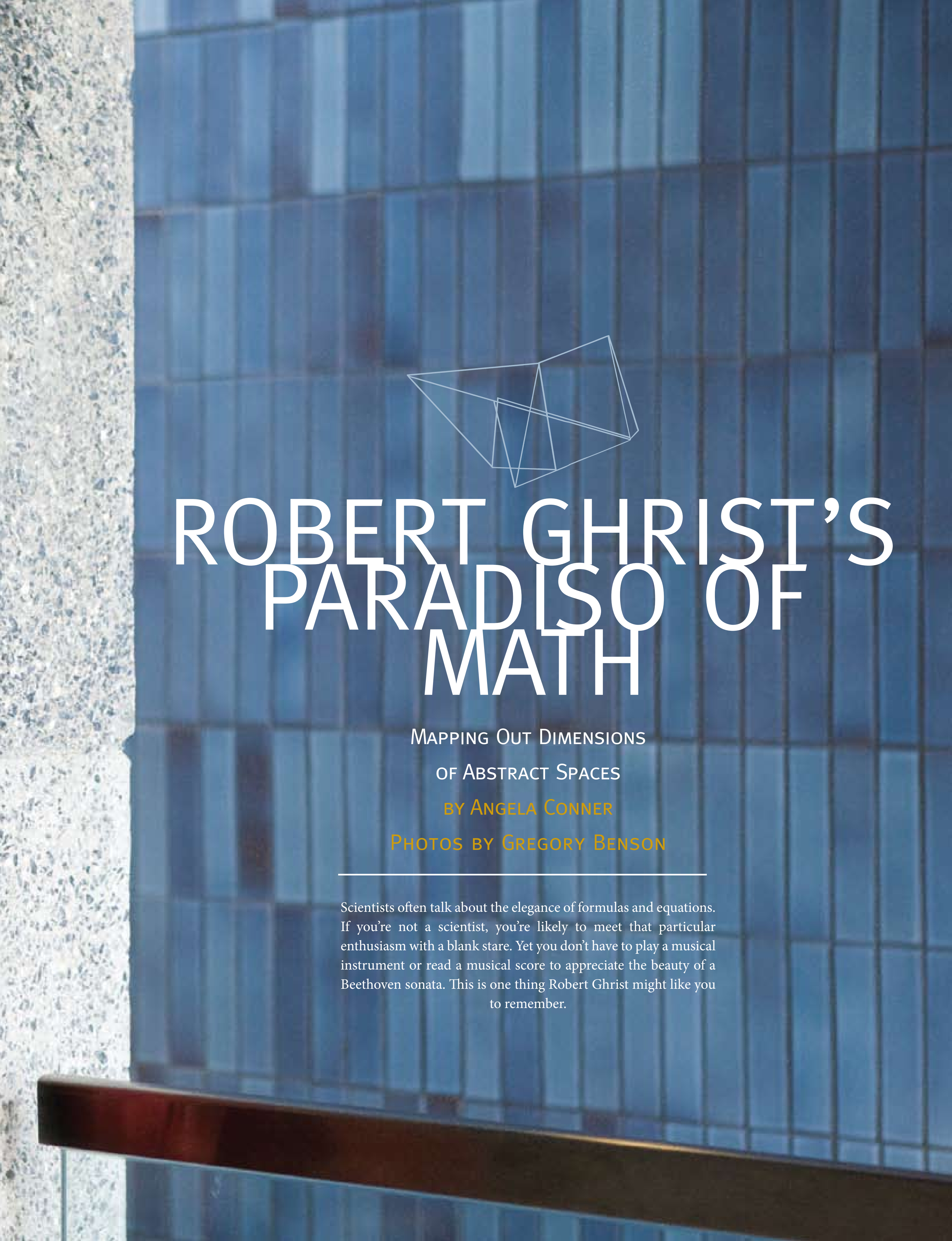


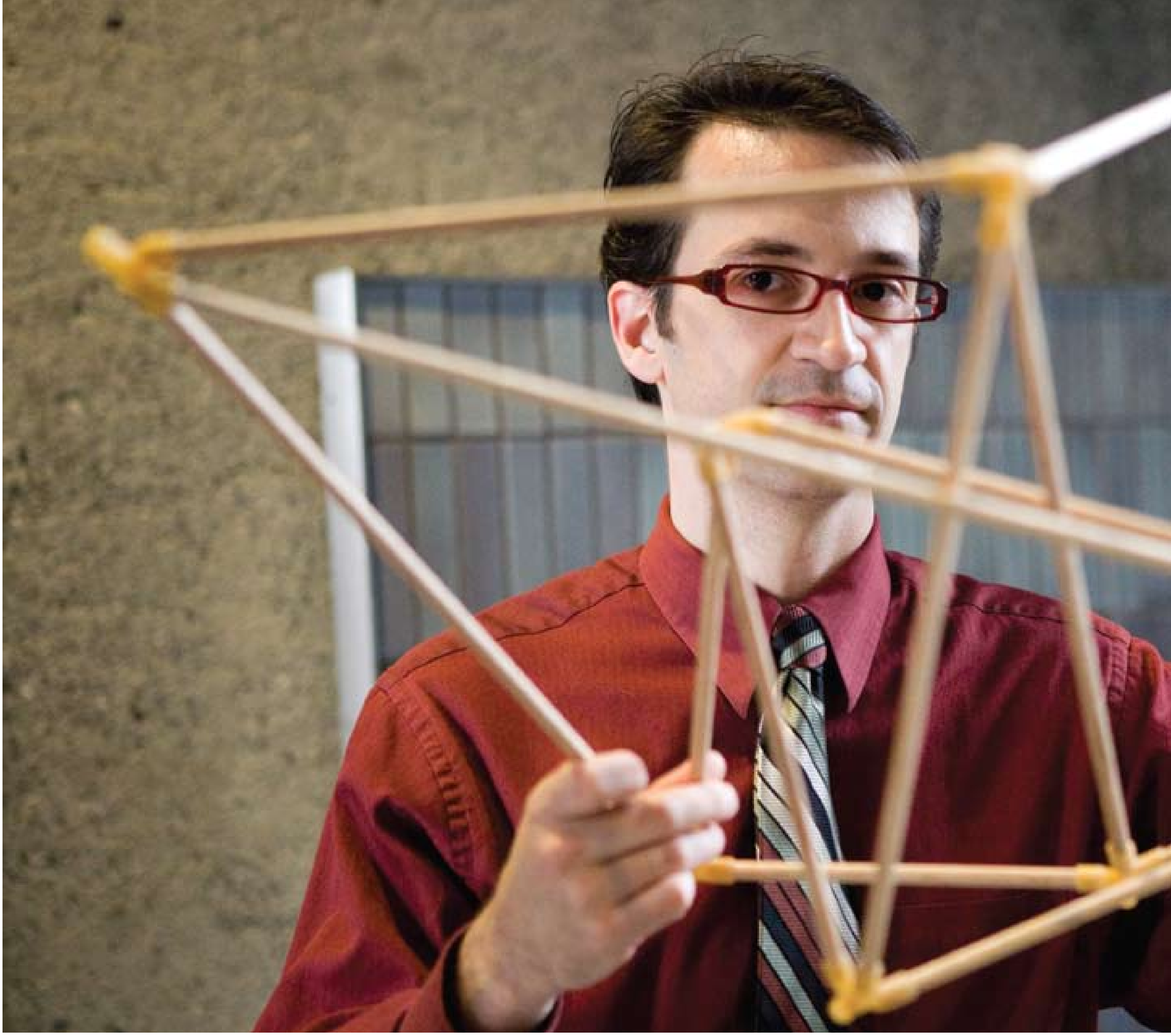
# ROBERT GHRIST'S PARADISO OF MATH

## Mapping Out Dimensions of Abstract Spaces

by Angela Conner

Photos by Gregory Benson

Scientists often talk about the elegance of formulas and equations. If you're not a scientist, you're likely to meet that particular enthusiasm with a blank stare. Yet you don't have to play a musical instrument or read a musical score to appreciate the beauty of a Beethoven sonata. This is one thing Robert Ghrist might like you to remember.



Ghrist joined Penn last summer as the Andrea Mitchell University Professor—the seventh Penn Integrates Knowledge Professor—and holds a joint appointment in the Department of Mathematics in the School of Arts and Sciences and in the School of Engineering and Applied Science. He's also a self-described amateur medievalist and is particularly fond of—some might say inspired by—Dante's *Divine Comedy*.

Let's talk about that for a moment. Most of us read Dante in college, perhaps even in high school, and were no doubt titillated by the *Inferno*'s creatively gory punishments, meted out to fit various sins. If we were so inclined, we may have enjoyed the poetry of the piece, and with a good guide we may have glimpsed the structure of the thing as a whole. But to truly appreciate the cosmology of Dante's world, one has to not only take apart the meticulous scaffolding that holds the whole thing together, but also delve into the complex allegories behind each image, the layering of meanings added together like a set of equations to represent what can't be easily represented.

This is not unlike the work Ghrist undertakes. "A mathematician is an engine for finding patterns," he says, and much of his current work is aimed at mapping out patterns in multi-dimensional abstract spaces that are much more difficult for us to visualize than Dante's concentric spheres. It's not hard to think in three dimensions, but fifteen, say, proves impossible. Not for a mathematician.

Suppose you were a bug living on the surface of a cardboard box. Locally, you would have two dimensions in which to move. But suppose you are an ambitious bug, and you want to know the shape of your cardboard cosmos. Is it a simple six-sided box? Or something more complex, a box with smaller boxes attached and a hole or two to crawl around in. You would have to make long excursions along your "globe," experiencing more dimensions than you're capable of keeping in your head, and return to your starting point to try to make sense of it all.

Increasingly, we are navigating similarly complex spaces. Think about when you surf the Internet. You begin at a specific site and move from site to site, hopping all over cyberspace. What does the shape of that space look like?

Ghrist thinks about these complex spaces, but his work is not just theoretical. As an applied mathematician, he says, "the particular style of what I try to do is take mathematical ideas that haven't found much application in the past and find a use for them." A lot of tools have been developed to study these abstract sorts of objects and Ghrist is finding them relevant to very practical problems in today's world.

Much of his current work involves sensors and their networks. Increasingly, our world is populated with these sensors: as a means to monitor security, to track forest fires or wildlife, to control the movements of robots on a factory floor. It's these moving sensors that Ghrist focuses on. Imagine them as many bugs on that complicated cardboard box. Each little bug gathers his local data while moving about his neighborhood, but how can he and his friends be sure they're covering the entire box? Similarly, think about a group of robots working to assemble a car. The path of a robot's arm as it completes a task maps out an abstract many-dimensioned space. Each sensor in its arm keeps track of local movements, but all the sensors in every robot need to coordinate globally to keep them from crashing into each other.

These are the problems Ghrist is solving. Using topology, a branch of math that grew out of geometry a hundred years ago, Ghrist is taking apart the complex shapes these networks form, "visualizing" with algebra and calculus what we can't see with our eyes, and putting them back together again to form a global picture of an environment. Once the global rendering is complete, engineers can use that information to discern if there are holes in a sensor network and where they are. They can keep robots from colliding.

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Ghrist is evangelical about the utility of mathematics. He believes that the tools that have been around for 100 years to study abstract objects and spaces have a surprising amount of relevance to solving very practical, contemporary problems. Opening a math book full of topology tools called sheaf-theory equations, most of us would groan at the seeming senselessness of what's on the page. Even those with quite a bit of education in engineering or applied science might have the same reaction, since, as Ghrist says, "mathematicians have done an amazing job of building very rarified machinery that's even harder to read than the *Divine Comedy*." But in these books, he finds tools that he can translate into a real-world engineering language. "I'm really optimistic," he says, "when I see how many math books there are that are hard to read because that means there's valuable stuff there that hasn't yet been converted to a language that's more useful."

Some of today's most pressing problems in engineering are being solved because Ghrist enjoys delving into the hard books. "I think if Dante were around today," he says, "he'd be delighted to know all the things that we've discovered about life and about the universe. And it would be delightful to see what kind of structure he would append to his cosmology, given our increased understanding of the world." You can bet Ghrist would like to map out that structure. ◆