

# Appendix

## A The Chilean “encaje” of the 1990s

The resumption of capital flows to emerging market economies after the Latin American debt crisis of the 1980s led to a surge in inflows into Chile that exerted upward pressure on the real exchange rate, created symptoms of overheating, and made the trade-off between different macroeconomic objectives increasingly difficult and costly. As a response, in 1991, the Chilean authorities established a capital account restriction in the form of an unremunerated reserve requirement. This capital control was an obligation to hold at the central bank an unremunerated fixed-term reserve deposit for a fraction of the capital that a private entity was bringing into the country. Hence, it was analogous to a tax per unit of time that declined with the permanence or maturity of the affected capital inflow (see Appendix A.1 for a detailed derivation of the equivalent tax rate).<sup>45</sup>

We focus our analysis on the *Chilean encaje* because it provides a useful laboratory for exploring the firm-and industry-level consequences of CCs for several reasons. First, the *Chilean encaje* was one of the most well-known examples of market-based capital controls, –i.e. reserve requirements, as opposed to administrative controls with which the authority limits some specific assets, and the market is not allowed to operate. During the 2000s, many countries, such as Colombia, Thailand, Peru and Uruguay, imposed CCs with similar features. Second, the *Chilean encaje* was economically relevant: the total equivalent reserve deposit represented 1.9 percent of GDP during the period 1991-1998, reaching 2.9 percent of GDP in 1997 and 30 percent of that year’s net capital inflows (Gallego et al. (2002)).<sup>46</sup> Finally, the CCs period in Chile was long enough to generate sufficient variation in the data to conduct the empirical analysis and to calibrate the model for the quantitative analysis. As Table A.1 shows, various features of the *Chilean encaje* were altered during its existence. These modifications, together with changes in the foreign interest rate, generated significant variability on the effective cost, or tax-rate-equivalent, of the CCs over time.

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<sup>45</sup>The equivalence with a debt tax can also be interpreted as if foreign investors were required to pay the central bank an up-front fee for borrowing from abroad, instead of making the unremunerated reserve deposit.

<sup>46</sup>In terms of the macroeconomic effects of *Chilean encaje*, the empirical evidence suggests that the more persistent and significant effect was on the time-structure of the capital inflows, which was tilted towards a longer maturity (see De Gregorio et al. (2000), Soto (1997), Gallego and Hernández (2003)). The policy also increased the interest rate differential (although without a significant long-run effect) and had a small effect on the real exchange rate, while there is no evidence on a significant effect on the total amount of capital inflows to the country.

Table A.1: Main changes in the administration of the *Chilean encaje*

Jun-1991	20% URR introduced for all new credit Holding period (months)= $\min(\max(\text{credit maturity}, 3), 12)$ Holding currency=same as creditor Investors can waive the URR by paying a fix fee (Through a repo agreement at discount in favor of the central bank) Repo discount= US\$ libor
Jan-1992	20% URR extended to foreign currency deposits with proportional HP
May-1992	Holding period (months)=12 URR increased to 30% for bank credit lines
Aug-1992	URR increased to 30% Repo discount= US\$ libor +2.5
Oct-1992	Repo discount= US\$ libor +4.0
Jan-1995	Holding currency=US\$ only
Sep-1995	Period to liquidate US\$ from Secondary ADR tightened
Dec-1995	Foreign borrowing to be used externally is exempt of URR
Oct-1996	FDI committee considers for approval productive projects only
Dec-1996	Foreign borrowing <US\$ 200,000 (500,000 in a year) exempt of URR
Mar-1997	Foreign borrowing <US\$ 100,000 (100,000 in a year) exempt of URR
Jun-1998	URR set to 10%
Sep-1998	URR set to zero

Note: URR=Unremunerated Reserve Requirement

Source: De Gregorio et al. (2000).

### A.1 Tax-equivalent of the Chilean *encaje*

Intuitively, capital controls alter the effective interest rate faced by domestic private agents abroad, depending on whether they want to save or borrow. If they want to save, the interest rate remains equal to the risk-free interest rate  $r^*$ . But, if they want to borrow, the effective interest rate they face is higher and given by  $r^* + \nu$ , where  $\nu$  is the tax-equivalent of the CC for funds borrowed with a  $g$ -months maturity. The methodology we describe below, based on the work of De Gregorio et al. (2000), constructs an estimate of  $\nu$  derived from a no-arbitrage condition that factors in the requirement to make the reserve deposit at the central bank.

To compute  $\nu$ , we first define  $r_g$  as the annual risk-free return that funds borrowed for  $g$ -months invested in Chile need to yield in order for an investor to make zero profits:  $r_g = r^* + \nu$ . Let  $u$  be the fraction of a foreign loan that an investor has to leave as an unremunerated reserve deposit and  $h$  the period of time that this deposit must be kept at the Central Bank. Then, if the investment period is shorter than the maturity of the deposit, i.e.,  $g < h$ , borrowing one dollar abroad at an annual rate of  $r^*$  to invest at an annual rate  $r_g$  in Chile for  $g$  months generates the following cash flows:

1. At  $t = 0$ , the entrepreneur can invest  $(1 - u)$  at  $r_g$ .
2. At  $t = g$ , repaying the foreign loan implies the following cash flow:  $-(1 + r^*)^{g/12}$ .
3. At  $t = h$ , the reserve requirement is returned generating a cash flow  $u$ .

Because of arbitrage, it follows that  $r_g$  must be a rate such that the investor is indifferent between investing at home and abroad (computing all values as of time  $h$ , when  $u$  is returned):

$$(1 - u)(1 + r_g)^{g/12}(1 + r^*)^{(h-g)/12} + u = (1 + r^*)^{h/12}.$$

Since  $r_g = r^* + \nu$ , we can use this expression to solve for  $\nu$  as the value that satisfies:

$$(1 + r^* + \nu)^{g/12} = \frac{(1 + r^*)^{g/12} - u(1 + r^*)^{(g-h)/12}}{1 - u}$$

If the investment horizon exceeds the term of the reserve requirement, i.e.,  $h > g$ , the investor has to decide, at the end of the  $h$ -month period, whether to maintain the reserve requirement in Chile or to deposit the amount outside the country. In order to obtain closed-form solutions, we assume that the investor deposits outside the country at the risk-free interest rate. Under this assumption, the previous arbitrage condition remains the same for longer investment horizons.

Using the approximation that  $(1 + j)^x \approx 1 + xj$ , the approximate tax-equivalent of the unremunerated reserve requirement is found by solving the following linear equation for  $\nu$ :

$$1 + gr^* - u(1 + (g - h)r^*) = (1 - u)(1 + g(r^* + \nu)),$$

which yields:

$$\nu = r^* \frac{u}{1 - u} \frac{h}{g}. \tag{A.1}$$

Based on the above description, computing  $\nu$  requires data on the evolution of the reserve requirement (the value of  $u$ ) and the length of the holding period for which the reserves had to remain at the central bank ( $h$ ). These are reported in Table A.1. We also need a proxy for the risk-free interest rate at which the borrowed funds could have been invested abroad  $r^*$ , for which we used the value used in the calibration of Section 5 of the paper and a value for the targeted maturity of the funds invested in Chile  $g$ , for which we used 12 months.

## B Solution Method

To solve for the model's recursive stationary equilibrium, we solve for aggregate prices  $\{w, p\}$ , final goods output  $\{y\}$ , entrepreneurs' decision rules  $\{c'(\tau, z), a'(\tau, z), n'(\tau, z), \tilde{m}'(\tau, z), p'_h(\tau, z), p'_f(\tau, z), y'_h(\tau, z), y'_f(\tau, z), d'(\tau, z), k'(\tau, z), e(\tau, z)\}$ , lump-sum taxes  $T(z)$ , and value functions  $v(\tau, z), v^{NE}(\tau, z), v^S(\tau, z), v^E(\tau, z)$  such that equilibrium conditions (1)–(5) of Section 3.4 hold.

The productivity process  $f(z)$  is discretized by means of a Gauss-Hermite quadrature algorithm. We include  $n_z = 10$  nodes and use the QWLOGN algorithm from Miranda and Fackler (2004). To solve the second-stage problems of exporters, non-exporters and switchers, we use analytic solutions.<sup>47</sup> The first-stage problems of exporters and switchers are solved using the endogenous grids method (EGM) proposed by Carroll (2006), and for non-exporters we use the discrete-choice augmented version of EGM developed by Iskhakov et al. (2017). The algorithm exploits the fact that the entrepreneurs' problems are effectively deterministic. Two properties of the model yield this outcome. First, productivity is stochastic only when firms are born, and is observed before they make their first-period decisions. Second, the Blanchard-Yaari OLG structure includes an insurance environment that allows entrepreneurs to perfectly diversify this risk.

The algorithm is as follows:

1. Initialize aggregate quantities and prices  $(w, p, y)$ .
2. Given a guess for  $(w, p, y)$ , solve the entrepreneur's problem for each  $z \in Z$  as follows:
  - (a) Initialize the entrepreneur's steady-state exporting status to  $\bar{e}(z) = 1$  if  $z > \hat{z}$  and  $\bar{e}(z) = 0$  otherwise, where  $\hat{z}$  is a guess for the highest  $z$  such that all entrepreneurs with productivity  $\hat{z}$  are non-exporters for all relevant  $m$ .<sup>48</sup>
  - (b) Given  $\bar{e}(z)$ , compute steady-state capital  $\bar{k}(z)$ , capital endowments for newborn firms  $\underline{k}(z) = \kappa \bar{k}(z)$ , and lump-sum taxes  $T(z)$  that balance the government budget  $T(z) = \rho p \underline{k}(z)$ .
  - (c) Define grids for future net-worth  $a'_\vartheta(z)$  for each exporting state  $\vartheta \in \Theta$ <sup>49,50</sup> and compute

<sup>47</sup>Note that the FOCs yield analytic expressions in all regions for the benchmark ALCC case. For region 1 in the ELCC case, where this is not the case, we solve for the capital policy function  $k'(\tau, z)$  using Newton's method.

<sup>48</sup>Note that low productivity entrepreneurs can be ruled out from exporting if they cannot afford the exporting fixed-cost in a non-exporting steady-state, that is,  $\bar{e}(z) = 0$  when  $\bar{m}_N(z) - (1 - \rho)\bar{a}_N(z) < \frac{w}{p}F$ .

<sup>49</sup> $\vartheta \in \Theta \equiv \{E, S, N\}$  denotes the entrepreneur's exporting-state—respectively exporters, switchers, and non-exporters—and  $\mathbb{1}_S(\vartheta)$  denotes an indicator function for switching. Since exporting is irreversible and delayed by one period, the entrepreneur's exporting-state  $\vartheta$  is described by:  $\vartheta_t = E$  if  $e_t = 1$ ;  $\vartheta_t = S$  if  $e_t = 0$  and  $e_{t+1} = 1$ ; and  $\vartheta_t = N$  otherwise.

<sup>50</sup>We define equally spaced grids  $A'_\vartheta(z)$  over  $[\kappa_0 \underline{k}(z), \kappa_1 \bar{a}_\vartheta(z)]$ , where  $\bar{a}_\vartheta(z)$  denotes the 2nd-stage policy functions'

the associated 2nd-stage grids for cash-on-hand  $\tilde{m}'_{\vartheta}(a', z)$ ; debt  $d'_{\vartheta}(a', z)$ ; and collateral- and debt- constraint multipliers— $\eta'_{\vartheta}(a', z)$  and  $\mu'_{\vartheta}(a', z)$ —using the FOCs.

- (d) Given grids  $a'_{\vartheta}(z)$ ,  $\tilde{m}'_{\vartheta}(a', z)$ ,  $d'_{\vartheta}(a', z)$ ,  $\eta'_{\vartheta}(a', z)$ , and  $\mu'_{\vartheta}(a', z)$  for  $\vartheta \in \Theta$ :<sup>51</sup>
- i. Solve for  $a'_E(m, z)$  and  $c_E(m, z)$  using the EGM to iterate on the 1st-stage exporter's Euler-equation. Compute the exporter's value function  $v^E(m, z)$  through iteration on the converged policy functions.
  - ii. Given  $a'_E(m, z)$ , solve for  $a'_S(m, z)$  and  $c_S(m, z)$  using the EGM on the switcher's 1st-stage Euler-equation. Compute the switcher's value function  $v^S(m, z)$  using  $v^E(m, z)$  and these policy functions.
  - iii. Given  $a'_S(m, z)$  and  $v^S(m, z)$ , solve for  $a'_N(m, z)$ ,  $c_N(m, z)$ ,  $e'_N(m, z)$ , and  $v^N(m, z)$  by applying the DC-EGM to the non-exporter's 1st-stage Euler-equation iteration, and compute the cash-on-hand switching threshold  $\hat{m}(z)$  given by  $v^S(\hat{m}, z) > v^N(\hat{m}, z)$ .
- (e) Given  $k_0(z) = \underline{k}(z)$ ,  $T(z)$ , and  $\hat{m}(z)$ , solve the entrepreneur's initial life-cycle states: compute the newborn firm's cash on hand  $m_0(z) = \underline{m}(z)$  and determine the initial exporting state  $\vartheta(z)$  using the switching threshold  $\hat{m}(z)$ .
- (f) Given initial states  $m_0(z) = \underline{m}(z)$  and  $\vartheta_0(z) = \vartheta(z)$ ; policy functions  $\{a'_{\vartheta}(m, z), m'_{\vartheta}(a', z)\}_{\vartheta \in \Theta}$ ; and the switching threshold  $\hat{m}(z)$ , map the solutions obtained for the state space  $(m, z, e)$  into  $(\tau, z)$  by recursive substitution as follows: When a firm is born ( $\tau = 0$ ), its choices are given by  $a'(0, z) = a'(\underline{m}(z), z)$  and  $m'(0, z) = m'(a'(0, z), z)$ , respectively. Its choices at age 1 are therefore  $a'(1, z) = a'(m'(0, z), z)$  and  $m'(1, z) = m'(a'(1, z), z)$ . Hence, for any age  $\tau$  the firm's choices are  $a'(\tau, z) = a'(m'(\tau - 1, z), z)$  and  $m(\tau, z) = m'(a'(\tau, z), z)$ . For  $0 \leq \tau < \hat{\tau}(\hat{m}(z))$ , use the non-exporter's decision rules, for  $\tau = \hat{\tau}(\hat{m}(z))$ , use the switcher's, and for  $\tau > \hat{\tau}(\hat{m}(z))$ , use the exporter's.
- (g) If  $e(T + 1, z) = 0$ , update  $\bar{e}(z) = 0$  and return to step 2b.

3. Construct the following system of equations  $\mathbf{h}(w, p, y) = \epsilon$  by using the market-clearing conditions of the problem:

$$\sum_{\tau} \sum_z n(\tau, z) \phi(\tau, z) + F \sum_z \hat{\tau}(\hat{m}(z)) f(z) - 1 = \epsilon_1,$$

steady-state kink conditional on exporting state  $\vartheta \in \Theta$ ,  $0 < \kappa_0 < 1$  and  $\kappa_1 > 1$ . In most applications we use  $n_{a'} = 20,000$  points and set  $\kappa_0 = 0.75$  and  $\kappa_1 = 1.25$ .

<sup>51</sup>The net-worth policy functions  $a'_{\vartheta}(m, z)$  are linearly interpolated and extrapolated, and the value functions  $v^{\vartheta}(m, z)$  are interpolated linearly and extrapolated using cubic splines where needed.

$$\sum_{\tau} \sum_z [c(\tau, z) + \rho \underline{k}(z) + x(\tau, z)] \phi(\tau, z) - y = \epsilon_2,$$

where  $c(\tau, z) = m(\tau, z) - (1 - \rho)a'(\tau, z) - \mathbb{1}_{\tau=\hat{\tau}(\hat{m}(z))}wF$  and  $x(\tau, z) = (1 - \rho)k'(\tau, z) - (1 - \delta)k(\tau, z)$ , and

$$\left[ \sum_{\tau} [y_h(\tau, z)^{\frac{\sigma-1}{\sigma}}] \phi(\tau, z) + y_m^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - y = \epsilon_3.$$

4. Solve for  $\mathbf{h}(w, p, y) \approx \mathbf{0}$  using Broyden's method, where the convergence criterion is set such that  $|\epsilon| \leq 1e - 12$ .
5. Check for convergence of the guess  $(w, p, y)$  used in Step 2 and the solutions from Step 4. If convergence fails, update the guess and return to Step 2.

## C Marginal Revenue Products

A firm's revenue is defined by the value of its sales:  $RV \equiv p_h y_h + p_f y_f$ . Hence, the MRPs of labor and capital are given by  $MRPN \equiv \delta RV / \delta n$  and  $MRPK \equiv \delta RV / \delta k$ , respectively. The results for the two MRPs used in conditions (15) and (16) are obtained as follows.

First, taking derivatives of  $RV$  with respect to  $n$  and  $k$ , we obtain:

$$MRPN = [p_h + y_h(\delta p_h / \delta y_h)](\delta y_h / \delta n) + [p_f + y_f(\delta p_f / \delta y_f)](\delta y_f / \delta n) \quad (\text{C.1})$$

$$MRPK = [p_h + y_h(\delta p_h / \delta y_h)](\delta y_h / \delta k) + [p_f + y_f(\delta p_f / \delta y_f)](\delta y_f / \delta k) \quad (\text{C.2})$$

Solving the demand functions faced by the entrepreneur (2)-(3) for  $p_h$  and  $p_f$ , respectively, yields  $p_h = (y_h/y)^{-1/\sigma} p$  and  $p_f = (y_f/y^*)^{-1/\sigma} p^*$ , and from these expressions we obtain:

$$\frac{\delta p_h}{\delta y_h} = \frac{-1}{\sigma} \left( \frac{y_h}{y} \right)^{-(\frac{1}{\sigma})-1} \frac{p}{y}, \quad \frac{\delta p_f}{\delta y_f} = \frac{-1}{\sigma} \left( \frac{y_f}{y^*} \right)^{-(\frac{1}{\sigma})-1} \frac{p^*}{y^*},$$

which multiplying by  $y_h$  and  $y_f$ , respectively, and simplifying yields:

$$\frac{\delta p_h}{\delta y_h} = \frac{-p_h}{\sigma}, \quad \frac{\delta p_f}{\delta y_f} = \frac{-p_f}{\sigma},$$

Substituting these expressions into (C.1)-(C.2) and simplifying using the equilibrium condition

$p_f = \zeta p_h$  we obtain:

$$MRPN = \frac{p_h}{\varsigma} \left( \frac{\delta y_h}{\delta n} + \zeta \frac{\delta y_f}{\delta n} \right), \quad MRPK = \frac{p_h}{\varsigma} \left( \frac{\delta y_h}{\delta k} + \zeta \frac{\delta y_f}{\delta k} \right), \quad (\text{C.3})$$

where, as defined in the paper,  $\varsigma = \sigma/(\sigma - 1)$ .

Now, differentiate the market-clearing condition  $y_h + \zeta y_f = zk^\alpha n^{1-\alpha}$  with respect to  $n$  and with respect to  $k$  to obtain:

$$\frac{\delta y_h}{\delta n} + \zeta \frac{\delta y_f}{\delta n} = z(1 - \alpha) \left( \frac{k}{n} \right)^\alpha, \quad \frac{\delta y_h}{\delta k} + \zeta \frac{\delta y_f}{\delta k} = z\alpha \left( \frac{n}{k} \right)^{1-\alpha}$$

Substituting these results into those obtained in (C.3) yields the expressions used in conditions (15) and (16) of the paper:

$$MRPN = \frac{p_h}{\varsigma} z(1 - \alpha) \left( \frac{k}{n} \right)^\alpha, \\ MRPK = \frac{p_h}{\varsigma} z\alpha \left( \frac{n}{k} \right)^{1-\alpha}.$$

## D Social Planner's Problem

We analyze the optimization problem of a utilitarian social planner. For simplicity, and since the planner will remove the distortions resulting from monopolistic competition in the domestic markets of intermediate goods, we assume that the planner participates in export markets as a price-taker. In addition, since the entry cost to become an exporter is assumed to represent administrative costs, we assume that the planner incurs only the physical cost of exporting (i.e., the iceberg costs) but not the entry costs. These two assumptions are inessential for the main result of the planner's problem, namely that there is no misallocation in capital and labor across firms.

The social planner's optimization problem is:

$$\max_{c_{t,\tau}(z), k_{t+1,\tau}(z), y_{t,\tau}^h(z), n_{t,\tau}(z), y_{t,\tau}^m, D_{t+1}} \sum_{t=0}^{\infty} \beta^t \left[ \sum_{\tau,z} u(c_{t,\tau}(z)) \phi(\tau, z) \right] \quad (\text{D.1})$$

subject to the following sequence of constraints for each  $t = 0, \dots, \infty$ :

$$\sum_{\tau, z} [c_{t, \tau}(z) + k_{t+1, \tau}(z) - (1 - \delta)k_{t, \tau}(z)] \phi(\tau, z) + \rho \sum_z k_0(z) = \left[ \sum_{\tau, z} y_{t, \tau}^h(z) \frac{\sigma-1}{\sigma} \phi(\tau, z) + y_{m, t}^m \right]^{\frac{\sigma}{\sigma-1}}, \quad (\text{D.2})$$

$$\sum_{\tau, z} n_{t, \tau}(z) \phi(\tau, z) = 1, \quad (\text{D.3})$$

$$\sum_{\tau, z} \frac{\eta^f(z)}{\zeta} \left[ z k_{t, \tau}(z)^\alpha n_{t, \tau}(z)^{1-\alpha} - y_{t, \tau}^h(z) \right] \phi(\tau, z) - \eta^m y_t^m = D_t - q D_{t+1}, \quad (\text{D.4})$$

where  $t$  denotes the time period,  $\tau$  the age of an agent and  $z$  the agent's productivity draw at birth.  $D_{t+1}$  denotes the planner's external borrowing (notice the planner is assumed not to face credit constraints) and  $\eta^f(z)$  and  $\eta^m$  are the world-determined relative prices in units of final goods at which the planner exports the domestic input varieties and imports foreign inputs, respectively (recall also that the debt is denominated in units of final goods).

Constraint (D.2) is the economy's resource constraint in final goods and carries the Lagrange multiplier  $\lambda_t^{SP}$ . Since there is no world trade in final goods, all domestic production is absorbed by domestic consumption, domestic investment and the planner's allocations of initial capital to newborn firms. Constraint (D.3) is the aggregate labor resource constraint with multiplier  $\omega_t^{SP}$ . Constraint (D.4) is the external resource constraint (with multiplier  $\psi_t^{SP}$ ), which equates the trade balance (exports minus imports of intermediate goods) with the change in the external debt position net of interest. In this constraint, the technological constraint on production of each input ( $y_{t, \tau}^h(z) + \zeta y_{t, \tau}^f(z) = z k_{t, \tau}(z)^\alpha n_{t, \tau}(z)^{1-\alpha}$ ) has been used to substitute for exports of domestically-produced inputs ( $y_{t, \tau}^f(z)$ ).

The first-order conditions of the planner's problem at each date  $t$  are:

$$\lambda_t = u'(c_{t, \tau}(z)), \quad (\text{D.5})$$

$$\frac{\lambda_t}{\psi_t} = \frac{\eta^f(z)}{\zeta} \left[ \frac{y_{t, \tau}^h(z)}{y_t} \right]^{1/\sigma}, \quad (\text{D.6})$$

$$\frac{\lambda_t}{\psi_t} = \eta^m \left[ \frac{y_t^m}{y_t} \right]^{1/\sigma}, \quad (\text{D.7})$$



$$\psi_t \frac{\eta^f(z)}{\zeta} (1 - \alpha) z k_{t,\tau}(z)^\alpha n_{t,\tau}(z)^{-\alpha} = \omega_t, \quad (\text{D.8})$$

$$\lambda_t = \beta \lambda_{t+1} \left[ (1 - \delta) + \frac{\psi_{t+1}}{\lambda_{t+1}} \frac{\eta^f(z)}{\zeta} \alpha z k_{t+1,\tau}(z)^{\alpha-1} n_{t+1,\tau}(z)^{1-\alpha} \right], \quad (\text{D.9})$$

$$\psi_t = \beta R \psi_{t+1}. \quad (\text{D.10})$$

Conditions (D.5), (D.8) and (D.9) have three important implications. First, it is evident from (D.5) that, at any date  $t$ , the planner allocates the same consumption to all agents, regardless of their age and productivity. Second, (D.8) and (D.9) imply that, also at any date  $t$ , there is no misallocation in labor and capital (the returns in units of final goods are equalized across all firms). To see this, use conditions (D.6), (D.8) and (D.9) to obtain these results:

$$\left[ \frac{y_{t,\tau}^h(z)}{y_t} \right]^{-1/\sigma} (1 - \alpha) z k_{t,\tau}(z)^\alpha n_{t,\tau}(z)^{-\alpha} = \frac{\omega_t}{\lambda_t},$$

$$\left[ \frac{y_{t+1,\tau}^h(z)}{y_t} \right]^{-1/\sigma} \alpha z k_{t+1,\tau}(z)^{\alpha-1} n_{t+1,\tau}(z)^{1-\alpha} = \frac{\lambda_t}{\beta \lambda_{t+1}} - 1 + \delta.$$

Moreover, these real returns are the same as the marginal revenue products of labor and capital under perfect competition. Recall that the demand functions for each input are given by  $y_{t,\tau}^h(z) = \left[ \frac{p_{t,\tau}^h(z)}{p_t} \right]^{-\sigma} y_t$ , hence  $\left[ \frac{y_{t,\tau}^h(z)}{y_t} \right]^{-1/\sigma} = \frac{p_{t,\tau}^h(z)}{p_t}$ , so the above results reduce to:

$$p_{t,\tau}^h(z) (1 - \alpha) z k_{t,\tau}(z)^\alpha n_{t,\tau}(z)^{-\alpha} = \frac{\omega_t}{\lambda_t} p_t,$$

$$p_{t+1,\tau}^h(z) \alpha z k_{t+1,\tau}(z)^{\alpha-1} n_{t+1,\tau}(z)^{1-\alpha} = \left[ \frac{\lambda_t}{\beta \lambda_{t+1}} - 1 + \delta \right] p_{t+1}.$$

The left-hand-sides of these expressions correspond to the marginal revenue products of labor and capital, respectively, as the solution of the monopolistic competition setup converges to perfect competition (i.e., as  $\sigma \rightarrow \infty$ ). In addition, the planner's shadow value of final goods  $\lambda_t$  should match  $p_t$  and the shadow value of labor  $\omega_t$  should match the wage  $w_t$  in the competitive equilibrium without distortions.

So far, we have established that the social planner's allocations support zero consumption dispersion and zero labor and capital misallocation across firms of different age and productivity at any given date, and also that the MRPs will be the same as in the equilibrium without monopolistic competition. To prove Proposition 2, however, we still have to show that the planner's MRPs are

constant over time and are the same as those of the decentralized equilibrium without financial frictions.

## D.1 Proof of Proposition 2

**Proposition 2** *If  $\beta R = 1$ , the marginal revenue products of capital and labor of the decentralized equilibrium without financial frictions (as  $\sigma \rightarrow \infty$ ) match the efficient real returns on capital and labor attained by a utilitarian social planner free of financial frictions. These MRPs are time-invariant, constant across firms regardless of age and productivity, and MRPK equals  $p(r + \delta)$ .*

*Proof.* To prove this proposition, we assume that (a) the initial capital allocations  $\underline{k}_0(z)$  are the same in the decentralized equilibrium and the planner's problem and (b) the exogenous world relative prices in units of final goods faced by the planner ( $\eta^f(z)$  and  $\eta^m$ ) are constant over time (since we are interested in stationary equilibria) and across ages (since for given  $z$  all producers sell the same input variety) and also support internal solutions for  $y_{t,\tau}^f(z)$  (these could be the competitive equilibrium prices as  $\sigma \rightarrow \infty$  such that  $\eta^f(z) = \zeta$ ).<sup>52</sup>

Since  $\beta R = 1$ , condition (D.10) implies that  $\psi_t = \psi_{t+1} = \bar{\psi}$  (the shadow value of the balance-of-payments equilibrium condition is constant across time, age and productivity). Because final goods are not traded internationally, however, there is no direct arbitrage of the domestic marginal rate of substitution in consumption ( $\lambda_t/\beta\lambda_{t+1}$ ) and the real interest rate  $R$ . But since the planner can borrow or save abroad to finance any gap between exports and imports of intermediate goods, there is an implicit no-arbitrage condition that follows from combining conditions (D.9) and (D.10), considering that  $\psi_t = \bar{\psi}$  for all  $t$ :

$$\frac{\lambda_{t+1}}{\lambda_t} \left[ \frac{\bar{\psi}\eta^f(z)}{\lambda_{t+1}\zeta} \alpha z k_{t+1,\tau}(z)^{\alpha-1} n_{t+1,\tau}(z)^{1-\alpha} + 1 - \delta \right] = R.$$

The term in the left-hand-side is the real return on investing capital to produce intermediate goods in units of final goods, which requires taking into account how the shadow value of final goods changes over time ( $\lambda_{t+1}/\lambda_t$ ), and  $R$  in the right-hand-side is the opportunity cost in units of final goods.

Next we show that  $\lambda_t$  is also constant over time. First, note that conditions (D.6) and (D.7)

<sup>52</sup>This result follows from the fact that  $\eta^f(z) \equiv p_{t,\tau}^f(z)/p$ ,  $p^f(z) = \zeta p^h(z)$  and as  $\sigma \rightarrow \infty$   $p^h(z) \rightarrow p$ . Since it also follows that  $p^f(z) \rightarrow p^*$ , it is also true that  $\eta^f(z) = p^*/p = \zeta$ .

imply:

$$\frac{y_{t,\tau}^h(z)}{y_t^m} = \left[ \frac{\eta^m \zeta}{\eta^f(z)} \right]^\sigma.$$

Then, factor out  $y_t^m$  from the CES production function of final goods to obtain

$$y_t = y_t^m \left[ \sum_{\tau,z} \left( \frac{y_{t,\tau}^h(z)}{y_t^m} \right)^{\frac{\sigma-1}{\sigma}} \phi(\tau, z) + 1 \right]^{\frac{\sigma}{\sigma-1}},$$

and then combine the two results to obtain:

$$\frac{y_t^m}{y_t} = \left[ \sum_z \left( \frac{\eta^m \zeta}{\eta^f(z)} \right)^{\sigma-1} f(z) + 1 \right]^{\frac{\sigma}{1-\sigma}}.$$

Hence, the ratio  $y_t/y_t^m$  is constant over time and then condition (D.7) yields:

$$\lambda_t = \bar{\psi} \eta^m \left[ \sum_z \left( \frac{\eta^m \zeta}{\eta^f(z)} \right)^{\sigma-1} f(z) + 1 \right]^{\frac{1}{1-\sigma}},$$

which implies that  $\lambda_t = \bar{\lambda}$  for all  $t$  (with  $\bar{\lambda}$  defined by the right-hand-side of the expression).

Since  $\lambda_t$  and  $\psi_t$  are constant, the no-arbitrage condition for returns on capital becomes:

$$\frac{\bar{\psi} \eta^f(z)}{\zeta} \alpha z k_{t+1,\tau}(z)^{\alpha-1} n_{t+1,\tau}(z)^{1-\alpha} = \bar{\lambda}(r + \delta),$$

and condition (D.8) can be rewritten as:

$$\frac{\bar{\psi} \eta^f(z)}{\zeta} (1 - \alpha) z k_{t,\tau}(z)^\alpha n_{t,\tau}(z)^{-\alpha} = \omega_t.$$

Note that the no-arbitrage condition implies that capital-labor ratios do not vary with  $\tau$  and  $t$  and the labor optimality condition implies that they cannot vary with  $z$  either (since the shadow value of labor is the same across firms). Hence, capital-labor ratios are constant over time, age and productivity. The planner's allocations move all firms to their optimal scales immediately. The common, time-invariant capital-labor ratio across firms of all ages and productivity is:

$$\overline{\left( \frac{k}{n} \right)} = \frac{\alpha}{1 - \alpha} \frac{\omega}{\bar{\lambda}(r + \delta)}.$$

The above results match the marginal revenue product conditions of the decentralized equi-

librium as it approaches the competitive equilibrium ( $\sigma \rightarrow \infty$ ). In this case, the decentralized equilibrium is efficient, the planner's shadow values of final goods and trade in intermediate goods must satisfy  $\bar{\lambda} = \bar{\psi} = p$ , the shadow value of labor must satisfy  $\omega = w$ , and the relative prices of the planner's problem and the decentralized equilibrium must satisfy  $\eta^f(z) = p^f(z)/p$ ,  $\eta^m = p^m/p$ , and  $p^f(z) = \zeta p^h(z)$ . Under these conditions, the above optimality conditions can be rewritten as follows:

$$p^h(z)\alpha z \left(\frac{\bar{k}}{n}\right)^{\alpha-1} = p(r + \delta),$$

$$p^h(z)(1 - \alpha)z \left(\frac{\bar{k}}{n}\right)^{\alpha} = w.$$

Hence, without financial frictions, the planner's optimality conditions and those of the decentralized equilibrium as  $\sigma \rightarrow \infty$  support the same conditions equating MRPN to  $w$  and MRPK to  $p(r + \delta)$  in all periods and across firms of different age and productivity.  $\square$

## E Earnings-based Collateral Constraint

In this Section of the Appendix we examine the implications of replacing the collateral constraint linked to assets (ALCC)  $qd_{t+1} \leq \theta k_{t+1}$  with an earnings-linked collateral constraint (ELCC):

$$p_{t+1}qd_{t+1} \leq \theta(p_{h,t+1}y_{h,t+1} + p_{f,t+1}y_{f,t+1} - w_{t+1}n_{t+1}). \quad (\text{E.1})$$

Intuitively, resources borrowed at  $t$  cannot exceed a fraction  $\theta$  of the firm's profits at  $t+1$ , which represent the net resources available for repaying. Considering this alternative formulation of credit constraints is interesting because recent empirical and theoretical studies have emphasized the relevance of credit constraints linked to cash flow.<sup>53</sup> This formulation is also equivalent to one in which gross resources (sales) are collateral but a fraction  $\theta$  of the wage bill is financed with working capital and both intertemporal debt and working capital financing are limited by the credit constraint ( $qd_{t+1} + \theta w_t n_t \leq \theta(p_{h,t}y_{h,t} + p_{f,t}y_{f,t})$ ).

Introducing the ELCC requires replacing the ALCC with the ELCC in the the second-stage

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<sup>53</sup>Lian and Ma (2020) show that about 80% of debt of U.S. non-financial firms is based on cash-flow constraints. Caglio et al. (2021) document a similar finding for SMEs. Drechsel (2022) and Li (2022) study the macroeconomic implications of these type of constraints.

optimization problem of entrepreneurs. For example, for a non-exporter, this problem becomes:

$$\tilde{m}'(a', z) = \max_{k', d', p'_h, n'} \left[ \frac{w' + \frac{p'_h^{1-\sigma}}{p'^{-\sigma}} y' - w' n' + p'(1-\delta)k' - p'd' - T(z)}{p'} \right]$$

$$\text{s.t.} \quad \left( \frac{p'_h}{p'} \right)^{-\sigma} y' = z k'^{\alpha} n'^{1-\alpha} \tag{E.2}$$

$$a' = k' - qd' \tag{E.3}$$

$$\hat{q}d' \leq \theta \left( \frac{p'_h^{1-\sigma}}{p'^{-\sigma}} y' - w' n' \right) / p'$$

$$q^* d' \leq 0$$

This formulation of the credit constraint alters the static effects on the determination of  $k'$  in region 1 of the capital decision rule of the second-stage optimization problem via two effects: First, an effect akin to lowering the fraction of pledgeable assets by the share of profits in the market value of capital ( $\pi/pk$ ). Second, a non-linear feedback effect because that share is decreasing in  $k'$  itself. In particular, the mapping from  $a'$  to  $k'$  is no longer  $k'(a') = a'/(1-\theta)$ . Instead,  $k'(a')$  solves this non-linear equation:

$$k' = \frac{a'}{1 - \theta \frac{\pi(k', z; w', p', y')}{p'k'}} \tag{E.4}$$

where  $\pi(k', z; w', p', y')$  is the entrepreneur's profit function.

The above result is derived as follows. First, using the definition of profits for a non-exporter,  $\pi' = p'_h y'_h - w' n'$ , and replacing it with the optimal demand function in monopolistic competition,  $p'_h = p' \left( \frac{y'}{y'_h} \right)^{\frac{1}{\sigma}}$ , and expression  $y'_h = z k'^{\alpha} n'^{(1-\alpha)}$ , we get:

$$\pi'(k', z; w', p', y') = \tilde{p}_{ne}(p, y) (z k'^{\alpha} n'^{(1-\alpha)})^{\frac{\sigma-1}{\sigma}} - w' n'$$

where  $\tilde{p}_{ne}(p, y) = p' y'^{\frac{1}{\sigma}}$ . Then, assuming  $k'$  is set by the collateral constraint (E.3), we can rewrite it as  $\hat{q}d' = \theta \pi'(k', z; w', p', y') / p'$ . Using this and condition (E.2) we obtain expression (E.4). Analogous for exporters, we can show that profits can be expressed as:

$$\pi'(k', z; w', p', y') = \tilde{p}_e(p, y) \left( z k'^{\alpha} n'^{(1-\alpha)} \right)^{\frac{\sigma-1}{\sigma}} - w' n' \tag{E.5}$$

where  $\tilde{p}_e(p, y) = [p^{\sigma} y + \tau^{1-\sigma} p^{*\sigma} y^*]^{\frac{1}{\sigma}}$ . Note that  $\tilde{p}_{ne}$  and  $\tilde{p}_e$  is a common price to all firms, as it is the price index of a firm's output of intermediate goods powered to  $1 - 1/\sigma$ .

In condition (E.4), the effect lowering the effective fraction of pledgeable assets is evident in that, for a given value of  $\theta$  and since  $0 < \pi'/p'k' < 1$ , region 1 in the ELCC case is analogous to that of the ALCC but with  $\theta$  reduced by the fraction  $\pi'/p'k'$ . The nonlinear effect follows from the fact that  $\pi(k', z; w', p', y')/k'$  is decreasing in  $k'$  so that as  $k'$  grows the constraint tightens endogenously, in the sense that a given  $a'$  yields a smaller  $k'$ .<sup>54</sup> It is also possible to show that this effect depends on the degree of monopolistic competition. In the limit under perfect competition, as  $\sigma \rightarrow \infty$ , it vanishes because profits become linear in capital (as in Buera and Moll (2015)) and hence  $\pi(k', z; w', p', y')/k'$  becomes independent of  $k'$ . Still, the first effect reducing the share of pledgeable assets remains.

Note two important properties of the capital decision rule in region 1 of the ELCC relative to the ALCC: First, in the ELCC, the coefficient of the decision rule depends on the full model solution (i.e. it responds to general equilibrium effects depending on how CCs affect  $(w, p, y)$  and hence profits). Second, since the *NCC* and *CC* regimes have different  $(w, p, y)$ , the region-1 decision rules are no longer the same at equilibrium (keeping  $(w, p, y)$  they are still the same).

The ELCC and ALCC also differ in their normative implications. The ELCC embodies pecuniary and non-pecuniary externalities by which individual firms do not internalize the effect of their borrowing decisions on aggregate variables  $(p, w, y)$  that alter borrowing capacity by affecting profits when the constraint binds. In contrast, the ALCC is not affected by externalities. The analysis of these externalities is an important topic for future research.

Switching collateral constraints also alters the dynamic and GE effects. The assets- and earnings-linked collateral constraints have different static effects because of how the latter alters the mapping from  $a'$  to  $k'$  in region 1, as explained above, and these differences affect dynamic and static effects. Changes in the tightness of the collateral constraint of firms in region 1 alter savings incentives and thus the rate at which firms grow their net worth and the time they spend in each of the different regions. The resulting changes in the stationary distribution of firms affect aggregate demand for goods and labor and thus change the GE effects on  $w, p, y$ , which affect the magnitude of misallocation and OSGs in each region.

In terms of how changing collateral constraints affects the effects of CCs (namely, how it

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<sup>54</sup>It can be shown that the ratio of profits to capital can be expressed as  $\frac{\pi(k, z; w, p, y)}{k} = \left[ \frac{\varsigma}{1-\alpha} - 1 \right] \left[ \frac{1-\alpha}{\varsigma} \right]^{\frac{\sigma}{1+\alpha(\sigma-1)}} \left[ \frac{z}{w^{1-\alpha}} \right]^{\frac{\sigma-1}{1+\alpha(\sigma-1)}} \frac{\tilde{p}_e^{\frac{\sigma}{1+\alpha(\sigma-1)}}}{(k)^{\frac{1}{1+\alpha(\sigma-1)}}}$ . This expression is obtained using the first-order condition for labor from (E.5) so we get  $(1-\alpha)\left(\frac{\sigma-1}{\sigma}\right)\tilde{p}_e(y_h + \tau y_f) = wn$ . Replacing this last equation into the profit function we get  $\hat{\pi} = \tilde{p}_e(zk^\alpha n^{1-\alpha})^{\frac{\sigma-1}{\sigma}} \left(1 - (1-\alpha)\frac{1}{\varsigma}\right)$ . Finally, replacing the optimal labor demand function we get the profits to capital expression as shown above. Detailed notes of this derivation are upon request.

affects the comparison across *NCC* and *CC* regimes, instead of just comparing ELCC v. ALCC for a given regime), in partial equilibrium and using the same parameter values for the ELCC and ALCC solutions, CCs have *weaker* static effects with the ELCC than the ALCC. The region-1 ray from the origin is flatter with ELCC than ALCC at the same  $\theta$ , because the ELCC's reduction in the effective fraction of capital pledgeable as collateral reduces the amount of  $k'$  that a given  $a'$  can sustain. But this flatter ray is the same for both *NCC* and *CC* regimes. Since the other regions are unchanged, it follows from visualizing the effect of making region 1 flatter in Figure 1, that the area where  $k'$  is lower under *CC* than *NCC* shrinks and in the new area there is a range of values of  $a'$  for which the difference in capital stocks (and hence in MRPKs) is smaller with *ELCC* than it was in the original area obtained with the *ALCC*. Hence, with the same calibration and considering only static effects at common  $(w, y, p)$ , the model with the ELCC yields smaller static misallocation effects than the ALCC. In the full numerical solution the effects may be even weaker or could be stronger, because changes in  $(w, y, p)$  yield differences in the region-1 decision rules under the each collateral constraint, and because those differences alter dynamic effects too.

The comparison across collateral constraints also needs to consider that the ELCC model ought to be calibrated to the same data targets as the ALCC, and hence the two model calibrations would differ. This re-calibration may in fact offset part of the reduction in effective pledgeable capital that we explained above under the assumption of the same  $\theta$  for ELCC and ALCC, thus pushing in the direction of making the results of the two collateral constraints more similar. This is because the ELCC needs to match the same 15-percent credit-GDP ratio as the ALCC, but since the reduction in effective pledgeable capital implies that in the *NCC* there is less credit with ELCC than ALCC at the same  $\theta$  of the ALCC benchmark calibration, the ELCC asks for a higher  $\theta$  to compensate and hit the 15-percent credit ratio target. The *NCC* static capital decision rules are  $k' = a'/(1-\theta)$  in the ALCC and  $k' = a'/[1-\theta^{ELCC}\pi(\cdot)/p'k']$  in the ELCC, and  $\theta$  and  $\theta^{ELCC}$  both need to support the same 15-percent aggregate credit-gdp ratio. Since  $\pi(\cdot)/p'k'$  is endogenous and varies with  $k'$ , the  $\theta^{ELCC}$  that does this cannot be solved by just setting it so that  $\theta = \theta^{ELCC}\pi(\cdot)/p'k'$ . This would work under perfect competition (and partial equilibrium), because  $\pi(\cdot)/p'k'$  is independent of  $k'$  and becomes a constant that depends on other parameters. But with monopolistic competition every firm would need a different  $\theta^{ELCC}$  to satisfy that condition at the same  $\theta$  of the ALCC.

The quantitative analysis shows that, using the same benchmark calibration and just swapping the ALCC for the ELCC, the model yields milder misallocation and welfare effects. This exercise yields a decrease in credit of 6pp, which turns out to be lower than in ALCC. Even if firms

spend more time in region 2 and 3, this is not enough to compensate the static effects explained in the previous paragraph. On the other hand, recalibrating the ELCC model (instead of imposing the same calibration of the ALCC benchmark), produces quantitative results that are not very different from those of the ALCC, because the re-calibration requires a higher value of  $\theta$  for the ELCC than the ALCC.

Table E.2 shows the calibration parameter values. Recalibrating the *NCC* regime for model with the ELCC requires re-setting  $\theta$  to 0.3481. The rest of the parameters remain close to the benchmark case. Moreover, the model continues to replicate closely the calibration targets as shown in table (E.3). Table E.4 show the aggregate effect of the policy. Output, final good prices and wages fall by a 0.46%, 0.29% and 0.89%, respectively. Investment and consumption fall by 1.21% and 0.52%, respectively. Finally, exports falls by 0.86% and of exporters falls by 6.90%. These magnitudes are slightly higher than the benchmark case.

Column (1) of Table E.5 shows the effects of CCs on misallocation and welfare. In this specification, the transition of firms towards their optimal scales is shorter, which ensures that the CC policy affects credit more markedly and credit over value added decreases by 6.37pp, around a 50% higher than before. As a result of the shorter transition, there is a larger mass of firms in regions 2 and 3 where the effects of CCs on misallocation are slightly stronger: aggregate misallocation increases by 0.67pp. As in the benchmark case, the change in misallocation is heterogeneous on the firm's productivity, its exporter status and OSG. The results are similar to the previous model in qualitative terms, although the effects are in general slightly higher.

Welfare losses are 0.33%, half of what we found in the ALCC model. Firms of all productivity level experience lower welfare losses compared to the benchmark model. This is due to the shorter transition that implies that firms reach sooner their optimal scale. Also,  $y$  falls less than in the benchmark model which causes a smaller reduction in profits. In terms of different productivities, the ELCC model shows the same qualitative pattern we find with the ALCC constraint.

Considering alternative policies, both the CC with lump sum transfers and LTV yield the same pattern as the ALCC model.



Table E.2: Parameter Values: Earnings-based constraint

Predetermined parameters				Calibrated parameters		
$\beta$	Discount factor	0.96	Standard	$\zeta$	Iceberg trade cost	3.8271
$\gamma$	Risk aversion	2	Standard	$\omega_z$	Productivity dispersion	0.4350
$\sigma$	Substitution elasticity	4	Leibovici (2021)	$F$	Sunk export entry cost	1.3993
$\delta$	Depreciation rate	0.06	Midrigan and Xu (2014)	$\theta^{NE}$	non-exporters collateral coefficient	0.3481
$\rho$	Death probability	0.08	Chilean data	$\theta_f$	Exporters collateral factor	1.0361
				$\alpha$	Capital intensity	0.4491
				$\kappa$	Fraction of steady-state capital as initial capital	0.4012

Table E.3: Moments: Earning-based constraint

Target Moment	Data (1990-1991) (1)	Model (No C.controls) (2)
Share of exporters	0.18	0.18
Average sales (exporters/non-exporters)	8.55	8.64
Average sales (age 5 / age 1)	1.26	1.24
Aggregate exports / sales	0.21	0.21
Aggregate credit / Value added	0.33	0.33
Aggregate capital stock / wage bill	6.60	6.53
$(\text{Investment} / \text{VA})_{\text{exporters}} / (\text{Investment} / \text{VA})_{\text{non-exporters}}$	1.84	1.84

Table E.4: Aggregate effects of the CC and LTV policies: Earning-based constraint

	Benchmark ( $\Delta\%$ ) (1)	Lump-sum ( $\Delta\%$ ) (2)	LTV ( $\Delta\%$ ) (3)
Exports	-0.86%	-0.12%	-1.17%
Share of exporters	-6.90%	5.75%	-1.62%
Domestic Sales	-0.71%	-0.20%	-0.22%
Investment	-1.21%	-1.55%	-1.19%
Consumption	-0.52%	-0.09%	-0.05%
Final goods output	-0.63%	-0.33%	-0.24%
Real GDP	-0.46%	-0.62%	-0.50%
Real wage	-0.61%	-0.49%	-0.53%
Wage	-0.89%	-0.01%	-0.45%
Price level (Real ex. rate)	-0.29%	0.49%	0.08%
Agg. credit/Value Added	-6.37pp	-6.10pp	-6.37pp

Table E.5: Effects of Capital Controls on Misallocation & Welfare  
(Earnings-linked Constraint)

	Capital Controls		Lump-sum		LTV	
	misallocation (1)	welfare (2)	misallocation (3)	welfare (4)	misallocation (5)	welfare (6)
<b>All firms</b>	0.61 <i>pp</i>	-0.33%	0.87 <i>pp</i>	0.09%	0.43 <i>pp</i>	-0.20%
<b>Exp. status</b>						
Exporters	0.93 <i>pp</i>	-1.08%	1.43 <i>pp</i>	2.15%	1.12 <i>pp</i>	0.10%
Non-exporters	0.55 <i>pp</i>	-0.30%	0.74 <i>pp</i>	0.02%	0.29 <i>pp</i>	-0.22%
<b>OSG</b>						
Large	0.64 <i>pp</i>	—	0.92 <i>pp</i>	—	0.46 <i>pp</i>	—
Small	0.18 <i>pp</i>	—	0.18 <i>pp</i>	—	0.04 <i>pp</i>	—
<b>Productivity</b>						
1	0.08 <i>pp</i>	-0.60%	0.08 <i>pp</i>	-0.49%	0.02 <i>pp</i>	-0.53%
2	0.25 <i>pp</i>	-0.58%	0.25 <i>pp</i>	-0.46%	0.06 <i>pp</i>	-0.52%
3	0.40 <i>pp</i>	-0.51%	0.39 <i>pp</i>	-0.39%	0.13 <i>pp</i>	-0.49%
4	0.57 <i>pp</i>	-0.36%	0.55 <i>pp</i>	-0.23%	0.24 <i>pp</i>	-0.41%
5	0.65 <i>pp</i>	-0.22%	0.61 <i>pp</i>	-0.05%	0.36 <i>pp</i>	-0.25%
6	0.69 <i>pp</i>	-0.36%	1.45 <i>pp</i>	0.61%	0.36 <i>pp</i>	-0.04%
7	0.45 <i>pp</i>	-0.78%	0.53 <i>pp</i>	-0.29%	0.96 <i>pp</i>	0.33%
8	0.41 <i>pp</i>	-0.86%	0.48 <i>pp</i>	-0.26%	0.99 <i>pp</i>	0.45%
9	0.39 <i>pp</i>	-0.89%	0.46 <i>pp</i>	-0.25%	1.00 <i>pp</i>	0.49%
10	0.38 <i>pp</i>	-0.89%	0.45 <i>pp</i>	-0.24%	1.00 <i>pp</i>	0.51%

## F Model with Domestic Credit Market

This Section of the Appendix examines a simplified version of the model that introduces a domestic credit market in which entrepreneurs can buy or sell bonds, so that they can optimally choose whether they prefer to invest in their own capital or effectively lend to other firms. The model is simplified by assuming that there are no exporters, no imported inputs, and no labor market. Firms use a fixed amount of labor  $\bar{n}$  so that effective productivity becomes  $\tilde{z} = z\bar{n}^{1-\alpha}$ , or alternatively we can think of  $\bar{n}$  as non-marketable land.

Individual holdings of domestic bonds are denoted  $b$ . The price of these bonds is  $q^b$  (with return  $R^b \equiv 1/q^b$ ). The foreign debt market is the same as in the model of the paper. The collateral constraint is now formulated in terms of the net bond position including foreign and domestic bonds:

$$qd' - q^b b' \leq \theta k'.$$

Net worth is now defined as:

$$a' = k' - qd' + q^b b'.$$

Hence, the collateral constraint in terms of net worth remains as before:

$$k' \leq \frac{a'}{1 - \theta}.$$

In principle, there would seem to be a portfolio choice involving  $d'$  and  $b'$ , but in fact, with one exception, the portfolio is always at the corners because of the following arguments:<sup>55</sup>

1. If  $R^b > \hat{R}$ , all firms that borrow always borrow from abroad, and therefore, there is no supply of domestic bonds. Hence, all the debt is in  $d'$  and  $b' = 0$  for all firms. Here, firms that have repaid their debt (i.e., attained  $a' = \bar{k}^{cc}(\tilde{z})$ ) move into region 3 and accumulate net worth along the ray  $k' = a'$  as assumed in the paper, because (a) there is no domestic debt market and (b) since the marginal return on saving exceeds  $R^*$  firms want to grow their net worth but can only allocate it to capital.
2. If  $R^* < R^b < \hat{R}$ , all firms that borrow always borrow in the domestic market and therefore there are no capital inflows. Hence, all the debt is in  $b'$  and  $d' = 0$  for all firms. At equilibrium,

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<sup>55</sup>The exception is when there is excess demand for credit in the domestic market at  $R^b = \hat{R}$ . In this case, the gap is covered by external borrowing at the aggregate level but the portfolio structure of domestic and foreign bonds of individual borrowers is undetermined.

some firms will borrow and have  $b' < 0$  and others will lend (save into bonds) and have  $b' > 0$  and this bond market must clear internally at the rate  $R^b$ . Thus, CCs move the economy to financial autarky.

3. If  $R^b = \hat{R}$ , the two bonds are perfect substitutes for borrowers, they are indifferent which one they use to borrow. The portfolio composition depends on whether at  $\hat{R}$  there is excess demand or supply of credit. If there is excess supply, since lenders cannot get  $\hat{R}$  by investing abroad, the domestic interest rate falls and thus  $R^b = \hat{R}$  cannot be an equilibrium. If there is excess demand, all domestic savers buy the domestic bonds they desire at  $\hat{q}$  and the excess over those that borrowers still want to sell are sold abroad paying the CCs tax, so that the price is still  $\hat{q}$ . We can assume that the domestic market opens first (since after all CCs are in place). Borrowers step in to borrow (sell bonds) and when the domestic bond demand is covered, the rest of borrowers can borrow from abroad. In this case, however, the portfolio structure of individual borrowers is undetermined. Their net position  $\hat{q}[d' - b']$  is well-defined, but the breakdown between  $b'$  and  $d'$  is not. Finally, if at  $\hat{R}$  the aggregate supply and demand of bonds are equal (recalling that lenders would always lend domestically since saving into international bonds pays  $R^*$  not  $\hat{R}$ ), there would be nobody left to borrow from abroad after the domestic market meets and thus  $d' = 0$ .
4. If  $R^b < R^*$ , firms that save would never want to save into domestic bonds, since the return is higher abroad, and therefore no firm would be able to borrow domestically at  $R^b$ . Moreover, since  $\beta R^* = 1$ , it must be that  $\beta R^b < 1$ , and thus dynamic effects will induce firms to always want to reduce their net worth. All firms would want to borrow inducing an excess demand for credit that would cause  $R^b$  to rise. Hence,  $R^b < R^*$  cannot be an equilibrium.

### F.1 Firms that save prefer buying domestic bonds than investing

We start the analysis by presenting a proposition that establishes that, for any  $R^* < R^b \leq \hat{R}$ , an entrepreneur with enough net worth to self-finance the pseudo-steady state of capital supported by CCs will prefer to save its additional net worth into domestic bonds (i.e., lend it to other firms) rather than accumulate more capital.

**Proposition F.1** Assume that  $R^* < R^b \leq \hat{R}$  ( $q^* > q^b \geq \hat{q}$ ), an entrepreneur with net worth  $a' \geq \bar{k}^{cc}(\tilde{z})$  increases its cash-on-hand more by investing its additional net worth into domestic bonds than by accumulating additional capital.

*Proof.* This proof shows that the entrepreneur's increase in cash-on-hand in response to an increase in  $a'$  is larger by investing the marginal net worth into bonds than into capital, because the marginal return of the former exceeds that of the latter.

Start with the case  $R^b = \hat{R}$  (domestic bonds yield the same as the interest rate with CCs). Since  $a' \geq \bar{k}^{cc}(\tilde{z})$ , the firm is not borrowing, and since  $R^b > R^*$ , the firm sets  $d = 0$  (saving abroad by setting  $d' < 0$  yields a smaller return than domestic bonds). The firm will then choose from one of two strategies: (i)  $b = 0$  if it sets  $k' = a'$  (this is the assumption in Region 3 of the analysis in the paper), or (ii)  $b = \hat{R}[a' - \bar{k}^{cc}(\tilde{z})]$  if it keeps its capital constant by setting  $k' = \bar{k}^{cc}(\tilde{z})$ .

Cash-on-hand is:

$$p'm' = p^h \tilde{z} k'^{\alpha} + p'(1 - \delta)k' + p'\hat{R}[a' - k']$$

Recall from the demand-determined output under monopolistic competition that  $p^h/p = [\tilde{z}k'^{\alpha}/y]^{-1/\sigma}$ , hence cash on hand simplifies to:

$$\begin{aligned} m' &= [\tilde{z}k'^{\alpha}/y]^{-1/\sigma} \tilde{z}k'^{\alpha} + (1 - \delta)k' + \hat{R}[a' - k'] \\ &= y^{1/\sigma} [\tilde{z}k'^{\alpha}]^{\frac{\sigma-1}{\sigma}} + (1 - \delta)k' + \hat{R}[a' - k'] \end{aligned}$$

The additional unit of  $a'$  is invested where it yields the larger increase in cash-on-hand, which can be determined by evaluating the total derivative of  $m'$  with respect to  $a'$  under each strategy. The total derivative of cash-on-hand is:

$$\frac{dm'}{da'} = y^{1/\sigma} \frac{\sigma-1}{\sigma} [\tilde{z}k'^{\alpha}]^{\frac{\sigma-1}{\sigma}} \alpha \tilde{z}k'^{\alpha-1} \frac{\partial k'}{\partial a'} + (1 - \delta) \frac{\partial k'}{\partial a'} + \hat{R} \left( 1 - \frac{\partial k'}{\partial a'} \right)$$

which using again  $p^h/p = [\tilde{z}k'^{\alpha}/y]^{-1/\sigma}$  reduces to:

$$\frac{dm'}{da'} = \frac{p^h}{p} \frac{\sigma-1}{\sigma} \alpha \tilde{z}k'^{\alpha-1} \frac{\partial k'}{\partial a'} + (1 - \delta) \frac{\partial k'}{\partial a'} + \hat{R} \left( 1 - \frac{\partial k'}{\partial a'} \right)$$

Since the marginal revenue product for a firm with productivity  $\tilde{z}$  and capital  $k'$  is  $MRPK(k', \tilde{z}) \equiv p^h \frac{\sigma-1}{\sigma} \alpha \tilde{z}k'^{\alpha-1}$ , we obtain that the total derivative is:

$$\frac{dm'}{da'} = \frac{MRPK(k', \tilde{z})}{p'} \frac{\partial k'}{\partial a'} + (1 - \delta) \frac{\partial k'}{\partial a'} + \hat{R} \left( 1 - \frac{\partial k'}{\partial a'} \right)$$

The additional  $m'$  earned by investing the extra unit of  $a'$  following strategy (i) that sets

$k' = a'$  is:

$$\frac{\partial m'}{\partial a'} = \frac{MRPK(a', \tilde{z})}{p'} + (1 - \delta)$$

and under strategy (ii) that sets  $k' = \bar{k}^{cc}(\tilde{z})$  is:

$$\frac{\partial m'}{\partial a'} = \hat{R} = \frac{MRPK(\bar{k}^{cc}(\tilde{z}), \tilde{z})}{p'} + (1 - \delta),$$

where the last equality follows from the optimality condition that defines the pseudo-steady state of capital  $\bar{k}^{cc}(\tilde{z})$  with CCs. Since  $a' \geq \bar{k}^{cc}(\tilde{z})$  and the MRPK is decreasing in  $k$  it follows that:

$$\left. \frac{\partial m'}{\partial a'} \right|_{k'=a'} \leq \left. \frac{\partial m'}{\partial a'} \right|_{k'=\bar{k}^{cc}(\tilde{z})},$$

which holds with equality only if  $a' = \bar{k}^{cc}(\tilde{z})$ . Hence, the firm that has attained  $a' = \bar{k}^{cc}(\tilde{z})$  still desires to increase  $a'$  because  $\hat{R} > R^*$  (so that  $\beta \hat{R} > 1$ ) but it will always prefer to keep capital constant and save at  $R^b$  than to invest in capital.

If  $R^b < \hat{R}$ , there is no borrowing from abroad and hence the economy is in financial autarky. There is something akin to region 2 but defined not by  $a' = \bar{k}^{cc}(\tilde{z})$  but by  $a' = \bar{k}^{R^b}(\tilde{z})$ , where  $\bar{k}^{R^b}(\tilde{z})$  is the pseudo steady-state of capital such that  $R^b = \frac{MRPK(\bar{k}^{R^b}(\tilde{z}), \tilde{z})}{p'} + (1 - \delta)$ . Then the same argument of the case with  $R^b = \hat{R}$  applies. Any firm with  $a' = \bar{k}^{R^b}(\tilde{z})$  will always prefer to save into domestic bonds at  $R^b$  keeping capital constant than investing into capital at  $k' = a'$  because the marginal return of the former strategy dominates that of the latter.  $\square$

The above result shows that Region 3 as presented in the paper can only exist if either (a) the domestic credit market under financial autarky is too small, in the sense that it yields an interest rate such that  $R^b > \hat{R}$ ; or (b) we assume restrictions that prevent firms from saving into the domestic bond market (i.e. domestic lending) at a rate higher than  $R^*$ . For instance, the government could tax domestic bond purchases so that savers can only earn  $R^*$ . This is reasonable under the interpretation that the CCs represent a form of financial repression, because by definition financial repression means that there are wedges that make interest rates on borrowing and saving different. Even relaxing this assumption so that the domestic bond market may exist, however, it does not follow that the static effects of CCs on misallocation are necessarily weaker than in the paper. The outcome depends on what interest rate is generated by the financial autarky equilibrium. This point is explained in detail in the next Section but for now consider the following intuition for two extreme cases.

On one hand, if  $R^b$  is negligibly higher than  $R^*$  it is clear that Region 3 disappears (because of what Proposition 1 proved). Firms never leave Region 2 after reaching it and Region 2 converges to Region 4, so the *NCC* and *CC* regimes would have nearly identical capital decision rules and therefore CCs would be nearly neutral. On the other hand, if  $R^b$  is negligibly lower than  $\hat{R}$ , the static effects would be stronger than in the paper because Region 2 is wider and there are no regions 3 and 4. Firms would never attain their efficient optimal scale. Instead, firms that are sufficiently old or have enough net worth converge to  $\bar{k}^{R^b}(\bar{z})$  and have permanently higher MRPK than the efficient one. Hence, understanding the financial autarky equilibrium is critical for determining whether the domestic credit market would strengthen or weaken the results produced by the model presented in the paper.

## F.2 Credit market equilibrium in financial autarky

We study next the general equilibrium of the model with domestic credit market and what it implies for misallocation relative to the results obtained with the benchmark model in the paper. To start the analysis, note that the arguments about portfolio choice of foreign and domestic bonds presented earlier imply that domestic borrowing emerges when CCs are introduced only if  $R^b \leq \hat{R}$ . Moreover, they also imply that when this happens all the borrowing is domestic and the economy moves to financial autarky. As we explain below, the case  $R^b = \hat{R}$  emerges only if by chance the autarky equilibrium yields a domestic interest rate equal to  $\hat{R}$ , and in this case we assume the domestic bond market opens first to support the equilibrium. Hence, the main case of interest for studying domestic debt is when  $R^b < \hat{R}$ . Before examining the implications of this case, we characterize the general equilibrium of the model.

The model is the same as in Section 3 of the paper, except for the following modifications. First, for simplicity, we assume that there are no exporters, no imported inputs, and no labor market. Firms use a fixed amount of labor  $\bar{n}$  so that effective productivity becomes  $\tilde{z} = z\bar{n}^{1-\alpha}$ , or alternatively we can think of this fixed labor as non-marketable land. The entrepreneurs' production technology hence becomes  $y_h = \tilde{z}k_h^\alpha$ . Since there are no imported inputs, the optimization problem of final goods producers becomes:

$$\max_{y_{h,t}(i), y_{m,t}} p_t y_t - \int_0^1 p_{h,t}(i) y_{h,t}(i) di,$$

$$\text{s.t.} \quad y_t = \left[ \int_0^1 y_{h,t}(i)^{\frac{\sigma-1}{\sigma}} di \right]^{\frac{\sigma}{\sigma-1}},$$

where  $p_t = [\int_0^1 p_{h,t}(i)^{1-\sigma} di]^{1/(1-\sigma)}$ . This problem yields the same demand functions for domestic inputs as in Section 3,  $y_{h,t}(i) = \left( \frac{p_{h,t}(i)}{p_t} \right)^{-\sigma} y_t$ .

Second, since we now allow for the possibility of domestic borrowing, the collateral constraint becomes:

$$qd_{t+1} - q^b b_{t+1} \leq \theta k_{t+1},$$

Keep in mind, however, that as implied by the results from earlier in this Section, when the domestic credit market operates the economy moves to financial autarky, so the relevant case for this analysis is when  $d_{t+1} = 0$ .

The value of an individual firm (assuming financial autarky) is:

$$v(m, \tilde{z}) = \max_{a'} \left[ u(m - (1 - \rho)a') + \tilde{\beta}v(\tilde{m}'(a', \tilde{z}), \tilde{z}) \right]$$

$$\tilde{m}'(a', \tilde{z}) = \max_{k', b', p'_h} \left[ \frac{\frac{p'_h{}^{1-\sigma}}{p'^{-\sigma}} y' + p'(1 - \delta)k' - p'd' - T(\tilde{z})}{p'} \right] \quad (\text{F.1})$$

$$\text{s.t.} \quad \left( \frac{p'_h}{p'} \right)^{-\sigma} y' = \tilde{z}k'^{\alpha} \quad a' = k' + q^b b' \quad k' \leq a'/(1 - \theta) \quad (\text{F.2})$$

### F.2.1 Static Effects in the Second-Stage Solution

The static effects of the collateral constraint are determined by the first-order conditions of the second-stage problem, which determines  $\tilde{m}'(a', \tilde{z})$ . These conditions simplify to:

$$\begin{aligned} MRPK &\equiv \frac{p'_h}{\varsigma} \alpha \tilde{z}(k')^{\alpha-1} = \left[ p'(r^b + \delta) + \eta(1 - \theta) \right] \\ \left( \frac{p'_h}{p'} \right)^{-\sigma} y &= \tilde{z}k'^{\alpha} \quad b' = R^b[a' - k'] \end{aligned}$$

where  $\varsigma = \sigma/(\sigma - 1)$  is the markup of price over marginal cost and  $\eta$  is the multiplier on the collateral constraint. When  $\eta > 0$ , the firm is borrowing and the capital and bond decision rules are:

$$k'(a') = [a'/(1 - \theta)], \quad b' = -R^b \frac{\theta}{1 - \theta} a',$$



When  $\eta = 0$ , the decision rules are:

$$k' = \bar{k}^{R^b}(\tilde{z}), \quad b' = R^b[a' - \bar{k}^{R^b}(\tilde{z})],$$

where  $\bar{k}^{R^b}(\tilde{z})$  is the capital stock at which  $MRPK(\tilde{z}, k') = p'(r^b + \delta)$ . The firm may still be borrowing, in which case  $a' < \bar{k}^{R^b}(\tilde{z})$  and  $b' < 0$ , otherwise the firm is saving and  $b' > 0$ .

The value of  $\bar{k}^{R^b}(\tilde{z})$  is given by:

$$\bar{k}^{R^b}(\tilde{z}) = \left[ \frac{\alpha y^{1/\sigma} \tilde{z}^{1/\varsigma}}{\varsigma(r^b + \delta)} \right]^{\frac{\varsigma}{\varsigma - \alpha}} \quad (\text{F.3})$$

which, using the Cobb-Douglas production function, implies that  $\bar{y}_h^{R^b}(\tilde{z}) = \left[ \frac{\alpha y^{1/\sigma}}{\varsigma(r^b + \delta)} \right]^{\frac{\varsigma}{\varsigma - \alpha}} \tilde{z}^{\varsigma/(\varsigma - \alpha)}$ . Since  $p^h/p = [y_h/y]^{-1/\sigma}$ , at this steady state the more productive firms have higher capital, higher output and lower prices.

## F.2.2 General equilibrium

The definition of this model's equilibrium is analogous to that of the model in the paper, except that we need to add the market-clearing condition of the domestic bond market. Aggregating over net worth and  $\tilde{z}$  using the stationary distribution  $\phi(a', \tilde{z})$ , the market-clearing condition is :

$$\sum_{a'} \sum_{\tilde{z}} \phi(a', \tilde{z}) b'(a', \tilde{z}) = 0.$$

Since  $R^b > R^*$  and  $\beta R^* = 1$ , the dynamic effect drives all firms to grow their net worth. At the threshold net worth  $\tilde{a}'(\tilde{z}) = \bar{k}^{R^b}(\tilde{z})$ , firms attain zero debt and become lenders/savers. All firms with  $a' < \tilde{a}'(\tilde{z})$  are borrowers and can be divided into two groups. First, in the interval  $0 \leq a' \leq (1 - \theta)\bar{k}^{R^b}(\tilde{z})$ , firms borrow  $b' = -R^b \theta a' / (1 - \theta)$ . Second, in the interval  $(1 - \theta)\bar{k}^{R^b}(\tilde{z}) < a' < \bar{k}^{R^b}(\tilde{z})$ , firms borrow  $b' = R^b[a' - \bar{k}^{R^b}(\tilde{z})] < 0$ . All firms with  $a' > \tilde{a}'(\tilde{z})$  are savers with  $b' = R^b[a' - \bar{k}^{R^b}(\tilde{z})] > 0$ . Hence, we can rewrite the market-clearing condition as expressing that the aggregate supply of bonds (aggregate debt) must equal the aggregate demand for bonds (aggregate credit). Thus, the negative of the sum of all negative bond positions must equal the

sum of all positive bond positions:

$$\sum_{a'=0}^{(1-\theta)\bar{k}^{R^b}(\tilde{z})} \sum_{\tilde{z}} \phi(a', \tilde{z}) \theta a' / (1-\theta) + \sum_{a'=(1-\theta)\bar{k}^{R^b}(\tilde{z})}^{\bar{k}^{R^b}(\tilde{z})} \sum_{\tilde{z}} \phi(a', \tilde{z}) [a' - \bar{k}^{R^b}(\tilde{z})] = \sum_{a' > \bar{k}^{R^b}(\tilde{z})} \sum_{\tilde{z}} \phi(a', \tilde{z}) [a' - \bar{k}^{R^b}(\tilde{z})]. \quad (\text{F.4})$$

The above condition can be rewritten in terms of the distribution of age and productivity:  $\phi(\tau, \tilde{z}) = \rho(1-\rho)^\tau f(\tilde{z})$ , where  $\rho$  is the probability of death and  $f(\cdot)$  is the pdf of firm productivity drawn at birth. Define  $\tau_1(\tilde{z})$  as the firm age threshold at which a firm of productivity  $\tilde{z}$  builds enough net worth to reach  $(1-\theta)\bar{k}^{R^b}(\tilde{z})$  (this is analogous to the vertex connecting regions 1 and 2 in the original model), and  $\tau_2(\tilde{z})$  as a similar age threshold at which net worth reaches  $\bar{k}^{R^b}(\tilde{z})$  (this is analogous to the vertex connecting regions 2 and 3 in the original model). The market-clearing condition can then be rewritten as:

$$\sum_{\tau=0}^{\tau_1(\tilde{z})} \sum_{\tilde{z}} (1-\rho)^\tau f(\tilde{z}) \theta a'(\tau, \tilde{z}; R^b) / (1-\theta) + \sum_{\tau=\tau_1(\tilde{z})+1}^{\tau_2(\tilde{z})} \sum_{\tilde{z}} (1-\rho)^\tau f(\tilde{z}) [a'(\tau, \tilde{z}; R^b) - \bar{k}^{R^b}(\tilde{z})] = \sum_{\tau=\tau_2(\tilde{z})+1}^{\infty} \sum_{\tilde{z}} (1-\rho)^\tau f(\tilde{z}) [a'(\tau, \tilde{z}; R^b) - \bar{k}^{R^b}(\tilde{z})]. \quad (\text{F.5})$$

In this expression,  $a'(\tau, \tilde{z}; R^b)$  denotes that  $a'$  changes with age and productivity and depends on the interest rate on bonds that firms took as given in solving their optimization problems. At the equilibrium interest rate,  $R^b$  needs to be such that this market clearing condition holds.<sup>56</sup> Note that all firms aged  $\tau > \tau_2(\tilde{z})$  continue to grow their net worth indefinitely, but as long as their net worth grows at a rate less than the exponential decay of  $(1-\rho)^\tau$ , the sum converges and aggregate demand for domestic bonds is well-defined even if very old firms have infinitely large bond positions.

The graph used to describe the static effects of CCs can be modified to draw a diagram that illustrates the equilibrium of the domestic bond market. The diagram shown in Figure F.1 assumes for simplicity that there is no collateral constraint and no differences in productivity. The shaded area in black represents the aggregate demand for domestic credit and the one in red the aggregate supply. To be an equilibrium interest rate,  $R^b$  must be such that the two are equal.<sup>57</sup> The threshold

<sup>56</sup>The aggregate variables  $p, y$  are also determinants of  $a'(\cdot)$  but are omitted for simplicity, and the market clearing conditions of the markets for intermediate goods and final goods are also part of the general equilibrium solution.

<sup>57</sup>Mathematically, the aggregate demand and supply of credit do not correspond to the entire shaded areas but to the part of them determined by the sums of the discrete elements formed by the optimal choices of net worth determined

$\hat{a}'$  is the value of  $a'$  at which the infinite (but converging) sum in the right-hand-side of the above market-clearing condition converges. This bounds how far to the right in the horizontal axis we need to go to pin down the supply of credit. If the area in black were bigger (smaller) than the area in red, there would be excess demand (supply) of credit and  $R^b$  would rise (fall). The equilibrium interest rate  $R^b$  is the financial autarky interest rate, because it is determined entirely within the domestic economy and all credit is financed internally.

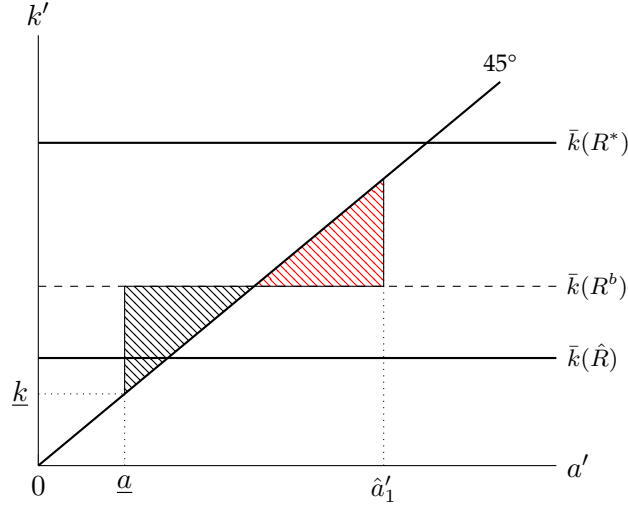


Figure F.1: Equilibrium in Domestic Bond Market

### F.2.3 Effects of capital controls on misallocation

Because of Proposition 1, if  $R^* < R^b < \hat{R}$ , the CCs cause the domestic credit market to emerge and the economy to move to financial autarky. Without CCs, the fact that  $R^* < R^b$  rules out borrowing at the autarky rate and all firms that borrow do so from abroad, and since  $\beta R^* = 1$ , firms stop growing net worth when they reach  $a' = \bar{k}^{R^*}(\tilde{z})$ , which is their optimal scale consistent with the world interest rate (or the rate of time preference since they are the same). Hence, without CCs all firms are borrowers that carry non-negative debt positions and they optimally choose to keep net worth, debt and capital constant when they reach their optimal scale. In contrast, with CCs,  $R^b \leq \hat{R}$  rules out any borrowing from abroad and therefore the economy moves to the financial autarky equilibrium. Moreover, as noted in the previous subsection, firms that reach  $a' = \bar{k}^{R^b}(\tilde{z})$  still want to grow their net worth, because  $\beta R^b > 1$ .

How do effects of CCs on misallocation vary because of the domestic credit market? As

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by the decision rule  $a'(\tau; R^b)$ .

Figure F.1 shows, one important result is that in this environment (with  $R^b \leq \hat{R}$ ) CCs cause permanent effects on misallocation for firms of all ages. In the model of the paper, when CCs are present, firms that build sufficient net worth reach the efficient capital stock  $\bar{k}^{R^*}(\tilde{z})$  and do not have misallocation, nor do they carry any debt or savings. But if the domestic credit market exists, CCs move the economy to financial autarky, firms stay with the lower capital stock given by  $\bar{k}^{R^b}(\tilde{z})$  permanently, and some firms are creditors and others debtors.

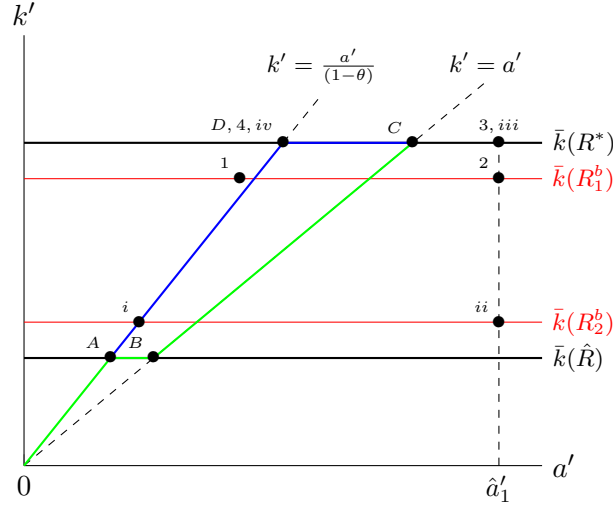


Figure F.2: Effects of Capital Controls with Domestic Bond Market

Whether CCs cause more or less misallocation in this setup with domestic credit market than in the model of the paper hinges on the value of  $R^b$ . Capital controls move the economy to financial autarky and because of this Region 3 disappears but Region 2 widens. Two possible (extreme) outcomes are illustrated in Figure F.2, which is again a variant of Figure 1 in the paper. The piece-wise linear function in blue is the capital decision rule without CCs and the one in green is the one with CCs and no domestic market, as in the paper. The magnitude of the effect of CCs on misallocation is reflected in the size of the overall decline in capital induced by the CCs, which is measured by the trapezoid formed by the vertexes A-B-C-D.

Consider now the case in which the domestic credit market exists and yields an interest rate  $R_1^b$  just above  $R^*$ . As Figure F.2 shows, the effect of CCs on misallocation is now reflected by the loss of capital measured by the trapezoid formed by the vertexes 1-2-3-4.<sup>58</sup> We have more firms in Region 2 than in the model of the paper but in this region the fall in capital and rise in

<sup>58</sup>The vertexes 3 and 4 are determined by the upper bound of net worth  $\hat{a}'$  at which the sum that defines the aggregate supply of bonds converges.

misallocation are small (since  $R^b$  and  $R^*$  are close). Firms that were in the original regions 2 and most of 3 have much less misallocation, and firms close to region 4 and in region 4 will have slightly more misallocation. Hence, the overall misallocation is likely to be smaller than in the model of the paper, as comparing the size of the trapezoids A-B-C-D and 1-2-3-4 suggests.

Now consider the case in which the domestic credit market yields an interest rate  $R_2^b$  just below  $\hat{R}$ . The effect of CCs on misallocation is now reflected by the loss of capital measured by the trapezoid formed by the vertexes i-ii-iii-iv. Again there are more firms in Region 2 but in this region misallocation is still large (just slightly smaller than in the model of the paper). Firms that were in the original region 2 and in region 3 close to 2 have slightly less misallocation but all the rest of firms in the original regions 3 and 4 have much higher misallocation. Hence, the overall misallocation is likely to be larger than in the model of the paper, as comparing the size of the trapezoids A-B-C-D and i-ii-iii-iv suggests. Thus, CCs may induce even stronger misallocation effects with than without a domestic credit market if the latter is relatively small (i.e., if it clears at an interest rate sufficiently close to  $\hat{R}$ ).

The severity of the CCs also matters. Given the financial autarky equilibrium, stricter CCs will yield larger misallocation effects in the model of the paper than in the model with domestic debt market.

## G Summary Statistics of Firm-level Panel and Macro Data

Table G.6: Summary statistics of firm-level panel

VARIABLES	N	mean	sd	min	max
	(1)	(2)	(3)	(4)	(5)
Payroll	91,374	0.384	1.429	0	80.36
Fixed Capital	91,374	2.226	30.41	0	5,717
Export Decision	91,374	0.200	0.400	0	1
TFP	91,374	2.152	0.149	-3.536	2.858
Int.Exp_Fixed_K	91,374	0.420	0.550	0	19.78
OSG	91,374	0.675	0.366	0	1

Note: Payroll and fixed capital are reported in millions of Chilean Pesos. The export status takes the value of zero when the firm does not export in the current period and 1 if it does export. TFP is calculated following the methodology of Wooldridge (2009). OSG is the percentage gap between the fixed capital of the firm and the year-industry average of fixed capital for firms that are older than 10 years old.

Table G.7: Summary Statistics: Macroeconomic Indicators 1990-2007

VARIABLES	(1) N	(2) mean	(3) sd	(4) min	(5) max
CC	18	0.881	1.109	0	2.649
Inflation	18	0.017	0.536	-0.626	1.887
RER_dev	18	-0.009	0.055	-0.082	0.113
Growth	18	0.055	0.028	-0.021	0.120
World Growth	18	3.054	1.000	1.369	4.476
Private Credit/GDP	18	0.613	0.107	0.442	0.743
Libor 12m	18	4.918	1.799	1.364	8.415

Note: Capital Controls are calculated following the methodology of De Gregorio et al. (2000). Inflation, RER\_dev, Growth and World Growth are from the Central Bank of Chile. RER\_dev is calculated as the yearly variation of the real exchange rate, which is defined as the inverse of the nominal exchange rate multiplied by an international price index relevant for Chile and deflated by the Chilean price index. The Private Credit to GDP ratio is from the Financial Structure Database (see Beck et al. (2000)). The 12-month Libor interest rate is obtained from the FRED Economic Data.

## H Robustness of empirical results

In this section, we conduct a set of tests that document the robustness of our empirical findings. In particular, we show that our results are robust to: (i) introducing the interaction of alternative macroeconomic controls with our firms' characteristics; (ii) winsorizing the top and bottom 1% observations of our database with respect to alternative dimensions—i.e., dependent variable, controls, and sectors' productivity; (iii) introducing alternative classifications of exporters, i.e., backward- and forward-looking; (iv) using data at the industry level instead of the firm level.

**Interaction with macroeconomic controls:** A potentially important concern is that the estimates of the interaction terms with CCs could be capturing the effect of an interaction between  $TFP_{ijt}$ ,  $OSG_{ijt}$  and  $Exp_{ijt}$  and other macroeconomic variables. To explore this issue, Table H.8 presents the results of a set of regressions adding to the baseline regression the interactions of a set of candidate macroeconomic variables (one at a time) with  $TFP_{ijt}$ ,  $OSG_{ijt}$  and  $Exp_{ijt}$ . The macro variables are: the LIBOR rate, inflation, growth, the real exchange rate, the ratio of private credit to GDP and world growth. All macroeconomic variables are lagged one period. Table G.7 presents the summary statistics of these variables. All the coefficients of the interactions of the CCs are similar in size, sign and significance when the macro control interactions are introduced.

Table H.8: Interaction with macroeconomic controls

VARIABLES	Libor (1)	Inflation (2)	Growth (3)	RER (4)	PrivCreditGDP (5)	WorldGrowth (6)
CC*TFP	0.890*** (0.121)	0.859*** (0.119)	1.007*** (0.127)	0.494*** (0.104)	1.052*** (0.126)	0.921*** (0.118)
CC*OSG	0.249*** (0.031)	0.255*** (0.031)	0.207*** (0.034)	0.286*** (0.034)	0.248*** (0.031)	0.258*** (0.031)
CC*Exp	0.211*** (0.030)	0.230*** (0.030)	0.139*** (0.033)	0.273*** (0.034)	0.202*** (0.032)	0.258*** (0.030)
Observations	91,374	91,374	91,374	91,374	91,374	91,374
R-squared	0.624	0.625	0.625	0.625	0.625	0.626
Controls	YES	YES	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES	YES	YES
Time FE	YES	YES	YES	YES	YES	YES

Note: This table examines the robustness of the interaction of CC with TFP, OSO and Exp on misallocation when introducing, one at a time, the interactions of macroeconomic variables and our variables of interest,  $TFP_{ijt}$ ,  $OSG_{ijt}$  and  $Exp_{ijt}$ . The macroeconomic variables under consideration are: the Libor rate, inflation, growth, RER, private credit.GDP and world growth. All macroeconomic variables are lagged one period. We include the interactions within the vector of controls for expositional reasons. All regressions include a constant term, firm and time fixed effects, and errors clustered at the firm level in parenthesis. \*\*\*, \*\*, and \* indicate significance at the 1%, 5%, and 10% level.

**Winsorize:** In order to make sure that potential outliers are not driving our results columns (1)-(3) of Table H.9 present a series of exercises where we run our baseline regression after winsorizing the top and bottom 1% observations of our database with respect to alternative dimensions. Column (1) presents the results when winsorizing the dependent variable; column (2) presents the results when winsorizing the control variables; and column (3) presents the results when winsorizing all the firms in sectors whose average productivity is on the top and bottom tails of the distribution. All our results are robust to the different winsorization exercises implying that they are not driven by outliers in terms the dependent variable, controls or sectors.

Table H.9: Winsorized samples, alternative definitions of exporters & industry-level results .

VARIABLES	Wins. MRPK (1)	Wins. Controls (2)	Wins. Sectors (3)	Backward-looking (4)	Forward-looking (5)	Industry level (6)
CC*TFP	0.855*** (0.126)	1.289*** (0.093)	0.902*** (0.130)	0.901*** (0.121)	0.897*** (0.121)	0.033 (0.133)
CC*Exp	0.229*** (0.019)	0.238*** (0.031)	0.234*** (0.030)	0.177*** (0.028)	0.156*** (0.029)	0.347*** (0.132)
CC*OSG	0.248*** (0.022)	0.263*** (0.031)	0.246*** (0.031)	0.234*** (0.031)	0.218*** (0.031)	1.260*** (0.133)
Observations	91,374	83,348	91,374	91,030	91,374	1,600
R-squared	0.624	0.630	0.622	0.623	0.624	0.595
Controls	YES	YES	YES	YES	YES	YES
Firm FE	YES	YES	YES	YES	YES	YES
Time FE	YES	YES	YES	YES	YES	NO
Industry FE	NO	NO	NO	NO	NO	YES

Note: This table examines the effect of the interaction of CC with TFP, OSG and Exp on misallocation while winsorizing the top and bottom 1% observations with respect to: (i) the dependent variable, column (1); (ii) the control variables, column (2); and (iii) the average productivity of the sector, column (3). Columns (4) and (5) present the results of the baseline regression while considering alternative definitions of exporters. Column (6) presents the results of our baseline regression when considering the 4-digit-industry as a unit of analysis. All regressions include a constant term, firm and time fixed effects, and errors clustered at the firm level in parenthesis (industry level for column (6)). \*\*\*, \*\*, and \* indicate significance at the 1%,5%, and 10% level.

**Alternative definition of exporters:** To make sure that our definition of exporters is not biasing our results columns (4) and (5) of Table H.9 replicates our baseline regression using two alternative classifications: a backward- and a forward-looking definition of exporters. The former, column (4), defines exporters as firms that report exports at least once in the previous two years, and aims at capturing that exporters can be differently affected as they typically have a higher level of capital in the steady state and are more productive. Since we do not observe the exporting decision prior to 1990, for the two first years we fix the export status to the respective firm's export status in 1992. The latter, column (5), defines exporters as firms that report exports at least once in the subsequent two years and aims at capturing that firms that want to export in the future might have to undertake more extensive investments today, thus being more exposed to CCs. Our results



are robust to both alternative classifications.

**Industry level regressions:** To wrap up this robustness analysis, we explore whether our firm-level findings also hold when considering the industry as the unit of analysis. To this end, we perform some additional computations. For the case of our dependent and control variables we calculate the period average at the 4-digit-industry-level. For the exporting status, TFP and OSG we create dummy variables that take the value of one when the industry's mean is above the mean of the whole distribution in 1990. Column (6) of Table H.9 presents the results of this estimation.

The results show that for exporters and OSG our insights also hold at the industry level: industries with a larger share of exporters and with a higher average OSG experience a more severe increase in misallocation. The coefficient for TFP, however is not significant. The effect on TFP as a relevant margin at the industry level is more difficult to identify as there is a high correlation at the industry level between the average TFP and the average MRPK. The fact that we are fixing the values of the dummy variables in 1990 is also a robustness to guarantee that our effects are not driven by these characteristics changing endogenously as a result of the introduction of the CC.

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