Cheating Because They Can: Social Networks and Norm Violators*

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Abstract

Settlers flocking to boomtowns on the American western frontier were faced with the same task that communities in weak states all across the globe face: self-governance. Reputation mechanisms and community punishment can enforce cooperation in these environments, but existing theories ignore the role of social networks in spreading relevant news. Explicitly building networks into a theory of informal governance produces two previously unforeseen insights. First, how easy it is to persuade a person to behave cooperatively depends on how central or peripheral the person is to his community. Some community members can even be so peripheral that persistent cheating makes sense for them despite the threat of a sullied reputation. Low but persistent levels of settler-native conflict can be understood as the result of rational choices rather than accidents. Second, accounting for social networks reveals that informal institutions face a tradeoff between being fully efficient and being egalitarian: efficient institutions must discriminate. Additionally, I argue that sweeping generalizations may arise as a heuristic for dealing with the relevant-but-more-taxing-to-calculate network position. This logic can help make sense of the prevalence of racism in the mining towns, and why only certain non-white settlers bore the brunt of it.


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1 Introduction

Many groups across the globe make use of informal institutions to enforce cooperation, sometimes to supplement a weak state or backlogged legal apparatus, sometimes to deal with behavior that has more to do with norms than laws, and sometimes simply to use institutions preferred to extant formal ones (see, e.g., Dixit, 2004; Landa, 1981; Ellickson, 1991). Using reputation mechanisms and community punishment can enforce cooperation well, but existing theories of such informal institutions ignore the role that social networks play in spreading the news that affects reputations and allows punishment to be carried out.

Building social networks into a theory of informal enforcement of cooperation reveals two new insights. First, how easy it is to persuade a person to behave cooperatively depends on how central or peripheral the person is to his community. Some community members can even be so peripheral that persistent cheating makes sense for them despite the threat of a sullied reputation. Certain networks permit some cheating in equilibrium, which provides an explanation of the persistence of real world cooperation failure that doesn’t rely on accidents or errors. Second, accounting for social networks reveals that, except in knife-edge cases, informal institutions are fundamentally unable to be both fully efficient and egalitarian. Institutions face a tradeoff between the two norms—groups that value efficiency must discriminate—and the extent of the trade depends on the social network of the people governed by the institutions.

While the arguments in this paper pertain to informal governance in general, in the bulk of the paper I focus on a particular context, 19th century boomtowns in the American West. The “wild west” is a weak state scenario which is uniquely well documented, so much so that we have some micro-level detail on cooperation and conflict, and even some indirect evidence of network structure. Settlers were tasked with policing behavior in their own communities and in interactions between settlers and neighboring Native Americans. Relations in these boomtowns provide suggestive support for the claims about the existence and nature of persistent cheating and the role that networks play.

Moreover, I argue that the wedge between efficiency and egalitarianism can help make sense of the prevalence of racism in the mining towns, and why only certain non-white settlers bore the brunt of it. Race roughly correlated with network position, exact network position was difficult to calculate as the population rapidly grew, so race served as a mental short cut for network position.
The results in the paper are derived from a game-theoretic model which shows how inter- and intra-group cooperation depends on networks along which word-of-mouth news spreads. I present a simple case of the model applied to the boomtown setting in the next section, followed by the general version of the model which relates cooperation and conflict to any network in the section after.

2 Informal Governance in Weak States

There is a small but growing literature which acknowledges the importance of social networks in the study of cooperation. Approaches range from considering particular equilibria on particular classes of networks (see, e.g., Dixit, 2003) to considering the most general results on the most general networks (see, e.g., Fudenberg and Yamamoto, 2010). The approach here falls somewhere in-between; my interest is in particular, realistic equilibria on general networks. Others taking the in-between route account for networks in constraining who interacts with whom (see, e.g., Nava and Piccione, 2013), whereas I allow players to interact with anyone but consider the role of networks in passing along relevant information.¹

Perhaps the work most similar to my own is Wolitzky (2013) which also considers general communication networks and their role in spreading news to foster cooperation. While our motivation is quite similar, a key difference is in the type of “cooperation” addressed. In Wolitzky (2013), players are “cooperative” if they make a contribution, potentially unboundedly large. Under this view of cooperation, the best possible punishment strategies are those that threaten the largest possible punishment to eek out the largest possible contribution, and these are the equilibria explored in Wolitzky (2013). Cooperation here is binary (and hence bounded)—players can be cooperative or not (by stealing or not, punching or not, being rude or not, etc.). When this is the cooperation of interest, the best punishment strategies are not necessarily those that threaten the largest possible punishment. Consequently I consider different equilibria (and discuss selection below).

As an additional departure from existing work, I consider the enforcement of both within-group and inter-group cooperation which is especially pertinent in cases of settlers who have neighboring populations with whom they must get along or ethnic groups who neighbor different ethnic groups and face the potential for conflict. I also address the possibility of partially

¹Cho (2011) considers information problems and resolves them with a public randomization device. My interest is in community enforcement without the aid of public devices, which seem unreliable at best in weak state contexts.
cooperative equilibria that result in persistent cheating.

In the next section, I introduce the model as it applies to a special class of networks. This is intended to simplify presentation, and to capture the particular context of relationships in boomtowns.

Considering weak state governance in the boomtown context is natural for three reasons. First, because the formal governing institutions on the frontier were often weak at best, groups of individuals were left to develop informal means of getting along. This was especially true in boomtowns that grew from zero to thousands of inhabitants in a short timespan. Second, more and more evidence suggests that settlers on the frontier did not face as much danger and chaos as had been earlier assumed or depicted on the silver screen (see McGrath, 1987). Instead, it seems that settlers by and large found ways to cooperate, and the violence and treachery that did occur was infrequent and on the margin, much like would be expected by the models presented here (see Anderson and Hill, 2004). Third, records of frontier violence and treachery on the 19th century American frontier are unusually detailed for a weak government setting. Such detail is especially important for a micro-oriented approach like the present one, in which the identity of offenders matters.

I present the general model for arbitrary networks in the section that follows, with full detail there and in the appendix.

3 Settler- Native Interactions

3.1 The Boomtown Context

Life on the frontier is filled with opportunities to impose costs on someone else for one’s own gain if one could get away with it. When interacting with fellow settlers, the shopkeeper could charge an unfairly high price for his own profit at the expense of the buyer. The lumber salesman could knowingly sell termite-ridden logs. The banker could siphon money out of an account. The issuer of a plot could issue a known dud. A prospective mining partner could conceal a find or jump a claim. Theft, bar brawls, and deceit are all possibilities. Interactions between settlers and native populations can take this form too—violence, theft, impinging on land, reneging on agreements and the like are all possibilities.

New towns are often outside the reach of established law enforcement tens or hundreds of miles away, leaving the enforcement of cooperation in the hands of the settlers. Groups on
the American western frontier had an interest in maintaining cooperation within the fledgeling communities, especially since attracting new settlers was key to their survival. Groups of settlers also had an interest in maintaining cooperative relations with other communities and nearby natives, evidenced by the numerous directives passed among community members for how to keep the peace and the various formal and informal peace treaties drawn up between settlers and natives.

According to the logic of Fearon and Laitin (1996), groups could successfully police their own if, say, all frontiersmen threaten to punish any settler who does wrong against either a fellow settler or a native, and likewise for policing within the group of natives. However, one hiccup to this enforcement plan is that learning about wrongdoing can be a slow process. Communications technology was poor at best on the nineteenth century frontier. While some communities in the American west had newspapers, and while occasionally a newspaper would print a story of someone within the community doing something wrong, by and large knowledge of misdeeds relied on word of mouth communication. Even if news did make it into a newspaper and even if the newspaper had a wide circulation, the process of manual typesetting, printing and delivering was far from instantaneous (Dary, 1998).

In a frontier community, if one settler does something uncooperative like stealing from another settler, some fellow settlers may learn of this offense immediately because they saw happen. Others may learn because someone who saw told them, or because someone who was told told them, and so on. While news may spread from person to person to person and eventually reach everyone, in the meantime, some may be out of the loop and so cannot join in punishing the offender. Someone who saw the robbery may have told the school teacher, who told her husband, who may eventually tell the bartender who will then tell the card players in the saloon. Until the husband has a chance to tell the bartender, though, neither the bartender nor the card players will know. If any of them run into the thief before they hear, they wouldn’t know to punish him. This network of communication—who tells whom—within each community determines how widely and quickly news reaches others, and so will bear on how many people could issue a punishment at a given time.

Our model, then, needs to account for interactions within and between the groups of settlers and natives, as well as the network along which news spreads within the two groups.
3.2 Modeling Newcomers to a Boomtown

As a stylized representation of the threats to cooperation that settlers face, suppose that every day, settlers interact with someone at random—perhaps they pass each other while traveling, or conduct a trade, or sit next to each other in the saloon. With probability $p$, that someone is a native, and with probability $1 - p$ it is a fellow settler. Suppose also that settlers know and recognize each other, natives know and recognize each other, but that settlers and natives do not know or recognize each other. When a settler encounters a native, say because he passes through the hunting grounds of a native, he merely knows he has encountered “a native,” not which one, nor could he describe which one well to his fellow settlers (and likewise for the native’s ability to recognize or describe the settlers to his fellow group members).

Interactions can be peaceful and mutually beneficial, or violent/uncooperative/sneaky where one can gain at the other’s expense. This incentive structure is captured by the prisoner’s dilemma with standard payoffs:

\[
\begin{pmatrix}
C & D \\
C & (1, 1 - \beta, \alpha) \\
D & (\alpha, -\beta, 0, 0)
\end{pmatrix}
\]

where $\alpha, \beta > 1$ and $\alpha - \beta > 1$, and everyone has common discount factor $\delta$.

Suppose also that settlers have a regular set of fellow settlers with whom they speak to and swap news. To keep things simple, suppose the swapping of news also happens once a day. The number of settlers may be low enough so that everyone knows everyone else, but who regularly communicates with whom may be constrained by a number of environmental, cultural or personal factors. For instance, the timing of arrival and the availability of land may make some lines of communication more likely than others. An initial core group of settlers may move in first and develop and occupy a stretch of road that becomes the main strip of the makeshift town. Newcomers may arrive to find the main strip full and have to establish plots of land outside the main strip. Those on the main strip may communicate with others on the main strip regularly as a result of close proximity and shared experiences, while those living in the outskirts may forge ties with only a few settlers living in the central part.

\[\text{In the general model, the frequency with which settlers meet to swap news is left variable.}\]
We can represent this pattern of communication with the network in Figure 1. Settlers 1 through 19 are the “central” settlers who all live along the main strip and communicate with all others along the main strip. Two additional settlers have moved in: settler 20 was a newcomer who forged a tie with a single central settler, and settler 21 was an even later newcomer who forged a tie with only the other newcomer. The new settlers, 20 and 21, can be considered the “peripheral” settlers. Links in the network indicate channels of communication between two settlers.\(^3\)

![Figure 1: Example communication network among 21 settlers, 19 of which communicate with each other regularly (are central).](image)

News of who does what spreads through the network. Settlers can punish fellow settlers whom they know committed misdeeds, and punishment lasts \(T_p\) days. The complete statement of the strategies can be found in the next section in Definition 5. Settlers learn that another settler has done something wrong by being the victim of the wrongdoing, by observing the wrongdoing, or by hearing about the wrongdoing from someone else in the network.

Of course the shape of the network determines how well this punishment scheme works by determining who knows about whom by when. For ease of illustration, take \(T_p = 1\), so that wrongdoing should incur punishment for a single day (with general \(T_p\) worked out in the appendix, as well as consideration of different speeds of communication). In the community in Figure 1, if settlers 2 through 19 do something wrong when interacting with a native—say, hunt on land allocated to the native—then nearly everyone else in the settler community will learn about this in time to punish.\(^4\) The single round of punishment could be carried out by any of

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\(^3\)In this example, the order of settlement and the geography of living patterns constrain who communicates with whom. While the development of a central strip followed by peripheral plot developments was a typical pattern in boomtown creation, the model is agnostic about why a communication network has a certain structure. Given that it does have a certain structure, we can make predictions about the prospects for cooperation.

\(^4\)Perhaps the neighbors in the network observe the offense in a large hunting party. Perhaps the hunter’s kill is visible and everyone knows where it came from. Perhaps the hunter brags about his kill. The network governs who
the others among 2 through 19 the next day (perhaps the offender has to turn over all of his kill
the next day). If settler 1 is the offender, even more fellow settlers would know since peripheral
settler 20 would too. If instead one of the peripheral settlers does the wrongdoing, only one (in
the case of 21) or two (in the case of 20) would know and so could exact the punishment the
next day. None of the others would know by then. The chances of the offender running into
someone who could punish him are slim.

Clearly, then, the newcomers who are peripheral to the community are in a better position
to get away with cheating in interactions than are the central settlers. Since no communication
channels connect settlers to natives, we can also identify the most tempting targets of the
peripheral settlers and make the following claim:

Claim 1 (Dangerous Newcomers). The interactions in which someone has the strongest
incentive to defect are those between the most peripheral settlers (here the newcomers) and any
natives, where the peripheral settlers have an incentive to defect on the natives.

This claim corresponds to the more general and precise Proposition 1 below. The newcomers
pose the greatest challenge to full inter- and intra-group cooperation.

3.3 Egalitarianism v. Efficiency

Each settler, based on his position in the group’s communication network, has a number of
days of punishment that would suffice to entice him to be cooperative with everyone, even the
natives. We can find the smallest number of days for each settler:

Claim 2 (Minimum Length of Punishment). Each settler has a minimum length that the
punishment phase would have to be in order to be persuaded to be a cooperator.

This claim corresponds to Lemma 1 below. Clearly more peripheral members have a weakly
(and often strictly) larger minimum since it takes longer for news about their misdeeds to spread
widely and hence to face sufficiently costly punishment. In the example of old and newcomer
settlers from above, we can fix a set of parameter values to show each settlers’ minimum pun-
ishment length, shown in parentheses in Figure 2.\(^5\)

\[^5\text{For illustration, fix } \alpha = 1.5, \beta = 1.05, \delta = .95, r = 1, \text{ and } p = .2. \text{ Given these values, the network implies a minimum } T_i \text{ for each player shown in Figure 2 that are required to incentivize compliance with the fully cooperative equilibrium. These parameters are defined in the next section.}\]
Figure 2: Minimum required punishment for each settler when $\alpha = 1.5, \beta = 1.05, \delta = .95, r = 1,$ and $p = .2.$

Only in knife-edge cases would all players require the same length of punishment (discussed in Proposition 2). This heterogeneity is a direct consequence of an irregular (and hence realistic) communication network and produces a tradeoff: the community could either coordinate on a length of punishment phase sufficiently long to encourage even the most peripheral player to be cooperative (threatening overkill punishment for most), or they could assign different punishment lengths to different players based on how much it would take to compel each person to be cooperative (punishing like offenses differently based on personal characteristics of the offender).

We can make this tradeoff more precise with the following definitions:

**Definition 1 (Egalitarian Punishment).** Call a punishment scheme *egalitarian* if it threatens like punishment for like offenses. If two people commit the same misdeed, egalitarian punishment requires that they receive the same punishment.

**Definition 2 (Fully Efficient Punishment).** Call a punishment scheme *fully efficient* if it is both efficient in equilibrium and obtains this in the most efficient way off the equilibrium path.

For example, a punishment scheme is fully efficient if it maintains full cooperation in a way that threatens destruction of the least value off the equilibrium path. Since punishment is value-destroying in in-group punishment schemes, this quality obtains when the least amount of punishment is threatened to accomplish the equilibrium behavior.\(^6\) This property is especially

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\(^6\)Note that this is a departure from refinements that find threatening the maximum punishment optimal. Maximal punishments make sense when agents’ action sets are unbounded from above. Here, a player is making a choice over
desirable in places with high chance of errors, accidents, drunkenness, short-tempers, miscommunication, as we could imagine a frontier boomtown to be.

In our example settler network, some are more central and well-connected, others are more peripheral. This heterogeneity introduces heterogeneity in minimum required punishments, which makes fully efficient punishment schemes discriminatory ones.

Claim 3 (Egalitarianism v. Full Efficiency). *Boomtowns with a central set of settlers and more peripheral newcomers cannot admit a punishment scheme that is both egalitarian and fully efficient.*

This claim corresponds to Proposition 2 below, which generalizes the claim to show that any network that admits heterogeneity in minimum required punishments (that is, almost all incomplete networks) cannot implement a punishment scheme that is both egalitarian and fully efficient.

Claim 4 (The Extent of the Tradeoff). *The wedge between egalitarianism and full efficiency is greater when the difference between the most and least peripheral settlers is greater. The wedge between egalitarianism and full efficiency disappears only in networks in which every settler is exactly equally peripheral/central.*

This claim corresponds to Corollary 1 below.

### 3.4 Persistent Settler-Native Cheating

Full cooperation can only be sustained when all players’ minimum punishment length is satisfied. Due to heterogeneity in minimum required punishment, there are situations in which low levels of inter-group cheating can persist in equilibrium. The issue is not simply that the full cooperation equilibrium isn’t sustainable for some communication networks. The issue is that there is a next-best equilibrium which is almost fully cooperative. Tolerating a few persistent cheaters can be stable in equilibrium.

Claim 5 (Persistent Cheating). *There exist communication networks and parameter values in which full inter and intra-group cooperation cannot be sustained, but almost full cooperation with the most peripheral settlers defecting on natives (and vice versa) is a sequential equilibrium.*

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two actions. Threatening infinite punishment would not make the desirable behavior any more desirable. This feature makes overkill punishments costly out of equilibrium without additional gain in equilibrium.
This claim corresponds to Proposition 3 below. In our example above, the newest settlers are the first candidates for the persistent cheaters.

Returning to our stylized example, it can be shown that there exists a sequential equilibrium in which settler 21 persistently cheats on the out group, but cooperates with his fellow settlers, and all other settlers are fully cooperative.\(^7\)

Settlers in these equilibria have a group among them (possibly with only a single member) which cheats the out-group whenever they have the chance. Despite fellow settlers’ best efforts at keeping the cheaters in line, the cheating settlers gain more by continuing to accept a little punishment in exchange for the gains from cheating.

The presence of persistent cheating in equilibrium is strictly a function of peripheral members in the communication network, and not of group size per se.

**Claim 6 (Size Alone Tells Nothing).** _Group size is not a sufficient statistic to predict ability to enforce full cooperation or the likelihood of persistent cheating in equilibrium._

This claim is a consequence of Proposition 1 below. If the size of the group is related to its ability to integrate newcomers, or its likelihood of having peripheral members, then size matters, but it matters via the structure of the communication network. Without knowing something about their communication networks, or perhaps how size relates to these networks, size alone is an insufficient metric for comparing groups’ cooperative prospects.

The next section presents the general model, precise strategies, and conditions for both the fully cooperative and the persistent cheating equilibria. A key result in this section is the generation of a network statistic that determines for any network whether cheating can occur in equilibrium and who the most likely cheater/s is/are. Readers uninterested in the technical detail can skip ahead to the discussion of the real frontier.

### 4 The General Model

The game is a generalization of Fearon and Laitin (1996), modified so that players communicate according to an arbitrary communication network. Let \(A\) and \(B\) be two groups with sets of players \(\{1, \ldots, n\}\) and \(\{n + 1, \ldots, 2n\}\), respectively. Define an infinitely repeated game \(G\) such

\[^7\text{Take the example from above, with } \alpha = 1.5, \beta = 1.05, \delta = .95, r = 1, \text{ and } p = .2, \text{ and set the common punishment length to } T^p = 2. \text{ At these values, all players but 21 prefer to play the complier strategy, and player 21 prefers to play the cheater strategy. For these values, the group cannot enforce full cooperation, but this almost-fully cooperative equilibrium in which one settler cheats is the next-best.}]

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that all players play one round of prisoners dilemma with an independently drawn random opponent each period who, with probability $p$, is from the different group, and with $1 - p$ is from the same group.\footnote{Groups are assumed to be the same size to make this matching possible. They could be different sizes, which would require us to consider \textit{pairs} of matching probabilities, bounded by group sizes.} Nature reveals to each player only his own pairing. Each round faces payoffs:

\[
\begin{pmatrix}
C & D \\
C & (1, 1 - \beta, \alpha) \\
D & (\alpha, -\beta, 0)
\end{pmatrix}
\]

where $\alpha, \beta > 1$ and $\frac{\alpha - \beta}{2} < 1$. Players have common discount factor $\delta < 1$.

Observation and communication within group $A$ occur according to a “communication network” defined by the pair $(g_A, A)$ with $n \times n$ adjacency matrix $g_A$ where $g_{i,j}^A = g_{j,i}^A = 1$ indicates a link between $i \neq j \in A$, and likewise $(g_B, B)$ for group $B$. For simplicity, I will refer to the networks as “$g^A$” and “$g^B$,” or simply as $g$ when the group identity is unimportant. Links in the networks are undirected and unweighted, and no links span the two groups. Networks are common knowledge within but not across groups. While players know their own network, they know merely the topology (but not node labeling) of the other group.

A group is a set of players who can all recognize each other: they can perfectly identify each other, describe each other, and would recognize each other if rematched. A group cannot recognize or describe out-group members. When matched with an out-group member, a player only knows he’s playing ‘someone’ from the other group. This scenario is the hard case for inter-group cooperation, and seems to be a reasonable description of some ethnic groups’ experience (Fearon and Laitin, 1996, p.727).

The communication network transmits relevant information about rounds (clarified below). Actions are observable by neighbors in the networks $g_A$ and $g_B$. Information spreads truthfully and deterministically through the network at a rate $r = \frac{\text{Degrees Spread}}{\text{Rounds Played}}$. When $r = 1$, news about rounds spreads one degree, to the immediate neighbors, before players are rematched and play again. When $r = 2$, news spreads two degrees, from person to person to person before the next round, and so on.

The repeated prisoner’s dilemma admits many equilibria, even in this case with random
matching, two groups and a network that transmits information. Here we will restrict attention to equilibria that exhibit the following properties:

1. Efficiency
   All in-group and out-group interactions are cooperative.

2. Use of the Network
   Strategies respond to the information that is spread through the network.

3. Presumption of Innocence
   When nothing is known about another player, he is presumed to be cooperative.

4. Full Efficiency
   Efficiency in equilibrium is enforced with the least damaging off-the-equilibrium-path threats.

Property (1) is naturally of interest when we want to know how groups can govern themselves well, though considering instances when (1) cannot be satisfied will be taken up below. Property (2) restricts our attention to equilibria in which players respond to the gossip being spread through their group, which is also natural given our interest in the role of the network. Property (3) is a refinement that has bite when information spreads along incomplete networks. Occasionally players will have heard nothing about an opponent. Treating such an opponent as if he were cooperative in these instances avoids potentially arduous and indeterminate inference problems (see, e.g., Goyal, 2007, chapter 3), is sensible for a group trying to maintain cooperation anyway, has normative appeal, and reduces the chance that errors or mistakes ruin an otherwise fully cooperative arrangement. Likewise, (4) buffers groups from the consequences of mistakes, errors, drunkenness, short tempers, etc. This condition is especially useful in an error-prone environment and has normative appeal in its lack of unnecessary vengeance.

I will also consider a fifth property:

5. Egalitarianism
   Like offenses are sentenced to like punishment.

Condition (5) is a strictly normative condition requiring that if two people commit the same misdeed, they receive the same punishment. It turns out that networks make full efficiency and egalitarianism impossible to satisfy simultaneously except in very rare cases.
4.1 Strategies

The equilibria of interest require players to keep track of whether and for how long in-group members deserve punishment. In-group punishment begins immediately after an offense in time $t$ and lasts through round $t + T^P$. Relevant information then includes who played which action using the most specific possible description of the person (his identity if speaking about an in-group member, his group membership if speaking about an out-group member), when the round occurred, and whether punishment of anyone involved was deserved as a result.\(^9\) Players pass relevant information along to their neighbors until it becomes irrelevant (after $T^P$ periods).

In-group policing requires each player to sort out who deserves punishment and who does not. Label three statuses a player can have going into a particular in-group match: “cooperator,” “unknown defector” and “known defector.” A player can be said to have “defected in $t$” if he played $D$ against a “cooperator,” an “unknown defector,” or an out-group member in $t$, or played $C$ against a “known defector” in $t$.

In order to define the statuses, we need an intermediate definition. For $i \in A$, the set $N_i = \{j \neq i \in A : g_{i,j}^A = 1\}$ is player $i$’s “neighborhood” and the $j \neq i \in A : g_{i,j}^A = 1$ are $i$’s “neighbors.” The neighborhood of $i$ contains all players who can be reached from $i$ in geodesic distance 1. Generalizing, we can define neighborhoods of size $k$:

**Definition 3 ($k$-neighborhood).** Player $i$’s “$k$-neighborhood” in network $g$, for $k \geq 2$, is the set

$$N^k_i = \left\{ \bigcup_{j \in N_{i}^{k-1}} N_j \right\} \setminus i.$$  

That is, $i$’s $k$-neighborhood is the set of all other players reachable from $i$ in paths of length $k$ (geodesic distance $k$). Information spread in the network is characterized below. Whenever I use the term “neighborhood” as opposed to “$k$-neighborhood,” I am referring to $N_i$.

Now define the statuses as follows:

**Definition 4 (Status Definitions).** A player $i$ is a “known defector” in a round against in-group opponent $j$ in $t$ if, in the last $T^P$ rounds, $i$ defected against $j$, or if in $t - l, l = 1 \ldots T^P$,

- $i$ defected against an rl-neighor of $j$, or if $i$ is a $k$-neighbor of $j$ and $i$ defected against any in-

\[^9\]Carefully specifying the content of messages in full notation is cumbersome and adds little—as the appendix makes clear, players carry this information in vectors of states relevant to their strategies and update these over time according to properties of the network.
or out-group member in \( t - l \) for \( k \leq r_l \leq r T^P \).

A player \( i \) is an “unknown defector” in a round against in-group opponent \( j \) in \( t \) if \( i \) is not in \( j \)’s \( r_l \)-neighborhood and in \( t - l \) defected against an in-group member also not in \( j \)’s \( r_l \)-neighborhood or against an out-group member for \( l = 1 \ldots T^P \).

A player \( i \) is a “cooperator” in a round against in-group opponent \( j \) in \( t \) if \( i \) is neither a known defector or unknown defector.

Now there two strategy profiles of interest. The first, Network In-Group Policing, can produce full cooperation in equilibrium:

**Definition 5 (Network In-Group Policing \( \sigma^{NWIGP} \)).** All players matched with in-group players play \( C \) if matched with a “cooperator” or “unknown defector” and \( D \) if matched with a “known defector.” All players matched with out-group players play \( C \).

A bit more machinery is necessary before establishing conditions for sequential equilibrium. The network matters because it determines who will have received news about whom before a certain amount of time. It will be convenient to use the following construction:

**Definition 6 (The Reach of News).** Define \( K_{i,j,t,l} \) to be the set of players in \( i \)’s group that know about his round with \( j \) that occurred in period \( t \) by period \( t + l \). Since news transmission is a function of the network, this set becomes, for \( j \in \text{outgroup} \),

\[
K_{i,j,t,l} = \left\{ \bigcup_{k \in N_{i,t-1}^l} N_k \right\} \setminus i \equiv N_{i,t}^l(g)
\]

and for \( j \in \text{ingroup} \),

\[
K_{i,j,t,l} = \left\{ \left\{ \bigcup_{k \in N_{i,t-1}^l} N_k \right\} \cup \left\{ \bigcup_{k \in N_{j,t-1}^l} N_k \right\} \right\} \setminus i \equiv N_{i,t}^l(g) \cup N_{j,t}^l(g).
\]

Now we can establish conditions for full cooperation under In-Group Policing, \( \sigma^{NWIGP} \):

**Proposition 1 (Full Cooperation).** \( \sigma^{NWIGP} \) is sequentially rational if and only if, given \( r \), \( p \), \( T^p \), \( g_A \) and \( g_B \), for all \( i \in A \cup B \):

\[
\frac{\alpha - 1}{\delta^{T^p}(1 - p)} \leq \frac{\#K_{i,j,t,T^p}}{N - 1}(1 + \beta)
\]
for \( j \) in the outgroup, and

\[
\frac{\beta}{\delta^T p(1 - p)} \leq \frac{\#K_{i,j,t,T_p}}{N - 1}(1 + \beta).
\]

(4)

for all \( i \) in the ingroup.

The appendix contains the proof, along with a discussion of beliefs that extend the behavior to sequential equilibrium.

It is clear from Proposition 1 that the players who are the most difficult to keep cooperating with punishment length \( T_p \) are those with the smallest \( \#K_{i,j,t,T_p} \), that is, those who have the smallest number of in-group members who learn about their offense in at most \( T_p \) rounds. These players may be impossible to keep cooperating, the logical conclusion of which is presented in Proposition 3.

Rearranging the conditions in Proposition 1, we can identify the minimum punishment that each player would require in order to be enticed to be cooperative. Clearly if the group were going to impose a single, egalitarian punishment and attain full cooperation, it must set \( T_p \) to be the maximum of these values.

**Lemma 1 (Minimum Punishment \( T_i \)).** A player \( i \)'s minimum punishment length \( T_i \) given network \( g \) and parameters \( \alpha, \beta, \delta, p \) and \( r \), is the \( T_i^* \) which satisfies

\[
\min_{j \neq i \in \{A,B\}} \left\{ \delta T_p \#K_{i,j,t,T_p} \right\} = \min \left\{ \frac{(\alpha - 1)(N - 1)}{(1 - p)(1 + \beta)}, \frac{\beta(N - 1)}{(1 - p)(1 + \beta)} \right\}.
\]

\( T_i \) is the punishment that is just long enough to entice \( i \) to be cooperative.\(^{10}\) Heterogeneity in this minimum makes satisfying both full efficiency and egalitarianism impossible.

**Proposition 2 (Elusive Fully-Efficient, Egalitarian Punishment).** A network \( g \) admits an equilibrium which is both fully-efficient and egalitarian with punishment length \( T \) only if \( T_1 = T_2 = \cdots = T_n = T \) for all players in \( A \), which is possible only if

\[
\#K_{1,j,t,T} = \#K_{2,j,t,T} = \cdots = \#K_{n,j,t,T}
\]

\(^{10}\)Strictly speaking, \( i \)'s minimum feasible punishment length is the smallest integer not less than \( T_i \), \( \lceil T_i \rceil \), since time is discrete. Some homogeneity in feasible punishment lengths will be introduced as an artifact of this discreteness. For instance, in the example above, settler 1’s \( T_1 \) is slightly less than settlers 2 through 19’s \( T_2 \ldots T_{19} \) since 1 has an additional neighbor, settler 20. However, because time is discrete, 1’s and 2 through 19’s minimum punishment are the same in practice for the chosen parameters. Even in this example, though, not all players have common feasible \( T_i \)s— the set of networks in which players have these in common is still extremely small.
for $1 \ldots n \in A$, $j \in B$.

Only networks in which every player’s $T$–neighborhood is the same size can produce a minimum punishment length $T$ common to all members, and hence have a fully-efficient and egalitarian punishment regime, an extremely small set of all possible networks.\(^{11}\) Outside of this knife-edge situation, a punishment can either be fully efficient or egalitarian (or neither), but not both.

It is straightforward to classify the extent of the tradeoff:

**Corollary 1 (The Size of the Wedge Between Full Efficiency and Egalitarianism).**

The tradeoff between full efficiency and egalitarianism becomes more severe as the following difference becomes greater:

$$\min_{i \in A}\{T_i\} - \max_{i \in A}\{T_i\}$$

The larger the difference between the smallest minimum required punishment in the group and the largest minimum required punishment in the group, the greater the excess of overkill punishment in securing egalitarianism, or the greater the disparity in inegalitarian punishments in securing full efficiency. This difference is exacerbated when some people are especially peripheral in the network compared to others who are especially central.

### 4.2 Persistent Cheating

If a group is unwilling, unable, or too impatient to threaten punishment phases sufficiently long to entice even the most peripheral players to cooperate, that group may still be able to enforce almost-full cooperation in equilibrium.

Under the right conditions, there exist sequential equilibria in which some players, the “cheaters,” cooperate with every in-group member and defect against the out-group, and the rest of the players, the “compliers,” cooperate with everyone except “cheaters” they’ve heard about.

\(^{11}\) Complete networks and those with rapid communication technology or very long punishment schemes relative to the network diameter ($rT_p \geq \text{diam}(g)$) trivially exhibit $T_p$-neighborhoods that are the same size for all $i$. When $rT_p < \text{diam}(g)$, networks with same size $T_p$-neighborhoods are a rarity. For an idea of how rare, take the case of supporting common punishment length $T_p = 1$. Networks which support this common punishment are those which are regular; regular networks with ten nodes comprise .009% of possible networks with ten nodes (see Sloane and Plouffe, 2014b,a).
Partial out-group deviance has some players, the "compliers," play as they would in $\sigma^{NWIGP}$ and some players, the "cheaters," follow $\sigma^{NWIGP}$ in in-group matches but always play $D$ in out-group matches:

**Definition 7 (Network Out-Group Cheating $\sigma^{NWOUT}$).** All players, "compliers" and "cheaters," play $C$ if matched with a "cooperator" or "unknown defector" and $D$ if matched with a "known defector. All "compliers" matched with out-group players play $C$. All "cheaters" matched with out-group players play $D$.

Now we can establish the conditions under which Out-Group Cheating secures almost-full cooperation.

**Proposition 3 (Out-Group Cheating).** $\sigma^{NWOUT}$ is sequentially rational for groups $A$ and $B$ with networks $g_A$ and $g_B$ given information speed $r$, discount factor $\delta$, and length of punishment phase $T^p$ if, $\forall i \in A \cup B$ playing the complier strategy $NWTFT$,

$$\frac{\alpha - 1}{\delta T^p (1 - p)} \leq \min \left\{ \frac{\#(K_{i,j,t,T^p})}{N - 1}(1 + \beta), \frac{\#K_{i,j,t,T^p}}{N - 1} \alpha \right\}$$

(5)

for $j$ in the outgroup, and

$$\frac{\beta}{\delta T^p (1 - p)} \leq \min \left\{ \frac{\#(K_{i,j,t,T^p})}{N - 1}(1 + \beta), \frac{\#K_{i,j,t,T^p}}{N - 1} \alpha \right\}.$$  

(6)

for all $i$ in the ingroup, and if, $\forall i \in A \cup B$ playing the cheater strategy $NWOUT$, when $T^p \leq \left\lceil \frac{1}{p} - 1 \right\rceil$,

$$\frac{\alpha - 1}{\delta T^p (1 - p)} \leq \min \left\{ \frac{\#(K_{i,j,t,T^p})}{N - 1}(1 + \beta), \frac{\#(K_{i,j,t,T^p})}{N - 1} \alpha \right\}$$

(7)

and

$$\frac{\beta}{\delta T^p (1 - p)} \leq \min \left\{ \frac{\#(K_{i,j,t,T^p})}{N - 1}(1 + \beta), \frac{\#(K_{i,j,t,T^p})}{N - 1} \alpha \right\}.$$  

(8)

for all $j$ in the ingroup, and when $T^p > \left\lceil \frac{1}{p} - 1 \right\rceil$,
\[
\frac{\alpha - 1}{\delta T^p (1 - p)} \leq \min \left\{ \frac{\#(K_{i,j,t,T^p} \setminus K_{i,j,t,T^p - \frac{1}{k}})}{N - 1}, \frac{\#(K_{i,j,t,T^p} \setminus K_{i,j,t,T^p - \frac{1}{k}})}{N - 1} \alpha \right\}
\]

and

\[
\frac{\beta}{\delta T^p (1 - p)} \leq \min \left\{ \frac{\#(K_{i,j,t,T^p} \setminus K_{i,j,t,T^p - \frac{1}{k}})}{N - 1}, \frac{\#(K_{i,j,t,T^p} \setminus K_{i,j,t,T^p - \frac{1}{k}})}{N - 1} \alpha \right\}
\]

for all \( j \) in the ingroup, and, for all values of \( T^p \) and \( p \),

\[
\alpha - 1 \geq \max \left\{ \sum_{l=1}^{T^p} \delta^l \frac{\#K_{i,j,t,l}}{N - 1} (1 + \beta), \sum_{l=1}^{T^p} \delta^l \frac{\#K_{i,j,t,l}}{N - 1} \alpha \right\}
\]

for \( j \) in the out group.

The proof, as well as a discussion of beliefs that extend the behavior to a sequential equilibrium, can be found in the appendix.\(^\text{12}\)

When there exists a sequential equilibrium according to \( \sigma^{NWOUT} \), there exists an ordering of players within each group determined by their network position and a cut such that all players below the cut will be deviants in equilibrium, and all players above the cut will be compliers in equilibrium.

**Lemma 2 (Separating Cheaters from Compliers).** If a non-degenerate equilibrium sustained by \( \sigma^{NWOUT} \) exists, then \( \exists \) an ordering of players \( i \in A \) according to a function \( f(K_{i,j,t,l}) \) and a cut \( K^{cut} \) such that all \( i \) with \( f(K_{i,j,t,l}) < K^{cut} \) play the cheaters strategy and all \( i \) with \( f(K_{i,j,t,l}) \geq K^{cut} \) play the compliers strategy.

The proof can be found in the appendix.

\(^\text{12}\)It is clear that the conditions in Proposition 1 and Proposition 3 are mutually exclusive, so for a set of parameters \( \alpha, \beta, \delta, g, r \) and \( T^p \), there cannot be both a fully cooperative and a partially cooperative equilibrium. If we take \( \alpha, \beta, g \) and \( r \) to be fixed by the group’s environment and \( T^p \) to be manipulable by the group, for some sets of parameters, a group for which there exists a partially cooperative Out-Group Cheating equilibrium could raise \( T^p \) and secure a fully cooperative In-Group Policing equilibrium instead. However, if \( \delta \) is too small, then raising \( T^p \) would not successfully secure an In-Group Policing equilibrium; the partially cooperative Out-Group Cheating equilibrium would be the best a group could do even if they were willing to increase punishments.
5 Persistent Cheating on the Real Frontier

The American frontier in the 19th century is a uniquely well-documented weak state scenario. It has become fashionable to correct the gun-slinging, bank-robbing image of the ‘Wild West’ by finding metrics on which the west rates as “a far more civilized, more peaceful and safer place than American society is today,” (Hollon and Crowe, 1974, p.x). Regardless of how the violence statistics actually stack up, formal governing institutions were largely absent and weak when present. In fact, revised histories of the frontier usually start with the observation that the West was surprisingly cooperative and peaceful despite the poor formal governing institutions (see, for example, Prassel, 1972; Anderson and Hill, 2004). Boomtowns were especially prone to ineffective governing institutions, in no small part because the growth of the towns outpaced formal governance (and sometimes the construction of a jail).\textsuperscript{13}

Aurora was a classic boomtown, situated in modern-day Nevada in the Sierra Nevada and growing from a few prospectors striking luck in the mines to thousands of residents in the span of a year (McGrath, 1987, p. 9). In the height of the boom, with 5,000 residents, “Aurora was bursting at the seams. Every hotel, lodging house, and miner’s cabin was jam-packed, and hundreds of people went without accommodations” (p.9). The miners quickly outstripped supplies, and nearby cattle ranchers capitalized on the new market by driving cattle in and setting up ranches in the hills around the town (p.17). The settlers occupied a very dense town center which filled first, and a sparser set of ranches in the hills populated by newcomers.

Of course, none of the land was actually previously unused, as the native Paiute used the land for foraging and growing roots. To live a peaceful existence, settlers then needed to cooperate both with fellow settlers and with the neighboring Paiute. As the settler population grew larger, relations between Aurora settlers and the Paiute grew tense. After some initial conflict between the white settlers and the Paiute, both parties met and drew up an informal treaty to promote inter-group peace (p.20).

If the framework presented here applies, we should see white settlers trying to keep other white settlers from violating the treaty, and likewise for the Paiute. We should also expect that if cheaters (treaty violators, troublemakers) exist, they should be the more peripheral group members, and that if some shock occurs which uniformly increases incentives to cheat, the most

\textsuperscript{13}Halaas (1981) notes that “in those mining districts where legally constituted law enforcement agencies were either ineffective or nonexistent, editors [of newspapers] encouraged the law-abiding population to use extralegal means of quieting chronic lawbreakers and violators of the public peace” (p.85).
peripheral group members should be the first to cheat.

Records are more detailed for the white settlers than for the Paiute, but what we see is consistent with the predictions. It does seem to be the case that in-group members tried to police their own among the settlers (p. 49). Despite the scarcity of evidence from within the Paiute, there are even records of Paiute counseling their own against making trouble with the settlers (p. 23).

The winter following the treaty, of 1862-63, was especially severe and created much greater scarcity of the resources supposed to be shared by the two groups (p. 20). Such a shock increased the incentives of everyone to steal from others in violation of the treaty. Those for whom the threat was previously just enough should be the first to find cheating profitable after the shock. Settlers more peripheral to (less tied into) the community’s network are the most likely candidates, according to the model.

While the exact network is unknown for either group, we can use the same argument about settlement patterns and centrality as above to try to infer the extent to which settlers were more or less peripheral to their group, and the extent to which the Paiute were more or less central to their group. The first documented instance of inter-group violence in violation of the treaty was by a Paiute known as “Joaquin Jim.” “Joaquin Jim” entered the record by name because he was known for being an outcast among his people (p. 21). ‘Outcast’ conveys rather clearly that he held a peripheral position within his group.

Accounts of cheating by the white settlers also suggest that it was the more peripheral who felt compelled to cheat first. The first settlers to violate the inter-group treaty were the more isolated ranchers living outside of the main core of town instead of the more densely packed prospectors in town (p. 18). Consistent with the framework presented here, those most isolated and peripheral were the more difficult to convince to respect the treaty and the most likely to commit inter-group cheating.

5.1 Racism as a Heuristic

To enforce cooperation in a fully efficient way, groups need to threaten each person with his minimally sufficient length of punishment. This requires knowing each person’s precise network position and will result in more peripheral people receiving longer punishment. Even scrap-

\[\text{\textsuperscript{14}}\] Also of note is that violence persisted as a string of isolated incidents through the fall and winter of 1863. Small-scale conflict can be durable, even if not permanent. Joaquin Jim acquired a band of followers who “remained at large and continued to prey on the unsuspecting traveler or prospector” (p. 40).
ping full efficiency, full cooperation at least requires knowing the minimally sufficient length of punishment of the most peripheral group member, which requires knowing his precise network position. The model takes for granted that everyone knows enough information to determine precise network positions for everyone.

In practice, we can imagine factors that would make knowing the whole network precisely more or less difficult. Clearly, the larger the group, the taxing it is to know, memorize, and calculate network position. Similarly, when group membership changes, it may be difficult to learn the position of new members. If someone cannot be sure how the new members connect to existing members, they cannot be certain of the existing members’ positions either (because they may or may not have ties to new members, which indirectly connect them to others, and so on.)

In the case of boomtowns, populations became large, and change in group membership was rapid. If too many newcomers were being added by the day to determine exactly who is connected to whom, how could settlers account for network position?

One work-around would be to stop trying to calculate any network position, and instead pick a punishment length so long that it would be likely to entice even the most peripheral newest settlers. Of course, this work-around would not be fully efficient, as it would threaten overkill punishment to the core, central group of settlers. In a highly error-prone environment like a dense, chaotic mining town, costs off-the-equilibrium-path could become real all too easily.

A workaround that would reduce the extent to which punishment is overkill would be to assign different punishment lengths to groups of settlers. If every settler’s network position could be exactly known, then the exact amount of needed punishment could be calculated. If the exact network position is imprecisely known or if calculating every settler’s position is too taxing, a coarsening could be applied. Two punishment lengths could be chosen: a short one that applies to the early-arriving settlers in the center, and a long one that applies to the rest. If the rest are more peripheral than the early-arriving settlers, then this discriminating punishment regime improves efficiency relative to a single, long punishment length for everyone.

The trick would be figuring out who is eligible for the short punishment and who is eligible for the long punishment, and ensuring that every settler in the community knows which punishment length applies to every other settler in the community. If the community is small enough, everyone may know when everyone else moved to town, so the timing of arrival, or location of homes could suffice. For large or rapidly growing populations, though, these features may not
perfectly distinguish everyone to everyone else. Differentiating punishment based on observable features would be an obvious solution.

If an observable characteristic of settlers correlated sufficiently well with network position, the observable feature would be a simple, if imprecise, way to distinguish which punishment length should apply. If most newcomers who occupy peripheral network positions share an observable characteristic, then assigning punishment based on that characteristic would be a simpler way to enforce cooperation than recalculating network position for each of those settlers.\textsuperscript{15}

In a mining town near to Aurora called Brody, a bit more information is available about the internal composition of the miners. Brody, also experiencing a speculative mining boom, attracted a wave of Chinese and Mexican settlers in addition to the early white European settlers. According to the model, settlers could have calculated the exact amount of punishment each new settler would require to be enticed to be cooperative. As it happened, Chinese settlers and Mexican settlers joined the existing settlers in different ways. The Chinese carved out a separate part of town at the far northern edge, and few learned English or desired to assimilate (McGrath, 1987, p. 124, 140). A reasonable conclusion would be that many of the Chinese settlers were peripheral to the town’s communication network. Conversely, the Mexican settlers dispersed throughout the town (McGrath, 1987, p.140). A reasonable conclusion would be that in general, the Mexican settlers were less peripheral to the town’s communication network.

Were the settlers to use a shortcut to distinguish treatment of offenders, race would have been a natural candidate. Moreover, since, in general, Chinese settlers occupied the more dangerous positions in the network, the Chinese would draw harsher treatment relative to the Mexican settlers. Indeed, the Chinese appear to have drawn much harsher treatment in Brody. Anti-Chinese Racism was rampant in mining towns. Brody even exhibited pressure to expel the Chinese. Such an exclusionary view was not uncommon in 1870s California, and we may be tempted to attribute it strictly to the wave of racism sweeping the frontier. On the other hand, such a view may have been at least reinforced by the perception, however unjust, that the Chinese miners must be subject to harsher punishment to keep peace. Putting the issue harshly, the \textit{Daily Free Press} printed in February 1880: “We reflect the sentiment of a large majority of the citizens of this coast when we say that we have no desire to see the Chinese ill-used or badly-treated in any way, but they are a curse to the people of the coast, and we do not want

\hspace{1cm}\textsuperscript{15}Calculating network position for each person separately would allow efficiency gains from having perfectly tailored punishments. However, even if exact network positions could be calculated for everyone, the mental costs of calculating, recalculating, and remembering these may outweighed the efficiency gains.
them here. They do not and cannot assimilate with Americans...” (McGrath, 1987, p.137).

While the historical record is not rich enough to offer a direct comparison of the placement of Chinese settlers and of Mexican settlers in the town’s communication network, it is a reasonable guess that the more dispersed Mexican settlers had more interaction with the other non-Mexican settlers. This would result in a less peripheral position in the communication network, and would mean Mexican settlers would not need especially long punishment for community enforcement to be persuasive to them. Consequently, there would have been less pressure to carve out separate punishment or attempt expulsion simply based on being Mexican. Indeed, the same level of venom towards and fear of Chinese settlers cannot be found in the historical record toward the Mexican settlers. Norton (1913, p. 286) summarizes (in the language of 1913): “[..] the full force of this anti-foreign persecution fell upon the unfortunate Chinaman. From the beginning, though well received, the Chinese had been a race apart. Their peculiar dress and pigtail marked them off from the rest of the population. Their camps at the mines were always apart from the main camps of white miners. This made it easier to turn upon them this hatred of outsiders.”

Attempts to expel or more harshly punish Chinese settlers can be understood as a coarse attempt to differentiate punishment length based on placement in the communication network.\(^6\) The peripheral and perhaps even insular nature of Chinese settlers made discriminating based on being Chinese a more effective heuristic than discriminating based on being Mexican.

6 Discussion

In weak states, the enforcement of cooperation is only as good as the information about who is and is not behaving cooperatively. Without a government or centralized authority who can reliably punish wrongdoing, the community members can undertake this role themselves if information about offenses becomes widely known. The reach of information depends on the communication network among community members.

Interactions with outsiders are especially difficult to police. If an outsider does something wrong, it can be nearly impossible to recognize the person or track down the right offender to issue punishment. If a community member does something wrong to an outsider, the community

\(^6\)Of course, a settler at the time may not have explained his attitude toward the Chinese as a “coarse attempt to differentiate punishment length based on placement in the communication network.” He might have understood his behavior as trying to contain the problem of dangerous people, and he may have arrived at this understanding because he was too lazy/preoccupied/unsophisticated to see that the correlation between race and risk was imperfect.
can threaten punishment to their own member, though since half of the participants to the offense are outsiders, weakly fewer insiders find out about the offense. All of this means that there are stronger incentives to act uncooperatively toward an outsider.

The net gains from acting uncooperatively toward an outsider vary based on a person’s position within his community’s communication network. The more peripheral a person is, the greater the incentive that person has to cheat and be uncooperative toward everyone, but especially outsiders. Some can be so peripheral that the group is unable to threaten enough punishment to make them cooperate. In some cases, those with the greatest incentive to cheat do so regularly, accepting their punishment and continuing to cheat. Note that this result demonstrates cheating in equilibrium. The cheaters did not make a mistake, nor do they cause the entire system to come crashing down.

Were a communication network designed by a social planner and imposed on a community, it could be made to spread news widely about everyone, boosting the chances that a group could enforce full cooperation. Of course, communication networks arise naturally and are subject to myriad constraints. Two people could live according to fundamentally different hours, never see each other, and so never communicate. Two people may belong to different clans and intentionally avoid communication with each other. Two people may live on different sides of a river which makes visiting to share news too arduous. Natural constraints on a communication network arise from many different sources.

Here I have considered a major constraint on communication in rapidly growing mining communities on the American frontier. Starting from almost no infrastructure, the best places to live are in a concentrated hub which, after some quick development, becomes a main strip. Once space along the strip is exhausted, settlement stretches out away from the concentrated core. These settlement patterns impact the settlers’ rhythm of life, including their set of contacts with whom they share news. If this settlement pattern results in the newest arrivals to the community being more peripheral in the community’s communication network, then the newcomers pose the biggest risk to inter-group cooperation.

The result that those most peripheral in the communication network pose the biggest risk to inter-group cooperation is general. Whether or not the most peripheral are in fact the newcomers depends on how exactly newcomers relate to the rest of the group. We could imagine exceptions—maybe a very gregarious newcomer goes out of his way to visit the public establishments on the main strip as often as possible despite not living nearby, forging communication ties with many
others in the community.

In fact, the danger that peripheral members pose to civil inter-group relations suggests that groups may adopt strategies for dealing with newcomers. If groups could fold the newcomers into the existing communication network thoroughly, they could ensure that newcomers are not peripheral and keep tabs on the would-be cheaters. Directly manipulating a communication network is not straightforward, though. The group could create opportunities for the newcomers to meet and form channels of communication with many others, and hope the opportunities create links in the communication network.

Groups face an alternate strategy as well. If connecting the newcomer to the rest of the network is infeasible or too costly, groups could instead restrict entry to the group. By creating barriers to entry, a group can avoid the problems of peripheral members—those at risk of being peripheral are simply not admitted.

This raises an important question: how stable is the persistent cheating equilibrium? If groups have a choice over the parameters of interest or can manipulate the network, then shouldn’t we expect to see full cooperation, at least over time? One interpretation of the partially cooperative equilibrium with persistent cheating alluded to above is that it can take hold in the presence of a shock, and can persist at least until sticky parameters can adjust.

Some have viewed population change as one such version of a shock that can disrupt cooperation. In the example above, it could be that settlers coordinate a punishment length that is long enough for the existing group, but then new settlers move in and are too peripheral to be motivated by the existing punishment regime. The group could re-coordinate, though that may be a long process.\footnote{Population change may be a more general source of peripheral members in networks. For instance, Freudenburg (1986) compares Colorado towns which have a relatively stable population to one which experiences a dramatic population boom. The rapidly growing town featured both a sparser density of acquaintances and more crime.}

While population change can generate a shock to the network, so could any number of environmental or external shocks that may block or remove links from a network. We also could imagine shocks to other parameters of the model that could result in a partially cooperative equilibrium. Damage to communications technology can slow the spread of news (reduce $r$), or expectations of future change can alter patience (reduce $\delta$). All of these parameters are mutable over time, but surely not that rapidly. Even if there is evolutionary pressure or social planners that favor full cooperation, shocks can occur and last long enough that new equilibria are coordinated and persist until the underlying parameters adjust.
7 Conclusion

Informal governance can keep neighboring groups cooperating with each other and internally. When the groups rely on news spread through word-of-mouth networks to learn about wrongdoing, cooperation can still be feasible, but just how feasible depends on the communication networks.

Real-world communication networks are messy. Some people are very tapped in, learning a lot and sharing a lot. Some people are more peripheral, learning a little and sharing a little. This heterogeneity is important to account for, because it renders some people much more difficult to entice to be cooperative. Successfully enforcing cooperation requires convincing these people to be cooperative. Ignoring networks, or assuming away their impact by treating everyone as perfectly informed, conceals a fundamental challenge of informal institutions: some people are better able to get away with misdeeds than others.

Building networks into a study of the informal enforcement of cooperation uncovers another difficulty in using community enforcement: fully efficient punishment regimes cannot, except in knife-edge cases, be egalitarian, and vice versa. Just how inegalitarian the fully efficient punishment regimes are, and how inefficient the egalitarian punishment regimes are, depends on the network. Attempts to close this gap by offering partially inefficient, partially inegalitarian punishment in the least analytically taxing way may result in overgeneralizations as heuristics. Racism in the mining towns on the American western frontier may have been, at least in part, a shortcut in the design of punishment schemes.
Appendix

Proof of Proposition 3

The proof of Proposition 3 is first because it will allow the proof of Proposition 1 as a special case.

Proof. Without loss of generality, consider group $A$. With respect to $i \in A$, call all $j \neq i \in A$ “in-group members” and all $j \in B$ “out-group members.” Let $C_t$ be the set of players in $A$ who would incur no punishment according to $\sigma^{NWOUT}$ in $t$ if the network were complete, and $\overline{C}_t$ be the set of players in $A$ who would incur punishment in $t$ according to $\sigma^{NWOUT}$ if the network were complete. Hence $C_t$ are the cooperators in $t$, $\overline{C}_t$ are the defectors in $t$, $C_t \cup \overline{C}_t = A$ and $\#(C_t \cup \overline{C}_t) = N$.

If network $g_A$ is incomplete, a player $i \in A$ may be unaware of at least one defection for at least some of the time. Let $C_{i,t}$ be the set of players that $i$ does not know to be defectors at time $t$, and $\overline{C}_{i,t}$ be the set of players that $i$ knows to be defectors at time $t$. Clearly $\overline{C}_{i,t} \leq C_t$ and once again $C_{i,t} \cup \overline{C}_{i,t} = N$.

Let $K_{i,j,t,l}$ be the set of players in $i$’s group that know about his round with $j$ that occurred in period $t$ by period $t+l$. $K_{i,j,t,l} = N_{i,t}^l \cup N_{j,t}^l \cup j$ when $i,j \in A$ or $i,j \in B$ and $K_{i,j,t,l} = N_{i,t}^l$ when $i \in A, j \in B$ or when $i \in B, j \in A$. Let $\overline{K}_{i,j,t,l}$ be the set of players in $i$’s group that do not know about his round with $j$ that occurred in $t$ by period $t+l$. For $i \in A$, $K_{i,j,t,l} \cup \overline{K}_{i,j,t,l} = A$ for any $j$.

If player $k$ does not know about any of player $i$’s defections, then we can say that to player $k$, $i$ is an unknown defector (which is to say that $k$ treats $i$ as if he were a cooperator).

In the strategy profile $\sigma^{NWOUT}$, call players who play NW TFT “compliers” and those who play NW OUT “cheaters.” There are 8 ways for compliers to defect and 8 ways for cheaters to defect. A complier $i$ can defect while he is a cooperator by playing $D$ against a cooperator/unknown defector (c1), playing $C$ against a known defector (c2), or playing $D$ against an out group member (c3). A complier $i$ can defect when he is a defector by playing $D$ against a cooperator/unknown defector when $i$ is unknown (d1), playing $D$ against a cooperator/unknown defector when $i$ is known (d2), playing $C$ against a defector when $i$ is unknown (d3), playing $C$ against a defector when $i$ is known (d4), or playing $D$ against the out group (d5). A cheater shares six of the complier’s eight ways to defect: c1, c2, d1, d2, d3, d4. In addition, a
cheater $i$ can defect while he is a cooperator by playing $C$ against the out group (c4) and while he is a defector by also playing $C$ against the out group (d6).

To establish sequential rationality, I will show that for any history and at any information set, a player prefers to comply with $\sigma_N^{W,OUT}$ given the above conditions.

**Incentives for Compliers**

First consider a complier $i$ who has defected most recently in $t-t^d$ and is contemplating defecting against in-group opponent $j$ whom $i$ does not know to be a defector in period $t$. In other words, $i$ is considering defecting via $d_1$. A defection in $t$ would result in punishment until $t+T^p$. He, being a defector, already expected punishment until $t+T^p-t^d$. Cooperating in $t$ is preferred iff

$$\alpha - 1 \leq \sum_{l=T^p-t^d+1}^{T^p-1} \delta^l (1-p) \left[ \frac{\#(C_{i,t+l} \cap K_{i,j,t,l})}{N-1} (1+\beta) + \frac{\#(\overline{C}_{i,t+l} \cap K_{i,j,t,l})}{N-1} \alpha \right]$$

$$+ \delta^{T^p} (1-p) \left[ \frac{\#(C_{i,t+T^p} \cap K_{i,j,t,T^p})}{N-1} (1+\beta) + \frac{\#(\overline{C}_{i,t+T^p} \cap K_{i,j,t,T^p})}{N-1} \alpha \right]$$

for $j$ in the ingroup where $C_{i,t+l}$ is the set of cooperators that $i$ believes he will know about in period $t+l$.\(^{18}\) This condition is hardest to satisfy in the set of histories in which $i$’s defection was in the previous period, i.e. where $t^d = 1$. The condition then becomes

$$\alpha - 1 \leq \delta^{T^p} (1-p) \left[ \frac{\#(C_{i,t+T^p} \cap K_{i,j,t,T^p})}{N-1} (1+\beta) + \frac{\#(\overline{C}_{i,t+T^p} \cap K_{i,j,t,T^p})}{N-1} \alpha \right]$$

for $i$ playing the complier strategy and $j$ in the ingroup, which is implied by the possibly slack sufficient condition:

$$\frac{\alpha - 1}{\delta^{T^p} (1-p)} \leq \min \left\{ \frac{\#K_{i,j,t,T^p}}{N-1} (1+\beta), \frac{\#K_{i,j,t,T^p}}{N-1} \alpha \right\}$$

(12)

for $i$ playing the complier strategy and $j$ in the ingroup.

To safeguard against $c_1$ defections, note that expected punishment from defecting via $c_1$ is the same as the expected punishment from defecting via $d_1$ when $t^d = T^p$, i.e. when the most

\(^{18}\)Strictly speaking, the condition as written holds for $t^d \geq 2$, and should be written without the first sum for $t^d = 1$. 

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recent defection was far enough in the past. The above makes clear that condition 12 is sufficient to prevent defections via c1.

Defections via d2 incur the same expected punishment but gain $\beta$ from playing $D$ rather than $C$ against an opponent’s $D$. The condition is then

$$\beta \leq \delta^{T^p}(1-p) \left[ \frac{#(C_{i,t+T^p} \cap K_{i,j,t+T^p})}{N-1}(1+\beta) + \frac{\#(\overline{C}_{i,t+T^p} \cap K_{i,j,t+T^p})}{N-1} \alpha \right]$$

for $j$ in the ingroup, which is implied by the possibly slack sufficient condition:

$$\frac{\beta}{\delta^{T^p}(1-p)} \leq \min \left\{ \frac{#K_{i,j,t+T^p}}{N-1}(1+\beta), \frac{#K_{i,j,t+T^p}}{N-1} \alpha \right\}. \quad (13)$$

Deviations according to c2, d3 and d4 are trivially not preferred. Deviating according to c2 and d3 would entail foregoing the gains from issuing punishment and in doing so, earning punishment in return. Deviating according to d4 would entail foregoing the chance to not incur a loss during punishment and earning punishment as a consequence.

Compliers deviate according to d5 when they play $D$ against the outgroup. Their expected punishment is the same as in d1 with the sole exception that the opponent was a member of the outgroup.\(^\text{19}\) The condition for compliers is then

$$\alpha - 1 \leq \delta^{T^p}(1-p) \left[ \frac{#(C_{i,t+T^p} \cap K_{i,j,t+T^p})}{N-1}(1+\beta) + \frac{\#(\overline{C}_{i,t+T^p} \cap K_{i,j,t+T^p})}{N-1} \alpha \right]$$

for $j$ in the outgroup, which is implied by the possibly slack sufficient condition:

$$\frac{\alpha - 1}{\delta^{T^p}(1-p)} \leq \min \left\{ \frac{#K_{i,j,t+T^p}}{N-1}(1+\beta), \frac{#K_{i,j,t+T^p}}{N-1} \alpha \right\}. \quad (14)$$

for $j$ in the outgroup.

By the same reasoning as above, satisfying this condition ensures that compliers also prefer not to deviate according to c3.

Hence if conditions 12, 13 and 14 are satisfied, no complier (player playing $NW\overline{TFT}$) has an incentive to deviate in any history given any beliefs.

\(^{19}\)The consequence is that the set of players who know, $#K$, will be smaller.
The incentives for cheaters

Consider a cheater assigned to play an in-group member in $t$ who is contemplating defecting according to $d_1$. As with condition 12, the binding case for the condition is a player who just defected in $t - 1$ and so the condition features only period $t + T_p$. However, while compliers expected facing no punishment in $t + T_p$ as their outside option, cheaters may expect non-zero punishment as their outside option in $t + T_p$ if they expect to play an outgroup member by then. The condition then depends on the net punishment cheating in $t$ yields, and the net punishment will be weakly higher the smaller $p$ is. When $T_p \leq \lceil \frac{1}{p} - 1 \rceil$ the condition becomes

$$\alpha - 1 \leq \delta^{T_p} (1 - p) \left[ \frac{(C_{i,t+T_p} \cap K_{i,j,t,T_p})}{N - 1} (1 + \beta) + \frac{(\overline{C}_{i,t+T_p} \cap K_{i,j,t,T_p})}{N - 1} \alpha \right]$$

for $i$ playing the cheater strategy and $j$ in the ingroup, and when $T_p > \lceil \frac{1}{p} - 1 \rceil$,

$$\alpha - 1 \leq \delta^{T_p} (1 - p) \left[ \frac{(C_{i,t+T_p} \cap (K_{i,j,t,T_p} \setminus \lceil \frac{1}{p} \rceil))}{N - 1} (1 + \beta) + \frac{(\overline{C}_{i,t+T_p} \cap (K_{i,j,t,T_p} \setminus \lceil \frac{1}{p} \rceil))}{N - 1} \alpha \right]$$

for $i$ playing the cheater strategy and $j$ in the ingroup. Simplifying to sufficient conditions by the same reasoning as above, we obtain, for $T_p \leq \lceil \frac{1}{p} - 1 \rceil$,

$$\frac{\alpha - 1}{\delta^{T_p} (1 - p)} \leq \min \left\{ \frac{(K_{i,j,t,T_p})}{N - 1} (1 + \beta), \frac{(K_{i,j,t,T_p})}{N - 1} \alpha \right\}$$

(15)

for $i$ playing the cheater strategy and $j$ in the ingroup, and when $T_p > \lceil \frac{1}{p} - 1 \rceil$,

$$\frac{\alpha - 1}{\delta^{T_p} (1 - p)} \leq \min \left\{ \frac{(K_{i,j,t,T_p} \setminus \lceil \frac{1}{p} \rceil)}{N - 1} (1 + \beta), \frac{(K_{i,j,t,T_p} \setminus \lceil \frac{1}{p} \rceil)}{N - 1} \alpha \right\}$$

(16)

for $i$ playing the cheater strategy and $j$ in the ingroup. Likewise, sufficient conditions to ensure that a cheater prefers to not defect via $d_2$ are:

$$\frac{\beta}{\delta^{T_p} (1 - p)} \leq \min \left\{ \frac{(K_{i,j,t,T_p})}{N - 1} (1 + \beta), \frac{(K_{i,j,t,T_p})}{N - 1} \alpha \right\}$$

(17)
for $i$ playing the cheater strategy and $j$ in the ingroup, and when $T^p > \left[ \frac{1}{p} - 1, \right.$

$$\frac{\beta}{\delta T^p (1 - p)} \leq \min \left\{ \frac{\#(K_{i,j,t,T^p \setminus K_{i,j,t,T^p - \left[ \frac{1}{p} \right]}})}{N - 1} (1 + \beta), \frac{\#(K_{i,j,t,T^p \setminus K_{i,j,t,T^p - \left[ \frac{1}{p} \right]}})}{N - 1} \alpha \right\}$$

(18)

for $i$ playing the cheater strategy and $j$ in the ingroup.

Again, the sufficient condition preventing defections via $d_1$ imply the sufficient conditions preventing defections via $c_1$, and defections via $c_2$, $d_3$ and $d_4$ are trivially not preferred. This leaves ensuring that cheaters prefer to cheat against the outgroup, i.e. that they do not want to deviate via $c_4$ or $d_6$.

Cheaters deviate according to $d_6$ when they play $C$ against the outgroup. Deviating avoids expected punishment. The amount of expected punishment avoided depends on how long it will be before the cheater has another opportunity to comply with his strategy and play $D$ against the outgroup, restarting punishment. These opportunities arise independently each period with probability $p$, which means in $t$, the expected number of rounds before the next opportunity is $\frac{1}{p}$. When opportunities arise sufficiently infrequently, that is when $T^p \leq \left[ \frac{1}{p} - 1, \right.$

$$\alpha - 1 \geq \sum_{l=1}^{T^p} \delta^l \left[ \frac{\#(C_{i,t}^c \cap K_{i,j,t,t})}{N - 1} (1 + \beta) + \frac{\#(C_{i,t}^c \cap K_{i,j,t,t})}{N - 1} \alpha \right]$$

for $j$ in the outgroup. When $T^p > \left[ \frac{1}{p} - 1, \right.$

it will suffice to note that condition 19 is sufficient for the case where $T^p > \left[ \frac{1}{p} - 1 \right.$ and is more slack the larger is $p$. The condition also implies the condition for $d_6$.

For all terms in the sum in condition 19 where $l < T^p$, the value of the term depends on beliefs as yet unspecified. However, it is possible to construct an upper bound for the right hand side of condition 19 that is independent of beliefs to produce a sufficient condition (i.e. a condition that accounts for worst case scenario beliefs). The right hand side is maximized when either full weight is placed on either the $1 + \beta$ or the $\alpha$ term, depending on whether $1 + \beta$ or $\alpha$ is
larger. Full weight is placed on the $1 + \beta$ term when $K_{i,j,t,l} \subset C_{i,t}^*$. Likewise, when $\alpha \geq 1 + \beta$, the right hand side is maximized when $\overline{C}_{i,t}^* \subset K_{i,j,t,l}$. This establishes the following sufficient condition for cheaters to prefer to cheat rather than deviate from their strategy (i.e. for $c_4$ and $d_6$):

$$\alpha - 1 \geq \max \left\{ \sum_{l=1}^{T_p} \delta_t^l \frac{\#K_{i,j,t,l}}{N-1} (1 + \beta), \sum_{l=1}^{T_p} \delta_t^l \frac{\#K_{i,j,t,l}}{N-1} \alpha \right\}$$  \hspace{1cm} (19)

for $j$ in the outgroup.

Hence so long as conditions 12 and 13 are satisfied for compliers playing in-group members, 14 is satisfied for compliers playing out-group members, conditions 15, 16, 17 and 18 are satisfied for cheaters playing in-group members, and 19 is satisfied for cheaters playing out-group members, no player has an incentive to deviate in any history given any beliefs and so $\sigma^{NW\text{OUT}}$ is sequentially rational. Since sequential rationality is independent of beliefs, any beliefs trivially extend the behavior to sequential equilibrium. \(\Box\)

**Proof of Proposition 1**

If everyone plays according to $NWTF$, then in the thought experiment in which players play according to their strategy in $t$, by period $t + T_p$, all players will be cooperators. That is, $C_{t+T_p} = N$. This means that $C_{i,t+T_p} \cap K_{i,j,t,T_p} = K_{i,j,t,T_p}$ and $\overline{C}_{i,t+T_p} \cap K_{i,j,t,T_p} = \emptyset$. It follows from the proof of the sequential rationality of $\sigma^{NW\text{OUT}}$ that necessary and sufficient conditions for all players playing $NWTF$ (i.e. for players all playing the complier strategy of $\sigma^{NW\text{OUT}}$) to be sequentially rational are, given $r$, $p$, $T_p$, $g_A$ and $g_B$, for all $i \in A \cup B$:

$$\frac{\alpha - 1}{\delta_{T_p}(1-p)} \leq \frac{\#K_{i,j,t,T_p}}{N-1} (1 + \beta)$$  \hspace{1cm} (20)

for $j$ in the outgroup, and

$$\frac{\beta}{\delta_{T_p}(1-p)} \leq \frac{\#K_{i,j,t,T_p}}{N-1} (1 + \beta).$$  \hspace{1cm} (21)

for all $i$ in the ingroup.

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\(^{20}\)For instance, if $i$ believes in $t+1$ that everyone is cooperative, this is satisfied.
Proof of Proposition 2

Proof. Let \( T \) be the common punishment length but suppose it is not the case that \( #K_{1,j,t,T} = #K_{2,j,t,T} = \cdots = #K_{n,j,t,T} \) for \( 1 \ldots n \in A \) and \( j \in B \). By definition, a fully-efficient punishment must threaten the least amount of punishment required to satisfy the conditions in Proposition 1. This means each person \( i \) must be threatened exactly his minimum sufficient punishment \( T_i \). A punishment is egalitarian if all players are threatened with the same punishment. Without loss of generality, suppose \( #K_{1,j,t,T} < #K_{2,j,t,T} = \cdots = #K_{n,j,t,T} \). Now \( \min_{j \neq 1 \in \{A,B\}} \{\delta^T #K_{1,j,t,T}\} < \min_{j \neq 2 \in \{A,B\}} \{\delta^T #K_{2,j,t,T}\} \). If \( T \) is minimally sufficient to entice \( 2 \ldots n \) to cooperate, then

\[
\min_{j \neq 2 \in \{A,B\}} \{\delta^T #K_{2,j,t,T}\} = \min \left\{ \frac{(a-1)(N-1)}{(1-p)(1+\beta)}, \frac{\beta(N-1)}{(1-p)(1+\beta)} \right\}, \text{ which implies } \min_{j \neq 1 \in \{A,B\}} \{\delta^T #K_{1,j,t,T}\} < \min \left\{ \frac{(a-1)(N-1)}{(1-p)(1+\beta)}, \frac{\beta(N-1)}{(1-p)(1+\beta)} \right\}, \text{ violating the conditions in Proposition 1 so } T \text{ is too low to keep 1 cooperative. If instead } T \text{ is minimally sufficient to entice 1 to cooperate, then } \min_{j \neq 1 \in \{A,B\}} \{\delta^T #K_{1,j,t,T}\} = \min \left\{ \frac{(a-1)(N-1)}{(1-p)(1+\beta)}, \frac{\beta(N-1)}{(1-p)(1+\beta)} \right\}, \text{ which implies } \min_{j \neq 2 \in \{A,B\}} \{\delta^T #K_{2,j,t,T}\} > \min \left\{ \frac{(a-1)(N-1)}{(1-p)(1+\beta)}, \frac{\beta(N-1)}{(1-p)(1+\beta)} \right\}, \text{ which means } \exists T' < T \text{ that would keep } 2 \ldots n \text{ cooperating. Hence, common } T \text{ does not admit both full efficiency and egalitarianism. Conversely, when } #K_{1,j,t,T} = #K_{2,j,t,T} = \cdots = #K_{n,j,t,T} \text{, if there exists a fully cooperative equilibrium with punishment length } T \text{, it is egalitarian, and if } T \text{ is the shortest punishment length that generates full cooperation, the equilibrium is also fully efficient.}

\[]

Proof of Lemma 2

Proof. Suppose a non-degenerate equilibrium sustained by \( \sigma^{NWOUT} \) exists but there is no such cut \( K^{cut} \) that separates cheaters from compliers. Call the set of players playing the compliers strategy in equilibrium \( COMP \) and the set of players playing the cheaters strategy in equilibrium \( CHEAT \). Let \( f(K_{i,j,t,l}) = \sum_{t=1}^{T_p} \delta^t \#K_{i,j,t,l} \) for \( j \) in the outgroup. Let \( k^{cut} = \min \left\{ \frac{(a-1)(N-1)}{1+\beta}, \frac{(a-1)(N-1)}{\alpha} \right\} \). By condition 19, \( f(K_{i,j,t,l}) \leq K^{cut} \forall i \in CHEAT \). Now by supposition there must exist a complier \( k \) such that

\[
f(K_{k,j,t,T'}) < \sup_{i \in CHEAT} \{f(K_{i,j,t,l})\}. \text{ But that must mean}
\]

\[
f(K_{k,j,t,T'}) < \min \left\{ \frac{(a-1)(N-1)}{1+\beta}, \frac{(a-1)(N-1)}{\alpha} \right\} \text{ which implies } \delta^{T_p} #K_{k,j,t,T'} < \min \left\{ \frac{(a-1)(N-1)}{1+\beta}, \frac{(a-1)(N-1)}{\alpha} \right\} \text{ which implies } (1-p)\delta^{T_p} #K_{k,j,t,T'} < \min \left\{ \frac{(a-1)(N-1)}{1+\beta}, \frac{(a-1)(N-1)}{\alpha} \right\}. \text{ However, by sufficient condition 12, if } k \text{ is a complier, it must be that } (1-p)\delta^{T_p} #K_{k,j,t,T'} \geq \min \left\{ \frac{(a-1)(N-1)}{1+\beta}, \frac{(a-1)(N-1)}{\alpha} \right\},
\]

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a contradiction.
References


