JOURNAL OF ECONOMIC THEORY 16, 116–121 (1977)

Disadvantageous Monopolies and Disadvantageous Endowments

1. The term "disadvantageous monopoly" has been introduced by Aumann [1] in the following context. Consider an exchange economy with two commodities (α, β) and three types of traders (a, u, v). The traders form a measure space consisting of a single atom $\{a\}$ and a nonatomic part, called the "ocean," divided with equal measures between types u and v. The atom a owns the initial supply of one commodity (is a monopolist for that commodity). Aumann constructs examples where the core of the economy consists of (i) a single competitive allocation y and (ii) a set of noncompetitive allocations, all of which are less favorable for the atom than y. In such a case, the atom reaches the best outcome in the core if it behaves competitively, disregarding its monopoly position. Aumann uses the term "disadvantageous monopoly" to describe such a situation; in [1], he gives examples of disadvantageous monopolies that are "disturbing by their utter lack of pathology" [1]. Aumann's Example B is presented through the Edgeworth box in his Fig. 6, reproduced here as Fig. 1. The indifference curves of type v are parallel straight lines. Those of type u (resp. a) have kinks with locus C_u (resp. C_a). The single competitive equilibrium is at point $y = (\frac{1}{2}, 1)$, with prices such that $p_\alpha = 2p_\beta$. The core consists of a set

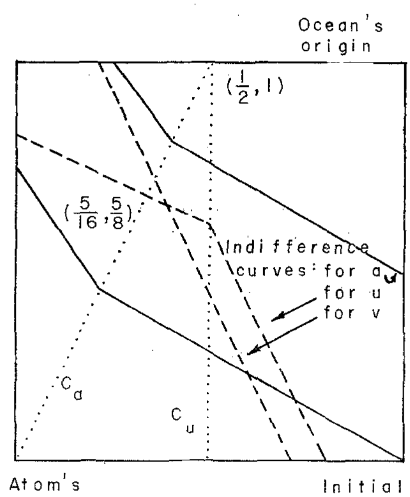

FIGURE 1

ISSN 0022-0531

of allocations assigning to the atom points in the line segment connecting y and $x_a = (\frac{5}{16}, \frac{5}{8})$. (For further details, see [1, pp. 6–7].)

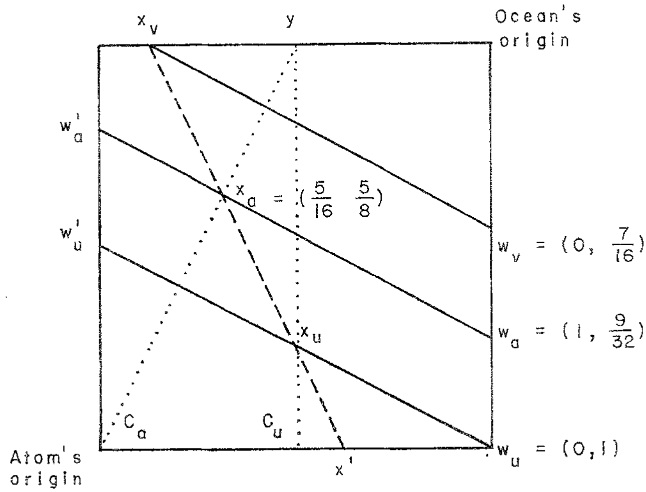

FIGURE 2

In this note, we first remark that every noncompetitive core allocation would appear as a competitive allocation if only some traders in the ocean were to transfer as a free gift a part of their initial endowment to the monopolist. This remark follows directly from Theorem A in Shitovitz's paper [6]. It is illustrated in Fig. 2, with reference to the core allocation assigning x_a to the atom. That allocation assigns to traders of type u (resp. v) the consumption x_u (resp. x_v) belonging to the indifference curve $x'x_uw_u'$ (resp. $x'x_v$). At prices such that $2p_\alpha = p_\beta$, the budget line of traders of type u is w_uw_u' and x_u is a best point for them in their budget set. Let now traders of type v transfer $\frac{9}{16}$ of their initial endowment to the atom. The budget line for type v is now w_vx_v, on which x_v is the best point. Because the measure of the atom is twice that of type v, the endowment of the atom after it receives the gift from v is at point w_a, and its budget line is w_aw_a'. Point x_a, belonging to the indifference curve $x'x_aw_a'$ is a best point on that budget line. Hence, (x_a, x_u, x_v) is a competitive allocation for the economy with initial resources defined by (w_a, w_u, w_v).[1] The gift from v to a has thus created a new competitive equilibrium which is better for v (and u), but worse for a, than the old competitive equilibrium y. We then say that the gift has generated a "disadvantageous endowment" for a. More formally:

[1] Many other price systems and initial endowments sustain (x_a, x_u, x_v) as a competitive allocation, but only those described in the text satisfy the conclusions of Shitovitz's theorem.

DEFINITION. In an exchange economy, a nonnegative transfer of initial resources (a "gift") from trader(s) T_1 to trader(s) T_2 generates a *disadvantageous endowment* if, after the gift, there exists a competitive allocation assigning to trader(s) T_2 a consumption less preferred than every consumption assigned to the same trader(s) by an allocation that was competitive before the gift.

DEFINITION. In an exchange economy with a single atom and a nonatomic part, the atom is a *disadvantageous monopoly* whenever the core of the economy consists of a single competitive allocation y and a set of noncompetitive allocations, all of which are less favorable for the atom than y.

PROPOSITION A. *Whenever the measure space of agents consists of a single atom $\{a\}$ and a nonatomic part, and the atom a is a disadvantageous monopoly, there exists a gift from the nonatomic part generating a disadvantageous endowment for a.*

This proposition asserts that disadvantageous endowments are no more unusual than disadvantageous monopolies. In fact, they are less unusual; we have indeed constructed examples of disadvantageous endowments in situations involving an atom that is not a disadvantageous monopolist.[2]

2. Does it follow that the atom should prefer to participate in the exchange economy described by Fig. 1 rather than in the exchange economy described by Fig. 2? Not necessarily. Indeed, starting from Fig. 1, *there exists a gift* from v to u *which is even more disadvantageous to a than the gift from v to a* described in Fig. 2. Such a gift is illustrated in Fig. 3, where the traders of type v have transferred to the traders of type u $\frac{9}{20}$ of their initial resources. The initial allocation is now (w_a, w_u, w_v) in Fig. 3. With prices such that $2p_\alpha = p_\beta$, the allocation (x_a, x_u, x_v) in Fig. 3 is a competitive allocation, with $x_a = (\frac{1}{5}, \frac{2}{5}) < (\frac{5}{16}, \frac{5}{8})$. It may thus be in a's interest to accept the initial allocation of Fig. 2 in order to prevent the gift from v to u leading to Fig. 3.

We have tried to verify the generality of this situation for a larger class of exchange economies. We succeeded only in proving that in an exchange economy with finitely many traders if there exists a gift from a set of traders T_1 to a single trader T_2 which generates a disadvantageous endowment

[2] One such example is obtained from an immediate extension of Aumann's Example B, by letting the measure of the atom drop from 1 to, say, $\frac{3}{4}$ and adding a nonatomic part identical to the atom, with measure $\frac{1}{4}$. It then follows from a theorem by Gabszewicz and Mertens [5] that the core shrinks to the single competitive allocation $(\frac{1}{2}, 1)$, leaving no room for any "disadvantageous monopoly" situation. Still, the transfer of resources from traders of type v to traders of type a, described in the second paragraph of this note, results in a "disadvantageous endowment" situation.

through a competitive allocation x, then there exists a gift from a subset of the traders in T_1 to other traders in T_1, and an allocation y which is competitive after this last gift and no more favorable for T_2 than x.

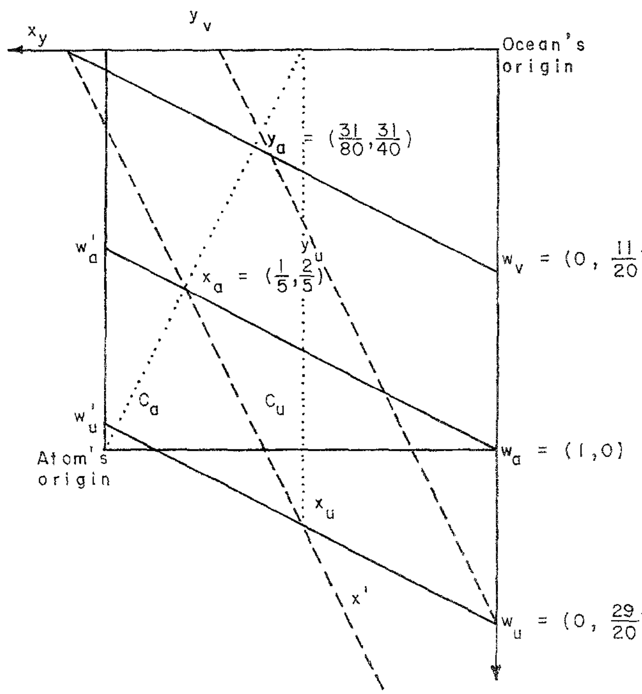

FIGURE 3

Remarks. (i) In the core of the economy described by Figs. 1 and 2, the atom gets anything on the line connecting $(\frac{5}{16}, \frac{5}{8})$ to $(\frac{1}{2}, 1)$. The gift from v to a changes the competitive allocation, but not the core. In the core of the economy described by Fig. 3, the atom gets anything on the line connecting $(\frac{1}{5}, \frac{2}{5})$ and $(\frac{31}{80}, \frac{31}{40})$. By a gift from v to u in the amount of $\frac{3}{5}$ (instead of $\frac{9}{20}$), the nonatomic part could restrict the core further, so that the atom would get at most $(\frac{7}{20}, \frac{7}{10})$, but still at least $(\frac{1}{5}, \frac{2}{5})$.

(ii) The possibility for a trader to replace a competitive equilibrium by another one more favorable for himself, through a free gift to another trader, has been recognized in [4]. An example with a similar flavor to the one given in [4] has more recently appeared in *The Journal of Mathematical Economics*, first with nonsmooth preferences [3] and again with smooth preferences [2].

3. In the examples above, the gifts generating disadvantageous

endowments always emanate from the nonatomic part of the traders. Once the possibility of "disadvantageous endowments" has been recognized, it is more likely that it will be used to advantage by monopolists than by traders behaving competitively. Indeed, it is somewhat unnatural to assume simultaneously (i) that traders behave competitively, and (ii) that the same traders become aware of the possibility of hurting the monopolist through a free gift. Moreover, there is nothing in the definition of disadvantageous endowments which guarantees a net gain for the "donors." More naturally, price or quantity setting monopolists might find it advantageous to make gifts, when these improve the exchange situation. Our last example shows that a price-setting monopolist may indeed find it advantageous to choose simultaneously a gift and a price, instead of choosing only a price.

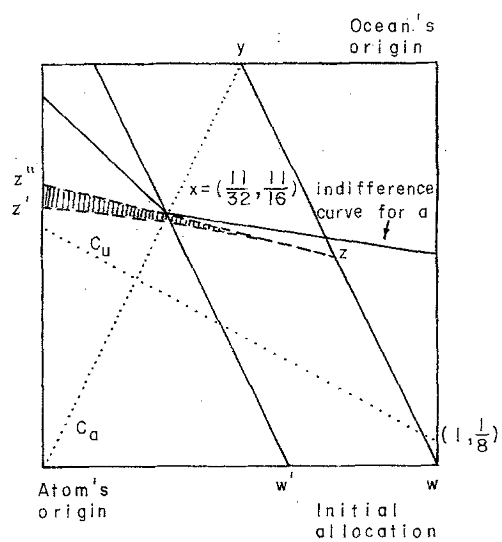

FIGURE 4

In Fig. 4, there is again a single atom a, with initial endowment (1, 0) and indifference curves slightly kinked along the line C_a; and there is a nonatomic part consisting of two types, u and v. Type v is exactly as in Aumann's Example B, with initial endowment (0, 1) and indifference curves which are parallel straight lines with $2\alpha + \beta =$ const; the line wzy in Fig. 4 is an example. Type u has initial endowment (0, 1) and indifference curves sharply kinked along the line C_u, with the more vertical segments (in the lower part of Fig. 4) parallel to the indifference curves of type v. Again, types u and v each have measure equal to half that of a.

With initial allocation at point w and preferences as described, the price setting monopolist is constrained to price ratios $2 \geqslant p_\alpha/p_\beta \geqslant 0.71$. At the price ratio 2, the ocean's demand is at any point on the line segment wz; at the price ratio 0.71, that demand is given by the point z'; at intermediate prices, it is given by points on the curve zz'.

If the atom kept the price ratio fixed at $p_\alpha/p_\beta = 2$, but transferred to the ocean part of its initial resources, the demand points would move along the line zz'' as the initial allocation moves along the horizontal axis.

The locus zz'' is in fact the envelop of all demands by the ocean given arbitrary gifts and prices; it can be shown geometrically that the line zz'' must dominate from above the curve zz'. Accordingly, free gifts widen the range of possible exchanges for the monopolist, from the locus zz' to the locus zz''. Under the assumed preferences, the best initial allocation for a nondiscriminating monopolist setting a price $p_\alpha/p_\beta = 2$ is at point w'. The allocation x obtained in that way dominates the price setting solution obtained without gifts, since this latter lies somewhere on the locus zz'. Furthermore, with the assumed preferences for $\{a\}$, both the solution x and the price setting solution are noncompetitive. The dominance of zz'' on zz' reveals that income effects are more powerful than substitution effects in generating demand for commodity α. There seems to be nothing pathological in the fact that a monopolist would use his knowledge of this fact to his own advantage.

References

1. R. J. Aumann, Disadvantageous monopolies, *J. Econ. Theory* **6** (1973), 1–11.
2. R. J. Aumann and B. Peleg, A note on Gale's example, *J. Math. Econ.* **1** (1974), 209–211.
3. D. Gale, Exchange equilibrium and coalitions: An example, *J. Math. Econ.* **1** (1974), 63–66.
4. J. Jaskold Gabszewicz and J. H. Drèze, Syndicates of traders in an exchange economy, *in* "Differential Games and Related Topics" (H. W. Kuhn and G. Szegö, Eds.), pp. 399–414, North-Holland, Amsterdam, 1971.
5. J. Jaskold Gabszewicz and J. -F. Mertens, An equivalence theorem for the core of an economy whose atoms are not "too" big, *Econometrica* **39** (1971), 713–721.
6. B. Shitovitz, Oligopoly in markets with a continuum of traders, *Econometrica* **41** (1973), 467–502.

Received: January 13, 1976; revised: November 13, 1976

Jacques H. Drèze
Jean Jaskold Gabszewicz
Andrew Postlewaite

CORE
De Croylaan 54
3030 Heverlee
Belgium

Printed in Belgium