Response to "Aristocratic Equilibria"

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A major point of Cole, Mailath, and Postlewaite (1992) is that the existence of nonmarket decisions leads naturally to multiple equilibria and, consequently, the possibility that ex ante identical societies might exhibit different patterns of economic growth. It was further argued that the multiplicity was not driven by technical aspects of the model but arose naturally from the economic structure of the problem. In that paper, we focused on two particular types of equilibria: wealth-is-status and aristocratic. Unfortunately, the argument in that paper for the existence of aristocratic equilibria is incorrect, as the comment by Steven Landsburg points out. Landsburg does provide a condition (condition 1: There is a positive measure of males with zero wealth) that guarantees the existence of aristocratic equilibria. The purpose of this response is, first, to argue that the existence of aristocratic equilibria in environments not covered by condition 1 is important and, second, to demonstrate by example that aristocratic equilibria do exist in some such environments.

The reason that aristocratic equilibria exist in a narrower set of environments than wealth-is-status equilibria is easy to understand. In wealth-is-status equilibria, women always accept the offer made by the wealthiest suitor, and the parents' matching (as opposed to saving) behavior has no impact on the ability of a son to match with a high-endowment woman. Loosely speaking, wealth-is-status equilibria involve "myopically" optimal or Markovian behavior (behavior that depends on only the payoff relevant aspects of history). Other equilibria, in contrast, require some degree of intergenerational enforcement, since in such equilibria some women will be accepting offers to match from men other than the wealthiest suitor. Such behavior can be optimal only if deviations from the prescriptions of the social norm result in future negative consequences (e.g., a loss of

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status so that sons of deviating parents may find that they can match only with the poorest women as prescribed in the aristocratic social norm). A central issue that arises in showing the existence of aristocratic equilibria is that while the norm prescribes a punishment for deviating from the social norm—future male offspring in this line will be matched with the poorest women of their generation—this punishment is of arbitrarily small concern to couples at the lower end of the status distribution: their male offspring will be matched with women who are close to the poorest even if these couples follow the social norm. The punishment may be insufficient to deter deviations by such couples.

Landsburg's condition 1 effectively avoids this problem by guaranteeing a mass of males who have no wealth and consequently cannot deviate from the social norm. This creates a uniform punishment per period, bounded away from zero, that can be imposed on *all* potential deviators with a nontrivial decision. Furthermore, since the punishment entails that all future males in this line will be affected in this way, the cost of this punishment can be made arbitrarily large by choosing a discount factor sufficiently close to one.

The argument just given has the flavor of the folk theorem in repeated games and relies on the existence of a sufficiently large punishment that can be imposed on deviators to ensure concurrence with a prescribed equilibrium. This raises the following question: Do there also exist social norms different from the wealth-is-status social norm that do not depend on punishments that are effectively the result of a supplementary assumption? In particular, in aristocratic equilibria in our earlier paper, the maximal possible punishment that can be imposed on agents of arbitrarily low status is necessarily arbitrarily small. Since these agents are not myopically optimizing in an aristocratic equilibrium, the nature in which the social norm is being enforced is necessarily different from that in equilibria for environments covered by Landsburg's condition 1.

If the capital distribution is atomless, aristocratic equilibria have the property that the costs of deviating for agents of arbitrarily low status are arbitrarily small, and these agents are choosing "myopically" suboptimal actions. When the capital distribution is atomless, aristocratic equilibria do not exist for many initial distributions of capital and utility functions of the wife's endowment. For example, if the initial distribution of capital satisfies $k_0(0) > 0$ and the utility function of the wife's quality, $v(j)$, is $v(j) = j$, then the zero-status parents find it profitable to deviate, just as the argument on p. 1111 that (14) fails suggests.[1] We now demonstrate that aristocratic equilib-

[1] As in our 1992 paper, denote by $L_t(j)$ the capital level that leaves the woman indifferent between matching with the man of status j with capital $k_t(j)$ and so main-

ria exist for the case $u(c) = \ln c$ (the limiting case as the intertemporal elasticity of substitution converges to one) and $k_0(i) = \kappa i$ (so that $k_0(0) = 0$). The log utility case has the advantage that the incentive constraint is time invariant. Standard calculations show that $\lambda^* = 1 - \beta$ and

$$V^*(k) = (1-\beta)^{-2}[(1-\beta)\ln(1-\beta) + \beta\ln\beta + \ln A + (1-\beta)\ln k] \quad \text{for } k > 0.$$

Our equation (14) (p. 1110) is a sufficient (and necessary) condition for the existence of aristocratic equilibria. Straightforward calculation yields $L_t(j) = k_t(j)e^{\beta v(j)}$. Rearranging (14) and using the definitions of V^* and L_t yield

$$V^*(k_{t-1}(i)) - V^*(k_{t-1}(j)) + u(Ak_{t-1}(j) - k_t(j)) - u(Ak_{t-1}(i) - L_t(j)) + \frac{\beta}{1-\beta}[v(i) - v(j)] \geq 0.$$

Substituting and canceling yield

$$\ln\left(\frac{i}{j}\right) + (1-\beta)\{\ln[(1-\beta)j] - \ln[i - \beta j e^{\beta v(j)}]\} + \beta[v(i) - v(j)] \geq 0. \tag{1}$$

We need to show that (1) holds for all feasible deviating offers by male i to a female $j \geq i$, where feasible offers require current consumption, $Ak_{t-1}(i) - L_t(j)$, to be positive; that is, the relevant region is all (i, j) satisfying $j \geq i > \beta j e^{\beta v(j)}$.

LEMMA. Aristocratic equilibria exist if the value of the match $v(j)$ is convex, $v(0) = 0$, $v'(j) > 0$ for $j > 0$, $\sup\{jv''(j)/v'(j): j \in [0, 1]\} \equiv M < \infty$,[2] and

$$1 > \beta \geq \max\left\{\frac{3}{4}, \frac{3M}{3M+1}, \frac{3v'(1)}{3v'(1)+1}\right\}.$$

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taining status for her son, and matching with a man with capital $L_t(j)$ and having a zero-status son. The intent of proposition 2 was to provide a condition on the wealth distribution so that low-status males could not save enough to induce a deviation, i.e., $L_t(j) \geq Ak_{t-1}(j)$. As Landsburg points out, this is impossible: The intuition is straightforward. For low-status females, $L_t(j)$ will be close to $k_t(j)$, and so $L_t(j) \geq Ak_{t-1}(j)$ must fail for j near zero since $k_t(j) = A(1 - \lambda^*)k_{t-1}(j) < Ak_{t-1}(j)$. Note that, when $k_0(0) > 0$, the zero-status parents suffer a zero first-order loss from increasing savings, while enjoying a first-order gain in the quality of their son's spouse. This is reminiscent of the distortions in separating equilibria in signaling games.

[2] The sup is finite, e.g., if $v(j) = j^n$, $n \geq 1$. This does not contradict the earlier observation that aristocratic equilibria do not exist if $v(j) = j$ when $k_0(0) > 0$. The inequality being verified assumes $k_0(0) = 0$.

Proof. Define

$$f(j; i, \beta) \equiv (1 - \beta)\{\ln[(1 - \beta)j] - \ln[i - \beta j e^{\beta v(j)}]\} - \ln j - \beta v(j) + \ln i + \beta v(i).$$

Then the desired inequality (1) is $f(j; i, \beta) \geq 0$. Now

$$f(i; i, \beta) = (1 - \beta)(\ln[(1 - \beta)i] - \ln\{[1 - \beta e^{\beta v(i)}]i\}) \geq 0.$$

We now argue that $f(j; i, \beta) \geq 0$ for all $j \geq i > \beta j e^{\beta v(j)}$ by showing $\partial f/\partial j \geq 0$ in that region. Differentiating f with respect to j yields

$$\frac{\partial f(j; i, \beta)}{\partial j} = \beta\left\{(1 - \beta)e^{\beta v(j)}\left[\frac{1 + \beta j v'(j)}{i - \beta j e^{\beta v(j)}}\right] - \frac{1}{j} - v'(j)\right\}.$$

This expression is nonnegative if and only if

$$\begin{aligned}&(1 - \beta)je^{\beta v(j)}[1 + \beta j v'(j)] \\ &- [i - \beta j e^{\beta v(j)}] - [i - \beta j e^{\beta v(j)}]jv'(j) \geq 0.\end{aligned} \tag{2}$$

The left-hand side equals

$$\begin{aligned}&je^{\beta v(j)} - i - [i - \beta j e^{\beta v(j)}]jv'(j) + (1 - \beta)\beta j^2 v'(j)e^{\beta v(j)} \\ &\geq je^{\beta v(j)} - i - [je^{\beta v(j)} - \beta j e^{\beta v(j)}]jv'(j) + (1 - \beta)\beta j^2 v'(j)e^{\beta v(j)} \\ &= je^{\beta v(j)} - i - (1 - \beta)^2 j^2 v'(j)e^{\beta v(j)} \\ &\geq j[e^{\beta v(j)} - 1 - (1 - \beta)^2 jv'(j)e^{\beta v(j)}] \equiv jh(j; \beta).\end{aligned}$$

Now, $h(0, \beta) = 0$. Differentiating h with respect to j yields

$$\frac{\partial h(j; \beta)}{\partial j} = \left\{\beta - (1 - \beta)^2\left[1 + \frac{jv''(j)}{v'(j)} + \beta j v'(j)\right]\right\}v'(j)e^{\beta v(j)}.$$

This expression is nonnegative since, under the bounds on β,

$$(1 - \beta)^2 < 1 - \beta \leq \frac{\beta}{3},$$

$$(1 - \beta)^2\frac{jv''(j)}{v'(j)} \leq (1 - \beta)^2 M < (1 - \beta)M \leq \frac{\beta}{3},$$

and

$$(1 - \beta)^2\beta j v'(j) < (1 - \beta)^2 v'(1) < (1 - \beta)v'(1) \leq \frac{\beta}{3}.$$

Thus $\partial h/\partial j \geq 0$, and so $h(j, \beta) \geq 0$, which in turn implies that the inequality (2), and so (1), holds. Q.E.D.

The statement of proposition 1 in our earlier paper excludes the log utility of joint consumption case since it requires that u be either bounded above or bounded below. Wealth-is-status equilibria do exist for the log utility case, however. The argument in the proof of proposition 1 when modified as follows demonstrates this. The argument there modified u to extend the domain to include negative consumptions, to be continuous on the extended domain, and to leave unchanged optimal choices. The crucial step of the modification was to show that arbitrarily small but positive consumption levels are never chosen. This holds for the log utility case as well since

$$\ln c + \frac{v(1)}{1-\beta} + \beta V^*(Ak_0) < V^*(k_0)$$

holds for sufficiently small c.[3] The remainder of the argument of proposition 1 applies without change.

In summary: (1) Wealth-is-status equilibria exist for all values of γ, the coefficient of risk aversion. (2) If the initial wealth distribution has $k_0(0) > 0$, with a zero measure set of males having initial wealth $k_0(0)$, aristocratic equilibria do not exist for any specification of the utility function of the wife's quality $v(j)$ that has $v'(0) > 0$. (3) If the initial wealth distribution has $k_0(0) = 0$, aristocratic equilibria may exist. A sufficient condition for the case $\gamma \geq 1$ is Landsburg's condition 1. (4) If the initial wealth distribution has $k_0(0) = 0$ and $\gamma = 1$, then aristocratic equilibria also exist when condition 1 is violated.

In Cole et al. (1995), we explore in more detail the implications of the difficulty in inducing individuals of very low status to follow a social norm. In that paper, we also analyze "hybrid" equilibria in which low-status individuals follow the wealth-is-status social norm, and high-status individuals follow a variant of the aristocratic social norm. Since low-status individuals follow the myopically optimal wealth-is-status social norm, they have no incentive to deviate. Moreover, the threat of a loss of status is a sufficient deterrent for the high-status "aristocratic" individuals.

References

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[3] For the general production case, the inequality is replaced by

$$\ln c + \frac{v(1)}{1-\beta} + \beta V^*(f(k_0)) < V^*(k_0),$$

where f is the production function and $V^*(k) \equiv \sup_{\{k_t\}} \Sigma\, \beta^t \ln[f(k_t) - k_{t+1}]$ is finite by hypothesis.