Active Courts and Menu Contracts*

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Abstract. We describe and analyze a contractual environment that allows a role for an active court. An active Court can improve on the outcome that the parties would achieve without it. The institutional role of the Court is to maximize the parties' welfare under a veil of ignorance.

This role of active Courts exists even if "menu contracts" are negotiated between the informed buyer and the uninformed seller at an ex-ante stage (before investment are sunk).

We find that if we maintain the assumption that one of the potential objects of trade is not contractible ex-ante, then active Courts can strictly enhance the contracting parties' ex-ante welfare by destroying the unique inefficient pooling equilibrium. If however we let all "widgets" be contractible ex-ante, then multiple equilibria obtain. In this case the role for an active Court is to ensure that inefficient pooling equilibria do not exist alongside the superior ones in which separation occurs.

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1. Introduction

In a recent paper (Anderlini, Felli, and Postlewaite 2011) we showed, by means of a simple example, that Courts that actively intervene in parties' contracts may improve on the outcome these parties could achieve without intervention. In particular if the role of the Court is to maximize the parties' welfare under the veil of ignorance, Court intervention can induce parties to reveal their private information and enhance their ex-ante welfare.

The example in Anderlini, Felli, and Postlewaite (2011) is one in which parties are asymmetrically informed when they write their ex-ante contract. The seller knows the value and cost associated with the widget she provides while the buyer is uninformed. If the seller is restricted to offering a simple trading contract (a price at which to trade the widget in question) without the Court's intervention she will offer the same price whatever the value and the cost of the widget. In other words, different types of seller offer the same trading contract and in equilibrium inefficient pooling arises. Court intervention that takes the form of a restriction on the price at which the parties can trade, induces the different types of seller to separate and reveal their private information. In so doing the inefficiency associated with the sellers' pooling is eliminated and ex-ante welfare increases.

In this paper we consider a different example from the one in Anderlini, Felli, and Postlewaite (2011). We first show that in this richer example if we follow Anderlini, Felli, and Postlewaite (2011) and restrict the contracts the informed party can offer to the uninformed one at the negotiation stage, Court intervention can improve the parties ex-ante welfare exactly as in Anderlini, Felli, and Postlewaite (2011). However, if we remove this restriction then multiple equilibria obtain. In this case an active Court still has an important role. It ensures that the inefficient pooling equilibria do not exist alongside the superior ones in which separation occurs.

In particular, the key type of contracts the informed party would like to offer to the uninformed one is a menu contract. This is a pooling contract across different types of the informed party that immediately becomes binding, and that contains an array of contractual arrangements. Which contractual arrangement applies is then left to an (incentive compatible) declaration by the informed party.

In two separate papers, Maskin and Tirole (1990, 1992) examine the general case of an "Informed Principal" problem. Among other insights, they point out that, under certain conditions a menu contract equilibrium may Pareto improve over other types of arrangements.

In our setup the informed party, the buyer, has private information and, ex-ante, makes a take-it-or-leave-it offer to the seller. Therefore he is an informed Principal. Our model in fact falls within the case of "Common Values" examined in Maskin and Tirole (1992). As in Maskin and Tirole (1992), it is then possible to construct equilibria of the game in which by means of a menu contract the informed parties can both pool when offering a contract to the uninformed party and reveal their private information in an incentive compatible way after the contract is accepted. Courts' intervention can select the equilibria supported by menu contracts that foster separation of the different types of the informed party.

1.1. Outline

The plan of the rest of the paper is as follows. In Section 2 we present the model under the assumption that menu contracts cannot be used. The equilibrium characterization of this model is presented in Section 3. We first consider the case in which Courts enforce everything the parties write and then introduce active Courts. In Section 4 we modify the model so as to allow parties, at the negotiation stage, to offer menu contracts. We then provide the equilibrium characterization of this new model first when Courts are passive enforcers and then when Courts are active. Section 5 briefly concludes. For ease of exposition all proofs have been gathered in the Appendix.[1]

[1] In the numbering of Propositions, Lemmas, equations and so on, a prefix of "A" indicates that the relevant item can be found in the Appendix.

2. The Model

2.1. Passive Courts

A buyer $\mathcal{B}$ and a seller $\mathcal{S}$ face a potentially profitable trade of three widgets, denoted w_1, w_2 and w_3 respectively.

Widgets w_1 and w_2 require a widget- and relationship-specific investment $I > 0$ on $\mathcal{B}$'s part. The buyer can only undertake one of the two widget-specific investments, The value and cost of both w_1 and w_2 are zero in the absence of investment, so that only one of them can possibly be traded profitably.

The cost and value of w_3 do not depend on any investment. To begin with assume that w_3 is *not contractible* at the ex-ante stage. Non-contractibility means that w_3 can be traded regardless of any ex-ante decision. In practice, in this case we can think of w_3 as being traded (or not) at the ex-post stage. When menu contracts are introduced the difference between w_3 being contractible or not at the ex-ante stage will become crucial. In the results presented in this Section it is not.

The buyer has private information at the time of contracting. He knows his type, which can be either α or β. Each type is equally likely, and the seller does *not* know $\mathcal{B}$'s type.

We take the cost and value of the three widgets to be as in the table below, where they are represented *net* of the cost of investment $I > 0$.[2] In each cell of the table, the left entry represents surplus, and the right entry represents cost (obviously the sum of the two gives the value to the buyer, net of investment cost).

	w_1	w_2	w_3
Type α	$\Delta_M,\ \ c_L$	$\Delta_H,\ \ c_L$	$-\Delta_H,\ \ c_H$
Type β	$\Delta_N,\ \ c_L$	$\Delta_L,\ \ c_L$	$\Delta_S,\ \ c_S$

(1)

We take these parameters to satisfy the following.

[2]The *gross* value is therefore computed as the sum of cost, surplus and I, while the *gross* cost is the cost value reported in table (1).

Assumption 1. *<u>Parameter Values</u>: The values of cost and surplus in the matrix in (1) satisfy*

(i) $0 < \Delta_L < \Delta_M < \Delta_H$

and

(ii) $\Delta_M + \Delta_H < \Delta_S$

and

(iii) $c_S + \Delta_H + \Delta_S + \dfrac{\Delta_M}{2} < c_H < \Delta_S + 2\ \Delta_M$

and

(iv) $0 < -\Delta_N < \Delta_H - \Delta_M - \Delta_L$

and

(v) $c_L < c_S$

The costs and values of the three widgets are *observable but not contractible*. We first restrict attention to an environment in which any contract between $\mathcal{B}$ and $\mathcal{S}$ can only specify the widget(s) to be traded, and price(s). Menu contracts are ruled out by assumption. The Court can only observe (verify) which one of w_1 or w_2 is specified in any contract, and whether the correct widget is traded or not as prescribed, and the appropriate price paid.

Assume that $\mathcal{B}$ has all the bargaining power at the ex-ante contracting stage, while $\mathcal{S}$ has all the bargaining power ex-post.

To sum up, the timing and relevant decision variables available to the trading parties are as follows.

The buyer learns his type *before* meeting the seller. Then $\mathcal{B}$ and $\mathcal{S}$ meet at the ex-ante contracting stage. At this point $\mathcal{B}$ makes a take-it-or-leave-it offer of a contract to $\mathcal{S}$, which $\mathcal{S}$ can accept or reject. A contract consists of a pair $s_i = (w_i, p_i)$, with $i = 1, 2$ specifying a single widget to trade and at which price. After a contract (if any) is signed, $\mathcal{B}$ decides whether to invest or not, and in which of the specific widgets.

After investment takes place (if it does), the bargaining power shifts to the seller and we enter the ex-post stage. At this point $\mathcal{S}$ makes a take-it-or-leave-it offer to $\mathcal{B}$ on whether to trade any widget not previously contracted on and at which price, which $\mathcal{B}$ can accept or reject. Without loss of generality, we can restrict $\mathcal{S}$ to make a take-it-or-leave-it offer to $\mathcal{B}$ on whether to trade w_3 and at which price p_3. After $\mathcal{B}$ decides whether to accept or reject $\mathcal{S}$'s ex-post offer (if any), production takes place. First $\mathcal{S}$ produces the relevant widgets and then he learns his cost.[3] Finally, delivery and payment occur according to contract terms. The Court's role is the one of a passive enforcer of the terms of the parties' ex-ante and ex-post contract.

2.2. Active Courts

The information of $\mathcal{B}$, $\mathcal{S}$ and the Court, and their bargaining power remain as described above. The timing, investment requirements and all the elements of the matrix in (1) also stay the same.

The Court announces a set of ex-ante contracts $\mathcal{U}$ which will be "upheld" and a set of ex-ante contracts $\mathcal{V}$ which will be "voided." There are two contracts in all to be considered, one of the type $s_1 = (w_1, p_1)$ and another of the type $s_2 = (w_2, p_2)$. We restrict the Court to be able to announce that certain contracts will be upheld or voided, *only according to the widget involved*. Therefore $\mathcal{U}$ and $\mathcal{V}$ are two mutually exclusive subsets of $\{s_1, s_2\}$ with $\mathcal{U} \cup \mathcal{V} = \{s_1, s_2\}$, so that effectively the Court's strategy set consists of a choice of $\mathcal{V} \subseteq \{s_1, s_2\}$.

We restrict the Court to make deterministic announcements; each contract is either in $\mathcal{V}$ or not with probability one.

If $\mathcal{V} = \emptyset$ so that all contracts are enforced, then the model is exactly as described in Subsection 2.1 above. If on the other hand one or two contracts are in $\mathcal{V}$, in the final stage of the game $\mathcal{B}$ and $\mathcal{S}$ are free to renegotiate the terms (price and delivery) of any widget in the voided contract, regardless of anything that was previously agreed.[4]

[3] The reason to assume that production costs are *sunk* before $\mathcal{S}$ learns what they are is to prevent the possibility of ex-post revelation games a la Moore and Repullo (1988) and Maskin and Tirole (1999).

[4] As well as negotiating the terms of trade for w_3, as before.

Notice that, by our assumptions on bargaining power, this means that $\mathcal{S}$ is free to make a take-it-or-leave-it offer to $\mathcal{B}$ of a price p_i at which any w_i with voided contract terms is to be delivered.[5]

The Court chooses $\mathcal{V}$ so as to maximize its payoff which equals the *sum* of the expected payoffs of $\mathcal{B}$ and $\mathcal{S}$.[6,7]

3. Equilibria with No Menu Contracts

3.1. Passive Court Equilibria

As we anticipated, when all contracts are enforced, inefficient pooling obtains in equilibrium.

Proposition 1. *Equilibrium With A Passive Court: Suppose the Court enforces all contracts, $\mathcal{V} = \emptyset$, and Assumption 1 holds. Then the unique equilibrium outcome of the model is that the two types of buyer pool with probability one: they both invest and trade w_2 at a price $p_2 = c_L$, and they both trade w_3 at a price $p_3 = \Delta_S + c_S$.*

The total amount of expected surplus (net of investment) in this case is given by $\dfrac{\Delta_S}{2} + \dfrac{\Delta_L}{2}$. By definition, this is also the Court's payoff.

The equilibrium outcome in Proposition 1 is inefficient in the sense that, in equilibrium w_3 is traded by the type α buyer; this trade generates a net surplus of $-\Delta_H$.

[5] Implicitly, this means that we are taking the view that "spot" trade is feasible ex-post even when contract terms are voided by the Court.

[6] Clearly, following a particular choice by the Court multiple equilibrium payoffs could ensue in the relevant subgame. When multiple equilibria arise in some relevant subgames, we deem something to be an equilibrium of the entire model when it is an equilibrium considering the Court as an actual player, complete with its equilibrium *beliefs*. For more on the distinction between a classical "planner" and a planner who is also a player see Baliga, Corchon, and Sjöström (1997).

[7] Throughout, by equilibrium we mean a Sequential Equilibrium (Kreps and Wilson 1982), or equivalently a Strong Perfect Bayesian Equilibrium (Fudenberg and Tirole 1991), of the game at hand. We do not make use of any further refinements. However, it should be pointed out that whenever we assert that something is an equilibrium outcome, then it is the outcome of at least one Sequential Equilibrium that passes the Intuitive Criterion test of Cho and Kreps (1987).

The reason separation is impossible to sustain as an equilibrium outcome with passive Courts is easy to see. In any separating equilibrium, it is clear that the type β buyer would trade w_3 ex-post for a price $p_3 = \Delta_S + c_S$. The type β buyer would also trade w_2 for a price $p_2 = c_L$ (this is in fact true in *any* equilibrium in which the Court does not void contracts for w_2). Given that the type β buyer trades both w_2 and w_3, the type α will always gain by deviating and pooling with the the type β buyer.

3.2. Active Court Equilibria

A Court that actively intervenes and voids contracts for w_2 will be able to induce separation between the two types of buyer and increase expected welfare.

Proposition 2. *Equilibrium With An Active Court: Suppose the Court is an active player that can choose $\mathcal{V}$ as above, and that Assumption 1 holds. Then the unique equilibrium outcome of the model is that the Court sets $\mathcal{V} = \{s_2\}$ and the two types of buyer separate: the type α buyer invests and trades w_1 at a price $p_1 = c_L$ and does not trade w_3; the type β buyer does not invest and only trades w_3 at a price $p_3 = \Delta_S + c_S$.*

The total amount of expected surplus (net of investment) in this case is given by $\frac{\Delta_S}{2} + \frac{\Delta_M}{2}$. By definition, this is also the Court's payoff.

When the Court voids contracts for either w_1 or w_2, the corresponding widget will not be traded in equilibrium. This would be true for completely obvious reasons if the Court's voiding makes the trade not *feasible*. It is also true when the Court allows in principle the trade of the widget ex-post acting as a minimal enforcement agency (see footnote 5 above). This is because a classic hold-up problem obtains in our model, driven by the relationship- and widget-specific investment. Given that the seller has all the bargaining power ex-post, unless an ex-ante contract is in place the buyer will be unable to recoup the cost of his investment and hence will not invest.

To see why the Court's intervention induces the two types of buyer to separate at the contract offer stage consider the incentives of the type α buyer to deviate from the separating equilibrium described in Proposition 2. With a passive Court, pooling with the type α buyer involves positive payoffs *both* in the trade of w_2 and in that of w_3 ex-post. Now that the Court renders the trade of w_2 impossible in equilibrium, the payoff to the type α buyer from deviating to pool with the type β buyer comes only from the ex-post trade of w_3. This decrease is enough to sustain the separating equilibrium of Proposition 2.

The Court's intervention has two direct effects. One is separation, so that the type α buyer no longer inefficiently trades w_3, and the other is the lack of trade of w_2. While the first increases expected welfare, the second reduces it. Overall expected welfare increases by $(\Delta_M - \Delta_L)/2$

4. Menu Contracts

Allowing menu contracts changes the terms on which we can justify Court intervention, but still provides a robust rationale for active Courts.

The effect of allowing menu contracts depends critically on whether we maintain the assumption that w_3 is not contractible ex-ante. If we do, Propositions 1 and 2 hold essentially unchanged.[8]

If on the other hand we allow ex-ante contracting on w_3, *as well as* menu contracts the picture changes. When menu contracts and ex-ante contracting on w_3 are both allowed, if the Court enforces all contracts, *multiple* equilibrium outcomes obtain. Pooling as in Proposition 1 is an equilibrium. However, the model also has an equilibrium in which a (non-trivial) menu contract is offered and the same separating outcome as in Proposition 2 obtains. Clearly, even in this case an active Court has a role in eliminating any possibility for the parties to inefficiently pool in equilibrium. The Court will step in when it expects inefficient pooling to occur.

In order to proceed, we need to be precise about two new elements of the model: the set of possible contracts when ex-ante contracting on w_3 is allowed, and the set

[8] When w_3 is contractible ex-ante, the prices at which each widget is traded, when w_3 is traded as well as w_1 or w_2, become indeterminate. The equilibrium trading and investment outcomes are as before. See Proposition 3 below.

of possible menu contracts built on the basis of these.

When w_3 can be contracted ex-ante, two types of contracts need to be considered (still abstracting from menu ones). For want of better terminology we label them *simple* and *bundle*. A simple contract, as before, consists of a pair $s_i = (w_i, p_i)$, with $i = 1, 2, 3$, specifying a single widget to trade and at which price.

A bundle contract consists of an offer to trade a specific widget w_i $i = 1, 2$ *and* the regular widget w_3 at prices p_i and p_3 respectively; a bundle contract is denoted by a triplet $b_{1,3} = (w_i, p_i, p_3)$.[9] So, as well as possible offers of s_3, $b_{1,3}$ and $b_{2,3}$, we now need to consider any possible choice of $\mathcal{V} \subseteq \{s_1, s_2, s_3, b_{1,3}, b_{2,3}\}$.

We also need to specify what a menu contract is. This is not hard to define. A menu ex-ante contract is a pair (m^α, m^β) with both m^α and m^β elements of $\{s_1, s_2, s_3, b_{1,3}, b_{2,3}\}$ if ex-ante contracting on w_3 is allowed, and just elements of $\{s_1, s_2\}$ if ex-ante contracting on w_3 is not allowed.[10] The interpretation is that m^α is the contract that rules if the Buyer announces that he is of type α after the contract is accepted and becomes binding, while m^β is the relevant arrangement if the Buyer announces that he is of type β.

With little loss of generality, we take $\mathcal{V} \subseteq \{s_1, s_2, s_3, b_{1,3}, b_{2,3}\}$ and $\mathcal{V} \subseteq \{s_1, s_2\}$, depending on whether ex-ante contracting on w_3 is allowed or not, even when menu contracts are allowed. In essence, we are restricting the Court to uphold or void on the basis of the applicable part of the menu (in other words on the basis of the part of the menu which rules as a result of the Buyer's declaration).

Proposition 3. *Menu Contracts and Non-Contractible w_3: Assume that menu contracts are allowed and that w_3 is not ex-ante contractible. Suppose that Assumption 1 holds.*

[9] There is no need to consider any other possible bundles since trading both w_1 and w_2 is never profitable. The two specific widgets are mutually exclusive since, by assumption, the buyer can only undertake one widget-specific investment.

[10] We restrict attention to pure strategy equilibria when menu contracts are allowed. That is, we do not allow the buyer to randomize across different menu contracts.

Then Propositions 1 and 2 still hold. In particular the equilibrium payoff of a passive Court is $\frac{\Delta_S}{2} + \frac{\Delta_L}{2}$ *while the equilibrium payoff of an active Court is* $\frac{\Delta_S}{2} + \frac{\Delta_M}{2}$.

We can now proceed to the case of w_3 contractible at the ex-ante stage.

Proposition 4. *Menu Contracts and Contractible w_3 – Passive Court: Assume that menu contracts are allowed and that w_3 is ex-ante contractible. Let Assumption 1 hold, and assume that the Court upholds all contracts, $\mathcal{V} = \emptyset$. Then:*

(i) *There is an equilibrium of the model in which the trading and investment outcome is as in Proposition 1. The menu contract in this equilibrium is degenerate in the sense that both types of buyer offer the same menu contract and $m^\alpha = m^\beta$. Both types of buyer invest in and trade w_2 and both types of buyer trade w_3. The total amount of expected surplus (net of investment) in this case is given by* $\frac{\Delta_S}{2} + \frac{\Delta_L}{2}$.

(ii) *There is an equilibrium of the model in which the trading and investment outcome is the same as in Proposition 2: the type α buyer invests in and trades w_1, and the type β buyer trades w_3. The menu contract in this equilibrium is non-degenerate in the sense that both types of buyer offer the same contract and $m^\alpha \neq m^\beta$. The type α buyer invests in and trades w_1, while the type β buyer does not invest in either w_1 or w_2, and trades w_3. The total amount of expected surplus (net of investment) in this case is given by* $\frac{\Delta_S}{2} + \frac{\Delta_M}{2}$.

(iii) *There is no equilibrium of the model in which the total amount of expected surplus (net of investment) exceeds* $\frac{\Delta_S}{2} + \frac{\Delta_M}{2}$.

We, finally, turn to the case of an active Court and show that active Courts do have a role when parties can negotiate menu contracts.

Proposition 5. *Menu Contracts and Contractible w_3 – Active Court: Assume that menu contracts are allowed and that w_3 is ex-ante contractible. Suppose that Assumption 1 holds. Suppose that the Court voids all contracts involving w_2. In other words suppose that $\mathcal{V} = \{s_2, b_{2,3}\}$.*

Then the unique equilibrium trading and investment outcome of the ensuing subgame is the same as in Proposition 2: the type α buyer invests in and trades w_1, and the type β buyer trades w_3.

Any equilibrium that sustains this equilibrium outcome is non-degenerate in the sense that both types of buyer offer the same menu contract and $m^\alpha \neq m^\beta$. The type α buyer invests in and trades w_1, while the type β buyer does not invest in either w_1 or w_2, and trades w_3.

In equilibrium, the total expected surplus (net of investment) is the maximum possible when menu contracts are allowed and the Court enforces all contracts, namely $\frac{\Delta_S}{2} + \frac{\Delta_M}{2}$.

5. Conclusions

In a world where contracts are incomplete (parties can write only simple trading contracts) active Courts can enhance the parties' ex-ante welfare by restricting the set of contracts they will enforce in equilibrium (Anderlini, Felli, and Postlewaite 2011).

This paper shows that active Courts have a role even in a world where parties can negotiate at an ex-ante stage more complex (menu) contracts.

The main finding is that the effect of menu contracts depends critically on whether w_3 (the widget whose cost and value do not depend on investment) is contractible ex-ante or not.

If w_3 is not contractible ex-ante then active Courts enhance the parties' ex-ante welfare by inducing them to reveal their private information and hence prevent inefficient pooling exactly as in Anderlini, Felli, and Postlewaite (2011).

If, on the other hand, w_3 is contractible ex-ante, in other words parties can write more complete contracts, than multiple equilibria emerge. When the Court does not intervene both separation and inefficient pooling are possible in equilibrium.

In the latter case the model still provides a robust rationale for Court intervention: when the Court steps in and voids contracts for w_2, the *only* possible equilibrium is the superior one involving separation. Court's intervention shrinks the equilibrium set, destroying, once again, the inefficient pooling equilibrium.

Appendix

Lemma A.1: *Consider either the model with passive Courts or any subgame of the model with active Courts following the Court's choice of $\mathcal{V}$. In any equilibrium of the model with passive Courts, or of the subgame, w_3 is traded with positive probability by the type β buyer. Moreover, the equilibrium price of w_3 is $p_3 = \Delta_S + c_S$.*

Proof: We distinguish four, mutually exclusive, exhaustive cases.

Consider first a possible separating equilibrium in which the two $\mathcal{B}$ types each offer a distinct contract at the ex-ante stage. In this case, at the ex-post stage it is a best reply for type β buyers to accept offers to trade w_3 at a $p_3 \leq \Delta_S + c_S$. Their unique best reply is instead to reject any offers to trade w_3 at any $p_3 > \Delta_S + c_S$. By standard arguments it then follows that in equilibrium it must be that w_3 is traded between $\mathcal{S}$ and type β buyers at a price $p_3 = \Delta_S + c_S$.

The second case is that of a possible pooling equilibrium in which both types of $\mathcal{B}$ offer the same ex-ante contract to $\mathcal{S}$ with probability 1. In this case the beliefs of $\mathcal{S}$ at the ex-post stage are that $\mathcal{B}$ is of either type with equal probability. The type β buyer's best reply to offers to trade w_3 at the ex-post stage is as in the previous case. It is a best reply for type α buyers to accept offers to trade w_3 at a $p_3 \leq c_H - \Delta_H$. Their unique best reply is to reject any offers to trade w_3 at any $p_3 > c_H - \Delta_H$. Since Assumption 1 (parts ii and iii) implies that $c_H - \Delta_H > \Delta_S + c_S$, it now follows by standard arguments that only two outcomes are possible in equilibrium: either w_3 is traded between $\mathcal{S}$ and both types of $\mathcal{B}$ at a price $p_3 = \Delta_S + c_S$, or w_3 is not traded at all because $\mathcal{S}$ does not make an offer that is accepted by either type of $\mathcal{B}$. The seller's expected profit from trading w_3 at $p_3 = \Delta_S + c_S$ is given by $\Delta_S + c_S/2 - c_H/2$, which is positive by Assumption 1 (parts i, ii and iii). Therefore, $\mathcal{S}$ will choose to offer to trade w_3 at $p_3 = \Delta_S + c_S$. Hence the conclusion follows in this case.

The third case is that of a possible semi-separating equilibrium in which the type β buyer offers a separating contract at the ex-ante stage with probability strictly between zero and one. In this case, the same logic of the first case applies to show that in equilibrium it must be the case that $\mathcal{S}$ and the type β buyers who offer the separating contract trade w_3 at $p_3 = \Delta_S + c_S$.

The fourth and last case is that of a possible semi-separating equilibrium in which the type β buyer offers a separating contract at the ex-ante stage with probability zero. Since some type α buyers are separating, there must be some contract that the type β buyer offers in equilibrium which is offered by the type α buyer with a strictly lower probability. Since the ex-ante probabilities of the two types of buyer are the same, there is some contract offered in equilibrium by the type β buyer such that the seller's beliefs after receiving the offer are that he is facing a type α buyer with probability $\nu \in (0, 1/2)$. After this contract is offered and accepted, in any Perfect Bayesian Equilibrium, the seller's beliefs when he contemplates making an offer to trade w_3 must also be that he faces the type α buyer with probability ν. Using the same logic as in the second case, only two possibilities remain. Either w_3 is traded at $p_3 = \Delta_S + c_S$, or $\mathcal{S}$ makes an offer that is not accepted. Given the beliefs we have described, the seller's expected profit from trading w_3 at $p_3 = \Delta_S + c_S$, is $\Delta_S + \nu c_S - \nu c_H$, which is positive using $\nu < 1/2$ and Assumption 1 (parts ii and iii). Hence $\mathcal{S}$ will choose to trade w_3 at $p_3 = \Delta_S + c_S$, and the conclusion follows in this case. ■

Lemma A.2: *Suppose that the Court enforces all contracts. Then in any equilibrium of the model w_2 is traded with probability one by the type β buyer at a price $p_2 = c_L$.*

Proof: Since the cost of w_2 is independent of $\mathcal{B}$'s type it is obvious that if it is traded, then it is traded at $p_2 = c_L$.

Suppose by way of contradiction that there exists an equilibrium in which with positive probability w_2 is not traded by the type β buyer. From Lemma A.1 we know that in this equilibrium some type β buyers trade w_3 at a price $p_3 = \Delta_S + c_S$. Therefore, type β buyers have a payoff of at most 0. (This follows from the fact that their expected profit from the w_3 trade is zero, and the maximum profit they can possibly make by trading w_1 is negative.) Consider now a deviation by the type β buyer to offering w_2 at $p_2 = c_L + \varepsilon$ with probability one. It is a unique best response for the seller to accept offers to trade w_2 at any $p_2 > c_L$. It then follows that the type β buyer can deviate to such offer and achieve a payoff of $\Delta_L - \varepsilon$. For ε sufficiently small this is clearly a profitable deviation for the type β buyer. ■

Lemma A.3: *Suppose that the Court enforces all contracts. Then in any equilibrium of the model the type α buyer offers a contract to trade w_1 with probability zero.*

Proof: Notice that by Lemma A.2 in any equilibrium the type β buyer trades w_2 with probability one. Suppose by way of contradiction that there exists an equilibrium in which the type α buyer separates with positive probability and offers a contract to trade w_1. In this case, the type α buyer's payoff must be Δ_M. This follows from the fact that, by separating, the type α buyer must be trading w_1 at a price $p_1 = c_L$ and, since he separates, $\mathcal{S}$ will not trade w_3 with him.

Suppose now that the type α buyer deviates to pool with the type β buyers who trade w_2 at $p_2 = c_L$ and then trade w_3 at $p_3 = \Delta_S + c_S$. By Lemmas A.1 and A.2 we know that the type β buyer behaves in this way with positive probability. Following this deviation the type α buyer's payoff is $\Delta_H + c_H - \Delta_H - \Delta_S - c_S$. The latter, by Assumption 1 (parts i and iii) is greater than Δ_M. Hence this is a profitable deviation for the type α buyer. ■

Lemma A.4*: Suppose that the Court enforces all contracts. Then in any equilibrium of the model w_2 is traded with probability one by the type α buyer at a price $p_2 = c_L$.*

Proof: Since the cost of w_2 is independent of $\mathcal{B}$'s type it is obvious that if it is traded, then it is traded at $p_2 = c_L$.

Suppose that the claim were false. Using Lemma A.3 we then know that, in some equilibrium, with positive probability the type α buyer trades neither w_1 nor w_2. By Lemma A.2 a type α buyer who does not trade w_2 actually separates from the type β buyer. Hence in any equilibrium in which with positive probability the type α buyer trades neither w_1 nor w_2 the type α buyer's payoff is at most zero. (The seller will not trade w_3 with him because of separation, and he makes no profit on either w_1 or w_2 since he does not trade them.)

As in the proof of Lemma A.3 the type α buyer has a profitable deviation from this putative equilibrium. He can pool with the type β buyers who trade w_2 at $p_2 = c_L$ and then trade w_3 at $p_3 = \Delta_S + c_S$. After this deviation the type α buyer's payoff is $\Delta_H + c_H - \Delta_H - \Delta_S - c_S$, which is positive by Assumption 1 (parts i and iii). ■

Lemma A.5*: Suppose that the Court enforces all contracts. Then in any equilibrium of the model w_3 is traded with probability one by both types of $\mathcal{B}$ at a price $p_3 = \Delta_S + c_S$.*

Proof: From Lemmas A.2 and A.4 we know that the two types of $\mathcal{B}$ pool with probability one at the ex-ante stage. The same reasoning as in the second case considered in the proof of Lemma A.1 now ensures that in equilibrium w_3 is traded with probability one by both types of $\mathcal{B}$ at a price $p_3 = \Delta_S + c_S$. ■

Proof of Proposition 1: The claim is a direct consequence of Lemmas A.2, A.4 and A.5. ■

Lemma A.6: *Consider the model with an active Court, and any of the subgames following the Court choosing a $\mathcal{V}$ that contains w_i, $i = 1, 2$. In any equilibrium of such subgames neither type of $\mathcal{B}$ invests in w_i, and hence it is not traded.*

Proof: If $w_i \in \mathcal{V}$ then the terms of its trade can be freely re-negotiated at the ex-post stage, when $\mathcal{S}$ makes a take-it-or-leave-it offer to $\mathcal{B}$, regardless of anything previously agreed.

Now suppose that in any equilibrium both types of $\mathcal{B}$ invest in $w_i \in \mathcal{V}$ with positive probability. Then by standard arguments in any equilibrium it must be that $\mathcal{S}$ offers to trade w_i at a price p_i that makes one of the two $\mathcal{B}$ types indifferent between accepting and rejecting the offer at the ex-post stage. But since $I > 0$ this must mean that one of the $\mathcal{B}$ types has an overall payoff equal to $-I$. Since either type of buyer can always guarantee a payoff of zero (by not investing and not trading) we can then conclude that in any equilibrium of any of these subgames it cannot be the case that both types of $\mathcal{B}$ invest in $w_i \in \mathcal{V}$ with positive probability.

Suppose then in any equilibrium only one type of $\mathcal{B}$ invests in $w_i \in \mathcal{V}$ with positive probability. Then by standard arguments in any equilibrium it must be that $\mathcal{S}$ offers to trade w_i at a price p_i that makes the type of buyer who is trading w_i indifferent between accepting and rejecting the offer at the ex-post stage. But since $I > 0$ this must mean that this type of $\mathcal{B}$ has an overall payoff equal to $-I$. Since, as before, this type of buyer can always guarantee a payoff of zero we can now conclude that in any equilibrium of any of these subgames it must be that neither type of $\mathcal{B}$ invests in $w_i \in \mathcal{V}$ with positive probability. ■

Lemma A.7: *Consider the model with an active Court. In any equilibrium of the subgame following the Court setting $\mathcal{V} = \{w_2\}$ the type α buyer trades w_1 with probability one.*

Proof: From Lemma A.6 we know that in this case neither type of $\mathcal{B}$ invests in w_2, and hence it is not traded.

Suppose that the type α buyer invests in w_1 and trades it. His payoff in this case is at least Δ_M. This is because clearly, in any equilibrium, p_1 is c_L, and at worst he is unable to trade w_3.

Suppose that instead the type α buyer does not invest in w_1 and hence does not trade it. Then his payoff is at most $c_H - \Delta_H - \Delta_S - c_S$. This is because, using Lemma A.1, at best he will be able to trade w_3 at a price $p_3 = \Delta_S + c_S$. Using Assumption 1 (part i and iii) we know that $\Delta_M > c_H - \Delta_H - \Delta_S - c_S$, and hence the argument is complete. ■

Lemma A.8: *Consider the model with an active Court. In any equilibrium of the subgame following the Court setting $\mathcal{V} = \{w_2\}$ the type β does not invest in either w_1 or w_2, separates from the type α buyer, and only trades w_3 at a price $p_3 = \Delta_S + c_S$.*

Proof: From Lemma A.6 we know that in this case neither type of $\mathcal{B}$ invests in w_2, and hence it is not traded.

Suppose that the type β buyer invests in w_1. Then his payoff must be negative. This is because, using Lemma A.1, he either trades w_3 at a price $p_3 = \Delta_S + c_S$ or does not trade w_3 (in either case the profit is zero), and using Lemma A.7 he trades w_1 at a price $p_1 = c_L$.

Since either type of buyer can always guarantee a payoff of zero (by not investing, making offers that must be rejected, and rejecting all ex-post offers) we can then conclude that the type β buyer does not invest in w_1.

Therefore, we know that the type β buyer does not invest in either w_1 or w_2. Using Lemma A.7 and the same reasoning as in the first case of Lemma A.1 we can now conclude that the type β buyer trades w_3 at a price $p_3 = \Delta_S + c_S$. ■

Lemma A.9: *Consider the model with an active Court. Suppose that the Court sets $\mathcal{V} = \{w_2\}$, then the two types buyer separate: the type α buyer invests in w_1 and only trades w_1 at a price of $p_1 = c_L$; the type β buyer does not invest in either w_1 or w_2 and only trades w_3 at a price $p_3 = \Delta_S + c_S$.*

By choosing $\mathcal{V} = \{w_2\}$ the Court achieves a payoff of $\dfrac{\Delta_S}{2} + \dfrac{\Delta_M}{2}$.

Proof: The claim is a direct consequence of Lemmas A.7 and A.8. ■

Lemma A.10: *Consider the model with an active Court. Suppose that the Court sets $\mathcal{V} = \{w_1\}$. Then the unique equilibrium outcome is that the two types of buyer pool with probability one: they both invest and trade w_2 at a price $p_2 = c_L$, and they both trade w_3 at a price $p_3 = \Delta_S + c_S$.*

By choosing $\mathcal{V} = \{w_1\}$ the Court achieves a payoff of $\dfrac{\Delta_S}{2} + \dfrac{\Delta_L}{2}$.

Proof: The proof essentially proceeds in the same way as the proof of Proposition 1. In fact by setting $\mathcal{V} = \{w_1\}$, the Court simply takes away the possibility that the parties may trade w_1 via Lemma A.6. The details are omitted for the sake of brevity. ■

Lemma A.11: *Consider the model with an active Court. Suppose that the Court sets $\mathcal{V} = \{w_1, w_2\}$. Then the two types of buyer pool: they do not invest in either w_1 or w_2 and they trade w_3 at $p_3 = \Delta_S + c_S$.*

By choosing $\mathcal{V} = \{w_1, w_2\}$ the Court achieves a payoff of $\dfrac{\Delta_S}{2} - \dfrac{\Delta_H}{2}$.

Proof: The claim follows immediately from Lemma A.6 using the same reasoning as in the second case of the proof of Lemma A.1. ■

Proof of Proposition 2: Using Assumption 1 (part i), the claim is an immediate consequence of Lemmas A.9, A.10 and A.11. ■

Proof of Proposition 3: Throughout the proof, we let $M_\alpha = (m_\alpha^\alpha, m_\alpha^\beta)$ and $M_\beta = (m_\beta^\alpha, m_\beta^\beta)$ denote the menu contract offers of the type α and the type β buyer respectively. We first show that Proposition 1 still holds. The two types of buyer must pool and trade both w_2 and w_3, yielding an equilibrium payoff for a passive Court of $\frac{\Delta_S}{2} + \frac{\Delta_L}{2}$.

There are three main cases to consider. The first is a possible equilibrium in which $M_\alpha \neq M_\beta$. In this case the two types of buyer would separate at the contract-offer stage. The same argument as in Proposition 1 can be used to establish that this cannot happen in any equilibrium of the model when the Court enforces all contracts. In other words, we conclude that there is no equilibrium of the model with passive Courts when menu contracts are allowed and w_3 is not contractible ex-ante in which $M_\alpha \neq M_\beta$.

The second case is that of a possible equilibrium in which $M_\alpha = M_\beta$ and $m_\alpha^\alpha = m_\alpha^\beta = m_\beta^\alpha = m_\beta^\beta$. In this case, the same argument as in Proposition 1 can be used to establish that the only possibility is that of an equilibrium in which the two types of buyer pool and trade both w_2 and w_3, yielding a Court equilibrium payoff of $\frac{\Delta_S}{2} + \frac{\Delta_L}{2}$.

The third case is that of $M_\alpha = M_\beta$, and $m_\alpha^\alpha \neq m_\alpha^\beta$ and $m_\beta^\alpha \neq m_\beta^\beta$. Let $m^\alpha = m_\alpha^\alpha = m_\beta^\alpha$ and $m^\beta = m_\alpha^\beta = m_\beta^\beta$.

Clearly, in equilibrium we need the "truth-telling" constraints to be satisfied: m^α and m^β must be such that the type α buyer does not prefer to declare that he is of type β, and, symmetrically, the type β buyer does not prefer to declare that he is of type α. We will show that these constraints are in fact impossible to satisfy.

Since $m^\alpha \neq m^\beta$, after declaring α, the buyer will be unable to trade w_3 since the seller's beliefs must be that he is facing a type α buyer with probability one. Moreover, after declaring β the buyer will trade w_3 ex-post at a price $p_3 = \Delta_S + c_S$. This is because the seller's beliefs in this case are that he is facing a type β buyer with probability one. There are four sub-cases to consider.

The first sub-case is that of m^α and m^β both being contracts for w_1, so that m^α and m^β differ only in the proposed prices. Let these be p_1^α and p_1^β respectively. Hence by declaring α, the type α buyer receives a payoff of $\Delta_M + c_L - p_1^\alpha$, while if he declares β he receives a payoff of $\Delta_M + c_L - p_1^\beta + c_H - \Delta_H - \Delta_S - c_S$. Therefore, to satisfy the truth-telling constraint for the type α buyer we need

$$p_1^\beta - p_1^\alpha \geq c_H - \Delta_H - \Delta_S - c_S \tag{A.1}$$

By declaring β, the type β buyer obtains a payoff of $\Delta_N + c_L + I - p_1^\beta$. If instead he declares to be of type α he obtains a payoff of$\Delta_N + c_L + I - p_1^\alpha$. Hence to satisfy the truth-telling constraint for the type β buyer we need

$$0 \geq p_1^\beta - p_1^\alpha \tag{A.2}$$

However, (A.1) and (A.2) cannot both be satisfied because of Assumption 1 (parts i and iii).

The second sub-case we consider is that of m^α and m^β both being contracts for w_2, so that m^α and m^β differ only in the proposed prices. Let these be p_2^α and p_2^β respectively. Reasoning in the same way as for the first case, the truth-telling constraint for the type α buyer implies

$$p_2^\beta - p_2^\alpha \geq c_H - \Delta_H - \Delta_S - c_S \tag{A.3}$$

while the truth-telling constraint for the type β buyer implies that

$$0 \geq p_2^\beta - p_2^\alpha \tag{A.4}$$

However, just as in the first case, (A.3) and (A.4) cannot both be satisfied because of Assumption 1 (parts i and iii).

The third sub-case is that of m^α and m^β being contracts for w_1 and w_2 respectively, with prices offered p_1^α and p_2^β. The truth-telling constraint for the type α buyer implies

$$p_2^\beta - p_1^\alpha \geq c_H - \Delta_M - \Delta_S - c_S \tag{A.5}$$

while the truth-telling constraint for the type β buyer tells us that

$$\Delta_L - \Delta_N \geq p_2^\beta - p_1^\alpha \tag{A.6}$$

However, (A.5) and (A.6) cannot both be satisfied because of Assumption 1 (parts i, iii and iv).

The fourth sub-case is that of m^α and m^β being contracts for w_2 and w_1 respectively, with prices offered p_2^α and p_1^β. The truth-telling constraint for the type α buyer can be written as

$$p_1^\beta - p_2^\alpha \geq c_H + \Delta_M - 2\Delta_H - \Delta_S - c_S \tag{A.7}$$

while the truth-telling constraint for the type β says that

$$\Delta_N - \Delta_L \geq p_1^\beta - p_2^\alpha \tag{A.8}$$

However, (A.7) and (A.8) cannot both be satisfied because of Assumption 1 (part i, iii and iv).

We conclude that there is no equilibrium of the model with passive Courts when menu contracts are allowed and w_3 is not contractible ex-ante in which $M_\alpha = M_\beta$, and $m_\alpha^\alpha \neq m_\alpha^\beta$ and $m_\beta^\alpha \neq m_\beta^\beta$.

Therefore, we have shown that Proposition 1 still holds. In any equilibrium of the model with passive Courts when menu contracts are allowed and w_3 is not contractible ex-ante the two types of buyer must pool and trade both w_2 and w_3, yielding an equilibrium payoff for a passive Court of $\frac{\Delta_S}{2} + \frac{\Delta_L}{2}$.

There remains to show that Proposition 2 still holds. When menu contracts are allowed and w_3 is not contractible ex-ante, in equilibrium, an active Court chooses $\mathcal{V} = \{s_2\}$ and its payoff is $\frac{\Delta_S}{2} + \frac{\Delta_M}{2}$.

Because of a standard hold-up problem caused by the relationship-specific investment (see for instance Lemma A.6 of AFP), in any of the subgames following the Court choosing a $\mathcal{V}$ that contains w_i, $i = 1, 2$, in equilibrium, neither type of $\mathcal{B}$ invests in w_i, and hence it is not traded.

It follows that without loss of generality whenever $\mathcal{V}$ equals either $\{s_1\}$ or $\{s_2\}$ we can restrict attention to menu contracts that specify the *same* widget in both components. Incentive-compatibility then ensures that any equilibrium menu contract would have to specify the *same* price for the single widget appearing in both menu entries. In other words, the only candidates for equilibrium are *degenerate* menus in which $m^\alpha = m^\beta$. Given this, the claim can be proved using the same argument used to prove Proposition 2 above. The details are omitted. ■

Proof of Proposition 4 (i): Take the degenerate menu offered by both types of buyer to be one that specifies $m^\alpha = m^\beta = s_2 = (w_2, c_L)$. In other words, the candidate equilibrium has the degenerate menu specifying that w_2 will be traded at a price $p_2 = c_L$, regardless of the buyer's announcement. Moreover, in the proposed equilibrium both types of buyer trade w_3 ex-post at a price $p_3 = \Delta_S + c_S$.

In the proposed equilibrium the type α buyer obtains a payoff of $c_H - \Delta_S - c_S$, the type β buyer obtains a payoff of Δ_L, and the seller obtains an expected payoff of $\Delta_S - c_H/2 + c_S/2$.

The argument proceeds in two steps. The first step is to show that neither type of buyer can profitably deviate from the proposed equilibrium by making an offer of a contract of the type s_1, s_2, s_3, $b_{1,3}$ or $b_{2,3}$. The second is to show that neither type of buyer can profitably deviate from the proposed equilibrium by offering a menu contract different from the equilibrium one.

The first step involves several cases.

Using the same argument as in the proof of Proposition 1 we already know that no type of buyer can profit from a unilateral deviation to offering any other simple contract of the type s_1 or s_2. Therefore, it only remains to show that no type of buyer can profit from a unilateral deviation to offering a contract of type s_3, $b_{1,3}$ or $b_{2,3}$.

It is easy to see that (see for instance Lemma A.1 of AFP), regardless of his beliefs, the seller will reject any off-path offer of an s_3 contract specifying a price $p_3' < \Delta_S + c_S$. (This is because $c_H - \Delta_H > \Delta_S + c_S$ by Assumption 1 (parts i and iii), and hence the seller will either trade w_3 ex-post at a price $p_3 = \Delta_S + c_S$ or will not trade it at all, depending on his beliefs.)

Now consider a possible deviation by the type α buyer to offering s_3 with a price $p_3' \geq \Delta_S + c_S$. In this case (a standard hold-up problem arises because of the relationship-specific investment, see for instance Lemma A.6 of AFP), he will not trade either w_2 or w_1. Hence his payoff after the deviation would be $c_H - \Delta_H - p_3'$. Therefore for this to be a profitable deviation we need $c_H - \Delta_H - p_3' > c_H - \Delta_S - c_S$. Since $p_3' \geq \Delta_S + c_S$, this is possible only if $\Delta_H < 0$, which is false by Assumption 1 (part i). We can the conclude that the type α buyer cannot profit from any deviation to offering a contract of the s_3 variety.

Next, consider a possible deviation from the type β buyer to offering s_3 with a price $p_3' \geq \Delta_S + c_S$. In this case (again, a standard hold-up problem arises because of the relationship-specific investment, see for instance Lemma A.6 of AFP), he will not trade either w_2 or w_1. Hence his payoff after the deviation would be $\Delta_S + c_S - p_3' \leq 0$. Since his payoff in the candidate equilibrium is positive, we conclude that the type β buyer cannot profit from any deviation to offering a contract of the s_3 variety.

The next case to consider is a possible deviation by the type α buyer to a bundle contract of the type $b_{2,3}$. Let the prices specified by the contract be denoted by p_2' and p_3'. For this to be a profitable deviation for the type α buyer we need $\Delta_H + c_L - p_2' + c_H - \Delta_H - p_3' > c_H - \Delta_S - c_S$, which implies $c_L + \Delta_S + c_S > p_2' + p_3'$. Let the seller's off-path beliefs, after receiving the offer of $b_{2,3}$, be that he is facing a type α buyer with probability $\nu \in [0, 1]$. For the seller to accept $b_{2,3}$ we need $p_2' - c_L + p_3' - \nu\, c_H - (1 - \nu)c_S \geq \max\{0, \Delta_S + c_S - \nu\, c_H - (1 - \nu)c_S\}$. This is because if he rejects the $b_{2,3}$ offer, then either w_3 will be traded at a price $p_3 = \Delta_S + c_S$, or will not be traded at all, depending on the seller's beliefs. But the last inequality implies $p_2' + p_3' \geq c_L + \Delta_S + c_S$. Hence we conclude that the type α buyer cannot profit from any deviation to offering a contract of the $b_{2,3}$ variety.

Consider now a possible deviation by the type β buyer to a bundle contract of the type $b_{2,3}$. Let the prices specified by the contract be denoted by p_2' and p_3'. For this to be a profitable deviation for the type β buyer we need $\Delta_L + c_L - p_2' + \Delta_S + c_S - p_3' > \Delta_L$, which implies $c_L + \Delta_S + c_S$

$> p_2' + p_3'$. Let the seller's off-path beliefs, after receiving the offer of $b_{2,3}$, be that he is facing a type α buyer with probability $\nu \in [0,1]$. For the seller to accept $b_{2,3}$ we need $p_2' - c_L + p_3' - \nu\ c_H - (1-\nu)c_S \geq \max\{0, \Delta_S + c_S - \nu\ c_H - (1-\nu)c_S\}$. This is because if he rejects the $b_{2,3}$ offer, then either w_3 will be traded at a price $p_3 = \Delta_S + c_S$, or will not be traded at all, depending on the seller's beliefs. But the last inequality implies $p_2' + p_3' \geq c_L + \Delta_S + c_S$. Hence we conclude that the type β buyer cannot profit from any deviation to offering a contract of the $b_{2,3}$ variety.

The next case we consider is that of a possible deviation by the type α buyer to offering a bundle contract of the $b_{1,3}$ variety. Let the prices specified by the contract be denoted by p_1' and p_3'. For this to be a profitable deviation for the type α buyer we need $\Delta_M + c_L - p_1' + c_H - \Delta_H - p_3' > c_H - \Delta_S - c_S$, which implies $\Delta_M + c_L + \Delta_S + c_S - \Delta_H > p_2' + p_3'$, which using Assumption 1 (part iv) in turn implies $\Delta_S + c_S - \Delta_L > p_2' + p_3'$. Let the seller's off-path beliefs, after receiving the offer of $b_{1,3}$, be that he is facing a type α buyer with probability $\nu \in [0,1]$. For the seller to accept $b_{1,3}$ we need $p_1' - c_L + p_3' - \nu\ c_H - (1-\nu)c_S \geq \max\{0, \Delta_S + c_S - \nu\ c_H - (1-\nu)c_S\}$. This is because if he rejects the $b_{2,3}$ offer, then either w_3 will be traded at a price $p_3 = \Delta_S + c_S$, or will not be traded at all, depending on the seller's beliefs. But the last inequality implies $p_1' + p_3' \geq \Delta_S + c_L + c_S$. Hence we conclude that the type α buyer cannot profit from any deviation to offering a contract of the $b_{1,3}$ variety.

The last case we need to consider to conclude the first step in the proof is that of a possible deviation by the type β buyer to offering a bundle contract of the $b_{1,3}$ variety. Let the prices specified by the contract be denoted by p_1' and p_3'. For this to be a profitable deviation for the type β buyer we need $\Delta_N - p_1' + \Delta_S + c_S - p_3' > \Delta_L$, which implies $\Delta_S + c_S + \Delta_N - \Delta_L > p_2' + p_3'$. Let the seller's off-path beliefs, after receiving the offer of $b_{1,3}$, be that he is facing a type α buyer with probability $\nu \in [0,1]$. For the seller to accept $b_{1,3}$ we need $p_1' - c_L + p_3' - \nu\ c_H - (1-\nu)c_S \geq \max\{0, \Delta_S + c_S - \nu\ c_H - (1-\nu)c_S\}$. This is because if he rejects the $b_{2,3}$ offer, then either w_3 will be traded at a price $p_3 = \Delta_S + c_S$, or will not be traded at all, depending on the seller's beliefs. But the last inequality implies $p_1' + p_3' \geq \Delta_S + c_L + c_S$. Hence we conclude that the type β buyer cannot profit from any deviation to offering a contract of the $b_{1,3}$ variety.

We have now ruled out the possibility that either type of buyer could profitably deviate from the proposed equilibrium by making an offer of a contract of the type s_1, s_2, s_3, $b_{1,3}$ or $b_{2,3}$. The second step in the argument rules out the possibility that either type of buyer can profitably deviate from the proposed equilibrium by offering a menu contract different from the equilibrium one. It involves considering several cases again.

Consider first the possibility that either type of buyer deviates to offering a degenerate menu with $m^\alpha = m^\beta$. In this case, the same argument we used in the first step clearly suffices to prove the claim.

Therefore, there remains to consider the case of some type of buyer deviating to offering a non-degenerate menu contract $M = (m^\alpha, m^\beta)$ with $m^\alpha \neq m^\beta$. Clearly in this case, without loss of generality, we can take it to be the case that the menu M satisfies the *truth-telling* constraints: m^α and m^β must be such that the type α buyer does not prefer to declare that he is of type β, and, symmetrically, the type β buyer does not prefer to declare that he is of type α. If this were not the case, the seller would believe that one of the two menu items will be chosen with probability one when the buyer announces his type. Therefore, the same argument as in the case of a degenerate menu would suffice to prove the claim.

It is convenient to classify the possible deviations to non-degenerate menus M that satisfy the truth-telling constraints into three mutually exclusive subsets. We say that a menu contract is of class α if it has the property that, if accepted, it constitutes a strictly profitable deviation (given truth-telling) from the proposed equilibrium for the type α buyer, but not for the type β buyer. The class of such menu contracts is denoted by $\mathcal{M}^\alpha$. We say that a menu contract is of class β if it has the property that, if accepted, it constitutes a strictly profitable deviation (given truth-telling) from the proposed equilibrium for the type β buyer, but not for the type α buyer. The class of such menu contracts is denoted by $\mathcal{M}^\beta$. We say that a menu contract is of class ω if it has the property that, if accepted, it constitutes a strictly profitable deviation (given truth-telling) from the proposed equilibrium for both the type α and the type β buyer. The class of such menu contracts is denoted by $\mathcal{M}^\omega$. Clearly, to conclude the proof it suffices to show that no type α buyer can profitably deviate by offering a menu $M \in \mathcal{M}^\alpha$, no type β buyer can profitably deviate by offering a menu $M \in \mathcal{M}^\beta$, and no buyer of either type can profitably deviate by offering a menu $M \in \mathcal{M}^\omega$.

Consider a possible deviation by a type α buyer to a menu $M \in \mathcal{M}^\alpha$. In this case, we assign off-path equilibrium beliefs to the seller that he is facing a buyer of type α with probability one. These beliefs clearly satisfy the Intuitive Criterion of Cho and Kreps (1987) (see footnote 7 above). The seller believes that the m^α component of M will apply with probability one after the buyer declares his type. It follows that the same argument used in the first step of this proof to show that the type α buyer cannot profitably deviate to a contract of type s_1, s_2, s_3, $b_{1,3}$ or $b_{2,3}$ now suffices to show that he cannot profit from a deviation to a menu $M \in \mathcal{M}^\alpha$.

Next, consider a possible deviation by a type β buyer to a menu $M \in \mathcal{M}^\beta$. In this case, we assign off-path equilibrium beliefs to the seller that he is facing a buyer of type β with probability one. These beliefs clearly satisfy the Intuitive Criterion of Cho and Kreps (1987) (see footnote 7 above). The seller believes that the m^β component of M will apply with probability one after the buyer declares his type. It follows that the same argument used in the first step of this proof to show that the type β buyer cannot profitably deviate to a contract of type s_1, s_2, s_3, $b_{1,3}$ or $b_{2,3}$ now suffices to show that he cannot profit from a deviation to a menu $M \in \mathcal{M}^\beta$.

Consider now a possible deviation by a type α buyer to a menu $M \in \mathcal{M}^\omega$. In this case, we assign off-path equilibrium beliefs to the seller that he is facing a buyer of type α with probability one. These beliefs clearly satisfy the Intuitive Criterion of Cho and Kreps (1987) (see footnote 7 above). The seller believes that the m^α component of M will apply with probability one after the buyer declares his type. It follows that the same argument used in the first step of this proof to show that the type α buyer cannot profitably deviate to a contract of type s_1, s_2, s_3, $b_{1,3}$ or $b_{2,3}$ now suffices to show that he cannot profit from a deviation to a menu $M \in \mathcal{M}^\alpha$.

Lastly, consider a possible deviation by a type β buyer to a menu $M \in \mathcal{M}^\omega$. As we specified above, in this case we assign off-path equilibrium beliefs to the seller that he is facing a buyer of type α with probability one. These beliefs clearly satisfy the Intuitive Criterion of Cho and Kreps (1987) (see footnote 7 above). The seller believes that the m^α component of M will apply with probability one after the buyer declares his type.

Recall that the argument used in the first step of this proof to show that the type β buyer cannot profitably deviate to a contract of type s_1, s_2, s_3, $b_{1,3}$ or $b_{2,3}$ applies *regardless* of the seller's off-path beliefs following the deviation. Therefore, that argument also suffices to now show that he cannot profit from a deviation to a menu $M \in \mathcal{M}^\omega$. ■

Proof of Proposition 4 (ii): Take the equilibrium non-degenerate menu contract to be $M = (m^\alpha, m^\beta)$ with m^α of the s_1 variety with a price $p_1 = \Delta_M + c_L - c_H + \Delta_S + c_S$ and m^β of the s_3 variety with a price $p_3 = \Delta_S - \Delta_M + c_S$.

In this candidate equilibrium the type α buyer gets a payoff (under truth-telling) of $\Delta_M + c_L - p_1 = \Delta_M + c_L - \Delta_M - c_L + c_H - \Delta_S - c_S = c_H - \Delta_S - c_S$, while the type β buyer obtains a payoff (under truth-telling) of $\Delta_S + c_S - p_3 = \Delta_S + c_S - \Delta_S + \Delta_M - c_S = \Delta_M$ and the seller gets an expected payoff (under truth-telling) of $(p_1 - c_L)/2 + (p_3 - c_S)/2 = \Delta_S - c_H/2 + c_S/2$. Crucially, notice that the type β buyer has a payoff strictly greater than the one he obtains in the equilibrium constructed in the proof of Proposition 4 (i). The type α buyer and the seller have the same payoffs as the ones they obtain in the equilibrium constructed in the proof of Proposition 4 (i).

We begin by verifying that the proposed equilibrium contract satisfies the necessary truth-telling constraints. The truth-telling constraint for the type α buyer can be written as

$$p_3 - p_1 \geq c_H - \Delta_H - c_L - \Delta_M \tag{A.9}$$

which is satisfied for $p_1 = \Delta_M + c_L - c_H + \Delta_S + c_S$ and $p_3 = \Delta_S - \Delta_M + c_S$ by Assumption 1 (part i).

The truth-telling constraint for the type β buyer can be written as

$$\Delta_S - \Delta_N \geq p_3 - p_1 \tag{A.10}$$

which is satisfied for $p_1 = \Delta_M + c_L - c_H + \Delta_S + c_S$ and $p_3 = \Delta_S - \Delta_M + c_S$ by Assumption 1 (part iii and iv).

Consider now a possible deviation by the type α buyer to offering a simple contract of the s_2 variety. At best, he would be able to get a payoff of $c_H - \Delta_S - c_S$. This is because the seller will not accept any offer to trade w_2 for a price below c_L, and the type α buyer, at best (depending on the seller's beliefs) will be able to trade w_3 ex-post for a price of $\Delta_S + c_S$. Since $c_H - \Delta_S - c_S$ is also his payoff in the proposed equilibrium, we conclude that the type α buyer cannot profit from a deviation to offering a simple contract of the s_2 variety.

Next, consider a possible deviation by the type β buyer to offering a simple contract of the s_2 variety. At best, he would be able to get a payoff of Δ_L. This is because the seller will not accept any offer to trade w_2 for a price below c_L, and the type β buyer, at best (depending on the seller's beliefs) will be able to trade w_3 ex-post for a price of $\Delta_S + c_S$. Since $\Delta_L < \Delta_M$, we conclude that the type β buyer cannot profit from a deviation to offering a simple contract of the s_2 variety.

All other possible deviations can be ruled out using the computations (including the off-path beliefs that they use) in the proof of Proposition 4 (i). This is because the equilibrium payoffs to both types of buyer in the equilibrium proposed here are at least as large as the payoffs that they receive in the equilibrium constructed there. ■

Proof of Proposition 4 (iii): Suppose that there were an equilibrium in which expected net surplus exceeds $\frac{\Delta_S}{2} + \frac{\Delta_M}{2}$. Then using Assumption 1 (parts i and ii) the equilibrium would have to be of one of the following three varieties. The first variety involves type α buyer trading w_2 only and the type β buyer trading w_1 and w_3. The second variety involves the type α buyer trading w_2 only and the type β buyer trading w_3 only. The third variety involves the type α buyer trading w_1 only and the type β buyer trading w_2 and w_3.

As in the proof of Proposition 3, throughout the argument we let $M_\alpha = (m_\alpha^\alpha, m_\alpha^\beta)$ and $M_\beta = (m_\beta^\alpha, m_\beta^\beta)$ denote the menu contract offers of the type α and the type β buyer respectively.

There are three main cases to consider. The first is a possible equilibrium in which $M_\alpha \neq M_\beta$. In this case the two types of buyer would separate at the contract-offer stage. Because of separation at the contract-offer stage we can take it to be the case that both M_α and M_β are degenerate menus, with $M_\alpha = (m_\alpha, m_\alpha)$ and $M_\beta = (m_\beta, m_\beta)$.

There are two possible ways to obtain an equilibrium of the first variety when $M_\alpha \neq M_\beta$. The first is that $m_\alpha = s_2$ and $m_\beta = s_1$, with the type β buyer trading w_3 ex-post. This possibility can clearly be ruled out in the same way as in the proof of Proposition 1. The second way is to have $m_\alpha = s_2$ and $m_\beta = b_{1,3}$. In such putative equilibrium, the type α buyer would obtain a payoff of Δ_H, since clearly the s_2 contract would have to specify $p_2 = c_L$. Notice also that, given separation, the seller can trade w_3 ex-post for a payoff of Δ_S if he rejects the type β buyer offer of $b_{1,3}$. It follows that the contract $b_{1,3}$ contains prices p_1 and p_3 such that $p_1 + p_3 = \Delta_S + c_L + c_S$. Therefore, by deviating to pooling with the type β buyer, the type α buyer would obtain a payoff of $c_H - \Delta_H - \Delta_S + \Delta_M - c_S$. Using Assumption 1 (part iii) this is a profitable deviation. Therefore we can conclude that the putative equilibrium is not viable.

A possible equilibrium of the second variety when $M_\alpha \neq M_\beta$ can be ruled out by noticing that in any case this will involve trading w_2 at a price $p_2 = c_L$ and w_3 at a price $p_3 = \Delta_S + c_S$. Therefore this possibility can clearly be excluded out in the same way as in the proof of Proposition 1.

There are two possible ways to obtain an equilibrium of the third variety when $M_\alpha \neq M_\beta$. The first is that $m_\alpha = s_1$ and $m_\beta = s_2$, with the type β buyer trading w_3 ex-post. This possibility can clearly be ruled out in the same way as in the proof of Proposition 1. The second way is to have $m_\alpha = s_1$ and $m_\beta = b_{2,3}$. In such putative equilibrium, the type α buyer would obtain a payoff of Δ_M, since clearly the s_1 contract would have to specify $p_1 = c_L$. Notice also that, given separation, the seller can trade w_3 ex-post for a payoff of Δ_S if he rejects the type β buyer offer of $b_{2,3}$. It follows that the contract $b_{2,3}$ contain prices p_2 and p_3 such that $p_2 + p_3 = \Delta_S + c_L + c_S$. Therefore, by deviating to pooling with the type β buyer, the type α buyer would obtain a payoff of $c_H - \Delta_S - c_S$. Using Assumption 1 (parts i and iii) this is a profitable deviation. Therefore we can conclude that the putative equilibrium is not viable.

The second case is that of a possible equilibrium in which $M_\alpha = M_\beta$ and $m^\alpha_\alpha = m^\beta_\alpha = m^\alpha_\beta = m^\beta_\beta$. Clearly, no equilibria of the first, second or third variety can be sustained in this case. This is because in all three varieties, the two types of buyer do not trade the same widget w_1 or w_2.

The third case is that of $M_\alpha = M_\beta$, and $m^\alpha_\alpha \neq m^\beta_\alpha$ and $m^\alpha_\beta \neq m^\beta_\beta$. Let $m^\alpha = m^\alpha_\alpha = m^\alpha_\beta$ and $m^\beta = m^\beta_\alpha = m^\beta_\beta$.

As in the proof of Proposition 3, in equilibrium we need the "truth-telling" constraints to be satisfied: m^α and m^β must be such that the type α buyer does not prefer to declare that he is of type β, and, symmetrically, the type β buyer does not prefer to declare that he is of type α. We will show that these constraints are in fact impossible to satisfy in any of the three varieties of equilibria.

Notice that, since $m^\alpha \neq m^\beta$, whenever m^α is a simple contract for either w_1 or w_2, after declaring α, the buyer will be unable to trade w_3 since the seller's beliefs must be that he is facing a type α buyer with probability one. Moreover, whenever m^β is a simple contract for either w_1 or w_2, after declaring β the buyer will trade w_3 ex-post at a price $p_3 = \Delta_S + c_S$. This is because the seller's beliefs in this case are that he is facing a type β buyer with probability one.

There are two ways to support a possible equilibrium of the first variety when $M_\alpha = M_\beta$, and $m^\alpha_\alpha \neq m^\beta_\alpha$ and $m^\alpha_\beta \neq m^\beta_\beta$. The first is with m^α and m^β being simple contracts for w_2 and w_1 respectively, with prices offered p^α_2 and p^β_1. The truth-telling constraint for the type α buyer can be written as

$$p_1^\beta - p_2^\alpha \geq c_H + \Delta_M - 2\Delta_H - \Delta_S - c_S \tag{A.11}$$

while the truth-telling constraint for the type β says that

$$\Delta_N - \Delta_L \geq p_1^\beta - p_2^\alpha \tag{A.12}$$

However, (A.11) and (A.12) cannot both be satisfied because of Assumption 1 (part i, iii and iv). The second is with m^α being a simple contract of the s_2 variety and m^β being a bundle contract of the $b_{1,3}$ variety with prices p_2^α, p_1^β and p_3^β respectively. The truth-telling constraint for the type α buyer can be written as

$$p_1^\beta + p_3^\beta - p_2^\alpha \geq c_H + \Delta_M - 2\Delta_H \tag{A.13}$$

while the truth-telling constraint for the type β says that

$$\Delta_S + c_S + \Delta_N - \Delta_L - c_L \geq p_1^\beta + p_3^\beta - p_2^\alpha \tag{A.14}$$

However, (A.13) and (A.14) cannot both be satisfied because of of Assumption 1 (parts i and iii).

When $M_\alpha = M_\beta$, and $m_\alpha^\alpha \neq m_\alpha^\beta$ and $m_\beta^\alpha \neq m_\beta^\beta$, to support an equilibrium of the second variety we would have to have m^α and m^β being simple contracts for w_2 and w_3 respectively, with prices offered p_2^α and p_3^β. The truth-telling constraint for the type α buyer implies

$$p_3^\beta - p_2^\alpha \geq c_H - c_L - 2\Delta_H \tag{A.15}$$

Using Assumption 1 (parts ii, iii and v), (A.15) implies that $p_3^\beta > p_2^\alpha$. If the seller rejects the menu contract, he will trade w_3 ex-post at a price of $\Delta_S + c_S$ with equal probability with either type of buyer. Hence by rejecting the offer the seller obtains an expected profit of $\Delta_S - c_H/2 + c_S/2$. By standard arguments the menu contract will leave $\mathcal{S}$ indifferent between accepting and rejecting. Hence

$$\frac{1}{2}(p_2^\alpha - c_L) + \frac{1}{2}(p_3^\beta - c_S) = \Delta_S - \frac{1}{2}c_L + \frac{1}{2}c_S \tag{A.16}$$

which together with $p_3^\beta > p_2^\alpha$ implies that $p_3^\beta > \Delta_S + c_S$. However, the latter implies that the type β buyer would get a negative profit from the putative menu contract equilibrium. This is not possible since he can always not invest and not trade and guarantee a payoff of zero.

There are two ways to support a possible equilibrium of the third variety when $M_\alpha = M_\beta$, and $m^\alpha_\alpha \neq m^\beta_\alpha$ and $m^\alpha_\beta \neq m^\beta_\beta$. The first is with m^α and m^β being simple contracts for w_1 and w_2 respectively, with prices offered p^α_1 and p^β_2, and the type β buyer trading w_3 ex-post. The truth-telling constraint for the type α buyer implies

$$p^\beta_2 - p^\alpha_1 \geq c_H - \Delta_M - \Delta_S - c_S \tag{A.17}$$

while the truth-telling constraint for the type β buyer tells us that

$$\Delta_L - \Delta_N \geq p^\beta_2 - p^\alpha_1 \tag{A.18}$$

However, (A.17) and (A.18) cannot both be satisfied because of Assumption 1 (parts i, iii and iv). The second is with m^α being a simple contract of the s_1 variety and m^β being a bundle contract of the $b_{2,3}$ variety with prices p^α_1, p^β_2 and p^β_3 respectively. The truth-telling constraint for the type α buyer can be written as

$$p^\beta_2 + p^\beta_3 - p^\alpha_1 \geq c_H - \Delta_M \tag{A.19}$$

On the other hand, the truth-telling constraint for the β type buyer implies that

$$c_L + \Delta_L + \Delta_S \geq p^\beta_2 + p^\beta_3 - p^\alpha_1 \tag{A.20}$$

However, inequalities (A.19) and (A.20) cannot be both satisfied because of Assumption 1 (parts i, iii, iv and v). ■

Proof of Proposition 5: We begin by arguing that the equilibrium constructed in the proof of Proposition 4 (ii) is still viable when the Court sets $\mathcal{V} = \{s_2, b_{2,3}\}$. This is straightforward since the Court now makes some deviations impossible. The remaining deviations can be shown not to be profitable in the same way as in the the proof of Proposition 4 (ii).

Given that $\mathcal{V} = \{s_2, b_{2,3}\}$, since a standard hold-up problem arises because of the relationship-specific investment (see for instance Lemma A.6 of AFP), we can be sure that in no equilibrium of the model will it be the case that either (or both) types of buyer will invest in w_2, and hence it will not be traded.

To show that the type α buyer investing in and trading w_1 and the type β buyer trading w_3 is the unique equilibrium outcome the following three varieties of equilibrium outcomes need to be ruled out. The first variety is one in which both types of buyer invest in and trade w_1. The second variety is one in which both types of buyer trade w_3. The third variety is one in which the type α buyer trades w_3, while the type β buyer invests in and trades w_1.

Consider an equilibrium of the first variety. This outcome cannot be sustained without using menu contracts in equilibrium. This can be proved using the same argument as in the proof of Proposition 2. For the same reason, this outcome cannot be sustained using menu contracts in an equilibrium in which the two types of buyer separate at the contract-offer stage by offering $M_\alpha \neq M_\beta$. Suppose that $M_\alpha = M_\beta$ and both menus are degenerate in the sense that $m^\alpha_\alpha = m^\beta_\alpha = m^\alpha_\beta = m^\beta_\beta$. In this case clearly we must have that the menu contracts specify $p_1 = c_L$. Hence, just as in the proof Proposition 2, the type β buyer has an incentive to deviate. Lastly, suppose that $M_\alpha = M_\beta$, and $m^\alpha_\alpha \neq m^\beta_\alpha$ and $m^\alpha_\beta \neq m^\beta_\beta$. Then, since both menu items must be simple contracts for w_1 the truth telling constraints trivially imply that $p^\alpha_1 = p^\beta_1$. Hence, in equilibrium $p^\alpha_1 = p^\beta_1 = c_L$, and therefore the type β buyer has an incentive to deviate as before.

Any equilibrium of the second variety can be ruled out in a completely analogous way as any equilibrium of the first variety. The details are omitted.

Consider now an equilibrium of the third variety. From the surplus and cost matrix in (1) it is evident that the sum of the payoffs of the two types of buyer and of the seller in any such equilibrium is negative. Hence at least one of the players will have a profitable deviation to not trading at all. ■

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