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Asymmetric Information Bargaining Problems with Many Agents

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A yes or no decision must be made about some issue. All agents must agree. The "Coase Theorem" asserts that the efficient outcome will always result. Suppose the value (positive or negative) that an individual attaches to an affirmative decision is privately known to that individual. It is proved, under very mild conditions, that with independent types, as the number of agents increases, the probability of an affirmative efficient decision goes to zero. An example in which it is common knowledge that an affirmative decision is efficient and yet the probability of such a decision goes to zero is given.

1. INTRODUCTION

There are many problems in which a group has to take a collective action and allocate individual costs or benefits of that action. The most pervasive example of such a problem is whether to undertake a public project and, if undertaken, how to distribute the costs of the project among the members of the group. Simple cost-sharing rules, such as equal division of the cost, may leave some individuals with negative net benefits (that is, the value of the project to this individual may be less than his or her share of the cost). There is sometimes a government that has the authority to compel participation, and the consequent sharing of the cost, in the project. In the absence of such a governmental authority (or similar quasi-governmental institution), each individual must agree to participate; in other words, each individual has veto power over any proposed plan.

While the most obvious problem in which agents are required to agree unanimously to changes from the status quo is the provision of a public good, other problems have essentially the same structure. Examples include oil field unitization, the enclosure of open fields and the drainage of marshland in 17–18th century France, a group of workers deciding on whether to leave their firm and establish a new venture (in which case the new wages must be agreed upon), the construction of a production facility generating pollution, and the establishment of a monopoly from separate operating units. We will present our model and discuss our results in terms of the provision of a public good; the model applies with no substantive changes to other variants of the bargaining problem. Section 5 contains a discussion of some of these other variants.

If the values the individuals place on the public project are public information, the constraint that each individual's net benefit be nonnegative will present no real problem. As long as the total benefits from the project are greater than the cost of the project, there will be many cost-sharing rules that leave each individual with a positive net benefit;

sharing the cost in proportion to the valuation of the public project is a simple example of such a rule. But for many problems, individuals' valuations are not public information. Individuals may have some information about others' valuations, but this information will constitute less than certain knowledge of the valuations.

Our intent is to examine the possibility of undertaking public projects in environments in which individuals' valuations are incompletely known and in which no agency can coerce individuals to participate in projects against their will. We will model the incomplete information about the agents' valuations in what has become a standard way: it is assumed that each agent's valuation is the realization of an independent random variable over possible valuations. The distribution of the random variable is common knowledge, but the realization is known only to the agent. Only in trivial cases will the minimum contribution by each agent (i.e. the contribution equal to the minimum possible valuation for that agent) be sufficient to cover the cost of the project. In general, at least some agents must contribute an amount greater than this minimum. But which agents? For many problems the only solution will be for those agents with relatively high valuations to contribute more than those agents with lower valuations. The problem is then to design a mechanism that will induce agents to reveal their valuations. Not surprisingly it is not always possible to design efficient mechanisms, that is, mechanisms that ensure that a project is undertaken with probability 1 in the case that it would be undertaken with certainty had the asymmetry in information not existed.

In general there will be mechanisms in which the probability that the project is undertaken is positive when it would be undertaken with certainty in the absence of the asymmetric information. We show, however, that when the minimum contribution by each agent is insufficient, this probability will go to zero as the size of the group gets large. Thus even if the total net benefits of undertaking the project are large (and increasing as the group gets large), the larger group makes the mechanism design problem more difficult, and asymptotically, impossible. The intuition for our result is as follows. In order to provide the good, some agents must contribute more than their minimum contribution. The presence of veto power implies an agent can lower his or her contribution by announcing a low willingness to pay. The only countervailing incentive not to lie is the possibility that the agent is "pivotal", in the sense that the announcement of a low valuation lowers the probability of provision. However, in a large economy, the probability that any agent is pivotal will be small. Thus, net utility must be nearly constant for all agents and so the probability of provision must be close to zero.

Perhaps more strikingly, if the agents' valuations are correlated, it can be *common knowledge* that the project should be undertaken, and yet as the economy gets large, the probability that the project is undertaken goes to zero (an example of such a case is given in Section 4 of the paper).

Our result is in strong accord with the many examples of the failure to undertake certain public projects, even though there appeared to be clear social gains from the projects (see Section 5). We leave to Section 5 a more detailed discussion of our results and the relationship to other literature.

We should mention here the closely related work of Rob (1989). Rob studies a model in which a firm must decide whether to build a polluting plant when each affected resident has a veto, so that compensation must be paid. The firm chooses the mechanism that maximizes expected profits. Rob proves that as the number of residents tends to infinity, the probability of building the plant and the ratio of realized to potential welfare under the firm's optimal mechanism tend to zero. The major difference between our work and Rob's is that in showing that the probability of provision converges to zero, we consider

all mechanisms, rather than just the firm's optimal mechanism. Also, since we do not require that all agents' valuations be drawn from the same distribution, our result is more general.

2. PUBLIC GOOD PROVISION MECHANISMS

We now introduce the formal elements of the model. The economy consists of n agents. The cost of providing the project is $C(n)$. Denote agent i 's valuation of the project by v_i . An agent's valuation is known only to that agent. The priors of the other agents about v_i are identical and given by the distribution function F_i with support $V_i \subset \mathbb{R}_+$.

Since the valuations are private information, a scheme that will separate agents according to their valuations must be devised. Suppose for the moment that there are only two possible valuations. The scheme needs to separate those agents with low valuations who are not willing to contribute substantially from those agents with high valuations who are willing to contribute substantially (if the alternative is no public project). In other words, the resulting contributions, or tax profile, must be incentive compatible: neither the high- nor the low-valuation agents can prefer the allocations the others receive to those they themselves receive. The problem then is how not to overtax the low-valuation agents while ensuring that the high-valuation agents prefer their own allocation. An agent who reveals himself to be a high type when all the other agents are revealing their types truthfully must be taxed at a rate that makes him as well off as if he had claimed to be low. If by claiming to be low, he can be sure of being taxed at the low rate, then all agents must be taxed the same. On the other hand, it may be possible to make provision of the public good a function of the number of those claiming low valuations, so that a high-type agent runs the risk of lowering the probability of provision when he claims to be high. In this case the tax of the high type need not be as low as that of a low type; it can be somewhat higher, since the high type will trade off the lower probability of provision with the lower tax that results from announcing low.

Rather than explicitly modelling the process by which provision is decided, we will study direct revelation mechanisms in which the probability of provision and the schedule of taxes are determined as functions of agents' reported valuations, denoted $\hat{v} \equiv (\hat{v}_1, \dots, \hat{v}_n)$. By the revelation principle (see, e.g. Myerson (1985)), the restriction to direct revelation mechanisms is without loss of generality; any outcome associated with an equilibrium of some process (game) will also be an equilibrium outcome of some revelation mechanism in which the agents report their private information (here, their valuations) truthfully. A *mechanism* is a pair (ρ, ξ) , where $\rho: \prod_i V_i \rightarrow [0, 1]$ gives the probability of provision as a function of the vector of reports and $\xi: \prod_i V_i \rightarrow \mathbb{R}_+^n$ gives the vector of taxes as a function of this vector of reports.¹ We will refer to any function mapping a vector of reports into $[0, 1]$ as a *provision rule*. Let E_{-i} denote the expectation operator with respect to $v_{-i} \equiv (v_1, \dots, v_{i-1}, v_{i+1}, \dots, v_n)$.

While the restriction to direct revelation mechanisms is without loss of generality, the assumption that the outcome space can be identified with the probability of provision and the vector of taxes is not. In particular, it does not adequately capture the scenario in which agents are bargaining through time and agents have *different* discount rates. The mechanisms studied in this paper *do* cover the scenario in which agents are bargaining through time and agents have a *common* discount rate. We discuss this issue in Section 5.1.

1. In this formulation taxes are only paid if the project is undertaken. The theorems in Section 4 are unaltered by allowing the mechanism to tax even if the project is not undertaken.

By assumption, an agent's valuation is private information, known only to him or her. For a mechanism to induce truthful reporting, it must be *incentive compatible*:

$$(IC) \quad E_{-i}\rho(v)(v_i - \xi_i(v)) \geq E_{-i}\rho(\hat{v}_i, v_{-i})(v_i - \xi_i(\hat{v}_i, v_{-i})), \quad \forall v_i, \hat{v}_i \in V_i.$$

Nothing in the definition of a mechanism guarantees that the proposed allocation is feasible in the sense that total taxes are sufficient to cover the costs of provision. There are two possible feasibility conditions. The first condition, relevant if the society does not have access to risk-neutral credit markets, is *ex post budget balance*:

$$(EXPBB) \quad \rho(v) \left(\sum_i \xi_i(v) - C(n) \right) \geq 0 \quad \forall v \in \Pi_i V_i.$$

The second and (potentially) weaker condition, relevant if the society does have access to risk-neutral credit markets, is *ex ante budget balance*:

$$(BB) \quad E\rho(v) \left(\sum_i \xi_i(v) - C(n) \right) \geq 0.$$

In the present context, for the continuum of types model of Section 3, *ex ante budget balance* implies *ex post budget balance* in the sense that for any mechanism satisfying incentive compatibility, *ex ante budget balance*, and interim individual rationality (see below), a contribution scheme exists that, with that mechanism's provision rule, preserves incentive compatibility and interim individual rationality and satisfies *ex post budget balance* (see Theorem 1).

We also require that no agent wish to veto provision. There are two possible voluntary participation, or *individual rationality*, conditions that we could impose. The stronger condition requires that no agent be made worse off by provision for any realization of the vector of the agents' valuations. This is an *ex post individual rationality* condition:

$$(EXPIR) \quad \rho(v)(v_i - \xi_i(v)) \geq 0, \quad \forall v \in \Pi_i V_i.$$

The weaker condition (which is the one we will impose) requires that no agent be made worse off by provision in expectation. This is *interim individual rationality*:

$$(INTIR) \quad E_{-i}[\rho(v)(v_i - \xi_i(v))] \geq 0, \quad \forall v_i \in V_i.$$

Low-valuation agents must be taxed at a low rate so that the participation constraint will not be violated. High-benefit agents can report that they are low-benefit in order to be taxed at a low level. In order to provide the correct incentives for a higher-benefit agent to report honestly, his tax must make him *strictly* better off with provision and the probability of provision must decrease if he reports that he has a lower benefit.

3. MECHANISMS WITH A CONTINUUM OF TYPES

We assume that v_i is independently distributed on $V_i \equiv [v_i, \bar{v}_i]$ with a strictly positive density, f_i . The techniques used in this section are similar to those used in Myerson (1981) and Myerson and Satterthwaite (1983). Let $\rho_i(v_i) \equiv E_{-i}\rho(v)$ and $\xi_i(v_i) \equiv E_{-i}\rho(v)\xi_i(v)$. An agent's utility in the mechanism (ρ, ξ) is given by $U_i(v_i) \equiv E_{-i}\rho(v)(v_i - \xi_i(v)) = \rho_i(v_i)v_i - \xi_i(v_i)$. Note that nothing is altered by interpreting ξ_i as the expected tax and allowing the mechanism to tax even if the project is not undertaken. In that case $U_i(v_i) \equiv E_{-i}(\rho(v)v_i - \xi_i(v))$ and a few expressions below must be changed appropriately. Incentive compatibility and interim individual rationality can be written as

$$(IC) \quad U_i(v_i) \geq \rho_i(\hat{v}_i)v_i - \xi_i(\hat{v}_i), \quad \forall v_i, \hat{v}_i \in [v_i, \bar{v}_i],$$

and

$$(INTIR) \quad U_i(v_i) \geq 0, \quad \forall v_i \in [\underline{v}_i, \bar{v}_i].$$

Since the first lemma and theorem are standard applications of mechanism-design techniques, their proofs are omitted.

Lemma 1. *The mechanism (ρ, ξ) is incentive compatible if, and only if, $\rho_i(v_i)$ is increasing in v_i for all i , and*

$$U_i(v_i) = U_i(\hat{v}_i) + \int_{\hat{v}_i}^{v_i} \rho_i(v) dv, \quad \forall v_i, \hat{v}_i \in [\underline{v}_i, \bar{v}_i]. \quad (1)$$

Incentive compatibility implies, from (1), that U_i is increasing in v_i , so $U_i(v_i) \geq 0$ is necessary and sufficient for interim individual rationality.

Theorem 1. *Suppose ρ is a provision rule such that ρ_i is increasing for each i . There exists a contribution scheme ξ such that (ρ, ξ) satisfies IC, BB and INTIR if, and only if,*

$$\int \cdots \int \left(\sum_i \{v_i - (1 - F_i(v_i))/f_i(v_i)\} - C(n) \right) \rho(v) \prod_k f_k(v_k) dv_k \geq 0. \quad (2)$$

Furthermore, if (2) is satisfied then ξ can be chosen so that EXPBB is satisfied.²

The *ex post efficient* provision rule is $\rho(v) = 1$ if $\sum_i v_i > C(n)$ and 0 otherwise, i.e. provide the public good if, and only if, there is sufficient social benefit to pay for the public good. It is easy to check that, even for $n = 2$, the *ex post efficient* provision rule violates (2). The *ex post inefficiency* of mechanisms that satisfy IC, INTIR, and BB is not surprising. Myerson and Satterthwaite (1983) have shown that there are no *ex post efficient* bargaining mechanisms in the canonical bilateral bargaining model in which the two traders have private valuations for the object.³

Call the term on the left hand side of (2) the *expected virtual surplus*. This is the expected surplus generated by the provision rule, ρ , after the agent valuations have been decreased by the factor “ $1 - (1 - F_i(v_i))/f_i(v_i)$ ”. This adjustment yields a smaller expected surplus and is due to the private nature of agents’ valuations. To illustrate this, consider mechanisms (ρ, ξ) that satisfy budget balance and *ex ante* individual rationality:

$$(EXAIR) \quad \int U_i(v_i) f_i(v_i) dv_i \geq 0.$$

Mechanisms that satisfy BB and EXAIR describe a scenario in which agents must decide on participation before they are aware of their own valuations and when the valuations are realized, they are public. Given a provision rule, ρ , a necessary and sufficient condition for there to be a contribution scheme, ξ , such that (ρ, ξ) satisfies BB and EXAIR is simply that the expected surplus generated by the provision rule be nonnegative, i.e.

$$\int \cdots \int \left(\sum_i v_i - C(n) \right) \rho(v) \prod_k f_k(v_k) dv_k \geq 0.$$

2. Given ρ , the contribution scheme expresses $\rho(v)(\xi_i(v) - C(n)/n)$, for $\rho(v) \neq 0$, by an equation similar to that used in Cramton, Gibbons, and Klemperer (1987). Details are in the CARESS working paper version of this paper.

3. Cramton, Gibbons, and Klemperer (1987) have extended this result. If n agents have private valuations for an object that is owned entirely by one of them, there is no *ex post efficient* mechanism for reallocating ownership of the object.

(Necessity is easy to check. Sufficiency follows from the payment scheme $\xi_i(v) = C(n)/n + v_i - \sum v_j/n$ for $\rho(v) \neq 0$.) If valuations are never private information, a provision rule is viable if, and only if, in ex ante terms it dominates (for the agents) the status quo. Thus, the adjustment factor in the expected virtual surplus captures the social cost (i.e. to the agents) of the private nature of the valuations. Ex ante domination is not sufficient for the viability of provision rules when valuations are privately known.

4. ASYMPTOTIC IMPOSSIBILITY

First we note that in the *replica* case the probability that the public good is provided is monotonically decreasing in the size of the economy.

Lemma 2. *Suppose v_i is identically distributed for all i and n , with density f and distribution function F . Assume the per capita cost of provision is constant. Then, for any mechanism satisfying IC, BB, and INTIR for the n -agent economy, there exists a mechanism for the m -agent economy, $m < n$, with the same probability of provision.*

Proof. Denote the constant per capita cost of provision by c . First, let (ρ, ξ) be an anonymous mechanism satisfying IC, BB and INTIR for the n -agent economy. From Theorem 1, we have that

$$\int \left(v_i - (1 - F(v_i))/f(v_i) - c \right) \rho_i(v_i) f(v_i) dv_i \geq 0. \quad (3)$$

Partition the agents into two groups, with m agents in the first group. We will denote the first group by a superscript 1, and the second group by a superscript 2. Thus $v = (v^1, v^2)$. Define $\rho^1(v^1) = E^2 \rho(v^1, v^2)$. Since $\rho_i^1(v_i^1) = \rho_i(v_i^1)$, (3) is trivially satisfied with ρ^1 replacing ρ . But (3) is equivalent to (2) for anonymous mechanisms and so there exists a contribution scheme ξ^1 such that (ρ^1, ξ^1) satisfies IC, BB and INTIR. Clearly (ρ^1, ξ^1) has the same probability of provision as (ρ, ξ) .

Given a non-anonymous mechanism, we can define an anonymous mechanism with the same probability of provision by first randomly choosing a permutation of the agents' names (each permutation has probability $1/n!$) and applying the non-anonymous mechanism to the relabeled economy. ||

In the following theorem, F_i^n is the distribution function of the i -th agent's valuation in the n -th economy of the sequence. The following theorem covers the regular case, i.e. when "virtual utility" is strictly increasing. Appendix 1 contains a more general result (due to a referee) which eliminates condition (i) and provides a rate of convergence.⁴ Note that weakening condition (i) is not a trivial exercise. In contrast to Myerson (1981), convexification of an appropriately chosen function does not allow the rest of the proof of the regular case to be applied without change.

The necessity of condition (ii) in the next theorem is illustrated by the following example. The n -th economy in the sequence is described by $C(n) = n$, with the i -th agent's valuation satisfying $v_i^n = 0$, $\bar{v}_i^n = 2$, $f_i^n(v_i) = 1/n$ for $v_i \in [0, 1]$ and $f_i^n(v_i) = (n-1)/n$ for $v_i \in (1, 2]$. Conditions (i), (iii), and (iv) hold. Consider the mechanism which specifies that agents reporting a valuation above 1 contribute 1 if the project is undertaken and the project is undertaken only if all agents report a valuation above 1. Note that all

4. The proof of Theorem 2 has the advantage that it uses substantially more elementary arguments than the proof in Appendix 1, as well as being in the spirit of the related work of Myerson and Satterthwaite (1983) and Cramton, Gibbons, and Klemperer (1987).

agents are pivotal. This mechanism is incentive compatible, individually rational, and budget balancing. The probability that the project is undertaken is given by $(1 - n^{-1})^n$, which converges to $e^{-1} > 0$. The necessity of condition (iv) is illustrated by taking $\bar{v}_i^n = 1 - n^{-1}$, $\bar{v}_i^n = 2 - n^{-1}$, $f_i^n(v_i) = 1$ for $v_i \in [1 - n^{-1}, 2 - n^{-1}]$, $C(n) = n$, and applying the same mechanism.

Theorem 2. *Suppose $\{C(n), F^n\}$ is a sequence of economies, where $F^n = (F_1^n, \dots, F_n^n)$, such that*

- (i) $v_i - (1 - F_i^n(v_i))/f_i^n(v_i)$ is strictly increasing in v_i ,
- (ii) there exists $\kappa > 0$ such that $f_i^n(v_i) > \kappa$,
- (iii) there exists v^* such that $\bar{v}_i^n < v^*$, and
- (iv) there exists $\varepsilon > 0$ such that $\sum_j \bar{v}_j^n + n\varepsilon < C(n)$,

for all i and n . The maximum probability that the public good is provided, $r(n) \equiv \sup \{E\rho(v) : \exists \xi, (\rho, \xi) \text{ satisfies IC, BB, and INTIR in the } n\text{-agent economy}\}$, converges to zero as n goes to infinity. Furthermore, if

- (v) there exists $\delta > 0$ such that $C(n) + n\delta < \sum_i E_i^n v_i$ for all n ,

the probability that the public good should be provided goes to one.

Proof. Observe that if $C(n) \geq \sum_i \bar{v}_i^n$, the public good cannot be provided by (BB) and (INTIR), so assuming $C(n) < \sum_i \bar{v}_i^n$ is without loss of generality. Let $\beta_i^n(v_i) \equiv v_i - (1 - F_i^n(v_i))/f_i^n(v_i)$, $c(n) \equiv C(n)/n$, and define, for $\alpha \geq 0$,

$$\rho^n(v; \alpha) \equiv \begin{cases} 1, & n^{-1} \sum_i \beta_i^n(v_i) + \alpha \geq c(n), \\ 0, & \text{otherwise.} \end{cases}$$

Arguments similar to Myerson and Satterthwaite (1983, Theorem 2) show that there exists an $\alpha^*(n) > 0$ so that $\rho^n(v; \alpha)$ satisfies (2) with equality, and that $\rho^n(v; \alpha^*(n))$ is the provision rule that maximizes the probability of provision. The maximum probability of provision is given by $r(n) \equiv E\rho^n(v; \alpha^*(n)) = \Pr[\sum_i \beta_i^n(v_i) + n\alpha^*(n) \geq C(n)]$.

Since the expected value of the virtual utility $\beta_i^n(v_i)$ is v_i , Chebyshev's inequality implies that $\Pr[\sum_i \beta_i^n(v_i) \geq C(n)]$ goes to zero. The remainder of the proof focuses on the behaviour of $\alpha^*(n)$.

Let $G_n(\alpha)$ be the per capita expected virtual surplus (i.e. the left-hand side of (2) divided by n) under $\rho^*(v; \alpha)$, i.e. $G_n(\alpha) = E[(\eta_n(v) - c(n))\rho^n(v; \alpha)]$, where $\eta_n(v) \equiv n^{-1} \sum_i \beta_i^n(v_i)$.

We now argue that as $n \rightarrow \infty$, $E\rho^n(v; \alpha^*(n)) \rightarrow 0$. Let σ^2 denote an upper bound on the variances of $\beta_i^n(v_i)$ for all i , which exists by (ii) and (iii). Suppose first that $\alpha^*(n) \rightarrow 0$ as $n \rightarrow \infty$. For large n , $\alpha^*(n) < \varepsilon$, and so applying Chebyshev's inequality, $\Pr[\eta_n(v) + \alpha^*(n) \geq c(n)] \leq \sigma^2/n(\varepsilon - \alpha^*(n))^2 \rightarrow 0$, as $n \rightarrow \infty$. Thus, if $\alpha^*(n) \rightarrow 0$ then $E\rho^n(v; \alpha^*(n)) \rightarrow 0$ as $n \rightarrow \infty$.

Suppose $E\rho^n(v; \alpha^*(n)) \not\rightarrow 0$, which implies that $\alpha^*(n) \not\rightarrow 0$. Since $\{\alpha^*(n)\}$ lies in a compact set and we are arguing to a contradiction, we can assume $\alpha^*(n)$ converges to some $\hat{\alpha} \neq 0$. Let $B(n)$ be the set of valuations for which there is provision, i.e. $B(n) \equiv \{v : \eta_n(v) - c(n) \geq -\alpha^*(n)\}$. For $m > 0$, partition $B(n)$ into $D(n)$, $A(n; m)$, and $A'(n; m)$, where $D(n) \equiv \{v : \eta_n(v) - c(n) \geq 0\}$, $A(n; m) \equiv \{v : 0 > \eta_n(v) - c(n) > -\alpha^*(n)/m\}$, and $A'(n; m) \equiv \{v : -\alpha^*(n)/m \geq \eta_n(v) - c(n) \geq -\alpha^*(n)\}$. Now (using Chebyshev's inequality), $\Pr[D(n)] \rightarrow 0$ as $n \rightarrow \infty$. Since $0 < G_n(0) \leq (n^{-1} \sum_i \bar{v}_i^n - c(n)) \Pr[D(n)]$, $G_n(0) \rightarrow 0$ as $n \rightarrow \infty$. Since $0 = G_n(\alpha^*(n)) =$

$$G_n(0) + \int \dots \int_{A(n)} (\eta_n(v) - c(n)) \prod_k f_k^n(v_k) dv_k,$$

where $A(n) = A(n; m) \cup A'(n; m)$, the second term is negative and converges to 0 as $n \rightarrow \infty$. The integrand is negative on $A(n)$, so

$$0 \cong \int \cdots \int_{A'(n; m)} (\eta_n(v) - c(n)) \Pi_k f_k^n(v_k) dv_k \rightarrow 0, \quad \text{as } n \rightarrow \infty.$$

But the above integral is less than $-\alpha^*(n)/m \Pr[A'(n; m)]$ and $\alpha^*(n) \rightarrow \hat{\alpha} \neq 0$, so $\Pr[A'(n; m)] \rightarrow 0$.

Thus $E\rho^n(v; \alpha^*(n)) \rightarrow 0$ requires $\Pr[A(n; m)] \rightarrow 0$ as $n \rightarrow \infty$. Let $\bar{\alpha}$ be an upper bound for $\alpha^*(n)$ and fix $m > 2\bar{\alpha}/\varepsilon$. Then $c(n) - n^{-1} \sum_i v_i^n - \alpha^*(n)/m > \alpha^*(n)/m > 0$, for all n . Applying Chebyshev's inequality again yields the sequence of inequalities: $\Pr[A(n; m)] \leq \sigma^2/n(c(n) - n^{-1} \sum_i v_i^n - \alpha^*(n)/m)^2 \leq m^2 \sigma^2/[n(\alpha^*(n))^2]$. The last term converges to zero, since m is fixed and, by hypothesis, $\alpha^*(n) \rightarrow \hat{\alpha} > 0$. This is a contradiction, and so $E\rho^n(v; \alpha^*(n)) \rightarrow 0$ as $n \rightarrow \infty$.

Finally, the probability that the public good should not be provided is

$$\Pr[n^{-1} \sum_i v_i < c(n)] \leq \Pr[|n^{-1} \sum_i (v_i - E_i^n v_i)| \geq n^{-1} \sum_i E_i^n v_i - c(n)] \leq s^2/[n\delta^2] \rightarrow 0,$$

where s^2 is an upper bound on the variances of v_i , so that the probability that the public good should be provided is going to 1 as $n \rightarrow \infty$. ||

The issue we now address is whether a subsidy can ameliorate the asymptotic inefficiency. If the *per capita* subsidy required to implement the ex post efficient provision rule becomes asymptotically small, then the asymptotic inefficiency displayed in Theorem 2 would not be of as great a concern as otherwise. It turns out that the *per capita* subsidy needed is not only bounded away from zero, but is equal to the difference between the *per capita* cost of provision and the average lowest possible valuation (when the *per capita* cost of provision is a constant and all agents have the same lowest possible valuation).

Corollary. Suppose $\{C(n), F^n\}$ is a sequence of economies satisfying conditions (i)-(iv) of Theorem 2. Let $r^*(n, s) = \sup\{E\rho(v): \exists \xi, (\rho, \xi) \text{ satisfies IC, BB and INTIR in the } n\text{-agent economy, when there is a per capita subsidy of } s\}$. If $r^*(n, s(n))$ does not converge to zero as $n \rightarrow \infty$, where $\{s(n)\}_n$ is a sequence of per capita subsidies, then $\limsup_{n \rightarrow \infty} s(n) - n^{-1}\{C(n) - \sum_i v_i^n\} \geq 0$. Furthermore, if condition (v) of Theorem 2 is satisfied, the ex post efficient provision rule can be implemented with a subsidy $\tilde{s}(n)$ satisfying $\lim_{n \rightarrow \infty} |\tilde{s}(n) - n^{-1}\{C(n) - \sum_i v_i^n\}| = 0$.

Proof. A total subsidy of $ns(n)$ implies a cost of provision for the n -agent economy of $C(n) - ns(n)$. Suppose $r^*(n, s(n))$ does not converge to zero as $n \rightarrow \infty$. Then, $\forall N, \forall \varepsilon > 0, \exists n > N$ such that $C(n) - ns(n) - \sum_i v_i^n \leq n\varepsilon$ (otherwise, by Theorem 2, $r^*(n, s(n)) \rightarrow 0$). That is, $\liminf_{n \rightarrow \infty} n^{-1}\{C(n) - \sum_i v_i^n\} - s(n) \leq 0$ and the first part of the corollary follows upon multiplication by -1 .

The ex post efficient provision rule can be implemented if

$$\int \cdots \int_{A(n)} (\sum_i \beta_i^n(v_i) - C(n) + S(n)) \Pi_k f_k^n(v_k) dv_k \geq 0,$$

where $\beta_i^n(v_i) = v_i - (1 - F_i^n(v_i))/f_i^n(v_i)$ and $A(n) = \{v_i, v_i \geq C(n)\}$. This inequality is equivalent to

$$S(n) \geq (\Pr[A(n)])^{-1} \left(C(n) - \sum_i v_i + \int \cdots \int_{A^c(n)} \left(\sum_i \beta_i^n(v_i) - C(n) \right) \Pi_k f_k^n(v_k) dv_k \right),$$

where $A^c(n)$ is the complement of $A(n)$. Defining $n\tilde{s}(n)$ to be the right hand side of the above inequality, a *per capita* subsidy of $\tilde{s}(n)$ will then implement the ex post efficient

provision rule. Since $\beta_i^n(v_i) - n^{-1}C(n)$ is uniformly bounded and, as $n \rightarrow \infty$, $\Pr[A(n)] \rightarrow 1$, the corollary is proved. \parallel

The degree of inefficiency can be even more dramatic if there is some correlation in agents' valuations: it can be *common knowledge* that the public good should be provided, and yet as the economy gets large the probability that the public good will be provided vanishes. Consider an economy consisting of m groups, each group containing $L+1$ agents. Each agent's valuation is either h or H , with $h < H$. No more than one agent in each group has a valuation of h . Valuations are independent across groups and each group has the same probability of having a low-valuation member. Suppose $(n-m)H + mh > C(n)$, so that it is common knowledge that the public good should be provided.

Agents are indistinguishable; in particular it is not possible to determine to which group any particular agent belongs. Assume $c \equiv C(n)/n$, the cost per agent of provision is constant, and $h < c$, i.e. it is not feasible to provide the public good and tax all agents as if they had the low valuation.

It is shown in Appendix 2 that, if there is a bound T (independent of n) on the tax that an individual can be forced to pay, the probability that the public good is provided goes to zero as the economy gets large. A bound on the maximum tax an individual can be forced to pay is a natural assumption to impose for some problems and in other contexts is automatically satisfied (see, e.g. the worker problem discussed in Section 5 and in Mailath and Postlewaite (1990)). If there is no bound on the maximum tax, it is possible to provide the public good (see Appendix 2).

5. DISCUSSION

5.1. Inefficiency as delay to agreement

A natural question is how the inefficiency described in the earlier sections is manifested in a game with time.⁵ Let $G^n \equiv (\Gamma^n, C(n), F_1^n, \dots, F_n^n)$, where Γ^n is a game form for n agents, $C(n)$ is the cost of provision in the n -agent economy, and F_i^n is the distribution function of agent i 's valuation in the n -agent economy. Agents bargain over the provision of the public good as well as the distribution of taxes. An outcome of G^n is either final agreement at time t on provision with agent i contributing x_i ,⁶ or no agreement (denoted ∞). Thus the outcome space is $\{Z \times \mathbb{R}_+^n\} \cup \{\infty\}$, where Z is the set of positive integers. Agent i 's payoff at the outcome (t, x_1, \dots, x_n) is $\delta^t(v_i - x_i)$, where δ is the common discount factor, and all agents' payoffs equal zero if there is no agreement. Observe that we can interpret the value agents receive from the project and their contributions either as one period quantities or as streams (in the latter case, v_i and x_i are the discounted values of these streams). Budget balance requires $\sum_i x_i \geq C(n)$.

As an implication of Theorem 2, we have that the expected time to agreement becomes arbitrarily long as the size of the economy increases. Suppose that $C(n)$ and F^n satisfy conditions (i)-(iv) of Theorem 2. From each G^n select an equilibrium, and let $p_n(T)$ be the probability that agreement is reached before or at T in the selected equilibrium of G^n . Then, for all T , $p_n(T) \rightarrow 0$ as $n \rightarrow \infty$. Furthermore, this convergence is uniform in the specification of game form, Γ^n , and choice of equilibrium. The proof is immediate.

5. We would like to thank Ariel Rubinstein for convincing us of the interest in the relationship between the dynamic-bargaining approach and the mechanism-design approach.

6. While we are restricting the class of games to those which require the voluntary participation of all agents, it is not necessary that all agents have a "vote" (move) at each period. The time of final agreement is the time at which provision occurs. There may be agents who only moved at time $t = 1$, and never again.

The difficulty with dealing with agents who have different discount factors is that it is no longer possible to identify the discounted value of the project in an equilibrium. In a particular equilibrium, different agents will value the project differently according to the degree to which they discount the future.

5.2. Applications of the main result

Our results are relevant to understanding many bargaining problems, several of which we now describe. The first three are examples in which conventional wisdom suggests a failure to provide public projects, even though there appeared to be clear social gains were the projects to be undertaken.⁷ The first is oil field unitization. Oil reservoirs usually lie under the land of more than one owner and each landowner has the legal right to any oil he can extract. In the absence of an agreement among the different parties extracting the oil, there is a clear incentive to extract quickly, which leads to higher extraction costs and a smaller total yield. The rate at which any firm can extract oil will depend upon the distribution of the oil pool across the affected properties. Firms acquire non-public information about this during the exploration stage, introducing private information regarding the value of reaching an agreement.⁸ Libecap and Wiggins (1985) compare unitization regulations in Oklahoma, Texas, and Wyoming (federal lands). Texas requires unanimous agreement for a field to be unitized, Oklahoma allows 63% of the firms involved in a field to impose a unitization agreement on the field, while federal regulations are designed to encourage unitization during exploration (when firms do not have any private information). Libecap and Wiggins analyze how the different rules under which unitization agreements are reached affects the likelihood that agreement will be reached (as well as why the different rules existed). The federal regulations are most successful at encouraging unitization and Texas is least successful. Libecap and Wiggins argue that, to a large extent, this is due to private information (see also Wiggins and Libecap (1985)).

The next two examples arise from a question economic historians have raised regarding what seems to be the differential ability to reach Pareto-improving agreements in France and England in the period 1600–1800. The enclosure of open fields and the drainage of marsh land seemed to have high social returns in both England and France (Grantham (1980) discusses enclosure and Rosenthal (1987, 1988) discusses the drainage of marshes). While enclosure and drainage played an important role in the growth of English agriculture in 1600–1800, they were conspicuously absent in France. While enclosure results in more efficient use of the land, certain members of the village lose the right of access and use of the field. Similarly, although drainage of a marsh results in more arable land, the marshes did have some value *qua* marshes and both villagers and the local lord had rights to the marsh. Furthermore, both enclosure and drainage involved expenditures (surveying and construction of new paths for enclosures; construction of

7. We are grateful to John McMillan and Jean-Laurent Rosenthal for bringing the first three examples to our attention.

8. In terms of our model, the public good in this problem is the unitization agreement. A referee pointed out that there is a potential problem in interpreting the individual rationality constraint of our model for this example. If the fact that someone vetoes a unitization agreement conveys some of his private information, this might alter the subsequent behaviour of the other firms. In this case, the relevant inequality would be that each firm's expected utility in the agreement be at least as large as its expected utility should it veto the agreement. It may be, of course, that behaviour following a veto is no different than that had no unitization agreement been considered. For example, if the firms' behaviour is that they pump oil as fast as they can in the absence of an agreement, then there is no problem with the IR constraint as we have formulated it. The reader should be aware, though, that the appropriate IR constraint is not obvious when a veto by one party can alter subsequent behaviour. This caveat applies to several of the following examples as well.

ditches and levees and the upkeep costs for drainage) which must be borne by somebody. It was also the case that the value of the property rights of the different participants was not common knowledge. The explanation for the difference in French and English experience appears to be that France throughout this period required unanimous consent by all affected parties, while England did not.

The following problem is a special case of the one discussed in Mailath and Postlewaite (1990). A firm in an industry employs a group of workers whose only barrier to leaving their current firm and starting a new venture is the necessity to get all the workers to agree to move (all the workers are needed for the group to have productive value and all the other factors of production are available in competitive markets). A worker's private information is about his or her reservation wage (how much he or she must be paid in order to leave the current firm), ρ indicates the probability of formation of the new venture, and ξ is the vector of wages in the new venture. The workers, as a group, have an incentive to create the new venture if the sum of their wages is less than the value of their contribution to the firm. However, Theorem 2 demonstrates that if the number of workers involved is large then it is very unlikely that the workers will be able to solve their bargaining problem.

Other bargaining problems include the construction of a polluting plant and the combination of competing firms into a monopoly. In the pollution application, $C(n)$ is the net revenue of the plant (after paying for costs of production, before compensating affected neighbours), the disutility of pollution is private information, and ξ is the vector of compensations.

To describe the monopoly application, suppose there are n firms in an industry, each firm having private information about its profits in the case that there is no monopoly. If a monopoly is established, then the firms as a group could earn more profits and the monopoly revenues must be shared among the firms. The results of this paper apply to the decision of whether to form a monopoly. A more complicated variant would involve determining, in addition, the level of production in the monopoly (this corresponds in the public good world to deciding on the level of provision, which is not addressed here). There are two possible individual rationality constraints in the monopoly application. The more severe (and the one used in this paper) is that, in the event of a veto, no cartel forms. The other individual rationality constraint would entail the non-vetoing firms still forming a cartel and competing with the vetoing firm. Clearly, firm profits are higher after a veto with the second individual rationality constraint than with the first and so monopolization should be easier to achieve with the first constraint. However, Theorem 2 shows that for large markets, monopolization is (asymptotically) impossible to achieve.

5.3. *Related literature*

The question of eliciting agents' information for making collective decisions has been studied in many forms. An important strand in that literature asks when there exists a mechanism for which either the dominant strategy equilibria or the Nash equilibria are efficient (see, e.g. the surveys by Groves and Ledyard (1987) and Postlewaite (1985)). Our framework differs from this strand in that we seek a mechanism that can do well for a given problem (that is, given a complete description of the agents and the probability distributions describing their valuations), while the literature referred to sought a mechanism that would do well for a large class of possible environments or problems.⁹ If a

9. The terms parametric and nonparametric are sometimes used to distinguish games whose outcome functions depend upon the data of the problem from those whose outcome functions do not.

particular problem had no asymmetry of information, our approach would be trivial. Since we allow the mechanism to depend upon the problem, we could simply choose an arbitrary cost-sharing rule that would give every agent positive net benefits; there will be such a rule if it is inefficient not to undertake the project. We then tell the agents that if everyone agrees the project will be undertaken using that cost-sharing rule and that the status quo will obtain otherwise. Unanimous agreement, resulting in undertaking the project, would then be an equilibrium. Allowing the mechanism to depend upon the problem eliminates the problem unless it is characterized by informational asymmetry.

In the literature that investigates the possibility of providing public goods in economies without asymmetric information, the paper that is closest in spirit to ours is that of Roberts (1976). Roberts considered sequences of economies with one public and one private good with an increasing number of agents. He showed that unless each agent's contribution to the provision of the public good goes to zero, the outcome cannot be "limiting incentive compatible". In Roberts' paper, limiting incentive compatible meant that as the number of agents went to infinity, the utility gain to an agent from misrepresenting his preferences went to zero. Roberts' results can be interpreted as saying that no outcome that provides any public good can be an ε -dominant strategy equilibrium, where ε goes to zero as the number of agents goes to infinity.

A literature closer to our work arose from work by d'Aspremont and Gerard-Varet (1979) which allows the mechanism to depend upon the problem, and introduces informational asymmetry in a manner similar to ours. While d'Aspremont and Gerard-Varet construct mechanisms that implement efficient outcomes, these mechanisms do not satisfy individual rationality. Our work demonstrates that individual rationality not only precludes efficiency, but that for large economies it effectively prevents any change in the status quo.

Other papers have addressed the issue of the asymptotic performance of mechanisms in the face of asymmetric information. Gresik and Satterthwaite (1989) show that in a particular private goods context, viz., bilateral trade with private valuations, ex post efficient allocations are asymptotically achievable with large numbers of agents. (The achievement of efficient outcomes with large numbers in the case they studied was actually quite simple; their main goal was to study the rate of convergence to an efficient outcome.) Cramton, Gibbons, and Klemperer (1987) analyse the problem of dissolving a partnership when agents have private values for the asset. Cramton and Palfrey (1990) characterize the set of cartel agreements possible in industries where firms have private information about costs. Both of these papers focus on the ex post efficient allocation rule, allowing for side payments. The asymptotic result in Cramton, Gibbons and Klemperer (1987) considers replications of partnerships, where an agent's share in the partnership is proportional to the inverse of the number of replications. Analogous to the core-convergence results in exchange economies, sufficient replication allows the partnership to be dissolved ex post efficiently. The replication reduces the distance of the existing share distribution from equal ownership (which can always be dissolved ex post efficiently). Cramton and Palfrey (1990) show that when there are sufficiently many firms in the industry, the monopoly outcome (which is the ex post efficient allocation) is unobtainable.

Finally, the intuition for our result provided in the introduction is reminiscent of the paradox of voting: since voting is costly (if only due to the opportunity cost of the time spent voting) and, in large electorates, it is unlikely that one vote will affect the outcome, why do people vote? Of course, voter turnout affects the probability that any

voter will be pivotal. Palfrey and Rosenthal (1985), in a model where voters are uncertain about the preferences and voting costs of other voters, show that for large electorates, the paradox of voting applies.

APPENDIX 1. A MORE GENERAL RESULT¹⁰

A more general formulation of an n -agent economy and provision mechanism is to first specify a probability space $(\Omega^n, \mathcal{F}^n, \mathcal{P}^n)$. This probability space will capture both the stochastic nature of agent's valuations as well as any randomness that the mechanism itself induces. Agent i 's valuation is given by $V_i^n: (\Omega^n, \mathcal{F}^n) \rightarrow \mathbb{R}$. Assume the random variables $\{V_i^n\}_{i=1}^n$ are independent for each n . Suppose first that all agents truthfully report their valuations. Agent i 's tax payment is given by $\xi_i^n: (\Omega^n, \mathcal{F}^n) \rightarrow \mathbb{R}$. Denote the event that the public good is provided by $X^n \in \mathcal{F}^n$. For $\omega \in \Omega^n$, agent i 's willingness to pay for the public good is given by $V_i^n(\omega)$ and his or her tax is $\xi_i^n(\omega)$. If $\omega \in X^n$ the public good is provided and if $\omega \notin X^n$ it is not provided. If taxes are only imposed when the public good is provided, $\omega \notin X^n$ implies $\xi_i^n(\omega) = 0$. Budget balance is given by

$$(BB^*) \quad \mathcal{P}^n(X^n)C(n) \leq E \sum_i \xi_i^n(\omega).$$

For each i , let \mathcal{F}_i^n be the smallest σ -algebra with respect to which V_i^n is measurable. Since agent i knows his or her own valuation, agent i conditions on the value of V_i^n , i.e. \mathcal{F}_i^n . Let $\rho_i^n: \Omega^n \rightarrow [0, 1]$ be a version of $\mathcal{P}^n\{X^n | \mathcal{F}_i^n\}$ and $\zeta_i^n: \Omega^n \rightarrow \mathbb{R}$ be a version of $E(\xi_i^n | \mathcal{F}_i^n)$. We can represent agent i 's utility from truthfully participating in the mechanism (assuming all the other agents are truthfully reporting) by $E[V_i^n I_{X^n} - \xi_i^n | \mathcal{F}_i^n] = \rho_i^n V_i^n - \zeta_i^n$, where I_A is the indicator function for the set A . Since the conditional probabilities ρ_i and expectations ζ_i are only unique almost everywhere, individual rationality and incentive compatibility hold only with probability one (rather than everywhere). Note that this problem did not arise in Section 2. There, ρ_i and ζ_i are obtained by taking expectations over the other agents valuations, an operation that is unambiguously defined everywhere. Individual rationality is given by

$$(INTIR^*) \quad \rho_i^n(\omega) V_i^n(\omega) - \zeta_i^n(\omega) \geq 0, \text{ almost all } \omega \in \Omega^n.$$

If agent i misrepresents his or her valuation when ω is the true state, then his/her utility is $\rho_i^n(\omega') V_i^n(\omega) - \zeta_i^n(\omega')$, for some ω' . Since the mechanism and the other agents do not know agent i 's valuation, incentive compatibility is given by

$$(IC^*) \quad \exists N \subset \Omega^n, \quad \mathcal{P}^n(N) = 0, \text{ such that } \forall \omega, \omega' \in \Omega^n \setminus N, \\ \rho_i^n(\omega) V_i^n(\omega) - \zeta_i^n(\omega) \geq \rho_i^n(\omega') V_i^n(\omega) - \zeta_i^n(\omega').$$

For $0 \leq \beta \leq 1$, define $v_i^n(\beta) = \inf \{x | \mathcal{P}^n(V_i^n \leq x) > \beta\}$; v_i^n is the inverse distribution function of V_i^n , with a tie-breaking rule.

The following theorem eliminates condition (i) from Theorem 2, as well as the assumption that the distributions are absolutely continuous. Assumption (A.1) is the counterpart of condition (iii), while Assumption (A.2) is the counterpart of conditions (ii) and (iv).

The proof formalizes the following argument. If the agents' valuations are independent, then the event X^n must be "nearly independent" of the valuations of most of them. That is, few agents can be pivotal. The agents who are not pivotal can avoid high taxes by reporting a low valuation (they will "free ride"). Thus, by incentive compatibility, these agents must pay low taxes regardless of their valuations. Only the small number of pivotal agents can be taxed at a rate above the minimum. Since these agents have bounded valuations, individual rationality prevents sufficient taxes from being raised from them alone to provide the public good.

Theorem. Let $\{(\Omega^n, \mathcal{F}^n, \mathcal{P}^n), X^n, C(n), (V_i^n, \xi_i^n)_{i=1}^n\}_n$ be a sequence of economies and provision mechanisms. Suppose that (BB^*) , $(INTIR^*)$, and (IC^*) are satisfied for all n . If

$$\exists v, w \text{ such that } \forall n, V_i^n: (\Omega^n, \mathcal{F}^n) \rightarrow [v, w], \text{ and} \tag{A.1}$$

$$\exists \gamma, \varepsilon > 0 \text{ such that } \forall n, C(n) > n\varepsilon + \sum_i v_i^n(\gamma), \tag{A.2}$$

then there exists a constant (independent of n and the sequence) K such that $\mathcal{P}^n(X^n) < n^{-1/4}K$. That is, the probability that the public good is provided goes to zero at least as fast as $n^{-1/4}$.

10. The result in this Appendix is due to a referee.

Proof. The superscript n will be suppressed until the end. Letting $p \equiv \mathcal{P}(X)$, we have that $p < 1$. [If $p = 1$, $\rho_i(\omega) = 1$ a.s. and so, by (IC*) and (INTIR*), $\zeta_i(\omega) \equiv v'(0)$ a.s., which violates (BB*).] Assume $p \neq 0$ (otherwise nothing to prove).

Set $a = -(p/(1-p))^{1/2}$, $b = ((1-p)/p)^{1/2}$ and define $f(\omega) = a$ if $\omega \notin X$ and $f(\omega) = b$ if $\omega \in X$. Since $Ef = 0$ and $Ef^2 = 1$, f is an element of $\mathcal{L}^2(\Omega, \mathcal{F}, \mathcal{P})$, the Hilbert space of square integrable random variables with zero mean and measurable with respect to \mathcal{F} (f is essentially the indicator function of X , normalized to have zero mean and unit variance). Now, in this space, conditioning on a σ -algebra is a projection operator and conditioning on independent σ -algebras yields orthogonal random variables. Letting $f_i \equiv E(f | \mathcal{F}_i)$, $\theta_i \equiv \|f_i\|$, and $f_i^* \equiv \theta_i^{-1} f_i$, we have that $\{f_i^*\}_{i=1}^n$ is an orthonormal set. By Bessel's inequality, $\|f\|^2 \geq \sum_i (\int f f_i^* d\mathcal{P})^2$. That is, $1 = \|f\|^2 \geq \sum_i \theta_i^{-2} (\int f f_i d\mathcal{P})^2 = \sum_i \theta_i^{-2} \|f_i\|^4 = \sum_i \|f_i\|^2$, since $f - f_i$ and f_i are orthogonal (f_i is the projection of f on the subspace $\mathcal{L}^2(\Omega, \mathcal{F}_i, \mathcal{P})$). Hence, no more than $n^{1/2}$ agents have $\|f_i\|^2 \geq n^{-1/2}$. (If X is independent of V_i , $f_i = Ef = 0$. Thus, X is nearly independent— $\|f_i\|^2 < n^{-1/2}$ —of the valuations of most— $n - n^{1/2}$ —of the agents.)

Since $[f - a]/(b - a)$ is the indicator function of X , $[f_i - a]/(b - a)$ is a version of ρ_i . Thus the symmetric difference of $C_i \equiv \{\omega : |f_i(\omega)| \geq n^{-1/8}\}$ and $C_i^* \equiv \{\omega : |\rho_i(\omega) - p| \geq n^{-1/8}/(b - a)\}$ is a null set. Now, $\|f_i\|^2 = Ef_i^2 \geq n^{-1/4} \mathcal{P}(C_i)$, so $\mathcal{P}(C_i^*) \leq n^{1/4} \|f_i\|^2$. Let $I \equiv \{i : \|f_i\|^2 < n^{-1/2}\}$, the set of agents whose valuations are nearly independent of X . (Since $\int \rho_i d\mathcal{P} = p$, for $i \in I$ the probability that agent i 's announcement has a significant impact on provision is $\mathcal{P}(C_i^*) < n^{-1/4}$.)

Letting $D_i \equiv \{\omega : V_i(\omega) \leq v_i(n^{-1/4})\}$ we have, from the definition of v_i , $\mathcal{P}(D_i) \geq n^{-1/4}$. Let N be the union of the exceptional sets in (INTIR*) and (IC*) for all i . Being the finite union of null sets, N itself is null. Then, if $i \in I$, $\mathcal{P}(D_i \setminus C_i^*) > 0$, and so there exists $\omega_i \in \Omega \setminus N$ such that $|\rho_i(\omega_i) - p| < n^{-1/8}/(b - a)$, and $V_i(\omega_i) \leq v_i(n^{-1/4})$.

At this point, (INTIR*) and (IC*) are used to bound $E\zeta_i$ and then (BB*) implies a bound on p . Consider first $i \in I$. Since i 's announcement is unlikely to have a significant impact on provision, (IC*) forces i 's taxes to be close to what he or she would pay when he or she does not value the project highly, which by (INTIR*) is a small tax. So, (INTIR*) implies $\zeta_i(\omega_i) \leq v_i(n^{-1/4})\rho_i(\omega_i)$. Combining with (IC*) yields $\zeta_i(\omega) \leq [\rho_i(\omega) - \rho_i(\omega_i)]V_i(\omega) + \rho_i(\omega_i)v_i(n^{-1/4})$ a.a. ω . Recalling that $|\rho_i(\omega) - p| < n^{-1/8}/(b - a)$ on $\Omega \setminus C_i^*$ and $\omega_i \notin C_i^*$, $|\rho_i(\omega) - \rho_i(\omega_i)| < 2n^{-1/8}/(b - a)$ on $\Omega \setminus C_i^*$. Since $V_i \leq w$, $\zeta_i(\omega) \leq 2wn^{-1/8}/(b - a) + [p + n^{-1/8}/(b - a)]v_i(n^{-1/4})$ for a.a. $\omega \in \Omega \setminus C_i^*$. Hence,

$$\begin{aligned} E\zeta_i &\leq [2wn^{-1/8}/(b - a) + [p + n^{-1/8}/(b - a)]v_i(n^{-1/4})\mathcal{P}(\Omega \setminus C_i^*) + w\mathcal{P}(C_i^*) \\ &< 2wn^{-1/8}/(b - a) + [p + n^{-1/8}/(b - a)]v_i(n^{-1/4}) + wn^{-1/4}. \end{aligned}$$

If $i \notin I$, $E\zeta_i \leq w$. Thus,

$$\begin{aligned} \sum_i E\zeta_i &< \sum_{i \in I} \{2wn^{-1/8}/(b - a) + [p + n^{-1/8}/(b - a)]v_i(n^{-1/4}) + wn^{-1/4}\} + \sum_{i \notin I} w \\ &\leq \sum_{i=1}^n \{2wn^{-1/8}/(b - a) + [p + n^{-1/8}/(b - a)]v_i(n^{-1/4}) + wn^{-1/4}\} + wn^{1/2} \\ &= [p + n^{-1/8}/(b - a)] \sum_i v_i(n^{-1/4}) + 2wn^{7/8}/(b - a) + wn^{3/4} + wn^{1/2} \\ &\leq p \sum_i v_i(n^{-1/4}) + n^{7/8}3w/(b - a) + wn^{3/4} + wn^{1/2} \\ &\leq p \sum_i v_i(n^{-1/4}) + 3wn^{7/8}/(b - a) + 2wn^{3/4}. \end{aligned}$$

By (BB*) and (A.2), $p(n\varepsilon + \sum_i v_i(\gamma)) < pC(n) \leq \sum_i E\zeta_i < p \sum_i v_i(n^{-1/4}) + 3wn^{7/8}/(b - a) + 2wn^{3/4}$, so for sufficiently large n (so that $n^{-1/4} < \gamma$), $n\varepsilon p < 3wn^{7/8}/(b - a) + 2wn^{3/4}$. Now, $(b - a)^{-1} = (p(1 - p))^{1/2}$. Hence, $n^{1/4}p < \{3wp^{1/2}n^{1/8} + 2w\}\varepsilon^{-1}$. Rearranging terms, $p^{1/2}n^{1/8}\{p^{1/2}n^{1/8} - 3w/\varepsilon\} < 2w/\varepsilon$, so that there exists K (independent of n) such that $p^{1/2}n^{1/8} < K^{1/2}$. Thus, $\mathcal{P}^n(X^n) < n^{-1/4}K$. \square

APPENDIX 2. AN EXAMPLE WITH CORRELATED VALUATIONS

The economy consists of m groups, each group containing $L + 1$ agents. Each agent's valuation is either h or H , with $h < H$. No more than one agent in each group has a valuation of h . Valuations are independent across groups and each group has the same probability of having a low-valuation member. Suppose $(n - m)H + mh > C(n)$, so that it is common knowledge that the public good should be provided. Agents are indistinguishable; in particular it is not possible to determine to which group any particular agent belongs. Assume $c \equiv C(n)/n$, the cost per agent of provision is constant, and $h < c$. There is a bound T (independent of n) on the tax that an individual can be forced to pay. Let p denote the probability that a particular group has one agent with valuation h . Let $k \equiv \#\{j : \hat{v}_j = h\}$, the number of reported lows. When all agents truthfully report their valuations, k is a binomial random variable with m trials and probability p of success on any trial.

Consider first anonymous mechanisms. (We will relax this restriction.) In these mechanisms, those agents who announce high are treated identically, and those who announce low are treated identically. The probability that the new organization is formed depends only on k , rather than on the full vector \hat{d} . Given a provision rule $\rho: \{0, 1, \dots, m\} \rightarrow [0, 1]$, we first consider mechanisms with the following tax scheme (we later argue that these are the only mechanisms we need consider):

$$\xi_i(\hat{d}) = \begin{cases} h + x(k), & \hat{d}_i = h, 1 \leq k \leq m, \\ T, & \hat{d}_i = h, k > m, \\ (C(n) - kh)/(n - k), & \hat{d}_i = H, \end{cases} \quad (\text{A.3})$$

where x satisfies $\sum_{1 \leq k \leq m} x(k) \Pr(k) \rho(k) = 0$. Mechanisms with a tax scheme given by (A.3) satisfy INTIR and BB (but not EXPBB).¹¹ The low types never want to report they are high types because $h < (C(n) - kh)/(n - k)$. The IC constraint for the highs is:

$$\sum_{k=0}^m \Pr(k) \rho(k) \left(H - \frac{(C(n) - kh)}{(n - k)} \right) \geq \sum_{0 \leq k \leq m-1} \Pr(k) \rho(k+1) (H - h - x(k+1)) + \Pr(m) \rho(m+1) (H - T). \quad (\text{A.4})$$

This implies:

$$\begin{aligned} & \sum_{1 \leq k \leq m} (\Pr(k-1) - \Pr(k)) \rho(k) x(k) + \left\{ \sum_k \Pr(k) (\rho(k) - \rho(k+1)) \right\} (H - h) + p^m \rho(m+1) (T - h) \\ & \geq \sum_{k=0}^m \Pr(k) \rho(k) \left(\frac{C(n) - nh}{(n - k)} \right). \end{aligned} \quad (\text{A.5})$$

The most probable number of agents with a low valuation will be denoted t and it is the unique integer satisfying $(m+1)p - 1 < t \leq (m+1)p$. Now, $\Pr(k-1) - \Pr(k) \leq (>) 0$ if $k \leq (>) t$. Thus,

$$\begin{aligned} \sum_{1 \leq k \leq m} (\Pr(k-1) - \Pr(k)) \rho(k) x(k) & \leq (T - h) \left\{ \sum_{1 \leq k \leq t} (\Pr(k) - \Pr(k-1)) + \sum_{t+1 \leq k \leq m} (\Pr(k-1) - \Pr(k)) \right\} \\ & = (T - h) \{ 2 \Pr(t) - (1 - p)^m - p^m \}. \end{aligned}$$

We also have:

Claim. The expression $\sum_k \Pr(k) (\rho(k) - \rho(k+1))$ is maximized by $\rho(k) = 1$ for $k \leq t$ and $= 0$ for $k > t$.

Proof. Note that $\Pr(k)$ is strictly increasing for $k \leq t-1$, strictly decreasing for $k \geq t$, and $\Pr(t) \geq \Pr(t-1)$. Let ρ be a provision rule that maximizes $\sum_k \Pr(k) (\rho(k) - \rho(k+1))$. Clearly, $\rho(0) = 1$ and $\rho(m+1) = 0$.

Given ρ , let $\hat{\rho}$ equal ρ , except at $k = t$ where $\hat{\rho}(t) = 1$, and at $k = t+1$ where $\hat{\rho}(t+1) = 0$. Then

$$\begin{aligned} & \sum_k \Pr(k) (\hat{\rho}(k) - \hat{\rho}(k+1)) - \sum_k \Pr(k) (\rho(k) - \rho(k+1)) \\ & = (1 - \rho(t)) (\Pr(t) - \Pr(t-1)) + \rho(t+1) (\Pr(t) - \Pr(t+1)) \geq 0. \end{aligned}$$

Since ρ is maximal, $\rho(t+1) = 0$ and if $\Pr(t) - \Pr(t-1) > 0$, $\rho(t) = 1$.

Now we claim that $\rho(k) = 1$ for $k \leq t-1$. Suppose there exist $k \leq t-1$ such that $\rho(k) < \rho(k+1)$ and let \hat{k} be the smallest such k . Let $\hat{\rho}$ equal ρ , except at $k = \hat{k}$ where $\hat{\rho}(\hat{k}) = \rho(\hat{k}+1)$. Then

$$\begin{aligned} & \sum_k \Pr(k) (\hat{\rho}(k) - \hat{\rho}(k+1)) - \sum_k \Pr(k) (\rho(k) - \rho(k+1)) \\ & = (\Pr(\hat{k}-1) - \Pr(\hat{k})) (\rho(\hat{k}) - \rho(\hat{k}+1)) > 0, \end{aligned}$$

and ρ cannot be maximal.

Similarly, it can be shown that $\rho(k) = 0$ for $k \geq t+1$. Finally, note that $\rho(t) \neq 1$ can only occur when $\Pr(t) = \Pr(t-1)$, and that in this case any value for $\rho(t)$ in $[0, 1]$ gives a maximal provision rule. \parallel

Thus, the left hand side of (A.5) is bounded above by $[2T + H - 3h] \Pr(t)$. But

$$\Pr(t) = \frac{m!}{(m-t)!t!} p^t (1-p)^{m-t} \leq \frac{m!}{(m-t)!t!} (t/m)^t (1-t/m)^{m-t}.$$

11. If there is an agent with no private information, adjusting his/her contribution by $-x(k)$ gives a mechanism which satisfies EXPBB.

Approximating using Stirling's formula shows that, for large m , the last term is of the same order of magnitude as $1/\sqrt{2\pi mp(1-p)}$. Thus as m gets large, the left-hand side of (A.5) gets small. The right hand side is greater than $\psi(m, \rho)(c-h)$, where $\psi(m, \rho)$ is the unconditional probability that the public good is provided, $\sum_k \Pr(k)\rho(k)$. Thus as m gets large, $\psi(m, \rho)$ tends to 0.

Generalizing the tax scheme by setting $\xi(\hat{v}) = (C(n) - kh)/(n-k) + y(k)$, when $\hat{v}_i = H$, with $\sum_k y(k) \Pr(k)\rho(k) = 0$ does not change the result, since this does not change the expected utility of a truthfully reporting agent with a high valuation and it is the high-valuation agents' incentive compatibility constraints which are binding.

It is a corollary of the above that the unconditional probability that the public good is provided goes to zero for bounded non-anonymous mechanisms as well. Suppose the converse were true. Let $\{(\rho^n, \xi^n)\}_n$ be a sequence of mechanisms such that the probability of provision does not go to zero. There are $n!$ possible permutations of the names of the agents in an n -agent economy. Let (ρ, ξ) be the mechanism for the n -agent economy obtained by randomly choosing a permutation of the names of the agents (each permutation has probability $1/n!$) and applying (ρ^n, ξ^n) to the *relabelled* economy. Since the probability that any permutation is chosen is independent of announcements, the new mechanism satisfies IC, EXPIR and BB. Furthermore the new mechanism is anonymous with the probability of provision not going to zero, a contradiction.

Remark. In general, in environments with correlated private information, the mechanism designer can take advantage of the correlation by introducing, from an agent's perspective, gambles whose distribution is affected by the strategy followed (see, for example, Cremer and McLean (1985, 1988)). In this example, a mechanism which achieves first best can easily be constructed if unbounded taxes are permitted.

If the mechanism is required to satisfy ex post individual rationality, then $x(k) \leq 0$, and so $x(k) = 0$. Then the bound on taxes can depend on m . As long as $T(m)$ does not grow exponentially, $\psi(m, \rho)$ tends to 0. In particular, the mechanism can impose the *full* burden on each of the low-valuation agents in the event that a deviation was detected (i.e. $T = C(n)$) without altering the conclusion of the example.

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