Social Norms, Savings Behavior, and Growth

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We argue that many goods and decisions are not allocated or made through markets. We interpret an agent's status as a ranking device that determines how well he or she fares in the nonmarket sector. The existence of a nonmarket sector can endogenously generate a concern for relative position in, for example, the income distribution so that higher income implies higher status. Moreover, it can naturally yield multiple equilibria. It is thus possible to explain differences in growth rates across countries without recourse to differences in underlying preferences, technologies, or endowments. Different social organizations lead to different reduced-form preferences, which lead to different growth rates.

To what purpose is all the toil and bustle of the world? . . . It is our vanity which urges us on. . . . It is not wealth that men desire, but the consideration and good opinion that wait upon riches. [ADAM SMITH, The Theory of Moral Sentiments]

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1092
I. Introduction

Many social scientists (including some economists) are troubled by the extent to which economists ignore relative position in society and status. Most economists are sympathetic to the concern that models that omit such factors may abstract from important determinants of economic behavior. The reluctance to include relative position in society and status in models stems in large part from a belief that if one is "allowed" to put status into agents' utility functions, then it is possible to explain anything. When agents care enough about some arbitrary property, then it should not be surprising that this property emerges. If allowing agents to care about status can explain everything, it has explained nothing. This paper is an attempt to introduce the concepts of status and social norms into economic models while avoiding the difficulty that doing so could eliminate the ability to discriminate between behavior that is consistent with our model and behavior that is inconsistent.

We begin with the assumption that there are goods and decisions that are not allocated or made through markets. Although people have strong preferences over whom they or their children mate with, the decision is not a market transaction (at least in most modern societies). One cannot simply pay an admission fee to join the board of trustees of a prestigious university or nonprofit organization, or to join a prestigious country club. Although large payments may result in an invitation to join such groups, willingness to make such a payment is not sufficient to guarantee inclusion. We assume that an agent's utility is affected by these nonmarket decisions as well as by the goods and services he acquires in the market and directly consumes and, further, that agents have identical utility functions over these variables. Agents will generally have conflicting preferences over the allocation of these nonmarket goods, and hence society must have some means of reconciling the differences. We interpret an agent's status as a ranking device that determines how well he or she fares with respect to the allocation of nonmarket goods.

The existence of nonmarket decisions can endogenously generate a concern for relative position in, for example, the income distribution so that higher income implies higher status (as Adam Smith and Madonna have suggested). Since the concern for relative position is generated endogenously, the relationship between relative position and market variables is not arbitrary. There are empirical restrictions that
must be satisfied. In particular, comparative statics (with respect to, e.g., the degree of income inequality) are, in principle, testable.

The second important point is that the existence of nonmarket decisions can naturally yield multiple equilibria. The allocation of market goods need not (indeed, if the multiplicity is nontrivial, will not) be identical in distinct equilibria. Thus multiplicity of equilibria can potentially explain how identical economies (in terms of preferences and technologies) can have different market allocations.

The ideas above are relevant to a broad range of models examining various economic questions. In this paper we apply them to a particular question that is of fundamental interest, economic growth. Economists have difficulty explaining the differences in growth rates across countries. In recent decades a number of Asian countries—Japan, Korea, and Taiwan, for example—have grown at extremely high rates; at the same time the United States has exhibited steady growth and a number of South American countries such as Argentina, Brazil, and Chile have grown very slowly. Any attempt to explain cross-country differences in growth must look at the incentives people in an economy have to expend effort and to invest resources in the production of goods and services. A standard approach consists in specifying preferences for the agents and a production technology available to those agents and deriving the growth path implied by optimizing behavior on the part of the agents. Typically, a representative agent model is studied in which the agent has access to a capital-consumption technology and the agent's decision consists of period-by-period choices of the level of labor effort and the fraction of output to consume, the remainder being the investment next period.\(^1\) This approach then seeks to explain differences across countries on the basis of differences in endowments, physical or human. These attempts have not been completely successful; there is typically a large residual that cannot be explained by such differences (see, e.g., Kormendi and Meguire 1985; Barro 1989).

It is tempting to explain these differences by positing differences in the underlying preferences or technologies across countries. For example, one might seek to explain the high savings rates in Japan and the corresponding low rate in the United States by assuming differences in the discount rates for people in the two countries. While such assumptions can bring the predictions of the model in line with the observed facts (as such assumptions reconcile almost any observations with the theoretical predictions of the model), such a model leaves unexplained the basis for these differences in prefer-

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\(^1\) This literature began with the nonoptimizing models of Solow (1956) and Swan (1956). More recent optimizing models include Cass (1965), Romer (1986), Rebele (1987), and Lucas (1988).
ences or technologies. In particular, if we start from the belief that, in the long run, technologies are the same and that the agents populating countries are, on average, essentially identical, it is incumbent on the modeler to explain how such differences arise.

One way of resolving this problem is to postulate that people have identical preferences over "deep" variables and that what we typically observe in economic problems is "reduced-form" preferences. For example, people in different societies may have identical utility functions over streams of consumption for themselves and their children, which give rise to different reduced-form, one-period utility functions for consumption because fewer children survive in one environment than in another. In this spirit, we investigate how different social organizations within societies can lead to different reduced-form preferences even when the underlying preferences are identical. Such different social organizations will lead to different reduced-form preferences, which in turn might result in different rates of growth.

The paper is structured as follows. In Section II we elaborate on how societies may differ with regard to the manner in which they allocate nonmarket goods. The specific distinction in social organization on which we have chosen to focus is the allocation of mates. Section III presents the basic voluntary matching model. Section IV discusses a stylized version of the model in which only savings and matching decisions are considered and analyzes the growth trajectories for economies with different stylized social structures. In Section V we discuss at greater length some of the issues that our model raises.

II. Social Decisions and Economic Models

The interaction between the organization of a society and its economic performance was once considered perhaps the fundamental question of political economy. At least since Karl Marx, it has been argued that there is an intimate relationship between the social and political organization of a society and its method of production. One could distinguish between those aspects of a society's organization that affect the reduced-form preferences and those aspects that affect the reduced-form technology. Reduced-form technology could vary, for example, if societies differed in their ability to generate cooperative behavior among economic agents. In future work we plan to address precisely such a question. In this paper we restrict attention to analyzing how reduced-form preferences might be affected by social organization.

The specific question we examine is how the means by which a
society makes nonmarket decisions can affect the market behavior of the agents. Examples of nonmarket decisions include the process by which decisions are made about the provision of public goods, the most attractive mate for oneself or one's child, a favorable location in church or at the dinner table, a respectful audience when one speaks, or the cemetery plot with the best view. In many cases these goods are consumed within a particular social context, which may explain why they are not allocated via a market mechanism. We assume that an agent's utility is affected by nonmarket decisions as well as by the goods and services he acquires in the market and directly consumes and, further, that agents have identical utility functions over these variables. As mentioned above, a society must have some means of reconciling conflicting preferences among agents; we interpret an agent's status as a ranking device that determines how well he fares with respect to nonmarket decisions.²

We are interested in examining how differing conventions for making nonmarket decisions interact with market decisions and which rules for conferring status on agents can be consistent with their market behavior. In order to make these conventions consistent with optimizing behavior, we want agents to take into account the effect of their decisions on their status. Thus our model will specify what the preferences are over market goods and nonmarket decisions, how nonmarket decisions are made as a function of the status levels of the agents, and how the allocation of status will change as a function of agents' decisions.³ An equilibrium will be a decision rule for agents such that, given these specifications, it is optimal for each agent to follow the prescribed rule when all other agents are following the rule.

By allowing for different social conventions in the allocation of nonmarket decisions, we have introduced a new dimension of indeterminacy of equilibria. If differing social conventions alter agents' incentives in making market decisions, there can be differing levels of economic activity. More specifically, if the relation between wealth and status differs across societies, we should expect this to be reflected in different levels of economic activity. We interpret a society in which higher wealth confers higher status as a society that exhibits greater

² In discussing the Industrial Revolution, Mokyr (1985) outlines a view of status that is close to ours. When describing Perkin's (1969) analysis of the Industrial Revolution, Mokyr writes that "status means here not only political influence and indirect control over the lives of one's neighbors, but also to which houses one was invited, what partners were eligible for one's children to marry, which rank one could attain (that is, purchase) in the army, where one lived, and how one's children were educated" (p. 18).

³ Status-enhancing actions can vary widely across cultures. Examples might include graduating from Harvard, having a title, possessing good table manners or witty conversational skills, or owning jewelry, a stylish wardrobe, a Ferrari, anything with Gucci on it, or, within the inner city, a pair of "Air Jordans" and a large boom box.
social mobility than a society in which status depends primarily on ancestry. Here social mobility refers to the nonmarket benefits of wealth rather than the equality of market opportunities. To the extent that one society is more socially mobile than another, agents in that economy have a greater incentive to acquire wealth.

As we have suggested, we believe that there are a number of nonmarket decisions that we could employ to construct the sort of model in which we are interested. The specific decision we shall use is a matching decision: our model will have men and women who will match and have preferences over the matches they will enter into. Roughly speaking, there may be different social norms about how people are “supposed” to match. An equilibrium will include a description of a social norm that agents have an incentive to follow. An agent’s decision about whether or not to follow the prescriptions of a social norm will be complicated by the fact that deviating from the prescription may have consequences for the agent’s offspring, about whom the agent cares.

Agents will have common preferences over potential matches. Relative success in the matching process will be determined by agents’ status. We shall be particularly interested in two stylized social norms: one in which agents’ status is inherited and another in which status is determined solely by relative wealth.

This paper departs sufficiently from past models that it is worth making a few points clear before presenting our model. First, we do not “put social status into the utility function.” If we were to say that an agent directly cared about his position in society and that his position depended on his relative wealth, we would obviously have savings behavior that differed from the case in which he cared only about his consumption. In our model, people care only about status because in equilibrium it may affect variables that enter the utility function. We emphasize that even when agents do not care directly about their position, there may be an “indirect utility” to status because in equilibrium it affects real variables. Note that a model in which one put status into the utility function would be silent on exactly the sort of cross-cultural comparisons that one would like to make.

We should also emphasize that the fundamental ideas about which we are writing are not new to us. As the quote at the beginning of the paper indicates, the notion that status is important in understanding economic decision making goes back (at least) to Adam Smith. We see our contribution not as the introduction of the concern about

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4 The advantage of this particular decision is that it requires few ad hoc assumptions (about such things as property rights). See also Sec. VA.

5 More recently, Akerlof (1976) and Basu (1989) have presented models in which an agent’s caste (or status) has economic implications and, moreover, the social norms supporting the caste structure form an equilibrium.
status into the analysis of growth but as the reintroduction of the
treatment of social organization into the analysis of economic growth
in a way that is compatible with the more formal models of growth
that have been the prevailing mode of analysis in the past several
decades. Given this view of our contribution, we have tried wherever
possible to maintain the form and substance of more traditional mod-els so as to highlight the differences that arise from integrating social
organizations with these prevailing models. We make no claim that
all our choices in modeling are obvious, or even best. Rather, our
model is constructed to get us to our goal: to incorporate status into
a growth model that has testable implications and fits, in at least broad
terms, the stylized facts as we see them.

III. The Basic Model

We focus on a simple multigenerational society in which at each pe-
period there is a continuum of each of two different types of one-
period-lived agents, men and women. The agents are matched into
pairs. Each pair will produce two offspring, one male and one female.
Besides the matching decision, agents make standard economic deci-
isions: what to consume or what to invest. The agents in this model
differ from neoclassical agents in that they care about the nature of
the mate with whom they pair.

The model is asymmetric with respect to men and women in two
important respects. First, women are endowed with a nontraded,
nonstorable good. (For concreteness, the reader can think of some
ability with which women are exogenously endowed.) The distribu-
tion of endowments of women is uniform on [0, 1] in each period.
The endowment of a daughter is independent of the mother’s endow-
ment. We use endowments to index the women so that woman j is
endowed with an amount j of the nontraded good. The men will be
indexed by i ∈ [0, 1].

The second asymmetry is that we assume that only the welfare of
the male offspring enters the pair’s utility function. We normalize so
that a male offspring inherits his father’s index, and we shall refer
to man i, his son, his son’s son, and so on as family line i.

Men and women have identical utility functions defined over joint
consumption c. (This is done to avoid distributional issues.) We as-
sume that the utility derived from joint consumption is given by a
constant relative risk aversion (CRRA) utility function with degree
of risk aversion γ, u(c) = (1 − γ)c^{1−γ}. The level of the woman’s
endowment, j, enters linearly into current utility. The model is clearly well defined for more general utility functions and the value
of the woman’s endowment. The existence theorem for one type of equilibrium is
level of their male descendants also enters linearly into each parent’s utility function, discounted by $\beta \in (0, 1)$.

The problem facing a couple is, given their capital (the bequest from the male’s parents), how much to consume and how much to bequeath to their son. Their son receives utility from the bequest in two distinct ways. First, it may affect the quality of his mate. Second, it affects the amount he and his male descendants can consume (and their mates, but the couple cares only about the male offspring). Parents will be willing to reduce current consumption if it sufficiently increases the quality of their son’s mate.

In order to describe equilibria, it is convenient to introduce the notion of status. The role of status in our model will be to rank the men. This ranking determines the outcome of the matching stage. A man’s status can depend on, among other things, his own wealth and the status and match of his parents. A social norm is a status assignment rule that indicates how a man’s status is determined and a prescription of status-dependent matching behavior. An equilibrium is a social norm together with a specification of market decisions such that no agent wishes to deviate from the social norm or specified market decisions.

We assume that matching is voluntary and takes place on the basis of complete current information. Let $P_i$ and $P_j$ denote the (reduced-form) preferences of men and women over mates. Men prefer women with higher values of $j$, all else equal. Of course, being successfully matched with a woman with a high $j$ (depending on the prevailing social norm) may result in a loss of status for the offspring, which $P_i$ will reflect. Women prefer men who can provide high current consumption (through their inheritance) or whose offspring will have high status or consumption. Here too, being successfully matched with a man who can provide high current consumption may result in a loss of status for the offspring, which $P_j$ will reflect.

We say that $m: [0, 1] \to [0, 1]$ describes a voluntary matching, where $m(i) = j \in [0, 1]$ is $i$’s match, if the following two conditions hold: (i) there does not exist $i \neq i' \in [0, 1]$ such that $m(i')P_i m(i)$ and $iP_{m(i')}i'$, with both preferences holding strictly; (ii) for any (measurable) set $B \subset m^{-1}[0, 1]$, $m(B)$ has the same Lebesgue measure (size) as $B$.

Condition i says that no pair prefers to be matched with each other rather than with their matches. Condition ii requires that any fraction of matched men be matched with that fraction of matched women (this condition is implied by one-to-oneness when there is a finite

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stated in the Appendix for the general case. We believe that most of the results of interest carry over to a much broader class of utility function than CARRA. We have chosen for expository purposes to keep the discussion in the text confined to this case.

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7 That is, these preferences take as fixed the behavior of all other agents.
number of agents). In a voluntary matching, all men and women receive a match (except possibly for a zero-measure set of women).

We shall not describe the details of the matching process. What is important is that each man can make an offer to match with any woman. Women will choose the most attractive offer if they receive more than one proposal. This concludes the matching round; in particular, there are no counterproposals by women to men and no offers from men reacting to the offers of other men. If only one proposal is received by a woman, then that proposal is accepted. If a man's proposal is unsuccessful, then that man is matched with a zero-endowment woman.  

IV. The Capital Accumulation Model

A. A Two-Period Example

Before considering the general infinite-horizon model, we shall present a two-period example that will illustrate some of the characteristics in which we are interested. While the finite horizon eliminates the multiplicity of equilibria that is at the heart of our model, the example is nevertheless of some independent interest.

Agents use capital for current consumption and savings. Output is produced according to

\[ c = Ak - k', \]

where \( k \) is the initial endowment capital, \( c \) is first-period consumption, \( k' \) is second-period capital, and \( A > 1 \) is a constant.

Matching takes place in the second period only. Each man must choose in the first period how much to consume out of first-period output, with the remainder being saved; in the second period all output is consumed jointly by the son and his mate. Initially, suppose that all men have the same initial endowment of capital in the first period, \( K \). Each man's utility function is given by

\[ u(c_0) + \beta[u(c_1) + f], \]

8 This is well defined as long as only a zero-measure set of men is unsuccessful. If there is a set of positive measure, then they receive no match. Note that in checking for equilibrium, we need be concerned only with unilateral deviations in the matching stage, which have zero measure. In a model with a finite number of agents, we can simply assign the woman with the lowest endowment to the unsuccessful man and reassign the other women, maintaining the order.

9 We have suppressed matching considerations in the first period since including first-period matching would have no effect on savings decisions. When all agents have equal wealth, any first-period allocation is an equilibrium.

10 Note that the only decision makers in this example are the men alive in the first period and the women; the sons affect the analysis only through the utility they provide their fathers.
where \( c_1 \) is the offspring pair's joint consumption in the second period, \( j \) refers to the endowment level of the son's mate, and \( u(\cdot) \) is CRRA with degree of relative risk aversion \( \gamma \). The utility function of the son's mate depends only on the pair's joint consumption in the second period and on its endowment level: \( u(c_1) + j \). Note that \( 0 \leq c_0 \leq AK \) and \( c_1 = A^2K - Ac_0 \).

Since the second period is the last period, if a woman receives multiple proposals, then the proposal from the wealthiest man is accepted (we can resolve any ties by assuming that the woman flips a coin). A man's status, then, is determined precisely by his capital relative to other men's capital, higher capital yielding higher status. The equilibrium matching thus depends only on the men's capital endowments of that period (which are determined by their fathers' bequests), matching the wealthiest man with the woman of highest endowment, and so on. A man's match in period 2 depends, then, only on his relative position in the capital distribution of period 2. An equilibrium is a description of consumption-savings decisions and matching behavior for the agents such that no agent has an incentive to deviate from the described behavior.

Since all the men have the same initial capital and can therefore imitate the behavior of any other man, all men must have the same utility level in equilibrium. At the same time, their sons will be matching with women of different endowment levels. Consider the problem of the man whose son matches with the woman with the lowest endowment, zero. His savings behavior cannot be distorted (from the level that is optimal when matching considerations are ignored) since any deviation in savings cannot reduce the quality of his match. Thus his maximized utility is given by

\[
V(0) = \max_{\lambda} u(AK\lambda) + \beta u(A^2K(1 - \lambda)).
\] (2)

Denote the optimal value of \( \lambda \) by \( \lambda(0) = [1 + (\beta A^{1-\gamma})^{1/\gamma}]^{-1} \). Note that the sign of \( \frac{\partial \lambda(0)}{\partial A} \) is the same as the sign of \( \gamma - 1 \), so that \( \gamma > 1 \) corresponds to the case in which the income effect dominates the substitution effect when there is a fall in the relative price of future consumption, and \( \gamma < 1 \) corresponds to the case in which the substitution effect dominates.

Now consider the equilibrium behavior of the man whose son matches with a woman whose endowment level is \( j \). It must be the case that this welfare level, \( V(j) \), is the same as \( V(0) \). This implies that his consumption fraction in the first period, \( \lambda(j) \), must be such that

\[
V(j) = u(AK\lambda(j)) + \beta[u(A^2K(1 - \lambda(j))) + j] = V(0),
\]
and $\lambda(j) < \lambda(0)$. This implies that $\lambda(j)$ must be such that

$$\lambda(j)^{1-\gamma} + \beta A^{1-\gamma}(1 - \lambda(j))^{1-\gamma} = [V(0) - \beta j(1 - \gamma)(AK)^{\gamma-1}.$$ 

Implicitly differentiating this expression yields

$$\frac{\partial \lambda(j)}{\partial j} = \frac{-\beta (AK)^{\gamma-1}}{\lambda(j)^{-\gamma} - \beta A^{1-\gamma}(1 - \lambda(j))^{-\gamma}}.$$ 

Since $\lambda(j) < \lambda(0)$, the denominator is positive and so $\partial \lambda/\partial j < 0$.

This example illustrates several points. First, matching considerations cause agents to increase their savings levels relative to their nonmatching or involuntary matching levels.\textsuperscript{11} Second, after the first period the income distribution is unequal. Note also that bequests as a function of $j$ are continuous and strictly increasing. Further, since we began with a degenerate income distribution, the men's savings behavior, conditional on their sons' match, is being distorted. This follows from the fact that in equilibrium each of the men is indifferent between his equilibrium choice and the savings level he would choose if his son were to be exogenously matched with the woman of zero endowment.

It is interesting to note the relationship between the initial endowment level, $K$, the curvature of agents' utility function for consumption, $\gamma$, and the degree to which matching considerations increase savings rates. If $\gamma > 1$, then the slope of bequests as a function of $j$ is increasing in $K$ and the competition for mates becomes more intense. The reverse occurs if $\gamma < 1$.

Suppose now that men's initial wealth is not constant but that man $i \in [0, 1/2]$ has initial capital level $K - \epsilon$ and $i \in [1/2, 1]$ has initial capital $K + \epsilon$, where $\epsilon > 0$. Note that the average level of capital is the same as before but that the distribution is now unequal.

The sons of those agents with the lower capital endowment will be matching in equilibrium with the lower half of the distribution of the women. Their equilibrium consumption fractions can be derived just as before. The welfare level of the agent who is in the lower half of the capital distribution and whose son matches in equilibrium with the zero-endowment woman has welfare level $V(0)$ given by (2) with $K - \epsilon$ replacing $K$. Let $V(j)$ be the welfare level of the man whose son, in equilibrium, matches with a woman of endowment level $j$. As before, we must have $V(j) = V(0)$ for $j < 1/2$. This in turn determines his agent's consumption rate and hence his son's capital level, $k(j)$.

Now consider the man who receives an initial endowment of $K + \epsilon$ and whose son's mate will, in equilibrium, have the lowest endow-

\textsuperscript{11}The man at the bottom of the matching hierarchy who matches with the least endowed woman is the only exception; his savings level is undistorted.
ment of those in the top half of the distribution, that is, \( \frac{1}{2} \). His equilibrium welfare level, \( V(\frac{1}{2}) \), is the value of the following maximization problem, where \( k^-(\frac{1}{2}) = \lim_{i \uparrow \frac{1}{2}} k(i) \):

\[
\max_{\lambda} u(A(K + \epsilon)\lambda) + \beta u(A^2(K + \epsilon)(1 - \lambda)) + \beta^{\frac{1}{2}}
\]

subject to \( \lambda \leq 1 - \frac{k^-(\frac{1}{2})}{(K + \epsilon)A} \).

This last restriction requires that the savings rate for this man be sufficiently high that his son’s capital level is at least as large as that of the son of any man in the lower half of the distribution. This guarantees that no man in the lower half of the capital distribution finds it profitable to reduce first-period consumption so that his son can match with a woman with endowment \( j \geq \frac{1}{2} \). Notice that for \( \epsilon \) large, this restriction will not bind.

The welfare level of any arbitrary man in the upper half of the initial capital distribution whose son mates in equilibrium with a woman of endowment level \( j \) (\( j \geq \frac{1}{2} \)) must be such that \( V(j) = V(\frac{1}{2}) \), where his initial consumption rate, \( \lambda(j) \), is adjusted so that this equality holds. Notice that the lower \( V(\frac{1}{2}) \) is, the lower \( V(j) \) must be and, hence, the lower \( \lambda(j) \) must be also.

As \( \epsilon \) gets larger and the restriction that the son with mate \( j = \frac{1}{2} \) be at least as wealthy as any son with mate \( j < \frac{1}{2} \) becomes less severe, the savings rates for men in the top half of the distribution fall. At the same time, this subset of men is wealthier (whereas those in the bottom half of the distribution are poorer), and this will tend to increase or decrease (decrease or increase) the extent of their sons’ competition for mates depending on whether \( \gamma \) is greater or less than one, respectively.

When we compare the two cases above, we see that men’s savings behavior depends not just on initial capital levels but also on the compactness of the distribution of wealth in the economy. This can be seen most simply by choosing initial distributions in the two cases such that the wealthier agents in the two-point distribution are just as well off as the agents in the one-point distribution. The average savings rate in any section of the top half of the distribution of second-period capital must be lower than the comparable fraction from the one-point capital level economy. This illustrates the general point that if we compare two men with the same initial income level in two different economies in which the equilibrium assignment rule for mates is based on wealth, then the agent from the economy with the more compact income distribution will tend to have a higher savings rate, all other things being equal.
B. The General Case

As above, agents use capital to fund current consumption and bequests to their male offspring.\textsuperscript{12} Output is again produced according to equation (1), \( c = Ak - k' \), where \( k \) is the amount of capital that the man brings to the match, \( k' \) is the bequest level, \( c \) is the amount of current joint consumption, and \( A > 1 \) is a constant.

Before we study the model with endogenous matching, it is useful to analyze the case of exogenous matching. Suppose that \( \{j_i\}_{i=0}^\infty \) is the sequence of matches for family line \( i \). Because of the recursive nature of the agents' utility functions, the problem facing the first member of family line \( i \) is effectively to solve\textsuperscript{13}

\[
\max_{\{k_i\}} \sum_{t=0}^{\infty} \beta^t [(1 - \gamma)^{-1} (Ak_i - k_{t+1})^{1-\gamma} + j_i]
\]

subject to \( k_0 = k_0(i), Ak_i \geq k_{t+1}, \) (3)

where \( k_0(i) \) is \( i \)'s capital endowment in period \( t = 0 \). Recall that \( u(c) = (1 - \gamma)^{-1}c^{1-\gamma} \). This is just the standard deterministic growth problem with an exogenous term to reflect the value of the matched woman. The first-order condition is

\[
(Ak_i - k_{t+1})^{-\gamma} = A\beta(Ak_{t+1} - k_{t+2})^{-\gamma}.
\] (4)

Note that \( A\beta > 1 \) implies that consumption (and hence the capital stock) is increasing over time. In what follows, we assume \( 1 < A\beta < A^\gamma \) (the second inequality is needed to ensure that the discounted value of utility from consumption is finite).

Let \( \lambda_t \) be the fraction of output that is consumed in period \( t, c_t/Ak_t \). Then \( \lambda_t \) satisfies

\[
(\lambda_tAk_t)^{-\gamma} = A\beta [(1 - \lambda_t)\lambda_{t+1}A^2k_t]^{-\gamma}.
\] (5)

This expression reduces to an expression that is independent of \( k \):

\[
\lambda_t^{\gamma+1} = \frac{\lambda_t^\gamma}{(1 - \lambda_t)^\gamma} A^{1-\gamma}\beta.
\] (6)

\textsuperscript{12} Since it is assumed that only the welfare of the male offspring enters the parents' utility function, parents will leave bequests only to their male offspring.

\textsuperscript{13} We are assuming here that the first member of a family line believes that the descendants will make the consumption-investment decisions that that member would choose if it was possible to dictate their choices. Since Bellman's principle of optimality applies, we have described a subgame perfect equilibrium of the extensive form game of consumption-investment choices with an infinite number of players (one player for each generation). It is possible that there are other equilibria, which we shall not discuss.
It is then a standard exercise to show that, in the solution to (3), the fraction of output consumed is time invariant and is given by \( \lambda^* = 1 - (A^{1-\gamma} \beta)^{1/\gamma} \). Note that \( \lambda^* \) is independent of \( k_n \), depending only on \( \gamma, \beta, \) and \( A \). The evolution of capital is governed by

\[
k_{t+n} = (1 - \lambda^*)^n A^n k_t, \quad \text{for } n \geq 0.
\]

(7)

The fact that the capital stock is increasing over time follows from our assumption that \( A\beta > 1 \), which implies that \( (1 - \lambda^*)A > 1 \).

The discounted value of utility from consumption is

\[
\sum_{t=0}^{\infty} \beta^t u(c_t) = (1 - \gamma)^{-1} \sum_{t=0}^{\infty} \beta^t [\lambda^*(1 - \lambda^*)^t A^{t+1} k_0]^{1-\gamma}
\]

\[
= \frac{(\lambda^* A k_0)^{1-\gamma}}{(1 - \gamma) (1 - \beta [(1 - \lambda^*) A]^{1-\gamma})}
\]

\[
= \frac{(\lambda^* A k_0)^{1-\gamma}}{(1 - \gamma) \lambda^*} \equiv V^*(k_0).
\]

(8)

Note that the sign of \( \partial \lambda^*/\partial A \) is the same as the sign of \( \gamma - 1 \), so that \( \gamma > 1 \) corresponds to the case in which the income effect dominates the substitution effect when there is a fall in the relative price of future consumption, and \( \gamma < 1 \) corresponds to the case in which the substitution effect dominates.

C. The Wealth-Is-Status Equilibrium

While there may be multiple equilibria when matching is voluntary, one equilibrium is particularly important, namely the equilibrium that arises when wealth determines status.\(^{14}\) If a woman receives multiple proposals, then the proposal from the wealthiest man is accepted (if there is a tie, then the woman flips a coin). The equilibrium matching thus depends only on the men’s capital endowments of that period (which are determined by their parents’ bequests), matching the wealthiest man with the woman of highest endowment, and so on. A man’s match in period \( t \) depends, then, only on his relative position in the capital distribution of period \( t \).

A wealth-is-status equilibrium is a description of consumption-bequest decisions and matching behavior for the agents such that no agent has an incentive to deviate from the described behavior in any situation. Whenever a woman receives multiple proposals from men, the pro-

\(^{14}\) Mokyr (1985, p. 18) provides the following citation from Perkin (1969, p. 85) on the social environment in England in the eighteenth century: “the pursuit of wealth was the pursuit of social status, not merely for oneself but for one’s family.”
posal from the wealthiest man is accepted (if there is a tie, then the woman flips a coin).

We shall assume that the initial endowment of capital $k_0: [0, 1] \rightarrow \mathbb{R}_+$ is a nondecreasing function of $i$. This assumption will be maintained throughout the remainder of the paper.\(^{15}\) Suppose that the bequest of capital in period $t$ is given by $k_t: [0, 1] \rightarrow \mathbb{R}_+$, where $k_t$ is a strictly increasing function. Further, suppose that in equilibrium all agents consume in such a way that the next-period capital bequest is given by $k_{t+1}(\cdot)$, also strictly increasing and differentiable on $(0, 1)$, so that the match in period $t + 1$ is $m(i) = i$. Consider family line $i$’s choice of bequest in period $t$. By increasing the bequest from $k_{t+1}(i)$ to an amount $k$ in the next period, the man of index $i$ will succeed in matching with any woman such that $j < k_{t+1}^{-1}(k)$. Let $m_{t+1}(k) = k_{t+1}^{-1}(k)$ denote the supremum of $i$’s match as a function of the bequest received. Then the problem facing an agent $i$ in period $t$ who inherits $k_t$ amount of capital is

$$\max_{0 \leq k_{t+1} \leq k_t} u(Ak_t - k_{t+1}) + m_t(k_t) + \beta V_{t+1}(k_{t+1}) \equiv V_t(k_t).$$  \hspace{1cm} (9)

If $V_{t+1}$ is differentiable, the first-order condition on bequests is given by $u'(Ak_t - k_{t+1}) = \beta V_{t+1}'(k_{t+1})$. From the envelope condition, $V_{t+1}'(k_{t+1}) = Au'(Ak_{t+1} - k_{t+2}) + m_{t+1}'(k_{t+1})$. Thus we get the following difference equation that describes a family line’s optimal capital accumulation decision:

$$u'(Ak_t - k_{t+1}) = \beta [Au'(Ak_{t+1} - k_{t+2}) + m_{t+1}'(k_{t+1})].$$  \hspace{1cm} (10)

This condition differs from (4), the corresponding condition for the case of exogenous matching, by the presence of a term reflecting matching. We can use (10) to illustrate the role of matching in the choice of bequest. Fix $k_t$ and $k_{t+2}$ and suppose that $k_{t+1}$ solves (10). Suppose now that the son’s match is more sensitive to the level of capital; that is, suppose that $m_{t+1}'$ were greater (equivalently, $k_{t+1}'$ decreases). Then in order for (10) still to hold, $u'$ must be smaller, which implies that period $t + 1$ capital must be larger. Thus we see that bequests will be larger when a son’s match is more sensitive to the level of capital on the margin. This will imply, all else being equal, that a more equal distribution of capital leads to a smaller share of current output devoted to current consumption, since a more equal distribution will be reflected in a larger $m_{t+1}'$.

If we reformulate (10) in terms of the fraction of current output consumed, $\lambda$, we get

\(^{15}\) This assumption is made without loss of generality in this section. The results in later sections in which we identify status with a person’s index might differ if this assumption is violated.
\[ (\lambda_t A k_t)^{-\gamma} = A \beta [(1 - \lambda_t) \lambda_{t+1} A^2 k_t]^{-\gamma} + \beta m'_{t+1} ((1 - \lambda_t) A k_t). \] (11)

This expression is in general not compatible with a steady-state rate of growth, that is, with \( \lambda_t = \lambda_{t+1} \). Notice also that since a pair's capital stock enters into this expression, a difference in capital stocks could cause different pairs to choose different fractions of their output to consume.

We show in the Appendix that wealth-is-status equilibria exist. In fact, the argument given there applies to a broader class of technologies and utility functions than considered here (all that is required is that the dynamic programming problem with exogenous matching be well behaved).

**Proposition 1.** Wealth-is-status equilibria exist.

**Proof.** See the Appendix, section A. Q.E.D.

We next describe several properties of a family line's optimal consumption-bequest decisions and wealth-is-status equilibria. In what follows, \( \{m_t\} \) will denote the sequence of matching functions that the family line will face given the choices of other family lines. Family line \( i \)'s optimal consumption ratio sequence is \( \{\lambda_t(i)\} \), yielding a sequence of capital bequests \( \{k_t(i)\} \); sometimes we shall write \( \lambda_t \) and \( k_t \). The proofs of the properties can be found in section B of the Appendix.

**Property 1.** In an equilibrium, if \( k_0(i) > k_0(i') \), then, for all \( t \), the optimal level of capital in period \( t \) for agents \( i \) and \( i' \) satisfies \( k_{t+1}(i) > k_{t+1}(i') \).

This property states that in an equilibrium, no family's relative position changes. Despite the fact that all family lines are taking into account how their savings behavior affects their relative ranking and are adjusting their behavior accordingly, the net effect of their decisions is that no family moves up or down in the ranking over time.

**Property 2.** For all family lines and all \( t \geq 0 \), \( \lambda_t \leq \lambda^* \).

In equilibrium, all families are consuming an amount less than or equal to what they would have consumed in the absence of matching concerns. That is, matching considerations cause agents to (weakly) increase their savings. In fact, the inequality is strict for all but a set of families of measure zero.\(^{16}\) Note that, as in the two-period example, the family line matching with the zero-endowment woman will not be distorted, that is, \( \lambda_t = \lambda^* \) for all \( t \).

**Property 3.** In any equilibrium, in every period, each bequest level is chosen by a zero measure of agents.

This property states that there are no atoms in the income distribu-

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\(^{16}\) Suppose not. Then since time is discrete so that the set of all dates is countable, there is a time \( t \) and a subset \( \{(i', i' + \epsilon)\} \) of family lines with a dense subset \( D \), such that \( \lambda_t = \lambda^* \), holding for all \( t \in D \). But this contradicts (11).
tion; at every period, the function relating family lines to capital, \( k_i(\cdot) \), is strictly increasing almost everywhere. Moreover, if \( k_0(\cdot) \) is strictly increasing, then the almost everywhere caveat can be dropped. This is a generalization of what we saw in the two-period example above. If there were an interval of families with equal wealth, that family matched with the least endowed woman could have obtained a positive increase in utility from matching with an arbitrarily small increase in savings.

Property 4. Suppose that \( k_0(\cdot) \) is a continuous function of \( i \). Given an equilibrium and any period \( t \), the capital distribution is strictly increasing, so that \( k_i(\cdot) \) is a continuous function of \( i \).

Property 5. If \( \gamma < 1 \), then, for all \( i \in [0, 1] \), \( \lambda_i(i) \to \lambda^* \) as \( t \to \infty \).

This property and the next describe the limiting behavior of family lines as time goes to infinity. Property 5 says that for the case of \( \gamma < 1 \) (recall that this is the case in which the substitution effect dominates), asymptotically, consumption-savings behavior is the same with and without matching considerations. In other words, in this case, the oversaving that results from the concern about relative positions asymptotically vanishes. On the other hand, if \( \gamma > 1 \), property 6 states that the effect of the concern for relative position increases as time goes on. It states that for any two family lines, either the poorer is saving a fraction of wealth that is going to one or the ratio of the wealth of the richer family to that of the poorer family is going to infinity.

Property 6. If \( \gamma > 1 \), then either \( \lambda_i(i') \to 0 \) or \( k_i(i')/k_i(i) \to \infty \) for all \( i < i' \) for all \( i' \in (0, 1] \) as \( t \to \infty \).

D. The Aristocratic Equilibrium

In this subsection, we examine a second type of equilibrium. In this equilibrium, which we call aristocratic, a man’s status is inherited. That is, his status is the same as his father’s as long as his father matched “appropriately.” In the initial period, there will be an exogenously given status assignment. An aristocratic equilibrium consists of this status assignment rule, consumption-bequest decisions, and matching behavior for the agents such that each man is voluntarily matched with the woman whose endowment equals the man’s status. If a man matches with a woman with an endowment not equal to his status, the family line from that point on has zero status; and no woman of positive endowment will match with a zero-status man when that status is newly acquired.\(^{17}\)

\(^{17}\)We are requiring only that it be optimal for a woman to follow status on the equilibrium path and for one generation off the equilibrium path.
Assume that initially the status of male $i$ is given by $i$. We could in general allow status to be allocated somewhat independently of the distribution of capital. Let $s$ denote the status of the father. The evolution of status is governed by

$$n(s,j) = \begin{cases} 
    s & \text{if } s = j \\
    0 & \text{otherwise.}
\end{cases} \quad (12)$$

If everyone obeys that aristocratic social norm (i.e., the status assignment rule and matching contingent on status), then a male from family line $i$ will always match with the female endowed with $j = i$, irrespective of his wealth. A family line is punished by having its status and, hence, match set to zero forever if one of the men in the line marries inappropriately.

If men match appropriately, then the value function is given by

$$V(k, s) = \max_{c, k'} u(c) + m(s) + \beta V(k', s) \quad \text{subject to (1).}$$

The first-order condition for bequests is given by $u'(A k - k') = \beta V'$. From the envelope condition, $V' = u'(A k - k')A$, and hence we have the following difference equation, which describes a family's optimal capital accumulation decision (family denotes the family line through the son):

$$u'(A k_t - k_{t+1}) = \beta u'(A k_{t+1} - k_{t+2})A.$$

An important feature of this equilibrium is that the path of the capital stock is identical to that in the involuntary matching model. In particular, the value function for a man of status level $s$ is given by

$$V(k, s) = V^*(k) + (1 - \beta)^{-1}s.$$

Thus when an aristocratic equilibrium exists, properties 2, 5, and 6 describe the relationship between the wealth-is-status and aristocratic equilibrium savings behavior.

The aristocratic status rule is consistent with equilibrium if at no time does family line $i$ wish to bequeath sufficient capital to its son so that he can induce a woman of endowment greater than $i$ to match. Let $L_t(j)$ be the capital level that leaves the woman indifferent between matching with the man of status $j$ with capital $k_t(j)$ and maintaining status for her son, and matching with a man with capital $L_t(j)$ and having a zero-status son. If the woman does match with the latter, the status of the man that the woman deviates to is irrelevant since offspring will have zero status; thus the capital level $L_t(j)$ is described by

$$V^*(L_t(j)) + j = V^*(k_t(j)) + \frac{1}{1 - \beta}j.$$
This implies that

\[ L_t(j) = \left[ k_t(j)^{1-\gamma} + \frac{\beta}{1-\beta} j(1 - \gamma)(\lambda^*)^\gamma A^{\gamma - 1} \right]^{1/(1-\gamma)} \]

Note that \( L_t(j) \) exceeds \( k_t(j) \) by an amount that compensates for the value of lost status. It is worth noting here that \( L_t(j) \) is close to \( k_t(j) \) for \( j \) close to zero.

A man \( i \) with capital \( k \) will not match with a woman \( j \) if \( V^*(k) + j \leq V^*(k) + [i/(1 - \beta)] \), that is,

\[ (1 - \beta)j \leq i. \quad (13) \]

Moreover, for any \( \beta \), there are men of sufficiently low status to make them willing to match with women that are higher than prescribed.

Consider the bequest decision of family line \( i \) in period \( t-1 \). Bequeathing \( L_t(j) \) will improve its son's match, but at a cost of reduced consumption today as well as lower status after two generations. Bequeathing \( k_t(i) \) yields a higher payoff than \( L_t(j) \) if

\[ V^*(k_{t-1}(i)) + \frac{i}{1 - \beta} \geq u(Ak_{t-1}(i) - L_t(j)) + i + \beta j + \beta V^*(L_t(j)). \quad (14) \]

Fix \( i' \in (0, 1) \). The following lemma shows that, for sufficiently high \( \beta \), neither the men nor the women with indexes above \( i' \) will deviate from the aristocratic social norm (it is not enough to ensure that the men with indexes above \( i' \) will follow the social norm; we need to ensure that women with endowments above \( i' \) will also reject offers from men with indexes below \( i' \)).

**Lemma.** Suppose \( \gamma > 1 \). Fix \( i' \in (0, 1) \). There exists \( \beta(i') \) such that, for \( \beta > \beta(i') \), (13) holds and any woman with endowment \( j > i' \) will not accept any offer from a man with index \( i \neq j \).

**Proof.** Clearly (13) holds for \( \beta \) sufficiently close to one. If woman \( j \) accepted an offer from man \( i \neq j \), then \( j \)'s son has status zero, the cost per period of which is greater than or equal to \( i' \). Since utility is bounded, there exists \( \tilde{c} \) such that, for all \( c > \tilde{c} \), \( \sup_{c'} u(c') - u(c) < i'/2 \). When family lines save in an aristocratic social norm, \( c_t(i) \uparrow \infty \). Moreover, \( c_t(i) \) is increasing in \( i \), for all \( t \). Therefore, there exists \( T \) such that, for all \( t > T \), \( c_t(i) > \tilde{c} \) for all \( i \geq i' \).

The utility to woman \( j \) from following the aristocratic social norm is \( V^*(j) + [j/(1 - \beta)] \). The utility to woman \( j \) of accepting an offer from \( i \neq j \) is less than

\[ T \sup_{\tilde{c}} u(\tilde{c}) + j + \sum_{\tau = T+1}^{\infty} \beta^\tau \sup_{\tilde{c}} u(\tilde{c}). \]
Thus the gain from the deviation from the aristocratic social norm is less than

\[
T \sup_{\tilde{c}} u(\tilde{c}) - \sum_{t=0}^{T} \beta^t u(c_\tau(j)) + j + \sum_{t=T+1}^{\infty} \beta^t [\sup_{\tilde{c}} u(\tilde{c}) - u(c_\tau(j))] - \frac{j}{1 - \beta}
\]

\[
\leq T \sup_{\tilde{c}} u(\tilde{c}) - \frac{1 - \beta^{T+1}}{1 - \beta} u(c_0(j)) - \frac{1 - \beta^T}{1 - \beta} \beta j - \frac{\beta^{T+1}}{1 - \beta} \left( j - i' \right)^2.
\]

For \( \beta \) sufficiently close to one, this last expression is negative. Q.E.D.

However, since low-status men and low-endowment women have very little to lose from deviations, the threat of a loss of status is not sufficient to maintain equilibrium: Since \( k_i(0) = L_i(0) \), the inequality (14) holds as an equality for \( i = j = 0 \). While the left-hand side is independent of \( j \), the partial derivative of the right-hand side with respect to \( j \) is given by \( \beta + (\beta V' - u')L' = \beta \) at \( i = j = 0 \). Thus (14) must fail for \( i = 0 \) and \( j \) near \( i \).\textsuperscript{18} However, if capital is sufficiently “spread out” at the lower tail and \( k_0(0) = 0 \), then it is impossible for sufficient wealth to be bequeathed: A man cannot save enough to induce a deviation next period by any woman with an endowment greater than or equal to his status level, that is, \( L_i(j) \geq Ak_{i-1}(j) \).

**Proposition 2.** Suppose \( k_0(0) = 0 \). Fix \( i' \in (0, 1) \) and suppose \( \beta \in (\beta(i'), 1) \). An aristocratic equilibrium exists if \( \gamma > 1 \) and

\[
k_{0(i)}^{\gamma} k_{0'(i)} < \frac{A^{2(\gamma - 1)}(1 - \lambda^*)^\gamma(\lambda^*)^{-1} \beta}{1 - \beta}
\]

for all \( i \leq i' \).

**Proof.** First observe that \( k_0(0) = 0 \) implies \( L_i(0) = k_i(0) = Ak_{i-1}(0) = 0 \). It is enough to show that \( dL_i(i)/di > d[Ak_{i-1}(i)]/di \) for all \( i < i' \) for all \( t \). Note that as \( t \to \infty \), \( k_i(i) \to \infty \) and \( L_i(i)/k_i(i) \to 1 \) for \( i \neq 0 \). Differentiating \( L_i(i) \) yields

\[
\frac{dL_i(i)}{di} = L_i(i)^\gamma \left\{ k_i(i)^{-\gamma(1 - \lambda^*)} \frac{d[Ak_{i-1}(i)]}{di} + \frac{\beta(\lambda^*)^\gamma A^{\gamma - 1}}{1 - \beta} \right\}
\]

\[
\geq (1 - \lambda^*) \frac{d[Ak_{i-1}(i)]}{di} + \frac{L_i(i)^\gamma \beta(\lambda^*)^\gamma A^{\gamma - 1}}{1 - \beta}.
\]

We now argue that the second term is larger than \( \lambda^* d[Ak_{i-1}(i)]/di \). By hypothesis,

\[
k_{0(i)}^{\gamma} k_{0'(i)} < \frac{A^{2(\gamma - 1)}(1 - \lambda^*)^\gamma(\lambda^*)^{-1} \beta}{1 - \beta}.
\]

\textsuperscript{18} Note that if instead the value of match \( j \) is \( v(j) \) and \( v'(0) = 0 \), then the argument fails. In fact, one can write down conditions for the general model (i.e., general \( u(\cdot) \), \( v(\cdot) \), and technologies) that yield the existence of aristocratic equilibria.
Since \( k_t(i) = [A(1 - \lambda^*)]^t k_0(i) \), this implies
\[
\frac{A^{2(\gamma - 1)}(1 - \lambda^*)^\gamma (\lambda^*)^{\gamma - 1} \beta}{1 - \beta} > [A(1 - \lambda^*)]^\gamma(1 - \lambda^*)^{-\gamma} k_{t-1}(i) - \gamma k_{t-1}'(i)
\]
\[
> k_{t-1}(i)^{-\gamma} k_{t-1}'(i),
\]
since \( A(1 - \lambda^*) > 1 \) and \( \gamma > 1 \). Note that the second term in (15) is larger than
\[
\frac{k_t(i)^\gamma \beta (\lambda^*)^\gamma A^{\gamma - 1}}{1 - \beta} = \frac{k_{t-1}(i)^\gamma (1 - \lambda^*)^\gamma \beta (\lambda^*)^\gamma A^{2\gamma - 1}}{1 - \beta}
\]
\[
= \lambda^* A \left( \frac{k_{t-1}(i)^\gamma A^{2(\gamma - 1)}(1 - \lambda^*)^\gamma \beta (\lambda^*)^{\gamma - 1}}{1 - \beta} \right)
\]
\[
= \lambda^* A k_{t-1}'(i),
\]
and we are done. Q.E.D.

Before going on, we shall discuss an important difference between the aristocratic equilibrium and the wealth-is-status equilibrium. Proposition 2 shows that for some distributions of initial capital, there will be an aristocratic equilibrium. The possibility that there might not be an aristocratic equilibrium stems from the fact that for some distributions of wealth and status, a low-status, high-wealth man might find it worthwhile to deviate from the social norm and make an offer to match with a woman of higher endowment than the social norm prescribed. By assumption about the matching process, the woman accepts the most desirable offer to match in the event that multiple offers are made. Sufficiently high wealth could more than compensate for the (assumed) loss of status that results from a woman’s deviation from the social norm. The difficulty in establishing an aristocratic equilibrium lies in assuring that when a man bequeaths to his son sufficiently high capital to induce a woman with a higher \( j \) to match, the benefits to the offspring do not exceed the cost to the parent. As the capital distribution becomes more “spread out,” the extra savings needed to induce a specific woman to deviate get larger. The inequality in proposition 2 that provides a sufficient condition for an aristocratic equilibrium to exist is essentially that the capital distribution is sufficiently spread out.

This discussion points out why the inequality in the proposition suffices to guarantee that a man will not have an incentive to deviate from the prescribed savings behavior, given that there have been no previous deviations. It should be noted that if the generations preceding a man have saved more than what is prescribed, the given man may not find it in his interest to follow the social norm. The question here is essentially the difference between a Nash equilibrium of a
game and a subgame perfect Nash equilibrium. An aristocratic equilibrium is essentially a Nash equilibrium in which each woman responds optimally to any first-round deviation by a man, but not necessarily a subgame perfect Nash equilibrium.

The reason that an aristocratic equilibrium may not be subgame perfect is that given a distribution of capital, if each generation of a given line saves enough, eventually a man in the line might have sufficient capital that he will find it optimal to deviate from the social norm. On the other hand, given any positive integer \( N \), there always exists a distribution of capital such that the number of agents that must deviate from equilibrium behavior before any agent finds it nonoptimal to follow the social norm exceeds \( N \). Thus despite the lack of “complete” subgame perfection, we find the aristocratic equilibria completely plausible.\(^{19}\)

Before closing this section we would like to discuss the aristocratic status assignment rule. In that rule, we have assumed that the son of a man who has deviated from his prescribed match was assigned status zero. It might seem that what drives the behavior of the agents is this “extreme” punishment. It is important to note that the existence of aristocratic equilibria does not depend on adjusting deviating men’s status levels to zero. Suppose that we altered the status assignment rule as follows. Following a deviation from the prescribed match, the male offspring in the deviating male’s line have their status reduced by a fixed positive proportion. Under this status assignment rule, for any \( i' \in (0, 1) \), a deviation in prescribed matching results in a proportionate reduction from matching each period equal to the proportionate reduction in status. As before, if wealth is sufficiently high and the discount factor is sufficiently close to one, no woman with an index higher than \( i \) could obtain an increase in utility from consumption that would offset the future decrease in matching utility. Thus no man with an index at least equal to \( i' \) would wish to deviate from the social norm. But if the income distribution is sufficiently flat (as in proposition 2), no man with an index below \( i' \) will have sufficient wealth to induce a deviation from a woman with a higher index.

Thus as long as the status assignment rule reduces the status of future generations by some positive amount following a deviation, there will still be equilibria of this modified aristocratic type. Of course, it is clear that if the amount that status is reduced is very small, the discount factor must be very close to one to assure that the annuity value of the loss of status is greater than the utility gain from

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\(^{19}\) See also the discussion of this issue in Kalai and Neme (1988) and Okuno-Fujiwara and Postlewaite (1990).
increased consumption. In addition, the capital distribution must be very spread out at the lower tail.

V. Discussion

A. The Role of Matching

Since the primary departure of our model from the classical growth model is our introduction of the matching process, it is worthwhile to go into some more detail about precisely how the matching process alters the equilibrium growth path for a particular society. In our model, the matching process may provide an incentive to save beyond that captured by standard models. The additional reward is the increased value of the matches of future male generations. A few comments on this matching process are in order.

First, as we emphasized in the Introduction, our goal has been to present a model that captures the idea that people have rational reasons to care about their relative position in an economy. The particular matching process we introduced is one of many ways the idea can be implemented. We believe that the incentives captured by the model are not unrealistic. One key advantage of our method of inducing a concern about relative position is that it does not require agents to be concerned with anything other than their family's outcome. As Duesenberry (1949) pointed out, introducing status directly into an agent's utility function seems to require a psychological theory to underpin the preference assumptions. The plausibility of the model is then tied to the plausibility of the psychological preference theory.

Second, our model derives as part of an equilibrium agents' concern with their rank in society. This is important because it provides substantial detail about how and why agents should care. In particular, the details of the model generate testable implications. Models such as the one presented here predict definite qualitative changes in economic behavior as the distribution of agents' characteristics changes; for example, other things being equal, tighter distributions of agents' characteristics induce greater deviations from that behavior than would have arisen had the ranking not mattered. This "fine detail" of the model allows an examination of the particular manner in which ranking matters that would not be possible in a model that puts rank directly into the utility function.

There is a third point regarding the model presented in this paper. One might think that it could be altered slightly to give a one-sex model with similar conclusions. For example, one can imagine a one-type model in which there are some goods that are allocated according to status rather than by markets (e.g., invitations to the White
House, church pews, or seats on university boards of trustees).\textsuperscript{20} If agents care about the goods to be allocated according to status, then whether or not the status of an individual depends on his wealth will affect his savings decisions. The difficulty with a model of this sort is that one would have to account for the person allocating the nonmarket goods: why is he giving away goods that people care about rather than selling them? To incorporate such decisions into a model would seriously increase its complexity. Our matching model captures these incentives while avoiding these difficulties.

B. The Implications of Relative Position Mattering

In the past several decades, an important theme in economics has been the importance of providing microeconomic foundations for macroeconomics. The argument has been that many (if not most) macroeconomic questions of interest can be analyzed and understood only if the individual agents in the economy were carefully modeled and their behavior was rooted in rational decision making. An important theme of this paper is to turn this argument around. Humans are social creatures. It is impossible to talk about an individual’s preferences and decision making in isolation; his effort choices, savings behavior, and buying habits depend on his environment. This paper provides a specific model in which an agent’s environment matters in describing his behavior. In a sense, this can be thought of as an initial attempt to provide “macroeconomic foundations for microeconomics.”

We think that such foundations are important in understanding several important economic questions. Consider, for example, standard consumption-savings models. In these models, agents’ incentives to save (invest) stem from the increased future consumption that would result. This is clearly an important—possibly the most important—incentive to save. But think for a moment about an already very rich agent such as Donald Trump.\textsuperscript{21} Why does he continue to work long days, endure substantial amounts of stress, and take enormous risks? Surely it cannot be that he is savoring the prospect of going to the grocery store with a looser budget constraint next year.

\textsuperscript{20} An example with no nonmarket goods that has been suggested to us is a housing market, where individuals care about their relative position in society because that determines the house they obtain in a competitive equilibrium. However, such a model cannot generate the type of multiplicity we are interested in. Moreover, as in the examples discussed in the text, we need to account for those who owned the houses originally.

\textsuperscript{21} This section was written prior to Trump’s recent financial difficulties. We have chosen to leave the example in since it underscores the risks we claim he took.
He seems to have more money than he could spend in several lifetimes. Even if we are wrong about Trump’s net worth, there clearly seem to be wealthy individuals that continue to work very hard and take large risks to increase their net worth. It is hard to reconcile such behavior with the underlying decision making in traditional growth models.

We propose that people like Trump continue to care about increasing their net worth because their utility depends not only on the absolute level of their wealth but also on their wealth relative to that of other very rich people: everyone would like to be the richest person in the world, even if the second richest has more money than can be sensibly spent. Taking account of this additional incentive to save could well change qualitatively the nature of optimizing behavior in a growth model. This paper has presented a model in which people may care about their relative position in society as well as their level of wealth. The model allows for societies to differ in the degree to which people may care about their relative position and, thus, differ in the severity of the “rat race of the rich” that results.

C. The Role of Nonmarket Decisions

In the models presented, the primary departure from more standard models was the inclusion of a matching decision about which the agents cared. We have stressed that this is just one (and perhaps not the most important) nonmarket decision that we could have incorporated into our model in order to have such nonmarket decisions affecting economic variables. Although the resulting model would be less tractable, there is no reason to believe that other nonmarket decisions would not play the same role.

The phenomena we have identified and analyzed—the existence of multiple equilibria that differ according to how a society allocates status—depend on there being some nonmarket decisions that matter. Further, if the decisions matter only slightly, they can have only a slight effect on the outcome. In the model as presented, we took as fixed and exogenously given the degree to which the nonmarket decision—matching—mattered. The match a man achieved could change his current-period utility by at most one unit, the maximum difference in utility across all possible current matches. To the extent that future matches might be affected by current choices, his utility could be changed by at most $1/(1 - \beta)$, the discounted value of the maximum change per period. We can easily determine the effect of changing the value of possible matches, say by multiplying the utility of all matches by some constant. If we did multiply the value of a match by a sufficiently small constant, it is clear that all equilibria give
approximately the same choices as in the exogenous matching case, since anything more than minor deviations will be more costly than the maximum benefit that could result from such a deviation. In summary, if the value of the nonmarket decisions that are affected by status is small, there may still be many equilibria, but they will all be approximated by the case in which social norms were ignored.

D. Income Distribution

Two remarks are in order regarding income distribution. First, as the wealth-is-status savings model makes clear, if relative position matters, this gives a greater incentive to save in equilibrium. The difference in savings behavior from the case in which matching considerations were ignored is in some (ill-defined) sense greater for higher incomes, all other things equal. In particular, there is no effect at all for the person with the lowest level of capital. Thus the effect of the introduction of matching into the consumption-savings decision will aggravate unequal distributions of income. The most extreme case of this arises when all agents begin with identical incomes. It is clear that even in this case, the only equilibrium paths give rise to unequal distributions of capital and that the inequality will increase over time.

The second remark is that in the context of our model, income distribution is of interest beyond the standard moral/ethical reasons. Here, the distribution of income is not just an end product of the economy. Rather, it feeds back into the economic analysis, changing agents' behavior in the future. As seen in the wealth-is-status savings model with $\gamma < 1$, when the capital distribution becomes sufficiently dispersed, the increased incentive to save disappears. Thus exogenous (or even endogenous, perhaps) changes in the income distribution may have an important effect on future economic performance.\(^\text{22}\)

E. Reduced Forms of the Model

As we have repeatedly stressed, we believe the model above to be of interest beyond the literal application to the particular matching context. We believe that people have a deep and general concern with their relative position within society and that their position sometimes depends on their economic decisions. Our model is one attempt to

\(^{22}\) Persson and Tabellini (1990) investigate a model in which the distribution of income affects the growth rate of an economy; growth rates depend on income distribution for different reasons in their paper, however.
formalize and analyze how the relationship between economic decisions and nonmarket decisions might arise. Having provided foundations for such a relationship, we may consider a reduced form of the model.

Suppose that the relevant equilibrium is the wealth-is-status equilibrium. If we were to ignore matching considerations and consider only the male population, we could consider the induced, reduced-form utility functions for men. A given man's utility function would then be \( \Sigma_{t=1,...,x} \beta^t[u(c_t) + r_t] \), where \( r_t \) is the man's ranking in the society in period \( t \). To the extent that one believes that our model captures real phenomena then, it provides the foundation for analyzing the reduced form in which the agents directly care about their ranking. This differs from the situation in which ranking enters into the "true" utility function in an important way: since the reduced form is derived from an equilibrium of the fully specified model, we can understand how different agents in different societies can look and behave differently in the reduced form by examining the fully specified model. We expect that such reduced-form models will be useful in reexamining questions of predicting agents' responses to shocks in the labor market or to exogenous changes in government policy such as income tax changes.

F. The Determination of Society's Norms

A fundamental question not addressed by this paper is the origin of the social norms governing a society. The model presented here shows the possibility of multiple equilibria when social norms are taken into account. It has nothing to say about which of the differing social norms that might exist actually arises. One would like a model that endogenizes the social norm. Such a model would help understand which social norms are likely to arise and under what circumstances. While we have no firm ideas about the form of such models, recent work in evolutionary models may provide the tools necessary for this task.

G. Wealth-Is-Status Equilibria

The focus of this paper has been an investigation of how different social norms in otherwise identical societies could result in significantly different economic performance in the societies. Toward this end, we investigate two different social norms that we believe can be considered to capture important aspects of actual societies. Although the two social norms we analyze have been given "equal time," we find the wealth-is-status equilibrium to be of particular interest for
two reasons. The first is theoretical. In the discussion of aristocratic
equilbria, we noted that these equilibria were not fully subgame
perfect and argued that such lack of full perfection did not diminish the
plausibility of these equilibria. It is true that social norms other than
wealth-is-status involve people altering what would otherwise be optimal
behavior because of future “sanctions” that follow deviations
from that norm. If shocks were incorporated into our model, there
is the possibility that such sanctions might, for some realizations of
the shocks, become “too weak” to effectively constrain people. Such
a situation could cause the given social norm to “break down” and be
replaced by the wealth-is-status social norm in which there is no con-
cern with future sanctions.\footnote{We thank a referee of this Journal for
comments that led to this discussion.} This argument is, of course, highly spec-
culative. To be convincing, one should write down a stochastic model
and derive the conditions under which a social norm would asymptot-
ically converge to the wealth-is-status social norm. We consider this
to be a fruitful topic of research.

The second reason we find the wealth-is-status social norm to be
particularly interesting is that we believe that it is at least partially
accurate for the United States. Tractable models that incorporate this
social norm can be constructed to determine how considerations of
relative standing would affect savings behavior, labor supply deci-
sions, and consumption behavior. In such models, decisions would be
affected by the distributions within society of (among other things)
wealth, income, and ability. In standard economic models, individual
decisions are independent of the distribution of these variables; thus
incorporating relative standing into standard economic models would
both be relatively straightforward and generate testable hypotheses.

Appendix

A. Existence of the Wealth-Is-Status Equilibrium

PROPOSITION 1. Suppose that the technology is given by \( c_t + k_{t+1} \leq f(k_t) \),
where \( f \) is continuous and increasing and \( f(k) > k \) for all \( k \). Suppose that
current utility is given by \( u(c) + v(j) \), where \( c \) is consumption and \( j \) is the
woman’s endowment. Assume that \( u \) is concave and continuous, \( u'(c) > 0 \) for
all \( c \), either \( u(c) \geq 0 \) for all \( c \) or \( u(c) \leq 0 \) for all \( c \), and \( v \) is continuous and
strictly increasing. If \( \sup_{k_t} \sum \beta^t u(f(k_t) - k_{t+1}) < \infty \) for all \( k_0 \), wealth-is-status
equilbria exist.

Proof. Following note 13, we shall treat a family line as an infinitely lived
player choosing a sequence \( \kappa = \{ k_t \}_{t \geq 1} \). Thus the game consists of a contin-
um of players, indexed by \( i \in [0, 1] \). Let \( K = \sup k_0(i) \). The set of actions
is \( \mathcal{X} \), the set of all sequences \( \{ k_t \} \) such that \( k_{t+1} \leq f(k_t), t \geq 1 \), and \( k_1 \leq f(K) \).
This is a subset of \( \Pi_{t=1}^{\infty} [0, K_t] \), where \( K_t = f^t(K) \). Define \( \bar{u}(c) = u(1) + (c - \)
1) $u'(1)$ if $c \leq 1$ and $u(c)$ if $c > 1$. Let $d(\kappa, \kappa')$ denote the metric on $X$ given by

$$d(\kappa, \kappa') = \sum_{i \geq 0} \beta^i |\bar{u}(f(k_i) - k_{i+1}) - \bar{u}(f(k'_i) - k'_{i+1})|.$$ 

If $\{\kappa^n\}$ is a Cauchy sequence, then so is $\{k^n\}$ for all $t$ (this follows from $u'(c_t) \geq u'(f_t(K)) > 0$ since $c_t \leq f_t(K)$). Hence, the metric space $(X, d)$ is complete. Let $M = \sup_{k \geq 0} \sum \beta^i u(f(k_i) - k_{i+1})$ for $k_0 = K$. Then since $\sup_{k \geq 0} \sum \beta^i u(f(k_i) - k_{i+1}) \leq M$ for all $k_0 \leq K$, $d(\kappa, \kappa') \leq 2 \max\{M, |u(1) - u'(1)|\}$, so that $(X, d)$ is also bounded and so compact. Let $\Delta(X)$ be the space of Borel probability measures on $(X, d)$ endowed with the weak convergence topology, with typical element $v$. Note that $\Delta(X)$ is metrizable and compact. Let $\Psi$ denote the space of continuous utility functions $U : X \times \Delta(X) \to \mathbb{R}$ endowed with the sup norm.

In order to apply Mas-Colell (1984), we need payoffs to be continuous functions of $\kappa$ and $v$, that is, elements of $\Psi$. Given $v$, let $F_t$ be the unconditional distribution of $t$th-period capital choices, $F_t(k) = v((k_1, k_2, \ldots): k_t \leq k)$. The difficulty is that payoffs are not continuous in $\kappa$ when $F_t$ has an atom. The proof proceeds by approximating agents' utility functions by continuous utility functions. The game with these continuous utility functions has an equilibrium. Taking limits and extracting a convergent subsequence yield a distribution, which by the maximum theorem will be an equilibrium of the original game.

If the utility from consumption is unbounded below (and so bounded above) and $k_0 > 0$, then there exists $c'$ such that, for all $0 \leq c < c'$,

$$u(c) + \frac{v(1)}{1 - \beta} + \sup_{c''} \frac{\beta u(c'')}{1 - \beta} < \frac{u(\bar{c})}{1 - \beta},$$

where $\bar{c} = \sup\{c: f(k_0) - c > k_0\}$. Thus in this case, any $c < c'$ is strictly dominated, and replacing $u$ by $u(c') + (c - c')u'(c')$ for $c \leq c'$ does not alter this. Moreover, with this interpretation, $u$ is a continuous function defined on negative consumptions. Similarly, in general we can extend $u$ to allow for negative consumption in a way that preserves continuity but that does not alter optimal choices.

Define

$$U_m(\kappa, v) = \sum_{i \geq 0} \beta^i \left[ u(f(k_i) - k_{i+1}) + v\left(\int_{-1/m}^{1/m} F_t(k_i + x)dx\right)^{1/m}\right].$$

The second term is the matching value of the convolution of $F_t$ with the distribution function of a uniform random variable on $[-m^{-1}, m^{-1}]$ (i.e., we are effectively smoothing by adding a small random component to the agent's $k$ choice). Thus, for each $t$, the second term is a continuous function of $k$ (uniformly in $F_t$). Moreover, if $F'_t(k) \to F_t(k)$ at all continuity points of $F_t$, then (since $F_t$ is monotone) the convergence is almost everywhere, and by Lebesgue's dominated convergence theorem, the integral converges. In order to show that $U_m$ is a continuous function on $X \times \Delta(X)$, it remains to consider

24 If $u$ rather than $\bar{u}$ is used to define the metric $d$, then for the case of CRRA preferences with $\gamma > 1$, $(X, d)$ is not bounded (since utility is not bounded below).

25 This is the payoff that results when each agent's choice is adjusted by an amount that is determined as an independent draw from some interval. Specifying payoffs in this way avoids the technical issues of a continuum of independent random variables.
\( \kappa \) and \( \kappa' \) and the utility due to consumption. Now
\[
\left| \sum_{i \geq 0} \beta^i u(f(k_i) - k_{i+1}) - \sum_{i \geq 0} \beta^i u(f(k'_i) - k'_{i+1}) \right|
\leq \sum_{i \geq 0} \beta^i |u(f(k_i) - k_{i+1}) - u(f(k'_i) - k'_{i+1})|,
\]
which can be made arbitrarily small by making \( d(\kappa, \kappa') \) small.

Let
\[
\Psi_m = \left\{ U \in \Psi : U(\kappa, \nu) = \sum_{i \geq 0} \beta^i \left[ u(f(k_i) - k_{i+1}) + \nu \left( \frac{1}{2m} \int_{-1/m}^{1/m} F_i(k_i + x) \, dx \right) \right] \right\}
\]
for some \( k_0 \in [0, K] \).

Note that the only difference between utility functions in \( \Psi_m \) is the initial capital stock (and, if necessary, the modification that is a function of \( k_0 \) described above). That is, different initial capital stocks, \( k_0(i) \), are reflected in different utility functions, not different constraint sets. The single-period utility function is such that it is never optimal to choose \( k_1 > f(k_0) \) (this is why it was necessary to extend \( u \) to negative consumptions).

Any probability measure on \( \Psi_m \) induces an initial distribution over capital stocks and vice versa. Let \( \mu_m \) be the measure reflecting \( k_0(\cdot) \), the initial capital distribution. By theorem 1 of Mas-Colell (1984), this game has a Nash equilibrium, that is, a measure \( \tau_m \) on \( \Psi \times X \) such that (where \( \tau_{m, \Psi} \) and \( \tau_{m, X} \) are the marginals on \( \Psi \) and \( X \), respectively) \( \tau_{m, \Psi} = \mu_m \) and \( \tau_m(\{(U, \kappa) : U(\kappa, \tau_{m, X}) \geq U(X, \tau_{m, X})\}) = 1 \). Denote by \( F_{t, m} \) the unconditional distribution of \( t \)th-period capital choices induced by \( \tau_m \).

It is a useful property of convolutions that if \( F \) has an atom at \( k \) of size \( \delta \), then for all \( \sigma \),
\[
(2\sigma)^{-1} \int_{-\sigma}^{\sigma} F(k + x) \, dx < F(k + \sigma) - \frac{\delta}{4}.
\]
This implies that, for fixed \( t \), the sup over the size of all atoms in \( F_{t, m} \) goes to zero as \( m \to \infty \). To see this, first note that the utility due to consumption is uniformly continuous on \( X \). Second, if \( F_{t, m} \) has an atom of size \( \delta \) at \( k \), then
\[
\frac{1}{2m} \int_{-1/m}^{1/m} F_{t, m}(k + 2m^{-1} + x) \, dx - \frac{1}{2m} \int_{-1/m}^{1/m} F_{t, m}(k + x) \, dx > \frac{\delta}{4},
\]
so that, for large \( m \), \( k + 2m^{-1} \) yields almost the same utility from consumption and the value of the gain in the match is bounded away from zero.

Given the sequence \( \{\tau_m\} \), there is a subsequence such that \( \tau_{m, X} \) converges. Denote the limit \( \tau_X^* \) and the sequence of capital distributions \( \{F_t^*\} \). Suppose that \( F_t^* \) has an atom at \( k \) for some \( t \) of size \( \delta > 0 \). Then for all \( \epsilon > 0 \), there exists \( M \) such that, for all \( m > M \), \( F_{t, m}(k + \epsilon) - F_{t, m}(k - \epsilon) > \delta/2 \). Then there exist \( k', k'' \) in the support of \( F_{t, m} \) such that \( F_{t, m}(k'') - F_{t, m}(k') \geq \delta/2 \) and \( 2\epsilon > k'' - k' > 0 \). Now,
\[
\frac{1}{2m} \left[ \int_{-1/m}^{1/m} F_{t, m}(k'' + 2m^{-1} + x) \, dx - \int_{-1/m}^{1/m} F_{t, m}(k' + x) \, dx \right]
\geq F_{t, m}(k'' + m^{-1}) - \frac{1}{2} [F_{t, m}(k') + F_{t, m}(k' + m^{-1})] \geq \frac{\delta}{4}.
\]
By choosing \( \epsilon \) sufficiently small and \( m > \epsilon^{-1} \), the consumption utility loss from choosing \( k'' + 2m^{-1} \) rather than \( k' \) can be made arbitrarily small. Since the matching gain is bounded away from zero, \( k' \) cannot be an optimal choice for any agent, a contradiction. Thus \( F_t \) has no atoms.

It remains to argue that \( \{F_t^*\} \) describes an equilibrium of the original game. We claim that, since the \( F_t \) are continuous for all \( t \), the convergence of \( U_m(\cdot, \tau^x) \) to \( U(\cdot, \tau^x) \) is uniform. In order to show this, fix \( \epsilon > 0 \) and \( T \) such that \( \sum_{t \leq T} \beta^t u(1) < \epsilon / 4 \). We demonstrate the uniformity of convergence for any finite number of periods, which is sufficient since the total discounted value of matching for the periods after \( T \) is less than \( \epsilon / 4 \). So suppose \( t < T \). Since \( F_t^* \) is continuous on \([0, K_t] \), there exists \( \delta > 0 \) such that \( |F_t^*(k') - F_t^*(k'')| < \epsilon / 4 \) whenever \( |k'' - k'| \leq \delta \). Now, there exists \( M_n \) such that if \( m > M_n \), then \( |F_t^*(n\delta) - F_{t,m}(n\delta)| < \epsilon / 4 \), for \( n = 0, \ldots, \delta^{-1}K_t \). Let \( M = \max\{M_n; n \leq \delta^{-1}K_t\} \). Now, suppose \( m > M \) and \( k \in [n\delta^{-1}, (n + 1)\delta^{-1}] \). Note that \( F_t^*(n\delta) + (\epsilon / 4) > F_t^*((n + 1)\delta) \geq F_t^*(n\delta) > F_t^*((n + 1)\delta) - (\epsilon / 4) \) and \( F_{t,m}((n + 1)\delta) + (\epsilon / 4) > F_{t,m}((n + 1)\delta) \geq F_{t,m}(n\delta) \geq F_{t,m}((n + 1)\delta) - (\epsilon / 4) \). Then \( |F_t^*(k) - F_{t,m}(k)| < \epsilon / 2 \) for all \( k \). Since payoff functions are converging uniformly, the limit of best-reply capital choices to \( \{F_t^*\} \) is a best reply in the limit, completing the argument. Q.E.D.

B. Proofs of Properties of Wealth-Is-Status Equilibria

Property 1

Suppose not. Let \( i \) and \( i' \) be two family lines in which a switch occurs and let \( t \) be the first period in which it occurs. The optimality of the agents' choices implies that (i)

\[
\begin{align*}
u(Ak_t(i) - k_{t+1}(i)) + m_t(k_t(i)) + \beta V(k_{t+1}(i)) & \\
\geq & \nu(Ak_t(i) - k_{t+1}(i')) + m_t(k_t(i')) + \beta V(k_{t+1}(i'))
\end{align*}
\]

and (ii)

\[
\begin{align*}
u(Ak_t(i') - k_{t+1}(i')) + m_t(k_t(i')) + \beta V(k_{t+1}(i')) & \\
\geq & \nu(Ak_t(i') - k_{t+1}(i)) + m_t(k_t(i')) + \beta V(k_{t+1}(i)).
\end{align*}
\]

Note that \( k_{t+1}(i') \) is feasible for \( i \) since it was feasible for \( i' \) and that \( k_{t+1}(i) \) is feasible for \( i' \) since it is less than the actual choice.

Conditions i and ii imply that

\[
\begin{align*}
u(Ak_t(i') - k_{t+1}(i')) + u(Ak_t(i) - k_{t+1}(i)) & \\
\geq & \nu(Ak_t(i') - k_{t+1}(i)) + u(Ak_t(i) - k_{t+1}(i')).
\end{align*}
\]

This contradicts our assumption that \( u(\cdot) \) is a monotonically increasing, strictly concave function, and hence switching is inconsistent with optimizing behavior. Q.E.D.

Property 2

First observe that for any family line, it is not optimal for there to exist a \( T \) such that, for all \( t \geq T, \lambda_t \geq \lambda^* \) with \( \lambda_t > \lambda^* \) for at least one period. Suppose otherwise. Then a unilateral deviation by a family line lowering the consumption ratio from \( \lambda_t \) to \( \lambda^* \) for all \( t \geq T \) increases the family line's utility from
consumption; their utility from matching cannot be lower since the subsequent capital sequence is at least as large as it was originally.

Now suppose that property 2 did not hold. From the previous paragraph, if $\lambda_i > \lambda^*$, then there exists a period $s$ such that $\lambda_i > \lambda^*$ and $\lambda_{i+1} \leq \lambda^*$. But this is inconsistent with equation (11) since

$$u'(\lambda_iAk_i) < u'(\lambda^*Ak_i) = \beta u'((1 - \lambda^*)\lambda^*Ak_i) < \beta u'((1 - \lambda_i)\lambda_{i+1}Ak_i)$$

and $m_{i+1}(\cdot) \geq 0$. Q.E.D.

Property 3

Suppose not. Assume that at some time $t \geq 1$ there is a set $A$ of positive measure of men who inherit the same level of capital. Let $B$ denote the set of women who match in equilibrium with a man from set $A$. Note that $\sup(B) > \int_B j d\mu$, so that the woman in $B$ with the highest endowment has a strictly greater endowment than the average woman in $B$. Then the family line in $A$ matched with a $B$ of lower endowment than the maximum could have achieved a discretely better match at time $t$ with an arbitrarily small increase in its bequest. Q.E.D.

Property 4

Let $t$ be the first period for which the equilibrium capital distribution is not strictly increasing. If $F_i$ is defined as in the proof of proposition 1, then there is an interval $(k', k'')$ in which $F_i$ is a constant, $F_i(k) > F_i(k'')$ for all $k > k''$, and $F_i(k) < F_i(k')$ for all $k < k'$. From property 1, there is no switching, so that the capital stock in period $t - 1$ of the family line that has stock just below $k'$ is just below the capital stock in period $t - 1$ of the family line with capital stock above $k''$. A small change in capital choice by either family line that does not violate the ordering cannot have a large impact on their matches. Note that if capital stocks are close, then the discounted value of consumption must be close. Then since $k''$ is not close to $k'$, one of the two family lines has a profitable deviation. Q.E.D.

Property 5

In equilibrium the quality of family line $i$'s match is unchanging and is equal to $i$. The cost to a family line in choosing $\lambda^*$ rather than $\lambda_i$ is the reduction in quality of future matches. This cost is less than $i/(1 - \beta)$; thus the gain in utility in any period that results from choosing $\lambda^*$ rather than $\lambda_i$ must also be less than $i/(1 - \beta)$. The consumption gain in period $t$ from setting $\lambda_i = \lambda^*$ is at least

$$k_{i}^{1 - \gamma} [u(\lambda^*A) + V^*((1 - \lambda^*)A) - [u(\lambda_iA) + V^*((1 - \lambda_i)A)]]$$

where $V^*$ is defined in equation (8). The term in braces is strictly positive. (Recall that the stationary consumption ratio $\lambda^*$ is the unique solution to the exogenous matching—equivalently pure consumption-investment—problem.) Since $k_i$ is growing at least at the rate $(1 - \lambda^*)A$, the only way the consumption gain can always be less than $i/(1 - \beta)$ is if the second term converges to zero. This implies that $\lambda_i \to \lambda^*$. Q.E.D.
Property 6

Note that the consumption cost to a family line $i$ of choosing an arbitrarily low consumption fraction, $\epsilon > 0$, at time $t$ is

$$(1 - \gamma)^{-1}[Ak_i(i)]^{1-\gamma}[\lambda_i(i)^{1-\gamma} - \epsilon^{1-\gamma}].$$

Fix $i$ and $i'$ with $i < i'$. Since $k_i \to \infty$ as $t \to \infty$, for the case of $\gamma > 1$, for any $\epsilon > 0$ there exists a time $T_\epsilon$ such that, for all $t > T_\epsilon$, the loss in utility from lowering consumption to $\epsilon$ is less than $i' - i$. Since there can be no switching in equilibrium, we must have $A(1 - \epsilon)k_i(i) < A[1 - \lambda_i(i')]k_i(i')$ for all $t > T_\epsilon$. This implies

$$\lim_{t \to \infty} \frac{[1 - \lambda_i(i')]k_i(i')}{k_i(i)} \geq 1.$$

That is, for any two family lines $i$ and $i'$ with $i' > i$, it must be the case that it eventually becomes infeasible for the family line $i$ to achieve the bequest level of the family line $i'$. Since the capital distribution is continuous, if $\lim_{t \to \infty} [k_i(i')/k_i(i)]$ is finite, then there exists a $\delta > 0$ such that $\lim_{t \to \infty} [k_i(i')/k_i(i' - \delta)] < 1 + \mu$ for any $\mu > 0$. If $\lambda_i(i')$ does not converge to zero, then the condition above will be violated and it will be optimal for family line $i' - \delta$ to acquire more capital than family line $i$ at some point in time. Q.E.D.

References