Manipulation via Endowments

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1. INTRODUCTION

A competitive allocation in an exchange economy has several desirable properties; it is (with minimal assumptions) Pareto optimal and it guarantees that each trader doesn't lose utility in the trade. Besides these properties of efficiency and individual rationality, however, there are undesirable properties such as its manipulability. There is the possibility that an agent can behave as though his utility function is other than the true utility function and achieve a better outcome (higher utility for him) than at a true competitive outcome. Hurwicz (1972) has shown that this manipulability isn't unique to the competitive process, but in fact is common to any reallocation scheme which achieves Pareto optimal, individually rational outcomes.

Besides manipulation via the agent's utility function, there is another type of manipulation, namely of the agent's endowment. In theory, an agent's endowment can be known to all people through inspection and therefore undetectable manipulation is impossible. In practice, however, it is often impossible to determine an agent's true endowment. Even if it were possible to determine true endowments, given a private ownership structure, there is no way of preventing an agent from destroying all or some portion of his endowment. In economies in which there are many agents, each of whom initially owns (at most) a small share of the total amount of a commodity the advantages of these types of manipulation would be small. It is straightforward to apply the techniques used by Roberts and Postlewaite (1976) to show (with similar assumptions as used there) that as an economy gets large agents have diminishing incentive to manipulate endowments.

However, in economies in which there are small numbers of agents, large agents, or coalitions of agents the manipulation of the competitive mechanism is theoretically possible. Situations in which farmers withhold from market or destroy portions of their crops are possible instances of coalitional attempts to manipulate via endowments. In international trade we normally must consider agents who are not insignificant and potentially can upset " competitive " allocations through manipulation. Because of this possible manipulation of the competitive mechanism, it becomes interesting to investigate the possibility of designing alternative mechanisms which are not manipulable.

In this paper we will show that the problem of manipulation by withholding endowments is not unique to the competitive mechanism, but rather is shared by any Pareto optimal, individually rational scheme of reallocating resources. We show that manipulation via destruction of endowments, however, can be avoided and give an example of a resource reallocation scheme which cannot be manipulated in this way. Finally, we consider manipulation by coalitions and show that if coalitions can " pre-trade ", that is, form a submarket among themselves, they may be able to enter the larger market in an improved position. We show that all Pareto optimal individually rational reallocation schemes are susceptible to this form of manipulation and discuss stability problems which therefore arise.

2. MODEL

We will consider only pure exchange economics with l commodities. A *trader* is characterized by

(i) a utility function $U: R^l_+ \rightarrow R_+$

(ii) an initial endowment $w \in R^l_+$

For the purposes of this paper we will assume that the utility function is strictly concave, continuous, and strictly increasing. An *economy* e is a collection of traders, i.e. $e = \{(U_1, w_1), ..., (U_n, w_n)\}$ where n is the number of traders in e. We denote by E the set of economies. A *resource allocation mechanism* (or mechanism for short) is a function Q on E which reallocates the resources in the economy, i.e.

$$Q(e) = (x_1, ..., x_n), x_i \in R^l_+, \quad i = 1, ..., n, \quad \text{and } \textstyle\sum_{i=1}^n x_i = \sum_{i=1}^n w_i.$$

A mechanism is *Pareto efficient* if $Q(e)$ is Pareto efficient (Pareto optimal) for all $e \in E$. A mechanism is *individually* rational if for $\forall e \in E$, $Q(e) = (x_1, ..., x_n)$, $U_i(x_i) \geqq U_i(w_i)$ for each $i = 1, ..., n$. For our purposes we consider only mechanisms which are Pareto efficient and individually rational, hence whenever we speak of a mechanism, we mean an individually rational and Pareto efficient resource allocation mechanism.

We will now ask how a trader might manipulate the outcome of a mechanism via manipulating his endowment in some way. One option a trader might have is to understate his endowment, i.e. withhold some of his endowment from the "market" and consume it himself. Formally we say that a mechanism d is *W-manipulable* (for withholding) if there exists $e, e' \in E$,

$$e = \{(U_1, w_1), ..., (U_n, w_n)\}$$

and

$$e' = \{(U_1, w_1'), ..., (U_n, w_n')\}$$

with

$$w_i = w_i', \text{ for } i \neq j, \; w_j' \leqq w_j, \; Q(e) = (x_1, ..., x_n), \; Q(e') = (x_1', ..., x_n')$$

$$\text{and } U_j(x_j' + (w_j - w_j')) > U_j(x_j).$$

In other words, if a mechanism is W-manipulable then in some economy e, at least one trader would have an incentive to withhold some of his endowment, make the economy appear to be e', and enjoy a higher utility (when taking into account the withheld commodities) than if he had not withheld any of his endowment. The higher utility is of course predicated upon no other agents withholding their endowments. All that is said is that true revelation of endowments is not a Nash equilibrium.

If we consider only those economies in which there is a unique competitive allocation, then the "competitive mechanism" which reallocates to that competitive allocation is indeed a mechanism (i.e. it is individually rational and Pareto efficient). The competitive mechanism can easily be shown to be W-manipulable through simple examples. But this flaw cannot be avoided by designing alternate mechanisms as is shown in the following theorem.

Theorem 1. *Any mechanism is W-manipulable.*

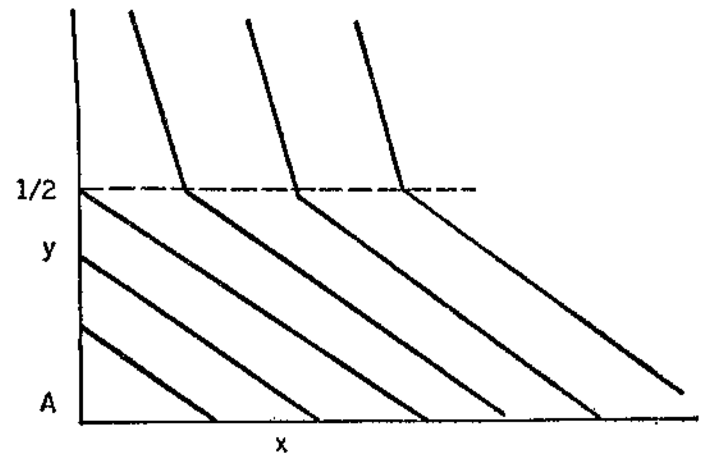
Proof. The theorem is proved by providing an economy with two traders and two commodities in which for any mechanism at least one of the two traders can withhold some of his endowment and achieve a higher utility than the mechanism would yield if he did not withhold.

Trader A has an initial endowment $w_A = (0, 1)$ and utility function

$$U_A(x, y) = \begin{cases} 3x + y & \text{if } y \geqq \frac{1}{2} \\ 3x + 6y - \frac{5}{2} & \text{if } y < \frac{1}{2}. \end{cases}$$

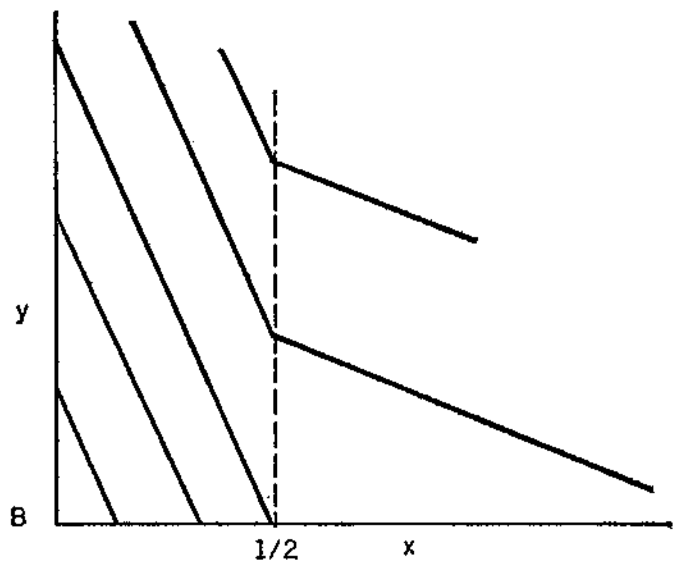
Trader B's initial endowment is $w_B = (1, 0)$ and his utility function is

$$U_B(x, y) = \begin{cases} 6x + 3y - \frac{5}{2} & \text{if } x \leqq \frac{1}{2} \\ x + 3y & \text{if } x > \frac{1}{2}. \end{cases}$$



$$U_A(x,y)=\begin{cases}3x+y & \text{if } y\geq 1/2\\ 3x+6y-5/2 & \text{if } y\leq 1/2\end{cases}$$

FIGURE 1A



$$U_B(x,y)=\begin{cases}6x+3y-5/2 & \text{if } x\leq 1/2\\ x+3y & \text{if } x\geq 1/2\end{cases}$$

FIGURE 1B

Sample indifference curves for the two traders are shown in Figure 1. Figure 2 shows an Edgeworth box with the endowment in the upper left-hand corner and the traders' indifference curves through the endowment. The utility functions are such that the Pareto optimal points are given by the line segments MO and ON. Hence individual rationality and Pareto efficiency require that whatever the mechanism is, it must choose a point on

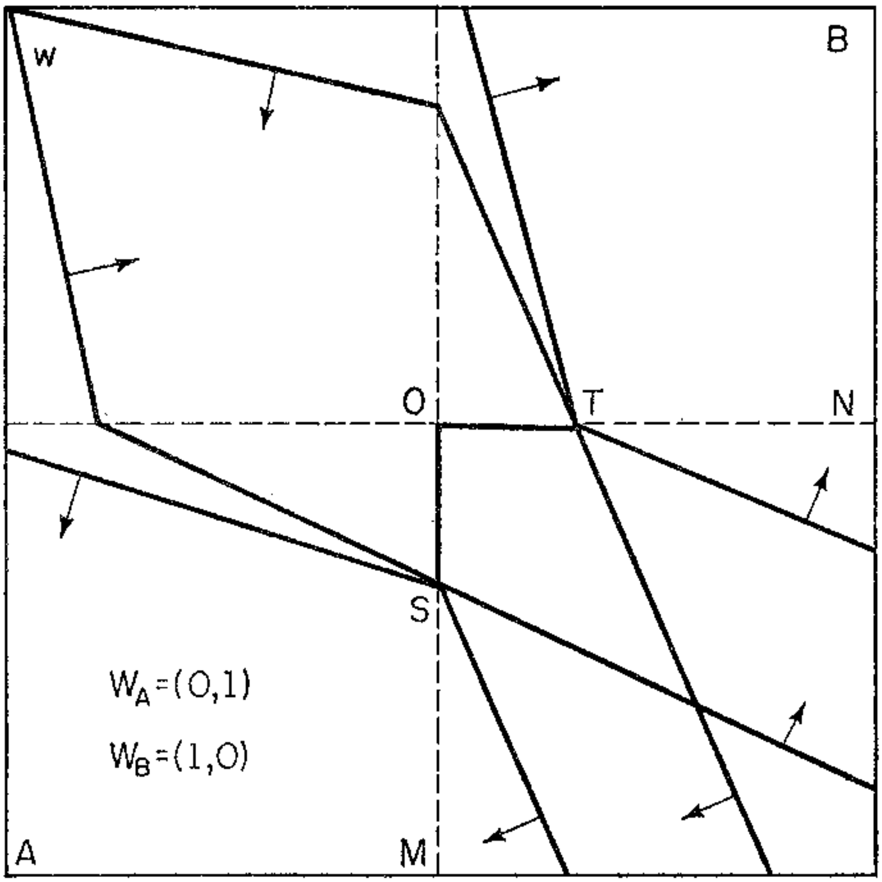


FIGURE 2

darkened line SOT. Suppose that the outcome of the mechanism is in SO, in which trader A gets $\frac{1}{2}$ unit of x and *at most* $\frac{1}{2}$ unit of y. If trader A witholds $\frac{1}{2}$ unit of y, Figure 3 shows the Edgeworth box for the economy as it appears to be. The individually rational Pareto optima are now MN in which trader A gets $\frac{1}{2}$ unit of x (as before) and at least $\frac{1}{4}$ unit of y. But when the $\frac{1}{2}$ unit of y he withheld is added in he ends up with $\frac{1}{2}$ unit of x and at least $\frac{3}{4}$ unit of y, which is strictly preferred by him to any point in SO. Similarly, if the outcome of the mechanism had been in OT, trader B could have withheld $\frac{1}{2}$ unit of x and been assured of a final outcome strictly preferred to any point in OT. Hence regardless of which point a mechanism picks at least one (and possibly both) traders will have an incentive to withhold. Thus any mechanism is W-manipulable. ||

The kinked, piecewise linear indifference curves are for ease of exposition. A " smoothed ", strictly convex version of the example would give rise to the same phenomenon.

It should be noted that withholding a portion of one's endowment is equivalent to a shift in one's preferences. Thus this result is quite similar to that of Hurwicz (1972) except that the type of preference manipulation is severely restricted.

The W-manipulation above, however may not be too serious a problem in some cases. If we think of a farmer who may try to manipulate the price of cotton by withholding 100 tons for his own consumption, he may find that the added utility of the 100 tons is quite

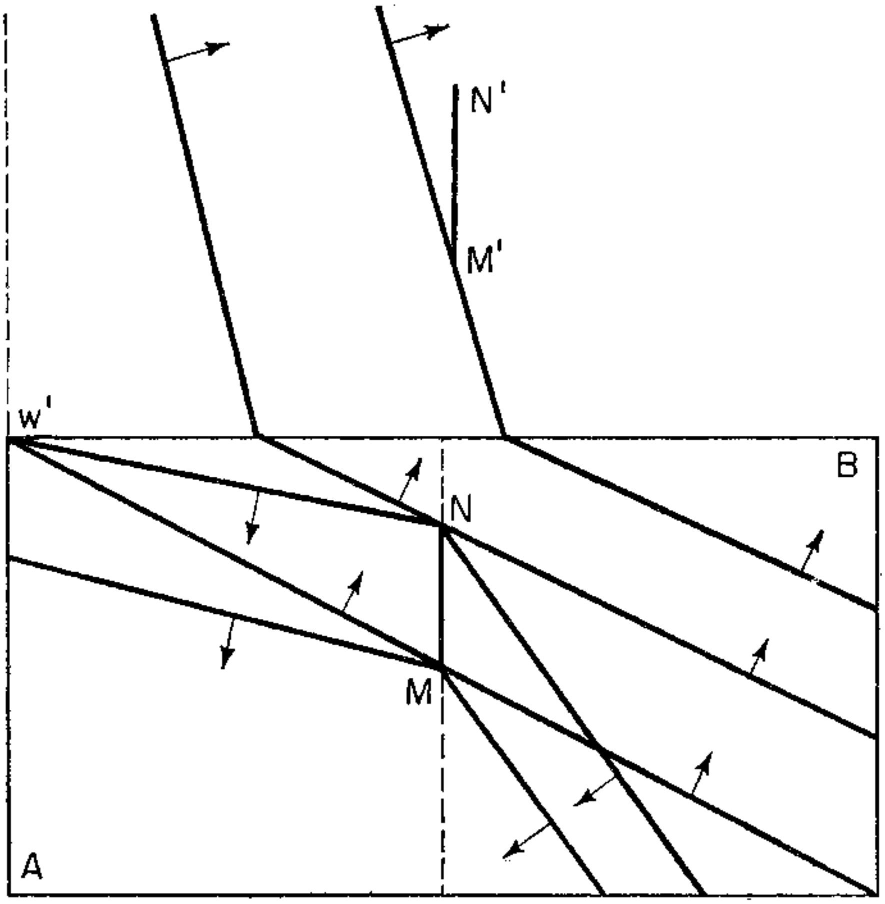


FIGURE 3

small. A more serious problem would arise if he could manipulate a mechanism by *destroying* some of his endowment, i.e. if a trader can destroy a portion of his endowment and achieve a higher utility. Formally we say that a mechanism Q is *D-manipulable* if there are

$$e,\, e' \in E,\; e = \{(U_1, w_1), \ldots, (U_n, w_n)\},\; e' = \{(U_1, w_1'), \ldots, (U_n, w_n')\},\; w_i = w_i',\; i \neq j,$$
$$w_j' \leqq w_j,\; Q(e) = (x_1, \ldots, x_n),\; Q(e') = (x_1', \ldots, x_n') \text{ and } U_j(x_j') > U_j(x_j).$$

This says that some trader can make an economy e appear to be e' (this time by destroying endowment, not withholding) and be better off than by not destroying endowment. If we look at the competitive mechanism (again restricted to those economies in which it is well-defined) it can be shown via examples that it is D-manipulable. However there exist some mechanisms which are *not* D-manipulable, as we will show. To show this we consider the set of feasible allocations for an economy $e = \{(U_1, w_1), \ldots, (U_n, w_n)\}$,

$$F(e) = \{x_1, \ldots, x_n \mid x_i \geqq 0 \;\; \textstyle\sum_{i=1}^n x_i = \sum_{i=1}^n w_i\}.$$

Since $F(e)$ is compact and the utility functions are continuous, there exists $\bar{x} \in F(e)$ such that $V(x)$: $\min_i [U_i(x_i) - U_i(w_i)]$ achieves a maximum at $\bar{x}$. For this $\bar{x}$, it must be that $U_i(\bar{x}_i) - U_i(w_i)$ are equal for each i, since by monotonicity and continuity we could take ε of some commodity from the trader with the largest " utility gain " and give it to the trader with the smallest and achieve a higher minimum gain. Similarly it is clear that $\bar{x}$ must be Pareto optimal, since if one trader's utility can be increased without decreasing any trader's utility, all can be increased. Lastly, the strict concavity of the utility functions yields uniqueness of such an $\bar{x}$. Thus we have shown the following lemma:

Lemma. *For any economy e, there exists a unique Pareto optimal, individually rational allocation $\bar{x}$ which gives maximal equal utility gains to each trader.*

If we define a mechanism $\gamma(e) = \bar{x}$ as in the lemma we have the following theorem:

Theorem 2. *γ is not D-manipulable.*

Proof. Let $e = \{(U_1, w_1), \ldots, (U_n, w_n)\}$
and

$$e' = \{(U_1, w_1'), \ldots, (U_n, w_n')\}$$

with $w_i = w_i'$, $i \neq j$, and $w_j' \leqq w_j$, $\gamma(e) = (x_1, \ldots, x_n)$, $\gamma(e') = (x_1', \ldots, x_n')$. By monotonicity $U_j(w_j') < U_j(w_j)$; hence if $U_j(x_j') > U_j(x_j)$,

$$U_j(x_j') - U_j(w_j') > U_j(x_j) - U_j(w_j).$$

Then by definition of γ,

$$U_i(x_i') - U_i(w_i') = U_i(x_i') - U_i(w_i) > U_i(x_i) - U_i(w_i),$$

and $U_i(x_i') > U_i(x_i)$ for all i. But this is impossible since γ is Pareto efficient and the utility functions are strictly increasing.[1]

It is clear that the mechanism γ depends not only on preferences, but on the utility representation of the preferences. We could however make it independent of the utility representation in the following manner. Whatever utility function a person has in the class of utility functions with the same underlying preferences, the mechanism " renormalizes " by substituting a particular utility representation of his preferences, for example, that utility function which gives $U(x) = |x|$ (euclidean norm of x) on the diagonal. We see then that for each such renormalization we get a different mechanism since the outcomes may change as the choice of utility functions changes. In this way we get a whole class of mechanisms which are not D-manipulable.

Having seen that W-manipulation cannot be prevented but that D-manipulation can, we turn to a last type of manipulation. As we suggested above, withholding as a means of manipulation may not prove to be a problem in at least some situations in that it may not be significantly different from destruction of endowment which can be prevented. However, a third alternative may occur to the farmer with his cotton crop, namely that it may behove him to trade some of his cotton (or other endowment) outside the " market structure ". It may be that there exists a group of traders who by trading can arrive at a new endowment such that the mechanism yields each trader in the group higher utility than if they had not traded. We say that a mechanism Q is *C-manipulable* (coalitionally-manipulable) if there exist

$$e, e' \in E, e = \{(U_1, w_1), \ldots, (U_n, w_n)\}, e' = \{(U_1, w_1'), \ldots, (U_n, w_n')\}$$

and S a subset of traders with

$$\textstyle\sum_{i \in S} w_i = \sum_{i \in S} w_i', \quad w_i = w_i', \quad i \notin S$$

$$Q(e) = (x_1, \ldots, x_n), Q(e') = (x_1', \ldots, x_n') \text{ and } U_i(x_i) > U_i(x_i') \quad \text{for all } i \in S.$$

We will show that there does not exist any mechanism which is not C-manipulable.

Lemma. *If a mechanism Q does not pick core points, it is C-manipulable.*

Proof. If $Q(e) = (x_1, \ldots, x_n)$ is blocked by $(y_1, \ldots, y_n)$ via a coalition S, then S could trade to $W' = (w'_1, \ldots, w'_n)$, $w'_i = w_i$, $i \notin S$ and $w'_i = y_i$, $i \in S$. The individual rationality of Q must then leave them at least as well off as their " new " endowment.

Theorem 3. *There does not exist any Q which picks core points which is not C-manipulable.*[2]

Proof. We first note that the competitive mechanism is C-manipulable,[3] i.e. there exist economies e, e' with unique competitive allocations x and x' respectively such that

(i) $w_i = w'_i, \quad i \notin S,$

(ii) $\sum_{i \in S} w_i = \sum_{i \in S} w'_i,$

(iii) $U_i(x'_i) > U_i(x_i), \quad \forall i \in S.$

If we replicate these economies in the manner of Debreu–Scarf (1963) the same manipulation is possible in e_n, e'_n, the n-fold replicas of e, e', by the coalition S_n of all agents of the types in S. But then it is straightforward to show that for any

$$y \in \text{core}\,(e_n),\ y' \in \text{core}\,(e'_n),^4\ |\ U_i(y_i) - U_i(x_i)\ |\ \text{and}\ |\ U_i(y'_i) - U_i(x'_i)\ |$$

can jointly be made arbitrarily small by suitable replication since the core is shrinking to the set of competitive allocations in both e and e'; hence for some replication we have

$$U_i(y'_i) > U_i(y_i), \quad \forall i \in S_n$$

and any

$$y_i \in \text{core}\,(e_n), \quad y'_i \in \text{core}\,(e'_n).$$

Corollary. *Any mechanism Q is C-manipulable.*

The C-manipulation with which we are presently concerned presents a different sort of problem than does W-manipulation or D-manipulation. W-manipulation is a concern in that it can upset the optimality of the final allocation. Given monotonicity, D-manipulation certainly upsets the optimality. C-manipulation on the other hand cannot disturb the optimality of the final outcome. C-manipulation is only a redistribution of the initial endowments, and since the final outcome is optimal with respect to this " new " economy, it must also be optimal with respect to the economy disregarding the manipulation.

It does affect the final distribution of utilities, however. The coalition S which can C-manipulate to increase the utility levels of its members does so at the expense of some traders outside the coalition, and these traders may form coalitions of their own.

The problem is then one of stability; for a given mechanism Q and any $e \in E$, are we certain that there is any outcome for which no coalition could C-manipulate to improve its members' final utilities?

The answer is no. If we look again at the proof of Theorem 3, we see that we have the existence of an economy e in which there is a coalition S which can trade its endowments so that every point in the core of the new economy is preferred by each member of S to any point in the core of the original economy. Hence if the outcome is in the core of the original economy, S will find it to its advantage to C-manipulate. But if the outcome is *not* in the core of the original economy, then as we saw in the lemma preceding Theorem 3, there must be a possibly different coalition T which could C-manipulate. Hence no outcome is " stable ".

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NOTES

1. This type of mechanism is similar in spirit to a method of distributing output from a production process in Hurwicz (1973).

2. J.-J. Laffont has independently arrived at a similar result.
3. Construction of such examples is a simple extension of examples in Drèze *et al.* (1977).
4. y and y' are symmetric allocations and are therefore independent of n.

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