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Barriers to Trade and Disadvantageous Middlemen: Nonmonotonicity of the Core*

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In applying cooperative game theory to economic problems of exchange, it is standard to assume that all logically possible coalitions may form. However, because of institutional, legal, or physical barriers, it may in fact be impossible for certain sets of agents to communicate or trade with one another directly. It would seem worthwhile to recognize this and to analyze the impact of such barriers to trade. In [3], Myerson introduced the concept of a game with a communications graph,[1] which is a graph whose nodes correspond to the players in a game and within which the presence or absence of a particular link indicates the possibility or impossibility of communication

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[1] Let $N = \{1, 2,..., n\}$. A graph on the set of nodes N is a collection of links $\mathbf{ij}$ where i and j are distinct elements of N and $\mathbf{ij} = \mathbf{ji}$. A graph is complete if it contains all the possible links. A path from node i to node j is a collection of nodes $i_1, i_2,..., i_k$ with $i_1 = i$, $i_k = j$, and $\mathbf{i}_r\mathbf{i}_{r+1}$ is a link in the graph for $r = 1, 2,..., k - 1$. A graph is connected if there is a path from any one node to any other node.

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between the relevant pair of players. Here we apply this approach to the problem of exchange. If a particular pair of traders are linked in the graph, direct trade between them is possible; if they are not linked, they cannot trade directly with one another, although it may be possible to trade indirectly through other linked traders.

The use of the graph structure on the set of traders permits the modeling of differing forms of market organization. For example, if the graph g on the set of traders is not connected, we are effectively looking at a system of autarkies, while if g is connected but not necessarily complete one obtains models of different forms of economic systems. For example, the complete graph in part (a) of Fig. 1 represents a system under which all agents are free to exchange directly with one another, while the graph in part (b) represents the existence of a middleman through which all trade flows. Figure 2 can be considered as representing two economies, where all foreign trade in one economy must flow through an export–import agency.

FIGURE 1

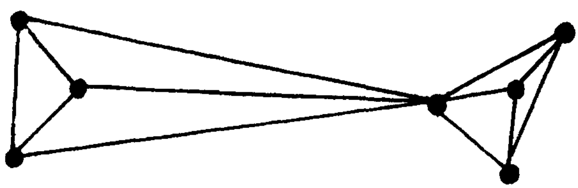

FIGURE 2

A particularly interesting aspect of the question of comparing different systems involves comparative statics analysis on the equilibria as the communications graph is changed, with a view to answering such questions as precisely who gains or loses when a link is introduced or deleted from g. A specific example of this involves the role of the middleman. Suppose that initially g is complete and then all links except those with one particular agent, say 1, are broken (as in Fig. 1). Since the graph is still connected, the set of Pareto optima is unchanged. However, no multiplayer coalitions not involving player 1 are connected in the second situation. Thus, the core allocations of the second economy will include those of the first. Intuitively, one might expect that the player in the middle would not do worse in this situation: all trade must flow through him, which ought to improve his position. Somewhat more formally, all coalitions including this player remain as before, while those excluding him are now powerless. The question is then whether he may in fact lose from having his special position.

The definition of "may lose" we adopt here involves a comparison of the core allocations to the middleman. Clearly, the core in the game with a middleman will contain that in the game where all coalitions are possible. We say then that the middleman may lose if there is a core allocation x in the middleman economy which is strictly worse for him than every core allocation of the unrestricted economy. Theorem 1 shows that in three-person games arising from situations of exchange, the middleman cannot lose. However, if we consider general three-person games, this result is not true: for some three-person games, if we do not allow the coalition of players 2 and 3 to form, every point entering the core may be no better or even strictly worse for player 1 than every point in the core of the original game. Moreover, in general n-person games arising from markets, there may be points entering the core which are worse for the middleman than any point in the core of the original game. However, in such cases, if preferences are strictly monotonic and the trade through the middleman is beneficial to the grand coalition, then some of the new core points will be better for the middleman.

An economic agent or trader is defined by his characteristic $a_i = (u^i, w^i)$, where u^i is a continuous, quasi-concave utility function over R_+^m and where $w^i \in R_+^m$. Here, m is the number of commodities and w^i is his initial endowment. Given n such traders a_i, $i \in N = \{1,..., n\}$, a communications graph is a graph g whose nodes correspond to the n traders. An economy E is then a pair (a, g), where $a = (a_1 ,..., a_n)$ and g is a communications graph. A coalition is a nonempty subset of N. A coalition S is connected if the restriction of g to S is connected. We assume that N is connected and that $\{i\}$ is connected for every $i \in N$. An allocation is a vector $x = (x^1,..., x^n)$, $x^i \in R_+^m$. An allocation x is feasible for the coalition S (S-feasible) if $\sum_{i \in S} x^i = \sum_{i \in S} w^i$ and S is connected. If x is feasible for N, we simply say that it is feasible. A coalition S can improve upon (block) an allocation x if S is connected and there exists an allocation y which is feasible for S and such that $u^i(y^i) > u^i(x^i)$ for every $i \in S$. The core of E consists of those feasible allocations which cannot be improved upon by any coalition.

Let $g_c = \{\mathbf{ij}: i, j \in N\}$ be the complete graph with n traders and let $g_m = \{\mathbf{1i}: i \in N, i \neq 1\}$. Then g_c represents free trade and g_m represents trading through a middleman (trader 1). Let $E_c = (a, g_c)$ and $E_m = (a, g_m)$ be the economies corresponding to these two structures. We are interested in comparing 1's payoffs at core allocations in E_c and E_m.

THEOREM 1. *Let $n = 3$ and suppose u^i is strictly monotonic for all i. For every $x \in \mathrm{Core}(E_m)$ there exists a $y \in \mathrm{Core}(E_c)$ with $u^1(x^1) \geqslant u^1(y^1)$.*

Thus, for every solution allocation x which arises when he is a middleman there is a solution allocation y in the economy with unrestricted trading which is no better than x: any point becoming a solution when he becomes a middleman is better than some solution under free trade.

To prove this theorem we use the methods of the theory of balanced cooperative games.

An n-person cooperative game is defined to be a collection $V = \{V_S\}_{\varnothing \neq S \subseteq N}$, where, for every nonempty $S \subseteq N$, V_S is a nonempty, strict subset of R^n which is closed and comprehensive (i.e., if $x \in V_S$, $y \in R^n$ and $y^i \leqslant x^i$ for every $i \in S$ then $y \in V_S$). We let $0_S = \{x \in V_S$: for some $y \in V_S$, $y^i > x^i$ for every $i \in S\}$. Then $\mathrm{Core}(V) = V_N \cap (\bigcap_{S \neq \varnothing} 0_S{}^c)$, where the superscript c denotes the complement of a set.

With an economy E we can associate an n-person cooperative game $V(E)$ as follows. We let u^i be a utility function for trader i normalized so that $u^i(w^i) = 0$. For $\varnothing \neq S \subseteq N$, define

$$V_S = \{v \in R^n \mid \text{there exists some } S\text{-feasible allocation } x \text{ with } v^i \leqslant u^i(x^i) \text{ for every } i \in S\} \text{ if } S \text{ is connected,}$$

and

$$V_S = R_-{}^n = \{v \in R^n \mid v^i \leqslant 0 \text{ for } i = 1, 2, \ldots, n\} \text{ if } S \text{ is not connected.}$$

A collection of coalitions $\{S_r\}_{r \in R}$ is called balanced (see [4]) if there is a collection of nonnegative real numbers $\{\delta_r\}_{r \in R}$ such that for every $i \in N$, $\sum_{r: i \in S_r} \delta_r = 1$.

LEMMA. *Let $E_c = (a, g_c)$ be an economy with strictly monotonic preferences and let V be an n-person game associated with it.*

1. *For every nonempty coalition S and for every $x, y \in V_S$, if $0 \leqslant y^j \leqslant x^j$ for every $j \in S$ and $y \neq x$, then $y \in 0_S$. Thus the Pareto surface of V_S contains no segments which are parallel to the axes of the players in S.*

2. *For every $x \in R^n$, $x \in \mathrm{Core}(V)$ if and only if for some $y \in \mathrm{Core}(E_c)$ $x = (u^1(y^1), u^2(y^2), \ldots, u^n(y^n))$.*

3. *For $x \in R^n$ let $T_x = \{S \subsetneqq N: x \in V_S\}$. If T_x is a balanced collection of sets with weights $\{\delta_S\}_{S \in T_x}$, and if $x \in 0_S$ for some $S \in T_x$ with $\delta_S > 0$, then there exists $y \in V_N$ with $y^i > x^i$ for every $i \in N$.*

Proof of the Lemma. Part 1 follows immediately from the monotonicity of the utility functions. Part 2 follows from the definitions of the cores and from part 1.

To prove part 3 we use a method due to Scarf [7]. Assume x satisfies the hypotheses of the lemma. For every $S \in T_x$ there exists an S-feasible allocation z_S with $x^i \leqslant u^i(z_S{}^i)$ for every $i \in S$. Also for some $S' \in T_x$, $z_{S'}$ can be chosen so that $x^i < u^i(z_{S'})$ for every $i \in S'$. Consider the allocation z defined by $z^i = \sum \delta_S z_S{}^i$, where the summation is over those $S \in T_x$ with $i \in S$. Observe that z^i is a convex combination of the $z_S{}^i$'s and therefore (by the quasi-concavity of the utility functions) $u^i(z^i) \geqslant x^i$ for every $i \in N$. Also, $u^i(z^i) > x^i$ for every $i \in S$. Let $y^i = u^i(z^i)$, $i \in N$. Since

$$\sum_{i\in N} z^i = \sum_{i\in N} \sum_{S\in T_x, i\in S} \delta_S z_S{}^i = \sum_{S\in T_x} \delta_S \sum_{i\in S} z_S{}^i \leqslant \sum_{S\in T_x} \delta_S \sum_{i\in S} \omega^i = \sum_{i\in N} \omega^i,$$

z is feasible and thus $y = (y^1,..., y^n) \in V_N$. By the monotonicity of the utility functions we can slightly perturb y to have $y^i > x^i$ for every $i \in N$.

Proof of Theorem 1. Let V be the game associated with E_c, and V' be that associated with E_m. We wish to show that for any $z \in \text{core } V'$ there exists $w \in \text{core } V$ with $z^1 \geqslant w^1$. To this end, suppose $z^1 \leqslant v^1$ for all $v \in B = V_{\{123\}} \cap 0^c_{\{12\}} \cap 0^c_{\{13\}} \cap 0^c_{\{1\}} \cap 0^c_{\{2\}} \cap 0^c_{\{3\}}$, $z \in B$, and $z \notin 0_{\{123\}}$. (Note that the core of V' is the set of Pareto optima in B.) If we can show that $z \notin 0_{\{23\}}$, then z must also be in the core of V. Then any point in the core V' is at least as good for 1 as some point in core V, namely z, and we are done.

Thus, to obtain a contradiction, we will suppose that $z \in 0_{\{23\}}$. Let $T = \{S \subsetneqq N: S \neq \varnothing, S \neq \{23\}, \text{ and } z \in V_S\}$. Then $1 \in \bigcup_{S\in T} S$, since otherwise the comprehensiveness of the V_S sets would imply that z does not minimize z^1 over B. If $\{1\} \in T$ then the proof of is completed because part 3 of the lemma would contradict the Pareto optimality of z ($\{1\}, \{23\}$ is a balanced collection). Then suppose $\{1\} \notin T$. We claim $i \in \bigcup_T S$, $i = 1, 2$. For example if $2 \notin \bigcup_T S$, i.e., $z \notin V_{\{2\}} \cup V_{\{12\}}$, then for some small enough $\epsilon > 0$, $z' = z - (0, \epsilon, 0)$ also minimizes z^1, while $z' \in B$, $z' \in 0_{\{123\}}$, and $z' \notin V_S$ for any $S \neq N$ with $2 \in S$. Therefore for some small enough $\delta > 0$, $z'' = z' + (0, 0, \delta)$ we have $z'' \in 0_{\{12\}\}}$ and $z'' \notin V_S$ for $\varnothing \neq S \neq N$ and $S \neq \{2, 3\}$. Thus z^1 is minimized at an interior point of B, which is impossible. So we assume without loss of generality that $\{12\} \in T$ and either $\{3\} \in T$ or $\{13\} \in T$. If $\{13\} \in T$ then we obtain a contradiction by the lemma ($\{12\}, \{13\}, \{23\}$ is a balanced collection). So we are left with the cases $T = \{\{12\}, \{3\}\}$ or $T = \{\{12\}, \{3\}, \{2\}\}$. In either of these two cases it is possible to transfer a small amount of utility from 1 to 2 while still staying in B, which again yields a contradiction. Thus, $z \notin 0_{\{23\}}$, so z is unblocked in V', and thus all points in the core of V' are at least as good for the middleman as some point in the core of V.

This result seems intuitive, and it is surprising that the proof is so involved. However, the following examples show that the analogue of this theorem is not true for general three-person cooperative games, or for market games with a larger number of players.

We first show an example of a three-person nonmarket, cooperative game in which the middleman may lose. Let ϵ be a small nonnegative real number and define the game V $(=V(\epsilon))$ as follows:

$$
\begin{aligned}
V_{\{i\}} &= \{v \in R^3\colon v^i \leqslant 0\} \qquad \text{for} \quad i = 1, 2, 3;\\
V_{\{123\}} &= \{v \in R^3\colon \text{for some } x \in \text{convex hull } \{A, B, C, F\},\ v \leqq x\};\\
V_{\{12\}} &= \{v \in R^3\colon v^1 + v^2 \leqq 0.01\};\\
V_{\{13\}} &= \{v \in R^3\colon v^1 + v^3 \leqq 1\};\\
V_{\{23\}} &= \{v \in R^3\colon \text{for some } x \in \text{convex hull } \{C, D, E\},\ v \leqq x\},
\end{aligned}
$$

where

$$
\begin{aligned}
&A = (1 + \epsilon, 0, 0), \qquad && B = (1, 1, 0), \qquad && C = (0, 0, 1),\\
&D = (0, 1, 0), && E = (0, \tfrac{1}{2}, \tfrac{3}{4}), && F = (0, 2, 0),\\
&G = (1, 0, 0).
\end{aligned}
$$

The core of $V(\epsilon)$ is $B = \{(1, 1, 0)\}$. When the coalition $\{2, 3\}$ is no longer allowed to block, the core becomes all of the triangle ABC with the exception of a small set of points near C. Thus the middleman is worse off at most of the points of the new core. Moreover, by letting $\epsilon \to 0$ we can make the proportion of the points where the middleman loses go to one. When $\epsilon = 0$

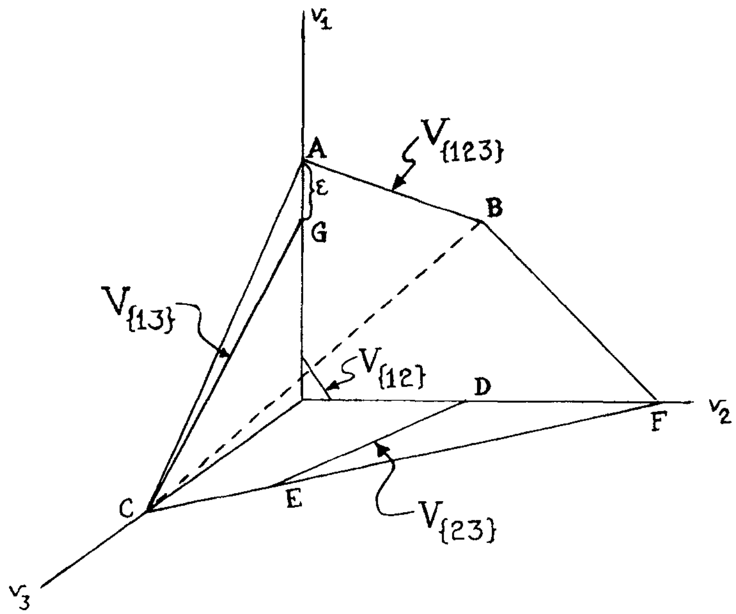

FIGURE 3

the middleman is never better off, and he is worse off at almost all the new points. Note, however, that when $\epsilon = 0$, V_N contains segments which are parallel to the v^2 axis. Note too that none of these games are balanced in the sense of Billera and Bixby [2], and so they could not come from markets with concave utility functions.

Thus, the condition that the game be balanced is crucial to the middleman not losing in three-person games. One might still hypothesize that Theorem 1 would continue to hold for market games with any finite number of players. However, the following example indicates that this is not the case.

Consider the economy with five goods and five people, A, B, C, D, and E, whose utility functions and endowments are as follows:

$$\begin{aligned}
U^A(x_1, x_2, x_3, x_4, x_5) &= x_1 x_2, & \omega^A &= (1, 0, 0, 0, 0),\\
U^B(x_1, x_2, x_3, x_4, x_5) &= x_1 x_2, & \omega^B &= (0, 1, 0, 0, 0),\\
U^C(x_1, x_2, x_3, x_4, x_5) &= (x_1 + x_5)\, x_3, & \omega^C &= (0, 0, 1, 0, 0),\\
U^D(x_1, x_2, x_3, x_4, x_5) &= (x_2 + x_5)\, x_4, & \omega^D &= (0, 0, 0, 1, 0),\\
U^E(x_1, x_2, x_3, x_4, x_5) &= (x_3 + x_4)(x_5 + 8), & \omega^E &= (0, 0, 0, 0, 1).
\end{aligned}$$

With all coalitions allowed, the coalition $\{A\ B\}$ can block any allocation unless $U^A + U^B \geqslant 1$. There are no feasible allocations for which this is possible except those in which A and B end up with all of the goods x_1 and x_2. Hence core allocations will be those in which the goods x_1 and x_2 are efficiently distributed among A and B and the goods x_3, x_4, and x_5 are divided among C, D, and E in such a way that no coalition can block the allocation.

Consider now the coalition $\{C\ E\}$. If C gets the bundle $(0, 0, \alpha, 0, 1)$ and E gets $(0, 0, 1 - \alpha, 0, 0)$, $U^C = \alpha$ and $U^E = 8(1 - \alpha)$. If $\alpha \geqslant 1/9$ the marginal rate of substitution for C of good 5 for good 3 is $1/\alpha \leqslant 9$. For E the marginal rate of substitution between goods 5 and 3 is $8/(1 - \alpha) \geqslant 9$, and hence this allocation of the goods of C and E is efficient with respect to them. Thus among the allocations which $\{C, E\}$ can block are any which give $U^C = \alpha \leqslant 1/9$ and $U^E < 8(1 - \alpha)$. Similarly $\{D, E\}$ can block any allocation which gives D, $U^D = \alpha \leqslant 1/9$ and $U^E < 8(1 - \alpha)$.

Now consider the coalition $\{C\ D\ E\}$. If C gets $(0, 0, 2\alpha, 0, 1/2)$ and D gets $(0, 0, 0, 2\alpha, 1/2)$, E gets $(0, 0, 2 - 4\alpha, 0)$. The utilities are $U^C = U^D = \alpha$ and $U^E = 8(2 - 4\alpha)$. If $\alpha \geqslant 1/18$, $MRS^C_{5,3} = (1/2)/2\alpha \leqslant 9/2$ while $MRS^E_{5,3} = 8/(2 - 4\alpha) > 9/2$. Thus the allocation is efficient with respect to C and E. The same argument shows that it is efficient with respect to D and E. It is trivially efficient with respect to C and D as well, and no reallocation among the three agents can make all of them better off. Thus this allocation between C, D, and E is efficient. Thus any allocation which gives utilities $U^C = U^D = \alpha \geqslant 1/18$ and $U^E = 8(2 - 4\alpha)$ cannot be blocked by $\{C\ D\ E\}$. However, if $\alpha > 1/3$ the coalition $\{C\ E\}$ (or $\{D\ E\}$) can block since the utility which E can be guaranteed while C gets $U^C = \alpha$ is $U^E = 8(1 - \alpha) > 8(2 - 4\alpha)$.

We now claim that at any core allocation, we must have $U^E \geqq 16/3$. To see this, note that if $U^E = \beta$, then $\{C\ D\ E\}$ can achieve $U^C = U^D = (16 - \beta)/32 = \alpha$ via the allocation given above, with $\alpha = 16 - \beta/32$. But if $\beta < 16/3$, this allocation is blocked by either $\{C\ E\}$ or $\{D\ E\}$, since in either E can get $8(1 - \alpha) = 8 - 8(16 - \beta)/32 = 4 + \beta/4 > \beta$, while the other member gets α, and it is then possible to redistribute so as to make both members strictly better off.

Now suppose we restrict blocking to only those coalitions containing E. Consider the allocation and associated utilities below:

$$
\begin{aligned}
x^A &= (0, 0, 0, 0, 0), & U^A &= 0,\\
x^B &= (0, 0, 0, 0, 0), & U^B &= 0,\\
x^C &= (1, 0, \tfrac{2}{3} + \epsilon, 0, \tfrac{1}{2}), & U^C &= 1 + \tfrac{3}{2}\epsilon,\\
x^D &= (0, 1, 0, \tfrac{2}{3} + \epsilon, \tfrac{1}{2}), & U^D &= 1 + \tfrac{3}{2}\epsilon,\\
x^E &= (0, 0, \tfrac{1}{3} - \epsilon, \tfrac{1}{3} - \epsilon, 0), & U^E &= \tfrac{16}{3} - 16\epsilon.
\end{aligned}
$$

We will show that for suitably small ϵ, this allocation is in the core as defined above of the corresponding game V'.

The coalitions $\{A\ E\}$, $\{B\ E\}$, and $\{A\ B\ E\}$ clearly cannot block this allocation. The coalitions $\{C\ E\}$ and $\{D\ E\}$ cannot block since these coalitions cannot give C or D utility greater than 1. The coalition $\{C\ D\ E\}$ similarly cannot block since they cannot simultaneously guarantee C and D utility greater than 1/2. We will now examine the marginal rates of substitution of C, D, and E to show that this distribution is efficient among them. $MRS^C_{5,3}$ is $(3/2)/(2/3 + \epsilon) < 9/4$, while $MRS^E_{5,3} = 8/(2/3 - 2\epsilon) > 12$. Thus this allocation is efficient with respect to C and E. Similarly it is efficient with respect to D and E and trivially so with respect to C and D. Further, no reallocation between the three agents dominates this allocation. Adding A and B to $\{C\ D\ E\}$ cannot yield a blocking coalition either. The only coalitions left to examine are those which contain C or D (but not both), E, and A and/or B.

Consider the coalition $\{A\ C\ E\}$. Can this coalition block? It will be enough to show that with their combined resources C and E cannot both improve upon their utilities in the proposed allocation. Consider the distribution $x^C = (1, 0, \alpha/2, 0, 1)$, $x^E = (0, 0, 1 - \alpha/2, 0, 0)$. This gives $U^C = \alpha$ and $U^E = 8(1 - \alpha/2)$. If $\alpha > 2/5$, $MRS^C_{5,3} = 2/(\alpha/2) = 1/\alpha < 5/2$ and $MRS^E_{5,3} = 8(1 - \alpha/2) > 5/2$, hence the distribution is efficient between them. In the proposed allocation we have $U^E = 16/3 - 16\epsilon > 8[1 - (1 + 3/2\epsilon)/2] = 8(1/2 - 3/4\epsilon)$ for $0 < \epsilon < 2/15$. Hence even with A's endowment, C and E cannot both improve upon the utilities in the proposed allocation. Clearly adding B to the coalition $\{A, C, E\}$ will not change this. Also it is clear that the same argument shows that the coalitions $\{B, D, E\}$ and $\{A, B, D, E\}$ cannot block the proposed allocation.

Thus the proposed allocation is in the core if only coalitions containing E are allowed to block. Yet this allocation is clearly worse for E than any previous core allocation (when all coalitions were allowed to block). In fact, his minimal utility over core allocations has gone down from 16/3 to $16/3 - 16(4/30) = 3.2$.

The preferences in this example are not strictly monotonic, as was assumed in Theorem 1. However, it is possible to modify the example so that this condition holds and yet there are still core allocations when E is a middleman which are worse for him than any point in the core of the original game. Specifically, consider modifying the above example by adding a term $r(\Sigma x_j)$ to each player's utility function, which then becomes strictly monotone for all $r > 0$. We show that for r sufficiently small, there are still core points when E is a middleman yielding $U^E < 16/3$.

Consider the allocation

$$\begin{aligned} x^A &= (\delta, \delta, 0, 0, 0), \\ x^B &= (\delta, \delta, 0, 0, 0), \\ x^C &= (1 - 2\delta, 0, 21/30, 0, 1/2), \\ x^D &= (0, 1 - 2\delta, 0, 21/30, 0, 1/2), \\ x^E &= (0, 0, 9/30, 9/30, 0). \end{aligned}$$

This corresponds to $\epsilon = 1/30$ in the example above, except that a small amount of x_1 and x_2 has been taken from C and D and given to A and B. Arguments similar to those used above show that for $0 < \delta < 1/84$, this allocation yields a utility vector that not only is in the core of the game V' with E as a middleman (and the original preference orderings), but also has the property that, for all coalitions $S \neq N$, it lies outside V_S. Note that $U^E \leqq 16/3$ at any such point.

Now, if we consider the games $V(r)$ and $V'(r)$ obtained by adding $r\Sigma x_j$ to each player's utility function, it is clear that for any point u^r in $V'_S(r)$ (resp., $V_S(r)$) there is some point $u \in V_S$ (resp., V'_S) with $u_i^r \leqq u_i + r \mid S \mid$, $i \in S$. Now consider sequences $\{V(r_k)\}$ and $\{V'(r_k)\}$, and let $u_k \in \text{core } V(r_k)$, $u_k \rightarrow u$. Then it is simple to show that $u \in \text{core } V$, so that for any $\epsilon > 0$ there exists K such that core $V(r_k)$ is contained in an ϵ neighborhood of core V for $k \geqslant K$. This in turn means that for any ϵ, the minimum of U^E on the core of $V(r_k)$ is at least $16/3 - \epsilon$ for all k large enough. If we can now show that for large enough k there are points in the core of $V'(r_k)$ which are arbitrarily close to the utility image of the allocation given above, we are done. To show this, take the utility image $\bar{u}$ of this allocation, and note that for k large enough (r_k small enough), $\bar{u}$ is not blocked by any $S \neq N$, since $V'_S(r_k)$ is within an ϵ-ball of V'_S. Now consider any Pareto optimal point u_k in $V'(r_k)$ with $u \leqq u_k$. (Such a point must exist since $U^i(x) \leqq U^i(x) + r\Sigma x_j$ for any

$x = (x_1, ..., x_5)$, with strict inequality if $x \neq 0$.) Since $V'_N(r_k)$ is within an ϵ-ball of V'_N, this sequence converges to u, and since u_k is unblocked in $V'(r_k)$, we are done.

It is worth noting, however, that although core $V'(r_k)$ contains points strictly worse for E than any in core $V(r_k)$, in contrast to the situation with V and V' it also contains points which are better for him than any in core $V(r_k)$. This is, in fact, generally true with strict monotonicity.

THEOREM 2. *Consider an economy where, for some trader i with strictly monotonic preferences, the initial endowment is not Pareto optimal for $S = N \sim \{i\}$. Then restriction of blocking to coalitions containing i must result in the addition of points to the core which i strictly prefers to any point in the core with unrestricted coalition formation.*

Proof. Consider a core allocation in the unrestricted game which gives the central trader utility $\bar{u}^i$ which is at least as high as any other core allocation At least one trader $j \neq i$ must have utility strictly higher than his initial endowment yields because ω is not Pareto optimal for $N \sim \{i\}$. Now consider a Pareto efficient allocation which gives i the maximal utility possible subject to the condition that the noncentral traders' utilities at least as big as their initial endowment utilities. Compactness of the feasible set and continuity of the utility functions guarantee that there is such an allocation and strict monotonicity implies that it yields $u^i > \bar{u}^i$. By construction, no coalition containing i can block this allocation; hence it is in the core when i is a middleman.

These examples of disadvantageous middlemen are reminiscient of the examples of disadvantageous monopolies or syndicates (see, e.g., [1]), although the phenomena do differ somewhat. Exploring the nature of the relationships between these two phenomena and, more generally, the monotonicity of the core would seem to be an important but difficult open problem. In any case, the rather surprising (to us, at least) nature of our results here would seem to indicate the value of further explorations of the communications graph approach.

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