Bank Runs as an Equilibrium Phenomenon

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A standard demand deposit contract in which individuals are entitled to their full deposit at any time provided the bank is solvent is analyzed in a context in which there are no exogenous events on which agents condition their behavior and a unique equilibrium involving a bank run with positive probability is shown to exist.

I. Introduction

Diamond and Dybvig (1983) have argued that uninsured demand deposit contracts provide liquidity but leave banks vulnerable to runs.[1] They show that the demand deposit contract can improve on a competitive market, but it also has a "bad" equilibrium, a bank run, in which depositors panic and withdraw their deposits immediately. This causes an interruption of the productive process, resulting in a worse outcome than the competitive equilibrium. Banks are thus vulnerable to runs because there are multiple equilibria, in one of which the public thinks the bank is going to fail, and indeed it fails. A potential problem with this approach is that bank runs should not be observed in equilibrium since no one would deposit anticipating a run. One way around this, as suggested by Diamond and Dybvig, is to suppose that the equilibrium is selected depending on a publicly observable random variable. Then agents may still deposit in the bank provided the probability of a run is small enough. The purpose of this

We are grateful to anonymous referees for helpful comments. Support from the National Science Foundation is gratefully acknowledged.

[1] Recent analyses of the topic include Jacklin (1983) and Chari and Jagannathan (1984).

[*Journal of Political Economy*, 1987, vol. 95, no. 3]

paper is to provide a somewhat different example in which there is a *unique* equilibrium that involves a positive probability of a bank run. In our example the bank run is associated with a Prisoner's Dilemma–type situation in which the agents withdraw their money from the bank not for consumption purposes but for reasons of self-interest. Our example has what we consider to be several attractive features. First, there are no exogenous events on which agents are conditioning their behavior, and, second, there is a unique equilibrium. The second feature implies that, unlike the Diamond-Dybvig model, there are *no* equilibria without the possibility of bank runs.

II. The Example

There are four periods, 0, 1, 2, and 3. The production process exhibits constant returns to scale. For each unit planted at period 0, there will be α units available after one period, β units available after two periods, and γ units available after three periods. For each unit left in period 1 of some amount planted in period 0 there will be β/α units in period 2 and γ/α units in period 3. For each unit left in period 2 of some amount planted in period 0 there will be γ/β units one period later. We assume that $\alpha < \beta < \gamma$, $\frac{1}{2} < \alpha < 1$, and $\gamma > 1$. If the production process is interrupted in period 1, one gets α, which is less than the initial investment 1. If the production process is not interrupted, one gets $\gamma > 1$. If it is interrupted in period 2, one gets β, which is larger than α but less than γ.

There are two agents, each of whom has one unit of endowment and will live for one, two, or three periods. At the beginning of period 1 agent i receives a signal $s^i \in S^i$, $S^i = \{s_1^i, s_2^i, s_3^i\}$, which indicates his or her life span (type). Suppose that there is a joint distribution P on $S^1 \times S^2$. Each agent has a Bernoulli utility for aggregate consumption $U: R_+ \to R$, which is strictly increasing. That is, if $x = \sum_{i=1}^{\tau} x_i$, where τ is the number of periods the agent will live and x_i the consumption in period i, the utility derived by the agent is $U(x)$. Thus each agent can get expected utility $U(1)$ by consuming his endowment without utilizing the production technology at all. If an agent plants all of the endowment, he then faces a random consumption depending on how long he will live. Using P, one can compute the expected utility that the agent gets, say $\bar{u}$ under this plan ($\bar{u}$ is the expected utility of autarchy). Suppose for the moment that $\bar{u} > U(1)$.

Now consider a "banking contract" between the two agents as described below. Each agent deposits an amount in a "bank," which plants the seeds. During periods 1 and 2, an agent can withdraw his deposit with no interest (penalty for early withdrawal). If the agent withdraws his deposit in period 3, he receives a share of the "profits" proportionate to his deposit. If, during any period, demands for withdrawal exceed assets, all assets will be distributed proportionate to withdrawal demands.

A strategy for an agent is a function σ^i, which indicates when agent i will withdraw his money from the bank for each possible signal, $\sigma^i: S^i \rightarrow A$, where $A = \{a_1, a_2, a_3\}$ and a_i indicates withdrawal in period i. Table 1 shows the consumption agents receive depending on their withdrawal times.

A Bayesian Nash equilibrium is a pair of strategies, one for each agent, and a pair of conjectures such that each agent's strategy is a best response to his conjecture about the behavior of the other agent and, furthermore, the conjectures are correct.

Suppose now that $\beta < 1$, $[\beta(2\alpha - 1)/\alpha] < \alpha < [\gamma(2\alpha - 1)/\alpha]$, and $1 < [\gamma(2\beta - 1)/\beta]$. In that case, it follows from table 1 and the fact that $U(\cdot)$ is strictly increasing that it is a dominant strategy for either agent to withdraw the money in period 1 if she finds out that she has a lifetime of one or two periods and to leave it until period 3 if she finds that she will live that long, provided that the expected utility of such plans is larger than the expected utility from autarchy. Suppose now that the signals the agents receive are perfectly correlated. If agent 1 learns that she is going to live for two periods, she knows that agent 2 will live for two periods too, but she will choose to withdraw in the first period since $\alpha < [\beta(2\alpha - 1)/\alpha]$ and $1 < \beta$. It is a Prisoner's Dilemma situation. If both were to withdraw in the second period, each would get β, which is larger than α. Agent 1 withdraws the money in period 1 not for consumption, since she is going to live for another period, but because it is the best she can do. It is thus a bank run. We have thus shown the following proposition.

PROPOSITION. Assume that $\beta < 1$, $[\beta(2\alpha - 1)/\alpha] < \alpha < [\gamma(2\alpha - 1)/\alpha]$, and $1 < [\gamma(2\beta - 1)/\beta]$. Then there is a unique Bayesian equilibrium (in dominant strategies) in which each agent plans to withdraw

TABLE 1

	Agent 2		
Agent 1	a_1	a_2	a_3
a_1	α, α	$1, \frac{2\alpha - 1}{\alpha}\beta$	$1, \frac{2\alpha - 1}{\alpha}\gamma$
a_2	$\frac{2\alpha - 1}{\alpha}\beta, 1$	β, β	$1, \frac{2\beta - 1}{\beta}\gamma$
a_3	$\frac{2\alpha - 1}{\alpha}\gamma, 1$	$\frac{2\beta - 1}{\beta}\gamma, 1$	γ, γ

the money in period 1 if he is of type 1 or 2 and to leave it until period 3 if he is of type 3.

Note that the result is independent of the correlation of the signals the agents receive since it is a dominant strategy result. Bank runs occur with positive probability. If p_{ij} is the ijth entry of the matrix $\mathbf{P}$ with probability $p_{22} + p_{12} + p_{21}$, at least one agent will withdraw his deposit for other than consumption purposes. With probability p_{22} both agents engage in this behavior.

Other possibilities arise for different values of the parameters. If $\beta > 1$ and $[\beta(2\alpha - 1)/\alpha] > \alpha$, then there is a unique equilibrium in which an agent of type i withdraws his deposit at time i, and there are no bank runs. If $\beta > 1$ but $[\beta(2\alpha - 1)/\alpha] < \alpha$, then there is always an equilibrium in which agents of type 1 or 2 withdraw at time 1; there may be other equilibria as well in this case. For example, if the signals are perfectly correlated, the strategies in which an agent of type i withdraws his deposit at time i form an equilibrium.

As an example of the proposition above, suppose that the agents have a utility function for aggregate consumption (sum of the consumption in each period)

$$U(x) = \begin{cases} 10x - 9 & \text{if } x \leq 1 \\ x & \text{otherwise.} \end{cases}$$

Let $\alpha = .9$, $\beta = .9 + \epsilon$, and $\gamma = 2.5$, where $.1 > \epsilon > 0$. Suppose that the signals are independently distributed. The probabilities that an agent will live one, two, or three periods are, respectively, $\frac{1}{2} - \delta$, δ, and $\frac{1}{2}$, where $\delta > 0$. Consumptions are given in table 2.

The expected utility of the unique equilibrium is $[0 + 1 + (20/9) + 2.5]/4 = 1.4306$. If an agent keeps her endowment and does not plant it, she gets $U(1) = 1$. If she plants the endowment, she faces a random consumption yielding approximately $(0 + 2.5)/2 = 1.25$ if ϵ and δ are small: $U(.9) = 0$, $U(2.5) = 2.5$. We see thus that the expected utility of the banking contract is larger than the expected utility of autarchy.

TABLE 2

	Agent 2		
Agent 1	a_1	a_2	a_3
a_1	.9, .9	1, $.8^+$	1, $\frac{20}{9}$
a_2	$.8^+$, 1	$.9^+$, $.9^+$	1, $\left(\frac{20}{9}\right)^+$
a_3	$\frac{20}{9}$, 1	$\left(\frac{20}{9}\right)^+$, 1	2.5, 2.5

III. Are Demand Deposit Contracts Optimal?

Optimality in the Diamond-Dybvig Model

Diamond and Dybvig (1983) define a bank deposit contract and a set of rules as to how much money people will receive if more people want to withdraw money from the bank than the bank has at a given time. These rules give rise to a game with incomplete information. The authors investigate the Bayes equilibria of this game and are interested in their optimality. They compare the equilibria of the game to the allocation that would arise if there were no asymmetric information, that is, if the types of the individuals were publicly observed. The authors state that their bank deposit contract can achieve the first-best allocation that would arise with no asymmetric information as an equilibrium of the resulting game. The notion of a bank run in the Diamond-Dybvig paper stems from the fact that there is a second equilibrium in this game, namely, one in which people withdraw from the bank despite the fact that they have no immediate need for the money. Diamond and Dybvig consider a run to be a change from the good equilibrium to the bank run equilibrium. They suggest that this could happen if the selection between the bank run equilibrium and the good equilibrium depended on some commonly observed random variable in the economy, for example, a sunspot that may occur with positive probability less than one. If the sunspot occurs, the agents select the bank run equilibrium, and if the sunspot does not occur, they select the good equilibrium. This suggests that the agents are taking actions based on two different random variables: the first is their type, and the second is the sunspot random variable. A simple way to model this would have been to let the space that agents use to condition their actions be the union of the two sets of states mentioned above. In this case the two separate equilibria in the Diamond-Dybvig model would become a single equilibrium. It would have two parts. The first part would be the equilibrium conditioned on not having seen a sunspot; this would be essentially the good equilibrium. The second part would be the equilibrium conditioned on there being a sunspot, which would correspond to the bank run equilibrium. The notion that the equilibrium in the Diamond-Dybvig paper is optimal clearly does not carry over to this reinterpretation. It is no longer the case that the first-best allocation arises for sure. It now depends on whether the sunspot occurs or does not. Diamond and Dybvig provide a modification of the demand deposit contract (suspension of convertibility), assuming a sequential service constraint, which allows withdrawals to be conditioned at most on the total amount of past withdrawals and which prevents runs when the proportion t of agents who want to withdraw early is known with certainty, that is, when there is no "social uncertainty" (if t is random and unobservable, then bank contracts cannot achieve optimal risk sharing—proposition 1 in Diamond and Dybvig).

Optimality in Our Example

The demand deposit contract we consider will not be optimal in general since it induces a Prisoner's Dilemma–type situation for agents of types 1 and 2. For example, if the signals are perfectly correlated, the optimal incentive-compatible contract will give α to type 1 in period 1, β to type 2 in period 2, and γ to type 3 in period 3, and nothing otherwise. It is incentive compatible; no type wants to pretend it is another type since $\alpha < \beta < \gamma$. The general case is similar.

One observes demand deposit contracts in the real world. The type of contract we are considering is a close approximation to what exists in common practice. We might then take the point of view that, since this contract exists and has existed for many years, it must be optimal among the contracts that people have considered. Thus there must be some aspects of reality that are missing from our model. It is our hypothesis that transactions costs and moral hazard considerations, compounded by heterogeneity in the population, may explain the existence and perdurability of standard demand deposit contracts.

One reason why standard demand deposit contracts are not optimal in general is that superior contracts can be found that take the form of a contingent demand contract. A contingent demand contract would be a contract in which, when an individual went to the bank to withdraw money, the amount of money she would receive would depend on how many other agents were attempting to withdraw money at the same time and how many were likely in the near future to demand money back as well. Such contracts are obviously much more costly to monitor and carry out than the simpler contracts we consider since they should include many contingencies and should treat different individuals differently in a world with asymmetric information.

A second reason for which demand deposit contracts will not be generally optimal, except in the case in which there is a risk-neutral agent, is that optimal contracts will typically have all agents bearing some of the risk involved in the underlying investment decisions. If we think of an investment yielding a return because of both the time value of money and the riskiness of the underlying venture, it seems plausible that one function of a bank is to separate the returns for these two reasons. There may be significant moral hazard problems in monitoring the underlying investment that make bank depositors unwilling to accept anything other than the least risky demand deposit contract. The contract that we consider is such a contract: the only risk that the depositors bear is that they will not be repaid their money in the situation in which it is physically impossible to repay them.

Finally, the presence of a heterogeneous population in terms of consumption and saving patterns with different basic preferences and attitudes toward risk, which are private information, may compound the problems of the optimal design of deposit contracts and may call for simple solutions as the standard deposit contract.

An attempt to build a more complicated model involving these or other aspects, so that we could endogenously derive an optimal contract with the characteristics of our demand deposit contract, is beyond the scope of this paper. Much work remains to be done in the explanation of standard demand deposit contracts and their role in bank runs. The implications for macroeconomic analysis are clearly important, as Diamond (1985) has remarked recently.

References

Chari, V. V., and Jagannathan, J. "Banking Panics, Information and Rational Expectations Equilibrium." Working paper. Evanston, Ill.: Northwestern Univ., 1984.

Diamond, Douglas W., and Dybvig, Philip H. "Bank Runs, Deposit Insurance, and Liquidity." *J.P.E.* 91 (June 1983): 401–19.

Diamond, Peter A. "Equilibrium without an Auctioneer." Paper presented at the Fifth World Congress of the Econometric Society. Boston, 1985.

Jacklin, Charles J. "Information and the Choice between Deposit and Equity Contracts." Working paper. Stanford, Calif.: Stanford Univ., Grad. School Bus., 1983.