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Notes, Comments, and Letters to the Editor

Disadvantageous Syndicates

By a *syndicate* of economic agents (see [2]), we mean a coalition of agents who have undertaken to operate as a single entity (e.g., a corporation or union). In the context of a formal economic or game model, a syndicate is treated as a single agent; no coalition is considered that contains some, but not all members of the syndicate. In the definition of core, for example, only the coalitions that either contain the syndicate or do not intersect it are considered (as potential "blocking" coalitions).

From the real world, as well as from economic theory, one is familiar with situations in which syndicates are economically advantageous. In fact, except for the direct costs of forming and operating such a syndicate, it appears difficult to imagine a situation in which syndication does not offer some advantages.

In our minds, though, the notion of the universal advantage of syndication has been called into some question by Aumann's examples of "disadvantageous monopolies" [1]. These are pure exchange economies with a nonatomic continuum of agents, in which there is a coalition S of traders whose syndication is "disadvantageous" in the following sense: When S is syndicated, there are core allocations that are worse for S than the core allocation x that is worst for S when it is unsyndicated. In some of the examples, moreover, none of the core allocations when S is unsyndicated are better for S than x. Under such circumstances, there seems to be considerable disincentive for the syndicate to form[1] (or incentive to disband, if formed). Economically, the examples are not transparent, and the conclusion seems so strange that Aumann (in his discussion) prefers to question the notion of core, rather than the advantageousness of syndication.

It is the purpose of this note partially to rehabilitate the core by presenting a simple example of a situation in which syndication is disadvantageous in Aumann's sense, but in which this disadvantageousness has obvious economic meaning and validity. Thus disadvantageous monopolies appear to constitute a valid economic phenomenon, not merely a peculiarity in the definition of the core.

[1] The examples are all the more surprising since the syndicate initially holds a corner on one of the two goods in the economy.

The rehabilitation is only partial because in our example there are only finitely many agents, and so in particular only finitely many unsyndicated agents. If one splits the unsyndicated agents into nonatomic continua, it turns out that the syndicate is no longer disadvantageous. In the traditional economic setup, one feels that a large part of the monopolist's advantage lies in the fact that he is facing a disorganized crowd of individually insignificant agents—i.e., a nonatomic continuum. To rehabilitate the core entirely, one would have to find an economically valid example of a disadvantageous monopoly, in which the syndicate faces a nonatomic continuum. This we have not succeeded in doing.

The example can be viewed either as a game or as a market (i.e., exchange economy). In the game there are five players, numbered 1–5. Set $M = \{1, 2\}$, $N = \{3, 4, 5\}$. Define the characteristic function v by

$$v(S) = \min(|S \cap M|, |S \cap N|/2),$$

where $|T|$ is the cardinality of T. There is precisely one point in the core, namely $(0, 0, \frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. If the members of N form a syndicate, we get a three-person game (with players $1, 2, N$) with characteristic function v^* defined by

$$v^*(S) = \begin{cases} 0, & \text{if } N \notin S \\ 1, & \text{if } S = \{1, N\} \text{ or } S = \{2, N\} \\ \frac{3}{2}, & \text{if } S = \{1, 2, N\}. \end{cases}$$

The core of this game is the convex hull of the four points $(0, 0, \frac{3}{2})$, $(\frac{1}{2}, 0, 1)$ $(0\ \frac{1}{2}, 1)$, and $(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$. Thus the most that the syndicate N can get in the core is $\frac{3}{2}$, and it may get less, whereas when unsyndicated N gets exactly $\frac{3}{2}$.

It is easily verified that v is completely balanced (i.e., every subgame has a nonempty core). Therefore it follows from a theorem of Shapley and Shubik [4] that it represents a market—in particular, a "side payment" (or "transferable utility") market. We can also describe such a market economy directly, without referring to [4]. Let A and B be completely complementary goods, i.e., useful only in equal quantities. Let players 1 and 2 initially hold one unit each of commodity A, and players 3–5 one-half unit each of commodity B. There is then an oversupply of commodity A, and therefore intense competition will develop between players 1 and 2 in the determination of terms of exchange with N. When N does not form a syndicate, this competition drives the payoff to 1 and 2 down to zero; any attempt by either player to get more will lead to the other one forming a coalition with two out of the three members of N and "underselling" him. But if the syndicate N forms, then though it can play off players 1 and 2 against each other as far as the first unit of payoff is concerned, the remaining half unit requires the cooperation of all three players (i.e., 1, 2, and N). The underselling cannot take place, and the result is core points with N receiving less than $\frac{3}{2}$.

If we split players 1 and 2 into nonatomic continua, the coalition N gets $\frac{3}{2}$ in the core whether or not it is syndicated; i.e., the syndicate ceases to be disadvantageous. Economically, what is happening is that not all the holders of commodity A are needed in order for society to obtain a total payoff of $\frac{3}{2}$. Thus competition develops among the holders of A whether or not N is syndicated, and this competition will drive their total payoff down to 0.

The utilities in the above example are nondifferentiable, but it is easy to find variants with differentiable utilities but essentially the same properties.[2]

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References

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[2] Samet [3] has proved that in a "side payment market" (see [4]) with a nonatomic continuum of agents, whose utility functions are differentiable, no coalition can lose by forming a syndicate; i.e., disadvantageous monopolies in Aumann's sense are impossible for differentiable side payment markets. Of course, our example bears this out.