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UNSTABLE PROSPERITY:  
HOW GLOBALIZATION MADE THE WORLD ECONOMY MORE VOLATILE

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Working Paper 30832  
<http://www.nber.org/papers/w30832>

NATIONAL BUREAU OF ECONOMIC RESEARCH  
1050 Massachusetts Avenue  
Cambridge, MA 02138  
January 2023

We are grateful to Andy Atkeson, Marco Del Negro, Luca Fornaro, Jean-Baptiste Michau, Alessandro Rebucci and Ludwig Straub for helpful comments and suggestions. We also acknowledge comments by participants at the following conferences: CEPR International Macroeconomics and Finance (IMF) Programme Meeting 2022, Emerging and Frontier Markets, organized by FLAR, the NBER and the Banco de la Republica de Colombia, New Global Challenges-Spillover Conference, organized by the BIS, BoE, ECB and IMF, the 2022 NBER Summer Institute, the Salento Macro Meeting in Lecce, the Seventh Annual FRB Dallas-Univ. of Houston-Bank of Mexico Conference in International Economics, and the XVII Meeting of the Central Bank Researchers Network organized by CELMA. The views expressed herein are those of the authors and do not necessarily reflect the views of the National Bureau of Economic Research.

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NBER Working Paper No. 30832  
January 2023  
JEL No. F34,F36,F62,F65

**ABSTRACT**

The sharp, secular decline in the world real interest rate of the past thirty years suggests that the surge in global demand for financial assets outpaced the growth in their supply. We argue that this phenomenon was driven by: (i) faster growth in emerging markets, (ii) changes in the financial structure of both emerging and advanced economies, and (iii) changes in demand and supply of public debt issued by advanced economies. We then show that the low-interest-rate environment made the world economy more vulnerable to financial crises. These findings are the quantitative predictions of a two-region model in which privately-issued financial assets (i.e., inside money) provide productive services but can be defaulted on.

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## 1 Introduction

Four key facts illustrated in Figure 1 highlight major changes in the world economy during the last three decades:

1. Emerging market economies (EMs) grew much faster than advanced economies (AEs). As shown in the first panel of the Figure, the GDP of EMs relative to that of AEs, measured in US dollars, rose from 28 to 68 percent between 1991 and 2020. Valuing GDP in PPP units, instead, the increase was from 57 to 125 percent. Thus, the growth in the relative size of emerging economies is evident with or without adjusting for real-exchange-rate movements.
2. The net foreign liabilities of advanced economies grew massively (a fact often labeled 'global imbalances'). As the second panel of the Figure shows, the net foreign assets (NFA) of advanced economies, as a share of their collective GDP, fell from close to zero at the beginning of the 1990s to about -20 percent in 2020.
3. Large changes in the financial structure of both emerging and advanced economies resulted in significant growth in credit to the private sector. The third panel of the Figure shows that private domestic credit as a percentage of GDP roughly tripled in EMs in the last 30 years and grew about half as much in AEs. Domestic credit as a share of GDP in EMs remains below that of advanced economies but the gap has narrowed markedly. This large expansion in worldwide financial intermediation could be driven by the growth in demand for financial assets and/or the growth in supply (i.e., issuance of liabilities). Whether demand or supply grew faster is important for determining the direction of the response of the equilibrium interest rate, which brings us to the last key fact.
4. The real interest rate fell sharply. The fourth panel of the Figure plots the ex-post real interest rate on U.S. long-term public debt, a proxy for the risk-free world interest rate. Starting from about 4 percent at the beginning of the 1990s, the real interest rate followed a declining trend reaching values close to zero at the end of 2020. Measures of *expected* real interest rates based on inflation expectations embedded in the pricing of inflation-indexed treasury bills also show significant

declines. The market yield on 10-year U.S. TIPS at constant maturity fell from 2.29 percent in January 2003 to -1 percent at the end of 2020.<sup>1</sup> This sharp drop in real interest rates suggests that the global demand for financial assets increased at a faster pace than the supply.

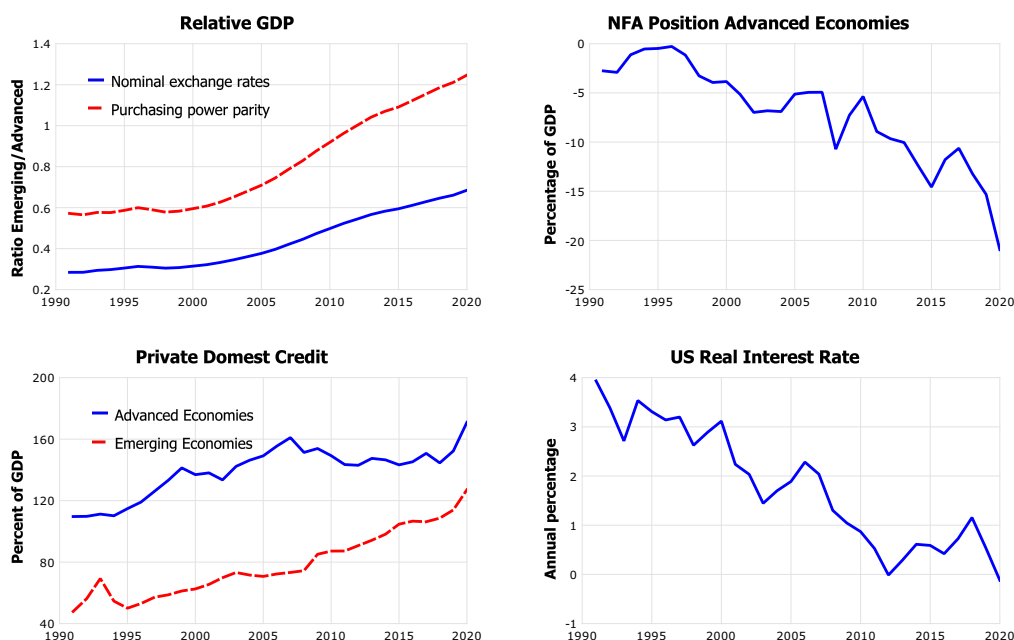


Figure 1: Real and Financial Trends in Advanced and Emerging Economies.

Note: **Emerging economies:** Argentina, Brazil, Bulgaria, Chile, China, Hong.Kong, Colombia, Estonia, Hungary, India, Indonesia, South Korea, Latvia, Lithuania, Malaysia, Mexico, Pakistan, Peru, Philippines, Poland, Romania, Russia, South Africa, Thailand, Turkey, Ukraine, Venezuela. **Advanced economies:** Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Ireland, Italy, Japan, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States. **Sources:** World Development Indicators (World Bank) and External Wealth of Nations database (Lane and Milesi-Ferretti (2018)).

The last three decades were also characterized by two trends that affected financial markets. The first panel of Figure 2 shows that emerging

<sup>1</sup>These data are available from FRED at [fred.stlouisfed.org/series/DFI10](https://fred.stlouisfed.org/series/DFI10).

economies increased sharply their holdings of foreign reserves (i.e., demand for public debt issued by AEs, particularly U.S. treasuries) in percentage of their GDP. The second panel shows that advanced economies increased their issuance of public debt, also in percentage of their GDP. These changes are important because a higher accumulation of FX reserves increases the demand for financial assets (pushing the interest rate down) while more AEs issuance of public debt increases the supply (pushing the interest rate up).<sup>2</sup>

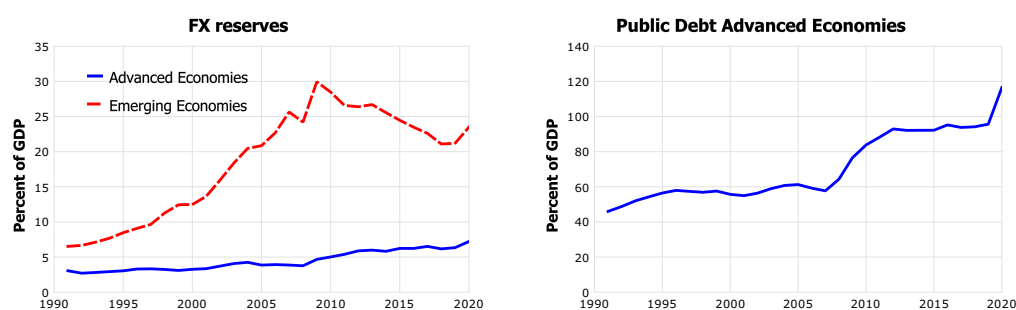


Figure 2: Foreign Exchange Reserves of Advanced and Emerging economies and Public Debt of Advanced economies.

Note: Data for FX reserves is from External Wealth of Nations database (Lane and Milesi-Ferretti (2018)). Data on public debt is from IMF Global Debt Database. We use the series Central Government Debt which is available for thirteen countries: Canada, Finland, France, Germany, Italy, Japan, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom, United States.

The trends documented in Figures 1 and 2 emerged during a period marked by financial globalization and a surge in the occurrence of financial crises. Well-established measures of de-jure and de-facto international capital mobility show the rapid progress of financial globalization as barriers to capital mobility were sharply reduced (see Chinn and Ito (2006)) and gross external assets and liabilities grew in a large number of countries (see Lane and Milesi-Ferretti (2007)).<sup>3</sup> The increase in the frequency of

<sup>2</sup>See Popper (2022) for a literature review of various channels through which foreign reserves interventions could have an impact on the economy.

<sup>3</sup>The latest update of the Chinn-Ito Index of financial openness is available at [web.pdx.edu/~ito/Chinn-Ito\\_website.htm](http://web.pdx.edu/~ito/Chinn-Ito_website.htm) and the latest update of the Lane-Milesi-Ferretti

financial crises is documented in well-known empirical studies (e.g. Reinhart and Rogoff (2009)). They show that there were no financial crises in advanced economies between 1940 and 1973 and only a handful between 1973 and 1990. Since then, between 15 and 20 crises have occurred, depending on the study one considers. Crises in emerging economies were also rare between 1940 and the onset of the sovereign debt crises of the 1980s, but the number of crises rose sharply after 1990 (see the survey by Sufi and Taylor (2021)).

This paper has two main goals. The first is to identify and measure the factors that caused the rise in net demand for financial assets—relatively to the growth in supply—and caused the drop in the world real interest rate. The second is to assess the implications of these changes for global financial and macroeconomic volatility.

We do this through the lens of a quantitative model with two regions, one representative of emerging economies and the other representative of advanced economies. In each region, there is a borrowing sector and a lending sector. Financial assets have features that make them akin to ‘inside money.’ They are issued by private agents—the debtors—and embody a ‘convenience yield’ to the holders—the creditors. The convenience yield arises from the services that financial assets provide in production.

A financial crisis occurs when the debt issued by borrowers is bigger than the liquidation value of their real assets. This generates haircuts in credit recovery and, therefore, causes wealth redistribution from creditors to debtors. The redistribution of wealth induced by a financial crisis is the central mechanism that causes real macroeconomic consequences. Importantly, the magnitude of these consequences depends on the changing structure of the financial sector, which in the model is driven by exogenous structural changes as well as endogenous general equilibrium adjustments.

We consider changes in three exogenous driving forces: (i) productivity, (ii) a structural parameter that affects the private demand for financial assets, and (iii) a structural parameter that affects the private supply of financial assets. Also, to gauge the importance of the changes in the public debt market, we consider exogenous changes in (iv) FX reserves and (v) public debt issued by AEs. We then use the model in conjunction with the data plotted in Figures 1 and 2 to identify and measure these changes over

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External Wealth of Nations database is available at [www.brookings.edu/research/the-external-wealth-of-nations-database](http://www.brookings.edu/research/the-external-wealth-of-nations-database).

the 1991-2020 period. Finally, we conduct counterfactual simulations to assess their contribution to the observed trends as well as to macroeconomic and financial volatility.

The counterfactual simulations show that the exogenous changes in productivity, financial structure and foreign reserves all contributed to increase macroeconomic and financial volatility. In contrast, the rise in public debt issued by advanced economies reduced them.

The mechanism behind these results can be described as follows: The changes in productivity, financial structure and FX reserves raised the demand for financial assets, relatively to the supply, causing the decline in the interest rate. The lower interest rate then caused the effective leverage ratio (i.e., the ratio of debt to the liquidation value of capital) to rise, which in turn increased financial and macroeconomic volatility. The increase in public debt, instead, raised the supply of financial assets and mitigated the decline in the interest rate.

The observed interest rate decline and NFA dynamics are key for the identification of the changes in financial structure. As mentioned above, the reduction in the interest rate indicates that the worldwide growth in demand for financial assets outpaced the growth in their supply. NFA dynamics are important for determining in which countries the demand for financial assets grew more than the supply. In particular, the fact that the net liabilities of advanced economies widened over the sample period indicates that the net demand for financial assets in these countries increased less than in emerging market economies.

**Related literature.** Our work is related to three important strands of literature: the literature on global imbalances, the literature on financial crises or Sudden Stops, and the literature on the growth of financial assets and corporate cash holdings.

Research on global imbalances proposes several theories to explain the growth in NFA positions of emerging economies. One explanation is based on the idea that emerging economies have a lower ability to create viable saving instruments for inter-temporal smoothing (Caballero, Farhi, and Gourinchas (2008)). Another explanation is that emerging economies have a higher demand for assets due to lower insurance, or lower financial development related to weaker enforcement (Mendoza, Quadrini, and Ríos-Rull (2009)) or because of higher uncertainty (Carroll and Jeanne (2009), An-

geletos and Panousi (2011), Song, Storesletten, and Zilibotti (2011), Sandri (2014), Bacchetta and Benhima (2015), Fogli and Perri (2015)). The first theory highlights cross-country heterogeneity in the supply of assets while the second emphasizes heterogeneity in the demand. In both cases, emerging economies turn to advanced economies for the acquisition of saving instruments. A third set of studies focuses on productivity differentials across emerging and advanced economies, including sectoral productivity differentials, and higher target NFA levels in emerging economies as a consequence of higher foreign reserves (e.g. Cova, Pisani, and Rebucci (2009)). It could also be the consequence of asset revaluation. Atkeson, Heathcote, and Perri (2022) shows that the large decline in the US NFA position after the global financial crisis is largely explained by the increase in price of assets held by foreigners in the United States.

Our model incorporates heterogeneity in both supply and demand for financial assets as well as productivity differentials between advanced and emerging economies. The aim of our paper, however, is not to explain why advanced economies are borrowing from emerging economies, which is the focus of the above referenced studies. Instead, it has two objectives that are relatively new in this literature: The first is to ‘measure’ the changes in the structural sources of demand and supply of financial assets. The second is to explore how this affected macroeconomic and financial volatility.

Various studies in the Sudden Stops literature examine the role of financial globalization, credit booms and high leverage as causing factors of financial crises. Examples include Calvo and Mendoza (1996), Caballero and Krishnamurthy (2001), Gertler, Gilchrist, and Natalucci (2007), Edwards (2004), Mendoza and Quadrini (2010), Mendoza and Smith (2014), Fornaro (2018).<sup>4</sup> Some of these studies emphasize mechanisms that cause financial crises because of equilibrium multiplicity due to self-fulfilling expectations as in Aghion, Bacchetta, and Banerjee (2001) and Perri and Quadrini (2018). Crises in our model also follow from periods of fast credit and leverage growth, and they are the result of self-fulfilling expectations. However, the mechanism that operates in our model differs in that it relies on the interaction between the inside-money-like role of financial assets for creditors with the debtors’ lack of commitment to repay. Financial crises have real effects because it redistributes wealth from creditors to debtors.

Several studies in the corporate finance literature document and pro-

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<sup>4</sup>See Bianchi and Mendoza (2020) for a survey of the literature.



vide explanations for the raising demand of financial assets. An example is the literature on the growing cash holdings of nonfinancial businesses (e.g., Riddick and Whited (2009), Busso, Fernández, and Tamayo (2016) and Bebczuk and Cavallo (2016)). Our model has a similar feature in that entrepreneurs hold positive positions in financial assets that expand as a result of faster growth of emerging economies and changes in financial structure in both emerging and advanced economies. Our focus, however, is on the macroeconomic implications. The increase in net demand for financial assets depresses the interest rate which in turn increases the incentives to leverage. While the higher leverage allows for sustained levels of financial intermediation and economic activity, it also makes both emerging and advanced economies more vulnerable to crises (global instability).

The remainder of the paper is organized as follows. Section 2 describes the model and characterizes the equilibrium. Section 3 uses the model in conjunction with the data plotted in Figure 1 to construct empirical series for productivity and the structural parameters that impact directly the demand and supply of financial assets. We then conduct counterfactual simulations to decompose the role played by changes in productivity and changes in financial structure for generating the observed trends. Section 4 analyzes the implications of the structural changes for macroeconomic and financial instability. Section 5 concludes.

## 2 Model

Consider a world economy that consists of two countries/regions indexed by  $j \in \{1, 2\}$ . Country 1 represents advanced economies and Country 2 emerging economies. In each country, there are three sectors: (i) an entrepreneurial sector that produces final output; (ii) a consolidated household/business sector that holds capital and supplies labor;<sup>5</sup> (iii) a public sector that holds financial assets in the form of FX reserves and, in Country 1, issues liabilities (public debt).

The reason we have two private sectors in each country is because they allow us to generate private borrowing and lending within and across countries. We can then have a clear distinction between the private ‘demand’ for

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<sup>5</sup>We interpret this sector as composed of firms that hold physical capital with high collateral value. In this sense, these firms are similar to households holding real estate: the availability of collateral allows both households and firms to borrow.

financial assets (from the sector with a positive financial position, the creditors) and the private ‘supply’ of financial assets (from the sector with a negative financial position, the debtors). The presence of the public sector allows us to study how the issuance of public debt and the accumulations of FX reserves affect the economies of the two countries.

Countries are heterogeneous in three key dimensions: (i) economic size, formalized by differences in aggregate productivity,  $z_{j,t}$ ; (ii) a financial parameter that affects directly the demand for financial assets,  $\phi_{j,t}$ ; and (iii) a financial parameter that affects directly the supply of financial assets,  $\kappa_{j,t}$ . They also differ in foreign reserves accumulated by each country’s government,  $FX_{j,t}$ , and in Country 1’s issuance of government debt,  $D_{p,t}$ .

Differences in economic size could be generated by other factors besides productivity (e.g., population, real exchange rates, etc.). For the questions addressed in this paper, however, these other factors are isomorphic to productivity differences. This will become clear in the next section. Productivity  $z_{j,t}$ , financial parameters  $\phi_{j,t}$  and  $\kappa_{j,t}$ , reserves  $FX_{j,t}$ , and Country 1’s public debt  $D_{p,t}$  are time varying but not stochastic. Their evolution over time is fully anticipated. The only source of uncertainty in the model derives from “sunspot” shocks that will be described later in this section.

## 2.1 Entrepreneurial sector

In each country, there is a unit mass of atomistic entrepreneurs that maximize the expected logarithmic lifetime utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_{j,t}),$$

where  $c_{j,t}$  is consumption in country  $j$  at time  $t$  and  $\beta$  is the discount factor common across entrepreneurs in both countries.

Entrepreneurs are business owners producing a single good with the technology described below. Although the model is presented as if final production is carried out by privately-owned businesses, we should think of the entrepreneurial sector broadly and including also some publicly-traded companies. Then, entrepreneurial consumption can be interpreted as dividend payments and the concavity of the utility function could derive from the risk aversion of managers and/or major shareholders. The concavity could also reflect, in reduced form, the cost associated with financial distress: even if shareholders and managers are risk-neutral, a convex

cost of financial distress would make the objective of the business concave. Since there are no idiosyncratic shocks in the model, we can focus on the representative entrepreneur in each country.

The production function takes the form

$$y_{j,t} = z_{j,t}^\gamma l_{j,t}^\gamma k_{j,t}^{1-\gamma}, \quad (1)$$

where  $z_{j,t}$  is total factor productivity,  $l_{j,t}$  is the input of labor, and  $k_{j,t}$  is physical capital rented from consolidated households/firms. In the long-run, productivity  $z_{j,t}$  grows in both countries at the common rate  $g - 1$ . In the short-run, however, the growth rate of productivity can deviate from its long-run value. As we will see, this is especially important for emerging economies.

Production also requires financial resources that increase with the scale of production. We proxy the production scale with the payments of rents and wages. Denoting by  $r_{j,t}$  the rental rate of capital and  $w_{j,t}$  the wage rate, producers face the constraint

$$m_{j,t} \geq \phi_{j,t} [r_{j,t} k_{j,t} + w_{j,t} l_{j,t}], \quad (2)$$

where  $m_{j,t}$  is the financial wealth of the entrepreneur.

A narrow interpretation of this constraint is that it represents working capital necessary for the advanced payment of a fraction  $\phi_{j,t}$  of the factor costs of capital and labor. However, we interpret the constraint more broadly based on several considerations. Although we specified the production function abstracting from intermediate stages of production, in reality firms also need to purchase intermediate goods which also require working capital. Besides the financing of advanced factor payments, financial wealth facilitates production through other channels that are not explicitly modelled here. For example, it provides insurance against earning risks and allows for smoother dividend payments. Higher financial wealth, then, makes entrepreneurs more willing to operate larger production scales. Also, firms with more favorable financial positions may find easier to hire workers, either because the risk of distress (which is associated with higher probability of lay off) is lower or because workers are able to negotiate higher wages. We will come back to this broader interpretation of  $m_{j,t}$  in the quantitative section of the paper.

The time-varying parameter  $\phi_{j,t}$  plays an important role in determining the demand for financial assets. The higher the value of  $\phi_{j,t}$ , the higher the entrepreneurs' holdings of  $m_{j,t}$ .

Financial wealth is in the form of bonds, which are liabilities issued by consolidated households/firms (either domestic or foreign) or by the public sector of advanced economies. The prices of private and public bonds differ—despite perfect capital mobility—because they are characterized by different repayment risks. In particular, while private bonds are defaultable, public bonds issued by advanced economies are always repaid in full. We denote by  $q_{j,t}$  the price of bonds issued by households/firms in country  $j$  at date  $t$ , and by  $q_{p,t}$  the price of public bonds issued by country 1.

The representative entrepreneur in country  $j$  enters period  $t$  with bonds issued by households/firms in country 1,  $b_{1,j,t}$ , bonds issued by households/firms in country 2,  $b_{2,j,t}$ , and government bonds issued by advanced economies,  $b_{p,j,t}$ . The first subscript denotes the bond issuer (country 1 or country 2 for private bonds, and  $p$  for public bonds), while the second subscript denotes the residence of the holder. In the event of default, entrepreneurs incur financial losses proportional to their ownership of private bonds (but not public bonds since they are risk-free).

Denote by  $\delta_{1,t}$  and  $\delta_{2,t}$  the fractions of private bonds repaid, respectively, by country 1 and country 2. The residual values of the two bonds are then  $\delta_{1,t}b_{1,j,t}$  and  $\delta_{2,t}b_{2,j,t}$ . The repayment fractions  $\delta_{1,t}$  and  $\delta_{2,t}$  are endogenous stochastic variables determined in general equilibrium. After their realization, which takes place at the beginning of the period, the entrepreneur's wealth becomes

$$m_{j,t} = \delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}.$$

This is the financial wealth that enters constraint (2).

After production, the end-of-period wealth is

$$a_{j,t} = m_{j,t} + z_{j,t}^\gamma l_{j,t}^\gamma k_{j,t}^{1-\gamma} - w_{j,t}l_{j,t} - r_{j,t}k_{j,t}.$$

This is in part allocated to consumption and in part to new bonds, in accordance to the budget constraint

$$c_{j,t} + q_{1,t}b_{1,j,t+1} + q_{2,t}b_{2,j,t+1} + q_{p,t}b_{p,j,t+1} = a_{j,t}. \quad (3)$$

While the production scale depends on  $m_{j,t}$  (through constraint (2)), portfolio decisions,  $b_{1,j,t+1}$ ,  $b_{2,j,t+1}$  and  $b_{p,j,t+1}$ , depend on  $a_{j,t}$ . To clarify the entrepreneurs' decision, it would be convenient to think of a period as divided in three subperiods:

1. **Subperiod 1 (Wealth Realization):** Entrepreneurs enter with financial assets  $b_{1,j,t}$ ,  $b_{2,j,t}$ ,  $b_{p,j,t}$ , and observe the repayments  $\delta_{1,t}$  and  $\delta_{2,t}$ . The residual wealth, after repayment, is  $m_{j,t} = \delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}$ .
2. **Subperiod 2 (Production Decision):** Given the residual wealth  $m_{j,t}$ , entrepreneurs choose the inputs of labor  $l_{j,t}$  and capital  $k_{j,t}$ . Market clearing determines the wage and rental rates,  $w_{j,t}$  and  $r_{j,t}$ .
3. **Subperiod 3 (Portfolio Decision):** The end-of-period wealth  $a_{j,t}$  is in part consumed,  $c_{j,t}$ , and in part saved in bonds,  $q_{1,t}b_{1,j,t+1}$ ,  $q_{2,t}b_{2,j,t+1}$  and  $q_{p,t}b_{p,j,t+1}$ .

The debt repayment in Subperiod 1 is determined by the decisions of households that we will characterize in the next section. Here, instead, we characterize the production and portfolio decisions made by entrepreneurs in Subperiods 2 and 3. We start with the optimal production decision.

**Lemma 2.1** *If constraint (2) is not binding, the inputs of production satisfy*

$$\begin{aligned} \gamma z_{j,t}^\gamma \left( \frac{k_{j,t}}{l_{j,t}} \right)^{1-\gamma} &= w_{j,t} \\ (1-\gamma) z_{j,t}^\gamma \left( \frac{k_{j,t}}{l_{j,t}} \right)^{-\gamma} &= r_{j,t}. \end{aligned}$$

*If constraint (2) binds, the inputs of production are*

$$\begin{aligned} l_{j,t} &= \left( \frac{\gamma}{\phi_{j,t} w_{j,t}} \right) m_{j,t}, \\ k_{j,t} &= \left( \frac{1-\gamma}{\phi_{j,t} r_{j,t}} \right) m_{j,t}. \end{aligned}$$

**Proof 2.1** *Appendix A.*

With a non-binding constraint, the entrepreneur chooses the input of production to equalize the marginal products of labor and capital. With constant return to scale, only the ratio of the production inputs are determined at the level of an individual entrepreneur. The scale of production

is determined only in aggregate. In this case the financial wealth of the entrepreneur,  $m_{j,t}$ , and the financial parameter  $\phi_{j,t}$  are irrelevant. This is because the entrepreneur has financial resources that are more than sufficient to hire the optimal input of labor and to rent the optimal input of capital. For optimality we mean the levels that equalize the marginal products to their costs. However, if the optimal inputs require more funds than available, then  $m_{j,t}$  constrains the scale of production. As a result, the inputs of labor and capital chosen by the entrepreneur increase in  $m_{j,t}$ . The fact that the entrepreneur is constrained in the use of  $l_{j,t}$  and  $k_{j,t}$  implies that the marginal products of these two inputs are higher than their costs, that is, the wage rate and the rental rate of capital.

Under what conditions is constraint (2) binding? In general, the constraint is binding when entrepreneurs have low financial wealth ( $m_{j,t}$  is low), the use of the production inputs requires more funds ( $\phi_{j,t}$  is high), and productivity is high ( $z_{j,t}$  is bigger). When productivity is high, entrepreneurs have more incentive to expand the scale of production, which requires more funds.

We characterize next the optimal portfolio choice that takes place in Subperiod 3.

**Lemma 2.2** *The optimal allocation of the end-of-period wealth is*

$$\begin{aligned} c_{j,t} &= (1 - \beta)a_{j,t}^i, \\ q_{1,t}b_{1,j,t+1} &= \beta\theta_{1,t}a_{j,t}, \\ q_{2,t}b_{2,j,t+1} &= \beta\theta_{2,t}a_{j,t}, \\ q_{p,t}b_{p,j,t+1} &= \beta(1 - \theta_{1,t} - \theta_{2,t})a_{j,t}, \end{aligned}$$

where  $\theta_{1,t}$  and  $\theta_{2,t}$  solve the first-order conditions

$$\begin{aligned} \mathbb{E}_t \left\{ \frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{1,t} \frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,t} \frac{\delta_{2,t+1}}{q_{2,t}} + (1 - \theta_{1,t} - \theta_{2,t}) \frac{1}{q_{p,t}}} \right\} &= 1, \\ \mathbb{E}_t \left\{ \frac{\frac{\delta_{2,t+1}}{q_{2,t}}}{\theta_{1,t} \frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,t} \frac{\delta_{2,t+1}}{q_{2,t}} + (1 - \theta_{1,t} - \theta_{2,t}) \frac{1}{q_{p,t}}} \right\} &= 1. \end{aligned}$$

**Proof 2.2** *Appendix B.*

Lemma 2.2 establishes that entrepreneurs split the end-of-period wealth between consumption and saving according to the fixed factor  $\beta$ . This property derives from the logarithmic specification of the utility function. A fraction  $\theta_{1,t}$  of saved wealth ( $\beta a_t$ ) is then allocated to private bonds issued by country 1, a fraction  $\theta_{2,t}$  to private bonds issued by country 2, and the remaining fraction  $1 - \theta_{1,t} - \theta_{2,t}$  is allocated to public (risk-free) bonds. The variables  $\theta_{1,t}$  and  $\theta_{2,t}$  change over time as recovery rates and bond prices vary. However, they are the same for entrepreneurs in country 1 and in country 2. This is indicated by the fact that  $\theta_{1,t}$  and  $\theta_{2,t}$  does not have the subscript  $j$ . Thus, entrepreneurs in both countries choose the same portfolio composition.<sup>6</sup>

## 2.2 Consolidated households/firms sector

In each country there is a consolidated sector with a unit mass of homogeneous households/firms. We think of this second type of firms as large owners of collateralizable assets (capital). In this sense they are similar to households who also own large collateralizable assets in the form of real estate. Entrepreneurial firms, instead, are more representative of businesses that own few collateralizable assets (zero for simplicity in the model). As we will see, the consolidated households/firms sector will borrow at equilibrium while the entrepreneurial sector will lend.

Households/firms maximize the utility

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left( e_{j,t} - z_{j,t} \frac{h_{j,t}^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}} \right),$$

where  $e_{j,t}$  is consumption,  $h_{j,t}$  is the supply of labor, and  $\nu$  is the elasticity of labor supply.

The assumption that the utility of households/firms is linear in consumption simplifies the characterization of the equilibrium. It allows us to derive analytic results without affecting, in important ways, the properties of the model that are central for the questions addressed in this paper. The dependence of the dis-utility of labor on  $z_{j,t}$  supports balanced growth.

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<sup>6</sup>Since  $\theta_{1,t}$  and  $\theta_{2,t}$  are without  $j$  subscript, the last three conditions in Lemma 2.2 are not simple accounting identities.

Households/firms make optimal plans to hold  $k_{j,t}$  units of capital that depreciates at rate  $\tau$ . To keep the model tractable, we assume that the aggregate supply of capital grows exogenously at the same rate as the long-run growth rate of the economy,  $g - 1$ . Therefore, capital in both countries evolves over time according to  $K_{j,t} = \bar{K} g^t$ . We interpret capital broadly including real estate and land. Households/firms trade capital among themselves at the market price  $p_{j,t}$  and rent it to domestic entrepreneurs at the rental rate  $r_{j,t}$ .

**Borrowing and default.** At the end of period  $t - 1$ , households/firms borrow  $d_{j,t}/R_{j,t-1}$  where  $R_{j,t-1}$  is the gross interest rate and  $d_{j,t}$  is the ‘promised’ repayment due at time  $t$ . At the beginning of time  $t$ , when the debt  $d_{j,t}$  is due, households/firms could default. In the event of default, creditors have the right to liquidate the capital  $k_{j,t}$ . However, the liquidation value at the beginning of period  $t$  could be insufficient to repay the loan.

Denote by  $\tilde{p}_{j,t}$  the liquidation price of capital at the beginning of period  $t$ . If the debt is bigger than the liquidation value of capital, that is,  $d_{j,t} > \tilde{p}_{j,t}k_{j,t}$ , the debt is renegotiated. Under the assumption that borrowers have all the bargaining power, the renegotiated debt is

$$\tilde{d}(d_{j,t}, \tilde{p}_{j,t}k_{j,t}) = \min \left\{ d_{j,t}, \tilde{p}_{j,t}k_{j,t} \right\} \quad (4)$$

After renegotiation, the market for capital returns to normal at the end of the period. The assumption of an immediate fresh-start is a simplification that makes the model tractable.

An important assumption is that there are states of nature in which the market for liquidated capital freezes and the liquidation price drops below its normal price  $p_{j,t}$ . More specifically, with probability  $\lambda$ , the liquidation price becomes  $\tilde{p}_{j,t} = \kappa_{j,t} < p_{j,t}$  while with probability  $1 - \lambda$  it remains at the normal price  $\tilde{p}_{j,t} = p_{j,t}$ . The variable  $\kappa_{j,t}$  is time-varying but exogenous.

Appendix D describes the mechanism that generates a freeze. The market structure described there allows for two self-fulfilling equilibria, one of which is characterized by the market freeze where the liquidation price drops to  $\kappa_{j,t}$ . The selection between the two equilibria in country  $j$  is done with the draw of a sunspot shock  $\varepsilon_j \in \{0, 1\}$ . The probability  $\lambda$  is then the exogenous probability that the draw of the sunspot shock is the one that leads to an equilibrium with a market freeze. Readers who are not interested in the micro-foundation of the market freeze can skip the appendix



without loss of continuity. What is essential is that the liquidation price  $\tilde{p}_{j,t}$  is  $\kappa_{j,t}$  with probability  $\lambda$  and  $p_{j,t}$  with probability  $1 - \lambda$ . The sunspot variables  $\varepsilon_1$  and  $\varepsilon_2$  are the only stochastic shocks in the model.

Issuance of new debt  $d_{j,t+1}$  carries a convex cost specified as

$$\varphi(d_{j,t+1}, \kappa_{j,t+1}k_{j,t+1}) = \eta \left[ \frac{\max\{0, d_{j,t+1} - \kappa_{j,t+1}k_{j,t+1}\}}{d_{j,t+1}} \right]^2 d_{j,t+1}. \quad (5)$$

Figure 3 provides a graphical illustration of this cost. As long as the debt repayment promised in the next period,  $d_{j,t+1}$ , exceeds the minimum liquidation value,  $\kappa_{j,t+1}k_{j,t+1}$ , the cost is zero. Beyond that point, the cost rises at a quadratic rate with debt  $d_{j,t+1}$ . This cost plays a similar role as a borrowing limit: it ensures that borrowing is bounded at equilibrium. The parameter  $\eta$  determines the speed with which the cost rises with debt (for given capital) and, therefore, the flexibility with which borrowing responds to changing market conditions (for example, the interest rate). For very high values of  $\eta$  we have, effectively, a hard borrowing constraint, that is,  $d_{j,t+1} \leq \kappa_{j,t+1}k_{j,t+1}$ .

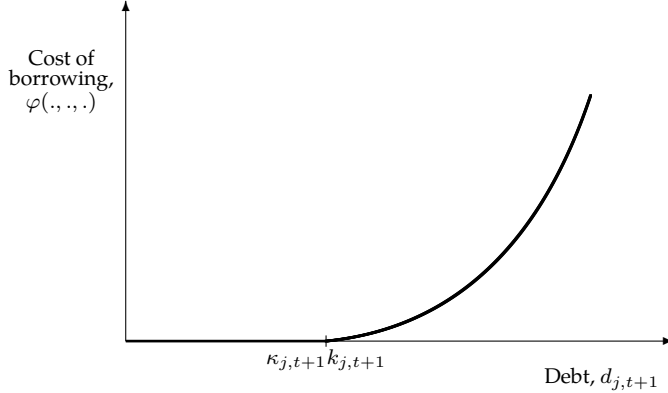


Figure 3: Convex cost of borrowing as a function of debt.

We can now write down the budget constraint for consolidated households/firms. After renegotiation, the budget constraint takes the form

$$\begin{aligned} \tilde{d}(d_{j,t}, \tilde{p}_{j,t}k_{j,t}) + p_{j,t}k_{j,t+1} + e_{j,t} + \varphi(d_{j,t+1}, \kappa_{j,t+1}k_{j,t+1}) = \\ w_{j,t}l_{j,t} + (r_{j,t} - \tau)k_{j,t} + p_{j,t}k_{j,t}g + \frac{d_{j,t+1}}{R_{j,t}} + T_{j,t}. \end{aligned}$$

The value of capital is multiplied by  $g$  because it grows exogenously at the same rate as the long-run growth rate of productivity. The new capital is added proportionally to the existing capital as a free endowment. The variable  $T_{j,t}$  denotes lump-sum transfers or taxes paid or levied by the government. Note that the full deterministic time path of  $\kappa_{j,t}$  for  $t = 0, \dots, \infty$ , is known. At date  $t$ ,  $\kappa_{j,t}$  matters for the repayment of the existing debt, while  $\kappa_{j,t+1}$  matters for the cost of issuing new debt.

The gross interest rate  $R_{j,t}$  depends on the individual borrowing decision. If the household/firm borrows more, relatively to the ownership of capital, the expected repayment rate could be lower in the next period. This will be reflected in a higher interest rate on newly issued bonds.

Denote by  $\bar{R}_{j,t}$  the *expected* gross return from holding the debt issued in period  $t$  and due at  $t + 1$ , by all households/firms in country  $j$ . This represents the aggregate expected market return from holding a diversified portfolio of debt issued by households/firms in country  $j$ . Since households/firms are atomistic and financial markets are competitive, the expected return on the debt issued by an ‘individual’ household/firm must be equal to the aggregate expected total return, that is,

$$\frac{d_{j,t+1}}{R_{j,t}} = \frac{1}{\bar{R}_{j,t}} \mathbb{E}_t \tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1} k_{j,t+1}). \quad (6)$$

The left-hand-side is the amount borrowed in period  $t$  while the right-hand-side is the expected repayment in period  $t + 1$ , discounted by the market return  $\bar{R}_{j,t}$ . Since the household/firm renegotiates the debt when  $d_{j,t+1} > \tilde{p}_{j,t+1} k_{j,t+1}$ , the actual repayment,  $\tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1} k_{j,t+1})$ , could differ from  $d_{j,t+1}$ . Competition in financial intermediation requires that the left-hand-side equals the right-hand-side of (6).

Equation (6) determines the interest rate  $R_{j,t}$  for an individual household/firm. It can also be viewed as determining an individual borrowing spread  $R_{j,t}/\bar{R}_{j,t} = d_{j,t+1}/\mathbb{E}_t \tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1} k_{j,t+1})$ . For a household/firm expected to repay in full with certainty, the spread is zero ( $R_{j,t}/\bar{R}_{j,t} = 1$ ). For one not expected to repay in full,  $R_{j,t}$  exceeds  $\bar{R}_{j,t}$ . The higher rate depends on how much the contracted repayment,  $d_{j,t+1}$ , is below the expected repayment after renegotiation, that is,  $\mathbb{E}_t \tilde{d}(d_{j,t+1}, \tilde{p}_{j,t+1} k_{j,t+1})$ . At equilibrium, all households/firms make the same decisions and they all borrow at the same rate. However, in order to characterize the optimal decision, we need to allow for individual deviations.

**First-order conditions.** As for entrepreneurs, households/firms make decisions sequentially. At the beginning of the period (Subperiod 1) they decide whether to default and renegotiate the debt. After that (Subperiod 2), they chose the supply of labor. Finally, at the end of the period (Subperiod 3), they choose the new debt. Appendix C describes the optimization problem and derives the following first-order conditions:

$$z_{j,t} l_{j,t}^{\frac{1}{\nu}} = w_{j,t}, \quad (7)$$

$$\frac{1}{R_{j,t}} = \beta + \Phi \left( \frac{d_{j,t+1}}{\kappa_{j,t+1} k_{j,t+1}} \right), \quad (8)$$

$$p_{j,t} = \beta \mathbb{E}_t \left\{ r_{j,t+1} - \tau + g p_{j,t+1} \right\} + \Psi \left( \frac{d_{j,t+1}}{\kappa_{j,t+1} k_{j,t+1}} \right). \quad (9)$$

Equation (7) sets the labor supply by equating the marginal dis-utility of labor to the wage rate. The typical wealth effect on labor supply is absent given the linear utility of consumption. However, we obtain a similar effect with the assumption that the dis-utility of labor increases with productivity  $z_{j,t}$ . In this way, even if the wage grows over time, the supply of labor remains stationary.

Equation (8) is the Euler equation for debt, while equation (9) is the Euler equation for capital. The functions  $\Phi(\cdot)$  and  $\Psi(\cdot)$  result from the expectation of future outcomes and the explicit functional forms are derived in Appendix C. In the model, the only source of uncertainty is the realization of sunspot shocks that could lead to lower future repayments of debt. Since the probability of default and future repayments, with and without default, are known in advance, we can calculate analytically the expected repayment. This is embedded in the two functions  $\Phi(\cdot)$  and  $\Psi(\cdot)$  as can be seen explicitly in Appendix C.

The important point is that the functions  $\Phi(\cdot)$  and  $\Psi(\cdot)$  are increasing in the ratio  $d_{j,t+1}/\kappa_{j,t+1} k_{j,t+1}$ , which is a measure of *effective* leverage (i.e., the ratio of debt over the minimum liquidation value of capital). Because  $\Phi(\cdot)$  is an increasing function, condition (8) posits a *negative* relationship between the expected return on the debt (the interest rate) and the effective leverage. At the same time, because  $\Psi(\cdot)$  is also increasing, condition (9) posits a *positive* relationship between the price of capital and leverage. Together, equations (8) and (9) imply that a decline in the interest rate is associated with an increase in leverage and an asset price boom.

### 2.3 Public sector

The government of country 1 issues risk-free bonds (public debt), and the governments of both countries hold some of these bonds as reserves. Governments also make lump-sum transfers to the consolidated households/firms sector to balance their budgets. The assumption that the transfers (or taxes if negative) are paid only to households/firms simplifies the analysis considerably given the linear utility of households/firms. The assumptions that public debt is only issued by advanced economies (country 1) and it is always repaid (no sovereign default) simplify the analysis further.

The reason we focus on public debt issued by advanced economies is in part justified by data availability: While data on public debt is available for many advanced economies, it is scarce for emerging economies. Independently of data availability, however, we think that modeling the issuance of public debt by advanced economies is more important than that of the emerging economies for the questions addressed in this paper, as we now discuss.

Sovereign default in advanced economies is rare and the public bonds issued by these countries are basically risk-free (at least as an aggregate of all public bonds issued by advanced economies). These features make the public debt of advanced economies very different from private debt, which is not risk-free. Because of low risk, the government bonds of advanced economies are very important for liquidity. The U.S. public debt, in particular, represents a large share of this market since it is the main instrument used for foreign reserves. Also, because the public debt of advanced economies is quite large, especially in recent years, it can be quite important for the economy.

Of course, governments in emerging economies also issue public debt. However, since the public debt of emerging economies is not risk-free and sovereign default is correlated with private default in these countries, for an investor there is no much difference between private and public debt issued by emerging economies. Also, the size of the public debt issued by emerging economies is significantly smaller than the public debt issued by advanced economies. Therefore, the quantitative general equilibrium implications should be less important.

The budget constraint of the government in country 1 (representative of advanced economies) is

$$FX_{1,t} + q_{p,t}D_{p,t+1} = q_{p,t}FX_{1,t+1} + D_{p,t} + T_{1,t}.$$

The left-hand-side is the sources of government funds, and it is the sum of two terms. The first is the value of foreign reserves accumulated in the previous period,  $FX_{1,t}$ . The second is the amount of funds raised with the issuance of new debt,  $D_{p,t+1}$ , which is sold at price  $q_{p,t}$ . The right-hand-side is the uses of government funds, and it is the sum of three terms. The first is the purchase of new reserves. The second is the repayment of the public debt issued in the previous period. The third is the transfer  $T_{1,t}$  to domestic households/firms. Notice that reserves are only in the form of public bonds issued by country 1. Therefore, what matters for country 1 and the market of public debt is the net debt<sup>7</sup>, that is,  $D_{p,t} - FX_{1,t}$ .

The budget constraint of the government in country 2 (representative of emerging economies) is

$$FX_{2,t} = q_{p,t}FX_{2,t+1} + T_{2,t}.$$

The variables  $D_{p,t}$ ,  $FX_{1,t}$  and  $FX_{2,t}$  are time varying but exogenous. In the quantitative exercise they will replicate the observed dynamics of public debt in advanced economies, and foreign exchange reserves in both advanced and emerging economies.

## 2.4 General equilibrium

Using capital letters to denote aggregate variables, the aggregate states include the bonds held by entrepreneurs,  $B_{1,1,t}$ ,  $B_{2,1,t}$ ,  $B_{p,1,t}$ ,  $B_{1,2,t}$ ,  $B_{2,2,t}$ ,  $B_{p,2,t}$ , and the sunspot shocks  $\varepsilon_{1,t}$  and  $\varepsilon_{2,t}$ . The aggregate debt issued by each country's households/firms in the previous period are  $D_{1,t} = B_{11,t} + B_{12,t}$  and  $D_{2,t} = B_{21,t} + B_{22,t}$ . The sequences of productivity,  $z_{1,t}$  and  $z_{2,t}$ , financial variables,  $\phi_{1,t}$ ,  $\phi_{2,t}$ ,  $\kappa_{1,t}$ ,  $\kappa_{2,t}$ , public debt and reserves,  $D_{p,t}$ ,  $FX_{1,t}$ ,  $FX_{2,t}$ , are also relevant for the equilibrium. Since these variables are deterministic and perfectly anticipated, their full sequence going from now into the future is part of the state space. We denote the sequence of a variable starting at time  $t$  and going to infinity with subscript  $t$  and subscript  $\infty$ . For

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<sup>7</sup>Technically, the reserves of country 1 are foreign assets, not the repurchase of its own public debt. However, since country 1 is the aggregation of all advanced economies, it is not possible to clearly distinguish  $D_{p,t}$  from  $FX_{1,t}$ . In reality, the reserves held by some advanced economies (for example European countries) could be in bonds issued by other advanced economies (for example, the US government). Once we aggregate all advanced economies, without netting out the reserves from the public debt, it looks like advanced economies issue public bonds and then repurchase the same bonds as reserves.

example,  $z_{j,t}^\infty$  represents the sequence of productivity for country  $j$  from time  $t$  to  $\infty$ . To use a compact notation, we denote the state vector as

$$\mathbf{s}_t \equiv (z_{1,t}^\infty, z_{2,t}^\infty, \phi_{1,t}^\infty, \phi_{2,t}^\infty, \kappa_{1,t}^\infty, \kappa_{2,t}^\infty, D_{p,t}^\infty, FX_{1,t}^\infty, FX_{2,t}^\infty, B_{1,1,t}, B_{2,1,t}, B_{p,1,t}, B_{1,2,t}, B_{2,2,t}, B_{p,2,t}, \varepsilon_{1,t}, \varepsilon_{2,t}).$$

Figure 4 sketches the steps leading to an equilibrium by dividing a period in the three subperiods as we did earlier.

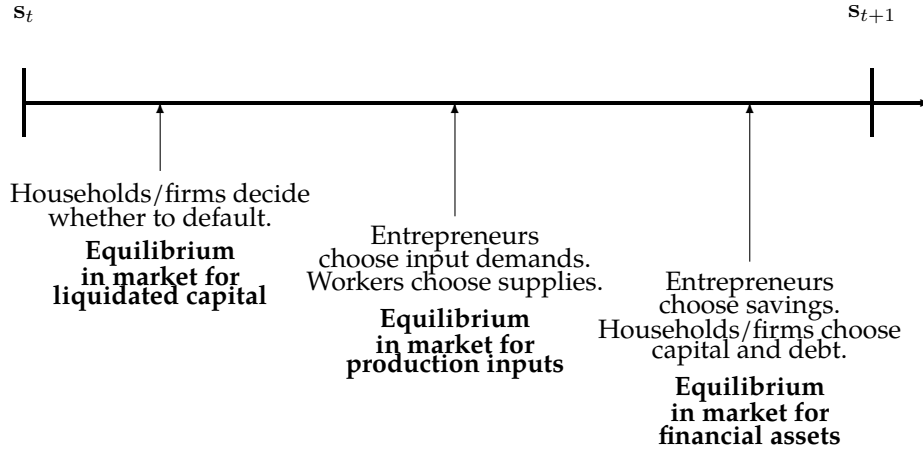


Figure 4: Timing within period  $t$ .

1. **Subperiod 1:** Given the realization of the sunspot shock  $\varepsilon_{j,t}$  and the consequent liquidation price  $\tilde{p}_{j,t}$ , households/firms choose whether to default. The renegotiated debt is

$$\tilde{D}_{j,t} = \begin{cases} \kappa_{j,t}K_{j,t}, & \text{if } D_{j,t} \geq \kappa_{j,t}K_{j,t} \text{ and } \varepsilon_{j,t} = 0 \\ D_{j,t}, & \text{otherwise} \end{cases}.$$

A financial crisis in our model has a fundamental cause—the level of debt—and a multiple equilibrium cause driven by sunspot shocks. Figure 5 plots the probability of a crisis as a function of debt,  $D_{j,t}$ . Given the stock of capital  $K_{j,t}$ , the probability of a crisis is zero when

the debt  $D_{j,t}$ , which is endogenous, is below the threshold  $\kappa_{j,t}K_{j,t}$ . Above this threshold the crisis probability becomes  $\lambda$ , that is, the likelihood that the draw of the sunspot shock is  $\varepsilon_{j,t} = 0$ . For values of  $D_{j,t}$  greater than  $p_{j,t}K_{j,t}$  the crisis probability is 1 because the liquidation value of capital is always smaller than the debt. This shows that a financial crisis is not just the result of a negative sunspot shocks but also the consequence of high leverage (the fundamental cause).

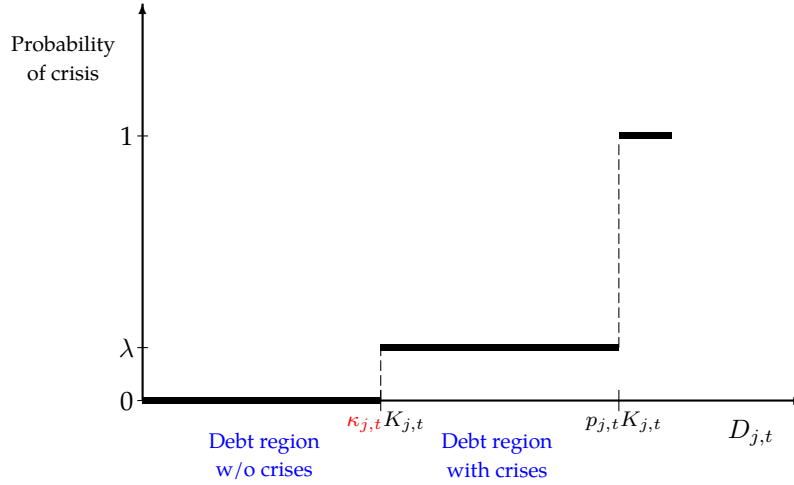


Figure 5: Probability of crisis: debt region with and without crises.

Given the default outcome, the post-default wealth of entrepreneurs is proportional to their holdings prior to default, that is,

$$M_{j,t} = \left( \frac{\tilde{D}_{1,t}}{D_{1,t}} \right) B_{1j,t} + \left( \frac{\tilde{D}_{2,t}}{D_{2,t}} \right) B_{2j,t} + B_{p,j,t}.$$

The terms in parenthesis are the repayment ratios for the private debt issued, respectively, by country 1 and country 2. The public debt issued by country 1, instead, is always repaid in full.

2. **Subperiod 2:** Given the post-default wealth  $M_{j,t}$ , entrepreneurs in country  $j$  choose the inputs of labor and capital, and households/firms choose their supplies. If constraint (2) is binding, the aggregate input demands in country  $j$  are obtained from the individual demands

derived in Lemma 2.1,

$$\begin{aligned} L_{j,t} &= \left( \frac{\gamma}{\phi_{j,t} w_{j,t}} \right) M_{j,t}, \\ K_{j,t} &= \left( \frac{1-\gamma}{\phi_{j,t} r_{j,t}} \right) M_{j,t}. \end{aligned}$$

If constraint (2) is not binding, instead, the aggregate demands for labor and capital are derived from the equalization of the corresponding marginal products to their prices,  $w_{j,t}$  and  $r_{j,t}$ . In this case, Lemma 2.1 shows that the two factor demands can be expressed as

$$\begin{aligned} L_{j,t} &= \left( \frac{\gamma}{\phi_{j,t} w_{j,t}} \right) M_{j,t}, \\ K_{j,t} &= \left( \frac{1-\gamma}{\phi_{j,t} r_{j,t}} \right) m_{j,t}. \end{aligned}$$

The aggregate supply of labor is derived from the household's first order condition (7). Imposing  $h_{j,t} = H_{j,t}$  and inverting we obtain

$$H_{j,t} = \left( \frac{w_{j,t}}{z_{j,t}} \right)^\nu.$$

The supply of capital is exogenous,  $K_{j,t} = \bar{K} g^t$ . Market-clearing in the labor and capital markets determines the wage rate  $w_{j,t}$ , the rental rate  $r_{j,t}$ , and employment  $L_{j,t} = H_{j,t}$  in each country.

### 3. Subperiod 3: The end-of-period wealth of entrepreneurs is

$$A_{j,t} = M_{j,t} + z_{j,t}^\gamma L_{j,t}^\gamma K_{j,t}^{1-\gamma} - w_{j,t} L_{j,t} - r_{j,t} K_{j,t}.$$

According to Lemma 2.2, a fraction  $1 - \beta$  is consumed while the remaining fraction  $\beta$  is saved in new bonds. A fraction  $\theta_{1,t}$  of the saved wealth is allocated to private bonds issued by country 1, a fraction  $\theta_{2,t}$  to private bonds issued by country 2, and the remaining fraction  $1 - \theta_{1,t} - \theta_{2,t}$  to public bonds issued by country 1. Households/firms choose new debt  $D_{j,t+1}$  and new holdings of capital  $K_{j,t+1}$ .

Market-clearing in the three financial markets requires:

$$B_{1,1,t+1} + B_{1,2,t+1} = D_{1,t+1}, \quad (10)$$

$$B_{2,1,t+1} + B_{2,2,t+1} = D_{2,t+1}, \quad (11)$$

$$B_{p,1,t+1} + B_{p,2,t+1} + FX_{1,t+1} + FX_{2,t+1} = D_{p,t+1}. \quad (12)$$



Because of capital mobility and cross-country heterogeneity, the net foreign asset positions of the two countries could be different from zero. For example, for country 1 we could have  $B_{1,1,t+1} + B_{2,1,t+1} + B_{p,1,t+1} + FX_{1,t+1} \neq D_{1,t+1} + D_{p,t+1}$ . Competition also implies that the price paid by entrepreneurs to purchase households/firms' debt is consistent with the interest rate, that is,

$$q_{j,t} = \frac{1}{R_{j,t}}.$$

Since  $\bar{R}_{j,t} = R_{j,t} \mathbb{E}_{t+1} \delta_{j,t+1}$ , the above condition relates the price of private bonds  $q_{j,t}$  to their expected return. A similar condition will be true for public bonds, that is,  $q_{p,t} = \frac{1}{\bar{R}_{j,t}}$ .

The supply of private bonds is derived from the borrowing decisions of households/firms. From the first order condition (8) we have

$$\frac{1}{\bar{R}_{j,t}} = \beta + \Phi \left( \frac{D_{j,t+1}}{\kappa_{j,t+1} K_{j,t+1}} \right).$$

Because at equilibrium  $\bar{R}_{j,t} = R_{j,t} \mathbb{E} \delta_{j,t+1}$  and  $q_{j,t} = 1/R_{j,t}$ , the condition can be rewritten as

$$q_{j,t} = \left[ \beta + \Phi \left( \frac{D_{j,t+1}}{\kappa_{j,t+1} K_{j,t+1}} \right) \right] \mathbb{E} \delta_{j,t+1}. \quad (13)$$

The market for capital must also clear, that is, the demand  $K_{j,t+1}$  must be equal to the exogenous supply  $\bar{K} g^{t+1}$ . The first-order condition (9) then provides a standard forward-looking condition that determines the end-of-period price  $p_t$ .

Because  $z_{j,t}$ ,  $\phi_{j,t}$ ,  $\kappa_{j,t}$ ,  $FX_{j,t}$ ,  $D_{p,t}$  are time-varying and households/firms can default, the economy does not reach a steady state but displays stochastic dynamics driven by the sunspot shocks. In particular, a realization of  $\varepsilon_{j,t} = 0$  could generate a drop in the liquidation value of capital (if the leverage of the country is sufficiently high), which in turn leads to a financial crisis with partial repayment of bonds. This redistributes wealth from lenders (entrepreneurs) to borrowers (households/firms). The decline in entrepreneurs' wealth  $M_{j,t}$  then, reduces the demand for labor and at equilibrium there is lower employment and production. This is the mechanism

through which financial crises have real macroeconomic consequences. A lower value of  $M_{j,t}$  also decreases the demand for capital which reduces the rental rate  $r_{j,t}$ . The lower return on capital then causes a drop in its price  $p_t$ . Therefore, financial crises impact negatively asset prices too.

## 2.5 Sequential property of the equilibrium

The particular structure of the model allows us to solve for the equilibrium at time  $t$  independently of future equilibria as if the model were static. The only exception is the price of capital in normal times,  $p_{j,t}$ . More specifically, given the states  $s_t$ , we can find the values of all equilibrium variables at time  $t$  (with the exception of  $p_{j,t}$ ) by solving the system of nonlinear equations listed in Appendix E. This allows us to solve the model sequentially. For example, to solve for the sequence of equilibria from  $t = 1991$  to  $t = 2020$ , we first solve for the equilibrium at  $t = 1991$ . We then solve for the equilibrium at  $t = 1992$ , and continue until  $t = 2020$ . Not being able to solve for  $p_{j,t}$  sequentially is not a problem because the *normal* price of capital does not enter the equation system listed in Appendix E.<sup>8</sup>

The sequential property of the equilibrium allows us to reduce the sufficient set of state variables. In general, the equilibrium depends on the whole time-varying sequences  $z_{j,t}, \phi_{j,t}, \kappa_{j,t}, FX_{j,t}, D_{p,t}$  from  $t$  to infinity. Because of the sequential property, however, equilibrium variables at time  $t$  are only affected by  $z_{j,t}, \phi_{j,t}, \kappa_{j,t}, \kappa_{j,t+1}, FX_{j,t+1}$  and  $D_{p,t+1}$ . Therefore, from now on, to characterize the equilibrium—except  $p_{j,t}$ —we redefine the sufficient set of state variables as

$$\mathbf{s}_t \equiv (z_{1,t}, z_{2,t}, \phi_{1,t}, \phi_{2,t}, \kappa_{1,t}, \kappa_{2,t}, \kappa_{1,t+1}, \kappa_{2,t+1}, FX_{1,t+1}, FX_{2,t+1}, D_{p,t+1}, B_{1,1,t}, B_{2,1,t}, B_{p,1,t}, B_{1,2,t}, B_{2,2,t}, B_{p,2,t}, \varepsilon_{1,t}, \varepsilon_{2,t}).$$

This property will be very useful for the quantitative application of the model where we use actual data to construct sequences of  $z_{j,t}, \phi_{j,t}, \kappa_{j,t}$ .

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<sup>8</sup>This property derives from the assumption that the liquidation price of capital under default is  $\tilde{p}_{j,t} = \kappa_{j,t}$ , which is exogenous. If  $\tilde{p}_{j,t}$  were a function of  $p_{j,t}$ , we would not be able to solve the model sequentially: to find the values of all other variables, we would need to solve for  $p_{j,t}$ , which is forward looking (see condition (9)). Relaxing the exogeneity of capital and/or the risk-neutrality of households could also break this property.

## 2.6 Other properties and remarks

Another property of the equilibrium worth noting is that the risk-free interest rate is lower than the rate of time preference (or, equivalently, the price of a risk-free bond is higher than the inter-temporal discount factor  $\beta$ ). In models with precautionary savings, this property holds because of the incentive to accumulate a buffer stock of savings for self-insurance. In our model, instead, entrepreneurs are willing to hold the (private) debt even if the interest rate is lower than their rate of time preference because of its inside money-convenience yield feature: it is a financial asset that facilitates production. This arises when constraint (2) is binding. Therefore, provided that the constraint is binding, entrepreneurs receive a benefit from holding bonds that is additional to the interests paid by the bonds.

The equilibrium property by which entrepreneurs are net savers and households/firms are borrowers is important for the macroeconomic consequences of a financial crisis. Because producers have a positive financial position, a crisis redistributes wealth away from producers. The drop in entrepreneurial net worth, then, causes a macroeconomic contraction. In an environment in which producers are net borrowers, a financial crisis characterized by lower repayments of debt would increase the net worth of producers and would have the opposite macroeconomic consequences.

Having producers with a positive financial position might seem counterfactual at first. However, it is consistent with the recent changes in the financial structure of US corporations. It is well known that during the last two-and-a-half decades, the corporate sector has increased its holdings of financial assets. This suggests that the proportion of financially dependent firms has declined over time, which is consistent with the empirical findings of Shourideh and Zetlin-Jones (2012) and Eisfeldt and Muir (2016).

The large accumulation of financial assets by producers—often referred to ‘cash’—is related to the significance of business savings. Although the rising savings of US corporations has attracted considerable attention in the literature (see, for example, Riddick and Whited (2009)), this is not just a US phenomena. Busso et al. (2016) document the share of savings done by firms both in advanced and emerging economies and present evidence that in Latin America this share is even larger than in advanced economies. The importance of business savings is also documented in Bebczuk and Cavallo (2016). Using data for 47 countries over 1995–2013, they show that the contribution of businesses to national savings is on average more

than 50%. The increase in corporate cash suggests that more and more firms borrow less than what is available to them. In this regard, we would like to point out that, in order for a firm to be financially unconstrained, it is not necessary to have a positive financial asset position. Firms with a negative financial asset position may still be unconstrained if they borrow less than their capacity. Our entrepreneurial sector captures the growing importance of firms that are not very dependent on external financing.<sup>9</sup>

At the same time, during the past three decades, we have witnessed a significant increase in household debt. Corporate debt has also increased, indicating that the nonfinancial sector has issued more debt while also accumulating financial assets. We conjecture, however, that there is significant heterogeneity among corporate firms and the increase in corporate debt has been driven by a subset of firms, most likely those that own a large amount of tangible assets. These firms are represented in the model by the consolidated households/firms sector.

### 3 Quantitative analysis

We now use the model to assess quantitatively how the changes in the model's exogenous driving forces affected financial and macroeconomic volatility over the past three decades. The quantitative implementation follows three steps:

1. Calibration of structural parameters.
2. Construction of sequences for  $z_{1,t}$ ,  $z_{2,t}$ ,  $\phi_{1,t}$ ,  $\phi_{2,t}$ ,  $\kappa_{1,t}$ ,  $\kappa_{2,t}$ .
3. Counterfactual simulations given the constructed sequences of  $z_{1,t}$ ,  $z_{2,t}$ ,  $\phi_{1,t}$ ,  $\phi_{2,t}$ ,  $\kappa_{1,t}$ ,  $\kappa_{2,t}$ , and the observed sequences of  $FX_{1,t}$ ,  $FX_{2,t}$  and  $D_{p,t}$ .

For the first two steps, we use data over the period 1991-2020, for both advanced economies (country 1) and emerging economies (country 2). The countries included in emerging and advanced economies are those listed in the note to Figure 1. The simulations conducted in the third step are also over the period 1991-2020.

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<sup>9</sup>See Kalemli-Ozcan, Sorensen, and Yesiltas (2012) for stylized facts about bank and firm leverage using internationally micro data.

### 3.1 Calibration of structural parameters

The model is calibrated to an annual frequency and the discount factor is set to  $\beta = 0.93$ , implying an annual intertemporal discount rate of about 7%. We set the elasticity of labor supply to  $\nu = 1$ , which is often used for the calibration of macroeconomic models.

The probability that the liquidation price of capital drops to  $\tilde{p}_{j,t} = \kappa_{j,t}$  (i.e., the probability of a negative sunspot shock  $\varepsilon = 0$ ) is  $\lambda = 0.04$ . This is within the range of estimates of crisis probabilities provided in the literature (see, for example, Bianchi and Mendoza (2018)). It implies that crises are low probability events, every twenty-five years on average. Since sunspot shocks are country-specific, that is, they are independent across countries, a *global* financial crisis is an even rarer event, with a probability of  $0.04 \times 0.04 = 0.0016$ .

We calibrate next the labor share parameter in the production function, which we set to  $\gamma = 0.6$ , and the depreciation rate, which we set to  $\tau = 0.08$ . These values are standard in the macro literature.

The last parameter we calibrate is the cost of borrowing  $\eta$ . Unfortunately, we have limited information to determine it. We set it to  $\eta = 0.1$  but we will later conduct a sensitivity analysis to gauge its role in our findings (see Appendix F). The full list of calibrated parameters is in Table 1.

Table 1: Parameter values.

<i>Description</i>	<i>Parameter</i>	<i>Value</i>
Discount factor	$\beta$	0.930
Share of labor in production	$\gamma$	0.600
Depreciation rate	$\tau$	0.080
Elasticity of labor supply	$\nu$	1.000
Probability of crises (low sunspot shock)	$\lambda$	0.040
Borrowing cost	$\eta$	0.100

### 3.2 Construction of sequences for $z_{j,t}, \phi_{j,t}, \kappa_{j,t}$

Differences in size and financial structure between the two regions are generated by the deterministic sequences  $z_{j,1991}^{2020}, \phi_{j,1991}^{2020}, \kappa_{j,1991}^{2021}$ , for  $j \in \{1, 2\}$ . We construct these sequences to replicate the empirical time series shown in Figure 1 over the period 1991-2020. As explained in Section 2.5, the se-

quential property of the equilibrium allows us to determine  $z_{j,1991}^{2020}$ ,  $\phi_{j,1991}^{2020}$ ,  $\kappa_{j,1991}^{2021}$  without knowing their values beyond 2020 (2021 for  $\kappa_{j,t}$ ).

We also need to assign the sequences of foreign exchange reserves,  $FX_{j,t}$ , and public debt in advanced economies,  $D_{p,t}$ . These are set to match the values observed in the data as percentages of GDP. Data for reserves is obtained from the *External Wealth of Nations* database (Lane and Milesi-Ferretti (2018)), while data for the public debt is from IMF's *Global Debt Database*. The Global Debt Database provides two series: 'Central Government Debt' and 'General Government Debt'. We use the former. Unfortunately, data for all years 1991-2020 are available only for thirteen of the advanced economies listed in the note to Figure 2. Hence, we compute the debt-to-GDP ratio for advanced economies by aggregating the data for these thirteen countries.

A complication in using the model to construct sequences of  $z_{j,t}$ ,  $\phi_{j,t}$ ,  $\kappa_{j,t}$  is that the resulting values depend on the stochastic realizations of the (sunspot) shocks  $\varepsilon_{1,t}$  and  $\varepsilon_{2,t}$ . Therefore, we have to choose particular sequences of  $\varepsilon_{1,t}$  and  $\varepsilon_{2,t}$  over the 1991-2020 period.

We choose the shock sequences based on the following assumptions. We assume that  $\varepsilon_{j,t} = 1$  (no crisis) in all simulated years with only few exceptions. For emerging economies it takes the value of zero in 1997 and 2009 ( $\varepsilon_{2,1997} = 0$  and  $\varepsilon_{2,2009} = 0$ ). These two years correspond, respectively, to the 1997 Sudden Stops in South-East Asia and to the Global Financial Crisis that started in 2008 and extended to 2009. Both crises had an impact on emerging economies. For advanced economies, instead, it takes the value of zero only in 2009 ( $\varepsilon_{1,2009} = 0$ ) reflecting, again, the Global Financial Crisis. It is important to point out that, even though we calibrate the model assuming a specific sequence of shocks, agents do not anticipate them and make decisions based on their stochastic distribution.

**Productivity  $z_{j,t}$ .** The productivity series  $z_{1,1991}^{2020}$  and  $z_{2,1991}^{2020}$  are constructed as Solow residuals from the production function. To do so, we need measures of production inputs and outputs. For output, we use GDP at nominal exchange rates, not PPP. Since movements in nominal exchange rates affect the purchasing power of a country in the acquisition of foreign assets, our productivity measure should also reflect movements in exchange rates. Another factor that contributes to differences in aggregate GDP is population growth. Since population is not explicitly included in the model, the

constructed sequences of productivity also capture changes in population.

Denote by  $P_{j,t}$  the nominal price index for country  $j$  expressed in US dollars. The price is calculated by multiplying the price in local currency by the dollar exchange rate. We can then define the nominal (dollar) aggregate output of country  $j$  as

$$P_{j,t}Y_{j,t} = P_{j,t}\hat{z}_{j,t}^\gamma L_{j,t}^\gamma K_{j,t}^{1-\gamma} N_{j,t},$$

where  $\hat{z}_{j,t}$  is actual productivity,  $L_{j,t}$  is per-capita employment,  $K_{j,t}$  is per-capita capital, and  $N_{j,t}$  is population. Notice that the above definition of output assumes that physical capital increases with population.

If we deflate the nominal GDP in both countries by the price index in country 1, we obtain

$$Y_{1,t} = \hat{z}_{1,t}^\gamma L_{1,t}^\gamma K_{1,t}^{1-\gamma} N_{1,t},$$

$$\frac{P_{2,t}Y_{2,t}}{P_{1,t}} = \left( \frac{P_{2,t}\hat{z}_{2,t}^\gamma}{P_{1,t}} \right) L_{2,t}^\gamma K_{2,t}^{1-\gamma} N_{2,t},$$

Thus, aggregate productivity in the model corresponds to

$$z_{1,t} = \hat{z}_{1,t} N_{1,t}^{\frac{1}{\gamma}},$$

$$z_{2,t} = \hat{z}_{2,t} \left( \frac{P_{2,t}N_{2,t}}{P_{1,t}} \right)^{\frac{1}{\gamma}}.$$

Since  $P_{2,t}$  is the dollar price of output in emerging-markets, the ratio  $P_{2,t}/P_{1,t}$  corresponds to the real exchange rate. The above expressions show that  $z_{1,t}$  and  $z_{2,t}$  also reflect cross-country differences in real exchange rates and population, in addition to actual TFP. The productivity sequences that we use in the model are calculated from the data as

$$z_{1,t} = \left( \frac{Y_{1,t}}{L_{1,t}^\gamma K_{1,t}^{1-\gamma}} \right)^{\frac{1}{\gamma}}, \quad (14)$$

$$z_{2,t} = \left( \frac{P_{2,t}Y_{2,t}/P_{1,t}}{L_{2,t}^\gamma K_{2,t}^{1-\gamma}} \right)^{\frac{1}{\gamma}}. \quad (15)$$

The numerator is total real GDP, deflated by the nominal price in advanced economies. If the real exchange rate of emerging economies appreciates, it will be reflected in higher relative productivity. Although this does not increase actual productivity, it raises the ability of these countries to purchase assets in advanced economies, which is important for the model's general equilibrium. Also notice that changes in relative prices could simply reflect movements in nominal exchange rates. Still, when the currencies of emerging economies appreciate, assets created in advanced economies become cheaper for emerging economies.

In order to use equations (14) and (15) to construct the productivity sequences, we need empirical counterparts for  $Y_{1,t}$ ,  $P_{2,t}Y_{2,t}/P_{1,t}$ ,  $L_{1,t}$ ,  $L_{2,t}$ ,  $K_{1,t}$ , and  $K_{2,t}$ . We got the empirical series from the World Bank's *World Development Indicators* (WDI).

The output variables  $Y_{1,t}$  and  $P_{2,t}Y_{2,t}/P_{1,t}$  are obtained by aggregating the GDP of advanced economies and the GDP of emerging economies, respectively, both expressed at constant US dollars. For the labor input  $L_{j,t}$  we use employment-to-population ratio (population over 15 years of age). The variable  $K_{j,t}$  grows in the model at the constant rate  $g - 1$ . Therefore, we can express the stock of capital as  $K_{j,t} = \bar{K}g^t$ , with  $\bar{K}$  is normalized to 1. Notice that the constant growth rate of capital is the same in the two regions. We set this rate to the average growth rate of aggregate GDP in advanced economies which is equal to 1.89% over the 1991-2020 period. We take this number as the long-run growth rate for both advanced and emerging economies (after convergence). The resulting productivity series are plotted in panel (a) of Figure 6.

**Financial structure  $\phi_{j,t}$  and  $\kappa_{j,t}$ .** The time-varying parameter  $\phi_{j,t}$  is important for the *demand* of financial assets, in the spirit of Mendoza et al. (2009): Higher values of  $\phi_{j,t}$  increase the demand because more financial assets are needed for production (working capital, etc.). The time-varying parameter  $\kappa_{j,t}$  is important for the *supply* of financial assets, in the spirit of Caballero et al. (2008): Higher values of  $\kappa_{j,t}$  increase the incentive for households/firms to borrow.

The sequences of  $\phi_{1,t}$ ,  $\phi_{2,t}$ ,  $\kappa_{1,t}$ ,  $\kappa_{2,t}$  are constructed so that the model replicates four empirical series over the period 1991-2020: (i) private domestic credit-to-GDP ratio in advanced economies, (ii) private domestic credit-to-GDP ratio in emerging economies, (iii) NFA position of advanced



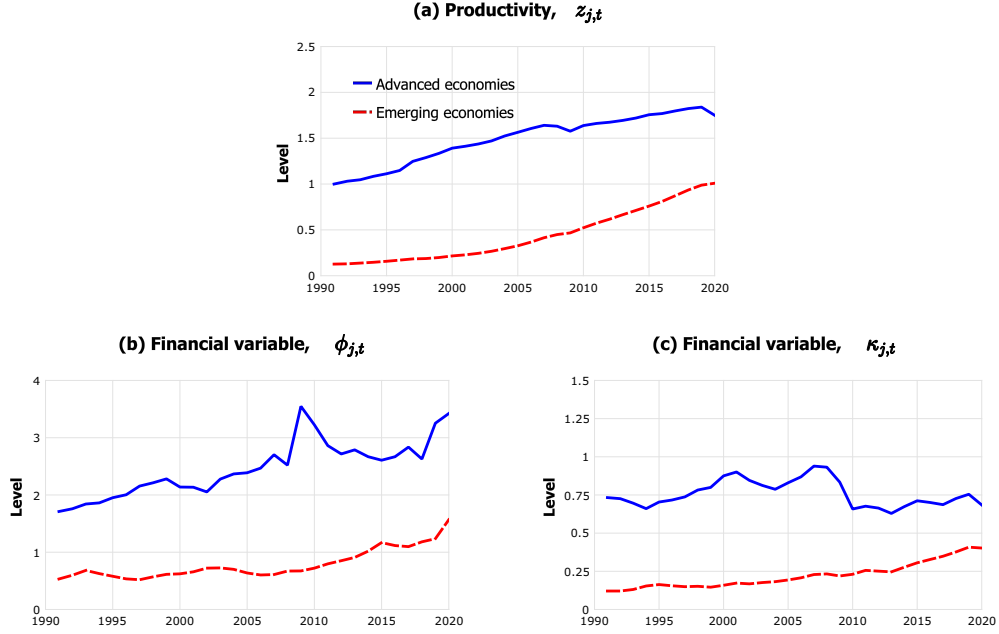


Figure 6: Computed productivity and financial variables series for advanced and emerging economies, 1991-2020.

economies, (iv) US risk-free real interest rate. These are the empirical series shown in the last three panels of Figure 1. The following equations describe the mapping from these four empirical targets to the corresponding variables in the model:

$$\text{Private Credit-to-GDP AEs} = \frac{q_{1,t}D_{1,t+1}}{Y_{1,t}}, \quad (16)$$

$$\text{Private Credit-to-GDP EEs} = \frac{q_{2,t}D_{2,t+1}}{Y_{2,t}}, \quad (17)$$

$$\begin{aligned} \text{NFA-to-GDP Advanced Economies} = & \\ & \frac{q_{1,t}B_{1,1,t+1} + q_{2,t}B_{2,1,t+1} + q_{p,t}B_{p,1,t+1} + q_{p,t}FX_{1,t+1} - q_{1,t}D_{1,t+1} - q_{p,t}D_{p,t+1}}{Y_{1,t}}, \end{aligned} \quad (18)$$

$$\text{US real interest rate} = \frac{1}{q_{p,t}} - 1. \quad (19)$$

The terms in the right-hand-side are equilibrium objects that we can compute from the model for a given set of values of  $\phi_{1,t}$ ,  $\phi_{2,t}$ ,  $\kappa_{1,t}$ ,  $\kappa_{2,t}$ ,  $\kappa_{1,t+1}$  and  $\kappa_{2,t+1}$ .<sup>10</sup> Given the sequential property of the equilibrium (see

<sup>10</sup>It also requires the constructed productivity series and the actual data on reserves

Section 2.5), we can find the equilibrium values of these variables in period  $t$  by solving the system of nonlinear equations listed in Appendix E.<sup>11</sup> After initializing  $\kappa_{1,1991}$  and  $\kappa_{2,1991}$ , we solve for  $\phi_{1,1991}$ ,  $\phi_{2,1991}$ ,  $\kappa_{1,1992}$  and  $\kappa_{2,1992}$  by applying two nested nonlinear solvers.<sup>12</sup> The inner solver finds the model's equilibrium given the values of  $\phi_{1,1991}$ ,  $\phi_{2,1991}$ ,  $\kappa_{1,1992}$  and  $\kappa_{2,1992}$  (as described in Appendix E). The outer solver then uses the results from the inner solver to check whether the equilibrium associated with the particular values of  $\phi_{1,1991}$ ,  $\phi_{2,1991}$ ,  $\kappa_{1,1992}$  and  $\kappa_{2,1992}$  satisfies conditions (16)-(19). If will then update the values of  $\phi_{1,1991}$ ,  $\phi_{2,1991}$ ,  $\kappa_{1,1992}$  and  $\kappa_{2,1992}$  until conditions (16)-(19) are satisfied. At this point we move to the next period and find the values of  $\phi_{1,1992}$ ,  $\phi_{2,1992}$ ,  $\kappa_{1,1993}$  and  $\kappa_{2,1993}$ . We continue until we have solved for all sample years 1991-2020.

Figure 7 provides a graphical intuition explaining how the above procedure yields the identification of the four time-varying financial parameters at a given date. The graph depicts the financial market equilibrium. The interest rate equalizes the global demand for assets (sum of the demands from both countries) to the global supply (sum of the supplies from both countries). Here demands and supplies contain both private and public components. More specifically, the demand for financial assets issued by advanced economies is given by  $q_{1,t}B_{1,1,t+1} + q_{2,t}B_{1,2,t+1} + q_{p,t}FX_{1,t+1}$  while the supply of these assets by advanced economies is  $q_{1,t}D_{1,t+1} + q_{p,t}D_{p,t+1}$ . On the other hand, the demand for financial assets issued by emerging economies is given by  $q_{1,t}B_{2,1,t+1} + q_{2,t}B_{2,2,t+1} + q_{p,t}FX_{2,t+1}$  while their supply is  $q_{2,t}D_{2,t+1}$ .

The parameters  $\phi_{j,t}$  and  $\kappa_{j,t+1}$  determine, respectively, the positions of the demand and supply curves in country  $j$ . Given the public demand for financial assets,  $FX_{j,t+1}$ , and the public supply,  $D_{p,t+1}$ , an increase in  $\phi_{j,t}$  shifts the *demand* of country  $j$  to the right while an increase in  $\kappa_{j,t+1}$  shifts the *supply* of country  $j$  to the right. To identify these four parameters we use the four circled variables: (i) the debt in country 1; (ii) the debt in country 2; (iii) the net foreign asset position of country 1; (iv) the world interest rate. As indicated in equations (16)-(19), the empirical counter-

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and public debt.

<sup>11</sup>As pointed out in Section 2.5, we can solve for all equilibrium variables at any time  $t$ , except for the normal price of capital  $p_{j,t}$ . However,  $p_{j,t}$  does not affect the equilibrium variables that are mapped to the four empirical targets listed in (16)-(19).

<sup>12</sup>As long as the realizations of the sunspot shock in 1991 are not those causing a crisis (which is our assumption), the values of  $\kappa_{1,1991}$  and  $\kappa_{2,1991}$  are irrelevant as initial states.

parts of these four variables are: (i) Private domestic credit in Advanced Economies; (ii) Private domestic credit in Emerging Economies; (iii) Net foreign asset position of Advanced Economies; (iv) US interest rate. A more detailed description of the data is provided at the bottom of Figure 1. The task that the two-nested-solver algorithm completes is to find the values of the four financial structure parameters so that the positions of the supply and demand curves in the two countries give rise to an equilibrium that matches the four empirical targets.

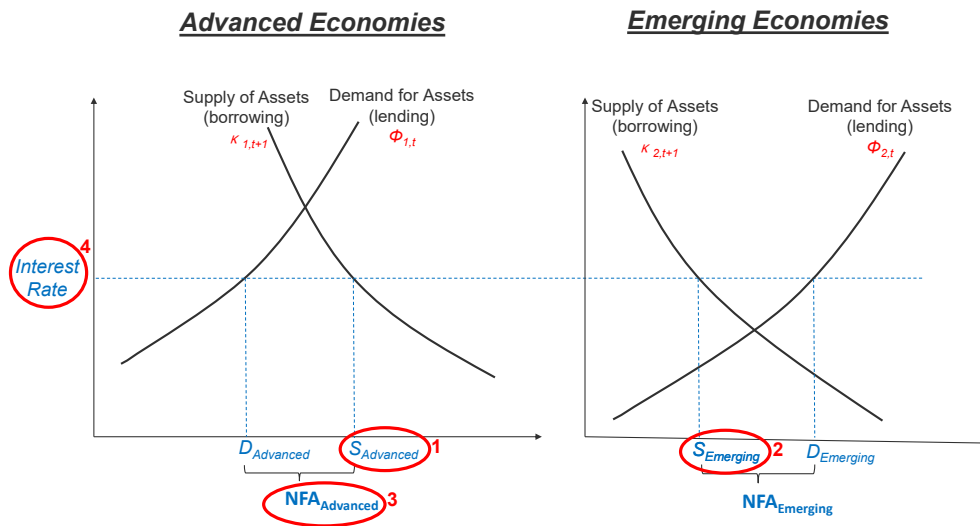


Figure 7: Identification of financial structure parameters.

Public debt and FX reserves are important because they are part of the demands and supplies of assets. For example, an increase in FX reserves, either from advanced economies or emerging economies, moves the demand for assets to the right, leading to a reduction in the world interest rate. On the other hand, an increase in public debt issued by advanced economies shifts the supply of assets from these economies to the right. This leads to an increase in the world interest rate.

The computed series are plotted in panels (b) and (c) of Figure 6. Panel (b) shows that, for advanced economies,  $\phi_{j,t}$  is significantly bigger than 1. This might seem inconsistent with the interpretation of  $m_{j,t}$  as working capital needed to finance input payments. However, as discussed earlier, we interpret the need for financial assets in production more broadly.

For example, the production function we used abstracts from intermediate inputs that also require working capital. There are also other channels through which financial wealth facilitates production even if they are not explicitly modelled here. Financial wealth could provide insurance against earning risks which allows for smoother dividend payments. Higher financial wealth, then, encourages entrepreneurs to operate larger production scales. Also, firms with more financial assets find easier to hire workers, either because the risk of financial distress is lower or because workers could negotiate higher wages.

Panel (b) shows that  $\phi$  trends upward in both advanced and emerging economies, but proportionally more in the latter. Panel (c) shows that  $\kappa_{j,t}$  has also increased in emerging economies, but not in advanced economies. Since higher values of  $\kappa_{j,t}$  raise the supply of assets, the computed series indicate that financial constraints in the private sector have been relaxed in emerging economies but not so much in advanced economies. However, even if financial constraints in advanced economies did not change much, private debt did increase endogenously in response to the lower interest rate. Also, the public debt issued by advanced economies, net of their accumulation of FX reserves, increased significantly as shown in Figure 2. Therefore, the total supply of financial assets (private plus public) still increased substantially in advanced economies.

### 3.3 Counterfactual simulations

In this section we explore how the changes in productivity and financial structure (shown in Figure 6) and the changes in reserves and public debt (shown in Figure 2) affected the observed macroeconomic dynamics. We do so by conducting counterfactual simulations in which we allow only one factor to change, while keeping the others fixed. We start with productivity.

**Faster growth in emerging economies.** We impose that  $\phi_{j,t}$  and  $\kappa_{j,t}$  remain constant at their 1991 values for the whole simulation period, while  $z_{j,t}$  takes the values shown in Figure 6. The detrended sequences of foreign reserves,  $FX_{j,t}$ , and public debt,  $D_{p,t}$ , also stay constant.<sup>13</sup> The series generated by this counterfactual simulation are plotted in Figure 8.

<sup>13</sup>These series are set to their 1990 values and then allowed to grow at the constant long-run growth rate  $g - 1$ . Hence, in the detrended model they are constant.

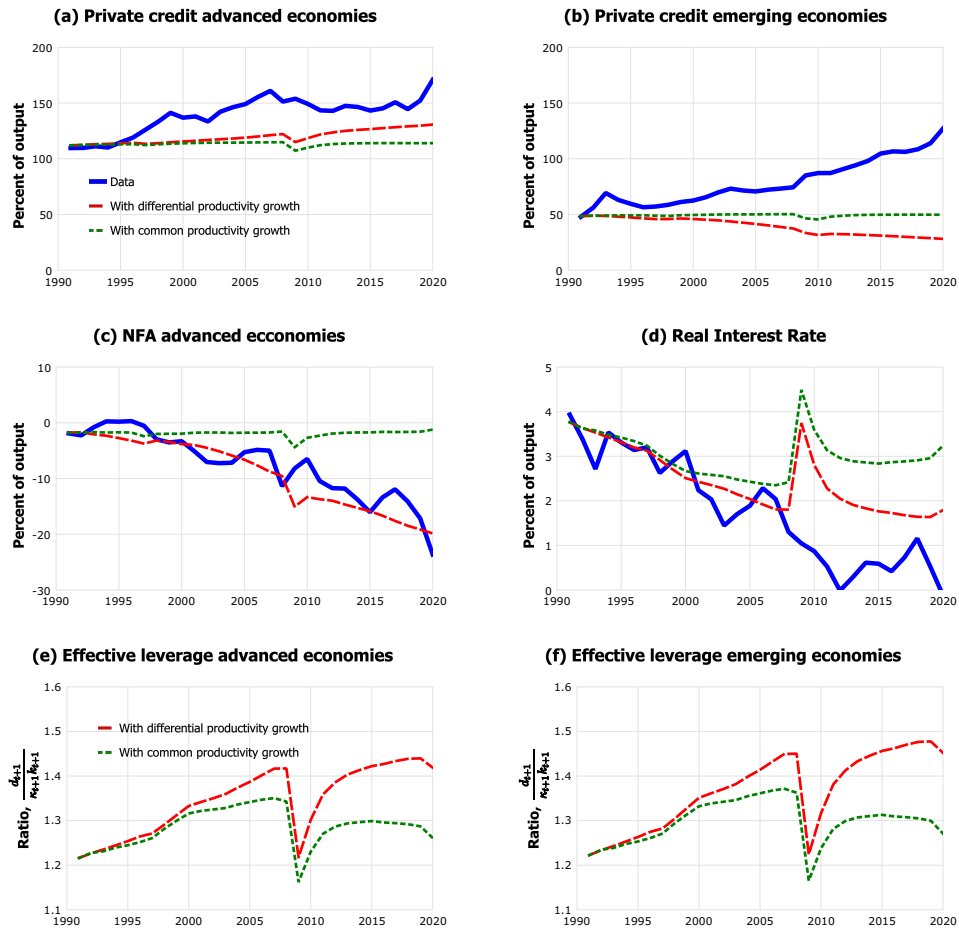


Figure 8: Counterfactual simulation when only productivity changes, 1991-2020.

Panels (a)-(d) plot domestic credit in advanced economies and emerging economies, NFA in advanced economies, and the risk-free real interest rate (which is common to the two regions). The continuous line is the actual data shown in Figure 1. By construction, this is also the series generated by the baseline model with productivity and financial variables taking the values plotted in Figure 6, and reserves and public debt taking the values plotted in Figure 2. The dashed line is the model-generated data when only productivity changes. The dotted line also plots model-generated data when only productivity changes, but imposing that productivity in emerging economies grows at the same (lower) rate as in ad-

vanced economies. The comparison of the dashed and dotted lines illustrates the importance of faster productivity growth in emerging economies.

Panel (c) shows that the faster growth in productivity experienced by emerging economies generates, by itself, a global imbalance that is similar to the data (the sharp NFA decline in advanced economies). From panel (d) we can see that it also explains a sizable share of the decline in the interest rate. The spike in the interest rate in 2009 is caused by the financial crisis. Remember that in these simulations we impose that in 2009 there is a negative sunspot shock mimicking the financial crisis. Panels (a) and (b) show that the growing size of emerging economies also generates some increase in the domestic credit of advanced economies (as a percentage of output), while it falls in emerging economies. However, the latter occurs because output (the denominator) grows faster than domestic credit (the numerator). The level of domestic credit does increase in emerging economies. Still, the plots show that the faster growth of emerging economies explains relatively little of the observed growth in credit in both regions.

Panels (e) and (f) plot the ‘effective’ leverage ratio (i.e., the ratio of private debt,  $D_{j,t+1}$ , to its recovery value in a financial crisis,  $\kappa_{j,t+1}K_{j,t+1}$ ). Except for the temporary drop after the financial crisis, the model predicts an increasing trend in response to the productivity changes experienced by the two regions (dashed line). This is directly related to the change in the interest rate: a lower interest rate is always associated with higher effective leverage (see condition (8)).

As we show later, the increase in effective leverage plays an important role in driving aggregate volatility. It is important to note, however, that the upward trend in leverage in both regions would not have emerged if emerging economies had grown at the same (lower) rate of advanced economies, as shown by the dotted line.

The main takeaway from this first counterfactual exercise is that the faster growth of emerging economies has been an important force for global imbalances and contributed to some of the decline in the world real interest rate. But why does faster EMs growth leads them to hold a *positive* NFA position and reduces the world interest rate? This may appear surprising because it differs from the prediction of a standard two-country neoclassical model. The mechanism can be described as follows.

In the standard neoclassical model, faster growth implies faster consumption growth and increased borrowing, which increases the interest rate. This is also true in our model for entrepreneurs since they have a

standard utility function. However, entrepreneurs also experience an increase in profits when the economy grows faster, part of which they save. The increase in entrepreneurial savings in fast-growing countries raises the demand for financial assets. Because households in both countries do not change the supply of financial assets at a given interest rate—since  $\kappa_{j,t}$  does not depend on productivity—the world interest rate must decline. The higher entrepreneurial savings in fast-growing emerging economies also implies an increase in their NFA position (and a fall in advanced economies).

To summarize, faster growth allows for higher profits that increase entrepreneurial wealth and, therefore, the demand for financial assets. But when  $\kappa_{j,t}$  does not change, the supply of financial assets at a given interest rate remains the same. To clear the market, then, the interest rate has to drop. The faster growth of entrepreneurial wealth in emerging economies also implies that part of that wealth will be invested abroad, generating global imbalances.

**Changes in financial structure.** To explore the importance of the changes in financial structure, we keep detrended productivity, reserves and public debt constant, while allowing for changes only in  $\phi_{j,t}$  and  $\kappa_{j,t}$ . More specifically, starting from the 1991 values, we impose that  $z_{j,t}$ ,  $FX_{j,t}$  and  $D_{p,t}$  all grow at the constant long-run rate  $g - 1 = 0.0189$ . This is the average GDP growth of advanced economies over the sample period 1991-2020. The financial parameters  $\phi_{j,t}$  and  $\kappa_{j,t}$  however, take the values shown in Figure 6. The simulated variables are plotted in Figure 9.

The changes in financial structure induced by changes in  $\phi_{j,t}$  are important for explaining the observed growth in financial intermediation (higher credit-to-GDP ratios) in both regions, as shown in panels (a) and (b). The changes in  $\phi_{j,t}$  also generate a decline in the NFA of advanced economies and in the real interest rate (see panels (c) and (d)). In contrast, changes in  $\kappa_{j,t}$  have the opposite effects (except in EMs they still induce higher credit).

When we consider the changes in both  $\phi_{j,t}$  and  $\kappa_{j,t}$ , we see that the effect of the former dominates for credit in advanced economies and the interest rate, so that the former rises and the latter drops (see panels (a) and (d)). In fact, allowing for changes in both financial structure parameters yields paths for credit in AEs and for the interest rate very close to those in the actual data.<sup>14</sup> The opposite happens with the NFA of advanced economies:

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<sup>14</sup>Recall that in this experiment, detrended productivity, reserves and public debt are

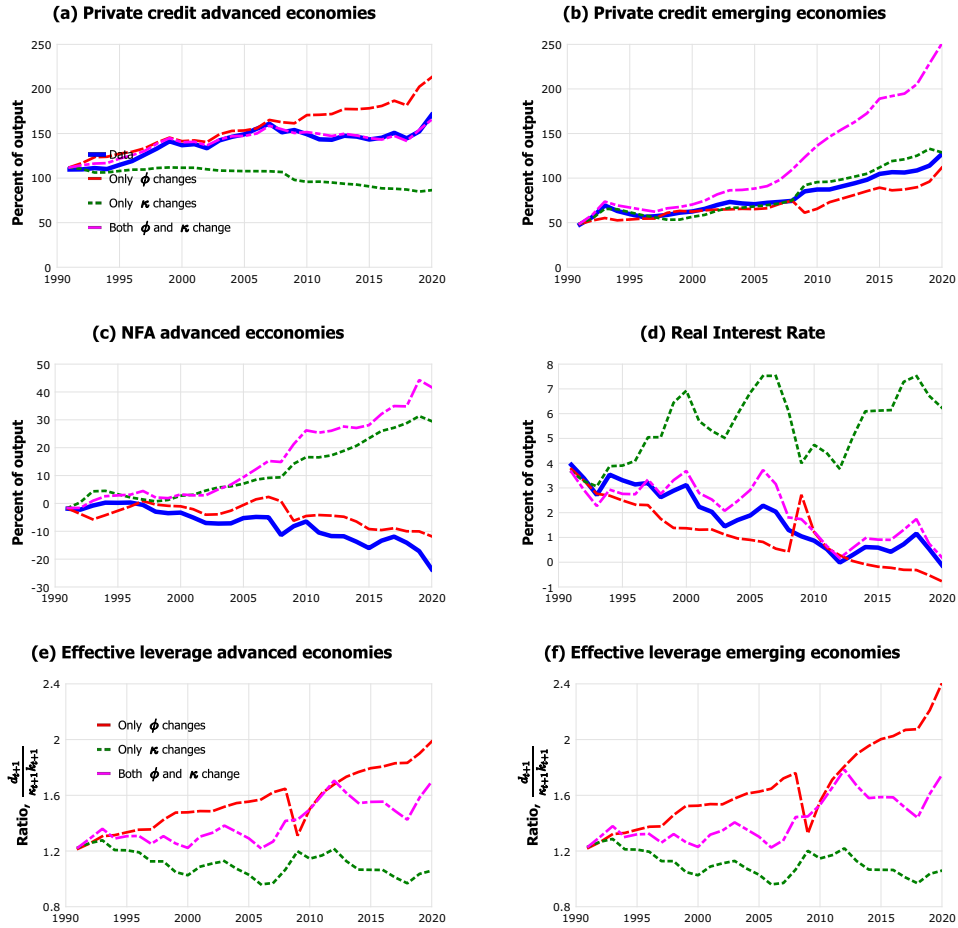


Figure 9: Counterfactual simulation when only the financial structure changes (either  $\phi_{j,t}$  or  $\kappa_{j,t}$ ), 1991-2020.

the effect of  $\kappa_{j,t}$  dominates so that NFA goes up instead of down (see panel (c)). For credit in emerging economies, since both  $\phi_{j,t}$  and  $\kappa_{j,t}$  push for higher credit, the combined effect is a stronger credit expansion, much larger than in the data.

Since the changes in  $\phi_{j,t}$  reduced the interest rate and the changes in  $\kappa_{j,t}$  increased it, these changes moved effective leverage in the opposite directions: higher with only changes in  $\phi_{j,t}$  and lower with only changes in  $\kappa_{j,t}$ . constant. Therefore, changes in  $\phi_{j,t}$  and  $\kappa_{j,t}$  are not sufficient to mimic the actual data.



However, since the impact of  $\phi_{j,t}$  dominates the impact of  $\kappa_{j,t}$ , the combined impact on effective leverage is positive.

**Accumulation of FX reserves.** We now explore the role of FX reserves accumulation. We keep detrended public debt, detrended productivity,  $\phi_{j,t}$  and  $\kappa_{j,t}$  constant.  $FX_{j,t}$ , instead, takes the values shown in Figure 2. The simulated variables are plotted in Figure 10.

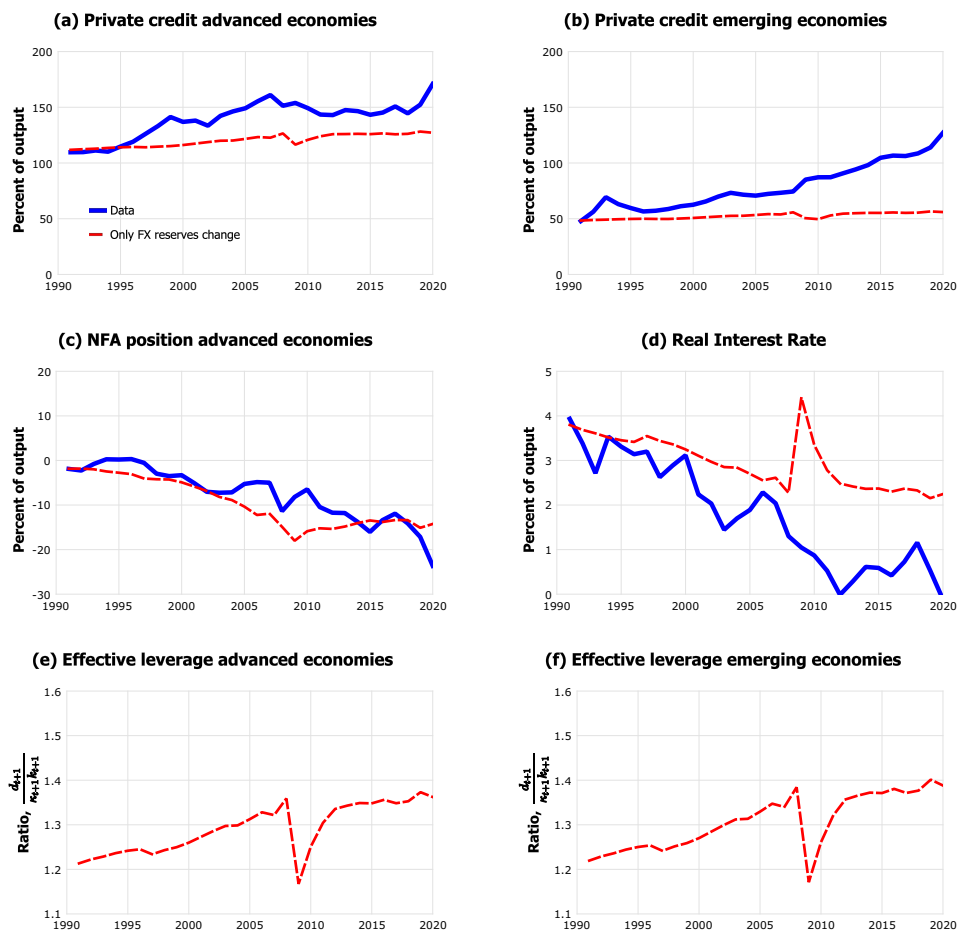


Figure 10: Counterfactual simulation when only the FX reserves change, 1991-2020.

The accumulation of foreign reserves has a significant impact on global imbalances and the world interest rate. It matches closely the fall in NFA

of advanced economies and explains roughly half of the fall in the interest rate (see panels (c) and (d)). Effective leverage rises sharply again because of the fall in the interest rate (panels (e) and (f)). Since the FX reserves of advanced economies remained relatively stable as a fraction of GDP, these effects are mostly induced by the accumulation of reserves by emerging economies. This surge in EMs foreign reserves by itself, however, cannot explain the growth in private credit in both advanced and emerging economies, as panels (a) and (b) show.

**The rising public debt of advanced economies.** We now keep detrended FX reserves, detrended  $z_{j,t}$ , and the parameters  $\phi_{j,t}$  and  $\kappa_{j,t}$  constant. The public debt issued by advanced economies  $D_{p,t}$ , instead, takes the values shown in Figure 2. The simulated variables are plotted in Figure 11.

The increase in the issuance of public debt generates a significant decline in the NFA of advanced economies (panel (c)). Since more debt is supplied, the interest rate has to rise in order to incentivize the purchase of the debt (panel (d)). Therefore, if the only change observed during this period was the increased borrowing of governments in advanced economies, we would have observed an increase in the interest rate, not the decline shown in the data. Notice that at some point the interest rate becomes flat in the graph. This is because, once the interest rate becomes equal to the inter-temporal discount rate ( $1/q_{p,t} = 1/\beta$ ), the linear utility of households/firms implies that they become indifferent between borrowing and lending. So, at that point, if there is not enough demand from entrepreneurs and governments, some of the unsold public debt will be purchased by households/firms. Essentially, when  $q_{p,t} = \beta$ , the demand for public debt becomes infinitely elastic.

The higher interest rate is associated with a decline in effective leverage (panels (e) and (f)). As leverage declines, we reach a point where households/firms will not default on the debt because it becomes smaller than the liquidation value of their assets. This arises exactly when the interest rate reaches the upper bound  $1/\beta$ . As we will see in the next Section, this reduces aggregate volatility.

Higher public debt in advanced economies also reduces private credit in both advanced and emerging economies (see panels (a) and (b)). This occurs because of the fall in leverage caused by the higher interest rate. Thus, the changes in  $D_{p,t}$  cannot explain the observed credit surges.

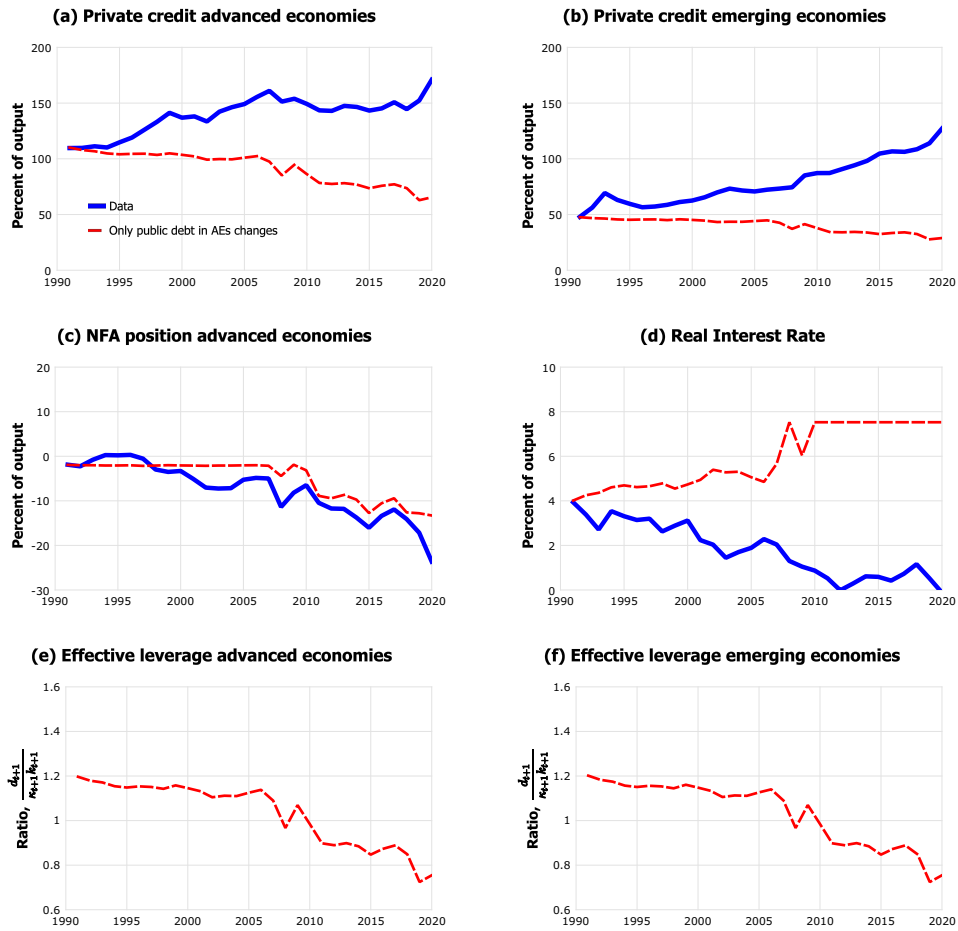


Figure 11: Counterfactual simulation when only the public debt of advanced economies changes, 1991-2020.

#### 4 Macroeconomic and financial volatility

In this section we explore the second key question addressed in this paper: How the faster growth of emerging economies, the changes in financial structure, and the changes in reserves and public debt impacted macroeconomic and financial stability.

To compute measures of volatility, we simulate the model for 130 years in response to random draws of the sunspot shocks ( $\varepsilon_{j,t} = 0$  with probability  $\lambda = 0.04$  and  $\varepsilon_{j,t} = 1$  with probability  $1 - \lambda = 0.96$ ). As explained

earlier, when  $\varepsilon_{j,t} = 0$  and leverage is sufficiently high, the liquidation price of capital drops to  $\kappa_{j,t}$  and the outstanding debt is renegotiated.

During the first 100 years of the simulation, productivity, foreign reserves and public debt grow at the same long-run rate  $g - 1 = 0.0189$  in both countries, and the financial structure parameters,  $\phi_{j,t}$  and  $\kappa_{j,t}$ , are kept constant at their 1991 values. The initial 100 years of simulation are used to derive the model's invariant distribution. The remaining 30 years correspond to the 1991-2020 period in which  $z_{j,t}$ ,  $\phi_{j,t}$ ,  $\kappa_{j,t}$ ,  $FX_{j,t}$  and  $D_{p,t}$  take the values plotted in Figures 2 and 6. Thus, we assume that the model starts at its stochastic steady state prior to 1991 (i.e., at the averages of the invariant distribution). The 130-years simulation is then repeated 10,000 times, each time with a new sequence of random draws of the sunspot shocks  $\varepsilon_{1,t}$  and  $\varepsilon_{2,t}$  over 130 years.

The repeated simulations generate 10,000 data points in each year. The mean of country  $j$  output in every year  $t$  is the “cross-sectional” arithmetic average computed as  $\bar{Y}_{j,t} = \frac{1}{10,000} \sum_{i=1}^{10,000} Y_{j,t}^i$ . We also compute the 5th and 95th percentiles of the 10,000 data points for each year. The difference between the two percentiles provides a measure of each country's output volatility. The 5th percentile for country  $j$ , denoted by  $P_{j,t}(5)$ , is the threshold value for which 5 percent of the 10,000 realizations of the variable are smaller than  $P_{j,t}(5)$ . Formally,  $\frac{1}{10,000} \sum_i^{10,000} \left( 1 \mid Y_{j,t}^i < P_{j,t}(5) \right) = 0.05$ . Similarly for the 95th percentile. We then construct a time-varying index of output volatility as the difference between the 5th and 95th percentiles, normalized by the mean of output,

$$VOL_{j,t} = \left( \frac{P_{j,t}(95) - P_{j,t}(5)}{\bar{Y}_{j,t}} \right) \times 100. \quad (20)$$

#### 4.1 The growth of emerging economies and volatility

Figure 12 plots the output volatility measures over the period 1991-2020. The figure also plots the effective leverage ratios  $d_{j,t}/\kappa_{j,t}k_{j,t}$ . Panels (a) and (b) are for the baseline model where productivity, financial structure, reserves and debt change over time. Panel (a) shows that volatility rose from about 2.0 percent to about 6 percent in advanced economies, and from about 1 percent to about 4 percent in emerging economies. Panel (b) confirms that the increase in volatility is directly related to the increase in average effective leverage, which rose from 1.2 to about 1.8 in both countries

(effective leverage is similar in both countries because they face the same world interest rate).

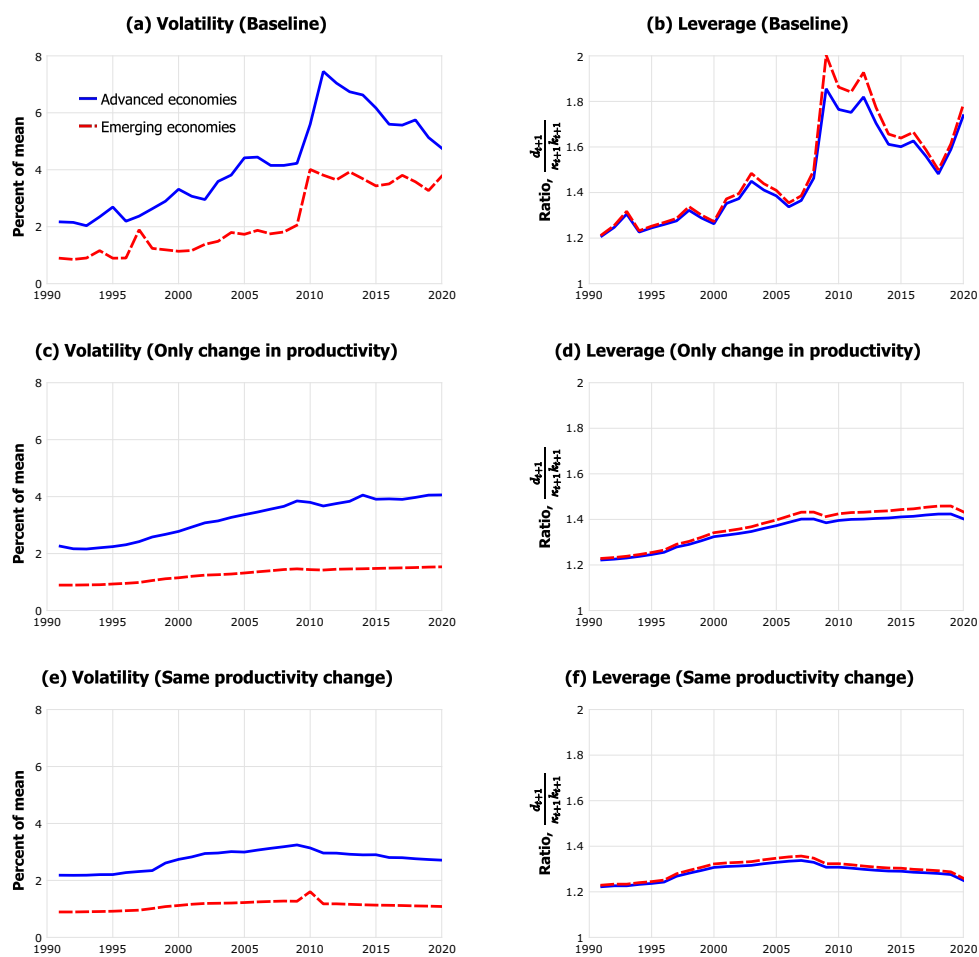


Figure 12: Output volatility and mean of effective leverage over the period 1991-2020. The volatility measure is the difference between the 5th and 95th percentiles as a percentage of the output mean. Effective leverage is the ratio of debt over the liquidation value of capital in a crisis.

As already mentioned, a financial crisis leads to debt restructuring which causes a redistribution of wealth from lenders (entrepreneurs) to borrowers (households/firms). The reduction in entrepreneurial wealth, then, reduces employment and production. Since the magnitude of the redistri-

bution increases with leverage, the model generates an increase in volatility as a consequence of the higher leverage.

Panels (c) and (d) show the importance of faster productivity growth in emerging economies, by allowing only for changes in  $z_{j,t}$ . This contributed significantly to the increase in volatility, particularly in advanced economies (see panel (c)). As before, the higher volatility resulted from a sharp rise in effective leverage in both countries (panel (d)).

It is important to emphasize that the increase in volatility would have been much smaller if emerging economies had experienced the same productivity growth as advanced economies. This is shown in panel (e), which plots the simulated series assuming that emerging economies experienced the same productivity growth as advanced economies. In this case, there is only a small change in volatility. This is because effective leverage does not change much (see panel (f)). Thus, the faster *relative* growth of emerging economies has been important for generating higher global macroeconomic and financial volatility even if it is not the only factor.

## 4.2 Changes in financial structure and volatility

Panels (a) and (b) of Figure 13 plot the volatility measure and the effective leverage ratio,  $d_{j,t}/\kappa_{j,t}k_{j,t}$ , when only the financial parameter  $\phi_{j,t}$  changes. Panels (c) and (d) plot the same variables when only  $\kappa_{j,t}$  changes, and panels (e) and (f) when both  $\phi_{j,t}$  and  $\kappa_{j,t}$  change.

The changes in  $\phi_{j,t}$  lead to a large increase in volatility in both countries (panel (a)) while the changes in  $\kappa_{j,t}$  lead to slight declines (panel (c)). Since the former are larger than the latter, allowing for the changes in both  $\phi_{j,t}$  and  $\kappa_{j,t}$  still generates significant increases in volatility (panel (e)). The mechanism works again through the changes in effective leverage. Changes in  $\phi_{j,t}$  only cause large increases in leverage (panel (b)) while changes in  $\kappa_{j,t}$  cause small declines (panel (d)). Thus, allowing for changes in both  $\phi_{j,t}$  and  $\kappa_{j,t}$  still generates higher leverage (panel (f)).

It is worth noting that the fact that changes in  $\kappa_{j,t}$  have small effects on volatility and leverage does not imply that this parameter is irrelevant for volatility. This is because  $\kappa_{j,t}$  is critical for the mechanism by which the increases in  $\phi_{j,t}$  cause higher volatility. Since  $\kappa_{j,t}$  is the liquidation value of capital when a crisis hits, if  $\kappa_{j,t}$  were the same as the market price  $p_{j,t}$ , the changes in  $\phi_{j,t}$  would have a much smaller impact on volatility.

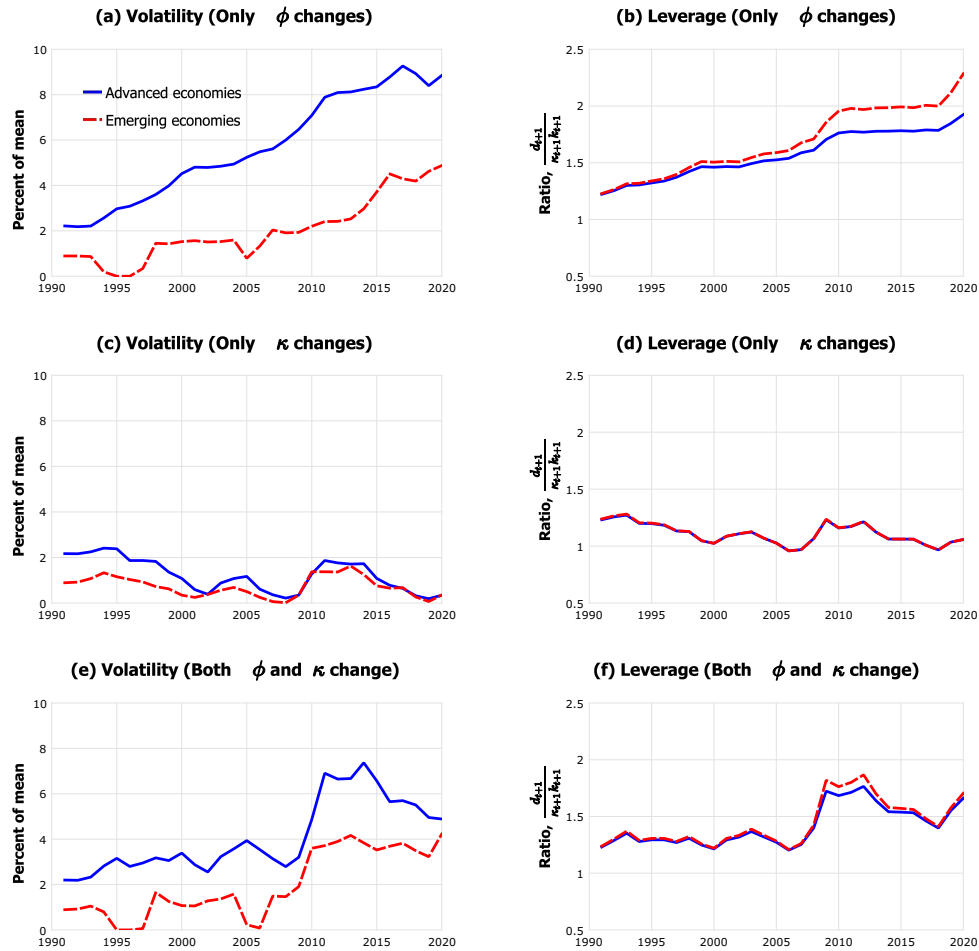


Figure 13: Output volatility and mean of effective leverage over the period 1991-2020. Only the parameters of the financial structure change. The volatility measure is the difference between the 5th and 95th percentiles as a percentage of the output mean. Effective leverage is the ratio of debt over the liquidation value of capital in a crisis.

### 4.3 Change in FX reserves, public debt and volatility

Figure 2 showed that emerging countries increased their holding of foreign exchange reserves. In the previous section we showed that this increased effective leverage by reducing the real interest rate. We can now see the impact on volatility. Panels (a) and (b) of Figure 14 show that the increase in FX reserves increased macroeconomic volatility, which is again connected

to the increase in effective leverage.

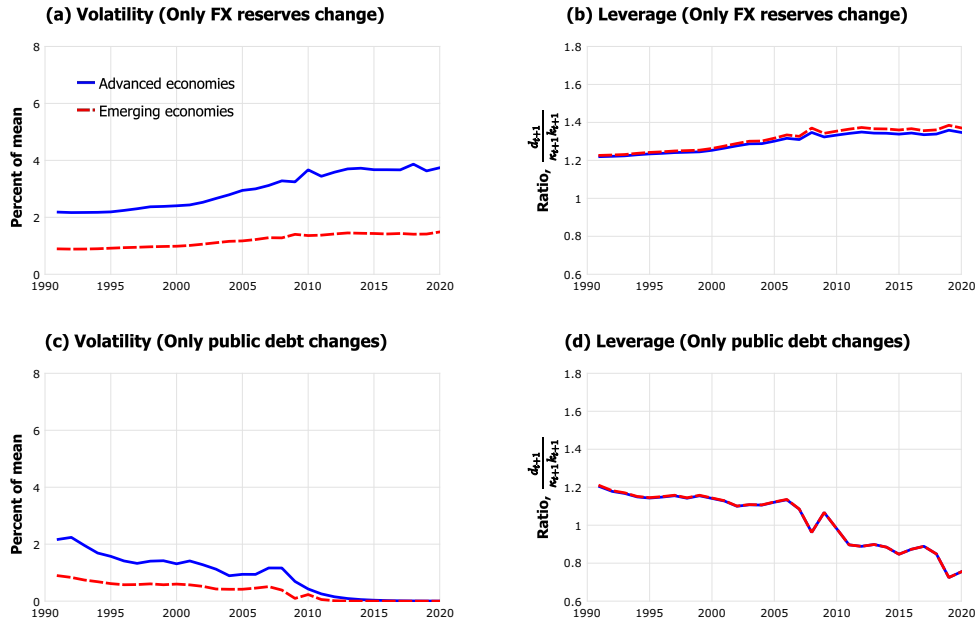


Figure 14: Output volatility and mean of effective leverage over the period 1991-2020. Only foreign reserves,  $FX_{j,t}$ , in panels (a) and (b) or public debt of advanced economies,  $D_{p,t}$ , in panels (c) and (d) change. The volatility measure is the difference between the 5th and 95th percentiles as a percentage of the output mean. Effective leverage is the ratio of debt over the liquidation value of capital in a crisis.

Panels (c) and (d) of Figure 14 plot volatility and effective leverage when only the public debt issued by advanced economies changes. We saw in the previous section that, when the supply of public debt increases, the interest rate rises, and this leads to lower effective leverage (see the bottom right-hand-side panel). Lower leverage, then, reduces volatility, as shown in panels (c) and (d). For lower levels of leverage, private debt is always repaid and, therefore, there are no financial crises. This is what happens toward the end of the sample where volatility goes to zero: without default the economy becomes deterministic. This arises when the interest rate becomes equal to the inter-temporal discount rate,  $1/\beta - 1$ .

Changes in reserves and public debt have spillovers that are worth noting. The increase in reserves, which is mainly driven by emerging markets, is “beneficial” to them in the sense that although leverage is about



the same as in advanced economies, the EMs volatility rises only slightly. In contrast, volatility in advanced economies doubles from 2 to 4 percent. Hence, higher demand for FX reserves in EMs has a negative spillover that increases volatility in advanced economies.

The large increase in public debt of advanced economies is beneficial to them because it neutralizes volatility. In addition, it has a positive spillover effect, because it reduces the volatility of emerging markets.

To summarize, the increased accumulation of foreign reserves by emerging economies raised macroeconomic and financial volatility, while the rise in public borrowing by advanced economies had a mitigating impact.

#### 4.4 Asset pricing implications

To conclude the volatility analysis, we examine the implications for the endogenous price of capital  $p_{j,t}$ . Recall that the key asset pricing condition is the Euler equation (9) that we rewrite here:

$$p_{j,t} = \beta \mathbb{E}_t \left\{ r_{j,t+1} - \tau + g p_{j,t+1} \right\} + \Psi \left( \frac{d_{j,t+1}}{\kappa_{j,t+1} k_{j,t+1}} \right).$$

The first term on the right-hand-side is the discounted expected next period value of capital, which consists of the cash-flow rental payout net of depreciation, plus the resale value. This would determine the asset price in a frictionless market. With frictions, however, there is a second component, captured by the function  $\Psi(\cdot)$ . This term derives from the fact that capital can be funded with debt but debt issuance incurs costs that are increasing in effective leverage. Hence, given the debt  $d_{j,t+1}$ , an increase in  $k_{j,t+1}$  lowers the cost of debt or, equivalently, keeping the marginal cost of debt fixed, higher capital allows for more borrowing.

The effect that capital has on the cost of borrowing is akin to what happens in models with a standard collateral constraint. In these models, part of the value of capital derives from its ability to relax the borrowing constraint. This is captured by the shadow value of relaxing the collateral constraint—the Lagrange multiplier—instead of  $\Psi(\cdot)$ .

We solve for the price of capital in advanced and emerging economies and compute its volatility measure in the same way we computed it for the other variables: The difference between the 5th and 95th percentile of the 10,000 repeated simulations, and express it as a percentage of the mean. Figure 15 plots the results.

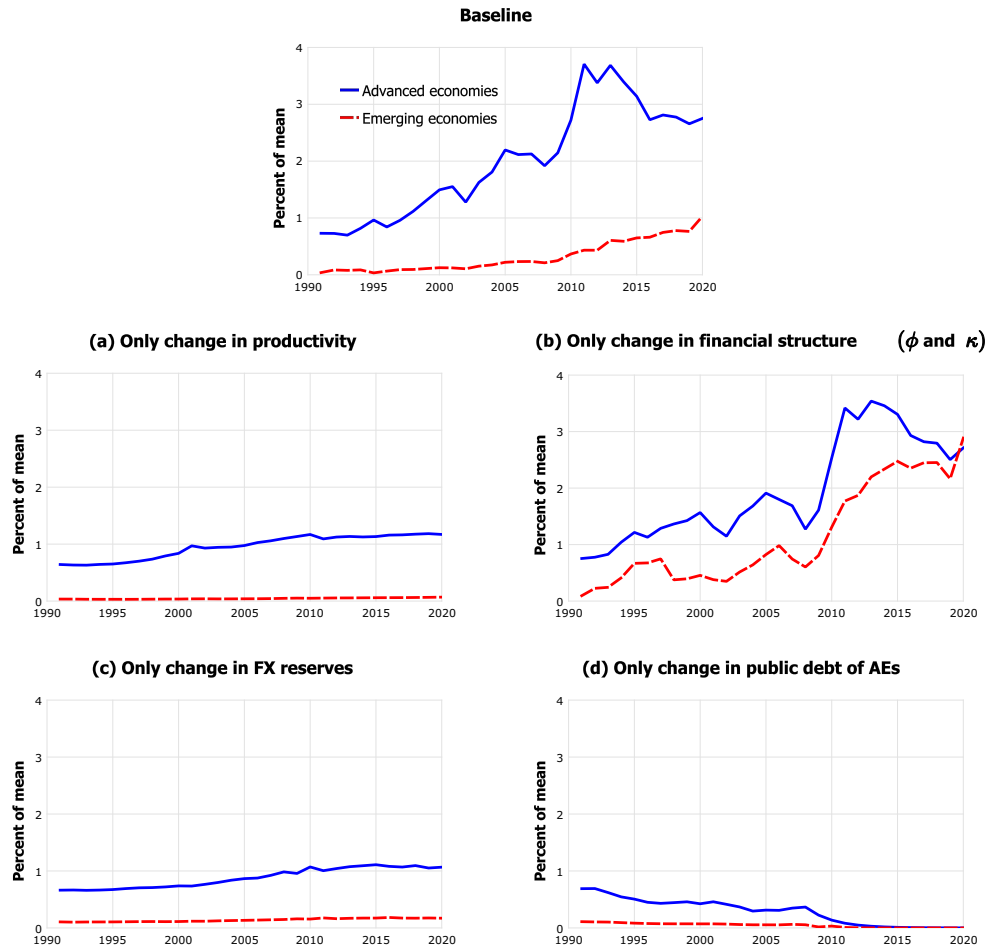


Figure 15: Asset price volatility over the period 1991-2020: baseline model and counterfactual simulations. The volatility measure is the difference between the 5th and 95th percentiles as a percentage of the price mean.

The results for the baseline case with changes in all of the model’s driving forces (top panel), show that asset price volatility increased sharply, particularly for advanced economies. The volatility of asset prices rose from about 0.8 (0.1) percent to around 3 (1) percent in advanced (emerging) economies.

Panels (a)-(d) show the contributions of faster productivity growth in emerging markets, changes in financial parameters  $\phi_{j,t}$  and  $\kappa_{j,t}$ , changes

in FX reserves and changes in the public debt of advanced economies. The changes in the financial parameters are the main culprit behind the increase in the volatility of asset prices. We infer that this is mainly due to the effect of higher  $\phi_{j,t}$  which caused effective leverage and therefore  $\Psi(\cdot)$  to rise.

## 5 Discussion and conclusion

An implication of the increased size of emerging economies is that, collectively, they are more influential in the world economy. The view that emerging markets are a collection of small open economies with negligible impact on advanced economies is no longer a useful approximation.

There are many channels through which emerging markets affect the rest of the world. In this paper, we emphasized one of these channels: the increased demand for financial assets traded in globalized capital markets resulting from faster growth in emerging economies, structural changes that increased demand for assets with productive value, and a surge in accumulation of foreign reserves in emerging economies. In particular, we showed that the worldwide increase in the demand for financial assets raises the incentives to leverage. On the one hand, this allows for an expansion of the financial sector with positive effects on real macroeconomic activity. On the other hand, it increases the fragility of the financial system, raising the probability and/or the consequences of a crisis.

From a policy perspective there is a trade-off: the benefit of an expanded financial system versus the potential cost of more severe crises. A similar mechanism also arises in models with asset price bubbles and borrowing constraints as in Miao and Wang (2011). Moreover, the model predicts significant spillovers by which the surge in reserves in emerging markets might have been a factor causing higher volatility in advanced economies, and the sharp increase in the public debt of advanced economies may have been a factor reducing volatility in emerging markets.

## Appendix

### A Proof of Lemma 2.1

The optimization problem of an entrepreneur in country  $j$  is

$$\max_{\{l_{j,t}, k_{j,t}, c_{j,t}, b_{1,j,t+1}, b_{2,j,t+1}, b_{p,j,t+1}\}_{t=0}^{\infty}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \ln(c_{j,t}) \quad (21)$$

subject to

$$\begin{aligned} m_{j,t} &= \delta_{1,t} b_{1,j,t} + \delta_{2,t} b_{2,j,t} + b_{p,j,t}, \\ m_{j,t} &= \phi_t \left( r_{j,t} k_{j,t} + w_{j,t} l_{j,t} \right), \\ a_{j,t} &= m_{j,t} + z_{j,t}^{\gamma} l_{j,t}^{\gamma} k_{j,t}^{1-\gamma} - w_{j,t} l_{j,t} - r_{j,t} k_{j,t}, \\ c_{j,t} &= a_{j,t} - q_{1,t} b_{1,j,t+1} - q_{2,t} b_{2,j,t+1} - q_{2,t} b_{2,j,t+1}. \end{aligned}$$

The first-order conditions for  $l_{j,t}$  and  $k_{j,t}$  are

$$\begin{aligned} \gamma z_{j,t}^{\gamma} l_{j,t}^{\gamma-1} k_{j,t}^{1-\gamma} &= (1 + \xi_{j,t} \phi_{j,t}) w_{j,t}, \\ (1 - \gamma) z_{j,t}^{\gamma} l_{j,t}^{\gamma} k_{j,t}^{-\gamma} &= (1 + \xi_{j,t} \phi_{j,t}) r_{j,t}, \end{aligned}$$

where  $\xi_{j,t}$  is the lagrange multiplier associated with the working capital constraint in the above optimization problem. If the constraint is not binding we have that  $\xi_{j,t} = 0$ , and the first-order conditions become

$$\begin{aligned} \gamma z_{j,t}^{\gamma} \left( \frac{k_{j,t}}{l_{j,t}} \right)^{1-\gamma} &= w_{j,t}, \\ (1 - \gamma) z_{j,t}^{\gamma} \left( \frac{k_{j,t}}{l_{j,t}} \right)^{-\gamma} &= r_{j,t}, \end{aligned}$$

If the constraint is binding,  $\xi_{j,t} > 0$ . Using the two first-order conditions together with the working capital constraint,  $m_{j,t} = \phi_t (r_{j,t} k_{j,t} + w_{j,t} l_{j,t})$ , we derive

$$\begin{aligned} l_{j,t} &= \left( \frac{\gamma}{\phi_{j,t} w_{j,t}} \right) m_{j,t}, \\ k_{j,t} &= \left( \frac{1 - \gamma}{\phi_{j,t} r_{j,t}} \right) m_{j,t}. \end{aligned}$$

## B Proof of Lemma 2.2

When the working capital constraint is binding, the inputs of labor and capital are linear functions of  $m_{j,t}$ . Therefore, the end of period wealth  $a_{j,t}$  is also linear in  $m_{j,t}$ . We can then write  $a_{j,t} = \pi_{j,t}m_{j,t}$  where the term  $\pi_{j,t}$  is a function of parameters and aggregate prices taken as given by an individual entrepreneur. When the working capital constraint is not binding, the end-of-period wealth  $a_{j,t}$  is not necessarily linear in  $m_{j,t}$ . However, in equilibrium, since profits are zero, the end-of-period wealth is just  $a_{j,t} = m_{j,t}$ , which is also linear in  $m_{j,t}$ . Therefore, we can use the expression  $a_{j,t} = \pi_{j,t}m_{j,t}$  independently of whether the working capital constraint is binding or not.

Since  $m_{j,t} = \delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}$ , we can write the end-of-period wealth at time  $t$  and at  $t + 1$  as

$$\begin{aligned} a_{j,t} &= \pi_{j,t}(\delta_{1,t}b_{1,j,t} + \delta_{2,t}b_{2,j,t} + b_{p,j,t}), \\ a_{j,t+1} &= \pi_{j,t+1}(\delta_{1,t+1}b_{1,j,t+1} + \delta_{2,t+1}b_{2,j,t+1} + b_{p,j,t+1}). \end{aligned}$$

We derive next the first-order conditions for Problem (21) with respect to  $b_{1,j,t+1}$ ,  $b_{2,j,t+1}$  and  $b_{p,j,t+1}$ ,

$$\frac{q_{1,t}}{c_{j,t}} = \beta \mathbb{E}_t \left( \frac{\pi_{j,t+1}\delta_{1,t+1}}{c_{j,t+1}} \right), \quad (22)$$

$$\frac{q_{2,t}}{c_{j,t}} = \beta \mathbb{E}_t \left( \frac{\pi_{j,t+1}\delta_{2,t+1}}{c_{j,t+1}} \right). \quad (23)$$

$$\frac{q_{p,t}}{c_{j,t}} = \beta \mathbb{E}_t \left( \frac{\pi_{j,t+1}}{c_{j,t+1}} \right). \quad (24)$$

We now guess that optimal consumption is a fraction  $1 - \beta$  of wealth,

$$c_{j,t} = (1 - \beta)a_{j,t}.$$

The saved wealth is allocated to private bonds issued by country 1 and by country 2 and public debt issued by country 2. Denoting by  $\theta_{1,j,t}$  and  $\theta_{2,j,t}$  the shares allocated to private bonds issued by country 1 and country 2, respectively, we have

$$q_{1,t}b_{1,j,t+1} = \theta_{1,j,t}\beta a_{j,t}, \quad (25)$$

$$q_{2,t}b_{2,j,t+1} = \theta_{2,j,t}\beta a_{j,t}, \quad (26)$$

$$q_{p,t}b_{p,j,t+1} = (1 - \theta_{1,j,t} - \theta_{2,j,t})\beta a_{j,t}. \quad (27)$$

We now multiply equation (22) by  $b_{1,j,t+1}$ , equation (23) by  $b_{2,j,t+1}$ , and equation (24) by  $b_{p,j,t+1}$ . Adding the resulting expressions and using the equations that define consumption and next period wealth, we obtain

$$q_{1,t}b_{1,j,t+1} + q_{2,t}b_{2,j,t+1} + q_{p,t}b_{p,j,t+1} = \beta a_{j,t}.$$

This is satisfied given (25)-(27). Since we have derived this condition from the Euler equations (22)-(24), we have proved that, if consumption is a fraction  $1 - \beta$  of wealth, the three Euler equations are satisfied. This verifies our guess.

We now replace the guess for  $c_{j,t}$  into equations (22) and (23), to obtain

$$\mathbb{E}_t \left\{ \frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{1,j,t} \frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,j,t} \frac{\delta_{2,t+1}}{q_{2,t}} + (1 - \theta_{1,j,t} - \theta_{2,j,t}) \frac{1}{q_{p,t}}} \right\} = 1. \quad (28)$$

$$\mathbb{E}_t \left\{ \frac{\frac{\delta_{2,t+1}}{q_{2,t}}}{\theta_{1,j,t} \frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,j,t} \frac{\delta_{2,t+1}}{q_{2,t}} + (1 - \theta_{1,j,t} - \theta_{2,j,t}) \frac{1}{q_{p,t}}} \right\} = 1. \quad (29)$$

These two conditions determine the shares of savings invested in the private bonds of the two countries. Since the conditions are the same for entrepreneurs in country 1 and in country 2, it must be that  $\theta_{1,1,t} = \theta_{1,2,t} = \theta_{1,t}$  and  $\theta_{2,1,t} = \theta_{2,2,t} = \theta_{2,t}$ . ■

### C First-order conditions for households/firms

The optimization problem of households/firms can be written recursively as

$$V(d, k) = \max_{l, c, d'} \left\{ e - z \frac{h^{1+\frac{1}{\nu}}}{1 + \frac{1}{\nu}} + \beta \mathbb{E} V(d', k') \right\},$$

subject to

$$\tilde{d}(d, \tilde{p}k) + pk' + e + \varphi(d', \kappa'k') = wh + (r - \tau)k + pkg + \frac{1}{R} \mathbb{E} \tilde{d}(d', \tilde{p}'k'),$$

where the function  $\tilde{d}(d, \tilde{p}k)$  is defined in (4) and the function  $\varphi(d', \kappa'k')$  in (5).

The first-order conditions with respect to  $h, d', k'$  are, respectively,

$$\begin{aligned} zh^{\frac{1}{\nu}} &= w, \\ \frac{1}{R} \mathbb{E} \left\{ \frac{\partial \tilde{d}(d', \tilde{p}'k')}{\partial d'} \right\} - \frac{\partial \varphi(d', \kappa'k')}{\partial d'} + \beta \mathbb{E} \left\{ \frac{\partial V(d', k')}{\partial d'} \right\} &= 0, \\ \frac{1}{R} \mathbb{E} \left\{ \frac{\partial \tilde{d}(d', \tilde{p}'k')}{\partial k'} \right\} - \frac{\partial \varphi(d', \kappa'k')}{\partial k'} + \beta \mathbb{E} \left\{ \frac{\partial V(d', k')}{\partial k'} \right\} &= p_{j,t}. \end{aligned}$$

The envelope conditions are

$$\begin{aligned} \frac{\partial V(d, k)}{\partial d} &= -\frac{\partial \tilde{d}(d, \tilde{p}k)}{\partial d}, \\ \frac{\partial V(d, k)}{\partial k} &= r - \tau + pg - \frac{\partial \tilde{d}(d, \tilde{p}k)}{\partial k}. \end{aligned}$$

Updating by one period and substituting in the first-order conditions for debt and capital we obtain

$$\frac{1}{R} = \beta + \frac{\frac{\partial \varphi(d', \kappa' k')}{\partial d'}}{\mathbb{E}\left\{\frac{\partial \tilde{d}(d', \tilde{p}' k')}{\partial d'}\right\}}, \quad (30)$$

$$p_{j,t} = \frac{1}{R} \mathbb{E}\left\{\frac{\partial \tilde{d}(d', \tilde{p}' k')}{\partial k'}\right\} + \beta \mathbb{E}\left\{r' - \tau + gp' - \frac{\partial \tilde{d}(d', \tilde{p}' k')}{\partial k'}\right\} - \frac{\partial \varphi(d', \kappa' k')}{\partial k'}. \quad (31)$$

We now derive the analytical expressions for the derivatives included in the above expressions, using the functional forms for the functions  $\tilde{d}(d, \tilde{p}k)$  and  $\varphi(d', \kappa' k')$  defined, respectively, in (4) and (5):

$$\begin{aligned} \frac{\partial \tilde{d}(d, \tilde{p}k)}{\partial d} &= \begin{cases} 0, & \text{if } d \geq \tilde{p}k \\ 1, & \text{otherwise} \end{cases} \\ \frac{\partial \tilde{d}(d, \tilde{p}k)}{\partial k} &= \begin{cases} \tilde{p}, & \text{if } d \geq \tilde{p}k \\ 0, & \text{otherwise} \end{cases} \\ \frac{\partial \varphi(d', \kappa' k')}{\partial d'} &= \begin{cases} 2\eta \left(1 - \frac{\kappa' k'}{d'}\right) \frac{\kappa' k'}{d'} + \eta \left(1 - \frac{\kappa' k'}{d'}\right)^2, & \text{if } d' \geq \kappa' k' \\ 0, & \text{otherwise} \end{cases} \\ \frac{\partial \varphi(d', \kappa' k')}{\partial k'} &= \begin{cases} -2\eta \left(1 - \frac{\kappa' k'}{d'}\right) \kappa', & \text{if } d' \geq \kappa' k' \\ 0, & \text{otherwise} \end{cases} \end{aligned}$$

We assume that the equilibrium is always characterized by  $d' \geq \kappa' k'$  and  $d' > \tilde{p}' k'$ . This will be the case in the parameterized model. Under this assumption,  $\tilde{p} = \kappa$  with probability  $\lambda$ . This is also the probability of default. The expected

values of the above derivatives can then be written as

$$\begin{aligned}\mathbb{E}\left\{\frac{\partial\tilde{d}(d,\tilde{p}k)}{\partial d}\right\} &= 1 - \lambda \\ \mathbb{E}\left\{\frac{\partial\tilde{d}(d,\tilde{p}k)}{\partial k}\right\} &= \lambda\kappa \\ \frac{\partial\varphi(d',\kappa'k')}{\partial d'} &= 2\eta\left(1 - \frac{\kappa'k'}{d'}\right)\frac{\kappa'k'}{d'} + \eta\left(1 - \frac{\kappa'k'}{d'}\right)^2 \\ \frac{\partial\varphi(d',\kappa'k')}{\partial k'} &= -2\eta\left(1 - \frac{\kappa'k'}{d'}\right)\kappa\end{aligned}$$

Using these expressions in the first-order conditions (30) and (31) we obtain

$$\frac{1}{R} = \beta + \Phi\left(\frac{d'}{\kappa'k'}\right), \quad (32)$$

$$p = \beta\mathbb{E}(r' - \tau + gp') + \Psi\left(\frac{d'}{\kappa'k'}\right), \quad (33)$$

where

$$\begin{aligned}\Phi\left(\frac{d'}{\kappa'k'}\right) &= \left(\frac{1}{1-\lambda}\right)\left[2\eta\left(1 - \frac{\kappa'k'}{d'}\right)\frac{\kappa'k'}{d'} + \eta\left(1 - \frac{\kappa'k'}{d'}\right)^2\right] \\ &= \left(\frac{1}{1-\lambda}\right)\eta\left[1 - \left(\frac{\kappa'k'}{d'}\right)^2\right], \\ \Psi\left(\frac{d'}{\kappa'k'}\right) &= \left[\lambda\Phi\left(\frac{d'}{\kappa'k'}\right) + 2\eta\left(1 - \frac{\kappa'k'}{d'}\right)\right]\kappa'.\end{aligned}$$

It is evident from these expressions that both functions are increasing in  $\frac{d'}{\kappa'k'}$ . In addition, taking derivatives we can verify that they are increasing in  $d'$  and decreasing in both  $k'$  and  $\kappa'$ . ■

## D Market for liquidated capital and equilibrium multiplicity

In the main body of the paper, we assumed that the liquidation price  $\tilde{p}_{j,t}$  can be either  $\kappa_{j,t}$  or  $p_{j,t}$  with constant probabilities  $\lambda$  and  $1 - \lambda$ . In this section, we describe the market structure that provides the micro-foundation for the determination of  $\tilde{p}_t$ . In this specification, there are multiple equilibria and  $\lambda$  represents the probability of a sunspot shock that selects one of two self-fulfilling equilibria.



The market for liquidated capital meets at the beginning of the period. We make two important assumptions about how this market operates.

**Assumption 1** *Capital can be sold to domestic households/firms or entrepreneurs. If sold to entrepreneurs, capital loses its functionality as a productive asset and it is converted to consumption goods at rate  $\kappa_{j,t}$ .*

This assumption formalizes the idea that capital may lose value when reallocated to non-specialized owners, provided that  $\kappa_{j,t}$  is sufficiently low. In order for capital to keep its functionality as a productive asset, it needs to be purchased by domestic households/firms, not foreign households/firms. With this assumption a crisis could be local, that is, it could take place in one country without spreading to the other country. However, even if a crisis takes place only in one country, it has real economic effects also in the other country due to the cross-country diversification of bond portfolios.

**Assumption 2** *Households/firms can purchase liquidated capital only if the liquidation value of their capital exceeds the debt obligations,  $d_{j,t} < \tilde{p}_{j,t}k_{j,t}$ .*

If a household/firm starts with liabilities bigger than the liquidation value of the owned assets, that is,  $d_{j,t} > \tilde{p}_{j,t}k_{j,t}$ , it will be unable to raise additional funds to purchase the liquidated capital. Potential investors know that the new liabilities (as well as the outstanding liabilities) are not collateralized, and the debt will be renegotiated immediately by households/firms after taking the new debt. We refer to a household/firm with  $d_{j,t} < \tilde{p}_{j,t}k_{j,t}$  as ‘liquid’ since it can raise extra funds at the beginning of the period. Instead, a household/firm with  $d_{j,t} > \tilde{p}_{j,t}k_{j,t}$  is ‘illiquid’.

To better understand Assumptions 1 and 2, consider the condition for not renegotiating,  $d_{j,t} \leq \tilde{p}_{j,t}k_{j,t}$ . Furthermore, assume that  $p_{j,t} > \kappa_{j,t}$ , that is, the price at the end of the period is bigger than the liquidation price when the market freezes. If this condition is satisfied, households/firms have the ability to raise funds to purchase additional capital. In turn, this ensures that the liquidation price is  $\tilde{p}_{j,t} = p_{j,t}$ . If  $d_{j,t} > \kappa_{j,t}k_{j,t}$  for all households/firms, however, there will be no households/firms capable of buying the liquidated capital. Then, liquidated capital can only be purchased by entrepreneurs at price  $\tilde{p}_{j,t} = \kappa_{j,t}$ .

This shows that the market price for liquidated capital depends on the financial decision of households/firms, which in turn depends on the liquidation price. This interdependence is critical for our argument because it can lead to self-fulfilling equilibria (i.e, it is what triggers financial crises in the model).

**Proposition D.1** *There exists multiple equilibria only if  $d_{j,t} > \kappa_{j,t}k_{j,t}$ .*

**Proof D.1** *At the beginning of the period, households/firms choose whether to renegotiate the debt. Given the initial states  $d_t$  and  $k_t$ , the renegotiation decision boils down to a take-it or leave-it offer made to creditors for the repayment of the debt.*

*Denote by  $\tilde{d}_t = \psi(d_t, k_t, \tilde{p}_t)$  the offered repayment. This depends on the individual liabilities,  $d_t$ , individual capital,  $k_t$ , and the price for liquidated capital,  $\tilde{p}_t$ . The price of the liquidated capital is the price at which the lender could sell the capital after rejecting the offer from the borrower. The best offer made by the household/firm is*

$$\psi(d_t, k_t, \tilde{p}_t) = \begin{cases} d_t, & \text{if } d_t \leq \tilde{p}_t k_t \\ \tilde{p}_t k_t, & \text{if } d_t > \tilde{p}_t k_t \end{cases}, \quad (34)$$

*which is accepted by creditors if they cannot sell at a price higher than  $\tilde{p}_t$ .*

*For the moment, we assume that the equilibrium is symmetric, that is, all households/firms start with the same ratio  $d_t/k_t$ . At this stage this is only an assumption. However, we will show below that households/firms do not have an incentive to deviate from the ratio chosen by other households/firms.*

*Given the assumption that the equilibrium is symmetric (all households/firms choose the same ratio  $d_t/k_t$ ), multiple equilibria arise if  $d_t/k_t \in [\kappa_t, p_t)$ . If the market expects that the liquidation price is  $\tilde{p}_t = \kappa_t$ , all households/firms are illiquid and they choose to renege on their liabilities (given the renegotiation policy (34)). As a result, there will be no households/firms that can purchase the liquidated capital of other households/firms. The only possible liquidation price that is consistent with the expected price is  $\tilde{p}_t = \kappa_t$ . On the other hand, if the market expects  $\tilde{p}_t = p_t$ , households/firms are liquid and, if one household/firm reneges, creditors can sell the liquidated assets to other households/firms at the liquidation price  $\tilde{p}_t = p_t$ . Therefore, it is optimal for households/firms not to renegotiate.*

*We now address the issue of whether individual households/firms have an incentive to deviate from the symmetric equilibrium and choose a different ratio  $d_t/k_t$  in the previous period  $t - 1$ . In particular, we need to show that, in the anticipation that the liquidation price could be  $\tilde{p}_t = \kappa_t$ , a household/firm does not find convenient to borrow less at time  $t - 1$  so that it could purchase the liquidated capital if the price drops to  $\kappa_t$ .*

*The first point to consider is that, in equilibrium, capital is never liquidated. The low liquidation price  $\kappa_t$  simply represents the threat value for creditors. However, in equilibrium all creditors accept the renegotiation offer and no capital is ever liquidated.*

*What would happen if there is a household/firm that is liquid and, therefore, has the ability to purchase the liquidated capital at a higher price than  $\kappa_t$ ? This would arise if a household/firm deviates from the symmetric equilibrium. In this case, debtors know that their creditors could liquidate the capital and sell it at a higher price than  $\kappa_t$ . Knowing this, debtors will offer a higher repayment and, as a result, capital is not liquidated. Potentially, this could drive the liquidation price to  $p_t$ . This shows that a household/firm cannot*

make any profit by remaining liquid. Therefore, there is no incentive to deviate from the symmetric equilibrium. ■

Assume that the equilibrium is symmetric. Then, all households/firms choose the same ratio  $d_t/k_t$  and multiple equilibria determined by self-fulfilling expectations of the liquidation price can exist. The proof above has shown that this requires  $d_t/k_t \in [\kappa_t, p_t)$ . On the one hand, if the market expects a liquidation price  $\tilde{p}_t = \kappa_t$ , all households/firms are illiquid and choose to renege on their liabilities. As a result, there are no households/firms that can purchase the liquidated capital and, therefore, the only liquidation price consistent with the expected price is  $\tilde{p}_t = \kappa_t$ . On the other hand, when the market expects  $\tilde{p}_t = p_t$ , households/firms are liquid and, if one household/firm reneges, creditors can sell the liquidated capital to other households/firms at price  $\tilde{p}_t = p_t$ , which makes it optimal not to renege.

When multiple equilibria are possible, that is, when we have  $d_{j,t} > \kappa_{j,t}k_{j,t}$ , the equilibrium is selected by a random draw of sunspot shocks. Let  $\varepsilon_{j,t}$  be a variable that takes the value of 0 with probability  $\lambda$  and 1 with probability  $1 - \lambda$ . If the condition for multiplicity is satisfied, agents coordinate their expectations on the low liquidation price  $\kappa_{j,t}$  if  $\varepsilon_{j,t} = 0$ . This implies that the probability distribution of the low liquidation price is

$$f_{j,t}(\tilde{p}_{j,t} = \kappa_{j,t}) = \begin{cases} 0, & \text{if } d_{j,t} \leq \kappa_{j,t}k_{j,t} \\ \lambda, & \text{if } d_{j,t} > \kappa_{j,t}k_{j,t} \end{cases}$$

The ratio  $d_{j,t}/\kappa_{j,t}k_{j,t}$  is the relevant measure of leverage. When it is sufficiently small, households/firms remain liquid even if the (expected) liquidation price is  $\kappa_{j,t}$ . But then the liquidation price cannot be low and the realization of the sunspot shock is irrelevant for the equilibrium. Instead, when leverage is high, households/firms' liquidity depends on the liquidation price. The realization of the sunspot shock  $\varepsilon_{j,t}$  then becomes important for selecting one of the two equilibria. When  $\varepsilon_{j,t} = 0$ —which happens with probability  $\lambda$ —the market expects that the liquidation price is  $\kappa_{j,t}$ , making the household's sector illiquid. On the other hand, when  $\varepsilon_{j,t} = 1$ —which happens with probability  $1 - \lambda$ —the market expects that households/firms are capable of participating in the liquidation market, validating the expectation of a higher liquidation price.

Notice that this argument is based on the assumption that  $\kappa_{j,t}$  is sufficiently low (implying a low liquidation price if the capital freezes). Also, the equilibrium value of capital without a freeze,  $p_{j,t}k_{j,t}$ , is always bigger than the debt  $d_{j,t}$ . Otherwise, households/firms would be illiquid with probability 1 and the equilibrium price is always  $\kappa_{j,t}$ . Condition (6) guarantees that this does not happen at

equilibrium: if the probability of default is 1, the anticipation of the renegotiation cost increases the interest rate, which deters households/firms from borrowing too much.

### E Equilibrium system of equations at time $t$

Given the values of  $\phi_{1,t}, \phi_{2,t}, \kappa_{1,t}, \kappa_{2,t}, \kappa_{1,t+1}, \kappa_{2,t+1}, FX_{1,t+1}, FX_{2,t+1}, D_{p,t+1}$ , and the stochastic states  $s_t$ , we can find the values of  $\delta_{j,t}, M_{j,t}, L_{j,t}, K_{j,t}, w_{j,t}, r_{j,t}, q_{j,t}, q_{p,t}, A_{j,t}, B_{j,1,t+1}, B_{j,2,t+1}, B_{p,1,t+1}, B_{p,2,t+1}, D_{j,t+1}, \theta_{1,t}$  and  $\theta_{2,t}$ , by solving the following system of equations:

$$\delta_{j,t} = \begin{cases} \min \left\{ 1, \frac{\kappa_{j,t} K_{j,t}}{D_{j,t}} \right\}, & \text{if } \varepsilon_{j,t} = 0 \\ 1, & \text{if } \varepsilon_{j,t} = 1 \end{cases} \quad (35)$$

$$M_{j,t} = \delta_{1,t} B_{1,t} + \delta_{2,t} B_{2,t} + B_{p,t} \quad (36)$$

$$L_{j,t} = \left( \frac{\gamma}{\phi_{j,t} w_{j,t}} \right) M_{j,t}, \quad (37)$$

$$K_{j,t} = \left( \frac{1-\gamma}{\phi_{j,t} r_{j,t}} \right) M_{j,t}, \quad (38)$$

$$L_{j,t} = \left( \frac{w_{j,t}}{z_{j,t}} \right)^\nu, \quad (39)$$

$$K_{j,t} = \bar{K} g^t, \quad (40)$$

$$A_{j,t} = M_{j,t} + z_{j,t}^\gamma L_{j,t}^\gamma K_{j,t}^{1-\gamma} - w_{j,t} L_{j,t} - (r_{j,t} + \tau) K_{j,t}, \quad (41)$$

$$B_{1,j,t+1} = \frac{\theta_{1,t} \beta A_{j,t}}{q_{1,t}}, \quad (42)$$

$$B_{2,j,t+1} = \frac{\theta_{2,t} \beta A_{j,t}}{q_{2,t}}, \quad (43)$$

$$B_{p,j,t+1} = \frac{(1 - \theta_{1,t} - \theta_{2,t}) \beta A_{j,t}}{q_{p,t}}, \quad (44)$$

$$1 = \mathbb{E}_t \left\{ \frac{\frac{\delta_{1,t+1}}{q_{1,t}}}{\theta_{1,t} \frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,t} \frac{\delta_{2,t+1}}{q_{2,t}} + (1 - \theta_{1,t} - \theta_{2,t}) \frac{1}{q_{p,t}}} \right\}, \quad (45)$$

$$1 = \mathbb{E}_t \left\{ \frac{\frac{\delta_{2,t+1}}{q_{2,t}}}{\theta_{1,t} \frac{\delta_{1,t+1}}{q_{1,t}} + \theta_{2,t} \frac{\delta_{2,t+1}}{q_{2,t}} + (1 - \theta_{1,t} - \theta_{2,t}) \frac{1}{q_{p,t}}} \right\}, \quad (46)$$

$$D_{j,t+1} = B_{j1,t+1} + B_{j2,t+1}, \quad (47)$$

$$q_{j,t} = \left[ \beta + \Phi \left( \frac{D_{j,t+1}}{\kappa_{j,t+1} K_{j,t+1}} \right) \right] \mathbb{E}_t \delta_{j,t+1}. \quad (48)$$

$$D_{p,t+1} = FX_{1,t+1} + FX_{2,t+1} + B_{p,1,t+1} + B_{p,2,t+1}. \quad (49)$$

Equation (35) defines the optimal renegotiation strategy (the fraction of the debt repaid). Equation (36) defines entrepreneurial wealth after default. Equations (37) and (38) are the demand for labor and capital from entrepreneurs, given the prices  $w_{j,t}$  and  $r_{j,t}$ , and their wealth  $M_{j,t}$ . Equations (39) and (40) are the supplies of labor and capital from households/workers. Equation (41) defines the end-of-period wealth of entrepreneurs after production. This is allocated to private bonds issued by the two countries and public bonds issued by country 1 as indicated in equations (42)-(44). Equations (45) and (46) are the conditions that determine the investment shares  $\theta_{1,t}$  and  $\theta_{2,t}$ . They are the Euler equations derived from the optimization problem of entrepreneurs. Equation (47) is equilibrium in the bond market. Equation (48) is the Euler equation for households/firms determining the price of bonds. The final equation (49) is the market equilibrium for public bonds.

The above system determines all equilibrium variables except the price of capital  $p_{j,t}$ . To solve for the price of capital we need to use condition (33) where the current price  $p_{j,t}$  depends on the future price  $p_{j,t+1}$ . This implies that we cannot solve for the equilibrium price in the current period without solving for the equilibrium in the future. This requires an iterative procedure. However, since the current price  $p_{j,t}$  does not affect other variables in the current period, we can use the above system to solve for the equilibrium in period  $t$  ignoring  $p_{j,t}$ . Notice that this would not be the case if the liquidation value of capital was a function of  $p_{j,t}$ .

## F Sensitivity to the cost of borrowing, $\eta$

In this section we conduct a sensitivity analysis with respect to the parameter  $\eta$ . This parameter determines the elasticity with which the cost of borrowing increases with debt. In all simulations presented in the paper, we set this parameter to 0.1. We now show how the results change when we double the value of this parameter, that is, we set  $\eta = 0.2$ .

After changing  $\eta$ , we repeat all quantitative exercises, including the construction of the time-varying parameters  $z_{j,t}$ ,  $\phi_{j,t}$  and  $\kappa_{j,t}$ . After changing  $\eta$ , the time-varying parameters are reconstructed to replicate the same empirical targets (domestic credit, NFA and interest rate).

Figure 16 plots effective leverage and output volatility in Advanced Economies when  $\eta = 0.1$  (left panels) and when  $\eta = 0.2$  (right panels). The same variables for Emerging Economies are plotted in Figure 17.

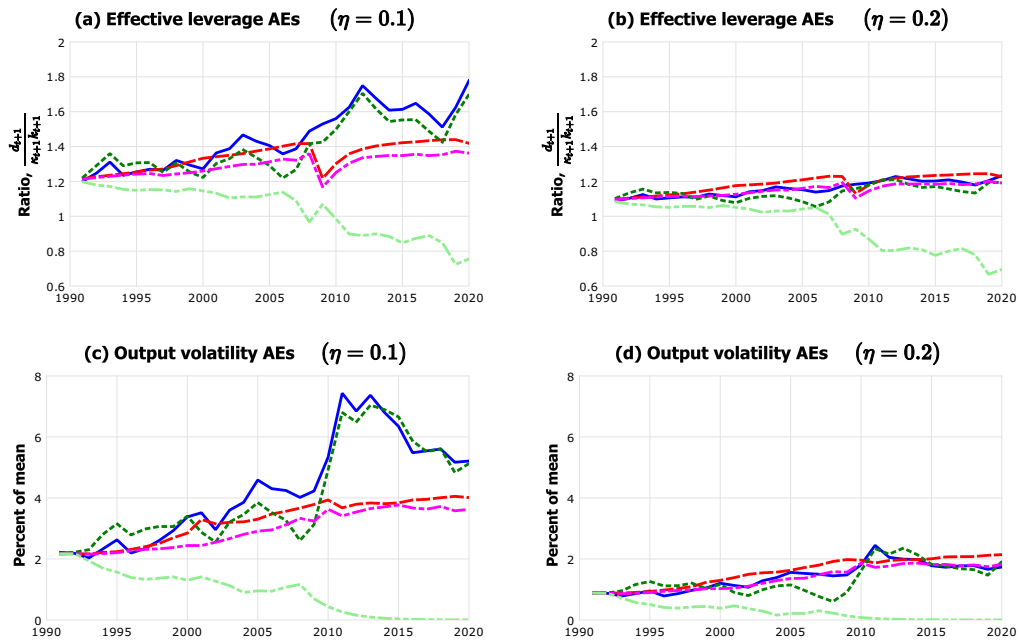


Figure 16: Sensitivity to cost of borrowing parameter  $\eta$  in Advanced Economies.

With a higher value of  $\eta$ , the cost of borrowing increases more rapidly with the stock of debt. As a result, leverage responds less to the structural changes. Panel (d) then shows that the increase in output volatility is smaller. Qualitatively, however, the predictions of the model do not change.

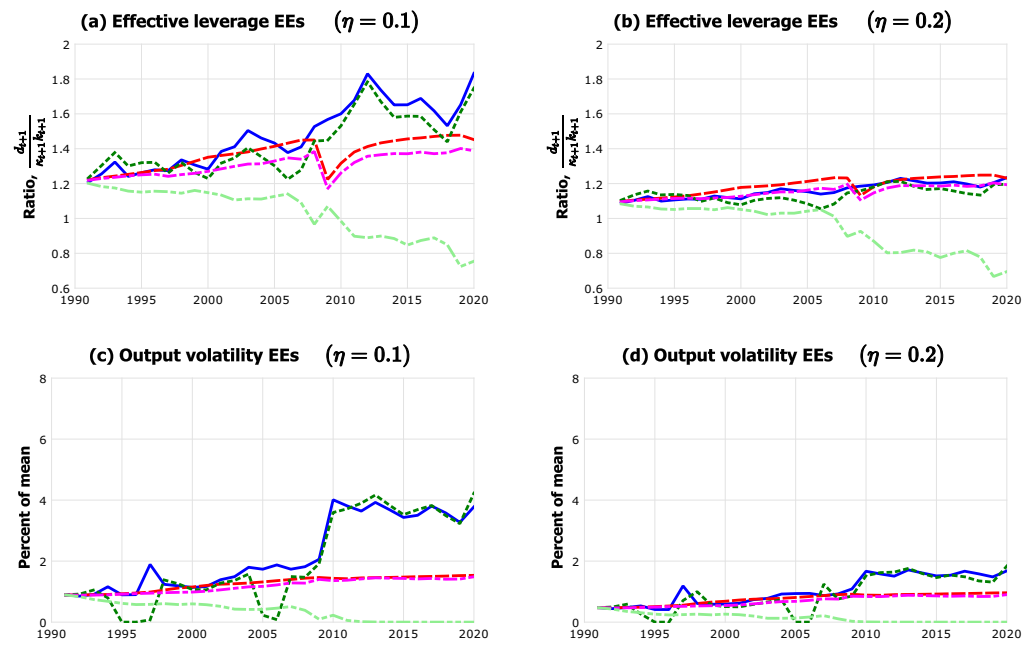


Figure 17: Sensitivity to cost of borrowing parameter  $\eta$  in Advanced Economies.

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