations rated Aa are judged to be of high quality and are subject to very low credit risk. |
| A | Obligations rate A are judged to be upper-medium grade and are subject to low credit risk. |
| Baa | Obligations rated Baa are judged to be medium-grade and subject to moderate credit and as such may possess certain speculative characteristics. |
| Ba | Obligations rated Ba are judged to be speculative and are subject to substantial credit risk. |
| B | Obligations rated B are considered speculative and are subject to high credit risk. |
| Caa | Obligations rated Caa are judged to be speculative of poor standing and are subject to very high credit risk. |
| Ca | Obligations rated Ca are very highly speculative and are likely in, or very near, default with some prospect of principal and interest. |
| C | Obligations rated C are the lowest rated and are typically in default, with little prospect for recovery of principal or interest. |

Note: Moody's appends numerical modifiers 1, 2, and 3 to each generic rating classification from Aa through Caa. The modifier 1 indicates that the obligation ranks in the higher end of its generic rating category, the modifier 2 indicates a mid-range ranking, and the modifier 3 indicates a ranking in the lower end of that generic rating category.

Table B2: Fitch International Credit Rating Scale

| Rating | Description |
|---|---|
| AAA | Highest credit quality. AAA ratings denote the lowest expectation of default risk. They are assigned only in cases of exceptionally strong capacity for payment of financial commitments. This capacity is highly unlikely to be adversely affected by foreseeable events. |
| AA | Very high credit quality. AA ratings denote expectations of very low default risk. They indicate very strong capacity for payment of financial commitments. This capacity is not significantly vulnerable to foreseeable events. |
| A | High credit quality. A ratings denote expectations of low default risk. The capacity for payment of financial commitments is considered strong. This capacity may, nevertheless, be more vulnerable to adverse business or economic conditions than is the case for higher ratings. |
| BBB | Good credit quality. BBB ratings indicate that expectations of default risk are currently low. The capacity for payment of financial commitments is considered adequate, but adverse business or economic conditions are more likely to impair this capacity. |
| BB | Speculative. BB ratings indicate an elevated vulnerability to default risk, particularly in the event of adverse changes in business or economic conditions over time; however, business or financial flexibility exists that supports the servicing of financial commitments. |
| B | Highly speculative. B ratings indicate that material default risk is present, but a limited margin of safety remains. Financial commitments are currently being met; however, capacity for continued payment is vulnerable to deterioration in the business and economic environment. |
| CCC | Substantial credit risk. Default is a real possibility. |
| CC | Very high levels of credit risk. Default of some kind appears probable. |
| C | Near default. A default or default-like process has begun, or the issuer is in standstill, or for a closed funding vehicle, payment capacity is irrevocably impaired. |
| RD | Restricted default. |
| D | D ratings indicate an issuer that in Fitch's opinion has entered into bankruptcy filings |

Note: Within rating categories, Fitch may use modifiers. The modifiers "+" or "-" may be appended to a rating to denote relative status within major rating categories. Such suffixes are not added to AAA ratings and ratings below the CCC category.

Unlike the III that is updated each semester, credit rating changes can occur at any time for an individual sovereign. In order to merge credit ratings data with the III, we use the credit rating that has been assigned the longest to a sovereign during a particular semester and merge that rating with the respective semester III reading. Since the III is a continuous variable and credit rating are a discrete variable (i.e. factor variable over the ordinal ratings labels), we visualize their correlation with box plots.

Table B3: Credit Agency Rating's First Issued Date

| Country | Moody's | Fitch |
|---|---|---|
| Argentina | 11/18/1986 | 5/28/1997 |
| Brazil | 11/18/1986 | 12/1/1994 |
| China | 5/18/1988 | 12/11/1997 |
| Colombia | 8/4/1993 | 8/10/1994 |
| Ecuador | 7/24/1997 | 11/8/2002 |
| Egypt | 10/9/1996 | 8/19/1997 |
| Gabon | | 10/29/2007 |
| India | 1/28/1988 | 3/8/2000 |
| Iran | | 5/10/2002 |
| Iraq | | 8/7/2015 |
| Kazakstan | 11/11/1996 | 11/5/1996 |
| Kuwait | 1/29/1996 | 12/20/1995 |
| Malaysia | 1/18/1986 | 8/13/1998 |
| Mexico | 12/18/1990 | 8/30/1995 |
| Oman | 1/29/1996 | |
| Qatar | 1/29/1996 | 3/6/2015 |
| Russia | 10/7/1996 | 10/7/1996 |
| Saudi Arabia | 1/29/1996 | 11/24/2004 |
| Venezuela | 12/29/1976 | 9/15/1997 |

Box plots are used to show the overall dispersion of a continuous variable over groups. In our case, the y-axis is the continuous III, and the x-axis is the agency's credit rating ranks. The credit rating ranks are ordered along the x-axis from highest to lowest credit risk (from left to right). The box plots then graphs the quartiles of III observations over each credit risk rating. The horizontal line across the middle of the box is the median. The second quartile is the region from the median line to the bottom of the box, while the third quarter is the region from the median line to the top of the box. The bottom end of the lower whisker is the smallest value excluding outliers and the top end of the upper whisker is the largest value excluding outliers. Outliers are plotted as dots above and below the whisker of the box. Outliers above the upper whisker are 1.5 times greater than the third quartile while outliers below the lower whisker are 1.5 times lower than the first quartile. Figure B1 plots the III over Moody's credit risk ratings, and figure B2 plots the III over Fitch credit risk ratings.

Figure B1: Moody's Long-Term Sovereign Credit Ratings over the III

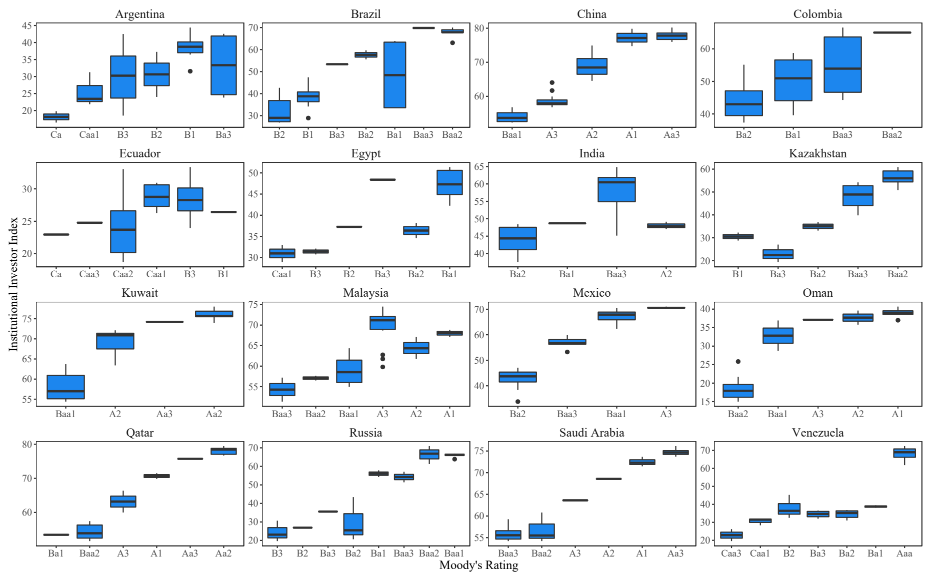
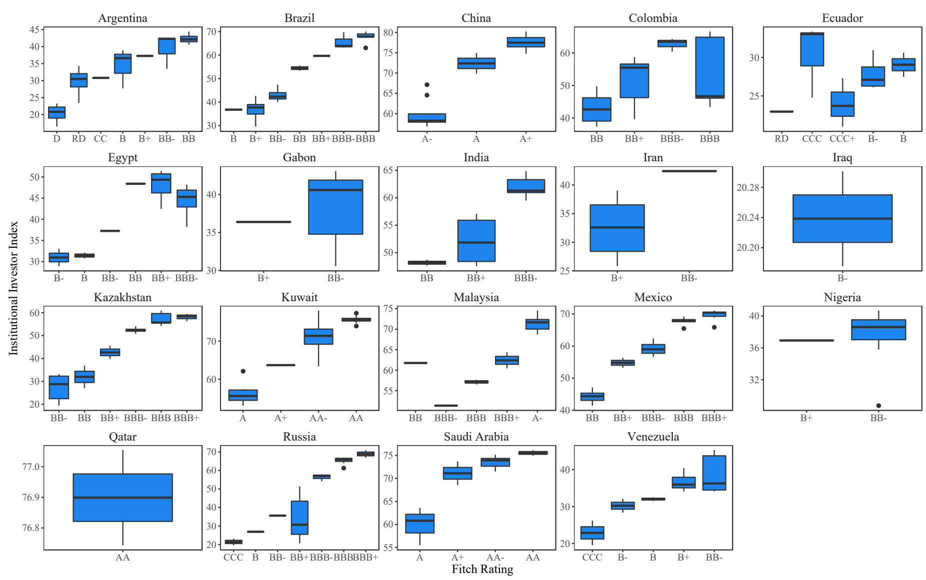



Figure B2: Fitch Long-Term Sovereign Credit Ratings over the III




We can see from the distributional characteristics of III over the Moody's and Fitch credit risk ratings that each sovereign's corresponding III measure tends to increase as its credit rating improves. This indicates that the III is correlated with credit ratings.

### B.2 Emerging Markets Bond Index (EMBI)

The Emerging Market Bond Index (EMBI) is JP Morgan's index of dollar denominated bonds issued for various emerging economies. It is one of the most widely used benchmarks of emerging market sovereign debt. The index comprises of US dollar-denominated Brady bonds, loans, and Eurobonds that have a face value of $500 million dollars or more and have a maturity greater than a year. The EMBI is quoted as a spread on sovereign debt over US treasuries, and the III is a measure of sovereign risk where 0 indicates high risk of default and 100 indicates low risk of default. Thus we expect to see these two move in opposite directions if the III is a good indicator of sovereign risk. In other words we expect the EMBI to rise as sovereign risk increases. Indeed we see in table B4 that the EMBI and III are negatively correlated, moving in the same direction to indicate sovereign risk.

Table B4: Correlation Between EMBI and III

| Country | Correlation |
|---|---|
| Angola | -0.570 |
| Argentina | -0.751 |
| Azerbaijan | 0.031 |
| Brazil | -0.789 |
| China | 0.312 |
| Colombia | -0.740 |
| Ecuador | -0.442 |
| Egypt | -0.642 |
| Gabon | -0.667 |
| India | -0.186 |
| Indonesia | -0.167 |
| Iraq | -0.163 |
| Kazakhstan | -0.293 |
| Malaysia | -0.434 |
| Mexico | -0.723 |
| Nigeria | -0.666 |
| Russian Federation | -0.686 |
| Venezuela | -0.629 |
| Vietnam | 0.146 |

Since the EMBI was introduced only in 1992, we have fewer observations of the EMBI than we have for the III. Following Erb et al. (1996), we can use the fact that the EMBI and the III are correlated with each other to extend the EMBI backwards so that it starts in the same year as the III for country i.

We use the following equation to build the index for each country:

$$EMBI_t = \alpha_0 + \alpha_1 III_t + \epsilon_t \tag{B1}$$

Suppose we have observations of the $EMBI$ for country $i$ starting at time $t$ through $T$ where $t < T$. We estimate (B1) using observations $t$ through $T$ of the $EMBI$ and $III$ for country $i$. Table B5 reports the estimates for $\alpha_1$ in (B1) for each country. We see that most country's estimate is negative and statistically significant. This implies that equation (B1) is an appropriate model to use to estimate values of the EMBI that are not available. We are then able to plug observation $III_{t-1}$ into the estimated model to calculate the fitted value for $EMBI_{t-1}$. Now we re-estimate (B1) using observations $t-1$ through $T$ of the $EMBI$ and $III$, and then plug observation $III_{t-2}$ into the newly estimated model to calculate the fitted value for $EMBI_{t-2}$. We continue this back-substitution until we have exhausted all observations of the III for country i. Our final output is an index of the EMBI re-constructed to the same time as the first observation of the III for country i. Figures of our reconstructed EMBI indices are available upon request.

Table B5: $\alpha_1$ Estimates on Observed Values of the EMBI

| Country | Slope Coefficient | Standard Error |
|---|---|---|
| Angola | $-61.18$ | 35.98 |
| Argentina | $-156.10^{***}$ | 20.91 |
| Azerbaijan | $1.44$ | 17.76 |
| Brazil | $-21.82^{***}$ | 2.59 |
| China | $1.75^{**}$ | 0.81 |
| Colombia | $-14.98^{***}$ | 2.24 |
| Ecuador | $-84.03^{***}$ | 26.61 |
| Egypt | $-14.14^{***}$ | 3.19 |
| Gabon | $-37.46^{***}$ | 10.79 |
| India | $-6.00$ | 12.96 |
| Indonesia | $-2.41$ | 2.95 |
| Iraq | $-5.01$ | 6.96 |
| Kazakhstan | $-18.56$ | 15.15 |
| Malaysia | $-7.38^{***}$ | 2.49 |
| Mexico | $-17.11^{***}$ | 2.49 |
| Nigeria | $-42.23^{***}$ | 7.88 |
| Russian Federation | $-41.69^{***}$ | 7.47 |
| Venezuela | $-74.32^{***}$ | 13.99 |
| Vietnam | $6.89$ | 10.43 |

*** $p<0.01$, ** $p<0.05$, * $p<0.1$

## C III, Oil Production and External Debt

Figure C1 plots the relationship between the III and oil production value to GDP ratio, for each country, over the period 1979-2010. One feature stands out from Figure C1: when oil production value to GDP ratio is high, the country risk index tends to improve. Note that there are countries where the correlation is not significant, such as Iran, United Kingdom, Egypt or Gabon.

Figure C1: Institutional Investor Index (X-Axis) and oil production value to GDP (%, Y-Axis).

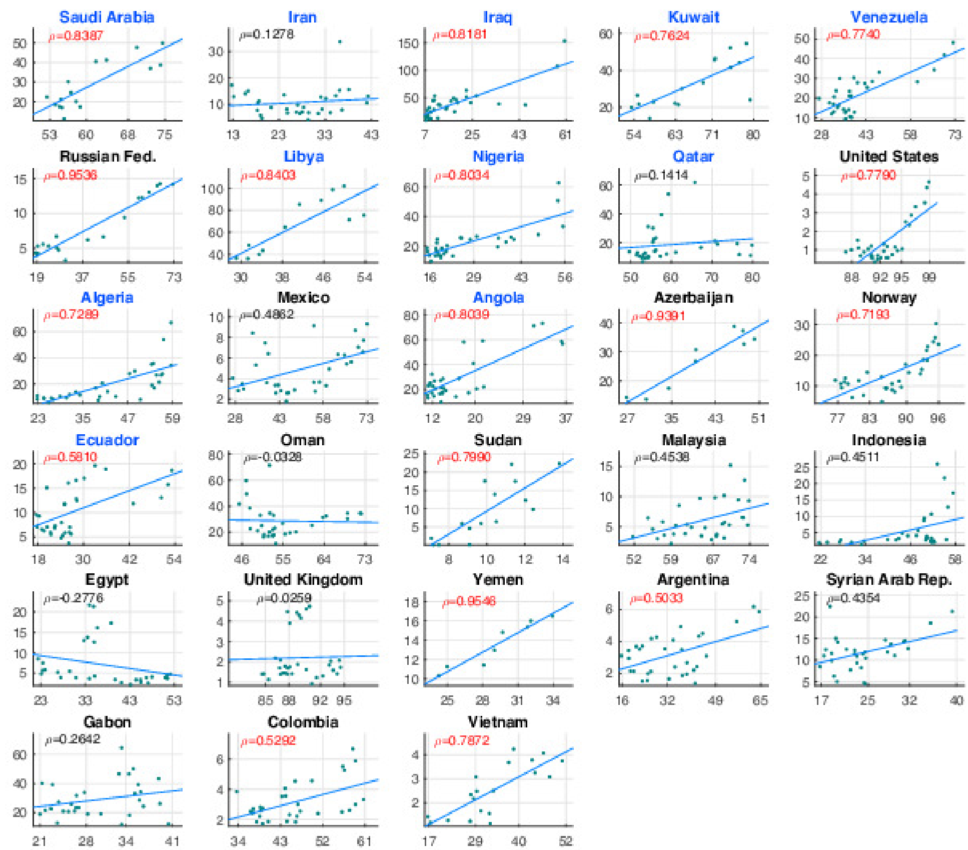
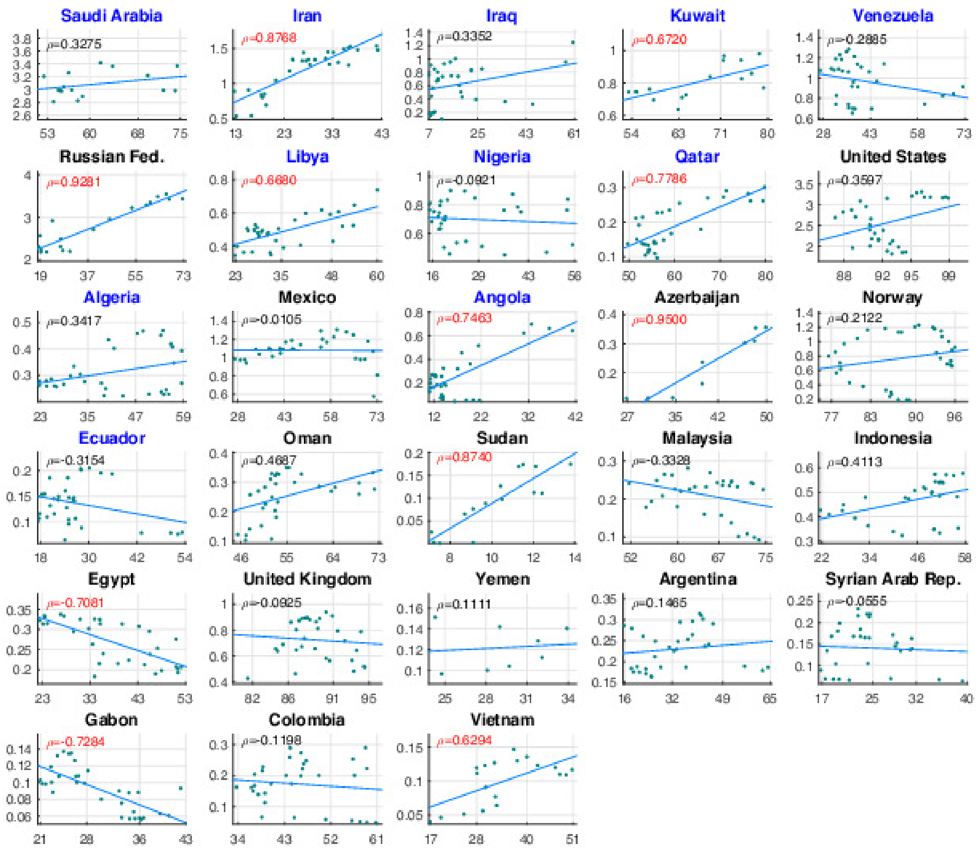


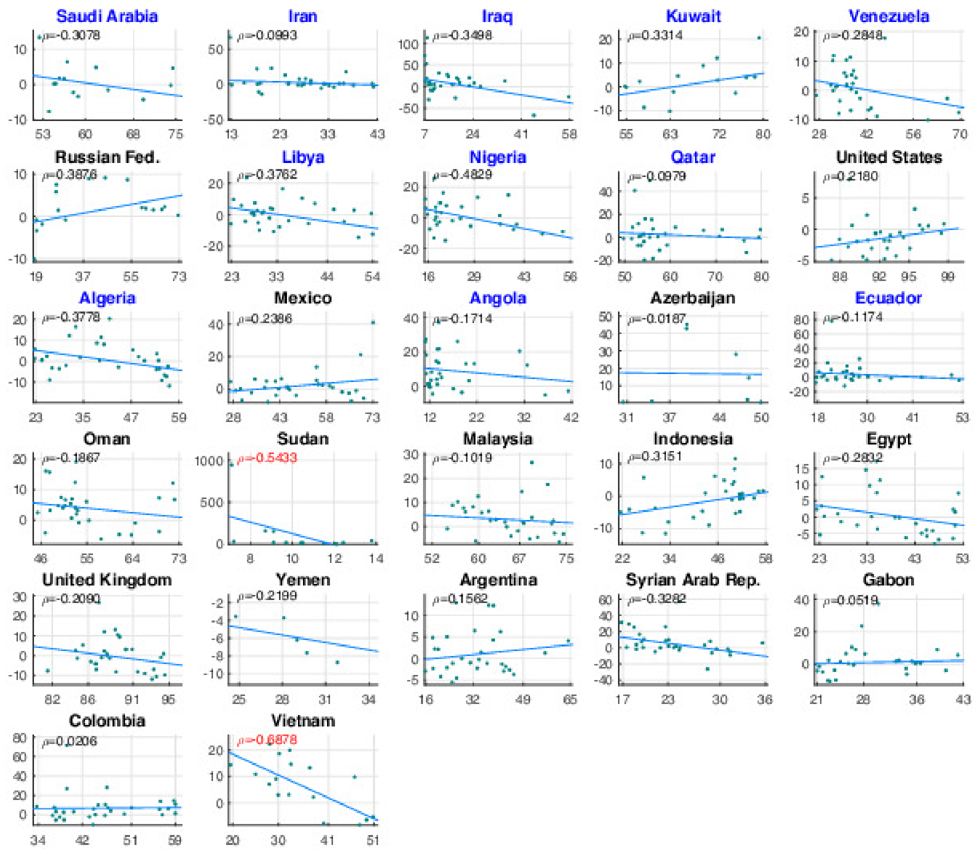

Figure C2 presents the III versus oil production (in billion barrels per year).

Figure C2: Institutional Investor Index (X-Axis) and oil production (billion barrels per year, Y-Axis).



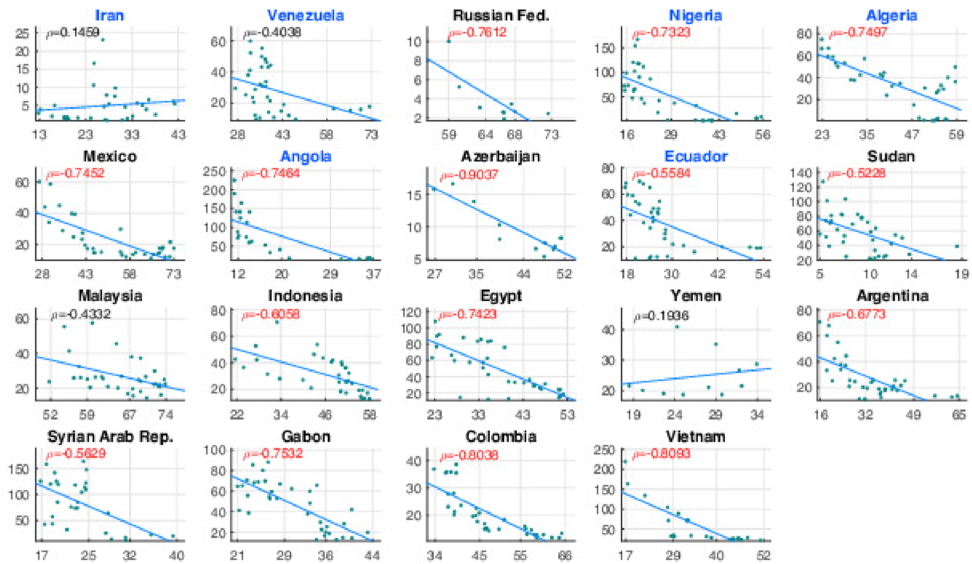

In this figure, absolute value of correlation coefficients greater than 0.5 are displayed in red. As we can see, there is not a clear pattern, since there are some countries for which the relationship is clearly positive, while for others it is negative or zero. This suggests that oil price is the "main driving force" behind changes in the country risk index (and not oil production). In Figure C3 we document the association between III and the oil production growth rate.

Figure C3: Institutional Investor Index (X-Axis) and oil production growth rate (%, Y-Axis).



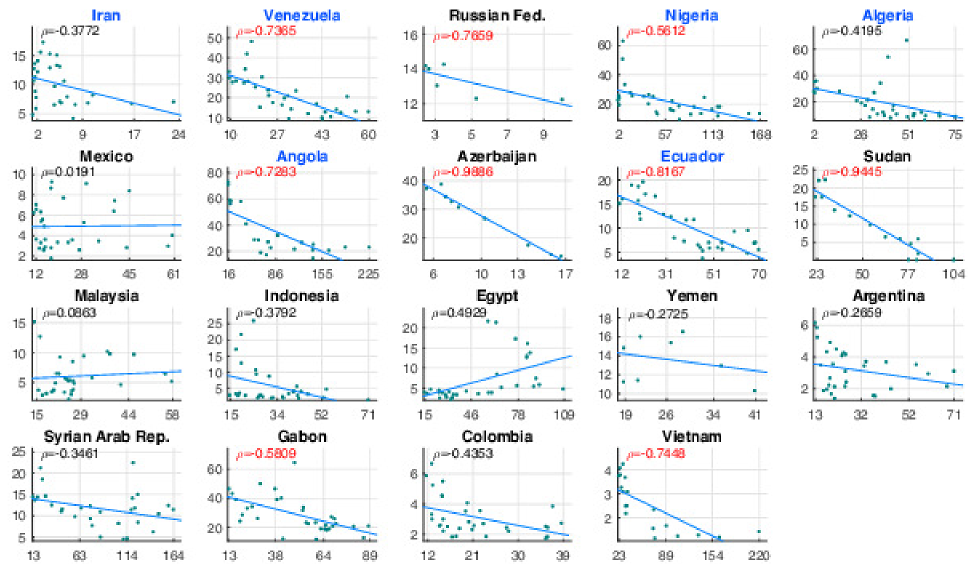

In this case, correlation coefficients lower than -0.5 are displayed in red. The results point in the direction that there is not any association between these two variables, although a negative relationship is observed for Sudan and Vietnam. Additionally, Figure C4 shows the relationship between the III and total public external debt to GDP ratio.

Figure C4: Institutional Investor Index (X-Axis) and total external public debt to GDP (%, Y-Axis).




Note that for most countries, correlation coefficients are displayed in red, which means that these are lower than -0.5. As we can see, III goes down when total public external debt increases. Additionally, Figure C5 shows the association between total external public debt to GDP ratio and oil production value to GDP ratio.

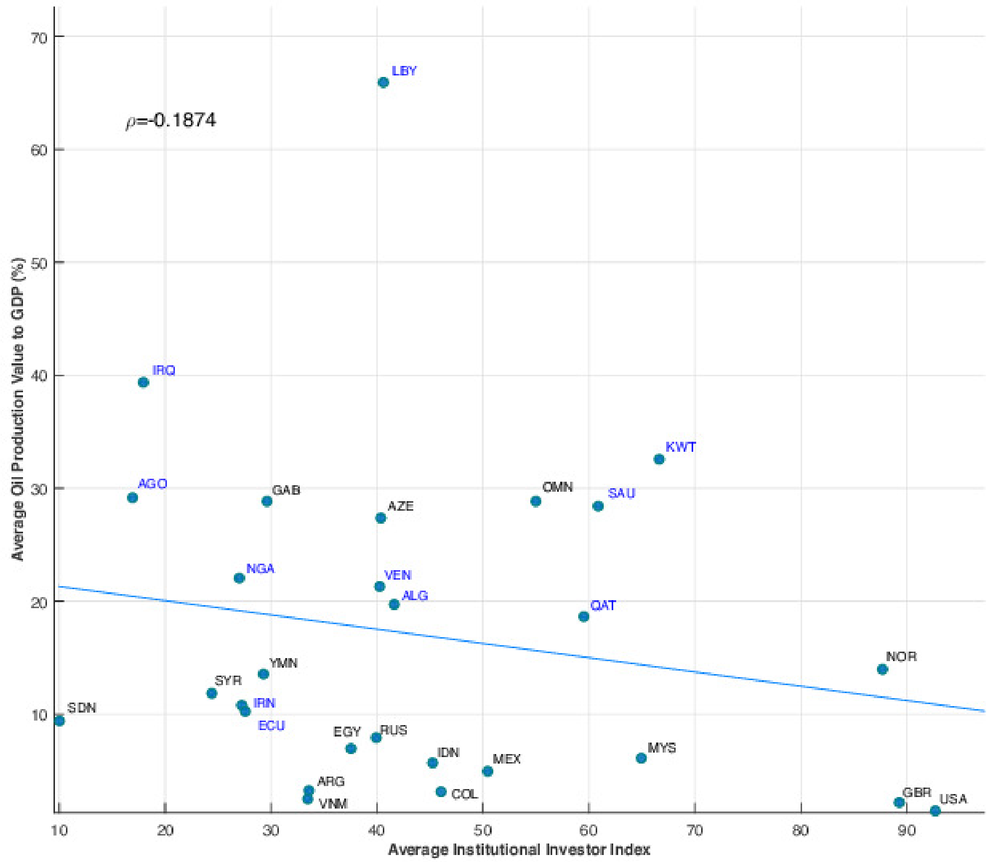
Figure C5: Total external public debt to GDP (%, X-Axis) and oil production value to GDP (%, Y-Axis).




As we can see, for 9 countries there is a negative correlation, which implies than when oil production value to GDP is high, total public external debt tends to be low. Nevertheless, such a contention is not reinforced by the rest of countries in the sample, since no significance is observed. Moreover, in the case of Egypt, the estimated coefficient shows almost a positive and statistically strong effect.

Moreover, Figure C6 plots the average III against average oil production value to GDP: In this case, we compute a low correlation coefficient (-0.187). The negative trend indicates that countries with high oil production value to GDP over time show a high country risk (or a low average III). It is important to mention that average oil production value to GDP may be low because historical GDP is very high when compared with the historical oil production value, such as in USA or Norway. Furthermore, this negative relationship may also be driven by exceptional cases such as Libya or Iraq, which have average oil production value to GDP of about 67 and 39 percent, and average III of about 41 and 17, respectively.

Figure C6: Average Institutional Investor Index (X-Axis) and average oil production value to GDP (Y-Axis): 1979-2010.



## D Panel Estimation Approach

Before proceeding to dynamic panel data models, we need to verify that all variables are integrated of the same order. In doing so, we have used the test of the panel unit root of Im et al. (2003) IPS henceforth), which is based on averaging individual unit root test statistics for panels. Specifically, they proposed a test based on the average of augmented Dickey-Fuller statistics (ADF henceforth) computed for each group in the panel. In accordance with some survey on panel unit root tests (such as those discussed in Banerjee (1999)), this test is less restrictive and more powerful than others that do not allow for heterogeneity in the autoregressive coefficient. IPS test permit solving serial correlation problem by assuming heterogeneity between units (in this case, countries) in a dynamic panel framework, as considered here. The basic equation of IPS test is as follows:

$$\Delta y_{it} = \alpha_i + \beta_i y_{it-1} + \sum_{j=1}^{p} \phi_{ij} \Delta y_{it-j} + \epsilon_{it} \tag{D1}$$

for $i = 1, 2, ..., N$ and $t = 1, 2, ..., T$, where $N$ refers to the number of countries in the panel and $T$ refers to the number of observations over time. In this case, $y_i$ stands for each variable under consideration in our model (for example, III, oil GDP or non-oil GDP), $\alpha_i$ is the individual fixed effect and $p$ is the maximum number of lags included in the test. The null hypothesis then becomes $\beta_i = 0$ for all $i$, against the alternative hypothesis, which is that $\beta_i < 0$ for some $i = 1, ..., N_1$ and $\beta_i = 0$ for $i = N_1 + 1, ..., N$, where $N_1$ denote the number of stationary panels. Therefore, IPS statistic can be written as follows:

$$\bar{t} = \frac{1}{N} \sum_{i=1}^{N} t_i^{ADF} \tag{D2}$$

where $t_i^{ADF}$ is the ADF t-statistic for country $i$, taking into account the country specific ADF regression, given by (D1). The $\bar{t}$ statistic has been shown to be normally distributed under $H_0$. Table D1 reports the outcome for the global sample of this test.

As we can see, each variable is integrated of order one. Once the order of stationary has been defined, we estimated a country risk equation on the basis of cross-country panel data. In particular, we focus on three estimation methods which are consistent when both $T$ and $N$ are large. At one extreme, the usual practice is either to estimate $N$ separate regressions

Table D1: Im et al. (2003) panel unit root test outcome: 1979-2010

| | Levels | | Logs | |
|---|---|---|---|---|
| | $t$-statistic | $P$-value | $t$-statistic | $P$-value |
| Inst. Inv. | 0.280 | 0.610 | 0.293 | 0.615 |
| Δ Inst. Inv. | -11.629 | 0.000 | -11.645 | 0.000 |
| Oil GDP | 5.286 | 1.000 | 0.680 | 0.752 |
| Δ Oil GDP | -11.972 | 0.000 | -13.776 | 0.000 |
| Non-oil GDP | 14.801 | 1.000 | 2.247 | 0.988 |
| Δ Non-oil GDP | -7.413 | 0.000 | -10.345 | 0.000 |
| Oil Reserves | 4.376 | 1.000 | 2.404 | 0.992 |
| Δ Oil Reserves | -13.954 | 0.000 | -14.352 | 0.000 |
| Ext. pub. debt to GDP | 1.113 | 0.867 | 3.727 | 1.000 |
| Δ Ext. pub. debt to GDP | -12.196 | 0.000 | -11.045 | 0.000 |
| NFA | 0.117 | 0.546 | . | . |
| Δ NFA | -9.364 | 0.000 | . | . |

Note: When computing NFA outcome, we excluded Iraq because of data limitations.

and compute the mean of the estimated coefficients across countries, which is called the Mean Group (MG) estimator. Pesaran & Smith (1995) show that the MG estimator will produce consistent estimates of the average of the parameters, but ignores the fact that certain parameters are the same across countries.

At the other extreme are the traditional pooled estimators (such as dynamic fixed effects estimators), where the intercepts are allow to differ across countries while all other coefficients and error variances are constrained to be the same. In this case, the model controls for all time-invariant differences between countries, so the estimated coefficient cannot be biased because of omitted time-invariant characteristics. An intermediate technique is the Pooled Mean Group (PMG) estimator, proposed by Pesaran et al. (1999), which relies on a combination of pooling and averaging of coefficients, allowing the intercepts, short-run coefficients and error variances to differ freely across countries, but the long-run coefficients are constrained to be the same.

Therefore, for the implementation of these methods we consider the following model:

$$III_{it} = \theta_{0i} + \theta_{1i}OilGDP_{it} + \theta_{2i}NonOilGDP_{it} + \theta_{3i}OilR_{it} + \theta_{4i}X_{it} + \theta_{5i}Default_{it} + \mu_i + \epsilon_{it} \quad \text{(D3)}$$

Again, each observation is subscripted for the country $i$ and the year $t$. In this case, $X \in \{ExtPubD, OilDisc, NFA\}$. The variable $III$ is the log of Institutional Investor's country credit ratings, $OilGDP$ is the log of oil GDP, $NonOilGDP$ is the log of non-oil GDP, $OilR$ is the log of oil reserves stock, $ExtPubD$ is the external public debt to GDP ratio, $OilDisc$ is the log of oil discoveries, $NFA$ corresponds to net foreign assets to GDP ratio, and $Default$ is a dummy variable that the country is in default. Additionally, $\mu_i$ is a set of country fixed effects (such as geographical or institutional factors) and $\epsilon_{it}$ is the idiosyncratic error term.

Now, with a maximum lag of one for all variables except $Default$, we construct the autorregresive distributive lag (ARDL) (1,1,1,1,1,1,0) dynamic panel specification of (D3):

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i}OilGDP_{it} + \delta_{11i}OilGDP_{i,t-1} + \delta_{20i}NonOilGDP_{it} +$$
$$\delta_{21i}NonOilGDP_{i,t-1} + \delta_{30i}OilR_{it} + \delta_{31i}OilR_{i,t-1} + \delta_{40i}X_{it} + \delta_{41i}X_{i,t-1} + \theta_{5i}Default_{it} + \mu_i + \epsilon_{it} \quad \text{(D4)}$$

Then, the error correction equation of (D4) is:

$$\Delta III_{it} = \phi_i \left( III_{i,t-1} - \hat{\theta_{0i}} - \hat{\theta_{1i}} OilGDP_{it} - \hat{\theta_{2i}} NonOilGDP_{it} - \hat{\theta_{3i}} OilR_{it} - \hat{\theta_{4i}} X_{it} - \hat{\theta}_{5i} Default_{it} \right) - \delta_{11i} \Delta OilGDP_{it} - \delta_{21i} \Delta NonOilGDP_{it} - \delta_{31i} \Delta OilR_{it} - \delta_{41i} \Delta X_{it} + \epsilon_{it} \quad \text{(D5)}$$

where

$$\hat{\theta_{0i}} = \frac{\mu_i}{1-\lambda_i}; \hat{\theta_{1i}} = \frac{\delta_{10i}+\delta_{11i}}{1-\lambda_i}; \hat{\theta_{2i}} = \frac{\delta_{20i}+\delta_{21i}}{1-\lambda_i}$$
$$\hat{\theta_{3i}} = \frac{\delta_{30i}+\delta_{31i}}{1-\lambda_i}; \hat{\theta_{4i}} = \frac{\delta_{40i}+\delta_{41i}}{1-\lambda_i}; \hat{\theta_{5i}} = \frac{\theta_{5i}}{1-\lambda_i}; \phi_i = -(1-\lambda_i)$$

In this case, $\phi_i$ is the error correction speed of adjustment parameter, and we would expect $\phi_i$ to be negative if the variables exhibit a return to long-run equilibrium[1].

[1]Replacing $\hat{\theta}_i$-parameters and $\phi_i$ in equation (D3) we get:

$$\Delta III_{it} = -(1-\lambda_i)\Big( III_{i,t-1} - \frac{\mu_i}{1-\lambda_i} - \frac{\delta_{10i}+\delta_{11i}}{1-\lambda_i} OilGDP_{it} - \frac{\delta_{20i}+\delta_{21i}}{1-\lambda_i} NonOilGDP_{it} - \frac{\delta_{30i}+\delta_{31i}}{1-\lambda_i} OilR_{it} - \frac{\delta_{40i}+\delta_{41i}}{1-\lambda_i} X_{it} - \frac{\theta_{5i}}{1-\lambda_i} Default_{it} \Big) - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}$$

Removing similar terms, the above expression is as follows:

$$\Delta III_{it} = -(1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} + (\delta_{40i}+\delta_{41i}) X_{it} + \theta_{5i} Default_{it} - \delta_{11i}\Delta OilGDP_{it} - \delta_{21i}\Delta NonOilGDP_{it} - \delta_{31i}\Delta OilR_{it} - \delta_{41i}\Delta X_{it} + \epsilon_{it}$$

Rewriting:

$$\begin{aligned} III_{it} - III_{i,t-1} = &- (1-\lambda_i) III_{i,t-1} + \mu_i + (\delta_{10i}+\delta_{11i}) OilGDP_{it} + (\delta_{20i}+\delta_{21i}) NonOilGDP_{it} + (\delta_{30i}+\delta_{31i}) OilR_{it} \\ &+ (\delta_{40i}+\delta_{41i}) X_{it} - \delta_{11i}(OilGDP_{it} - OilGDP_{i,t-1}) - \delta_{21i}(NonOilGDP_{it} - NonOilGDP_{i,t-1}) \\ &- \delta_{31i}(OilR_{it} - OilR_{i,t-1}) - \delta_{41i}(X_{it} - X_{i,t-1}) + \theta_{5i} Default_{it} + \epsilon_{it} \end{aligned}$$

Again, simplifying this equality we obtain:

$$III_{it} = \lambda_i III_{i,t-1} + \delta_{10i} OilGDP_{it} + \delta_{11i} OilGDP_{i,t-1} + \delta_{20i} NonOilGDP_{it} + \delta_{21i} NonOilGDP_{i,t-1} + \delta_{30i} OilR_{it} + \delta_{31i} OilR_{i,t-1} + \delta_{40i} X_{it} + \delta_{41i} X_{i,t-1} + \theta_{5i} Default_{it} + \mu_i + \epsilon_{it}$$

Note that this expression is equivalent to (D4). For a long-run relationship to exist, we require that $\phi \neq 0$.

Table D2: Hausman test outcome: 1979-2010

| | Model (1) | | Model (2) | | Model (3) | |
|---|---|---|---|---|---|---|
| | $\chi^2$-stat | P-value | $\chi^2$-stat | P-value | $\chi^2$-stat | P-value |
| MG vs. DFE | 0.02 | 1.000 | 0.01 | 1.000 | 0.06 | 1.000 |
| PMG vs. DFE | 0.03 | 1.000 | 0.03 | 1.000 | 0.03 | 1.000 |
| MG vs. PMG | 4.42 | 0.491 | 5.05 | 0.537 | 8.99 | 0.174 |

### D.1 Estimation results

In this subsection we estimate the PMG, MG and DFE estimators for model (D5). In order to obtain reliable estimators and seeking to maintain a large data sample, we include information for China, India, and Brazil since these countries have large proven oil reserves, although these have not been oil net exporters in the time interval considered here. When deciding about model selection, we apply the Hausman test to see whether there are significant differences among these three estimators. The null of this test is that the difference between DFE and MG, DFE and PMG or PMG and MG is not significant. Consider, for example, the test between DFE and PMG. If the null is not rejected, the DFE estimator is recommended since it is efficient. The alternative is that there is a significant difference between PMG and DFE, and the null is rejected. Specifically, the Hausman statistic is:

$$H = \left(\beta_{DFE} - \beta_{PMG}\right)' \left[\text{var}(\beta_{DFE}) - \text{var}(\beta_{PMG})\right]^{-1} \left(\beta_{DFE} - \beta_{PMG}\right) \sim \chi^2$$

where $\beta_j$ is the vector of coefficients and $\text{var}(\beta_j)$ is the covariance matrix of $\beta_j$, estimated using the $j-$technique, for $j$ =DFE, PMG. Under the null hypothesis, $H$ has asymptotically the $\chi^2$ distribution. Table D2 reports the results of Hausman test, in which Model (1) corresponds to equation (D5), excluding $NFA$ from $X_i$, while Model (2) excludes $Default$. Model (3) includes all variables in $X_i$ into the regressors.

Under the current specification, the hypothesis that the country risk equation (equation (D5)) is adequately modeled by a PMG or MG model is resoundingly rejected. In general, when considering Model (1) the results in table D2 suggest that it is not possible to reject the null hypothesis of the homogeneity restriction on regressors (in the short and long run), since P-values are both 1, which indicates that DFE is more efficient estimator than MG and

PMG, respectively. Notice that this conclusion holds for Model (2) and Model (3), because P-values associated to these tests are 1. Because of this, we choose to employ the DFE estimator.

The results for the unbalanced panel are found in Section 2 of the paper, and for robustness purposes Table D3 shows the results for the balanced panel.

Table D3: BALANCED PANEL- Dynamic Fixed Effects Regression Results for Institutional Investor Index

| | Δ Inst. Investor Index | | |
|---|---|---|---|
| | Model (1) | Model (2) | Model (3) |
| **Convergence Coefficient** | | | |
| Inst. Investor Index (-1) | -0.233*** | -0.234*** | -0.236*** |
| | (0.032) | (0.032) | (0.032) |
| **Short-Run Coefficients** | | | |
| Δ Oil Production | 0.012 | 0.011 | 0.019 |
| | (0.032) | (0.031) | (0.031) |
| Δ Non-Oil GDP | 0.112 | 0.071 | 0.061 |
| | (0.075) | (0.076) | (0.075) |
| Δ Oil Reserves | 0.028 | 0.027 | 0.025 |
| | (0.024) | (0.023) | (0.023) |
| Δ Ext. pub. debt to GDP | -0.082 | -0.220** | -0.238*** |
| | (0.073) | (0.091) | (0.090) |
| Δ Oil Discoveries | -0.003 | -0.002 | -0.002 |
| | (0.004) | (0.004) | (0.004) |
| Δ NFA | | -0.193*** | -0.187*** |
| | | (0.056) | (0.055) |
| **Long-Run Coefficients** | | | |
| Oil Production | 0.217** | 0.256*** | 0.250*** |
| | (0.085) | (0.085) | (0.083) |
| Non-Oil GDP | 0.529*** | 0.480*** | 0.495*** |
| | (0.174) | (0.172) | (0.169) |
| Oil Reserves | -0.217*** | -0.195*** | -0.196*** |
| | (0.060) | (0.061) | (0.060) |
| Ext. pub. debt to GDP | -1.024*** | -1.436*** | -1.205*** |
| | (0.217) | (0.275) | (0.274) |
| Default | -0.225** | | -0.213** |
| | (0.091) | | (0.088) |
| Oil Discoveries | 0.026 | 0.021 | 0.022 |
| | (0.024) | (0.024) | (0.023) |
| NFA | | -0.067 | -0.082 |
| | | (0.148) | (0.146) |
| Constant | -2.577** | -2.219* | -2.355** |
| | (1.218) | (1.205) | (1.194) |

Standard errors in parentheses
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

## E Oil Price Upswings and Downswings

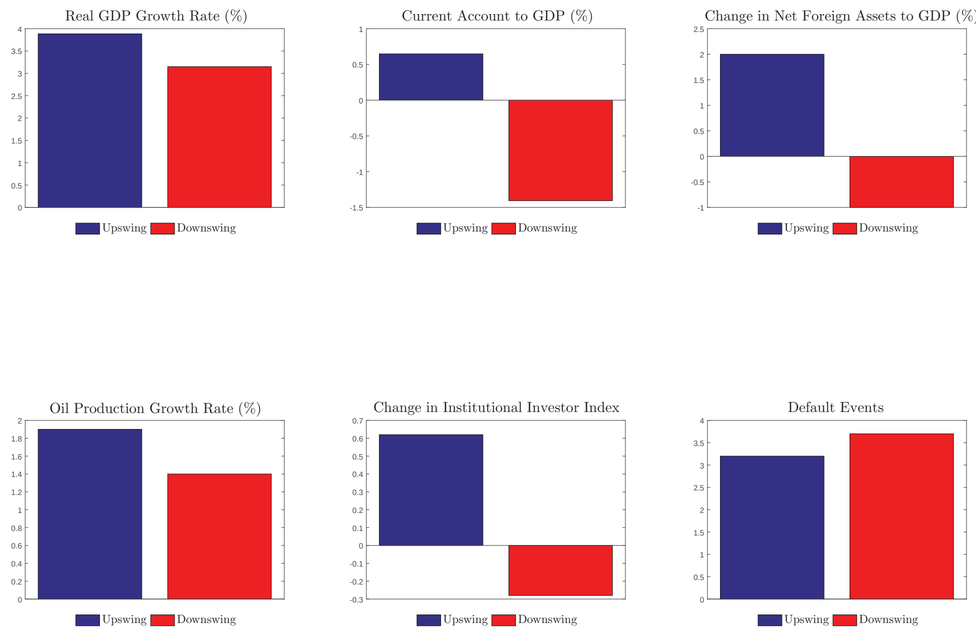
Table E1: Oil Price Upswings and Downswings

| Downswings | | Upswings | |
|---|---|---|---|
| Period | Number of Months | Period | Number of Months |
| NOV 75 - OCT 78 | 36 | NOV 78 - JAN 81 | 27 |
| FEB 81 - JUL 86 | 66 | AUG 86 - JUL 87 | 12 |
| AUG 87 - NOV 88 | 16 | DEC 88 - OCT 90 | 23 |
| NOV 90 - DEC 93 | 38 | JAN 94 - OCT 96 | 34 |
| NOV 96 - DEC 98 | 26 | JAN 99 - SEP 00 | 21 |
| OCT 00 - DEC 01 | 15 | JAN 02 - JUL 08 | 79 |
| AUG 08 - MAY 10 | 22 | | |
| TOTAL | 219 | TOTAL | 196 |

Figure E1 provides a complementary view of the association between oil-price movements and macroeconomic fluctuations to that provided by business cyce moments. It shows the differential performance of macro variables across oil price upswings and downswings. To construct this figure we divided our panel dataset into two groups, one for all years in which oil prices rose (oil-price upswings) and one for all years in which they fell (oil-price downswings). Table E1 shows how each year in the time-series corresponds to a downswing or an upswing. We then averaged the changes in the different macroeconomic variables over the upswings and downswings and provide in Figure E1 plots of the average change in each macro variable over the upswings and downswings of oil prices.

Oil price upswings are associated with higher growth in GDP and oil production, trade balance improvement, and lower sovereign risk (higher III). Likewise, oil price downswings are associated with lower growth in oil extraction and GDP, trade balance deterioration, and higher sovereign risk.

Figure E1: Oil Price Swings and Macro Performance




## F Are all Oil Exporting Countries Price Takers?

This appendix examines whether the countries in our sample are price takers in the world market of oil.[2] We examine causality between a country's extraction and oil prices using two strategies, both in a bivariate context. First, we test on the levels, using a modified version of the Granger causality test proposed by Toda & Yamamoto (1995). Second, we test causality using the Granger test on the first differences of both series.

For the causality test a modified Wald test (MWALD) is used as proposed by Toda & Yamamoto (1995) that avoids the problems associated with the ordinary Granger causality test by ignoring any possible non-stationary or cointegration between series when testing for causality. [3]The Toda & Yamamoto (1995) approach fits a standard vector autoregressive

[2]We are grateful to Norberto Rodriguez-NiÒo from the Banco de la República de Colombia for his assistance with this analysis.

[3]As quoted from Wolde-Rufael (2005) "... given that unit root and cointegration tests have low power against

model in the levels of the variables (rather than the first differences, as the case with Granger causality tests) thereby minimizing the risks associated with the possibility of wrongly identifying the order of integration of the series.

The basic idea of this approach is to artificially augment our bivariate VAR or order k, by the maximal order of integration, one in this case. Once this is done, a (k+1)-th order of VAR is estimated and the coefficients of the last one lagged vector is ignored. The application of the Toda & Yamamoto (1995) procedure ensures that the usual Wald test statistic for Granger causality has the standard asymptotic distribution hence valid inference can be done.

Lag length for VAR are chosen based on information criteria (Akaike, Schwarz and Hanna-Quinn), when there is not agreement between those indicators, pormanteau (bivariate Lung-Box statistic) test is used to decide. This statistics joint with its P-values are contained and third and four columns of tables F1 and F2.

### F.1 Data

We used monthly data of crude oil for the 20 major exporting countries; the sample period cover from January 2002 to November 2016. The data source is Joint Oil Data Initiative (JODI) Database (available at http://www.jodidb.org/TableViewer/tableView.aspx). For Colombia, the figures have source Banco de la Rep˙blica and are based on DIAN-DANE. Units are thousand barrels per period. Exports the top 20 countries accounted for approximately 96% of reported crude oil exports at the JODI base in 2015.

### F.2 Results

Unit root test results (not presented here but available up to request) show that all the variables are integrated of order one.

Table F1 shows the results for the TY test. It is worth to remain that the null hypothesis in this as next table is that of non-causality. Table F2 presents results for Granger causality test, for the series in differences. Results in both tables coincide signaling oil exports from

the alternative, these tests can be misplaced and can suffer from pre-testing bias (see Pesaran et al. (2001); Toda & Yamamoto (1995)). Moreover, as demonstrated by Toda & Yamamoto (1995), the conventional F-statistic used to test for Granger causality may not be valid as the test does not have a standard distribution when the time series data are integrated or cointegrated."

United Arab Emirates, Oman, Brazil and Azerbaijan causing (in Granger sense) oil prices. TY shows that exports from Canada also G-cause prices, and model in differences indicated that Kuwait G-cause oil prices.

Table F1: Taro-Yamamoto test results for series in levels

| Country | Lag | Lung-Box | | Jarque-Bera | | Taro-Yamamoto | | |
|---|---|---|---|---|---|---|---|---|
| | | Q-Stat | P-Value | Stat | P-Value | Statistic | P-Value | Decision |
| Saudi Arabia | 2 | 26.75 | 0.32 | 164.12 | 0.00 | 1.47 | 0.48 | |
| Russia | 2 | 30.14 | 0.18 | 75.23 | 0.00 | 1.90 | 0.39 | |
| Iraq | 2 | 28.48 | 0.24 | 50.22 | 0.00 | 1.28 | 0.53 | |
| U. Arab Emir. | 2 | 29.43 | 0.20 | 31.25 | 0.00 | 17.32 | 0.00 | Cause |
| Canada | 2 | 26.31 | 0.34 | 70.50 | 0.00 | 7.30 | 0.03 | Cause |
| Nigeria | 2 | 17.33 | 0.83 | 13.42 | 0.01 | 0.99 | 0.61 | |
| Kuwait | 2 | 21.64 | 0.60 | 23.36 | 0.00 | 1.20 | 0.55 | |
| Angola | 4 | 23.88 | 0.09 | 17.30 | 0.00 | 7.86 | 0.10 | |
| Venezuela | 2 | 23.61 | 0.48 | 46.25 | 0.00 | 5.17 | 0.08 | |
| Iran | 2 | 27.83 | 0.27 | 66.94 | 0.00 | 5.00 | 0.08 | |
| Mexico | 2 | 21.95 | 0.58 | 14.50 | 0.01 | 4.19 | 0.12 | |
| Norway | 3 | 18.43 | 0.56 | 6.47 | 0.17 | 3.45 | 0.33 | |
| Oman | 2 | 18.92 | 0.76 | 4320.42 | 0.00 | 9.10 | 0.01 | Cause |
| Brasil | 7 | 3.17 | 0.53 | 24.80 | 0.00 | 16.69 | 0.02 | Cause |
| Azerbaijan | 2 | 20.98 | 0.64 | 1171.78 | 0.00 | 13.11 | 0.00 | Cause |
| Uni. Kingdom | 2 | 28.88 | 0.22 | 22.93 | 0.00 | 0.10 | 0.95 | |
| Algeria | 2 | 29.15 | 0.21 | 12.62 | 0.01 | 4.89 | 0.09 | |
| Qatar | 2 | 20.04 | 0.69 | 127.50 | 0.00 | 1.84 | 0.40 | |
| USA | 3 | 21.69 | 0.36 | 332.23 | 0.00 | 1.19 | 0.76 | |
| Colombia | 3 | 25.90 | 0.17 | 13.44 | 0.01 | 0.81 | 0.85 | |

Table F2: Granger tets results for series in diferences

| Country | Lag | Lung-Box | | Jarque-Bera | | Taro-Yamamoto | | |
|---|---|---|---|---|---|---|---|---|
| | | Q-Stat | P-Value | Stat | P-Value | Statistic | P-Value | Decision |
| Saudi Arabia | 7 | 6.69 | 0.15 | 49.06 | 0.00 | 10.77 | 0.15 | |
| Russia | 6 | 14.45 | 0.07 | 132.17 | 0.00 | 7.56 | 0.27 | |
| Iraq | 2 | 34.37 | 0.08 | 6.76 | 0.15 | 3.41 | 0.18 | |
| U. Arab Emir | 6 | 9.38 | 0.31 | 71.06 | 0.00 | 18.78 | 0.00 | Cause |
| Canada | 6 | 12.82 | 0.12 | 5.86 | 0.21 | 7.58 | 0.27 | |
| Nigeria | 1 | 37.67 | 0.10 | 25.24 | 0.00 | 0.33 | 0.57 | |
| Kuwait | 6 | 6.57 | 0.58 | 14.00 | 0.01 | 13.63 | 0.03 | Cause |
| Angola | 6 | 8.01 | 0.43 | 342.62 | 0.00 | 10.84 | 0.09 | |
| Venezuela | 1 | 26.57 | 0.54 | 16.27 | 0.00 | 2.14 | 0.14 | |
| Iran | 2 | 34.39 | 0.08 | 95.29 | 0.00 | 2.96 | 0.23 | |
| Mexico | 2 | 28.31 | 0.25 | 32.99 | 0.00 | 2.65 | 0.27 | |
| Norway | 2 | 32.64 | 0.11 | 20.19 | 0.00 | 3.26 | 0.20 | |
| Oman | 6 | 10.13 | 0.26 | 13053.21 | 0.00 | 26.42 | 0.00 | Cause |
| Brazil | 7 | 8.94 | 0.06 | 265.77 | 0.00 | 18.39 | 0.01 | Cause |
| Azerbaijan | 2 | 32.15 | 0.12 | 1029.34 | 0.00 | 12.68 | 0.00 | Cause |
| Uni. Kingdom | 6 | 14.49 | 0.07 | 27.07 | 0.00 | 5.27 | 0.51 | |
| Algeria | 2 | 33.76 | 0.09 | 7.44 | 0.11 | 3.82 | 0.15 | |
| Qatar | 6 | 7.20 | 0.51 | 87.55 | 0.00 | 12.24 | 0.06 | |
| USA | 3 | 35.85 | 0.02 | 33.07 | 0.00 | 2.56 | 0.46 | |
| Colombia | 2 | 29.64 | 0.20 | 18.90 | 0.00 | 1.43 | 0.49 | |

## G Model Variants under Commitment

We analyze here three variants of the model under the assumption that the planner is committed to repay. The planner's optimization problem is characterized in a generic form that allows us to capture cases in which the planner accesses world financial markets facing with either a given bond pricing function that depends on the planner's debt and reserves) or a constant world real interest rate, and a case in which the planner operates under financial autarky. The latter coincides with the solution of the default payoff if default triggers permanent exclusion from credit markets.

The generic planner's problem in sequential form is the following:

$$\max_{c_t, x_t, b_{t+1}, s_{t+1}} E_t \sum_{t=0}^{\infty} \beta^t u\left(c_t\right) \tag{G1}$$

s.t.

$$c_t + e\left(x_t, s_t\right) = y_t + p_t x_t - q\left(s_{t+1}, b_{t+1}\right) b_{t+1} + b_t \tag{G2}$$

$$s_{t+1} = s_t - x_t + \kappa \tag{G3}$$

$$x_t \geq 0 \tag{G4}$$

$$x_t \leq s_t + \kappa. \tag{G5}$$

The first constraint is the resource constraint, where $q\left(s_{t+1}, b_{t+1}\right)$ is an ad-hoc pricing function of bonds that is assumed to be the equilibrium pricing function of the model with default and satisfies the following assumptions: $q\left(\cdot\right)$ is continuously differentiable, strictly concave and increasing in $b_{t+1}$ for $b_{t+1} \in [-\overline{b}(s_{t+1}), 0]$, where $-\overline{b}(s_{t+1})$ is the threshold debt above which default is certain for a given $s_{t+1}$ (i.e., $D(\overline{b}(s_{t+1}), s_{t+1})$ includes all $(y_{t+1}, p_{t+1})$ pairs, which exists because of Proposition 1), with $q\left(\cdot\right) = q^*$ for $b_{t+1} \geq 0$ and $q\left(\cdot\right) = 0$ for $b_{t+1} \leq \overline{b}(s_{t+1})$. $q\left(\cdot\right)$ is also increasing and concave in $s_{t+1}$ for $s_{t+1} \in [\tilde{s}(b_{t+1}), s_t + \kappa]$, where $\tilde{s}(b_{t+1}) = max[s_t + \kappa - s_t (p_t/\psi)^{(1/\gamma)}, \overline{s}(b_{t+1})]$, $s_t + \kappa - s_t(p_t/\psi)^{(1/\gamma)}$ is the minimum $s_{t+1}$ needed for profits to be non-negative, and $\overline{s}(b_{t+1})$ is the threshold oil reserves below which default is certain for a given $b_{t+1}$ (i.e., $D(b_{t+1}, \overline{s}(b_{t+1}))$ includes all $(y_{t+1}, p_{t+1})$ pairs, which exists because of Proposition 4). We also assume that $\overline{b}(s_{t+1})$ is increasing in $s_{t+1}$ and $\overline{s}(b_{t+1})$ is increasing in $b_{t+1}$. In addition, we assume shocks are i.i.d so that $q(\cdot)$ is independent of $p_t$ and $y_t$. The second constraint is the law of motion of reserves. The third and fourth constraints are the feasibility boundaries of oil extraction.

The first-order conditions are:

$$\lambda_t = u'(c_t) \tag{G6}$$

$$\lambda_t \left[p_t - e_x(x_t, s_t)\right] + \psi_t^l = \mu_t + \psi_t^u \tag{G7}$$

$$u'(c_t)\left[p_t - e_x(x_t, s_t) + q_s(s_{t+1}, b_{t+1})\, b_{t+1}\right] + \psi_t^l - \psi_t^u = \beta E_t\left[u'(c_{t+1})\left(p_{t+1} - e_x(x_{t+1}, s_{t+1}) - e_s(x_{t+1}, s_{t+1})\right) + \psi_{t+1}^l\right] \tag{G8}$$

$$u'(c_t)\left[q(s_{t+1}, b_{t+1}) + q_b(s_{t+1}, b_{t+1})\, b_{t+1}\right] = \beta E_t\left[u'(c_{t+1})\right]. \tag{G9}$$

where $\lambda_t$ is multiplier on the resource constraint, $\mu_t$ is the multiplier on the law of motion of reserves, and $\psi_t^h$ and $\psi_t^l$ are the multipliers on the upper and lower feasibility constraints on oil extraction.

Defining the planner's return on bonds as $R^b(s_{t+1}, b_{t+1}) \equiv \frac{1}{q(t+1)+q_b(t+1)b_{t+1}}$, which is decreasing in $b_{t+1}$ (i.e. the planner's real interest rate increases with debt) because of the assumed properties of $q(\cdot)$, the Euler equation for bonds (eq (G9)) implies:[4]

$$u'(c_t) = R^b(s_{t+1}, b_{t+1})\, \beta E_t\left[u'(c_{t+1})\right]. \tag{G10}$$

Notice that, as implied by the definition of $R^b$, in evaluating the marginal benefit of borrowing in the right-hand-side of this expression, the planner internalizes that borrowing more (i.e. making $b_{t+1}$ "more negative") increases the cost of borrowing.

The rate of return on oil extraction is defined as $R_{t+1}^O \equiv \frac{q_{t+1}^O + d_{t+1}^O}{q_t^O}$, where $q_t^O$ is the asset price of oil defined as $q_t^O \equiv p_t - e_x(t) + \Delta\tilde{\psi}_t$ (where $\Delta\tilde{\psi}_t \equiv \tilde{\psi}_{t+1}^l - \tilde{\psi}_{t+1}^h$ and $\tilde{\psi}_t^i = \psi_t^i / u'(t)$ for $i = h, l$) and $d_{t+1}^O$ is the dividend from oil extraction at $t+1$ defined as $d_{t+1}^O \equiv -e_s(t+1) + \tilde{\psi}_{t+1}^h$. Notice that $d_{t+1}^O > 0$ because $e_s(t+1) < 0$ and $\tilde{\psi}_{t+1}^h \geq 0$. The Euler equation for oil reserves (eq. (G8)) can then be rewritten as:

$$u'(c_t)\left[1 + \frac{q_s(s_{t+1}, b_{t+1})\, b_{t+1}}{q_t^O}\right] = \beta E_t\left[u'(c_{t+1})\, R_{t+1}^O\right]. \tag{G11}$$

The left-hand-side of this expression shows that in evaluating the marginal cost of accumulating additional reserves, the planner internalizes the fact that higher $s_{t+1}$ increases the price of bonds, so that if it plans to issue debt ($b_{t+1} < 0$), the higher price at which it can

[4]The derivative of $R^b(\cdot)$ w.r.t. $b_{t+1}$ is $R_b^b(\cdot) = \frac{-\left(2q_b(\cdot)+q_{bb}(\cdot)b_{t+1}\right)}{\left(q(\cdot)+q_b(\cdot)b_{t+1}\right)^2}$, and the properties that $q(s_{t+1}, b_{t+1}) = q^*$ for $b_{t+1} \geq 0$ and $q(s_{t+1}, b_{t+1})$ is strictly concave and increasing in $b_{t+1}$ for $b_{t+1} \in [-\bar{b}(s_{t+1}), 0]$ imply that $-\left(2q_b(\cdot) + q_{bb}(\cdot)b_{t+1}\right) > 0$ and hence $R_b^b(\cdot) < 0$ in that same interval.

be sold reduces the marginal cost of building reserves. Hence, we can also express the Euler equation of reserves redefining the rate of return on oil to impute this extra gain:

$$u'(c_t) = \beta E_t \left[ u'(c_{t+1}) \tilde{R}^O_{t+1} \right], \tag{G12}$$

where $\tilde{R}^O_{t+1} \equiv \frac{q^O_{t+1} + d^O_{t+1}}{\left[q^O_t + q_s(s_{t+1}, b_{t+1}) b_{t+1}\right]}$ is the rate of return on oil inclusive of the benefit of higher reserves increasing the price at which newly-issued debt is sold.

The above Euler equation can be used to solve forward for the asset price of oil. To this end, rewrite the equation as follows:

$$q^O_t + z_t = E_t \left[ \frac{\beta u'(c_{t+1})}{u'(c_t)} \left( q^O_{t+1} + d^O_{t+1} \right) \right] \tag{G13}$$

where $z_t \equiv q_s(t) b_{t+1}$ and $q_s(t)$ is the derivative with respect to reserves of the price of bonds sold at date $t$, which is a function of $(b_{t+1}, s_{t+1})$. Notice $z_t \leq 0$ because $q_s(\cdot) > 0$ for $b_{t+1} < 0$ and otherwise $q_s(\cdot) = 0$. Adding and subtracting $z_{t+1}$ to $q^O_{t+1}$ in the right-hand-side of this equation and solving forward yields:

$$q^O_t + z_t = E_t \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} [d^O_s - z_s] \right] > 0 \tag{G14}$$

The expression in the right-hand-side is positive because marginal utility is positive, $d^0_s > 0$ and $z_s \leq 0$. It follows then that $q^O_t + z_t > 0$, and since $z_s \leq 0$ we obtain $q^O_t > -z_t \geq 0$. Thus, the asset price of oil equals the expected present discounted value (discounted with the planner's stochastic discount factors) of the revenue stream composed of oil dividends plus the marginal revenue of selling bonds at a higher price when reserves increase. Or, the asset price of oil with this marginal revenue imputed, $\tilde{q}^0_t$ equals the expected present discounted value of the stream of oil dividends with the stream of these marginal revenues included $\tilde{q}^O_t = E_t \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} \tilde{d}^O_s \right]$, where $\tilde{d}^O_s \equiv d^O_s - z_s$.

Combining the Euler equations for bonds and reserves yields the following expression for the excess return on oil (the oil risk premium):

$$E_t \left[ R^o_{t+1} \right] - R^b_{t+1}(s_{t+1}, b_{t+1}) \left[ 1 + \frac{q_s(t+1) b_{t+1}}{q^O_t} \right] = -\frac{\text{cov}_t \left( u'(c_{t+1}), R^o_{t+1} \right)}{E_t \left[ u'(c_{t+1}) \right]}. \tag{G15}$$

The left-hand-side is the excess return relative to the yield on bonds inclusive of the effect of higher reserves on the resources generated by borrowing. Defined in this way, the excess return takes the standard form of an equity premium determined by the covariance of the planner's marginal utility and the rate of return on oil. Defining the return on oil with the effect of higher reserves increasing bond prices imputed, the excess return is:

$$E_t\left[\tilde{R}^o_{t+1}\right] - R^b_{t+1}\left(s_{t+1}, b_{t+1}\right) = -\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), \tilde{R}^o_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]}. \tag{G16}$$

We explore next three cases of this generic setup. First, a case in which the economy is in permanent financial autarky but can export oil. Second, a small-open-economy case in which the economy has access to a world credit market at a constant, exogenous price of bonds $q^*$, which is akin to an RBC model with oil extraction. Third, a case in which the planner faces the exogenous bond pricing function $q(b_{t+1}, s_{t+1})$. In each instance we discuss results with and without uncertainty.

### G.1 Financial Autarky

Consider first the case in which the economy is in financial autarky and there is no uncertainty. The Euler equation of reserves implies:

$$\frac{q^o_{t+1} + d^o_{t+1}}{q^o_t} = \frac{u'\left(c_t\right)}{\beta u'\left(c_{t+1}\right)}. \tag{G17}$$

In turn, solving forward this condition yields a standard asset-pricing condition by which the asset price of oil equals the present discounted value of oil dividends discounted with the intertemporal discount factors:

$$q^O_t = \sum_{s=t+1}^{\infty} \beta^{s-t}\frac{u'(s)}{u'(t)} d^O_s \tag{G18}$$

Note that since $d^0_s > 0$ and $u'(s), u'(t) > 0$, it follows that $q^O_t > 0$.

In this case, the optimal extraction and reserves plans equate $R^o_t$ with the endogenous domestic real interest rate represented by the intertemporal marginal rate of substitution, each represented by the left- and right-hand-side of the reserves Euler equation, respectively. Oil extraction and reserves are used to smooth consumption.

The deterministic steady state is characterized by these two conditions:

$$\beta\left(q^{Oss} + d^{Oss}\right) = q^{Oss} \Rightarrow \frac{d^{Oss}}{q^{Oss}} = \rho,$$

$$x^{ss} = \kappa,$$

where $\rho$ is the rate of time preference. Using the definitions of $d^O$ and $q^O$ and assuming an internal solution for extraction yields the following steady-state equilibrium condition:

$$-e_s(ss) = \rho \left[ p^{ss} - e_x(ss) \right].$$

Using the functional form for extraction costs, $e = \psi \left(\frac{x_t}{s_t}\right)^\gamma x_t$, the above condition becomes:

$$\gamma\psi \left(\frac{\kappa}{s}\right)^{1+\gamma} = \rho \left[ p^{ss} - (1+\gamma)\psi \left(\frac{\kappa}{s}\right)^\gamma \right]$$

which can be rewritten as:

$$\psi \left(\frac{\kappa}{s}\right)^\gamma \left[ \gamma \left(\frac{\kappa}{s}\right) + \rho(1+\gamma) \right] = \rho p^{ss}. \tag{G19}$$

The steady state oil reserves $s^{ss}$ is the value of $s$ that solves the above equation. Since the left-hand-side is a decreasing, convex function of $s$, the condition determines a unique value of $s^{ss}$ that rises as $p^{ss}$ falls. Hence, a permanent decline in oil prices causes a permanent increase in oil reserves.

In the stochastic version of this setup, the planner uses oil reserves for self insurance, since there are no state-contingent claims to hedge oil-price shocks and no credit market of non-state-contingent international bonds. The Euler equation becomes: $u'(c_t) = \beta E_t \left[ R^O_{t+1} u'(c_{t+1}) \right]$. The asset price of oil is still positive and given by $q^O_t = E_t \left[ \sum_{s=t+1}^{\infty} \beta^{s-t} \frac{u'(s)}{u'(t)} d^O_s \right]$. Because of self insurance, the long-run average of reserves in this economy will be larger than $s^{ss}$ (i.e., the planner builds a buffer stock of precautionary savings in the form of oil reserves).

In Appendix I, we present the recursive formulation of this financial autarky setup and derive key properties of the associated dynamic programming problem. In particular, we show that non-negativity of oil profits and a coefficient $\psi$ in the extraction cost function larger than the largest realization of $p$ guarantee that the decision rule on reserves $s'(s, p, y)$ is increasing is $s$ and that the lower bound on $s_{t+1}$ (i.e., the upper bound on $x_t$) is never binding.

## G.2 Exogenous $q$

Consider next the small-open-economy case with a constant, world-determined real interest rate such that $R^b(s_{t+1}, b_{t+1}) = R^*$. Without uncertainty, the Euler equations for bonds and reserves yield the following no-arbitrage condition for the real returns on bonds and oil:

$$R^o_{t+1} = \frac{u'(c_t)}{\beta u'(c_{t+1})} = R^*. \tag{G20}$$

Using the law of motion of reserves and the definitions of the asset price of oil and oil dividends, this no-arbitrage condition yields the following condition (assuming an internal solution for $x_t$ for simplicity):

$$\frac{p_{t+1} - e_x\left(s_{t+1} - s_{t+2}, s_{t+1}\right) - e_s\left(s_{t+1} - s_{t+2}, s_{t+1}\right)}{p_t - e_x\left(s_t - s_{t+1}, s_t\right)} = R^*. \tag{G21}$$

This is a second-order difference equation in $s$ that pins down the optimal decisions for $\{x_t, s_{t+1}\}_{t=0}^{\infty}$ as functions of oil prices and reserves only (and the parameter values of the extraction cost function and $R^*$). Hence, this setup is akin to the deterministic small-open-economy model with capital accumulation in which there is "Fisherian separation" of the investment and production decisions from the consumption and savings plans. Here, the same happens with the optimal plans for oil extraction and accumulation of oil reserves: they are determined independently of those for consumption and debt.

Assuming $\beta R^* = 1$, consumption is perfectly smooth for all $t$, while reserves and extraction follow the dynamics governed by the above second-order difference equation. The sovereign adjusts bond holdings as necessary so that consumption is perfectly smooth while extraction follows its transitional dynamics towards its steady state. This determines the present value of oil income net of extraction costs, and given that the perfectly smooth level of consumption is determined so as to satisfy the intertemporal resource constraint (i.e. the present value of constant consumption equals the present value of oil plus non-oil GDP plus initial bond holdings).

Since $e(\cdot)$ is increasing in $x_t$ and decreasing in $s_t$, the above condition implies that, when $p_{t+1}$ rises relative to $p_t$, the planner reallocates extraction from $t$ to $t+1$ (i.e. increases the accumulation of reserves at $t$). This is a key incentive that is also a work in the model with default risk, but there it interacts with the planner's incentives to default and to affect the price of issuing new debt by adjusting reserves. As we demonstrate in Appendix G, default incentives strengthen when oil prices are low and the set of pairs of income and oil prices at which default is preferable shrinks as reserves grow.

This model's deterministic steady state is analogous to the one of the financial autarky case, except that the net world real interest rate $r^* = R^* - 1$ replaces the rate of time preference. Hence, the condition pinning down the deterministic steady state of reserves becomes:

$$\psi\left(\frac{\kappa}{s}\right)^{\gamma}\left[\gamma\left(\frac{\kappa}{s}\right) + r^*(1+\gamma)\right] = r^* p^{ss}.$$

As in the case of financial autarky, there is a unique deterministic steady state for $s^{ss}$ and it increases as the steady-state price of oil falls.

The stochastic version of the model yields a standard equity-premium expression for the excess return on oil:

$$E_t\left[R^O_{t+1}\right] - R^*_{t+1} = -\frac{\text{cov}_t\left(u'\left(c_{t+1}\right), R^O_{t+1}\right)}{E_t\left[u'\left(c_{t+1}\right)\right]},$$

This is also analogous to the expression that a standard small-open-economy RBC model would yield. Bonds are ued for self-insurance (i.e., borrowing incentives are weakened by the precautionary savings motive) and extraction and reserves play the role of investment and capital. The asset price of oil is again positive and is now given by $q^O_t = E_t\left[\sum_{s=t+1}^{\infty}(R^*)^{-(s-t)}d^O_s\right]$. Fisherian separation does not hold strictly, because the excess return on oil depends on the marginal utility of consumption, but it holds approximately because equity premia in this class of models are small (as is typical of standard consumption asset pricing models). Hence, the asset price of oil is approximately independent of consumption and savings decisions.

### G.3 Endogenous $q$

The third case takes into account the ad-hoc bond pricing function. Without uncertainty, the Euler equations for bonds and reserves (eqs. (G10) and (G11)) imply the following no-arbitrage condition:

$$R^O_{t+1} = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right)\left[1 + \frac{q_s\left(s_{t+1}, b_{t+1}\right)b_{t+1}}{q^O_t}\right]. \tag{G22}$$

Using the alternative definition of the returns on oil that imputes the effect of reserves on bond prices, and since the planner arbitrages returns on bonds and oils against the intertemporal marginal rate of substitution, we obtain that:

$$\tilde{R}^O_{t+1}\left(s_{t+1}, b_{t+1}\right) = \frac{u'\left(c_t\right)}{\beta u'\left(c_{t+1}\right)} = R^b_{t+1}\left(s_{t+1}, b_{t+1}\right). \tag{G23}$$

It follows from these conditions that this model's deterministic steady state is pinned down by a two-equation nonlinear system in $\left(b^{ss}, s^{ss}\right)$ formed by $\tilde{R}^O\left(s^{ss}, b^{ss}\right) = 1/\beta$ and $R^b\left(s^{ss}, b^{ss}\right) = 1/\beta$. The asset price of oil is still positive in this economy, and is simply determined by the deterministic version of eq. (G14).

The conditions that characterize the equilibrium of this economy under uncertainty are the ones provided in the generic characterization of the setup. Equations (G10), (G11), (G14) and (G15) are, respectively, the Euler equations for bonds and reserves, the oil asset-pricing equation and the oil risk premium. This economy is akin to the RBC-like case where there is no default risk, except that in this case the interest rate rises as bonds and/or reserves fall, whereas in the RBC case it remains constant. It also differs in that the planner chooses bonds and reserves internalizing how those choices affect the price of bonds and thus the cost of borrowing, but all of this is done under commitment to repay. Intuitively, it is as if the government acts as a monopolist when it sells its debt.

## H Theoretical Results on Debt, Reserves & Country Risk

This Section of the Appendix derives theoretical results about how country risk and default incentives are affected by the debt position, oil reserves and the realizations of non-oil GDP and oil prices. These results show the extent to which existing results from the sovereign default literature extend to the model we proposed, and provide insights about how oil reserves and oil prices interact with country risk and default incentives. Extending the analysis of standard default models is not straightforward, because in those models the default payoff is exogenous to the sovereign's actions, whereas in our model it depends on the sovereign's optimal plans for oil reserves. As we explain below, this is particularly important for deriving results related to how default sets respond to oil reserves, what contracts are feasible under repayment when default is possible, and how shocks to $y$ and $p$ affect default incentives.

Since some of the propositions rely on conjectures, impose parameter restrictions (i.i.d shocks, $\lambda = 0$, $\hat{p} = p$), and provide only sufficiency conditions, we evaluated numerically both the conjectures and the propositions in the calibrated model. As reported in Table H1, all the propositions and conjectures hold in 100 percent of the possible model evaluations that apply to each, except for Conjecture 2 which holds in 98 percent of the corresponding evaluations.

Table H1: Validation of Propositions and Conjectures in the Baseline Model

| **Conjecture or Proposition** | **Case** | **Holds in %** | **Max. Error** |
|---|---|---|---|
| **Conjecture 1*** | Repayment | 100 | |
| | Default | 100 | |
| **Conjecture 2** | | 98 | $\tilde{c}^{nd}\left(b, s^2, p, y\right) - \tilde{c}^{nd}\left(b, s^1, p, y\right) = -0.2$ |
| **Conjecture 3** | | 100 | |
| **Proposition 1** | | 100 | |
| **Proposition 2** | $s$ | 100 | |
| | $s'$ | 100 | |
| **Proposition 3** | Repayment | 100 | |
| | Default | 100 | |
| **Proposition 4** | | 100 | |
| **Proposition 5** | | 100 | |
| **Proposition 6** | | 100 | |

*Note:* *This conjecture is evaluated computing oil asset prices as the expected present value of dividends

We also evaluated the non-negativity of profits included in Conjecture 1 and the trade balance conditions that are part of Propositions 5 and 6 (see Table H2).[5] Profits are strictly positive for all optimal decision rules of $s'$ under repayment and default. The trade balance conditions of Propositions 5 and 6 hold 97 and 100 percent of all model evaluations, respectively. Removing the trade balance conditions, the main results of those propositions, namely that default incentives strengthen at lower $y$ (Proposition 5) or lower $p$ (Proposition 6), both hold 100 percent of the model evaluations. Thus, in our calibrated numerical solution, lower oil prices and lower non-oil GDP *always* strengthen default incentives.

[5]We also checked whether the boundary conditions for $x$ (or $s'$) bind and found that they are never binding.

Table H2: Additional Conditions on the Validation of Propositions and Conjectures in the Baseline Model

| Condition or Proposition | Case | Validation | Holds in % | Max. Error |
|---|---|---|---|---|
| **Trade balance condition** | Proposition 5 | $tb\left(b^1, s^1, b\right) \geq M\left(s^1, s, p\right) - M\left(\tilde{s}^2, s, p\right)$ for $y_2 \in D\left(b, s\right)$ | 97 | $-0.05^*$ |
| | Proposition 6 | $tb\left(b^1, s^1, b\right) \geq M\left(s^1, s, p_2\right) - M\left(\tilde{s}^2, s, p_2\right)$ for $p_2 \in D\left(b, s\right)$ | 100 | |
| **Reserves choice condition** | Proposition 6 | $s^1 \leq \tilde{s}^2$ | 100 | |
| **Proposition 5** | Without trade balance condition | For all $y_1 < y_2$, and $y_2 \in D\left(b, s\right)$ then $y_1 \in D\left(b, s\right)$ | 100 | |
| **Proposition 6** | Without trade balance condition or $s^1 \leq \tilde{s}^2$ | For all $p_1 < p_2$, and $p_2 \in D\left(b, s\right)$ then $p_1 \in D\left(b, s\right)$ | 100 | |
| **Profits in optimal decisions** | Repayment | $M^{nd}\left(s'^{nd}\left(s, p, y\right), s, p, y\right) > 0$ | 100 | |
| | Default | $M^{d}\left(s'^{d}\left(s, p, y\right), s, p, y\right) > 0$ | 100 | |
| $s^{nd}\left(b, s, p, y\right)$ **boundaries hit** | Lower bound | $s^{nd}\left(b, s, p, y\right) = \left(s + \kappa\right) - s(p/\psi)^{(1/\gamma)}$ | 0 | |
| | Upper bound | $s^{nd}\left(b, s, p, y\right) = s + k$ | 0 | |
| $s^{d}\left(s, p, y\right)$ **boundaries hit** | Lower bound | $s^{d}\left(b, s, p, y\right) = \left(s + \kappa\right) - s(p/\psi)^{(1/\gamma)}$ | 0 | |
| | Upper bound | $s^{d}\left(b, s, p, y\right) = s + k$ | 0 | |

*Note:* $^*$The Max. Error is computed as $tb\left(b^1, s^1, b\right) - \left[M\left(s^1, s, p\right) - M\left(\tilde{s}^2, s, p\right)\right]$

$^{**}$The Max. Error is computed as $tb\left(b^1, s^1, b\right) - \left[M\left(s^1, s, p_2\right) - M\left(\tilde{s}^2, s, p_2\right)\right]$

$^{**}$The Max. Error is computed as $s^1 - \tilde{s}^2$

For the analysis that follows, we define these functions:

(a) Profits from oil extraction under repayment and default (using the law of motion of reserves to express oil extraction as a function $x(s', s) = s - s' + \kappa$):

$$M^{nd}(s', s, p) \equiv px(s', s) - e(x(s', s), s), \quad M^{d}(s', s, p) \equiv h(p)x(s', s) - e(x(s', s), s).$$

(b) Asset prices of oil under repayment and default:[6]

$$q^{Ond}(s', s, p) \equiv p - e_x(x(s', s), s), \quad q^{Od}(s', s, p) \equiv h(p) - e_x(x(s', s), s).$$

(c) Trade balance under repayment:

$$tb(b', s', b, y, p) \equiv q\left(b', s', y, p\right) b' - b.$$

(d) Consumption under repayment and default:

$$c^{nd}(b', s', b, s, y, p) \equiv y - A + M^{nd}(s', s, p) - tb(b', s', b, y, p), \quad c^{d}(s', s, y, p) \equiv y - A + M^{d}(s', s, p).$$

Next, we postulate three conjectures that are used later to prove some of of the propositions in this Appendix:

[6]In Appendix F, we showed that in a model without default risk $p - e_x(x(s', s), s)$ is equal to the asset price of oil (i.e., the expected present value of oil dividends discounted with the sovereign's stochastic discount factors) for internal solutions of $x$ and it is always positive.

**Conjecture 1.** ***Asset prices of oil are positive under repayment and default.***

*$q^{Ond}(s', s, p), q^{Od}(s', s, p) > 0$ for all $p$, $s \in [\underline{s}, \overline{s}] = \{s : \underline{s} \leq s \leq \overline{s}\}$, and $s'$ in the interval $(s + \kappa) - s(p/\psi)^{(1/\gamma)} \leq s' \leq (s + \kappa)$, where $s' \geq s + \kappa - s(p/\psi)^{(1/\gamma)}$ is implied by the upper bound of $x$ above which profits are negative and $s' \leq s + \kappa$ is the upper bound of reserves if $x = 0$.*

Appendix F shows that this conjecture is an equilibrium outcome for three variants of the model in which the sovereign can commit to repay (i.e., financial autarky and a small open economy facing either a constant real interest rate or an exogenous interest rate function with the qualitative features of the equilibrium interest rate of a model with default). This is because the equilibrium asset price of oil equals the expected present value of the stream of (non-negative) oil dividends discounted with the stochastic discount factor of the sovereign. Assuming $\lambda = 0$, it can also be proven that $q^{Od}(\cdot) > 0$ is an equilibrium outcome in the model with default, because with permanent exclusion the planner's dynamic programming problem is the same as that with commitment to repay under financial autarky.[7]

**Conjecture 2.** ***If default is possible for some state $(b, \tilde{s}, y, p)$, the optimal consumption choice under repayment is nondecreasing in $s$ in the interval $\underline{s} \leq s \leq \tilde{s} \leq \overline{s}$.***

*For all $s^1, s^2 \in [\underline{s}, \tilde{s}]$ and $s^1 \leq s^2$, $\hat{c}^{nd}(b, s^2, y, p) \geq \hat{c}^{nd}(b, s^1, y, p)$, where optimal consumption under repayment is: $\hat{c}^{nd}(b, s, y, p) \equiv y - A + M^{nd}(s'(b, s, y, p), s, y, p) - tb(b'(b, s, y, p), s'(b, s, y, p), b, y, p)$, and $b'(b, s, y, p), s'(b, s, y, p)$ are the bonds and reserves decision rules under repayment, respectively.*

This conjecture is also an equilibrium outcome if the sovereign is committed to repay. It is a standard result that follows from consumption being increasing in wealth but proving this property is not straightforward in the model with default, because it requires properties of decision rules under repayment that are difficult to establish since the optimization problem under repayment retains the option to default in the future and is not differentiable.

**Conjecture 3.** ***If default on outstanding debt is optimal at a given level of existing reserves for some realizations of income and oil prices, all the available contracts for new debt and choices of oil reserves under repayment yield a trade balance at least as large as the difference in oil profits between repayment and default.***

*If for some $(b, s)$ the default set is non-empty $D(b, s) \neq \oslash$, then for $(y, p) \in D(b, s)$ there are no contracts $\{q(b', s', y, p), b', s'\}$ available such that $tb(b', s', b, y, p) < M^{nd}(s', s, p) - M^d(s^d(s, y, p), s, p)$,*

[7] We showed in Appendix F that under financial autarky and assuming an internal solution for $x$, $q^O_t = E_t \sum_{j=1}^{\infty} \beta^j u'(t + j)/u'(t)[-e_s(t + j)]$. This corresponds to $q^{Od}(s', s, p)$ if the probability of re-entry is zero.

*where $s^d(s, y, p)$ is the optimal choice of reserves under default.*

This conjecture is related to Proposition 2 in Arellano (2008). She shows that, assuming i.i.d. shocks, $\lambda = 0$, and no default income costs, if the default set is non-empty for $b$ then there are no contracts $\{q(b'), b'\}$ under repayment that can yield more net resources for current consumption than the resources available under default. Under default, resources are determined by the *exogenous* realization of $y$, which is the same under repayment, so this result implies also that there are no contracts that can yield a trade deficit. In our model, however, the debt contracts may need to entail a trade surplus in order to match the property that they cannot generate more net resources for current consumption than what the *endogenous* choice of oil profits generates under default. This is clearer if we consider that Conjecture 3 implies: $tb(b', s', b, y, p) \geq M^{nd}(s', s, p) - M^d(s^d(s, y, p), s, p)$. If profits under repayment are larger than under default (which is the case if a lower $s'$ is chosen under repayment, since Proposition 2 below shows that profits are decreasing in $s'$), all available debt contracts generate trade surpluses at least as large as the amount by which oil profits under repayment exceed those under default. A zero trade balance is not sufficient to guarantee that there are fewer net resources for consumption under repayment.[8]

**Proposition 1.** ***The repayment payoff is non-decreasing in $b$ and default sets shrink as $b$ rises (i.e. grow as debt rises).***

*For all $b^1 \leq b^2$, $v^{nd}(b^2, s, y, p) \geq v^{nd}(b^1, s, y, p)$. Moreover, if default is optimal for $b^2$ ($d(b^2, s, y, p) = 1$) for some states $(s, y, p)$ then default is optimal for $b^1$ for the same states $(s, y, p)$ (i.e. $D(b^2, s) \subseteq D(b^1, s)$ and $d(b^1, s, y, p) = 1$)*

*Proof.* This proof follows Arellano (2008).

1. From the definition of $D(\cdot)$ and $d(b^2, s, y, p) = 1$ it follows that $v^d(s, y, p) \geq v^{nd}(b^2, s, y, p)$ $\forall \{y, p\} \in D(b^2, s)$, hence:

$$v^d(s, y, p) \geq v^{nd}(b^2, s, y, p) \geq u\left(c^{nd}(b', s', b^2, s, y, p)\right) + \beta E\left[V(b', s', y', p')\right] \quad \forall (b', s')$$

[8] We can show that Conjecture 3 holds as a proposition under the sufficiency condition that, if the default set is not empty for a pair $(b, s)$, there are no available debt contracts under repayment with associated choices of oil reserves that are smaller than the reserves chosen under default (i.e., the planner cannot generate more resources by setting $s'$ lower in repayment than in default). However, this condition fails in the majority of the state space of the numerical solution with the baseline calibration.

2. It follows that, since $b^1 \leq b^2$ implies $c^{nd}(b', s', b^2, s, y, p) \geq c^{nd}(b', s', b^1, s, y, p)$, the continuation values for $b^1 \leq b^2$ satisfy:

$$u\left(c^{nd}(b', s', b^2, s, y, p)\right) + \beta E\left[V(b', s', y', p')\right] \geq u\left(c^{nd}(b', s', b^1, s, y, p))\right) + \beta E\left[V(b', s', y', p')\right],$$

for all $(b', s')$, which implies that $v^{nd}(b, s, y, p)$ is nondecreasing in $b$.

3. It follows from 1. and 2. that $v^d(s, y, p) \geq v^{nd}(b^2, s, y, p) \geq v^{nd}(b^1, s, y, p)$, hence $v^d(s, y, p) \geq v^{nd}(b^1, s, y, p)$ which implies $\{y, p\} \in D(b^1, s)$ and thus $d(b^1, s, y, p) = 1$.

□

**Proposition 2.** ***If asset prices of oil are positive, oil profits are increasing in $s$, for given $s'$, and decreasing in $s'$, for given $s$.***

*Given Conjecture 1, oil profits under repayment and default are increasing in $s \in [\underline{s}, \overline{s}]$, namely $M_s^{nd}(\cdot), M_s^d(\cdot) > 0$, and decreasing in $s' \in [s + \kappa - s(p/\psi)^{(1/\gamma)}, s + \kappa]$, namely $M_{s'}^{nd}(\cdot), M_{s'}^d(\cdot) < 0$.*

*Proof.* We show first that profits are increasing in $s$, and then that they are decreasing in $s'$.

1. The derivatives of oil profits with respect to $s$ under repayment and default are $M_s^{nd}(\cdot) = p - e_x(x(s', s), s) - e_s(x(s', s), s)$ and $M_s^d(\cdot) = h(p) - e_x(x(s', s), s) - e_s(x(s', s), s)$.

2. Since $q^{Ond}(s', s, p) = p - e_x(x(s', s), s)$ and $q^{Od}(s', s, p) = h(p) - e_x(x(s', s), s)$, the derivatives can be rewritten as $M_s^{nd}(\cdot) = q^{Ond}(s', s, p) - e_s(x(s', s), s)$ and $M_s^d(\cdot) = q^{Od}(s', s, p) - e_s(x(s', s), s)$ respectively.

3. Since $q^{Ond}(s', s, p), q^{Od}(s', s, p) > 0$ and $e_s(x(s', s), s) < 0$ for $s \in [\underline{s}, \overline{s}]$, it follows that $M_s^{nd}(\cdot) = q^{Ond}(s', s, p) - e_s(x(s', s), s) > 0$ and $M_s^d(\cdot) = q^{Od}(s', s, p) - e_s(x(s', s), s) > 0$.

4. The derivatives of oil profits with respect to $s'$ under repayment and default are $M_{s'}^{nd}(\cdot) = -p + e_x(x(s', s), s))$ and $M_s^d(\cdot) = -h(p) + e_x(x(s', s), s)$.

5. Since $q^{Ond}(s', s, p) = p - e_x(x(s', s), s)$ and $q^{Od}(s', s, p) = h(p) - e_x(x(s', s), s)$, the derivatives can be rewritten as $M_s^{nd}(\cdot) = -q^{Ond}(s', s, p))$ and $M_s^d(\cdot) = -q^{Od}(s', s, p)$ respectively.

6. Since $q^{Ond}(s', s, p), q^{Od}(s', s, p) > 0$, it follows that $M_{s'}^{nd}(\cdot) = -q^{Ond}(s', s, p) < 0$ and $M_{s'}^d(\cdot) = -q^{Od}(s', s, p) < 0$.

□

**Proposition 3.** ***The default and repayment payoffs are non-decreasing in*** $s$***.***

*For all* $s^1, s^2 \in [\underline{s}, \overline{s}]$ *and* $s^1 \leq s^2$*,* $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$ *and* $v^d(s^2, y, p) \geq v^d(s^1, y, p)$*.*

*Proof.* This proof uses the consumption functions $c^{nd}(b', s', b, s, y, p), c^d(s', s, y, p)$.

1. Since $s^1 \leq s^2$, the result that oil profits are increasing in $s$ (Proposition 2) and the definitions of the consumption functions imply that $c^{nd}(b', s', b, s^2, y, p) \geq c^{nd}(b', s', b, s^1, y, p)$ and $c^d(s', s^2, y, p) \geq c^d(s', s^1, y, p)$ for all $(b', s')$. Hence, the continuation values for $s^1 \leq s^2$ satisfy:

$$\begin{aligned} v^{nd}(b, s^2, y, p) &\geq u\left(c^{nd}(b', s', b, s^2, y, p)\right) + \beta E\left[V(b', s', y', p')\right] \\ &\geq u\left(c^{nd}(b', s', b, s^1, y, p))\right) + \beta E\left[V(b', s', y', p')\right], \end{aligned}$$

   for all $(b', s')$, which implies that $v^{nd}(b, s^2, y, p) \geq v^{nd}(b, s^1, y, p)$. Hence, $v^{nd}(b, s, y, p)$ is nondecreasing in $s$.

2. Similarly, the default payoffs satisfy:

$$\begin{aligned} v^d(s^2, y, p) &\geq u\left(c^d(s', s^2, y, p)\right) + \beta E\left[\lambda V(0, s', y, p) + (1-\lambda)v^d(s', y', p')\right] \\ &\geq u\left(c^d(s', s^1, y, p))\right) + \beta E\left[\lambda V(0, s', y, p) + (1-\lambda)v^d(s', y', p')\right], \end{aligned}$$

   for all $s'$, which implies that $v^d(s^2, y, p) \geq v^d(s^1, y, p)$. Hence, $v^d(s, y, p)$ is nondecreasing in $s$.

□

**Proposition 4.** ***Default sets shrink as*** $s$ ***rises (i.e. grow as reserves fall).***

*Assume* $\hat{p} = p$ *and* $\lambda = 0$ *for simplicity. For all* $s^1, s^2 \in [\underline{s}, \overline{s}]$ *and* $s^1 \leq s^2$*, if default is optimal for* $s^2$ $(d(b, s^2, y, p) = 1)$ *for some states* $(b, y, p)$*, then default is optimal for* $s^1$ *for the same states* $(b, y, p)$ *(i.e.* $D(b, s^2) \subseteq D(b, s^1)$ *and* $d(b, s^1, y, p) = 1$*).*

*Proof.* We show first that this proposition is valid if the decision rules for oil reserves under default and repayment are such that $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$, and then we show that this condition holds under Conjecture 2.[9] The proof also requires Conjectures 1 and 3.

1. Since $d(b, s^2, y, p) = 1$ implies $v^d(s^2, y, p) - v^{nd}(b, s^2, y, p) \geq 0$ and both $v^{nd}(b, s, y, p)$ and $v^d(s, y, p)$ are nondecreasing in $s$, in order for $v^d(s^1, y, p) - v^{nd}(b, s^1, y, p) \geq 0$ (i.e. $d(b, s^1, y, p) = 1$), we need to show that when $s$ falls, the default payoff falls as much or less than the repayment payoff: $v^d(s^2, y, p) - v^d(s^1, y, p) \leq v^{nd}(b, s^2, y, p) - v^{nd}(b, s^1, y, p)$.

2. Using the definition of $v^d(b, s, p)$ and since $s^d(s^2, y, p)$ is the optimal reserves choice under default when $s = s^2$, it follows that the difference $v^d(s^2, y, p) - v^d(s^1, y, p)$ satisfies this condition:

$$\begin{aligned} &v^d(s^2, y, p) - v^d(s^1, y, p) \leq \\ &u\left(c^d(s^d(s^2, y, p), s^2, y, p)\right) + \beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') + (1-\lambda) v^d(s^d(s^2, y, p), y', p')\right] \\ &-u\left(c^d(s^d(s^2, y, p), s^1, y, p)\right) + \beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') - (1-\lambda) v^d(s^d(s^2, y, p), y', p')\right] \end{aligned}$$

which reduces to:

$$v^d(s^2, y, p) - v^d(s^1, y, p) \leq u\left(c^d(s^d(s^2, y, p), s^2, y, p)\right) - u\left(c^d(s^d(s^2, y, p), s^1, y, p)\right)$$

3. Using the definition of $v^{nd}(b, s, p)$ and since $b'(b, s^1, y, p)$, $s'(b, s^1, y, p)$ are the bonds and reserves decision rules under repayment when reserves are $s = s^1$, respectively, it follows that the difference $v^{nd}(b, s^2, y, p) - v^{nd}(b, s^1, y, p)$ satisfies this condition:

$$\begin{aligned} &v^{nd}(b, s^2, y, p) - v^{nd}(b, s^1, y, p) \geq \\ &u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^2, y, p)\right) + \beta E\left[V(b'(b, s^1, y, p), s'(b, s^1, y, p), y', p')\right] \\ &-u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right) + \beta E\left[V(b'(b, s^1, y, p), s'(b, s^1, y, p), y', p')\right] \end{aligned}$$

which reduces to:

$$\begin{aligned} &v^{nd}(b, s^2, y, p) - v^{nd}(b, s^1, y, p) \geq \\ &u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^2, y, p)\right) - u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right) \end{aligned}$$

[9] Conjecture 2 could be replaced with the assumption that $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$ and the last step of the proof would be unnecessary, but Conjecture 2 is more reasonable because it states a familiar property of consumption decision rules (i.e. that they are increasing in wealth) and only with respect to consumption under repayment, whereas $s^d(s^2, y, p) \leq s'(b, s^1, y, p)$ refers to decision rules for reserves under default with higher $s$ v. repayment with lower $s$.

4. The results in 3. and 4. imply the following sufficiency condition for $v^d(s^2,y,p) - v^d(s^1,y,p) \leq v^{nd}(b,s^2,y,p) - v^{nd}(b,s^1,y,p)$:

$$u\left(c^d(s^d(s^2,y,p),s^2,y,p)\right) - u\left(c^d(s^d(s^2,y,p),s^1,y,p)\right) \leq$$
$$u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^2,y,p)\right) - u\left(c^{nd}(b'(b,s^1,y,p),s'(b,s^1,y,p),b,s^1,y,p)\right),$$

which, using the definitions of $c^{nd}(\cdot)$ and $c^d(\cdot)$ and noting that since $\hat{p}=p$ we can write the profit functions as $M^d(\cdot) = M^{nd}(\cdot) = M(\cdot)$, can be rearranged as follows:

$$\begin{aligned} u&\left(y - A + M(s^d(s^2,y,p),s^2,p)\right) \\ &- u\left(y - A + M(s'(b,s^1,y,p),s^2,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)\right) \\ &\leq u\left(y - A + M(s^d(s^2,y,p),s^1,p)\right) \\ &- u\left(y - A + M(s'(b,s^1,y,p),s^1,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p))\right), \end{aligned}$$

and using this notation $\tilde{y}^2 \equiv y - A + M(s^d(s^2,y,p),s^2,p)$, $\tilde{y}^1 \equiv y - A + M(s^d(s^2,y,p),s^1,p)$, $z^2 = M(s'(b,s^1,y,p),s^2,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p) - M(s^d(s^2,y,p),s^2,p)$, $z^1 = M(s'(b,s^1,y,p),s^1,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)) - M(s^d(s^2,y,p),s^1,p)$ it can be re-written as:

$$u\left(\tilde{y}^2\right) - u\left(\tilde{y}^2 + z^2\right) \leq u\left(\tilde{y}^1\right) - u\left(\tilde{y}^1 + z^1\right),$$

5. The strict concavity of $u(\cdot)$ implies that the above condition holds if we can show that $\tilde{y}^2 > \tilde{y}^1$ and $z^1 \leq z^2 \leq 0$. Since $M_s(\cdot) > 0$ as shown in Proposition 2, it follows that $\tilde{y}^2 > \tilde{y}^1$. Conjecture 3 implies that if the default set for $(b,s)$ is not empty, then all the contracts available under repayment are such that $M(s',s,p) - tb(b',s',b,y,p) - M(s^d(s,y,p),s,p) \leq 0$, therefore $z^1, z^2 \leq 0$. Hence, $z^1 \leq z^2 \leq 0$ holds if

$$M(s^d(s^2,y,p),s^2,p) - M(s^d(s^2,y,p),s^1,p) \leq M(s'(b,s^1,y,p),s^2,p) - M(s'(b,s^1,y,p),s^1,p).$$

Since $M_{ss'}(\cdot) \geq 0$, it follows that the above condition holds if the reserves decision rules under default and repayment are such that $s^d(s^2,y,p) \leq s'(b,s^1,y,p)$.[10]

[10] Given the functional form of $e(x,s)$, it is straightforward to show that $M^{nd}_{ss'}(\cdot) = M^d_{ss'}(\cdot) = e_x(\cdot)\gamma(s' - \kappa)/(xs)$. Moreover, we show in Appendix I that under financial autarky (or under default with permanent exclusion), the optimal reserves decision rule is increasing in reserves if $p^{max} < \psi$ (i.e. if the highest realization of oil prices is smaller than the coefficient $\psi$ of the extraction costs function). Hence, $min(s'-x) = s[1-(p/\psi)^{1/\gamma}]$ and therefore $M^{nd}_{ss'}(\cdot) = M^d_{ss'}(\cdot) > 0$.

6. Finally, we show that a sufficiency condition for $s^d(s^2,y,p) \leq s'(b,s^1,y,p)$ to hold is that $\hat{c}^{nd}(b,s^2,y,p) \geq \hat{c}^{nd}(b,s^1,y,p)$, which holds because of Conjecture 2. To show this, note first that because of Conjecture 3 (if the default set for $(b,s^1)$ is not empty) $tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)) \geq M(s'(b,s^1,y,p),s^1,p)-M(s^d(s^1,y,p),s^1,p)$, and hence $M(s^d(s^1,y,p),s^1,p) \geq M(s'(b,s^1,y,p),s^1,p)-tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)$. Moreover, in the optimization problem under full financial autarky of Appendix F (which is the same as the default problem since $\lambda = 0$) $dM(s',s,p)/ds > 0$.[11] Hence, these two result imply that:

$$\begin{aligned} M(s^d(s^2,y,p),s^2,p) &> M(s^d(s^1,y,p),s^1,p) \\ &\geq M(s'(b,s^1,y,p),s^1,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p), \end{aligned}$$

therefore:

$$\begin{aligned} &y - A + M(s^d(s^2,y,p),s^2,p) \geq \\ &\qquad y - A + M(s'(b,s^1,y,p),s^1,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p) \end{aligned}$$

Since $u(c)$ is increasing in $c$:

$$\begin{aligned} &u\Big(y - A + M(s^d(s^2,y,p),s^2,p)\Big) \geq \\ &\qquad u\left(y - A + M(s'(b,s^1,y,p),s^1,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)\right) \end{aligned}$$

Add $\beta E\left[\lambda V(0,s^d(s^2,y,p),y',p') + (1-\lambda)v^d(s^d(s^2,y,p),y',p')\right]$ to both sides of the above expression and simplify using the definition of $v^d(s^2,y,p)$:

$$\begin{aligned} v^d(s^2,y,p) \geq\ & u\left(y - A + M(s'(b,s^1,y,p),s^1,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)\right) \\ &+\beta E\left[\lambda V(0,s^d(s^2,y,p),y',p') + (1-\lambda)v^d(s^d(s^2,y,p),y',p')\right] \end{aligned}$$

Subtracting $v^{nd}(b,s^2,y,p)$ from both sides yields:

$$\begin{aligned} &v^d(s^2,y,p) - v^{nd}(b,s^2,y,p) \geq \\ &\quad u\left(y - A + M(s'(b,s^1,y,p),s^1,p) - tb(b'(b,s^1,y,p),s'(b,s^1,y,p),b,y,p)\right) \\ &\quad - v^{nd}(b,s^2,y,p) + \beta E\left[\lambda V(0,s^d(s^2,p),y',p') + (1-\lambda)v^d(s^d(s^2,y,p),y',p')\right], \end{aligned}$$

[11] From the definition of $M(s',s,p)$ it follows that $dM/ds = q^{Od}(s',s,p)[1 - \partial s^d(\cdot)/\partial s] - e_s(\cdot) > 0$, because $q^O d(s',s,p) > 0$, $e_s(\cdot) < 0$ and we conjecture that $\partial s^d(\cdot)/\delta s < 1$ for local stability (Appendix I proves that $\partial s^d(\cdot)/\delta s > 0$).

which using the definitions of $v^{nd}(b, s^2, y, p)$ and $c^{nd}(b', s', b, s, y, p)$ can be written as:

$$v^d(s^2, y, p) - v^{nd}(b, s^2, y, p) \geq u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right)$$
$$-\left[u\left(c^{nd}(b'(b, s^2, y, p), s'(b, s^2, y, p), b, s^2, y, p)\right) + \beta E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p')\right]\right]$$
$$+\beta E\left[\lambda V(0, s^d(s^2, y, p), y', p') + (1-\lambda)v^d(s^d(s^2, y, p), y', p')\right],$$

and rearranging terms in the above expression yields:

$$u\left(c^{nd}(b'(b, s^2, y, p), s'(b, s^2, y, p), b, s^2, y, p)\right) - u\left(c^{nd}(b'(b, s^1, y, p), s'(b, s^1, y, p), b, s^1, y, p)\right)$$
$$+\beta E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p') - \lambda V(0, s^d(s^2, y, p), y', p') - (1-\lambda)v^d(s^d(s^2, y, p), y', p')\right]$$
$$\geq v^{nd}(b, s^2, y, p) - v^d(s^2, y, p).$$

Since $\lambda = 0$, and using the definition of the optimal consumption decision rule $\hat{c}^{nd}(b, s, y, p)$, this expression can be written as:

$$u\left(\hat{c}^{nd}(b, s^2, y, p)\right) - u\left(\hat{c}^{nd}(b, s^1, y, p)\right)$$
$$+\beta E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p') - v^d(s^d(s^2, y, p), y', p')\right]$$
$$\geq v^{nd}(b, s^2, y, p) - v^d(s^2, y, p).$$

This inequality holds because $d(b, s^2, y, p) = 1$ implies that the right-hand-side of this expression is non-positive ($v^{nd}(b, s^2, y, p) - v^d(s^2, y, p) \leq 0$) while the left-hand-side is non-negative because: (a) Conjecture 2 and the fact that $u(c)$ is increasing in $c$ imply that $u\left(\hat{c}^{nd}(b, s^2, y, p)\right) \geq u\left(\hat{c}^{nd}(b, s^1, y, p)\right)$, and (b) $E\left[V(b'(b, s^2, y, p), s'(b, s^2, y, p), y', p')\right]$ $-E\left[v^d(s^d(s^2, y, p), y', p')\right] \geq 0$ by the definition of $V(\cdot)$.

□

**Proposition 5.** ***If the trade balance is sufficiently large, default incentives strengthen as non-oil GDP falls.***

*Assuming i.i.d shocks, $\lambda = 0$ and $\hat{p} = p$, for all $y_1 < y_2$, if $y_2 \in D(b, s)$ and $tb(b^1, s^1, b) \geq M(s^1, s, p) - M(\tilde{s}^2, s, p)$ (where $b^1 \equiv b'(b, s, y_1, p)$, $s^1 \equiv s'(b, s, y_1, p)$ are the optimal choices of bonds and reserves under repayment with $y_1$ and $\tilde{s}^2 \equiv s^d(s, y_2, p)$ is the optimal reserves choice under default with $y_2$), then $y_1 \in D(b, s)$.*

*Proof.* This proof aims to extend Proposition 3 in Arellano (2008), but for this model it requires a lower bound condition on the trade balance linked to the optimal decision rules of reserves under repayment with $y_1$ v. under default with $y_2$.

1. If $y_2 \in D(b,s)$ and denoting $b^2 \equiv b'(b,s,y_2,p)$, $s^2 \equiv s'(b,s,y_2,p)$ the optimal choices of bonds and reserves when $y = y_2$ under repayment, it follows that by definition:

$$u\left(y_2 - A + M^d(\tilde{s^2}, s, p)\right) + \beta E[v^d(\tilde{s^2}, y', p')] \geq$$
$$u\left(y_2 - A + M^{nd}(s^2, s, p) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')]$$

2. To establish that $y_2 \in D(b,s) \Rightarrow y_1 \in D(b,s)$ it is sufficient to show that, denoting $\tilde{s}^1$ as reserves chosen when $y = y_1$ under default, the following holds: $u\left(y_2 - A + M^{nd}(s^2, s, p) - tb(b^2, s^2, b)\right)+$
$\beta E[V(b^2, s^2, y', p')] -$
$\left[u\left(y_1 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$
$u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y_1 - A + M^d(\tilde{s}^1, s, p)\right) + \beta E[v^d(\tilde{s}^1, y', p')]\right]$

3. Given that $(b^2, s^2)$ maximizes the repayment payoff with $y_2$ and $\tilde{s}^1$ maximizes the default payoff with $y_1$, the following two conditions hold:

$$u\left(y_2 - A + M^{nd}(s^2, s, p) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')]$$
$$\geq \left[u\left(y_2 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right]$$

$$u\left(y_1 - A + M^d(\tilde{s}^1, s, p)\right) + \beta E[v^d(\tilde{s}^1, y', p')] \geq \left[u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$$

4. Using the results in 3., the condition in 2. holds if:

$\left[u\left(y_2 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] -$
$\left[u\left(y_1 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$
$u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$

5. The above expression simplifies to:

$$u\left(y_2 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right) - u\left(y_1 - A + M^{nd}(s^1, s, p) - tb(b^1, s^1, b)\right)$$
$$\geq u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) - u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right),$$

which adding and subtracting $M^d(\tilde{s}^2, s, p)$ inside the argument of the repayment utilities and rearranging yields:

$$u\left(y_2 - A + M^d(\tilde{s}^2, s, p)\right) - u\left(y_2 - A + M^d(\tilde{s}^2, s, p) + z(y_1)\right)$$
$$\leq u\left(y_1 - A + M^d(\tilde{s}^2, s, p)\right) - u\left(y_1 - A + M^d(\tilde{s}^2, s, p) + z(y_1)\right),$$

where $z(y_1) \equiv M^{nd}(s^1, s, p) - tb(b^1, s^1, b) - M^d(\tilde{s}^2, s, p)$. The above inequality holds because: (a) the utility function is increasing and strictly concave, (b) $y_2 > y_1$ and (c) $z(y_1) < 0$ because of the assumption that $tb(b^1, s^1, b) \geq M^{nd}(s^1, s, p) - M^d(\tilde{s}^2, s, p)$.

□

**Proposition 6.** ***If the trade balance is sufficiently large and reserves chosen under default at high oil prices exceed those chosen under repayment at low prices, default incentives strengthen as oil prices fall.***

*Assuming i.i.d shocks, $\lambda = 0$ and $\hat{p} = p$, for all $p_1 < p_2$ and $p_2 \in D(b, s)$, if $tb(b^1, s^1, b) \geq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$ and $s^1 \leq \tilde{s}^2$ (where $b^1, s^1$ are the optimal bonds and reserves choices under repayment in state $(b, s, y, p_1)$ and $\tilde{s}^2$ is the optimal reserves choice under default in state $(s, y, p_2)$, then $p_1 \in D(b, s)$.*

*Proof.* This proof follows a similar strategy as that of Proposition 5. Again it requires a lower bound condition on the trade balance, but now linked to the optimal decision rules of reserves under repayment with $p_1$ v. under default with $p_2$, and it also requires optimal reserves under default with $p_2$ to exceed those under repayment with $p_1$.

1. If $p_2 \in D(b, s)$ and denoting $(b^2, s^2)$ and $\tilde{s}^2$ as the optimal choices of bonds and reserves when $p = p_2$ under repayment and default, respectively, it follows that by definition::

$$u\left(y - A + M(\tilde{s}^2, s, p_2)\right) + \beta E[v^d(\tilde{s}^2, y', p')] \geq$$
$$u\left(y - A + M(s^2, s, p_2) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')],$$

   where the profit functions under default and repayment are the same because $\hat{p} = p$.

2. To establish that $p_2 \in D(b, s) \Rightarrow p_1 \in D(b, s)$ it is sufficient to show that, denoting $(b^1, s^1)$ and $\tilde{s}^1$ as the bonds and reserves chosen when $p = p_1$ under repayment and default, respectively, the following holds:

$$u\left(y - A + M(s^2, s, p_2) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')] -$$
$$\left[u\left(y - A + M(s^1, s, p_1) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$$
$$u\left(y - A + M(\tilde{s}^2, s, p_2)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y - A + M(\tilde{s}^1, s, p_1)\right) + \beta E[v^d(\tilde{s}^1, y', p')]\right]$$

3. Given that $(b^2, s^2)$ maximizes the repayment payoff with $p_2$ and $\tilde{s}^1$ maximizes the default payoff with $p_1$, the following two conditions hold:

$$u\left(y - A + M(s^2, s, p_2) - tb(b^2, s^2, b)\right) + \beta E[V(b^2, s^2, y', p')]$$
$$\geq \left[u\left(y - A + M(s^1, s, p_2) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right]$$

$$u\left(y - A + M(\tilde{s}^1, s, p_1)\right) + \beta E[v^d(\tilde{s}^1, y', p')] \geq \left[u\left(y - A + M(\tilde{s}^2, s, p_1)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$$

4. Using the results in 3., the condition in 2. holds if:

$$\left[u\left(y - A + M(s^1, s, p_2) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] -$$
$$\left[u\left(y - A + M(s^1, s, p_1) - tb(b^1, s^1, b)\right) + \beta E[V(b^1, s^1, y', p')]\right] \geq$$
$$u\left(y - A + M(\tilde{s}^2, s, p_2)\right) + \beta E[v^d(\tilde{s}^2, y', p')] - \left[u\left(y - A + M(\tilde{s}^2, s, p_1)\right) + \beta E[v^d(\tilde{s}^2, y', p')]\right]$$

5. The above expression simplifies to:

$$u\left(y - A + M(s^1, s, p_2) - tb(b^1, s^1, b)\right) - u\left(y - A + M(s^1, s, p_1) - tb(b^1, s^1, b)\right)$$
$$\geq u\left(y - A + M(\tilde{s}^2, s, p_2)\right) - u\left(y - A + M(\tilde{s}^2, s, p_1)\right),$$

which adding and subtracting $M(\tilde{s}^2, s, p_2)$ and $M(\tilde{s}^2, s, p_1)$ to the arguments of the repayment utility in the left- and right-hand-sides, respectively,and rearranging yields:

$$u\left(y - A + M(\tilde{s}^2, s, p_2)\right) - u\left(y - A + M(\tilde{s}^2, s, p_2) + z(p_2)\right)$$
$$\leq u\left(y - A + M(\tilde{s}^2, s, p_1)\right) - u\left(y - A + M(\tilde{s}^2, s, p_1) + z(p_1)\right)$$

where: $z(p_1) = M(s^1, s, p_1) - tb(b^1, s^1, b) - M(\tilde{s}^2, s, p_1)$ and $z(p_2) = M(s^1, s, p_2) - tb(b^1, s^1, b) - M(\tilde{s}^2, s, p_2)$. The above inequality holds because: (a) the utility function is increasing and strictly concave, (b) $M(\tilde{s}^2, s, p_2) > M(\tilde{s}^2, s, p_1)$ since profits are increasing in $p$, (c) $z(p_2) \leq 0$ because of the assumption that $tb(b^1, s^1, b) \geq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$, and (d) $z(p_1) \leq z(p_2)$ because $s^1 \leq \tilde{s}^2$ (note that $z(p_1) \leq z(p_2) \leftrightarrow M(s^1, s, p_1) - M(\tilde{s}^2, s, p_1) \leq M(s^1, s, p_2) - M(\tilde{s}^2, s, p_2)$ or $M(\tilde{s}^2, s, p_2) - M(\tilde{s}^2, s, p_1) \leq M(s^1, s, p_2) - M(s^1, s, p_1)$ and using the functional form of $M(.)$ this yields $(p_2 - p_1)(s - \tilde{s}^2 + \kappa) \leq (s - s^1 + \kappa)(p_2 - p_1)$, which implies that $s^1 \leq \tilde{s}^2$).

□

## I Dynamic Programming Problem under Financial Autarky

The dynamic programming problem of the planner under financial autarky, which corresponds also to the default payoff and decision rules when $\lambda = 0$, can be written as follows:

$$V^d(s,p,y) = \max_{s' \in \Gamma(s)} \left\{ F\left(s,s',p,y\right) + \beta E\left[V^d\left(s',p',y'\right)\right]\right\}$$
$$F\left(s,s',p,y\right) \equiv u\left(y - A + p(s - s' + \kappa) - e\left(s - s' + \kappa, s\right)\right)$$
$$\Gamma(s) \equiv \left\{s' : 0 \le s' \le s + \kappa\right\},$$

with first-order condition:

$$[s'] : u_c(t)\left(p - e_x(\cdot)\right) = \beta V^d_{s'}\left(s',p',y'\right)$$

or

$$-F_{s'}\left(s,s',p,y\right) = \beta V^d_{s'}\left(s',p',y'\right).$$

This Appendix shows that the period-payoff $F(s,s',p,y)$ of the above problem satisfies standard properties analogous to those of the textbook neoclassical Ramsey model, with oil reserves taking the place of the capital stock. In particular, we show that $F(s,s',p,y)$ is continuously differentiable in $(s,s')$, strictly increasing (decreasing) in $s$ $(s')$, and strictly concave in $(s,s')$. We also show that the optimal decision rule $s'(s,p,y)$ is increasing in $s$. These properties, together with the assumptions that $F(\cdot)$ is bounded and $\Gamma(s)$ is a nonempty, compact-valued, monotone, and continuous correspondence with a convex graph, ensure that the value function $V^d(\cdot)$ that solves the above Bellman equation exists and the solution is unique, and that $V^d(\cdot)$ is strictly concave, strictly increasing and continuously differentiable.[12] The proofs of these properties are analogous to those of the textbook Ramsey model and therefore are omitted here. Existence and uniqueness follow from the contraction mapping theorem. The proof that $V^d(\cdot)$ is increasing requires $F(\cdot)$ to be increasing and $\Gamma(s)$ to be monotone, the proof that $V^d(\cdot)$ is concave requires $F(\cdot)$ to be concave, and proving the differentiability of $V^d(\cdot)$ requires $F(\cdot)$ to be continuously differentiable and concave.

[12]We also assume a standard, twice-continuously-differentiable, increasing and concave utility function. The CRRA utility function that defines $F(\cdot)$ in the numerical solution satisfies these properties but is unbounded. It can be transformed into a bounded function with a piece-wise truncation at an arbitrary small but positive consumption level (see Suen (2009). "Bounding the CRRA Utility Functions," Working Papers 200902, University of California at Riverside, Department of Economics).

**1.** $F(\cdot)$ **is strictly increasing in** $s$ **(**$F_s(\cdot) > 0$**) and decreasing in** $s'$ **(**$F'_s(\cdot) < 0$**).**

To prove these two properties, recall that $e_s(\cdot) < 0$ and that we showed in the sequential solution of the autarky model of Appendix F that the asset price of oil is positive for internal solutions of $x$, hence $p - e_x(\cdot) > 0$. By differentiating $F(\cdot)$ with respect to $s$ and $s'$ we obtain:

$$F_s(\cdot) = u_c(\cdot)(p - e_x(\cdot) - e_s(\cdot)) > 0,$$

$$F_{s'}(\cdot) = u_c(\cdot)(-p + e_x(\cdot)) = -u_c(\cdot)(p - e_x(\cdot)) < 0.$$

**2.** $F(\cdot)$ **is continuously differentiable.**

To prove that $F(\cdot)$ is continuously differentiable, we need to show that: (a) $F(\cdot)$ is continuous in its domain and (b) $F_s(\cdot)$ and $F_{s'}(\cdot)$ exist and are continuous in their domain. For this proof, consider the above expressions for $F_s(\cdot)$ and $F_{s'}(\cdot)$ and express the extraction cost and its derivatives as functions of $s$ and $s'$ using the law of motion $x = s - s' + \kappa$ as follows:

$$e(s', s) = \psi \frac{(s - s' + \kappa)^{1+\gamma}}{s^{\gamma}}$$

$$e_x(s', s) = (1+\gamma)\psi \left(\frac{s - s' + \kappa}{s}\right)^{\gamma} = (1+\gamma)\psi \left(1 - \frac{(s' - k)}{s}\right)^{\gamma}$$

$$e_s(s', s) = -\gamma\psi \left(\frac{s - s' + \kappa}{s}\right)^{1+\gamma} = -\gamma\psi \left(1 - \frac{(s' - k)}{s}\right)^{1+\gamma},$$

where $e_x(\cdot)$ and $e_s(\cdot)$ are continuous in the domain given by $0 \le s' \le s + k$ and $s > 0$ with the following upper and lower bounds:

$$e_x(0, s) = 0, \quad e_s(0, s) = 0$$

$$e_x(s + k, s) = (1+\gamma)\psi \left(\frac{s + k}{s}\right)^{\gamma}, \quad e_s(s + k, s) = -\gamma\psi \left(\frac{s + k}{s}\right)^{1+\gamma}$$

If in addition, oil profits are required to be non-negative, which is analogous to the non-negativity constraint on consumption (or the Inada condition in $u(c)$) in the texbook Ramsey model, the domain of the cost function and its derivatives requires $px \ge e(\cdot)$. Moreover, if oil revenue is the only income or $y - A \le 0$, the Inada condition would imply that negative profits are never optimal and profits must always be sufficient to sustain $c > 0$. Using again the law of motion $x = s - s' + \kappa$, we obtain that with non-negative profits the lower bound of $s'$ becomes $s' \geq \kappa + s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right]$ instead of $s' > 0$. Hence, the domain of $s'$ becomes $\kappa + s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s+\kappa$.

The functions:

$$F(\cdot) = u\left(y - A + p(s - s' + \kappa) - e\left(s - s' + \kappa, s\right)\right)$$

$$F_s(\cdot) = u_c\left(y - A + p(s - s' + \kappa) - e\left(s - s' + \kappa, s\right)\right) \\ \times \left[p - (1+\gamma)\,\psi\left(1 - \frac{(s'-\kappa)}{s}\right)^{\gamma} + \gamma\psi\left(1 - \frac{(s'-\kappa)}{s}\right)^{1+\gamma}\right],$$

$$F_{s'}(\cdot) = -u_c\left(y - A + p(s - s' + \kappa) - e\left(s - s' + \kappa, s\right)\right)\left[p - (1+\gamma)\,\psi\left(1 - \frac{(s'-\kappa)}{s}\right)^{\gamma}\right],$$

are continuous and exist in the domain defined by $\kappa + s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s+\kappa$ and $s > 0$.

**3.** $s'(s,p,y)$ **is increasing in** $s$**.**

From the first-order condition for $s'$, this property requires that $-F_{s'}(\cdot) = u_c(\cdot)\,(p - e_x(\cdot))$ be decreasing in $s$, since $V^d_{s'}(\cdot)$ is independent of $s$. Thus, we need to show that $\frac{\partial - F_{s'}(\cdot)}{\partial s} < 0$.

$$\frac{\partial - F_{s'}(\cdot)}{\partial s} = \left[p - e_x(\cdot)\right]\left[u_{cc}(\cdot)\{p - e_x(\cdot) - e_s(\cdot)\}\right] + u_c(t)\left\{-\left[e_{xx}(\cdot) + e_{xs}(\cdot)\right]\right\}.$$

Since $e_s(\cdot) < 0$, $p - e_x(\cdot) > 0$, $u_c(\cdot) > 0$ $u_{cc}(\cdot) < 0$, the above expression is negative if $\{-\left[e_{xx}(\cdot) + e_{xs}(\cdot)\right]\} < 0$. To determine the sign of this expression, use the functional form $e(x,s) = \psi\frac{x^{1+\gamma}}{s^{\gamma}}$ to show that the derivatives $e_{xx}(\cdot)$ and $e_{xx}(\cdot)$ can be expressed as follows:

$$e_{xx}(x,s) = \gamma(1+\gamma)\,\psi\frac{x^{\gamma}}{s^{\gamma}}x^{-1} = e_x(\cdot)\,\gamma x^{-1} > 0,$$

$$e_{xs}(x,s) = -\gamma(1+\gamma)\,\psi\frac{x^{\gamma}}{s^{\gamma}}s^{-1} = -e_x(\cdot)\,\gamma s^{-1} < 0.$$

Using these expressions, we obtain:

$$\{-\left[e_{xx}(t) + e_{xs}(t)\right]\} = \left\{-\left[e_x(\cdot)\,\gamma\left(x^{-1} - s^{-1}\right)\right]\right\} < 0 \quad \text{if } x < s,$$

and using $x = s - s' + \kappa$, the condition $x < s$ implies $s - s' + \kappa < s$ which reduces to:

$$s' > \kappa.$$

Hence, $s'(s,p,y)$ is increasing in $s$ if the choice of reserves always exceeds oil discoveries. Since the non-negativity of profits requires $s' \geq \kappa + s\left[1-\left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right]$ and existing reserves satisfy $s > 0$, the condition $s' > \kappa$ is implied by the non-negativity of profits if $p^{max} < \psi$ (i.e., $\psi$ is larger than the largest realization of oil prices so that $p/\psi$ is always less than 1). This result also implies that the upper bound on $x$ never binds (since $s'$ is always strictly positive because $s' > \kappa > 0$).

**4. $F(\cdot)$ is strictly concave**

To show that $F(\cdot)$ is strictly concave, let $H(\cdot)$ be the Hessian matrix of $F(\cdot)$ defined as

$$H(\cdot) = \begin{bmatrix} F_{ss}(\cdot) & F_{ss'}(\cdot) \\ F_{s's}(\cdot) & F_{s's'}(\cdot) \end{bmatrix}$$

$F(\cdot)$ is strict concave if $H(\cdot)$ is negative definite. That is

- $F_{ss}(\cdot) < 0$
- $F_{ss}(\cdot)\,F_{s's'}(\cdot) - F_{ss'}(\cdot)\,F_{s's}(\cdot) > 0$

$$F_{ss}(\cdot) = [p - e_x(\cdot) - e_s(\cdot)]\,u_{cc}(\cdot)\,[p - e_x(\cdot) - e_s(\cdot)] + u_c(\cdot)\,[-e_{xx}(\cdot) - e_{sx}(\cdot) - e_{xs}(\cdot) - e_{ss}(\cdot)]$$

Recall

$$e(x,s) = \psi\frac{x^{1+\gamma}}{s^{\gamma}} e_x(x,s) = (1+\gamma)\,\psi\left(\frac{x}{s}\right)^{\gamma} e_s(x,s) = -\gamma\psi\left(\frac{x}{s}\right)^{1+\gamma}$$

Where

1. $e_{xx}(x,s) = \gamma(1+\gamma)\,\psi\frac{x^{\gamma}}{s^{\gamma}}x^{-1} = e_x(\cdot)\,\gamma x^{-1} > 0$
2. $e_{xs}(x,s) = -\gamma(1+\gamma)\,\psi\frac{x^{\gamma}}{s^{\gamma}}s^{-1} = -e_x(\cdot)\,\gamma s^{-1} < 0$
3. $e_{sx}(x,s) = -\gamma(1+\gamma)\,\psi\left(\frac{x}{s}\right)^{1+\gamma}x^{-1} = e_s(\cdot)\,(1+\gamma)\,x^{-1}$
4. $e_{ss}(x,s) = \gamma(1+\gamma)\,\psi\left(\frac{x}{s}\right)^{1+\gamma}s^{-1} = -e_s(\cdot)\,(1+\gamma)\,s^{-1}$

Additionally, from 3. we can obtain:

$$e_{sx}(x,s) = -\gamma(1+\gamma)\,\psi\left(\frac{x}{s}\right)^{1+\gamma}x^{-1} = -e_x(\cdot)\,\gamma s^{-1}$$

Using $-e_x(\cdot)\gamma s^{-1} = e_s(\cdot)(1+\gamma)x^{-1}$,

$$e_x(\cdot) = -e_s(\cdot)\frac{(1+\gamma)}{\gamma}x^{-1}s,$$

1. $e_{xx}(x,s) = e_x(\cdot)\gamma x^{-1} = -e_s(\cdot)(1+\gamma)x^{-2}s$
2. $e_{xs}(x,s) = -e_x(\cdot)\gamma s^{-1} = e_s(\cdot)(1+\gamma)x^{-1}$
3. $e_{sx}(x,s) = e_s(\cdot)(1+\gamma)x^{-1} = e_{xs}(x,s)$
4. $e_{ss}(x,s) = -e_s(\cdot)(1+\gamma)s^{-1}$

Then

$$F_{ss}(\cdot) = [p - e_x(\cdot) - e_s(\cdot)]\,u_{cc}(\cdot)\,[p - e_x(\cdot) - e_s(\cdot)] + u_c(\cdot)\{-[e_{xx}(\cdot) + 2e_{xs}(\cdot) + e_{ss}(\cdot)]\}$$

$$\begin{aligned} F_{ss}(\cdot) &= [p - e_x(\cdot) - e_s(\cdot)]\,u_{cc}(\cdot)\,[p - e_x(\cdot) - e_s(\cdot)] \\ &\quad + u_c(\cdot)\left\{-\left[\left\{-e_s(\cdot)(1+\gamma)x^{-2}s\right\} + 2\left\{e_s(\cdot)(1+\gamma)x^{-1}\right\} + \left\{-e_s(\cdot)(1+\gamma)s^{-1}\right\}\right]\right\} \end{aligned}$$

$$F_{ss}(\cdot) = \overset{\ominus}{u_{cc}}(\cdot)[p - e_x\overset{\oplus}{(\cdot)} - e_s(\cdot)]^2 + \overset{\oplus}{u_c}(\cdot)\left\{e_s(\cdot)\overset{\ominus}{(1}+\gamma)\left[x^{-2}s - 2x^{-1} + s^{-1}\right]\right\}$$

For $F_{ss}(\cdot) < 0$ to hold, $\left[x^{-2}s - 2x^{-1} + s^{-1}\right]$ must be positive

$$\begin{aligned} x^{-2}s - 2x^{-1} + s^{-1} &> 0 \\ \frac{1}{x}\left(\frac{s}{x} - 2\right) + \frac{1}{s} &> 0 \\ \frac{1}{x}\left(\frac{s}{x} - 2\right) &> -\frac{1}{s} \\ \left(\frac{s}{x} - 2\right) &> -\frac{x}{s} \\ \left(\frac{s}{x} + \frac{x}{s}\right) &> 2 \\ s^2 + x^2 - 2sx &> 0 \end{aligned}$$

$$(s-x)^2 > 0$$

$$(s'-k)^2 > 0$$

Which holds in domain of

$$k + s\left[1 - \left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s + k$$

$$s > 0$$

Finally for $F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot) > 0$

$$F_{s'}(\cdot) = -u_c(\cdot)\,(p - e_x(\cdot))$$

$$F_{s's'}(\cdot) = -\left(-p + e_x(\cdot)\right) u_{cc}(\cdot)\,(p - e_x(\cdot)) + \{-u_c(\cdot)\,[-e_{xs'}(\cdot)]\}$$

$$F_{s's'}(\cdot) = -\left(-p + e_x(\cdot)\right) u_{cc}(\cdot)\,(p - e_x(\cdot)) + \{u_c(\cdot)\,[e_{xs'}(\cdot)]\}$$

$$e_x(x,s) = (1+\gamma)\,\psi\left(\frac{s - s' + k}{s}\right)^{\gamma}$$

$$e_{xs'}(\cdot) = -\gamma(1+\gamma)\,\psi\left(\frac{x}{s}\right)^{\gamma} x^{-1} = -\gamma e_x(\cdot)\, x^{-1} = -e_{xx}(\cdot) < 0$$

$$F_{s's'}(\cdot) = u_{cc}(\cdot)\,(p - e_x(\cdot))^2 - \{u_c(\cdot)\,[e_{xx}(\cdot)]\}$$

And

$$F_s(\cdot) = u_c(\cdot)\,(p - e_x(\cdot) - e_s(\cdot))$$

$$F_{ss'}(\cdot) = [(-p + e_x(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot) - e_s(\cdot))] + [u_c(\cdot)\,(-e_{xs'}(\cdot) - e_{ss'}(\cdot))]$$

$$F_{ss'}(\cdot) = [(-p + e_x(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot) - e_s(\cdot))] - [u_c(\cdot)\,(e_{xs'}(\cdot) + e_{ss'}(\cdot))]$$

$$F_{ss'}(\cdot) = [(-p + e_x(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot) - e_s(\cdot))] - [u_c(\cdot)\,(-e_{xx}(\cdot) - e_{sx}(\cdot))]$$

$$F_{ss'}(\cdot) = [-(p - e_x(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot) - e_s(\cdot))] + [u_c(\cdot)\,(e_{xx}(\cdot) + e_{sx}(\cdot))]$$

And

$$F_{s's}(\cdot) = -(p - e_x(\cdot) - e_s(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot)) - u_c(\cdot)\,(-e_{xx}(\cdot) - e_{xs}(\cdot))$$

$$F_{s's}(\cdot) = -(p - e_x(\cdot) - e_s(\cdot))\, u_{cc}(\cdot)\,(p - e_x(\cdot)) + u_c(\cdot)\,(e_{xx}(\cdot) + e_{xs}(\cdot))$$

Let

$$M \equiv [p - e_x(\cdot) - e_s(\cdot)]$$

$$q^o \equiv [p - e_x(\cdot)]$$

$$A \equiv [e_{xx}(\cdot) + 2e_{xs}(\cdot) + e_{ss}(\cdot)]$$

$$B \equiv [e_{xx}(\cdot)]$$

$$C \equiv (e_{xx}(\cdot) + e_{sx}(\cdot))$$

Rewriting the system

$$F_{ss}(\cdot) = u_{cc}(\cdot)\, M^2 - u_c(\cdot)\, A$$

$$F_{s's'}(\cdot) = u_{cc}(\cdot)(q^o)^2 - u_c(\cdot)B$$

$$F_{ss'}(\cdot) = -u_{cc}(\cdot)Mq^o + u_c(\cdot)C$$

$$F_{s's}(\cdot) = -u_{cc}(\cdot)Mq^o + u_c(\cdot)C$$

Operating $F_{ss}(\cdot)F_{s's'}(\cdot)$

$$F_{ss}(\cdot)F_{s's'}(\cdot) = \left\{u_{cc}(\cdot)M^2 - u_c(\cdot)A\right\}\left\{u_{cc}(\cdot)(q^o)^2 - u_c(\cdot)B\right\}$$

$$F_{ss}(\cdot)F_{s's'}(\cdot) = [u_{cc}(\cdot)]^2[Mq^o]^2 - u_{cc}(\cdot)u_c(\cdot)BM^2 - u_{cc}(\cdot)u_c(\cdot)A[q^o]^2 + [u_c(\cdot)]^2 AB$$

And $F_{ss'}(\cdot)F_{s's}(\cdot)$

$$F_{s's}(\cdot)F_{ss'}(\cdot) = \{-u_{cc}(\cdot)Mq^o + u_c(\cdot)C\}\{-u_{cc}(\cdot)Mq^o + u_c(\cdot)C\}$$

$$F_{s's}(\cdot)F_{ss'}(\cdot) = [u_{cc}(\cdot)]^2[Mq^o]^2 - 2u_{cc}(\cdot)u_c(\cdot)CMq^o + [u_c(\cdot)]^2C^2$$

So $F_{ss}(\cdot)F_{s's'}(\cdot) - [F_{ss'}(\cdot)]^2 > 0$

$$\begin{aligned}F_{ss}(\cdot)F_{s's'}(\cdot) - F_{ss'}(\cdot)F_{s's}(\cdot) = {} & [u_{cc}(\cdot)]^2[Mq^o]^2 - u_{cc}(\cdot)u_c(\cdot)BM^2 - u_{cc}(\cdot)u_c(\cdot)A[q^o]^2 + [u_c(\cdot)]^2AB \\ & - [u_{cc}(\cdot)]^2[Mq^o]^2 + 2u_{cc}(\cdot)u_c(\cdot)CMq^o - [u_c(\cdot)]^2C^2 > 0\end{aligned}$$

$$F_{ss}(\cdot)F_{s's'}(\cdot) - F_{ss'}(\cdot)F_{s's}(\cdot) = -u_{cc}(\cdot)u_c(\cdot)\left[BM^2 - 2CMq^o + A(q^o)^2\right] + [u_c(\cdot)]^2\left[AB - C^2\right]$$

Replacing $\left[AB - C^2\right]$

$$AB = [e_{xx}(\cdot)]^2 + 2e_{xs}(\cdot)e_{xx}(\cdot) + e_{ss}(\cdot)e_{xx}(\cdot)$$

$$C^2 = [e_{xx}(\cdot)]^2 + 2e_{xx}(\cdot)\, e_{sx}(\cdot) + [e_{sx}(\cdot)]^2$$

$$\left[AB - C^2\right] = [e_{xx}(\cdot)]^2 + 2e_{xs}(\cdot)\, e_{xx}(\cdot) + e_{ss}(\cdot)\, e_{xx}(\cdot) - [e_{xx}(\cdot)]^2 - 2e_{xx}(\cdot)\, e_{sx}(\cdot) - [e_{sx}(\cdot)]^2$$

$$\left[AB - C^2\right] = e_{ss}(\cdot)\, e_{xx}(\cdot) - [e_{sx}(\cdot)]^2$$

Recall

1. $e_{xx}(x,s) = e_x(\cdot)\,\gamma x^{-1} = -e_s(\cdot)(1+\gamma)\, x^{-2} s$
2. $e_{xs}(x,s) = -e_x(\cdot)\,\gamma s^{-1} = e_s(\cdot)(1+\gamma)\, x^{-1}$
3. $e_{sx}(x,s) = e_s(\cdot)(1+\gamma)\, x^{-1} = e_{xs}(x,s)$
4. $e_{ss}(x,s) = -e_s(\cdot)(1+\gamma)\, s^{-1}$

$$\left[AB - C^2\right] = \left\{-e_s(\cdot)(1+\gamma)\, s^{-1}\right\}\left\{-e_s(\cdot)(1+\gamma)\, x^{-2} s\right\} - \left[e_s(\cdot)(1+\gamma)\, x^{-1}\right]^2$$

$$\left[AB - C^2\right] = \left\{[e_s(\cdot)]^2 (1+\gamma)^2 x^{-2}\right\} - \left[e_s(\cdot)(1+\gamma)\, x^{-1}\right]^2$$

$$\left[AB - C^2\right] = \left\{[e_s(\cdot)]^2 (1+\gamma)^2 x^{-2}\right\} - \left\{[e_s(\cdot)]^2 (1+\gamma)^2 x^{-2}\right\}$$

$$\left[AB - C^2\right] = 0$$

So the expression $F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot)$ is redefined as,

$$F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot) = -u_{cc}(\cdot)\, u_c(\cdot)\left[BM^2 - 2CMq^o + A\,(q^o)^2\right]$$

Then, since $-u_{cc}(\cdot)\, u_c(\cdot) > 0$, $F_{ss}(\cdot)\, F_{s's'}(\cdot) - F_{ss'}(\cdot)\, F_{s's}(\cdot) > 0$ holds if,

$$\left[BM^2 - 2CMq^o + A\left(q^o\right)^2\right] > 0$$

Let

$$Z \equiv \frac{M}{q^o}$$

$$\left[BZ^2 - 2CZ + A\right] > 0$$

Solving the inequality

$$Z > \frac{2C \pm \sqrt{4C^2 - 4AB}}{2B}$$

$$Z > \frac{C \pm 2\sqrt{C^2 - AB}}{B}$$

As we show $AB - C^2 = 0$

$$Z > \frac{C}{B}$$

Replacing $Z \equiv \frac{M}{q^o}$

$$\frac{M}{q^o} > \frac{C}{B}$$

Since $M = q^o - e_s\left(\cdot\right)$

$$\frac{q^o - e_s\left(\cdot\right)}{q^o} > \frac{C}{B}$$

$$1 - \frac{e_s\left(\cdot\right)}{q^o} > \frac{C}{B}$$

$$1 - \frac{C}{B} > \frac{e_s\left(\cdot\right)}{q^o}$$

$$\frac{B-C}{B} > \frac{e_s(\cdot)}{q^o}$$

Since $\frac{e_s(\cdot)}{q^o} < 0$ it is sufficient to show $B - C > 0$

Recall

$$B \equiv [e_{xx}(\cdot)]$$

$$C \equiv (e_{xx}(\cdot) + e_{sx}(\cdot))$$

Then

$$B - C > 0$$

$$e_{xx}(\cdot) - e_{xx}(\cdot) - e_{sx}(\cdot) > 0$$

$$-e_{sx}(\cdot) > 0$$

Recall $e_{sx}(\cdot) = e_s(\cdot)(1+\gamma)x^{-1}$

$$-e_s(\cdot)(1+\gamma)x^{-1} > 0$$

Since $e_s(\cdot) < 0$, the condition holds in domain of

$$k + s\left[1 - \left(\frac{p}{\psi}\right)^{\frac{1}{\gamma}}\right] \leq s' \leq s + k$$

$$s > 0$$

# J Business Cycle Moments by Country

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| | | | **Defaulters** | | | | |
| **1. Algeria** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.12147 | 1 | 0.60593 | 0.86634 | 0.84749 |
| Non-oil GDP | 0 | 0.049395 | 0.47551 | -0.23628 | -0.094443 | 0.11386 | 0.52334 |
| GDP | 0 | 0.024752 | 1 | 0.12147 | 0.15821 | 0.34198 | 0.73414 |
| Oil production | 0 | 0.053372 | 0.3074 | 0.18849 | 0.25201 | 0.14864 | 0.63562 |
| Consumption | 0 | 0.033709 | 0.50335 | 0.030535 | 0.026005 | 0.21056 | 0.56724 |
| Gross oil output | 0 | 0.20108 | 0.2301 | 0.42278 | 0.23747 | 0.19343 | 0.25491 |
| Trade balance to GDP | 0.051308 | 0.094454 | 0.096215 | 0.26867 | 0.47205 | 0.0025333 | 0.74715 |
| Institutional Investor Index | 42.7099 | 12.2003 | 0.34198 | 0.86634 | 0.38626 | 1 | 0.93769 |
| Debt to GDP | 0.32898 | 0.22219 | -0.39214 | -0.76632 | -0.79507 | -0.73833 | 0.92074 |
| Reserves | 9.9381 | 1.471 | 0.15821 | 0.60593 | 1 | 0.38626 | 0.89187 |
| Total Debt | 51.4296 | 28.23 | -0.3002 | -0.82656 | -0.82417 | -0.72306 | 0.91817 |
| **2. Angola** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.1278 | 1 | 0.43503 | 0.77286 | 0.84749 |
| Non-oil GDP | 0 | 0.14135 | 0.5386 | -0.13 | -0.00055829 | 0.025451 | 0.29789 |
| GDP | 0 | 0.077409 | 1 | 0.1278 | 0.076171 | 0.093347 | 0.65645 |
| Oil production | 0 | 0.12037 | 0.63629 | 0.047291 | 0.10218 | 0.11615 | 0.6129 |
| Consumption | 0 | 0.029976 | -0.13614 | 0.018858 | 0.057124 | -0.054983 | 0.29984 |
| Gross oil output | 0 | 0.22578 | 0.45932 | 0.3572 | 0.075461 | 0.13198 | 0.33259 |
| Trade balance to GDP | 0.16158 | 0.10714 | 0.18158 | 0.21714 | -0.00080032 | 0.030024 | 0.1676 |
| Institutional Investor Index | 19.1445 | 9.5761 | 0.093347 | 0.77286 | 0.80215 | 1 | 0.91153 |
| Debt to GDP | 0.73997 | 0.64742 | -0.53112 | -0.69736 | -0.51013 | -0.64947 | 0.83586 |
| Reserves | 4.6649 | 3.0696 | 0.076171 | 0.43503 | 1 | 0.80215 | 0.90737 |
| Total Debt | 72.7056 | 59.0513 | -0.19186 | -0.66452 | -0.28245 | -0.60741 | 0.70937 |
| **3. Argentina** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.034086 | 1 | 0.31432 | 0.26231 | 0.84749 |
| Non-oil GDP | 0 | 0.064923 | 0.99358 | 0.0042762 | -0.30157 | 0.56915 | 0.60668 |
| GDP | 0 | 0.061199 | 1 | 0.034086 | -0.31465 | 0.5831 | 0.6004 |
| Oil production | 0 | 0.051743 | 0.30411 | -0.091604 | 0.18964 | 0.50553 | 0.78416 |
| Consumption | 0 | 0.067635 | 0.95562 | 0.017355 | -0.35319 | 0.61901 | 0.59212 |
| Gross oil output | 0 | 0.17838 | 0.13156 | 0.39824 | -0.077621 | 0.27276 | 0.17326 |
| Trade balance to GDP | 0.023568 | 0.039506 | -0.72317 | 0.14959 | 0.45455 | -0.56829 | 0.67007 |
| Institutional Investor Index | 32.8994 | 11.0973 | 0.5831 | 0.26231 | 0.035197 | 1 | 0.71106 |
| Debt to GDP | 0.3049 | 0.18544 | -0.64625 | -0.37286 | 0.28406 | -0.59689 | 0.64215 |
| Reserves | 2.4436 | 0.35208 | -0.31465 | 0.31432 | 1 | 0.035197 | 0.80351 |
| Total Debt | 51.6696 | 28.6498 | -0.6713 | -0.15385 | 0.41493 | -0.63168 | 0.67686 |

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| **4. Brazil** | | | | | | | |
| Oil price | 0 | 0.18202 | -0.024834 | 1 | 0.49806 | 0.79039 | 0.84749 |
| Non-oil GDP | 0 | 0.031904 | 0.995 | -0.049718 | -0.005606 | 0.13673 | 0.53178 |
| GDP | 0 | 0.030964 | 1 | -0.024834 | -0.00017989 | 0.12743 | 0.51617 |
| Oil production | 0 | 0.098616 | -0.042723 | -0.049722 | 0.043541 | -0.17762 | 0.74246 |
| Consumption | 0 | 0.023024 | 0.68973 | -0.050018 | -0.016978 | 0.15284 | 0.62325 |
| Gross oil output | 0 | 0.19681 | -0.14609 | 0.36011 | 0.066779 | 0.007495 | 0.19901 |
| Trade balance to GDP | 0.0062973 | 0.02584 | -0.14629 | -0.072047 | -0.34401 | -0.46856 | 0.77803 |
| Institutional Investor Index | 44.1188 | 14.4936 | 0.12743 | 0.79039 | 0.7969 | 1 | 0.91109 |
| Debt to GDP | 0.17167 | 0.098496 | -0.18291 | -0.39401 | -0.81942 | -0.78011 | 0.92278 |
| Reserves | 6.4396 | 4.5441 | -0.00017989 | 0.49806 | 1 | 0.7969 | 0.91461 |
| Total Debt | 60.4659 | 16.342 | -0.16848 | -0.19718 | 0.28742 | -0.12239 | 0.61726 |
| **5. Ecuador** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.051057 | 1 | 0.58312 | 0.5854 | 0.84749 |
| Non-oil GDP | 0 | 0.037235 | 0.54629 | -0.21933 | 0.017453 | 0.14004 | 0.25751 |
| GDP | 0 | 0.020902 | 1 | 0.051057 | 0.056465 | 0.28225 | 0.45286 |
| Oil production | 0 | 0.097024 | 0.39881 | 0.033802 | 0.050543 | 0.052916 | -0.046454 |
| Consumption | 0 | 0.028389 | 0.77422 | 0.023455 | 0.02264 | 0.22543 | 0.32885 |
| Gross oil output | 0 | 0.20848 | 0.24602 | 0.37527 | 0.037589 | 0.11712 | 0.34209 |
| Trade balance to GDP | -0.013223 | 0.029856 | -0.094194 | 0.21103 | -0.067087 | 0.045609 | 0.24675 |
| Institutional Investor Index | 27.7376 | 9.1089 | 0.28225 | 0.5854 | -0.022518 | 1 | 0.79698 |
| Debt to GDP | 0.42338 | 0.21561 | -0.22603 | -0.88703 | -0.6235 | -0.57583 | 0.89807 |
| Reserves | 3.0755 | 2.1566 | 0.056465 | 0.58312 | 1 | -0.022518 | 0.8648 |
| Total Debt | 54.1452 | 27.8287 | -0.18635 | -0.80185 | -0.62826 | -0.58214 | 0.89795 |
| **6. Gabon** | | | | | | | |
| Oil price | 0 | 0.18202 | -0.042988 | 1 | 0.07798 | 0.63564 | 0.84749 |
| Non-oil GDP | 0 | 0.12207 | 0.48721 | -0.23237 | -0.053168 | 0.067017 | 0.14097 |
| GDP | 0 | 0.049374 | 1 | -0.042988 | -0.0037056 | -0.026527 | 0.42249 |
| Oil production | 0 | 0.10043 | 0.44186 | 0.047309 | 0.009268 | -0.12596 | 0.64414 |
| Consumption | 0 | 0.059532 | 0.34465 | -0.032275 | -0.011857 | 0.1533 | 0.15307 |
| Gross oil output | 0 | 0.2277 | 0.17888 | 0.35005 | -0.057871 | -0.10572 | 0.31161 |
| Trade balance to GDP | 0.19825 | 0.12625 | 0.070059 | 0.47261 | 0.48684 | -0.21946 | 0.65996 |
| Institutional Investor Index | 30.9989 | 6.8734 | -0.026527 | 0.63564 | -0.36294 | 1 | 0.87933 |
| Debt to GDP | 0.46043 | 0.22137 | -0.019771 | -0.87982 | 0.035243 | -0.71143 | 0.88292 |
| Reserves | 1.4738 | 0.82694 | -0.0037056 | 0.07798 | 1 | -0.36294 | 0.93084 |
| Total Debt | 54.4104 | 25.9217 | -0.055267 | -0.83435 | 0.078494 | -0.72078 | 0.83663 |

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| **7. Indonesia** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.053508 | 1 | -0.11502 | 0.45531 | 0.84749 |
| Non-oil GDP | 0 | 0.046084 | 0.89693 | -0.034757 | -0.070217 | 0.54932 | 0.57237 |
| GDP | 0 | 0.03981 | 1 | 0.053508 | -0.083444 | 0.59106 | 0.65606 |
| Oil production | 0 | 0.042957 | 0.20162 | -0.09781 | -0.012229 | 0.081818 | 0.46736 |
| Consumption | 0 | 0.033207 | 0.75728 | 0.022585 | -0.04999 | 0.3798 | 0.51527 |
| Gross oil output | 0 | 0.18137 | 0.31897 | 0.39463 | 0.064644 | 0.22136 | 0.16729 |
| Trade balance to GDP | 0.029811 | 0.034131 | -0.59189 | -0.25534 | -0.1687 | -0.73348 | 0.57345 |
| Institutional Investor Index | 46.395 | 10.1564 | 0.59106 | 0.45531 | 0.22458 | 1 | 0.89033 |
| Debt to GDP | 0.31423 | 0.13998 | -0.45537 | -0.80597 | 0.2782 | -0.50932 | 0.74508 |
| Reserves | 6.2538 | 2.2037 | -0.083444 | -0.11502 | 1 | 0.22458 | 0.85514 |
| Total Debt | 39.2984 | 19.4416 | -0.62056 | -0.59725 | -0.21956 | -0.86968 | 0.82975 |
| **8. Iran** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.15857 | 1 | 0.40507 | 0.054683 | 0.84749 |
| Non-oil GDP | 0 | 0.073453 | 0.60377 | -0.050171 | -0.10405 | -0.015032 | 0.3807 |
| GDP | 0 | 0.073615 | 1 | 0.15857 | 0.098755 | 0.35133 | 0.28258 |
| Oil production | 0 | 0.12585 | 0.86464 | 0.095334 | 0.19691 | 0.40041 | 0.07954 |
| Consumption | 0 | 0.060214 | 0.73043 | 0.21046 | 0.030794 | 0.28754 | 0.61748 |
| Gross oil output | 0 | 0.24475 | 0.69768 | 0.35528 | 0.18975 | 0.30947 | 0.17973 |
| Trade balance to GDP | -0.0076219 | 0.078019 | 0.29908 | 0.097042 | 0.50352 | 0.55082 | 0.5453 |
| Institutional Investor Index | 27.2039 | 7.7952 | 0.35133 | 0.054683 | 0.68527 | 1 | 0.83778 |
| Debt to GDP | 0.042387 | 0.04314 | 0.038605 | -0.49961 | -0.11957 | 0.1153 | 0.77461 |
| Reserves | 97.671 | 32.9913 | 0.098755 | 0.40507 | 1 | 0.68527 | 0.8819 |
| Total Debt | 25.8154 | 14.863 | -0.38321 | -0.37097 | -0.52886 | -0.62742 | 0.74972 |
| **9. Iraq** | | | | | | | |
| Oil price | 0 | 0.18202 | -0.010138 | 1 | -0.019418 | 0.68839 | 0.84749 |
| Non-oil GDP | 0 | 0.17174 | 0.26918 | -0.047664 | -0.097586 | 0.085655 | 0.029526 |
| GDP | 0 | 0.18404 | 1 | -0.010138 | -0.029922 | 0.080636 | 0.16095 |
| Oil production | 0 | 0.45619 | 0.71465 | -0.061324 | 0.13592 | 0.079961 | 0.58805 |
| Consumption | 0 | NaN | NaN | NaN | NaN | NaN | NaN |
| Gross oil output | 0 | 0.45053 | 0.70353 | 0.10428 | 0.13296 | 0.11704 | 0.44848 |
| Trade balance to GDP | 0.045435 | 0.15317 | 0.11515 | 0.091827 | 0.50183 | -0.00022145 | 0.46848 |
| Institutional Investor Index | 18.662 | 13.346 | 0.080636 | 0.68839 | -0.44838 | 1 | 0.80853 |
| Debt to GDP | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| Reserves | 95.4903 | 33.2975 | -0.029922 | -0.019418 | 1 | -0.44838 | 0.86665 |
| Total Debt | 109.4799 | 98.7461 | -0.020384 | -0.24542 | -0.10036 | -0.2338 | 0.5729 |

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| **10. Mexico** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.092198 | 1 | -0.60096 | 0.72544 | 0.84749 |
| Non-oil GDP | 0 | 0.032551 | 0.92799 | -0.011324 | 0.02944 | 0.24804 | 0.32856 |
| GDP | 0 | 0.030111 | 1 | 0.092198 | 0.017065 | 0.20485 | 0.34017 |
| Oil production | 0 | 0.077306 | 0.55276 | 0.0087796 | 0.033767 | -0.090297 | 0.50157 |
| Consumption | 0 | 0.031051 | 0.93123 | 0.12556 | -0.056781 | 0.27185 | 0.41967 |
| Gross oil output | 0 | 0.19909 | 0.45499 | 0.3799 | -0.048464 | 0.10932 | 0.21382 |
| Trade balance to GDP | -0.0022552 | 0.03185 | -0.22031 | 0.030631 | 0.38487 | -0.47624 | 0.7678 |
| Institutional Investor Index | 52.5603 | 14.5972 | 0.20485 | 0.72544 | -0.80462 | 1 | 0.88926 |
| Debt to GDP | 0.22286 | 0.12516 | -0.35314 | -0.209 | 0.60275 | -0.72996 | 0.87573 |
| Reserves | 34.5215 | 18.1988 | 0.017065 | -0.60096 | 1 | -0.80462 | 0.91667 |
| Total Debt | 46.7147 | 11.6141 | -0.4022 | -0.22242 | 0.38067 | -0.61867 | 0.72589 |
| **11. Nigeria** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.17039 | 1 | 0.63731 | 0.87623 | 0.84749 |
| Non-oil GDP | 0 | 0.069396 | 0.32629 | -0.037719 | -0.055911 | 0.1627 | 0.16864 |
| GDP | 0 | 0.055495 | 1 | 0.17039 | 0.10299 | 0.19702 | 0.70152 |
| Oil production | 0 | 0.089818 | 0.62918 | 0.07489 | 0.091378 | 0.0054419 | 0.48725 |
| Consumption | 0 | 0.11459 | 0.5226 | 0.088171 | 0.05639 | 0.13622 | 0.25979 |
| Gross oil output | 0 | 0.21086 | 0.43751 | 0.38738 | 0.18211 | 0.15793 | 0.27889 |
| Trade balance to GDP | 0.071467 | 0.05457 | 0.20338 | 0.096052 | 0.24205 | -0.19605 | 0.14642 |
| Institutional Investor Index | 28.5439 | 12.419 | 0.19702 | 0.87623 | 0.35007 | 1 | 0.88691 |
| Debt to GDP | 0.29304 | 0.26822 | 0.070585 | -0.84592 | -0.64236 | -0.75736 | 0.89848 |
| Reserves | 23.6913 | 8.6542 | 0.10299 | 0.63731 | 1 | 0.35007 | 0.92064 |
| Total Debt | 66.8785 | 54.7299 | -0.045511 | -0.78722 | -0.71858 | -0.65879 | 0.87998 |
| **12. Russia** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.2719 | 1 | 0.55311 | 0.80289 | 0.84749 |
| Non-oil GDP | 0 | 0.062358 | 0.94979 | 0.23707 | 0.2257 | 0.30558 | 0.63282 |
| GDP | 0 | 0.067967 | 1 | 0.2719 | 0.19922 | 0.27066 | 0.61568 |
| Oil production | 0 | 0.062511 | 0.88223 | 0.1946 | 0.22979 | 0.26284 | 0.6484 |
| Consumption | 0 | 0.048853 | 0.76437 | 0.27278 | 0.23107 | 0.32728 | 0.7311 |
| Gross oil output | 0 | 0.21767 | 0.81236 | 0.3303 | 0.13583 | 0.22346 | 0.32 |
| Trade balance to GDP | 0.09276 | 0.044501 | 0.26678 | -0.043356 | -0.33218 | -0.078509 | 0.64272 |
| Institutional Investor Index | 44.0281 | 20.5408 | 0.27066 | 0.80289 | 0.59573 | 1 | 0.92518 |
| Debt to GDP | 0.20467 | 0.13006 | -0.59291 | -0.66382 | -0.60589 | -0.76039 | 0.77786 |
| Reserves | 58.4132 | 9.512 | 0.19922 | 0.55311 | 1 | 0.59573 | 0.64457 |
| Total Debt | 38.8104 | 31.7584 | -0.016431 | -0.69739 | -0.45332 | -0.86177 | 0.77349 |

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| **13. Sudan** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.10269 | 1 | 0.74135 | 0.54717 | 0.84749 |
| Non-oil GDP | 0 | 0.038045 | 0.52293 | -0.13108 | 0.059501 | 0.23654 | 0.25786 |
| GDP | 0 | 0.051153 | 1 | 0.10269 | 0.14085 | 0.4381 | 0.57484 |
| Oil production | 0 | 0.72661 | 0.36305 | -0.0084862 | -0.082255 | 0.16519 | 0.49178 |
| Consumption | 0 | 0.051675 | 0.79625 | 0.07848 | 0.11023 | 0.34266 | 0.55617 |
| Gross oil output | 0 | 0.76737 | 0.40793 | 0.069806 | -0.052558 | 0.20926 | 0.38924 |
| Trade balance to GDP | -0.033098 | 0.036949 | 0.21206 | 0.45423 | 0.52063 | 0.41664 | 0.33045 |
| Institutional Investor Index | 9.435 | 2.4371 | 0.4381 | 0.54717 | 0.46449 | 1 | 0.78572 |
| Debt to GDP | 0.57049 | 0.30479 | -0.25856 | -0.68829 | -0.71035 | -0.82402 | 0.75686 |
| Reserves | 1.9854 | 2.2537 | 0.14085 | 0.74135 | 1 | 0.46449 | 0.85812 |
| Total Debt | 151.2396 | 97.4567 | -0.13478 | -0.57317 | -0.58696 | -0.77624 | 0.59737 |
| **14. Venezuela** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.10281 | 1 | 0.43225 | 0.50023 | 0.84749 |
| Non-oil GDP | 0 | 0.091015 | 0.66055 | -0.12132 | -0.056011 | 0.15224 | 0.35908 |
| GDP | 0 | 0.058581 | 1 | 0.10281 | -0.024891 | 0.33555 | 0.53322 |
| Oil production | 0 | 0.06455 | 0.59643 | -0.011309 | 0.13291 | 0.28042 | 0.55666 |
| Consumption | 0 | 0.061368 | 0.85383 | 0.15415 | -0.12276 | 0.32052 | 0.59518 |
| Gross oil output | 0 | 0.19167 | 0.41066 | 0.38726 | -0.005015 | 0.25628 | 0.23017 |
| Trade balance to GDP | 0.058102 | 0.077381 | -0.32614 | -0.037448 | -0.32518 | -0.12908 | 0.36541 |
| Institutional Investor Index | 40.0426 | 11.0146 | 0.33555 | 0.50023 | -0.30061 | 1 | 0.79414 |
| Debt to GDP | 0.30327 | 0.12997 | -0.21128 | -0.81101 | -0.38869 | -0.43579 | 0.85081 |
| Reserves | 81.6334 | 68.6858 | -0.024891 | 0.43225 | 1 | -0.30061 | 0.7755 |
| Total Debt | 38.5001 | 16.0959 | -0.41222 | -0.15469 | 0.58401 | -0.69161 | 0.75068 |
| **15. Vietnam** | | | | | | | |
| Oil price | 0 | 0.18202 | -0.063041 | 1 | 0.46626 | 0.73752 | 0.84749 |
| Non-oil GDP | 0 | 0.024728 | 0.84349 | -0.18059 | -0.066845 | 0.13148 | 0.66492 |
| GDP | 0 | 0.016654 | 1 | -0.063041 | -0.13467 | 0.18874 | 0.6636 |
| Oil production | 0 | 0.34284 | -0.7322 | 0.0038434 | 0.0055359 | -0.16185 | 0.35955 |
| Consumption | 0 | 0.023801 | 0.54768 | 0.11043 | -0.19696 | 0.33064 | 0.67519 |
| Gross oil output | 0 | 0.41887 | -0.64234 | 0.15838 | -0.03544 | -0.032436 | 0.34928 |
| Trade balance to GDP | -0.068455 | 0.047577 | -0.2561 | 0.30688 | 0.67755 | 0.04364 | 0.55626 |
| Institutional Investor Index | 35.8785 | 9.7368 | 0.18874 | 0.73752 | 0.35861 | 1 | 0.84578 |
| Debt to GDP | 0.76606 | 0.89076 | -0.44188 | -0.32025 | -0.17845 | -0.43097 | 0.73128 |
| Reserves | 1.05 | 1.2941 | -0.13467 | 0.46626 | 1 | 0.35861 | 0.66284 |
| Total Debt | 68.1786 | 48.4276 | -0.1675 | -0.39461 | -0.16143 | -0.70647 | 0.55072 |

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| **16. Yemen** | | | | | | | |
| Oil price | 0 | 0.18202 | -0.0444 | 1 | -0.12154 | -0.13002 | 0.84749 |
| Non-oil GDP | 0 | 0.094879 | 0.31162 | -0.14784 | -0.045711 | -0.13788 | 0.29921 |
| GDP | 0 | 0.027804 | 1 | -0.0444 | -0.025765 | 0.46008 | 0.17201 |
| Oil production | 0 | 0.3665 | 0.057148 | -0.0039444 | 0.35407 | 0.063962 | 0.33813 |
| Consumption | 0 | NaN | NaN | NaN | NaN | NaN | NaN |
| Gross oil output | 0 | 0.42209 | 0.011548 | 0.15062 | 0.33423 | 0.1877 | 0.41198 |
| Trade balance to GDP | -0.06614 | 0.035184 | 0.23654 | -0.021483 | -7.29e-17 | 0.2341 | -0.11407 |
| Institutional Investor Index | 26.062 | 4.6396 | 0.46008 | -0.13002 | 0.11777 | 1 | 0.804 |
| Debt to GDP | 0.533 | 0.36362 | -0.026591 | -0.67409 | 0.25251 | 0.059943 | 0.84729 |
| Reserves | 3.3666 | 1.1196 | -0.025765 | -0.12154 | 1 | 0.11777 | 0.62868 |
| Total Debt | 76.747 | 47.0205 | -0.031907 | -0.57385 | 0.19513 | -0.042079 | 0.6939 |
| | | | **Non-Defaulters** | | | | |
| **17. Azerbaijan** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.21181 | 1 | 0.62293 | 0.42364 | 0.84749 |
| Non-oil GDP | 0 | 0.14018 | 0.87752 | 0.084549 | 0.16139 | 0.16346 | 0.46703 |
| GDP | 0 | 0.14569 | 1 | 0.21181 | 0.28169 | 0.17802 | 0.61574 |
| Oil production | 0 | 0.16765 | 0.84413 | 0.13629 | 0.088299 | 0.30632 | 0.65709 |
| Consumption | 0 | 0.15356 | 0.86902 | 0.17327 | 0.13545 | 0.10888 | 0.46498 |
| Gross oil output | 0 | 0.28027 | 0.78874 | 0.29466 | 0.27724 | 0.29143 | 0.52646 |
| Trade balance to GDP | 0.051233 | 0.25159 | 0.66768 | 0.69511 | 0.53064 | 0.51948 | 0.75444 |
| Institutional Investor Index | 43.047 | 8.9829 | 0.17802 | 0.42364 | 0.27873 | 1 | 0.74146 |
| Debt to GDP | 0.097518 | 0.039908 | -0.37481 | -0.13065 | -0.2358 | -0.62979 | 0.71651 |
| Reserves | 5.0593 | 2.8241 | 0.28169 | 0.62293 | 1 | 0.27873 | 0.80556 |
| Total Debt | 15.5461 | 5.9003 | -0.2411 | -0.47873 | -0.59967 | -0.4971 | 0.77035 |
| **18. China** | | | | | | | |
| Oil price | 0 | 0.18202 | -0.036086 | 1 | -0.47398 | 0.84405 | 0.84749 |
| Non-oil GDP | 0 | 0.032768 | 0.96633 | -0.10071 | -0.28496 | 0.28244 | 0.67975 |
| GDP | 0 | 0.030865 | 1 | -0.036086 | -0.27416 | 0.3101 | 0.68824 |
| Oil production | 0 | 0.027673 | 0.57384 | -0.060982 | -0.072393 | 0.19003 | 0.57637 |
| Consumption | 0 | 0.017956 | 0.16566 | 0.061627 | 0.10645 | 0.047153 | 0.41269 |
| Gross oil output | 0 | 0.17455 | -0.030224 | 0.41976 | -0.12395 | 0.09737 | 0.11062 |
| Trade balance to GDP | 0.020206 | 0.027838 | -0.096362 | 0.21084 | -0.21181 | 0.19404 | 0.75323 |
| Institutional Investor Index | 65.2294 | 8.445 | 0.3101 | 0.84405 | -0.55666 | 1 | 0.90512 |
| Debt to GDP | 0.066389 | 0.046554 | 0.05866 | -0.78616 | 0.61028 | -0.79031 | 0.9377 |
| Reserves | 21.1141 | 2.8723 | -0.27416 | -0.47398 | 1 | -0.55666 | 0.80603 |
| Total Debt | 19.4289 | 11.9003 | -0.082252 | 0.52356 | -0.098074 | 0.57411 | 0.87207 |

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| **19. Colombia** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.023415 | 1 | -0.47787 | 0.77307 | 0.84749 |
| Non-oil GDP | 0 | 0.027483 | 0.93808 | -0.048825 | 0.29114 | 0.33976 | 0.69423 |
| GDP | 0 | 0.025402 | 1 | 0.023415 | 0.34775 | 0.32465 | 0.72272 |
| Oil production | 0 | 0.12805 | 0.13207 | -0.22519 | 0.38682 | 0.0021588 | 0.63081 |
| Consumption | 0 | 0.026964 | 0.87274 | -0.016224 | 0.40004 | 0.40695 | 0.79227 |
| Gross oil output | 0 | 0.17366 | 0.2131 | 0.27031 | 0.1534 | 0.10656 | 0.08262 |
| Trade balance to GDP | -0.017071 | 0.034053 | -0.36393 | -0.028673 | -0.29959 | -0.35708 | 0.73896 |
| Institutional Investor Index | 47.6329 | 9.3616 | 0.32465 | 0.77307 | -0.1543 | 1 | 0.86378 |
| Debt to GDP | 0.20429 | 0.076394 | -0.35729 | -0.51183 | -0.079178 | -0.76895 | 0.88924 |
| Reserves | 1.7719 | 0.74994 | 0.34775 | -0.47787 | 1 | -0.1543 | 0.8471 |
| Total Debt | 33.028 | 8.3529 | -0.42967 | -0.18933 | 0.44906 | -0.26016 | 0.78438 |
| **20. Egypt** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.00098939 | 1 | -0.18737 | 0.28561 | 0.84749 |
| Non-oil GDP | 0 | 0.04091 | 0.69954 | -0.2254 | -0.20686 | 0.14379 | 0.56325 |
| GDP | 0 | 0.021784 | 1 | 0.00098939 | -0.29075 | 0.23407 | 0.65191 |
| Oil production | 0 | 0.042294 | 0.3223 | -0.1803 | -0.032266 | -0.14919 | 0.42761 |
| Consumption | 0 | 0.014297 | 0.44185 | -0.047312 | -0.018467 | -0.05899 | 0.33011 |
| Gross oil output | 0 | 0.17533 | 0.14789 | 0.38401 | -0.051724 | 0.056272 | 0.030664 |
| Trade balance to GDP | -0.076504 | 0.041234 | -0.15671 | -0.094604 | 0.11054 | 0.52514 | 0.75274 |
| Institutional Investor Index | 37.2455 | 9.2005 | 0.23407 | 0.28561 | -0.43256 | 1 | 0.92446 |
| Debt to GDP | 0.4809 | 0.27319 | 0.040232 | -0.30082 | 0.035121 | -0.74259 | 0.9244 |
| Reserves | 3.8619 | 0.78127 | -0.29075 | -0.18737 | 1 | -0.43256 | 0.60942 |
| Total Debt | 96.6575 | 22.9784 | -0.28934 | 0.0050562 | 0.072672 | -0.59136 | 0.85361 |
| **21. India** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.11474 | 1 | -0.12386 | 0.74406 | 0.84749 |
| Non-oil GDP | 0 | 0.014836 | 0.98202 | 0.063684 | -0.013067 | 0.22294 | 0.31528 |
| GDP | 0 | 0.015102 | 1 | 0.11474 | 0.022112 | 0.22986 | 0.3558 |
| Oil production | 0 | 0.10639 | 0.2196 | -0.071514 | -0.038504 | 0.088205 | 0.32216 |
| Consumption | 0 | 0.013859 | 0.69138 | 0.021225 | -0.12051 | 0.1975 | 0.50988 |
| Gross oil output | 0 | 0.20193 | 0.26979 | 0.33757 | 0.18667 | 0.158 | 0.1558 |
| Trade balance to GDP | -0.022496 | 0.017243 | -0.2384 | -0.75674 | 0.029634 | -0.88982 | 0.83753 |
| Institutional Investor Index | 50.6239 | 7.5071 | 0.22986 | 0.74406 | 0.083211 | 1 | 0.92655 |
| Debt to GDP | 0.14051 | 0.070829 | -0.15753 | -0.80208 | 0.3202 | -0.85013 | 0.94031 |
| Reserves | 5.0273 | 1.269 | 0.022112 | -0.12386 | 1 | 0.083211 | 0.81426 |
| Total Debt | 65.0445 | 15.2329 | -0.26017 | -0.078496 | 0.51251 | 0.10151 | 0.8038 |

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| **22. Kazakhstan** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.23532 | 1 | 0.64444 | 0.78508 | 0.84749 |
| Non-oil GDP | 0 | 0.072999 | 0.86562 | 0.14236 | 0.30521 | 0.13985 | 0.50661 |
| GDP | 0 | 0.07723 | 1 | 0.23532 | 0.23419 | 0.18287 | 0.67725 |
| Oil production | 0 | 0.074574 | 0.82865 | 0.13551 | -0.10057 | 0.1705 | 0.5863 |
| Consumption | 0 | 0.094576 | 0.93877 | 0.24272 | 0.3022 | 0.1595 | 0.65169 |
| Gross oil output | 0 | 0.22091 | 0.69801 | 0.31615 | 0.15733 | 0.18155 | 0.29574 |
| Trade balance to GDP | 0.051226 | 0.085722 | 0.095318 | 0.77938 | 0.56057 | 0.88558 | 0.7302 |
| Institutional Investor Index | 39.8525 | 16.1737 | 0.18287 | 0.78508 | 0.56789 | 1 | 0.90127 |
| Debt to GDP | 0.080684 | 0.059606 | -0.70165 | -0.59714 | -0.70868 | -0.56454 | 0.82654 |
| Reserves | 17.139 | 11.913 | 0.23419 | 0.64444 | 1 | 0.56789 | 0.84319 |
| Total Debt | 15.7604 | 8.0704 | -0.21697 | -0.60581 | -0.42935 | -0.75388 | 0.6264 |
| **23. Kuwait** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.095057 | 1 | -0.15293 | 0.84603 | 0.84749 |
| Non-oil GDP | 0 | 0.09616 | 0.36697 | -0.14248 | 0.0068189 | 0.012673 | 0.1878 |
| GDP | 0 | 0.074964 | 1 | 0.095057 | 0.04395 | 0.059636 | 0.46165 |
| Oil production | 0 | 0.36662 | 0.21689 | -0.047836 | 0.11408 | 0.23503 | 0.20665 |
| Consumption | 0 | NaN | NaN | NaN | NaN | NaN | NaN |
| Gross oil output | 0 | 0.37911 | 0.3281 | 0.15145 | 0.093032 | 0.27573 | 0.055362 |
| Trade balance to GDP | 0.16462 | 0.27861 | 0.14304 | 0.60546 | 0.054319 | 0.79931 | 0.55377 |
| Institutional Investor Index | 64.7681 | 9.8694 | 0.059636 | 0.84603 | -0.014489 | 1 | 0.8388 |
| Debt to GDP | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| Reserves | 93.5138 | 11.0614 | 0.04395 | -0.15293 | 1 | -0.014489 | 0.83333 |
| Total Debt | 37.5747 | 43.8855 | -0.012044 | -0.64745 | 0.18429 | -0.88504 | 0.70237 |
| **24. Libya** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.024853 | 1 | 0.5899 | 0.76233 | 0.84749 |
| Non-oil GDP | 0 | 0.20823 | 0.76677 | -0.077611 | 0.0064743 | 0.097558 | -0.017665 |
| GDP | 0 | 0.21593 | 1 | 0.024853 | 0.021208 | 0.16785 | -0.21674 |
| Oil production | 0 | 0.21329 | 0.88271 | 0.084693 | 0.064272 | 0.26356 | -0.12773 |
| Consumption | 0 | NaN | NaN | NaN | NaN | NaN | NaN |
| Gross oil output | 0 | 0.29694 | 0.64816 | 0.31326 | 0.098807 | 0.32712 | 0.14199 |
| Trade balance to GDP | 0.13715 | 0.19328 | 0.39705 | 0.29135 | 0.18841 | 0.38285 | 0.484 |
| Institutional Investor Index | 35.8695 | 9.8611 | 0.16785 | 0.76233 | 0.44969 | 1 | 0.83257 |
| Debt to GDP | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| Reserves | 30.4673 | 9.4812 | 0.021208 | 0.5899 | 1 | 0.44969 | 0.91999 |
| Total Debt | 33.4442 | 25.0854 | -0.24876 | -0.78284 | -0.55695 | -0.71205 | 0.88186 |

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| **25. Malaysia** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.11833 | 1 | -0.050013 | 0.72303 | 0.84749 |
| Non-oil GDP | 0 | 0.039384 | 0.93874 | 0.015836 | 0.43921 | 0.50438 | 0.65475 |
| GDP | 0 | 0.037482 | 1 | 0.11833 | 0.36499 | 0.53318 | 0.61325 |
| Oil production | 0 | 0.048213 | -0.078619 | -0.12784 | -0.25106 | -0.15053 | 0.34382 |
| Consumption | 0 | 0.051255 | 0.89881 | 0.22268 | 0.24793 | 0.60973 | 0.67068 |
| Gross oil output | 0 | 0.1813 | 0.44124 | 0.38397 | -0.1751 | 0.28339 | 0.21367 |
| Trade balance to GDP | 0.094175 | 0.099975 | -0.43144 | 0.069919 | 0.11728 | -0.22257 | 0.8577 |
| Institutional Investor Index | 65.5677 | 6.4491 | 0.53318 | 0.72303 | 0.24617 | 1 | 0.84407 |
| Debt to GDP | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| Reserves | 3.4504 | 0.54123 | 0.36499 | -0.050013 | 1 | 0.24617 | 0.79896 |
| Total Debt | 54.9042 | 21.0568 | -0.48645 | 0.0056616 | -0.49443 | -0.21253 | 0.91002 |
| **26. Oman** | | | | | | | |
| Oil price | 0 | 0.18202 | -0.14766 | 1 | -0.082668 | 0.55695 | 0.84749 |
| Non-oil GDP | 0 | 0.11348 | 0.64532 | -0.24098 | 0.0098537 | 0.0078003 | 0.27049 |
| GDP | 0 | 0.045174 | 1 | -0.14766 | 0.042412 | 0.038155 | 0.57016 |
| Oil production | 0 | 0.051852 | 0.49367 | -0.29461 | 0.1083 | -0.070816 | 0.68927 |
| Consumption | 0 | 0.029801 | 0.16066 | -0.074973 | -0.024208 | 0.009143 | 0.22586 |
| Gross oil output | 0 | 0.16562 | -0.0034674 | 0.36529 | -0.045746 | 0.050327 | -0.0084406 |
| Trade balance to GDP | 0.15867 | 0.082516 | -0.30959 | 0.65739 | 0.090286 | 0.38578 | 0.38047 |
| Institutional Investor Index | 56.5927 | 8.4563 | 0.038155 | 0.55695 | 0.70443 | 1 | 0.90708 |
| Debt to GDP | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| Reserves | 4.6242 | 1.0347 | 0.042412 | -0.082668 | 1 | 0.70443 | 0.91068 |
| Total Debt | 19.1806 | 11.3073 | 0.21107 | -0.91822 | -0.037765 | -0.6253 | 0.8681 |
| **27. Qatar** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.093372 | 1 | 0.61133 | 0.77492 | 0.84749 |
| Non-oil GDP | 0 | 0.085857 | 0.79249 | -0.019189 | -0.023466 | 0.023461 | 0.5578 |
| GDP | 0 | 0.07296 | 1 | 0.093372 | -0.044694 | 0.068474 | 0.51897 |
| Oil production | 0 | 0.084106 | 0.21994 | 0.1162 | 0.005421 | 0.095879 | 0.26424 |
| Consumption | 0 | NaN | NaN | NaN | NaN | NaN | NaN |
| Gross oil output | 0 | 0.20399 | 0.29474 | 0.41535 | 0.03571 | 0.18514 | 0.26541 |
| Trade balance to GDP | 0.29213 | 0.13982 | 0.091686 | 0.52198 | 0.55354 | 0.46285 | 0.74384 |
| Institutional Investor Index | 61.4436 | 10.2191 | 0.068474 | 0.77492 | 0.90632 | 1 | 0.92607 |
| Debt to GDP | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| Reserves | 9.6466 | 8.1516 | -0.044694 | 0.61133 | 1 | 0.90632 | 0.89421 |
| Total Debt | 35.9414 | 17.3465 | 0.14565 | -0.45207 | -0.1992 | -0.37198 | 0.77213 |

| | Mean | Standard Dev. | Corr(i,GDP) | Corr(i,Oil Price) | Corr(i,Reserves) | Corr(i,III) | Acorr |
|---|---|---|---|---|---|---|---|
| **28. Saudi Arabia** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.093562 | 1 | -0.076044 | 0.88576 | 0.84749 |
| Non-oil GDP | 0 | 0.17502 | 0.13306 | -0.15895 | -0.20685 | -0.11394 | 0.20709 |
| GDP | 0 | 0.082459 | 1 | 0.093562 | 0.29007 | -0.0019226 | 0.64228 |
| Oil production | 0 | 0.14206 | 0.97993 | 0.066585 | 0.31121 | -0.037623 | 0.5881 |
| Consumption | 0 | 0.030485 | 0.050334 | 0.08688 | 0.0092598 | 0.14608 | 0.47002 |
| Gross oil output | 0 | 0.23984 | 0.69878 | 0.35196 | 0.2622 | 0.090359 | 0.48944 |
| Trade balance to GDP | 0.12552 | 0.13254 | 0.5343 | 0.51524 | 0.4465 | 0.223 | 0.79947 |
| Institutional Investor Index | 64.7 | 8.7672 | -0.0019226 | 0.88576 | -0.30642 | 1 | 0.84223 |
| Debt to GDP | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| Reserves | 235.7527 | 42.8149 | 0.29007 | -0.076044 | 1 | -0.30642 | 0.89578 |
| Total Debt | 51.7547 | 36.5915 | -0.060756 | -0.80723 | -0.025926 | -0.70775 | 0.91232 |
| **29. Syria** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.084082 | 1 | 0.18712 | 0.26454 | 0.84749 |
| Non-oil GDP | 0 | 0.081494 | 0.36677 | -0.11875 | 0.090388 | -0.002332 | 0.29537 |
| GDP | 0 | 0.10387 | 1 | 0.084082 | 0.094381 | 0.57556 | 0.55672 |
| Oil production | 0 | 0.27056 | 0.81357 | 0.085588 | 0.041599 | 0.58598 | 0.53976 |
| Consumption | 0 | 0.10915 | 0.86763 | 0.10062 | 0.040483 | 0.57075 | 0.4668 |
| Gross oil output | 0 | 0.36232 | 0.67032 | 0.27306 | 0.029237 | 0.55464 | 0.43158 |
| Trade balance to GDP | -0.069884 | 0.10411 | 0.2943 | -0.37683 | 0.30248 | 0.41571 | 0.72445 |
| Institutional Investor Index | 23.5469 | 5.1509 | 0.57556 | 0.26454 | 0.45601 | 1 | 0.70706 |
| Debt to GDP | 0.082614 | 0.07366 | -0.76169 | 0.063321 | 6.1314e-17 | -0.56729 | 0.61438 |
| Reserves | 2.1469 | 0.43027 | 0.094381 | 0.18712 | 1 | 0.45601 | 0.91607 |
| Total Debt | 118.7856 | 51.1906 | -0.35132 | -0.66738 | -0.33057 | -0.63724 | 0.84869 |
| **30. United Arab Emirates** | | | | | | | |
| Oil price | 0 | 0.18202 | 0.23249 | 1 | -0.21656 | 0.64213 | 0.84749 |
| Non-oil GDP | 0 | 0.11159 | 0.65725 | -0.017325 | -0.36335 | -0.023349 | 0.4086 |
| GDP | 0 | 0.063566 | 1 | 0.23249 | -0.14334 | 0.089487 | 0.44501 |
| Oil production | 0 | 0.097505 | 0.10854 | 0.095805 | 0.38152 | -0.016545 | 0.67709 |
| Consumption | 0 | 0.12904 | 0.29147 | 0.10815 | -6.4027e-17 | 0.25673 | 0.50686 |
| Gross oil output | 0 | 0.22406 | 0.4791 | 0.37622 | 0.18714 | 0.14032 | 0.30607 |
| Trade balance to GDP | 0.17334 | 0.11149 | -0.034899 | 0.35753 | 3.8804e-17 | 0.080296 | 0.7576 |
| Institutional Investor Index | 64.8673 | 7.2239 | 0.089487 | 0.64213 | 0.3734 | 1 | 0.89042 |
| Debt to GDP | NaN | NaN | NaN | NaN | NaN | NaN | NaN |
| Reserves | 82.8153 | 28.1244 | -0.14334 | -0.21656 | 1 | 0.3734 | 0.86403 |
| Total Debt | 8.6445 | 5.2709 | -0.23925 | 0.3964 | 0.2324 | 0.51979 | 0.83988 |

# K VAR

## K.1 Reduced-Structural-IRFS

To identify the conditional long-run effects of today's reserves on the long-run bond's price, we estimate a Vector Autoregressive Model (VAR) with the model generated data. Specifically, we estimate the response of long-run the bond's to a shock on the oil price on price using the linearized version of the model's decision rules and dynamics of the exogenous processes.

we aim to estimate the structural VAR model described by,

$$
\begin{aligned}
p_t &= a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p \\
y_t &= a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y \\
b_{t+1} &= a_5 s_t + a_6 b_t + a_7 p_t + a_8 y_t + c_3 + \Phi_1 history_t + \psi_1 transition_t + \gamma_1 redemption_t \\
s_{t+1} &= a_9 s_t + a_{10} b_t + a_{11} p_t + a_{12} y_t + c_4 + \Phi_2 history_t + \psi_2 transition_t + \gamma_2 redemption_t,
\end{aligned}
$$

and, independently, the linearized rule for the bond's price, $q_t$,

$$
q_t = a_{13} s_t + a_{14} b_t + a_{15} p_t + a_{16} y_t + c_5 + \Phi_3 history_t + \psi_3 transition_t + \gamma_3 redemption_t,
$$

where $b_{t+1}$ and $s_{t+1}$ refer to the decisions on debt and reserves at time $t$ for time $t+1$, $(b_t, s_t, p_t, y_t)$ are the state variables at time $t$, $c_i$ are constant terms and three dummy variables to control for the default and exclusion periods ($history_t$), the transition towards a default ($transition_t$), and the Sovereign's redemption to the financial markets ($redemption_t$).

The treatment of the dummy variables is as follows: $history_t$ takes value of one when the Sovereign optimally decides to default and when is excluded of the financial markets without the exogenous signal of redemption and zero otherwise. During these periods, bond's price, $q_t$, debt at time $t$, $b_t$, and decision of debt for $t+1$, $b_{t+1}$, take value of zero. $transition_t$ takes value of one when the Sovereign's bond's price fall below a threshold and is typically associated to periods prior to a default event and zero otherwise. During these periods, bond's price, $q_t$ fall between 0.8 and 0.9, but the Sovereign holds debt $t$, $b_t$, and is still able to issue new debt, $b_{t+1}$. Finally, dummy variable $redemption_t$ takes value of one when the Sovereign is excluded from the financial markets but receives the signal of redemption for the next period. During these periods, bond's price takes a positive value, but the Sovereign does not hold debt, $b_t = 0$ and starts the next period with zero debt, $b_{t+1} = 0$

$$history_t = \begin{cases} 1 & q_t = 0, \quad b_t = 0, \quad b_{t+1} = 0 \\ 0 & otherwise, \end{cases}$$

$$transition_t = \begin{cases} 1 & q_t \in (0.8, 0.9), \quad b_t \leq 0, \quad b_{t+1} \leq 0 \\ 0 & otherwise, \end{cases}$$

$$redemption_t = \begin{cases} 1 & q_t \geq 0.9, \quad b_t = 0, \quad b_{t+1} = 0 \\ 0 & otherwise, \end{cases}$$

As we know the VAR of the exogenous variables, oil price $(p_t)$ and non-oil GDP $(y_t)$ introduced in section (x),

$$\begin{aligned} p_t &= a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p \\ y_t &= a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y, \end{aligned}$$

We can replace it on the structural VAR (ennumerate) and obtain a reduced-form VAR as,

$$\begin{aligned} b_{t+1} &= a_5 s_t + a_6 b_t + a_7 \left(a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p\right) + a_8 \left(a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y\right) \\ &\quad + c_3 + \Phi_1 history_t + \psi_1 transition_t + \gamma_1 redemption_t \\ s_{t+1} &= a_9 s_t + a_{10} b_t + a_{11} \left(a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p\right) + a_{12} \left(a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y\right) \\ &\quad + c_4 + \Phi_2 history_t + \psi_2 transition_t + \gamma_2 redemption_t, \end{aligned}$$

and for the single bond's price equation,

$$q_t = a_{13}s_t + a_{14}b_t + a_{15}\left(a_1p_{t-1} + a_2y_{t-1} + c_1 + \epsilon_t^p\right) + a_{16}\left(a_3p_{t-1} + a_4y_{t-1} + c_2 + \epsilon_t^y\right)$$
$$+ c_5 + \Phi_3 history_t + \psi_3 transition_t + \gamma_3 redemption_t,$$

grouping and simplifying similar terms, we can rewrite the reduced-form VAR as,

$$b_{t+1} = a_5s_t + a_6b_t + A_1p_{t-1} + B_1y_{t-1} + C_1 + \Phi_1 history_t + \psi_1 transition_t + \gamma_1 redemption_t + \xi_t^b$$
$$s_{t+1} = a_9s_t + a_{10}b_t + A_2p_{t-1} + B_2y_{t-1} + C_2 + \Phi_2 history_t + \psi_2 transition_t + \gamma_2 redemption_t + \xi_t^s,$$

and the bond's price pricing rule,

$$q_t = a_{13}s_t + a_{14}b_t + A_3p_{t-1} + B_3y_{t-1} + C_3 + \Phi_3 history_t + \psi_3 transition_t + \gamma_3 redemption_t + \xi_t^q,$$

where, $A_1 = (a_7a_1 + a_8a_3)$, $A_2 = (a_{11}a_1 + a_{12}a_3)$, $A_3 = (a_{15}a_1 + a_{16}a_3)$, $B_1 = (a_7a_2 + a_8a_4)$, $B_2 = (a_{11}a_2 + a_{12}a_4)$, $B_3 = (a_{15}a_2 + a_{16}a_4)$, $C_1 = (a_7c_1 + a_8c_2 + c_3)$, $C_2 = (a_{11}c_1 + a_{12}c_2 + c_4)$, $C_3 = (a_{15}c_1 + a_{16}c_2 + c_5)$, $\xi_t^b = (a_7\epsilon_t^p + a_8\epsilon_t^y)$, $\xi_t^s = (a_{11}\epsilon_t^p + a_{12}\epsilon_t^y)$, $\xi_t^q = (a_{15}\epsilon_t^p + a_{16}\epsilon_t^y)$.

By estimating the reduced-form VAR and the single equation for the bond's price, we obtain,

Table K1: Reduced-Form VAR for $(b_{t+1}, s_{t+1})$

| | Debt (t+1) | Reserves (t+1) |
|---|---|---|
| Reserves (t) | -0.005*** | 0.981*** |
| | (0.000) | (0.000) |
| Debt (t) | 0.460*** | -0.074*** |
| | (0.014) | (0.008) |
| Oil Price (t-1) | -0.245*** | -0.191*** |
| | (0.008) | (0.005) |
| Non-Oil GDP (t-1) | 0.051*** | 0.024*** |
| | (0.007) | (0.004) |
| History | 0.103*** | 0.020*** |
| | (0.002) | (0.001) |
| Transition | -0.000*** | 0.005*** |
| | (0.002) | (0.001) |
| Redemption | 0.039*** | -0.006*** |
| | (0.004) | (0.002) |
| Constant | 0.147*** | 0.419*** |
| | (0.013) | (0.008) |
| Observations | 8999 | 8999 |
| R-squared | 0.850 | 0.998 |

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table K2: Single equation for $q_t$

| | Bond's Price (t) |
|---|---|
| Reserves (t) | 0.001*** |
| | (0.0000) |
| Debt (t) | 0.037*** |
| | (0.0015) |
| Oil Price (t-1) | 0.009*** |
| | (0.0009) |
| Non-Oil GDP (t-1) | 0.007*** |
| | (0.0008) |
| History | -0.992*** |
| | (0.0003) |
| Transition | -0.146*** |
| | (0.0003) |
| Redemption | -0.000 |
| | (0.0005) |
| Constant | 0.969*** |
| | (0.0015) |
| Observations | 8999 |
| R-squared | 0.999 |

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

To obtain the structural parameters $a_7, a_8, a_{11} a_{12}, a_{15,} a_{16}$, we use the known parameters of the exogenous variables VAR, $a_1, a_2, a_3, a_4$ and solve the system of equations $(A_1, B_1)$, $(A_2, B_2)$, $(A_3, B_3)$, obtaining,

Table K3: Structural VAR for $(b_{t+1}, s_{t+1})$

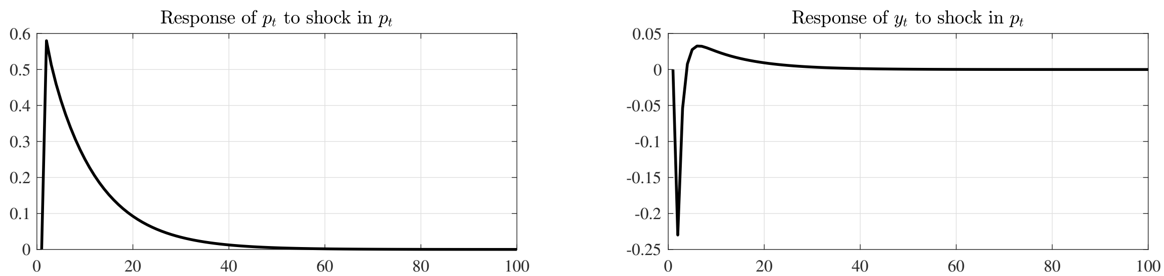
| | Debt (t+1) | Reserves (t+1) |
|---|---|---|
| Reserves (t) | -0.005 | 0.981 |
| Debt (t) | 0.460 | -0.074 |
| Oil Price (t) | -0.282 | -0.217 |
| Non-Oil GDP (t) | 0.170 | 0.088 |
| History | 0.103 | 0.020 |
| Transition | -0.000 | 0.005 |
| Redemption | 0.039 | -0.006 |
| Constant | 0.147 | 0.419 |
| Observations | 8999 | 8999 |

Table K4: Structural Single equation for $q_t$

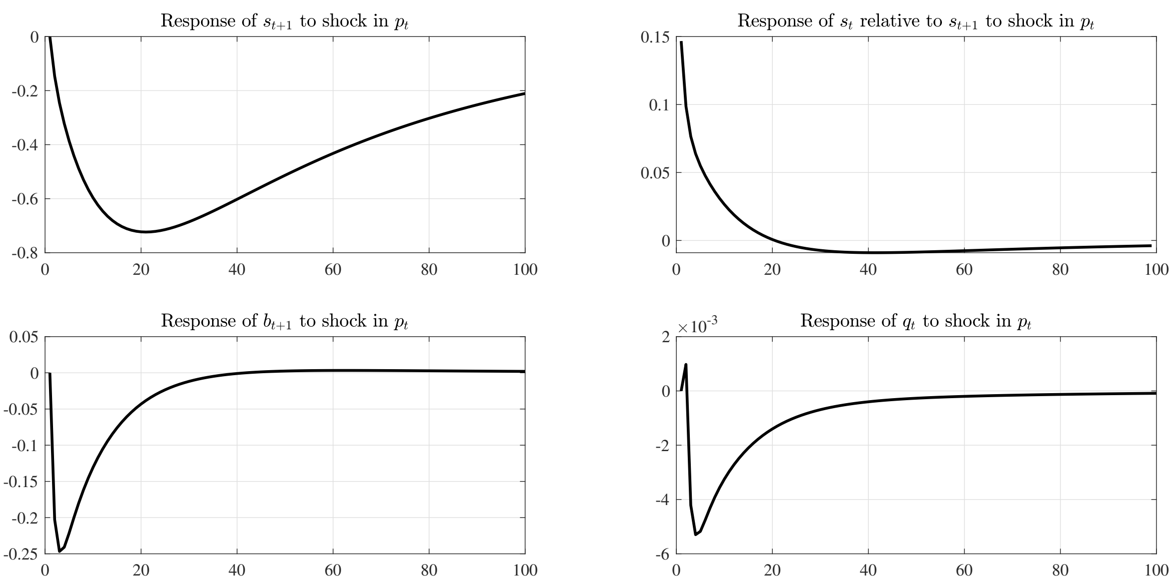
| | Bond's Price (t) |
|---|---|
| Reserves (t) | 0.001 |
| Debt (t) | 0.037 |
| Oil Price (t) | 0.009 |
| Non-Oil GDP (t) | 0.018 |
| History | -0.992 |
| Transition | -0.146 |
| Redemption | -0.000 |
| Constant | 0.969 |
| Observations | 8999 |

Figure K1: SVAR non-cumulative Response to a shock in oil price

a) Response of exogenous variables in the baseline model



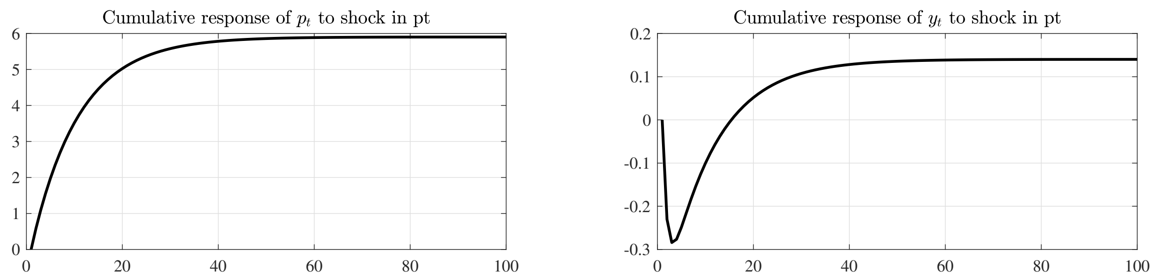

b) Response of endogenous variables in the baseline model



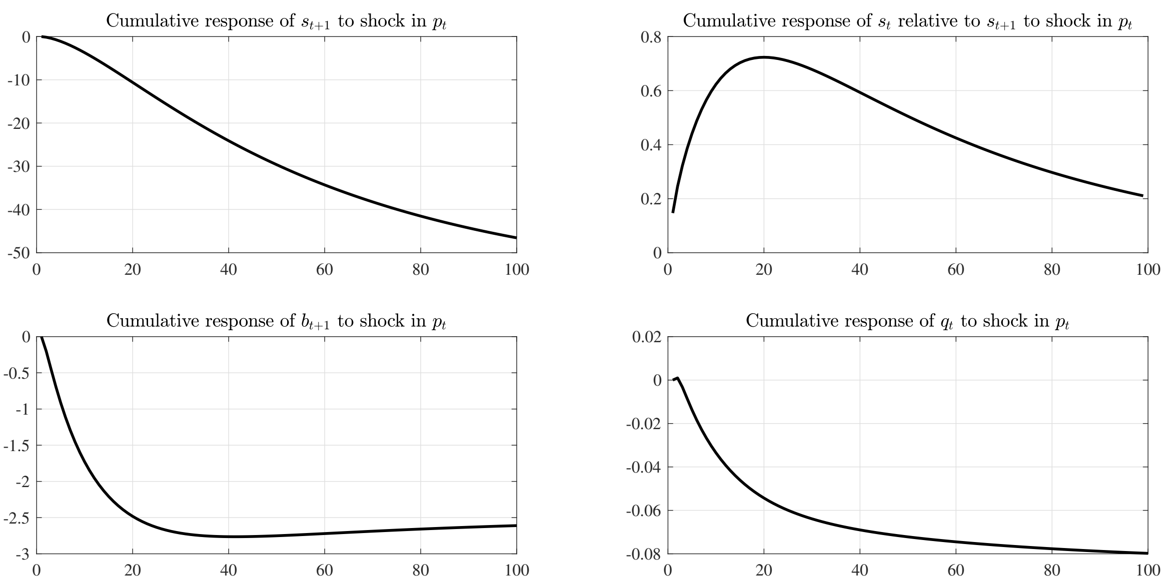

Note:

Figure K2: SVAR cumulative Response to a shock in oil price

a) Response of exogenous variables in the baseline model




b) Response of endogenous variables in the baseline model




Note:

## K.2 Reduced-IRFS

To identify the conditional long-run effects of today's reserves on the long-run bond's price, we estimate a Vector Autoregressive Model (VAR) with the model generated data. Specifically, we estimate the response of long-run the bond's to a shock on the oil price on price using the linearized version of the model's decision rules and dynamics of the exogenous processes.

We estimate the response of the decision variables $(b_{t+1}, s_{t+1})$, and the bond's price, $q_t$, to a shock on the oil price, $\epsilon_t^p$, in the system of linear-equations,

$$
\begin{aligned}
p_t &= a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p \\
y_t &= a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y \\
b_{t+1} &= a_5 s_t + a_6 b_t + a_7 p_t + a_8 y_t \\
s_{t+1} &= a_9 s_t + a_{10} b_t + a_{11} p_t + a_{12} y_t, \\
q_t &= a_{13} s_t + a_{14} b_t + a_{15} p_t + a_{16} y_t,
\end{aligned}
$$

Since we know the VAR of the exogenous variables, oil price $(p_t)$ and non-oil GDP $(y_t)$ introduced in section (x),

$$
\begin{aligned}
p_t &= a_1 p_{t-1} + a_2 y_{t-1} + c_1 + \epsilon_t^p \\
y_t &= a_3 p_{t-1} + a_4 y_{t-1} + c_2 + \epsilon_t^y,
\end{aligned}
$$

we only have to find the coefficients $a_5$ to $a_{16}$. To do so, we estimate a reduced-form VAR model,

$$
\begin{aligned}
b_{t+1} &= a_5 s_t + a_6 b_t + a_7 p_t + a_8 y_t + c_3 + \Phi_1 history_t + \psi_1 transition_t + \gamma_1 redemption_t + \epsilon_t^b \\
s_{t+1} &= a_9 s_t + a_{10} b_t + a_{11} p_t + a_{12} y_t + c_4 + \Phi_2 history_t + \psi_2 transition_t + \gamma_2 redemption_t + \epsilon_t^s,
\end{aligned}
$$

and, independently, the linearized rule for the bond's price, $q_t$,

$$
q_t = a_{13} s_t + a_{14} b_t + a_{15} p_t + a_{16} y_t + c_5 + \Phi_3 history_t + \psi_3 transition_t + \gamma_3 redemption_t + \epsilon_t^q,
$$

where $b_{t+1}$ and $s_{t+1}$ refer to the decisions on debt and reserves at time $t$ for time $t+1$, $(b_t, s_t, p_t, y_t)$ are the state variables at time $t$, $c_i$ are constant terms and three dummy variables to control for the default and exclusion periods ($history_t$), the transition towards a default ($transition_t$), and the Sovereign's redemption to the financial markets ($redemption_t$). The innovations terms, $\left(\epsilon_t^b, \epsilon_t^s, \epsilon_t^q\right)$, could be interpreted as linearization errors since we are estimating the linear version of a non-linear model.

The treatment of the dummy variables is as follows: $history_t$ takes value of one when the Sovereign optimally decides to default and when is excluded of the financial markets without the exogenous signal of redemption and zero otherwise. During these periods, bond's price, $q_t$, debt at time $t$, $b_t$, and decision of debt for $t+1$, $b_{t+1}$, take value of zero. $transition_t$ takes value of one when the Sovereign's bond's price fall below a threshold and is typically associated to periods prior to a default event and zero otherwise. During these periods, bond's price, $q_t$ falls between 0.8 and 0.9, but the Sovereign holds debt $t$, $b_t$, and is still able to issue new debt, $b_{t+1}$. Finally, dummy variable $redemption_t$ takes value of one when the Sovereign is excluded from the financial markets but receives the signal of redemption for the next period. During these periods, bond's price takes a positive value, but the Sovereign does not hold debt, $b_t = 0$ and starts the next period with zero debt, $b_{t+1} = 0$

$$history_t = \begin{cases} 1 & q_t = 0, \quad b_t = 0, \quad b_{t+1} = 0 \\ 0 & otherwise, \end{cases}$$

$$transition_t = \begin{cases} 1 & q_t \in (0.8, 0.9), \quad b_t \leq 0, \quad b_{t+1} \leq 0 \\ 0 & otherwise, \end{cases}$$

$$redemption_t = \begin{cases} 1 & q_t \geq 0.9, \quad b_t = 0, \quad b_{t+1} = 0 \\ 0 & otherwise, \end{cases}$$

By estimating the reduced-form VAR (ennumerate) and the single equation for the bond's price, we obtain,

Table K5: Reduced-Form VAR for $(b_{t+1}, s_{t+1})$

| | Debt (t+1) | Reserves (t+1) |
|---|---|---|
| Reserves (t) | -0.008*** | 0.981*** |
| | (0.000) | (0.000) |
| Debt (t) | 0.350*** | -0.016*** |
| | (0.003) | (0.003) |
| Oil Price (t) | -0.386*** | -0.189*** |
| | (0.002) | (0.002) |
| Non-Oil GDP (t) | 0.207*** | 0.030*** |
| | (0.003) | (0.003) |
| History | 0.041*** | -0.009*** |
| | (0.001) | (0.001) |
| Transition | -0.024*** | -0.009*** |
| | (0.001) | (0.001) |
| Redemption | 0.008*** | -0.021*** |
| | (0.002) | (0.002) |
| Constant | 0.189*** | 0.427*** |
| | (0.006) | (0.006) |
| Observations | 8999 | 8999 |
| R-squared | 0.960 | 0.998 |

Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Table K6: Single equation for $q_t$

| | Bond's Price (t) |
|---|---|
| Reserves (t) | 0.001*** |
| | (0.0000) |
| Debt (t) | 0.031*** |
| | (0.0009) |
| Oil Price (t) | 0.005*** |
| | (0.0005) |
| Non-Oil GDP (t) | 0.025*** |
| | (0.0008) |
| History | -0.991*** |
| | (0.0003) |
| Transition | -0.144*** |
| | (0.0003) |
| Redemption | 0.000 |
| | (0.0005) |
| Constant | 0.969*** |
| | (0.0015) |
| Observations | 8999 |
| R-squared | 0.999 |

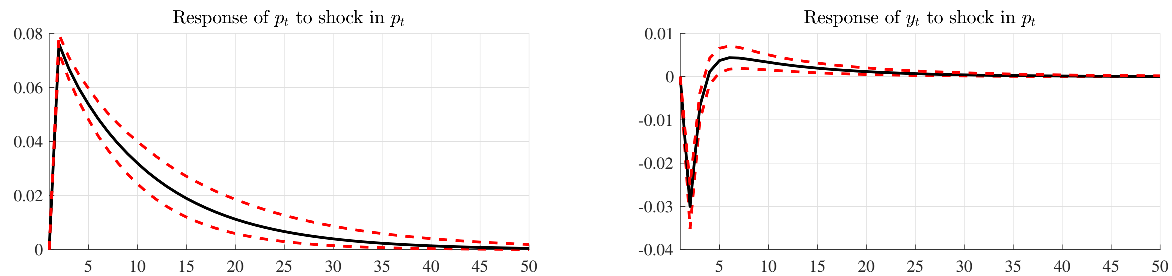
Standard errors in parentheses

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

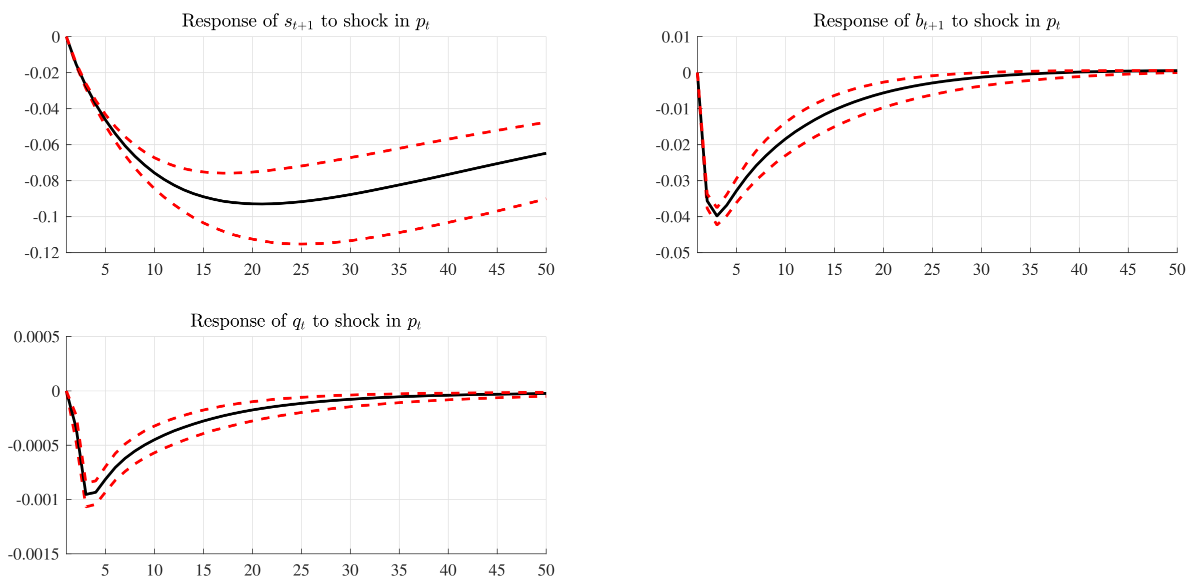
And generating the IRFs using the system of equations (ennumerate), we obtain,

Figure K3: Non-cumulative Response to a shock in oil price

a) Response of exogenous variables in the baseline model



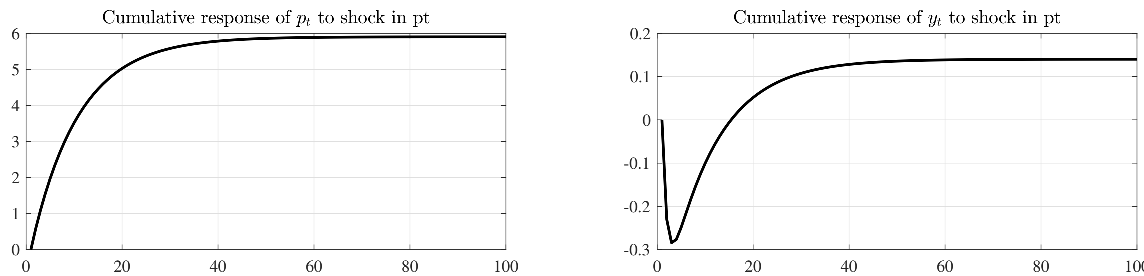

b) Response of endogenous variables in the baseline model



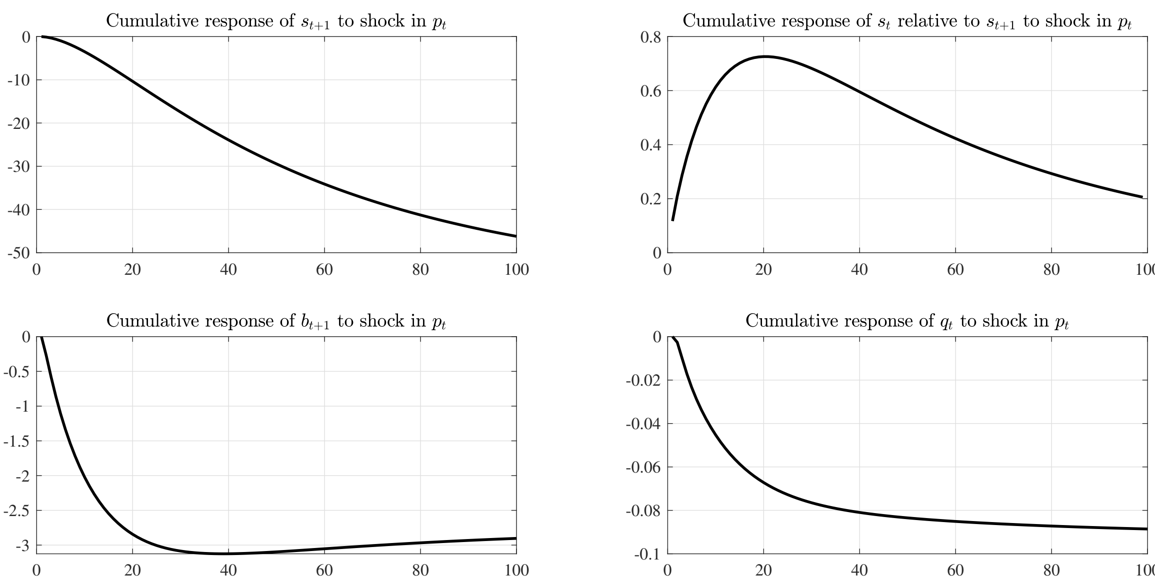

Note:

Figure K4: C umulative Response to a shock in oil price

a) Response of exogenous variables in the baseline model




b) Response of endogenous variables in the baseline model




Note:

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