Deterministic and Stochastic Dynamics of Bed Load Tracer Particles in a Coarse Grained River

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Field site: North East Puerto Rico, Luquillo Critical Zone Observatory

Mameyes River – 300 RFID tracer rocks
(150 placed in 2010, 150 placed in 2011)

USGS Stream Gages

15 minute instantaneous discharge measured upstream and correlated to field site location.
Field site: The Mameyes River

Coarse grained bed

20 m

0.5 m

Measured in the field (repeat surveys)

Displacement

Position → Mobile / Immobile

RFID tag

Tracer cobble

32 mm

10 cm

Grain size distributions

Two populations of Tracer particles.

[Similar D50 for stream and tracers]
Single flood scale: Fraction of mobile tracers & defining the threshold of motion.

\[ \tau_b = \rho g h S \rightarrow U^*_c = \sqrt{\frac{\tau_b}{\rho}} \]

Quantifying the Hydrograph – Assumptions.
- Normal flow – Flow is steady both spatially and temporally.

Boundary shear stress (Pa) Shear velocity (m/s)

- Observed nonlinear relationship in field. Contrary to linear relationship observed in laboratory [Lajeunesse et al., 2010].
Nondimensional Displacement lengths

- Laboratory results [Lajeunesse et al., 2010] show that modal step lengths scale linearly with excess shear velocity.

- Field displacement lengths for single floods are single Step lengths.
Multi flood scale: Dimensionless flood impulse & mean displacement

- Mean displacement data collapse onto linear relationship using dimensionless impulse.
- Dimensionless impulse accounts for hydrograph unsteadiness.

Dimensionless impulse – integral of excess shear velocity normalized by grain size.

\[ t_J = \frac{\int (U_* - U_{*c}) \, dt}{D_{50}} \]

Mean displacement (for all permutations of tracer surveys)

\[ <X/D> = 0.046t_J + 10.2, \quad R^2 = 0.98 \]
Random walk framework: Step lengths & waiting times

Normal Diffusion – Variance of particle position is proportional to time.

\[ \sigma^2 \sim t \]  

Einstein, 1905

Anomalous Diffusion \[ \sigma^2 \sim t^\gamma \]

Super Diffusion \[ \gamma > 1 \]

Sub Diffusion \[ \gamma < 1 \]

Determine scaling from steps & rest distributions

Step lengths (measured)

Tracer particles display thin tailed step lengths.

Waiting times

Heavy tailed waiting times observed in flume due to burial and excavation of particles [Martin et al., 2012].

CDF of particle waiting times

\[ P(t_w > t) = t^{-0.85} \]

[Martin et al., 2012]

Expect heavy tailed waiting times for tracer particles undergoing burial.
Asymmetric random walks: Dimensionless impulse & tracer dispersion

Drift in mean displacement results in an asymmetric random walk process.

Observed dispersion is **Super Diffusive**.

- Can infer scaling of particle waiting times from asymmetric framework.
- Particle waiting times are calculated to be heavy tailed (exponent $\sim 2.4$).
- With heavy tailed waiting times immobile particles appear to take Levy flights upstream.
Summary

Inherent unsteadiness in hydrograph

\[ t_J = \int \frac{(U* - U_{*c})}{D_{50}} dt \]

Single floods to Series of floods

Dimensionless Impulse

Thin tailed steps + Heavy tailed waiting times + Asymmetric random walk → Super diffusion

Thin tailed steps

Heavy tailed waiting times

Asymmetric random walk

Buried particle + Mobile particle → Super diffusion

Burial + Excavation

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