# Premuneration Values and Investments in Matching Markets*

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**Abstract**

We analyze a model in which agents make investments and then match into pairs to create a surplus. The agents can make transfers to reallocate their pretransfer ownership claims on the surplus. Mailath, Postlewaite, and Samuelson (2013) showed that when investments are unobservable, equilibrium investments are generally inefficient. In this paper we work with a more structured model that is sufficiently tractable to analyze the nature of the investment inefficiencies. We provide conditions under which investment is inefficiently high or low and conditions under which changes in the pretransfer ownership claims on the surplus will be Pareto improving, as well as examine how the degree of heterogeneity on either side of the market affects investment efficiency.

**Keywords:** Directed search, matching, premuneration value, prematch investments, search.

**JEL codes:** C78, D40, D41, D50, D83

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# Contents

# 1 Introduction

## 1.1 Motivation

How are heterogeneous workers matched with heterogeneous firms? What determines the division of the resulting surplus? When will outcomes be efficient? We are interested in these questions in the context of a market in which workers and firms first make productivity-enhancing investments, and then match into pairs to produce a surplus.

It is a familiar result that if workers and firms cannot contract prior to making investments, then market power at the matching stage can lead to inefficient investments. However, Cole, Mailath, and Postlewaite (2001) show that if the matching market is competitive, then efficient two-sided investments are consistent with equilibrium.[1] However, the results of Cole, Mailath, and Postlewaite (2001) and Peters and Siow (2002) depend crucially on there being complete information about investments. Mailath, Postlewaite, and Samuelson (2013) study an economy similar to that in Cole, Mailath, and Postlewaite (2001), but with the difference that workers' investments are not observable when workers and firms match. The results are also different: except in the extreme case that firms' pretransfer values from a match are independent of the worker with whom they match, investments will not be efficient.

Our interest is in the nature of the inefficiency. Will investments be inefficiently low, or can they be inefficiently high? How does the magnitude of ex ante heterogeneity of workers affect the inefficiency, and are there policy interventions that might ameliorate the inefficiencies? How does the allocation of property rights to the surplus affect investments? Mailath, Postlewaite, and Samuelson's (2013) model is too general to answer these questions. We address these questions here in the context of a more structured model.

## 1.2 Investment and Matching Markets

The agents in our analysis can be interpreted in many ways—we opened our earlier paper by referring to firms and workers, but we could just as well

[1]Peters and Siow (2002) show that efficiency also holds in a nontransferable utility setting (more specifically, when transfers are not possible). Nöldeke and Samuelson (2015) extend the results of Cole, Mailath, and Postlewaite (2001) and Peters and Siow (2002) to general nontransferable utility settings.

think of students and universities, men and women, lawyers and clients, and so on. For concreteness, we henceforth refer to them as laboratories and researchers, terms chosen with the view that either side may own the output of the relationship, and that payments may flow either way.

We examine a market with a large set of laboratories of differing sophistication on one side and an analogous set of researchers of different abilities on the other side. Researchers first have the opportunity to invest in human capital, and then laboratories and researchers are matched into pairs.[2] Each pair produces a surplus, arising (for now) from the patents they create.

The patents that arise out of a laboratory/researcher match may belong to the laboratory, or may belong to the researcher, or ownership may be shared. When there is no uncertainty about matching-relevant characteristics it makes no difference for most problems whether the patents belong to the laboratory or to the researcher. We expect laboratories to hire researchers in the former case and researchers to buy or rent laboratories in the latter case. In either case, a monetary payment from the party that owns the patents to the other party delivers the equilibrium division of the surplus. For any change in the distribution of patent ownership, there is an offsetting change in the equilibrium monetary transfer between agents preserving the equilibrium welfare distribution and investments.[3] In particular, outcomes with efficient investments exist no matter who owns the patents. If laboratories own the patents, for example, competition among laboratories to hire talented researchers will ensure that the latter capture the returns from their investments and hence face efficient investment incentives.

If researchers' match-relevant characteristics are unobservable, initial ownership will play a central role in the efficiency of investments and in the final welfare distribution.[4] Laboratories now cannot observe a researcher's investment, precluding the enhanced competition that facilitates the researcher's capture of the returns on her investment when laboratories own

[2]In order to focus on the implications of unobservable researcher investments, we assume in much of the paper that laboratories' investments are fixed (in Section 4, we also allow laboratories to make investments).

[3]See Cole, Mailath, and Postlewaite (2001) and Mailath, Postlewaite, and Samuelson (2013, Section 6.1 and Appendix E) for details.

[4]This is reminiscent of the Coase theorem (Coase, 1960): in the absence of bargaining frictions (such as asymmetric information), bargaining will result in an efficient allocation irrespective of the original allocation of property rights. On other hand, in the presence of asymmetric information, the possibility of reaching an efficient agreement depends on the original allocation of property rights (see, for example, Cramton, Gibbons, and Klemperer (1987)). However, the similarity is superficial, since the Coase theorem ignores investments that may be taken before bargaining (Grossman and Hart, 1986).

the patents, and potentially leading to inefficient investments. More importantly, increasing the share of the patents owned by the researcher then provides incentives to invest more efficiently, giving rise to a link between initial ownership and investments that is the focus of this paper, and that can have unexpected implications.

### 1.3 Premuneration Values

In general, the surplus generated by a match will be a composite of many different items in addition to patents, with the ownership of these various items split between the laboratory and researcher in different ways. The researcher's value of the match includes the value of the human capital she accumulates at the laboratory, as well as the value from contacts she makes at the laboratory. The researcher may also derive utility from laboratory parties and social opportunities, but may derive disutility from exerting costly effort. The laboratory's value of the match may include the prestige of employing a noted researcher, as well as the accumulation of organizational capital that will be of use in other research endeavors, but may also include the costs of training the researcher. In addition, some of the value from the researcher's contacts may accrue to the firm, perhaps because they make it easier to hire additional researchers.

Rather than itemize all the elements that comprise the surplus in the match between the laboratory and researcher, we take as the primitive the aggregate match value to each of the agents in the absence of any transfers. Mailath, Postlewaite, and Samuelson (2013) call these values *premuneration values* (from *pre* plus the Latin *munerare*, to give or pay). The total surplus in a match is then simply the sum of the matched parties' premuneration values. The premuneration values determine the division of the surplus *in the absence of transfers.* In equilibrium, of course, there typically will be transfers. What is central to our problem is that any transfers that reallocate surplus are determined *after* investments have been made.

We find that premuneration values matter.[5] More specifically:

- When researchers do not own all the surplus from a match, they invest less than is efficient; their investments and payoffs increase as their premuneration value increases.

[5] Liu, Mailath, Postlewaite, and Samuelson (2014) examine a finite matching model with incomplete information but no investments in which premuneration values also play a central role.

- Laboratories' equilibrium payoffs increase as researcher premuneration values increase if the latter are small, and then decrease. In particular, increasing the share owned by researchers can lead to more investment, with the laboratories enjoying some of the fruits of that investment. In addition, there is competition among researchers for laboratories, and researchers who own more of the surplus find all laboratories more valuable. This intensifies the competition for laboratories, leading to higher market prices for laboratories. When the share of the surplus owned by researchers is small, both sides can gain by having premuneration values allocate more of the surplus to researchers.

- When both sides invest, but researchers' attributes are unobservable, laboratories invest more than is efficient when researchers invest close to their efficient levels (which occurs when researchers are more heterogeneous).

- The increase of researcher investments and payoffs as their premuneration values increase depends on the ex ante heterogeneity of researchers. If researchers are identical, competition will ensure that all surplus goes to laboratories, and in this case the premuneration values of the researchers is irrelevant. But heterogeneity among researchers will attenuate researchers' competition for laboratories and researchers will accordingly get positive surplus in equilibrium. Their equilibrium welfare then increases in their premuneration values and increases in the heterogeneity of their investment costs.

It is a familiar result that inefficiencies can arise when the characteristics of the agents on one side of the market cannot be observed. The important finding is that the equilibrium allocation depends upon premuneration values, sometimes counterintuitively. Premuneration values matter whenever there are unobserved exogenous attributes or unobserved investments, though we focus on the latter. Investments in human capital are especially difficult to verify, bringing any market for skilled labor within the scope of our model.

Our analysis assumes that laboratories cannot learn researchers' attributes. In an online appendix, we examine the researcher-investment case on which the paper is focussed, but allow laboratories to learn the attributes of researchers at a cost. Changes in premuneration values can have further surprising effects on which laboratories become informed and on the resulting division of the surplus.

### 1.4 Why Do We Care?

The finding that premuneration values matter would be relatively innocuous if we could simply redesign them appropriately. Indeed, in the absence of any obstacles, the specification of premuneration values would simply be part of an optimal contracting problem between a researcher and a laboratory. However, we believe premuneration values are often determined by the legal and institutional environment, and so should serve from the researchers' and laboratory's point of view as exogenous points of departure for their contracting problem.

Efficiency may yet simply be a matter of designing premuneration values appropriately, even if this is a legal rather than contract design problem. However, some (perhaps many) configurations of premuneration values may be impossible to achieve. Laws that prohibit selling one's human capital or prohibit relinquishing legal rights may preclude some allocations. Measurement or collection problems may preclude others. Premuneration values often include future returns, requiring future costly actions and hence moral hazard problems that preclude reallocation.[6]

To illustrate the difficulties in redesigning premuneration values, consider a match between a student and a university. While at the university, the student acquires knowledge and skills that lead to higher lifetime earnings and a greater satisfaction in life after school. She may also make contacts that will be important in her career, and she may be a regular at campus parties and generally enjoy the social life of the university. Each of these increases the student's value of the match, and consequently the surplus in the match. The university may derive value from the contribution to its ranking caused by her stellar SAT score, from touting the student's background and her ability to play the saxophone as additions to its diverse and artistically rich community, as well as from claiming her as a graduate when she achieves fame and fortune. The university's value of these items also contributes to the surplus of the match. Each side owns some of these components, in the sense that the value of that component accrues to them. Some components might be owned by either side, depending on circumstances, but others are inextricably linked to a particular side. We might be able to reallocate the ownership of the student's future income stream, perhaps by financing her education with income-contingent loans, but there are obvious limits in the possible shifting. There is no obvious way to reallocate her utility from partying.

[6] See Mailath, Postlewaite, and Samuelson (2013, Sections 1.4 and 6.5) for a discussion of this issue.

### 1.5 Related Literature

Other papers have also investigated the relationship between the incentives for efficient investments and subsequent bargaining.[7] Acemoglu and Shimer (1999) analyze a worker-firm model in which firms (only) make ex ante investments. If wages are determined by post-match bargaining, a standard hold-up problem induces firms to underinvest. The hold-up problem disappears if workers have no bargaining power, but then there is excess entry on the part of firms. Acemoglu and Shimer show that efficient outcomes can be achieved if the bargaining process is replaced by wage-posting on the part of firms, followed by competitive search. de Meza and Lockwood (2010) examine an investment and matching model that gives rise to *excess* investment. Their overinvestment possibility rests on a discrete set of investment choices and the presence of bargaining power in a noncompetitive post-investment stage.

In contrast, the competitive post-investment markets of Cole, Mailath, and Postlewaite (2001) and Peters and Siow (2002) lead to efficient two-sided investments. Our analysis shows that this efficiency rests on both ex post competition and complete information, with the latter allowing prices to be conditioned on both worker and firm characteristics. Gall, Legros, and Newman (2006, 2009) and Bhaskar and Hopkins (2011) examine an alternative class of models in which information is complete and hence different prices can be set for different workers, but inefficiencies arise out of limitations on the ability to reallocate the surplus in a match via transfers, including limiting cases in which no transfers can be made. In contrast to these models, monetary transfers allow us to achieve any division of the surplus between a pair of matched agents.

Moving from complete-information to incomplete-information matching models typically gives rise to issues of either screening, as considered here, or signaling. Cole, Mailath, and Postlewaite (1995), Hopkins (2012), Hoppe, Moldovanu, and Sela (2009) and Rege (2008) analyze models incorporating signaling into matching models with investments.[8]

[7] Early literature suggesting that frictionless, competitive search might create efficient investment incentives include Hosios (1990), Moen (1997), and Shi (2001). Eeckhout and Kircher (2010) provide an extension to asymmetric information, while Masters (2011) examines a model with two-sided investments.

[8] The inability to observe workers' characteristics forces a firm to offer the same payment to all workers. Firms setting the "impersonal prices" of Bulow and Levin (2006) similarly offer the same price to all workers, but Bulow and Levin offer a motivation in terms of institutional constraints rather than incomplete information, including the possibility that firms may be able and desirous of committing to such prices in order to secure a more

## 2 The Model

There is a unit measure of researchers whose types (names) are indexed by $\rho$ and are distributed uniformly on $[0, 1]$, and a unit measure of laboratories whose types are indexed by $\lambda$ and distributed uniformly on $[0, 1]$. For ease of reference, researchers are female and laboratories male.

At the first stage, each researcher chooses an attribute $r \in \mathbb{R}_+$. Each laboratory is characterized by an attribute $\ell$, where for convenience we take the attribute of laboratory $\lambda$ to be fixed at $\ell = \lambda$. Following the attribute choices, researchers and laboratories match, with a researcher with attribute $r$ receiving a premuneration value of $h_R(r, \ell)$ from a match with a laboratory with attribute $\ell$, and with the laboratory's premuneration value from the same match denoted by $h_L(r, \ell)$. The second-stage values depend only on the attributes of the researcher and laboratory, $r$ and $\ell$, and not on their underlying types.

Researcher attributes are costly, with researcher $\rho$ paying a cost of $c(r, \rho)$ to acquire attribute $r$. We assume

1. $h_R$ and $h_L$ are both $\mathcal{C}^2$, increasing in $r$ and $\ell$, and

$$\frac{\partial^2 h_R}{\partial r \partial \ell} > 0 \quad \text{and} \quad \frac{\partial^2 h_L}{\partial r \partial \ell} \geq 0,$$

2. $c$ is $\mathcal{C}^2$, strictly increasing, and convex in $r$, with $c(0, \rho) = \partial c(0, \rho)/\partial r = 0$, and

$$\frac{\partial^2 c}{\partial r \partial \rho} < 0,$$

and

3. there exists $\bar{r} > 0$ such that for all $r > \bar{r}$ and for all $\ell$, and all $\rho$,

$$h_R(r, \ell) + h_L(r, \ell) - c(r, \rho) < 0.$$

This is the model of Mailath, Postlewaite, and Samuelson (2013), with the restriction that laboratory attributes are exogenous (in our earlier paper, attributes are treated symmetrically on the two sides of the market).

lucrative equilibrium.

## 2.1 Efficient Allocations

The cost function $c(r, \rho)$ satisfies a single crossing condition, ensuring that researchers with larger indices choose larger attributes. The supermodularity of the premuneration values then implies that efficient matching is positive assortative on index, so that researcher $\rho$ matches with laboratory $\lambda = \rho$. Finally, the investment choice $r^e(\rho)$ in a match between researcher $\rho$ and laboratory $\lambda = \rho$ (and hence with attribute $\ell = \lambda$) maximizes

$$h_R(r, \ell) + h_L(r, \ell) - c(r, \rho) = h_R(r, \rho) + h_L(r, \rho) - c(r, \rho).$$

If the researchers' investments are observable, then this model is a special case of the complete-information model described in Section 6.1 of Mailath, Postlewaite, and Samuelson (2013). The appropriate equilibrium concept for the complete-information case is a personalized pricing equilibrium, which is the counterpart of Cole, Mailath, and Postlewaite's (2001) ex post contracting equilibrium in the current setting. When investments are two-sided, coordination failures can give rise to inefficient personalized price equilibria, but in the current one-sided investment setting, every personalized price equilibrium of the complete-information case is efficient. Section 3.2 presents the notion of a *personalized pricing equilibrium* and the attendant efficiency result for the parametric example considered in Section 3. The following subsection defines a *matching equilibrium* for the incomplete-information case that is our primary interest.

## 2.2 Equilibrium

Matching takes place in a competitive market. Laboratories' attributes are observable and priced, with $p(\ell)$ denoting the price of a laboratory with attribute $\ell$. Researchers' attributes are not observable to laboratories, hence the price of laboratory with attribute $\ell$, $p(\ell)$, is the same to all researchers. Given a price function $p$, each researcher optimally chooses her attribute and the laboratory with whom she wishes to match. That is, researcher $\rho$ solves

$$\max_{\ell, r} \; h_R(r, \ell) - p(\ell) - c(r, \rho). \tag{1}$$

We denote by $r_R : [0, 1] \to \mathbb{R}_+$ the function describing the attributes chosen by researchers and by $\ell_R : [0, 1] \to [0, 1]$ the function describing the laboratories chosen by researchers.

The function $\ell_R$ is *market-clearing* if it is one-to-one, onto, and every set of researchers $\mathcal{R}$, is mapped to a set of equal size of laboratories.[9] Given a price function $p$ and researcher behavior $r_R$ and $\ell_R$ (where $\ell_R$ is market clearing), the payoff to laboratory $\ell$ is $h_L(r_R(\ell_R^{-1}(\ell)), \ell) + p(\ell)$.

**Definition 1** *A price function $p$ and researcher choices $(\ell_R, r_R)$ constitute a* matching equilibrium *if*

1. *for every $\rho \in [0,1]$, the choice $(\ell_R(\rho), r_R(\rho))$ solves the researcher-optimization problem (1),*
2. *every researcher and laboratory earns nonnegative payoffs, and*
3. *$\ell_R$ is market-clearing.*

The second property of equilibrium is an *individual rationality* requirement, ensuring that all agents prefer participation to not participating. This equilibrium notion is the analogue in our setting (since laboratories do not choose attributes) of Mailath, Postlewaite, and Samuelson's (2013) uniform price equilibrium.

We begin by identifying three useful properties of a matching equilibria (the proofs are in Appendix A).[10] The first is a direct implication of market clearing:

**Lemma 1** *Every equilibrium price function $p$ is strictly increasing and continuous.*

The researchers' cost functions satisfy a single-crossing condition, giving the following lemma.

**Lemma 2** *Every equilibrium researcher attribute-choice function $r_R$ is strictly increasing.*

[9]Formally, if $\mu$ is Lebesgue measure and $\mathcal{R}$ is a measurable set of researcher types, then $\mu(\mathcal{R}) = \mu\{\ell | \ell = \ell_R(\rho)$ for some $\rho \in \mathcal{R}\}$. Our assumption that laboratory attributes are exogenously and uniformly distributed on an interval allows us to avoid various technical issues that arise with a continuum of agents in two-sided investment models; see Mailath, Postlewaite, and Samuelson (2013, Section 3.2) for a discussion.

[10]Analogues of Lemmas 1–3 should hold in the two-sided investment model of Mailath, Postlewaite, and Samuelson (2013), but the two-sided investments and associated more complicated notion of matching feasibility preclude the simple arguments used here. We expect an argument mimicking the existence argument from Mailath, Postlewaite, and Samuelson (2013) to yield existence of matching equilibria in the current context. Much of this paper studies a parametric example for which existence is immediate.

The supermodularity of the surplus function ensures that matching is assortative.

**Lemma 3** *Every equilibrium researcher laboratory-choice function* $\ell_R$ *is given by*

$$\ell_R(\rho) = \rho.$$

### 2.3 Underinvestment

Mailath, Postlewaite, and Samuelson (2013) showed that in general, attribute investments in uniform price equilibria are inefficient. Inefficiencies arise from coordination failures (since both sides invest) and from a lack of full appropriability of the returns from investment. Unfortunately, that model is too general to permit a more precise determination of the nature of the inefficiency.

By assuming that one side's attributes are exogenously determined, we both avoid the possibility of coordination failures, and can unambiguously determine the direction of inefficiency. This gives us our fundamental underinvestment result, arising from the researchers' inability to capture the full social return on investments.

**Proposition 1** *Suppose the first derivative of social surplus with respect to* $r$ *has a unique strictly positive zero for all* $\rho > 0$*. If* $h_L$ *has a strictly positive derivative with respect to* $r$*, then in any matching equilibrium* $(p, (\ell_R, r_R))$*, for almost all* $\rho > 0$*, the equilibrium investment* $r_R(\rho)$ *is lower than the efficient investment.*

The assumption that $dh_L/dr > 0$ indicates that the a laboratory's premuneration value is increasing in the investment of a researcher with whom the laboratory is matched. This assumption most obviously fails when $h_L$ is identically zero, so that all of the value created by a match is captured in the researcher's premuneration value. In Section 3, this is the only way this assumption can fail.

**Proof.** Let

$$f(r;\rho) := \frac{\partial h_R(r,\rho)}{\partial r} + \frac{\partial h_L(r,\rho)}{\partial r} - \frac{\partial c(r,\rho)}{\partial r}$$

denote the first derivative of the social surplus. Then, $f(\,\cdot\,;\rho)$ is $\mathcal{C}^1$ on $\mathbb{R}_+$

and

$$f(r;\rho)\begin{cases} >0, & r<r^e(\rho), \\ =0, & r=r^e(\rho), \\ <0, & r>r^e(\rho). \end{cases}$$

Suppose $(p,(\ell_R,r_R))$ is a matching equilibrium. Then, as $r_R(\rho)$ is interior, it satisfies the first order condition (using Lemma 3)

$$\frac{\partial h_R(r,\rho)}{\partial r}-\frac{\partial c(r,\rho)}{\partial r}=0.$$

Since $h_L$ has a strictly positive derivative with respect to $r$, $f(r_R(\rho),\rho)>0$, and so $r_R(\rho)<r^e(\rho)$. ■

# 3 A More Structured Model

## 3.1 The Premuneration Values and Cost Function

We next analyze equilibrium investments in more detail for a class of premuneration and cost functions. The premuneration values of a match, as a function of the attributes $\ell$ and $r$ of the agents in the match, are

$$h_R(r,\ell)=\theta r\ell \tag{2}$$

and

$$h_L(r,\ell)=(1-\theta)r\ell, \tag{3}$$

so that the total surplus in a match is given by

$$v(\ell,r)=\ell r.$$

The parameter $\theta$ describes the researcher's premuneration value share of the surplus, while $(1-\theta)$ describes the laboratory's share. From Section 2, the basic properties we need are that the premuneration values are increasing in both attributes and are supermodular, i.e., have a positive cross derivative. The surplus function $\ell r$ is a special case of a Cobb-Douglas production function, and exhibits these properties while being simple enough to exhibit closed-form solutions. The constant share embodied in (2)–(3) ensures that these properties are inherited by the premuneration values. Premuneration shares may be constant if researchers and laboratories own different components of the value of a match (e.g., the researcher may own her accumulated human capital, while the laboratory may own the value of patents produced in the interaction) that arise in constant proportions across interactions.

Our choice of cost function is guided by our desire to examine the effects of changes in the strength of competition in the market for researchers as well as changes in premuneration values. We assume the cost of attribute $r \in \mathbb{R}_+$ to researcher $\rho > 0$ is given by

$$c(r, \rho) = \frac{r^{2+k}}{(2+k)\rho^k}, \qquad k \in \mathbb{R}_+.$$

We constrain $\rho = 0$ to choose $r = 0$. When $k = 0$, researchers are homogeneous in the sense that all have the same cost. We can thus expect fierce competition between researchers for laboratories. When $k > 0$, researchers are heterogeneous, with higher $\rho$ researchers having a lower cost of acquiring any level of the attribute. As $k$ increases, so does the curvature of the cost function. In particular, the marginal cost of attribute $r = \rho$ for researcher $\rho$ remains fixed at $r$ as $k$ varies, but the slope of the marginal cost function through this point increases as does $k$. As a result, researcher $\rho$ becomes increasingly reluctant to stray from the attribute $r = \rho$ as $k$ increases. This makes it less attractive for researcher $\rho$ mimic the attribute and laboratory choice of researcher $\rho' \neq \rho$, thus dampening the competition between researchers for laboratories.

The cost function has the property that efficient researcher investments are independent of $k$. This allows us to study how the effects of changes in premuneration values vary with $k$, and how these changes affect the efficiency of investments.

### 3.2 Complete Information and Efficient Outcomes

As a point of comparison, we first consider the complete information scenario mentioned in Section 2.1 in which researchers' investments are observable. Matching takes place in a competitive market, characterized by a *personalized* price function $p(\ell, r)$ specifying, for any laboratory attribute $\ell$ and researcher attribute $r$, the payment from the laboratory to the research if the pair form a match. When laboratories cannot distinguish between researchers, the equilibrium requirement that each laboratory chooses a researcher reduces to individual rationality. Under complete information, laboratories can distinguish researchers and equilibrium requires that the laboratory chosen by a researcher chooses that researcher.[11] The appropriate

[11] The complete information scenario is a special case of Cole, Mailath, and Postlewaite (2001), and their notion of *ex post contracting equilibrium* applied to the current setting

competitive equilibrium notion under complete information is *personalized pricing equilibrium*:

**Definition 2** *A personalized price function* $\tilde{p}$ *and researcher choices* $(\tilde{\ell}_R, \tilde{r}_R)$ *constitute a* personalized pricing equilibrium *if*

1. *for every* $\rho \in [0,1]$*, the choice* $(\tilde{\ell}_R(\rho), \tilde{r}_R(\rho))$ *maximizes*

$$h_R(r,\ell) - \tilde{p}(\ell, r) - c(r,\rho),$$

2. *for all* $\ell \in \tilde{\ell}_R([0,1])$,

$$\tilde{r}_R(\ell) \in \arg\max_r h_L(r,\ell) + \tilde{p}(\ell, r),$$

3. *every researcher and laboratory earns nonnegative payoffs, and*

4. $\tilde{\ell}_R$ *is market-clearing.*

Personalized pricing equilibria exist and are, because investments are one-sided, efficient.[12] Efficient researcher attribute choices have a particularly simple form. First, strict supermodularity of the surplus function $\ell r$ implies that, for any strictly increasing researcher attribute choice function, total surplus is maximized under assortative matching. Second, the cost function for researchers is decreasing in researcher index $\rho$, so for any researcher attribute distribution, the minimum cost of obtaining that distribution is for the attribute choice function $r_R$ to be (weakly) increasing. Thus, total net surplus is maximized when the matching on indices $\lambda$ and $\rho$ is positively assortative: laboratory $\lambda$ will be matched with researcher $\rho = \lambda$. Total net surplus is thus maximized when the net surplus for each such matched pair is maximized. For the $\rho$-matched pair of laboratory and researcher, the surplus-maximization problem is (since laboratory $\lambda = \rho$ has attribute $\ell = \rho$)

$$\max_r \ \rho r - \frac{r^{2+k}}{(2+k)\rho^k}. \tag{4}$$

The first-order condition is

$$\rho = \frac{r^{1+k}}{\rho^k},$$

yields the same outcomes as personalized pricing equilibrium.

[12] The proof of efficiency follows that of Cole, Mailath, and Postlewaite (2001, Lemma 2).

immediately implying

$$r = \rho. \tag{5}$$

Hence, efficiency requires $r_R(\rho) = \rho$ and $\ell_R(\rho) = \rho$. As we indicated earlier, the efficient allocation does not depend on $k$, the degree of heterogeneity of the researchers.

It is straightforward to verify that the equilibrium personalized pricing function is

$$\tilde{p}(r, \ell) = \frac{1}{2}\ell^2 - (1 - \theta) r \ell.$$

The personalized prices are important in achieving positive assortative matching on index and efficiency. Researcher $\rho$ would prefer to match with laboratory $\ell = \rho + \varepsilon$ if researcher $\rho$ could do so while retaining her current investment and trading at the equilibrium price that appears in the match between researcher $\rho + \varepsilon$ and laboratory $\ell = \rho + \varepsilon$. But the personalized prices preclude this. Researcher $\rho$ can match with laboratory $\ell = \rho + \varepsilon$ only if she boosts her investment to match that of researcher $\rho + \varepsilon$ or pays a higher price, which researcher $\rho$ prefers not to do.

Personalized pricing is impossible when researcher attributes are unobservable. Each laboratory is now characterized by a single price at which it stands ready to hire all willing researchers. If we attach to each laboratory the price at which it trades in the personalized price equilibrium, the result will not be an equilibrium. Now nothing deters researcher $\rho$ from matching with some laboratory $\ell = \rho + \varepsilon$, and the market does not clear. Equilibrium matching under incomplete information is still positive assortative (Lemma 3), so there is no inefficiency in matching, but this sorting in general requires equilibrium prices to increase more slowly in laboratories' types than in the complete-information case. This attenuates the incentives for reseachers to invest, leading to inefficient investments.

### 3.3 Incomplete Information Matching Equilibrium

We turn now to the structure of the matching equilibrium. First, suppose that the equilibrium price of laboratories is differentiable, a supposition that will be validated by the equilibrium we construct.[13] Researcher $\rho$'s problem

[13] A standard revealed preference argument shows that in fact every equilibrium price function is differentiable, and so the equilibrium investment function is unique. This is true even when the bottom index for researchers and laboratories is strictly positive, so that the surplus at the bottom is strictly positive. In the latter case, there are multiple equilibrium price functions (though they only differ by a constant).

is to choose $\ell$ and $r$ to maximize

$$\theta\ell r - p(\ell) - c(r,\rho) \;=\; \theta\ell r - p(\ell) - \frac{r^{2+k}}{(2+k)\rho^k}.$$

The first order conditions are

$$\theta\ell = \frac{r^{1+k}}{\rho^k} \tag{6}$$

and

$$\theta r = p'(\ell). \tag{7}$$

In equilibrium, researcher $\rho$ is matched with laboratory $\ell = \rho$, hence from (6) we have that in equilibrium

$$r_R(\rho) = \rho \cdot \theta^{\frac{1}{1+k}}. \tag{8}$$

For all $\theta \in (0,1)$, $\theta^{\frac{1}{1+k}} \in (0,1)$, and hence $r_R(\rho) < \rho$; for $\theta = 1$, $r_R(\rho) = \rho$.

For any given researcher $\rho$, $r_R(\rho)$ is increasing in both $k$ and $\theta$. As $\theta$ increases, the researcher has a larger share of the surplus, and hence has an increased incentive to invest; when $k$ increases, less of a researcher's benefit is competed away, giving researchers further reason to increase their investment.

Combining the two first order conditions (7) and (8) gives

$$p'(\ell) = \ell \cdot \theta \cdot \theta^{\frac{1}{1+k}}$$

and hence

$$p(\ell) = \tfrac{1}{2}\ell^2 \cdot \theta^{\frac{2+k}{1+k}} \tag{9}$$

(the constant of integration is set so that $p(0) = 0$, as required by the individual-rationality requirement that payoffs be nonnegative).

Summarizing the above discussion, we have the following proposition.

**Proposition 2** *There is a unique matching equilibrium, with researcher investment function given by (8) and price function by (9). Investment is therefore below the efficient level of $r = \rho$ unless $\theta = 1$.*

**Remark 1** A natural conjecture is that the pervasive inefficiency reflected in Proposition 1 and the investment function (8) simply reflects that we have given laboratories too meager an arsenal of contracting weapons. As discussed in Section 1.4, we view the specification of premuneration values as part of the contracting environment, just as are the specification of the surplus and cost functions. The price function $p(\ell)$ already allows the contract between a researcher and a laboratory to be conditioned on all of the observable variables in their interaction. However, instead of simply posting a price, we could allow laboratories to force the researchers who approach them to play a direct revelation game, announcing a type $\hat{r}$, with the price $p(\ell, \hat{r})$ then depending on both the laboratory's price and the researcher's announced type. This brings the laboratory no new flexibility. The researcher's payoff depends on her announcement only through its effect on the price, and every researcher approaching laboratory $\ell$ would name that type $\hat{r}$ that elicits the most favorable price. ◆

### 3.4 Payoffs

Given the equilibrium choices, laboratory $\lambda$'s payoff given $\theta$ and $k$ is (inserting the equilibrium investment function to obtain the second line and the equilibrium price function to obtain the third)

$$\begin{aligned} u_L(\theta, k, \lambda) &\equiv (1-\theta)\ell r + p(\ell) \\ &= (1-\theta)\lambda(\lambda\theta^{\frac{1}{1+k}}) + p(\ell) \\ &= (1-\theta)\lambda^2\theta^{\frac{1}{1+k}} + \tfrac{1}{2}\lambda^2\theta^{\frac{2+k}{1+k}} \\ &= \tfrac{1}{2}\theta^{\frac{1}{1+k}}(2-\theta)\lambda^2. \end{aligned} \tag{10}$$

We are interested in identifying conditions under which the laboratory's payoff increases when the researcher's share of the surplus, $\theta$, increases. From (10), the laboratory's payoff is increasing in its share of the surplus when $\frac{d}{d\theta}\theta^{\frac{1}{1+k}}(2-\theta) < 0$. This derivative is given by

$$\begin{aligned} \frac{d}{d\theta}\theta^{\frac{1}{1+k}}(2-\theta) &= \tfrac{1}{1+k}\theta^{\frac{-k}{1+k}}(2-\theta) - \theta^{\frac{1}{1+k}} \\ &= \theta^{\frac{-k}{1+k}}\left[\tfrac{1}{1+k}(2-\theta) - \theta\right]. \end{aligned}$$

Thus the sign of $du_L/d\theta$ is the same as the sign of $\frac{1}{1+k}(2-\theta)-\theta$, that is, of $2-(2+k)\theta$.

Figure 1 shows the region in which laboratories' payoffs increase as the researchers' premuneration values increase: $(\theta, k)$ combinations that are below and to the left of the curved line are situations in which the laboratories' payoff increases when the researchers' premuneration values increase.

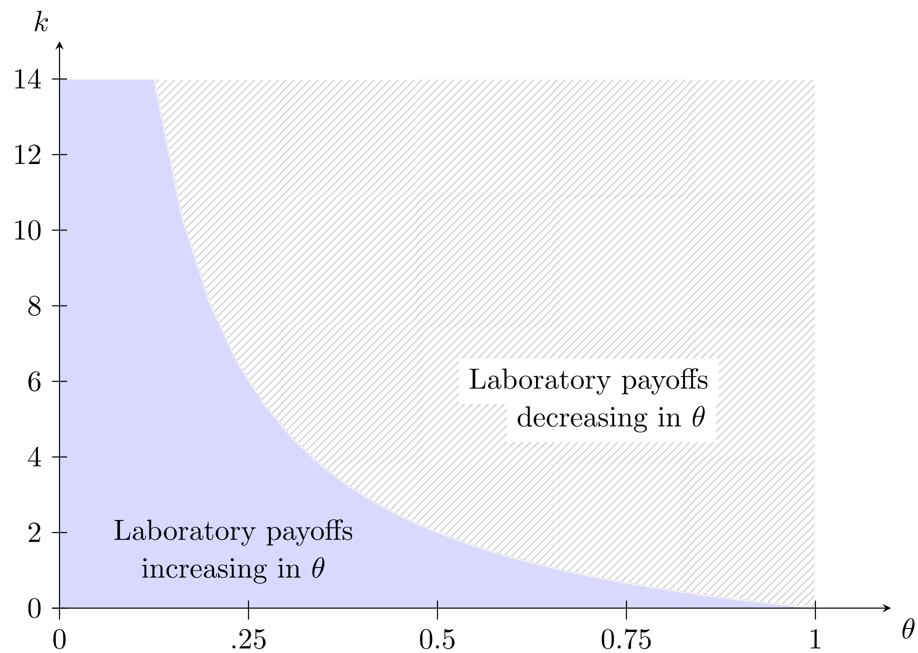


Figure 1: Parameter regions for which laboratory payoffs are increasing or decreasing in laboratory premuneration values.

Above the line, laboratories' payoffs decrease as researchers' premuneration value increases. Hence, the line represents the optimal premuneration values from the laboratory's perspective. In summary:

**Proposition 3** *Laboratories' equilibrium payoffs are first increasing in $\theta$, the researchers' premuneration value share, are maximized at $2/(2+k)$, and then are decreasing in $\theta$.*

For $k = 0$, the laboratory's payoff is increasing for all $\theta$, that is, laboratories' payoffs are maximized when premuneration values assign all the surplus to researchers. When $k = 0$, researchers are identical and so the competition for laboratories is the most intense, with researchers bidding away all rents in the competition for higher attribute laboratories. Since laboratories ultimately capture all the surplus through market competition, they do best when total surplus is maximized, which is when $\theta = 1$.

For positive but small $k$, the laboratories' payoffs are maximized with $\theta$ near, but less than, 1. When $\theta < 1$, researchers' attribute choices will be less than the attribute choices that maximize total net surplus. This is nevertheless optimal for laboratories since they will not capture the entire surplus in the market given that competition among researchers is imperfect when $k > 0$. As $k$ increases, competition among researchers decreases as researchers become more heterogeneous (and their choices become more efficient) and the researcher share of the surplus that maximizes laboratory payoff decreases, approaching zero as $k$ gets large.

The researcher's payoff can be calculated as the total net surplus minus the laboratory's payoff. The total net surplus for a matched pair $\rho = \lambda$ is

$$\theta^{\frac{1}{1+k}}\rho^2 - \frac{(\theta^{\frac{1}{1+k}}\rho)^{2+k}}{(2+k)\rho^k} = \rho^2\theta^{\frac{1}{1+k}}\left[1 - \tfrac{1}{(2+k)}\theta\right].$$

From (10), the laboratory's payoff is $\rho^2\theta^{\frac{1}{1+k}}(1 - \frac{1}{2}\theta)$, so the researcher's payoff is

$$\begin{aligned} u_R(\theta, k, \rho) &\equiv \rho^2\theta^{\frac{1}{1+k}}\left[1 - \tfrac{1}{2+k}\theta\right] - \rho^2\theta^{\frac{1}{1+k}}\left(1 - \tfrac{1}{2}\theta\right) \\ &= \theta^{\frac{1}{1+k}}\rho^2\left[1 - \tfrac{\theta}{2+k} - 1 + \tfrac{\theta}{2}\right] \\ &= \frac{k\theta^{\frac{2+k}{1+k}}}{2(2+k)}\rho^2. \end{aligned} \tag{11}$$

**Proposition 4** *Researchers' equilibrium payoffs increase in $\theta$, i.e., as researchers' premuneration value share increases.*

Thus, both researchers' and laboratories' payoffs increase, as the researcher's premuneration value increases, in the solid shaded region in Figure 1.

### 3.5 The Impact of Competition on Payoffs

We next investigate the effect of changes in the heterogeneity of researchers, via changes in $k$, on payoffs. The equilibrium payoffs of laboratories and researchers are given by (10) and (11). Figure 2 illustrates these payoffs as a function of $k$.

As $k$ increases, researchers' investments increase toward the efficient level, increasing the value created in each equilibrium match. The price function $p(\ell)$ also increases. Increasing $k$ dampens the competition between laboratories, suggesting that prices for matching with laboratories should decrease, but it also leads to higher investments, making matches more valuable.

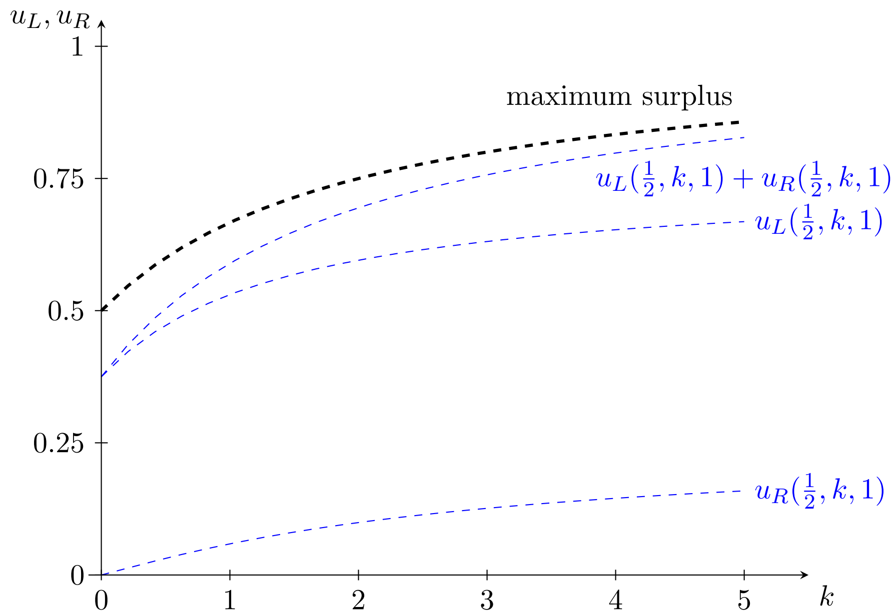


Figure 2: Payoffs for the scenario in which the researchers choose attributes for $\theta = \frac{1}{2}$ and $\rho = \lambda = 1$. Since researcher (respectively, laboratory) payoffs for index $\rho$ (resp., $\lambda$) are proportional to $\rho^2$ (resp., $\lambda^2$), these also represent the proportionality factors for the other indices. The maximum surplus can be calculated to be $(k+1)/(k+2)$.

Researchers' payoffs increase in $k$, reflecting the enhanced investment incentives of reduced competition and reduced investment costs. Laboratories' payoffs increase with $k$. A larger value of $k$ makes researchers more heterogeneous, and hence dampens their competition for laboratories, seemingly to the latter's deficit. However, this is outweighed by the enhanced researcher investment incentives of increasing $k$.

In the limit, when $k = 0$, all researchers are identical, giving rise to fierce competition that allows laboratories to capture all the surplus:

$$\lim_{k \to 0} u_R(\theta, k, \rho) = 0$$

$$\text{and} \qquad \lim_{k \to 0} u_L(\theta, k, \lambda) = \frac{\theta}{2}(2 - \theta)\lambda^2.$$

At the other extreme, as $k \to \infty$, researchers have increasingly different values for any particular laboratory, dampening their competition. We then get efficient attribute choices, but the premuneration values still matter in terms of the division:

$$\lim_{k\to\infty} u_R(\theta,k,\rho) = \frac{\theta\rho^2}{2}$$

$$\text{and} \qquad \lim_{k\to 0} u_L(\theta,k,\lambda) = \left(1-\frac{\theta}{2}\right)\lambda^2.$$

# 4 Laboratories Also Invest

Researchers' investments are inefficiently low when laboratory premuneration values are not degenerate. To understand the source and nature of the inefficiency, this section maintains our previous information structure (laboratories' attributes are commonly known but researchers' attributes are not), but now laboratories as well as researchers choose attributes. Our model contains the model of Section 3 (in which only researchers choose attributes) as the limiting case where laboratories become arbitrarily heterogeneous.

Once again, we find that if $\theta < 1$, so that laboratories have nontrivial premuneration values, then researchers' investments are inefficiently low. In addition, we find that laboratories investments are in general also inefficient, though this inefficiency reflects forces that can be quite different from those that shape researcher investments. When $\theta$ (and as a result, researchers' premuneration values) is large, laboratories *overinvest*.

Our model also contains as a limiting case a setting in which researchers' attributes are fixed and only laboratories choose attributes, though researchers' attributes are still unobservable. Here, we find that the laboratories' investments are inefficiently large, no matter what the value of $\theta$.[14] Hence, it is the *unobservability* of researcher's attributes that causes inefficient investments, regardless of who makes the investment.

## 4.1 The Premuneration Values and Cost Functions

As before, matching takes place in a competitive market, with laboratory attributes observable and priced. We use the diacritic ˆ to distinguish the equilibrium prices, attribute choices and payoffs here from their analogs in Section 3.

We retain the assumption that premuneration values are given by $h_R(r,\ell) = \theta r\ell$ for researchers and $h_L(r,\ell) = (1-\theta)r\ell$ for laboratories, and hence that

[14]The model is continuous: for any value of $\theta$, if researchers are sufficiently heterogeneous, then laboratories overinvest.

the total surplus in a match is given by $v(\ell, r) = \ell r$.

The cost to researcher $\rho$ of attribute $r$ is again given by

$$c(r, \rho) = \frac{r^{2+k}}{(2+k)\rho^k}.$$

The cost of attribute $\ell \in \mathbb{R}_+$ to laboratory $\lambda$ is similarly given by

$$\psi(\ell, \lambda) = \frac{\ell^{2+\kappa}}{(2+\kappa)\lambda^k}, \qquad \beta, \kappa \in \mathbb{R}_+.$$

### 4.2 Efficient Outcomes

An efficient outcome again exhibits positive assortative matching. The efficient attribute choices $r_e(\rho)$ and $\ell_e(\lambda)$ maximize, for any $\rho = \lambda$,

$$r\ell - \frac{r^{2+k}}{(2+k)\rho^k} - \frac{\ell^{2+\kappa}}{(2+\kappa)\rho^\kappa}.$$

The first-order conditions for this maximization can be written as

$$\ell\rho^k = r^{1+k} \qquad (12)$$

$$r\rho^\kappa = \ell^{1+\kappa}. \qquad (13)$$

The efficient choices are independent of $k$ and $\kappa$, and researcher $\rho$ to choose attribute $r = \rho$ and for laboratory $\lambda$ to choose attribute $\ell = \lambda$. We denote the efficient choices by $\hat{r}^e$ and $\hat{\ell}^e$.

### 4.3 Incomplete Information Matching Equilibrium

The market is characterized by a price function $\hat{p}$, with $\hat{p}(\ell)$ identifying the price at which any research can buy a match with attribute $\ell$. Given the price function $\hat{p}$, researcher $\rho$ chooses $(r, \ell)$ to maximize

$$\theta\ell r - \hat{p}(\ell) - c(r, \rho).$$

We denote by $\hat{r}_R, \hat{\ell}_R : [0, 1] \to \mathbb{R}_+$ the functions describing the researcher and laboratory attributes selected by researchers.

Laboratories choose attributes given $(\hat{p}, \hat{r}_L)$, where $\hat{r}_L : \mathbb{R}_+ \to \mathbb{R}_+$ is the *matching function* that specifies the attribute $\hat{r}_L(\ell)$ of the researcher that the market matches to a laboratory with attribute $\ell$. Laboratory $\lambda$ chooses $\ell \in \mathbb{R}_+$ to maximize

$$(1-\theta)\ell\hat{r}_L(\ell) + \hat{p}(\ell) - \psi(\ell, \lambda).$$

We denote by $\hat{\ell}_L : [0,1] \to \mathbb{R}_+$ the function describing the laboratories' attribute choices.

The specification of a matching equilibrium is similar to that of Section 3, though we must now specify investments on both sides of the market.[15]

**Definition 3** *A price function $\hat{p}$, matching function $\hat{r}_L$, and strictly increasing attribute choices $(\hat{r}_R, \hat{\ell}_R, \hat{\ell}_L)$ constitute a* matching equilibrium *if*

1. *$(\hat{r}_R(\rho), \hat{\ell}_R(\rho))$ is an optimal pair of attribute choices for researcher $\rho$, for all $\rho \in [0,1]$,*
2. *$\hat{\ell}_L(\lambda)$ is an optimal laboratory attribute for laboratory $\lambda$, for all $\lambda \in [0,1]$,*
3. *every researcher and laboratory earns nonnegative payoffs, and*
4. *markets clear: $\hat{r}_L(\hat{\ell}_R(\rho)) = \hat{r}_R(\rho)$ for all $\rho \in [0,1]$ and $\hat{\ell}_R(\lambda) = \hat{\ell}_L(\lambda)$ for all $\lambda \in [0,1]$.*

Appendix A contains the calculations behind the following:

**Proposition 5** *A matching equilibrium is given by the collection $(\hat{p}, \hat{r}_L, \hat{r}_R, \hat{\ell}_R, \hat{\ell}_L)$, where*

$$\begin{aligned}
\hat{p}(\ell) &= \frac{1}{2}\theta\frac{\zeta}{\xi}\ell^2, \\
\hat{r}_L(\ell) &= \frac{\zeta}{\xi}\ell, \\
\hat{r}_R(\rho) &= \zeta\rho, \\
\hat{\ell}_R(\rho) &= \xi\rho, \quad \textit{and} \\
\hat{\ell}_L(\lambda) &= \xi\lambda,
\end{aligned}$$

[15] As in the initial model, we are able to avoid many technical details. In particular, our notion of equilibrium assumes that $\hat{\ell}_R$ and $\hat{\ell}_L$ are strictly increasing; these properties can be deduced from the general model of Mailath, Postlewaite, and Samuelson (2013). Given these assumptions, market clearing requires $\hat{r}_L(\hat{\ell}_R(\rho)) = \hat{r}_R(\rho)$ and $\hat{\ell}_R(\lambda) = \hat{\ell}_L(\lambda)$.

In the equilibrium we analyze, the range of $\ell_R$ is an interval starting at 0, and so we need place no further restrictions on $\hat{r}_L$ (though setting $\hat{r}_L(\ell) = \hat{r}_R(1)$ for $\ell > \hat{\ell}_R(1)$ would be natural). A central concern of Mailath, Postlewaite, and Samuelson (2013) is the appropriate treatment of matches when an attribute is chosen outside the range of putative equilibrium attributes and the set of such attributes does not form an interval.

*with the constants $\zeta$ and $\xi$ given by*

$$\zeta = \theta^{\frac{\kappa+1}{k\kappa+k+\kappa}}(2-\theta)^{\frac{1}{k\kappa+k+\kappa}}$$

$$\textit{and} \qquad \xi = \theta^{\frac{1}{k\kappa+\kappa+k}}(2-\theta)^{\frac{k+1}{k\kappa+k+\kappa}}.$$

*Equilibrium payoffs are given by*

$$\hat{u}_R(\theta,k,\kappa,\rho) = \frac{k}{2(2+k)}\theta^{\frac{(\kappa+1)(k+2)}{k\kappa+k+\kappa}}(2-\theta)^{\frac{k+2}{k\kappa+k+\kappa}}\rho^2$$

*and*

$$\hat{u}_L(\theta,k,\kappa,\lambda) = \frac{\kappa}{2(2+\kappa)}\theta^{\frac{\kappa+2}{k\kappa+k+\kappa}}(2-\theta)^{\frac{(k+1)(\kappa+2)}{k\kappa+k+\kappa}}.$$

We now compare the equilibrium investments to the efficient investments $\hat{r}^e(\rho) = \rho$ and $\hat{\ell}^e(\lambda) = \lambda$. It is a straightforward calculation that $\zeta < 1$ as long as $\theta < 1$. Hence, once again, researchers underinvest. Underinvestment on the part of researchers is a robust result, and reflects familiar holdup reasoning. As long as $\theta < 1$, the researcher does not capture all of the marginal gain of an investment, and so underinvests.

The situation for laboratories is more involved. For small values of $\theta$, we have $\xi < 1$, and hence laboratories also underinvest. Researchers' investments are very small when $\theta$ is small, and hence so are the returns to laboratories' investments. As a result, laboratories underinvest. However, for larger $\theta$, we have $\xi > 1$, and laboratories overinvest relative to the efficient level. The boundary value of $\theta$ above which laboratories overinvest depends only on $k$, the parameter of the *researchers'* cost function, and decreases as $k$ increases (see Figure 3). Thus, laboratories overinvest when researchers are more heterogeneous.

Laboratories overinvest because of researchers' response to their investments. Consider laboratory $\lambda$'s equilibrium investment. It is higher than the efficient level, so why doesn't the laboratory decrease its investment?

In the calculation of the efficient investment level, we know that an efficient outcome must match agents assortatively on index. If a laboratory's investment is too high, we can decrease the investment *keeping the matching fixed*, and thereby increase the surplus. In contrast, in the market equilibrium, a laboratory that decreased its investment level from the equilibrium level would find that the researcher's attribute that the laboratory is matched with decreases. It is this concern for the quality of the researcher (which it doesn't observe) with whom it is matched that makes it optimal for laboratories to invest more than the efficient level.

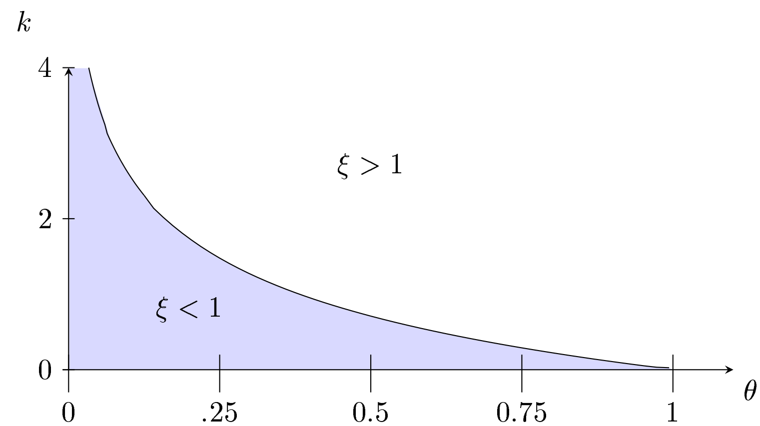


Figure 3: Laboratories overinvest when researchers are more heterogeneous (i.e., for sufficiently large $k$). Laboratories overinvest if $\xi > 1$ and underinvest if $\xi < 1$.

The intuition for laboratories' overinvestment is quite general, as long as laboratories' premuneration values increase with the attribute of their matched partner. Laboratories want higher-attribute researchers, and are willing to pay for them. But when they cannot directly observe researchers' attributes, they cannot simply pay for higher-attribute researchers by accepting lower prices to match, since that would be equally attractive to all researchers. But they can increase their attractiveness to matched partners by investing more. This makes a laboratory more attractive to all researchers, but more so for higher attribute researchers. Hence, a laboratory can combine an increase in their attribute with an increase in their price that will screen potential researchers so that only higher attribute researchers will find the combination attractive.

## 4.4 The Effects of Competition

As $k$ increases, the range of values of $\theta$ for which laboratories overinvest expands. This naturally directs our attention to the effect on investment incentives as $k$ and $\kappa$, and hence the degree of competition between researchers and laboratories, vary.

First, we hold $k$ fixed and consider the limits with respect to $\kappa$:

$$\lim_{\kappa\to\infty}\zeta=\theta^{1/(1+k)}, \qquad \lim_{\kappa\to 0}\zeta=[\theta(2-\theta)]^{1/k},$$
$$\lim_{\kappa\to\infty}\xi=1, \qquad \text{and} \qquad \lim_{\kappa\to 0}\xi=\theta^{1/k}(2-\theta)^{(k+1)/k}.$$

As $\kappa$ grows large, the cost function for laboratory $\lambda$ becomes increasing sharply curved around the attribute $\ell=\lambda$, and the attribute cost function approaches a function that is kinked at the value $\ell=\lambda$. This pushes laboratory $\lambda$'s attribute choice to $\ell=\lambda$, and returns us to the one-sided investment case of Section 3. The value $\zeta$ approaches $\theta^{1/(1+k)}$, duplicating the solution from Section 3.

As $\kappa\to 0$, the laboratories become more homogeneous. The difference in limit behavior of the researchers reflects the limit behavior of the laboratories in the following sense: if $k$ is such that the laboratories overinvest (relative to the efficient level), then $\lim_{\kappa\to\infty}\zeta<\lim_{\kappa\to 0}\zeta$, and conversely. Moreover, as laboratories become more homogeneous, their equilibrium payoffs converge to zero as competition allows the researchers to capture all the surplus.

Second, we fix $\kappa$ and take limits with respect to $k$, giving

$$\lim_{k\to\infty}\zeta=1, \qquad \lim_{k\to 0}\zeta=\theta^{(\kappa+1)/\kappa}(2-\theta)^{1/\kappa},$$
$$\lim_{k\to\infty}\xi=(2-\theta)^{1/(1+\kappa)} \qquad \text{and} \qquad \lim_{k\to 0}\xi=[\theta(2-\theta)]^{1/\kappa}.$$

In this case, as $k\to\infty$, researcher $\rho$'s investment approaches $r=\rho$ (as $\lim_{k\to\infty}\zeta=1$), and we obtain the one-sided investment case in which researcher attributes are fixed and laboratories invest. Moreover, $\lim_{k\to\infty}\xi=(2-\theta)^{1/(1+k)}>1$, and hence laboratories overinvest. Again, we see that the inefficiency arises our of the unobservability of researchers' attributes rather than the nature of investment. However, when only laboratories invest, we have overinvestment, arising out of laboratories' attempts to attract better researchers (at higher prices) by increasing their investments.

As researchers become homogenous (i.e., $k\to 0$), laboratories necessarily underinvest (consistent with our intuition from the previous section).

Third, we take limits as $k=\kappa$, giving

$$\lim_{k=\kappa\to\infty}\zeta=1, \qquad \lim_{k=\kappa\to 0}\zeta=0,$$
$$\lim_{k=\kappa\to\infty}\xi=1, \qquad \text{and} \qquad \lim_{k=\kappa\to 0}\xi=0.$$

Here we approach the efficient outcome as $k=\kappa$ gets large and no investments as $k=\kappa$ vanish. However, care should be taken in the interpretation of these limits, since the order matters. For example, $\lim_{\kappa\to\infty}\lim_{k\to\infty}\zeta = 1 \neq 0 = \lim_{k\to\infty}\lim_{\kappa\to\infty}\zeta$. The first order reflects a scenario where researchers are more heterogenous than laboratories, and the second the reverse.

## 5 Discussion

Our basic result is that researchers underinvest. To keep our discussion simple, we focus on the case in which researchers only invest. In equilibrium, researcher $\rho$ chooses investment

$$r_R(\rho) = \rho \cdot \theta^{\frac{1}{1+k}}, \tag{14}$$

while the efficient investment is $r = \rho$. It is apparent from (14) that the underinvestment problem is most severe in markets in which researchers' premuneration values are relatively low ($\theta$ is small), and researchers are relatively homogeneous and consequently compete aggressively ($k$ is small). An important part of researchers' premuneration value is the human capital they acquire in the course of a match. We can accordingly expect underinvestment to be especially problematic in occupations in which researchers (workers) acquire relatively little human capital. Similarly, for given premuneration values, underinvestment will be severe in occupation in which workers learn a specific skill (e.g., passing a certification exam that is a prerequisite for performing some duty), and workers who have acquired that skill are largely substitutes. In this case, those who invest will compete away the benefits of the acquired skill, with the benefits of the increased efficiency accruing to the firms.

Our model not only indicates when underinvestment is likely to be particularly problematic, but also allows us to examine how one might address this underinvestment. We first consider subsidizing investments. An investment subsidy is a program that transforms a researcher's index $\rho$ into the index $\hat{\rho}$, where

$$\hat{\rho} = b_0 + b_1\rho.$$

If $\hat{\rho} > \rho$, then the cost of investment for this researcher has been decreased. Setting $b_0 = 0$ has the effect of multiplying the cost function by $b_1^{-k}$, and hence gives simply a proportionate reduction in every cost. Setting $b_0 > 0$ and letting $b_1$ diminish reduces the sensitivity of costs to the type of researcher. Indeed, as $b_1 \to 0$, the researchers become identical, while as $b_1 \to \infty$, the researchers become arbitrarily heterogeneous.

Policies that increase $b_0$ confer a cost reduction that is not directly related to a researcher's type. We might think of the investment costs as reflecting primarily the costs of higher education, including monetary costs, opportunity costs, and the disutility of attached to seemingly endless problem sets. Policies to decrease the costs across the board might include tuition subsidies, free access to junior colleges for all, subsidized loans to replace lost earnings, and enhanced K-12 education programs to make the work more bearable.[16]

Policies that increase $b_1$ have a differential impact on high-ability researchers. These might include steps that make a college education more effective, such as acquiring new technology for teaching or adopting effective teaching methods, creating internship and research programs, enhancing libraries, and so on. These policies may have effects on all, but may benefit some researchers more than others. Suppose, for example, that investment consists of sitting in the library and memorizing formulas. Suppose that all researchers are equally proficient at the task—everyone can memorize the same number of formulas per hour—but that researchers vary in the disutility they get from sitting in the library and memorizing. Then making the library more pleasant, perhaps by installing comfortable chairs, free coffee, more flexible opening hours, and air conditioning, will lower the cost of investing to all workers, but will do so differentially. The workers who invest most (those who spend the most time in the library) will be subsidized more, with a greater efficiency gain than if the extra cost were somehow distributed evenly across all workers. Alternatively, suppose researchers differ in ability to memorize formulas, with the higher ability researchers able to memorize more per hour than the lower ability, and suppose also that all researchers have the same opportunity cost of time. Here, merit-based scholarships will better target the higher investing workers than increased amenities.

One might also encourage researcher investments by enhancing laboratories' attributes. Larger values of $\ell$ increase the surplus, in turn giving rise to enhanced incentives for researchers to invest. We could accordingly think of programs that transform laboratory investment $\ell = \lambda$ into $\hat{\ell}$, where

$$\hat{\ell} = a_0 + a_1 \lambda.$$

As $a_1 \to 0$, the laboratories become more homogenous, while as $a_1 \to \infty$, the laboratories become arbitrarily heterogeneous. One might think of subsi-

[16]Increasing $b_0$ makes researchers more homogeneous, leading to enhanced competition that shifts payoffs towards laboratories. There is then an argument for taxing laboratories to pay for the subsidies.

dizing the computerization of laboratory operations, or supporting research into new techniques.

Since we continue to have single crossing, market clearing still requires researcher $\rho$ be matched with laboratory $\lambda$, which implies that researcher of type $\hat{\rho}$ be matched with laboratory with attribute

$$\hat{\ell} = a_0 - \frac{a_1 b_0}{b_1} + \frac{a_1}{b_1}\hat{\rho} =: \alpha + \beta\hat{\rho}.$$

It is straightforward to verify that the matching equilibrium researcher attribute choice function is given by

$$\hat{r}_R(\hat{\rho}) = \theta^{1/(1+k)} r^e(\hat{\rho}),$$

where

$$\hat{r}^e(\hat{\rho}) := [(\alpha + \beta\rho)\rho^k]^{1/(1+k)}$$

is the efficient attribute choice.

As one would expect, changing the costs of investments changes researchers' efficient attribute choice. However subsidizing either researcher investment costs or laboratory attributes does not affect the relative underinvestment of researchers in the matching equilibrium. One may identify other reasons for subsidizing investments, but closing the equilibrium inefficiency in investments is not one of them.

One might focus on the outputs rather than inputs of a match, considering taxes and subsidies on the portions of the surplus accruing to the researcher and laboratory. Indeed, the unfettered ability to tax or subsidize premuneration values provides a perfect remedy for underinvestment. One need only impose a hundred-percent tax on laboratories' premuneration values, with a corresponding subsidy on researchers' premuneration values, in order to ensure efficient investments. A tax of one hundred percent sounds draconian, but notice that if $k$ is not too large (i.e., researchers are not too heterogeneous, which is precisely the circumstances in which underinvestment is particularly problematic), then laboratory payoffs under such a program will be very close to the that which they would receive if $\theta$ was set so as to maximize their payoff.

The difficulty is that the same legal and institutional obstacles the preclude arbitrarily rearranging premuneration values may make it impractical to tax them. The government may find it difficult to tax human capital, Instead, it is likely that we can tax some but not all of the factors that determine $\theta\ell r - p(\ell)$ for the researcher and $(1-\theta)\ell r + p(\ell)$ for the laboratory. Examining this optimal tax problem would require a yet more detailed model, delving into the details of how premuneration values are determined. Notice, however, that the basic problem is to increase investments. A proportional subsidy to the payoff $\theta\ell r - p(\ell)$ is equivalent to a cost subsidy, and hence (as we have seen) will not close the efficiency gap. Instead, we will need subsidies to researchers whose marginal value increases as premuneration values increase, building in a degree of regressiveness.

We have obtained clean comparative static results with respect to important aspects of prematch investment behavior by putting structure on the surplus function, premuneration values and cost functions. It is clear that the precise nature of the results depends on that structure. However, the value of the model extends beyond these particular results, with the framework underlying the analysis allowing us to identify which aspects of a particular problem can be important, and how they are likely to qualitatively affect investment decisions.

# A Proofs

**Proof of Lemma 1.** En route to a contradiction, suppose $p$ is not strictly increasing. Then there are two laboratories $\ell' < \ell$ satisfying $p(\ell') \geq p(\ell)$. But then no researcher will choose laboratory $\ell'$ — why pay just as much or more for an inferior laboratory? Hence, $\ell_R$ cannot be market clearing. Continuity similarly follows from the observation that if the function $p$ takes an upward jump at $\ell'$, then there will be an interval of laboratories $(\ell', \ell'+\varepsilon)$ that will be unchosen by researchers, again contradicting our assumption that $\ell_R$ is market clearing. ∎

**Proof of Lemma 2.** We first argue that $r_R$ is weakly increasing. Suppose not, so that there exist researchers $\hat{\rho} > \rho$ such that $\hat{r} = r_R(\hat{\rho}) < r_R(\rho) = r$. Since researchers are optimizing in their attribute and laboratory choices,

$$h_R(r, \ell_R(\rho)) - p(\ell_R(\rho)) - c(r, \rho) \;\geq\; h_R(\hat{r}, \ell_R(\hat{\rho})) - p(\ell_R(\hat{\rho})) - c(\hat{r}, \rho)$$

and

$$h_R(\hat{r}, \ell_R(\hat{\rho})) - p(\ell_R(\hat{\rho})) - c(\hat{r}, \hat{\rho}) \;\geq\; h_R(r, \ell_R(\rho)) - p(\ell_R(\rho)) - c(r, \hat{\rho}),$$

which when added together, gives

$$c(r, \rho) + c(\hat{r}, \hat{\rho}) \;\leq\; c(\hat{r}, \rho) + c(r, \hat{\rho}),$$

a contradiction.

We now argue that $r_R$ is strictly increasing. If $r_R$ is not strictly increasing, there exist $\hat{\rho} > \rho$ such that $\ell_R(\hat{\rho}) = \hat{\ell}$ and $\hat{r} = r_R(\hat{\rho}) = r = r_R(\rho)$, and so $\hat{\ell}$ is an optimal choice for both $\hat{\rho}$ and $\rho$ at $r$. Since $\ell_R$ is market clearing, we can assume $\ell_R(\hat{\rho}) = \hat{\ell} > 0$. But this implies that $\rho$'s choice of $r = \hat{r}$ must be optimal given $\hat{\ell}$. But this is impossible (since the marginal cost of attributes is strictly decreasing in $\rho$). ■

**Proof of Lemma 3.** We first argue that in equilibrium, the researcher laboratory-choice function is strictly increasing. Let $\hat{\rho} > \rho$ and hence, from Lemma 2, $\hat{r} = r_R(\hat{\rho}) > r_R(\rho) = r$. We need to show that $\hat{\ell} = \ell_R(\hat{\rho}) > \ell_R(\rho) = \ell$. Suppose this is not the case. Then since researchers with attributes $r$ and $\hat{r}$ are optimizing in their choice of laboratories, we have

$$\begin{aligned} & h_R(r,\ell) - p(\ell) \geq h_R(r,\hat{\ell}) - p(\hat{\ell}) \\ \text{and}\quad & h_R(\hat{r},\hat{\ell}) - p(\hat{\ell}) \geq h_R(\hat{r},\ell) - p(\ell), \end{aligned}$$

which when added, give

$$h_R(r,\ell) + h_R(\hat{r},\hat{\ell}) \geq h_R(r,\hat{\ell}) + h_R(\hat{r},\ell),$$

which is impossible if $\hat{\ell} \leq \ell$ when $\hat{r} > r$.

The conclusion of the lemma then follows from equilibrium $\ell_R$ being a strictly increasing and measure-preserving map from $[0,1]$ onto $[0,1]$: Fixing $\rho \in [0,1]$, and recalling footnote 9, we have $\mu\{\ell | \ell = \ell_R(\hat{\rho}) \text{ for } \hat{\rho} \in [0,\rho]\} = \mu([0,\ell_R(\rho)]) = \ell_R(\rho)$, and so $\rho = \mu([0,\rho]) = \ell_R(\rho)$. ■

**Proof of Proposition 5.**

We conjecture that there is an equilibrium in linear strategies:

$$\hat{r}_R(\rho) = \zeta\rho \quad \hat{\ell}_R(\rho) = \xi\rho, \quad \text{and} \quad \hat{\ell}_L(\lambda) = \xi\lambda.$$

The last equality of the coefficients in $\hat{\ell}_R$ and $\hat{\ell}_L$ comes from market clearing. The first order conditions are

$$\theta\ell = \frac{r^{1+k}}{\rho^k}, \tag{A.1}$$

$$\theta r = \hat{p}'(\ell), \quad \text{and} \tag{A.2}$$

$$(1-\theta)[\hat{r}(\ell) + \ell\hat{r}'(\ell))] + \hat{p}'(\ell) = \frac{\ell^{\kappa+1}}{\lambda^\kappa}. \tag{A.3}$$

We can eliminate $\hat{p}'$ by combining (A.2) and (A.3) to get

$$(1-\theta)[\hat{r}(\ell) + \ell\hat{r}'(\ell))] + \theta\hat{r}(\ell) = \frac{\ell^{\kappa+1}}{\lambda^\kappa}. \tag{A.4}$$

Substituting the conjectured linear equilibrium functional forms into (A.1) gives

$$\theta\xi\rho = \frac{(\zeta\rho)^{1+k}}{\rho^k} \implies \theta\xi = \zeta^{1+k}.$$

From (A.4), we get

$$\begin{aligned}(1-\theta)2\frac{\zeta}{\xi} + \frac{\theta\zeta}{\xi} &= \frac{(\xi\lambda)^\kappa}{\lambda^\kappa} \\ \implies (2-\theta)\zeta &= \xi^{\kappa+1} \\ \implies (2-\theta)\zeta &= \left(\frac{\zeta^{1+k}}{\theta}\right)^{\kappa+1} \\ \implies \zeta^{k\kappa+k+\kappa} &= \theta^{\kappa+1}(2-\theta)\end{aligned}$$

which implies

$$\zeta = \theta^{\frac{\kappa+1}{k\kappa+k+\kappa}}(2-\theta)^{\frac{1}{k\kappa+k+\kappa}}, \tag{A.5}$$

and so

$$\begin{aligned}\xi = \frac{\zeta^{k+1}}{\theta} &= \frac{\theta^{\frac{(\kappa+1)(k+1)}{k\kappa+k+\kappa}}(2-\theta)^{\frac{k+1}{k\kappa+k+\kappa}}}{\theta} \\ &= \theta^{\frac{1}{k\kappa+\kappa+k}}(2-\theta)^{\frac{k+1}{k\kappa+k+\kappa}}.\end{aligned} \tag{A.6}$$

The expressions for equilibrium payoffs are the result of a straightforward calculation. ■

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# Premuneration Values and Investments in Matching Markets

## Supplementary Appendix: Endogenizing Information

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## S.1 Introduction

Our analysis has assumed that laboratories could not learn researchers' attributes. This section examines the researcher-investment case on which the paper is focussed, but allows laboratories to learn the attributes of researchers at a cost. We will see that changes in premuneration values can have surprising effects on which laboratories become informed and on the resulting division of the surplus.

## S.2 Endogenous-Information Equilibria

We suppose that, by incurring a cost $\kappa > 0$, any given laboratory can acquire the ability to observe the attribute of each researcher. We can think of $\kappa$ as the cost of hiring an agent who can test any applicant or the cost of installing a testing procedure. Assume that laboratories make their decisions of whether to become informed and researcher choose their investments simultaneously.[4]

If $\kappa$ is sufficiently large, the gain in efficiency would not warrant a laboratory incurring the cost to become informed. On the other hand, for $\kappa$ small, it is generally not an equilibrium for all laboratories to remain uninformed. To illustrate, suppose all laboratories are uninformed and that researchers choose attributes according to (8). If a laboratory deviates by becoming informed, it then can target any available researcher attribute, i.e., any attribute in the set $[0, r_R(1)] = [0,\ \theta^{1/(1+k)}]$. Suppose a laboratory

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[4]This ensures that a laboratory cannot induce a change in researcher investment behavior by deciding to become informed.

$\lambda < \theta^{\frac{1}{1+k}}$ with (by assumption) $\ell = \lambda$ becomes informed and then offers a price $p$ to the researcher with attribute $r = \ell$, i.e., to a researcher of type $\rho = \lambda\theta^{-1/(1+k)}$.[5] Since the price is simply a transfer between the two agents, such an offer is a profitable deviation if and only if the surplus generated by the resulting match, $\ell r - \kappa$ exceeds the ex post equilibrium payoffs of the two agents (using (10)-(11) for the first equality):

$$\begin{aligned}
&u_L(\theta, k, \lambda) + u_R(\theta, k, \lambda\theta^{-1/(1+k)}) + c(\lambda, \lambda\theta^{-1/(1+k)}) \\
&\qquad = \left[\frac{1}{2}\theta^{1/(1+k)}(2-\theta) + \frac{k}{2(2+k)}\theta^{k/(1+k)} + \frac{1}{(2+k)}\theta^{k/(1+k)}\right]\lambda^2 \\
&\qquad = \frac{\theta^{1/(1+k)}}{2}\left[2 - \theta + \theta^{(k-1)/(1+k)}\right]\lambda^2 =: g(\theta)\lambda^2.
\end{aligned}$$

A straightforward calculation verifies the inequality $g(\theta) < 1$ for all interior $\theta$ (in particular, $g(1) = 1$, $g'(1) = 0$ and $g$ is concave).

Thus for $\kappa > 0$ but not too large, in equilibrium, some laboratories will choose to become informed. However, it is clear that not *all* laboratories will choose to become informed, since laboratories with types near 0 cannot under any circumstance generate sufficient surplus to cover the cost $\kappa$.

A natural hypothesis is that for positive but not too large $\kappa$, there will be a *hybrid* equilibrium characterized by a threshold $\widetilde{\lambda}$ with laboratories $\lambda > \widetilde{\lambda}$ incurring the cost to become informed and laboratories with $\lambda < \widetilde{\lambda}$ not incurring the cost.

In such an equilibrium, informed laboratories are priced by a function $\hat{p} : [\widetilde{\lambda}, 1] \times \mathbb{R}_+ \to \mathbb{R}_+$, where $\hat{p}(\ell, r)$ is the price paid by researcher of attribute $r$ to laboratory of attribute $\ell$. We extend $\hat{p}$ to $[0, 1] \times \mathbb{R}_+$ to cover uninformed laboratories by requiring $\hat{p}$ to be independent of $r$ for $\ell < \tilde{\lambda}$. Researcher $\rho$ maximizes

$$\max_{\ell, r} \theta\ell r - \hat{p}(\ell, r) - \frac{r^{2+k}}{(2+k)\rho^k}. \tag{S.1}$$

**Definition S.1** *A price function $\hat{p}$, cutoff $\tilde{\lambda} \in [0, 1]$, and researcher choices $(\ell_R, r_R)$ constitute a* hybrid equilibrium *if*

1. *for every $\rho \in [0, 1]$, the choice $(\ell_R(\rho), r_R(\rho))$ solves (S.1),*
2. *for every $\ell \in [0, \tilde{\lambda}]$, for all $r$ and $r'$, $\hat{p}(\ell, r) = \hat{p}(\ell, r')$,*
3. *no laboratory $\lambda \in [1, \tilde{\lambda})$ strictly prefers to be informed at a cost of $\kappa$,*

[5]The bound $\lambda < \theta^{1/(1+k)}$ ensures that $r = \ell$ is feasible, i.e., $r < \theta^{1/(1+k)}$.

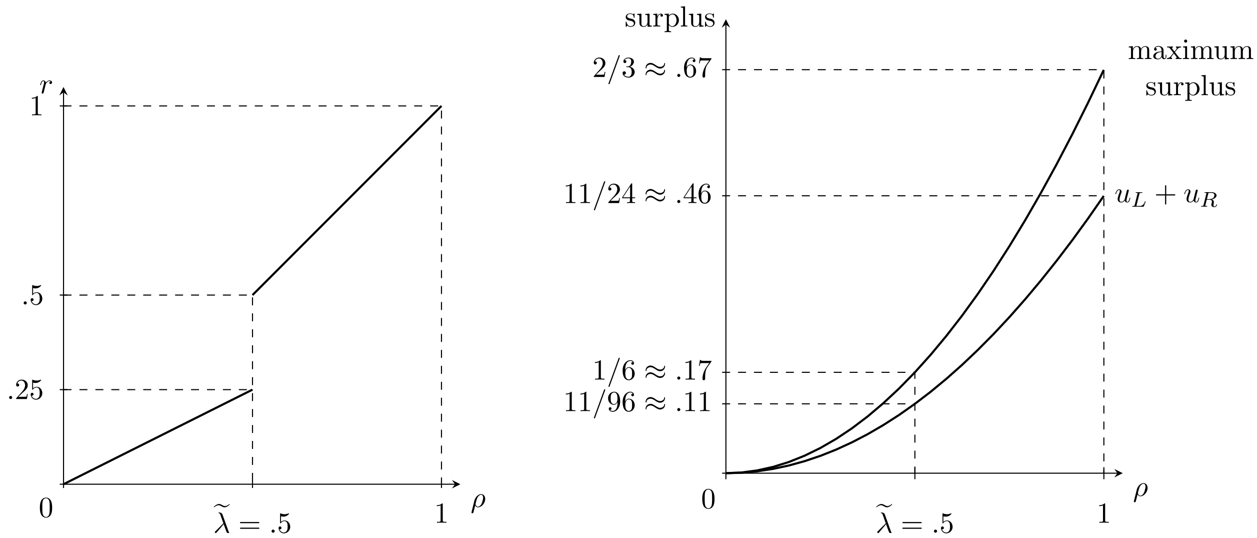


Figure S.1: The researcher attribute choice function for the case $k = 1$, $\kappa = \frac{5}{96}$, $\theta = \frac{1}{4}$, and $\widetilde{\lambda} = \frac{1}{2}$ is illustrated on the left. At $\widetilde{\lambda} = \frac{1}{2}$, the efficiency gain from efficient investments equals $\frac{5}{96}$.

4. *no laboratory $\lambda \in [\tilde{\lambda}, 1]$ strictly prefers to be uninformed,*

5. *every researcher and laboratory earns nonnegative payoffs, and*

6. *$\ell_R$ is market-clearing.*

Intuitively, higher type researchers choose higher attributes, and match with higher attribute laboratories. Market clearing then implies that researchers $\rho \in [\tilde{\lambda}, 1]$ match with informed laboratories and so choose efficient investments.

## S.3 An Example

We present here a hybrid equilibrium for the case in which $\kappa = \frac{5}{96}$, $\theta = \frac{1}{4}$ and $k = 1$ (thus $\theta^{\frac{1}{1+k}} = \frac{1}{2}$), and with switch point $\widetilde{\lambda} = \frac{1}{2}$, and then examine its comparative statics. See Section S.5 for the analysis of general parameter values that underlies our discussion here.

The left panel of Figure S.1 shows the researchers' investment levels, which jump at $\widetilde{\lambda}$ as researchers switch from the investments appropriate for matching with uninformed laboratories (described in (8)) to the efficient levels appropriate for matching with informed laboratories. Despite this discontinuity in investments, the payoffs of both researchers and laboratories must be continuous as their indices move across index $\widetilde{\lambda}$, since otherwise an agent just on the low-payoff side of $\widetilde{\lambda}$ would have an incentive to make the same investment as that of an agent just on the other side (high-payoff) of $\widetilde{\lambda}$. This joint indifference implies that at the switch point $\widetilde{\lambda}$ the gain in surplus equals the cost $\kappa$ of becoming informed. Figure S.1 (right panel) shows that the threshold pair $\tilde{\lambda} = \frac{1}{2}$ gives an efficiency gain of $\frac{5}{96}$, which equals the assumed value of $\kappa$.

The equilibrium price function is given by

$$\hat{p}(\ell, r) = \begin{cases} \frac{1}{16}\ell^2, & \text{if } \ell < \frac{1}{2}, \\ \frac{1}{2}\ell^2 - \frac{3}{4}r\ell + \frac{7}{192}, & \text{if } \ell \geq \frac{1}{2}. \end{cases}$$

For $\ell < \frac{1}{2}$, $\hat{p}(\ell, r)$ is given by (9), while for $\ell \geq \frac{1}{2}$, the price function is determined by the requirement that payoffs are continuous at $\frac{1}{2}$ and that efficient investments are optimal for the high index researchers. A researcher choosing an uninformed laboratory $\ell = \frac{1}{2}$ pays a price of $\frac{1}{64}$. The price paid by a researcher choosing $r = \frac{1}{2}$ to an informed laboratory $\ell = \frac{1}{2}$ is lower, taking the *negative* value of $-\frac{5}{192}$, compensating the researcher for the upward jump in investment from $\frac{1}{4}$ to $\frac{1}{2}$. Figure S.2 illustrates the resulting payoff functions, which have a kink but not a discontinuity at $\frac{1}{2}$.

## S.4 Comparative Statics

If the fixed cost of information $\kappa$ decreased, the threshold $\widetilde{\lambda}$ that determines which laboratories decide to invest would decrease, until the net surplus increase that is a consequence of the threshold laboratory's becoming informed again equals $\kappa$.

More interesting is the role of premuneration values in determining who becomes informed, and the resulting payoffs. As $\theta$ decreases, researchers' investments decrease, and hence the inefficiency associated with any matched pair increases. The threshold for laboratories to become informed must then decrease, in order for the gain from becoming informed to be equal to $\kappa$. Hence, the extent of information acquisition increases as the researchers' premuneration value share decreases.

Not only does the threshold change in response to changes in $\theta$, but the division of the surplus between laboratories and researchers is affected. If all laboratories are informed (such as would arise if $\kappa = 0$), investments

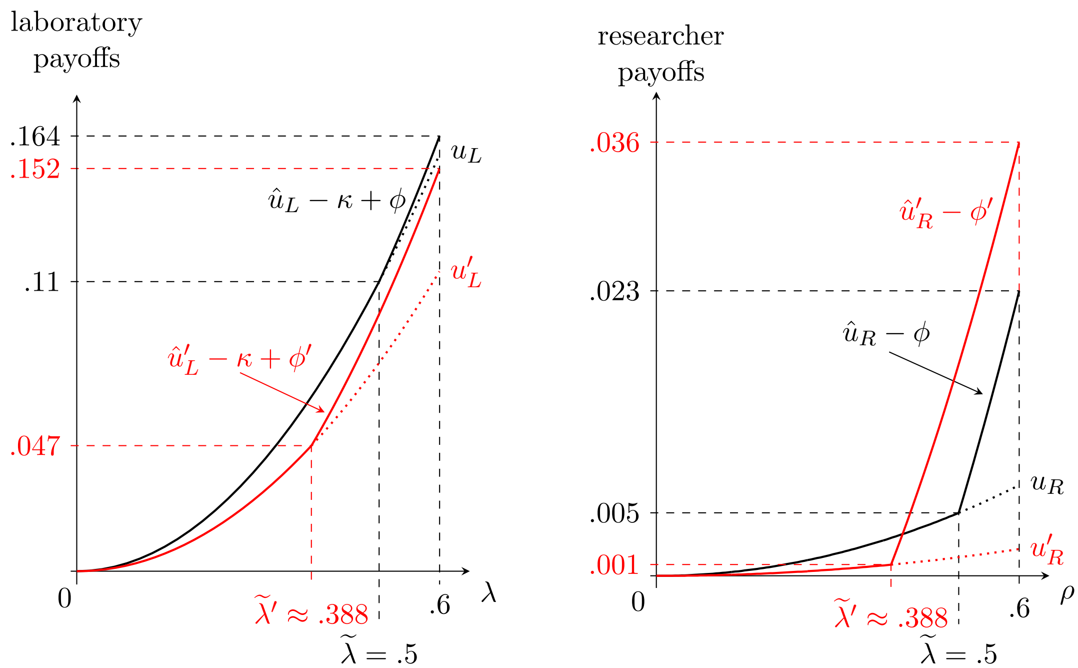


Figure S.2: Payoffs in the hybrid equilibrium for the case $k = 1$, for $\lambda \leq .6$. The cost of becoming informed is $\kappa = \frac{1}{6} - \frac{11}{96} = \frac{5}{96}$. Two values of $\theta$ are illustrated, $\theta = \frac{1}{4}$ (which implies $\widetilde{\lambda} = \frac{1}{2}$) and $\theta = \frac{1}{9}$ (which implies $\widetilde{\lambda}' \approx .388$). The expressions for $\theta = \frac{1}{9}$ are indicated by a prime. For $\lambda$ below $\widetilde{\lambda}$, laboratory payoffs are given by $u_L$, while for indices above $\widetilde{\lambda}$, they are given by $\hat{u}_L - \kappa + \phi$. For $\rho$ below $\widetilde{\lambda}$, researcher payoffs are given by $u_R$, while for indices above $\widetilde{\lambda}$, they are given by $\hat{u}_R - \phi$. The constant in the price function (S.4) is $\phi = \frac{7}{192}$ for $\theta = \frac{1}{4}$, and $\phi' = \frac{65}{2688} \approx .024$ for $\theta = \frac{1}{9}$.

are efficient and laboratory and researcher payoffs are *independent* of $\theta$. In contrast, when $\kappa > 0$, as illustrated in Figure S.2, the premuneration values affect the location of the threshold, and so affect all agents' payoffs, including those involving fully informed laboratories. For example, under the lower premuneration value share of $\theta = \frac{1}{9}$, all researchers matched with uninformed laboratories have a lower payoff than under $\theta = \frac{1}{4}$. However, all researchers matched with informed laboratories under $\theta = \frac{1}{4}$ are strictly better off under the lower premuneration value share of $\theta = \frac{1}{9}$. Moreover, all laboratories prefer the scenario of the higher researcher premuneration value share of $\frac{1}{4}$.

Finally, hybrid equilibria do not exist for all parameters, and in particular do not exist if researchers' premuneration values are too large. If we fix $\kappa$, laboratories close to $\lambda = 1$ will have vanishingly small possible gains from acquiring information as $\theta$ goes to 1, and hence will choose *not* to become informed.

## S.5 Calculations

This section presents the calculations behind the hybrid equilibrium of the preceding sections. Let

$$\bar{\kappa}(\theta) := \frac{1}{(2+k)}\left[1 + k - (2+k)\theta^{1/(1+k)} + \theta^{(2+k)/(1+k)}\right].$$

Since $\bar{\kappa}(1) = 0$ and $\kappa'(\theta) < 0$, we have $\bar{\kappa}(\theta) > 0$ for all $\theta \in [0, 1)$.

**Proposition S.1** *Suppose*

$$2(2-\theta)\bar{\kappa}(\theta) > \theta^{1/(1+k)}(1-\theta)^2. \tag{S.2}$$

*For any* $\kappa \in (\theta^2\bar{\kappa}(\theta), \bar{\kappa}(\theta))$*, satisfying*

$$\frac{1}{2} \geq \kappa\left[1 - \frac{1}{2\bar{\kappa}(\theta)}\right] + \sqrt{\frac{\kappa}{\bar{\kappa}(\theta)}}, \tag{S.3}$$

*there exists a hybrid equilibrium with switch point*

$$\widetilde{\lambda} = \sqrt{\frac{\kappa}{\bar{\kappa}(\theta)}}.$$

For all $\theta$, the condition (S.2) fails for sufficiently large $k$. Since, $\lim_{k\to\infty} \bar{\kappa}(\theta) = 0$, as researcher heterogeneity becomes large (and so researcher choices become efficient), the critical cost of becoming informed must converge to zero. If $k = 1$, then condition (S.2) simplifies to

$$f(\theta) := 8 - 4\theta + \theta^{1/2}\left\{16\theta - 15 - 5\theta^2\right\} > 0.$$

The function $f$ has one root $\tilde{\theta} \approx 0.629$ in the open interval $(0, 1)$, with $f(\theta) > 0$ for $\theta < \tilde{\theta}$ and $f(\theta) < 0$ for $\theta > \tilde{\theta}$.

For $k = 1$ and $\theta = \frac{1}{4}$, $\bar{\kappa}(\frac{1}{4}) = \frac{5}{24}$, and so $\kappa = \frac{5}{96}$ is in the interval $(\theta^2\bar{\kappa}(\theta), \bar{\kappa}(\theta))$ and implies $\widetilde{\lambda} = \frac{1}{2}$. These parameter values satisfy $\widetilde{\lambda} > \theta$, (S.2) and (S.3).

### S.5.1 Informed Laboratories

This subsection characterizes the behavior and payoffs for the laboratories that are informed and the researchers with whom they match. We are interested in the case in which laboratories with indices in the interval $[\widetilde{\lambda}, 1]$ are informed, and (in equilibrium) match with researchers with the same set of indices. In this subsection, we accordingly suppose that researcher and laboratory indices are uniformly distributed on the interval $[\widetilde{\lambda}, 1]$.

For appropriate values of $\phi$, the price function

$$\hat{p}(\ell, r) = \phi + \frac{\ell^2}{2} - (1-\theta)\ell r \tag{S.4}$$

will clear markets with researcher $\rho$ choosing the efficient $\ell = \rho$ and $r = \rho$. In particular, researcher $\rho$'s payoff from $\ell$ and $r$ is

$$\theta\ell r - \hat{p}(\ell, r) - \frac{r^{2+k}}{(2+k)\rho^k} = \ell r - \phi - \frac{1}{2}\ell^2 - \frac{r^{2+k}}{(2+k)\rho^k}.$$

Maximizing the payoff yields $\ell = \rho$ and $r = \rho$ (the efficient choices), and a payoff value of

$$\frac{k}{2(2+k)}\rho^2 - \phi =: \hat{u}_R(\theta, k, \rho) - \phi.$$

Laboratory payoffs under $\hat{p}$ are given by

$$\frac{1}{2}\lambda^2 + \phi =: \hat{u}_L(\theta, k, \lambda) + \phi.$$

Note that the payoff functions $\hat{u}_R$ and $\hat{u}_L$ are defined to exclude the $\phi$ surplus reallocation. Moreover,

$$\hat{u}_R(\theta, k, \rho) + \hat{u}_L(\theta, k, \rho) - (u_R(\theta, k, \rho) + u_L(\theta, k, \rho)) = \bar{\kappa}(\theta)\rho^2. \tag{S.5}$$

If $\widetilde{\lambda} = 0$, then individual rationality implies $\phi = 0$, and the equilibrium is unique. If $\widetilde{\lambda} > 0$, there is a one parameter family of equilibrium price functions, indexed by

$$\phi \in [-\hat{u}_L(\theta, k, \widetilde{\lambda}),\ \hat{u}_R(\theta, k, \widetilde{\lambda})] = \left[\frac{-\widetilde{\lambda}^2}{2},\ \frac{k\widetilde{\lambda}^2}{2(2+k)}\right].$$

All these price functions induce the same efficient attribute choices, but imply different divisions of the surplus.

The total net surplus of the pair with index $\rho$ is

$$\frac{k}{2(2+k)}\rho^2 + \frac{1}{2}\rho^2 = \frac{(1+k)}{(2+k)}\rho^2,$$

which is the maximum (i.e., efficient) value of the $\rho$-match surplus.

### S.5.2 Equilibrium

Fix $\widetilde{\lambda} \in [0, 1]$ and consider a putative equilibrium in which laboratories with $\lambda \leq \widetilde{\lambda}$ are uninformed and laboratories with $\lambda > \widetilde{\lambda}$ choose to be informed. Within each region, researchers will be choosing attributes increasing in index, and researcher $\rho$ will be matched with laboratory $\lambda = \rho$. Thus, researcher $\rho \leq \widetilde{\lambda}$ will choose $r = \theta^{1/(1+k)}\rho$ and be matched with the uninformed laboratory $\lambda = \rho$. Researcher $\rho > \widetilde{\lambda}$ will choose $r = \rho$ and be matched with the informed laboratory $\lambda = \rho$.

In this putative equilibrium, the set of chosen researcher attributes is $[0,\ \widetilde{r}] \cup (\widetilde{\lambda},\ 1]$, where $\widetilde{r} = \theta^{1/(1+k)}\widetilde{\lambda} < \widetilde{\lambda}$.

Recalling (S.5), for $\kappa < \bar{\kappa}(\theta)$, we choose $\widetilde{\lambda} \in (0, 1)$ so that the ex ante efficient surplus from the match of researcher $\widetilde{\lambda}$ and laboratory $\widetilde{\lambda}$ exactly exceeds the uninformed laboratory equilibrium match surplus by $\kappa$:

$$\kappa = \hat{u}_R(\theta, k, \widetilde{\lambda}) + \hat{u}_L(\theta, k, \widetilde{\lambda}) - \ u_R(\theta, k, \widetilde{\lambda}) - u_L(\theta, k, \widetilde{\lambda}) = \bar{\kappa}(\theta)\widetilde{\lambda}^2. \quad \text{(S.6)}$$

The pricing constant

$$\phi := \frac{k}{2(2+k)}\left[1 - \theta^{(2+k)/(1+k)}\right]\widetilde{\lambda}^2$$

in (S.4) makes laboratory $\widetilde{\lambda}$ indifferent between being informed and not:

$$\hat{u}_L(\theta, k, \widetilde{\lambda}) + \phi - \kappa = u_L(\theta, k, \widetilde{\lambda}).$$

This immediately implies that researcher $\widetilde{\lambda}$ is also indifferent *ex ante* between being matched with an informed or uninformed laboratory. We should then be able to simply "paste" the informed laboratory equilibrium for $\lambda \geq \widetilde{\lambda}$ to the uninformed laboratory equilibrium for $\lambda < \widetilde{\lambda}$.

### S.5.3 Researcher Incentives to Deviate

For the researchers, we need only verify that researchers with indices below (respectively, above) $\widetilde{\lambda}$ prefer to be matched with uninformed (respectively, informed) laboratories rather than choosing a sufficiently high (respectively, low) attribute to be matched with an informed (respectively, uninformed) laboratory. But this follows from the single crossing property on the cost function together with the implied indifference for researcher $\widetilde{\lambda}$.

### S.5.4 Laboratory Incentives to Deviate

Turning to the laboratories, there are two potentially profitable types of deviations. The first is that a laboratory with index $\lambda < \widetilde{\lambda}$ may find it profitable to be informed. The second is that a laboratory with index $\lambda > \widetilde{\lambda}$ may find it profitable to be uninformed.

**Do Uninformed Laboratories Wish to be Informed?** Consider first a deviation by a laboratory $\lambda \leq \widetilde{\lambda}$ to becoming informed and targeting a researcher with attribute $r \leq \widetilde{r}$. The attribute $r$ is chosen by researcher $\rho = \theta^{-1/(1+k)}r$, and matches with $\lambda = \rho = \theta^{-1/(1+k)}r$, paying a price of $\rho^2\theta^{(2+k)/(1+k)}/2 = \theta^{k/(1+k)}r^2/2$. The resulting ex post payoff is the researcher's share of the surplus less the price, $\theta \times \theta^{-1/(1+k)}r^2 - \theta^{k/(1+k)}r^2/2 = \theta^{k/(1+k)}r^2/2$. An offer of a price $p$ satisfying

$$\theta^{k/(1+k)}r^2/2 < \theta\lambda r - p$$

will induce the researcher to accept the deviating offer. Such an offer is profitable for the laboratory if

$$u_L(\theta, k, \lambda) < (1-\theta)\lambda r + p - \kappa.$$

Thus, there is a $p$ for which the deviation by the laboratory is strictly profitable if, and only if,

$$\begin{aligned}\kappa &< \lambda r - \theta^{k/(1+k)}r^2/2 - u_L(\theta, k, \lambda)\\ &= \lambda r - \theta^{k/(1+k)}r^2/2 - \frac{1}{2}\theta^{1/(1+k)}(2-\theta)\lambda^2 =: \Delta(\lambda, r).\end{aligned}$$

For $\lambda \le \theta\widetilde{\lambda}$, $\Delta(\lambda,\cdot)$ is maximized at $r = \theta^{-k/(1+k)}\lambda \le \widetilde{r}$, and has value $\theta^{-k/(1+k)}\lambda^2(1-\theta)^2/2$. Note that $\theta^{-k/(1+k)}\theta\widetilde{\lambda} = \widetilde{r}$. Moreover, for $\lambda \in [\theta\widetilde{\lambda}, \widetilde{\lambda}]$, $\Delta(\lambda, \widetilde{r})$ is uniquely maximized at $\lambda = \widetilde{\lambda}/(2-\theta)$. This implies that the maximum of $\Delta(\lambda, r)$ over $(\lambda, r) \in [0, \widetilde{\lambda}] \times [0, \widetilde{r}]$ is achieved at $(\widetilde{\lambda}/(2-\theta), \widetilde{r})$. Thus, there is no strictly profitable deviation if

$$\kappa \ge \Delta(\widetilde{\lambda}/(2-\theta), \widetilde{r}) = \frac{\theta^{1/(1+k)}\widetilde{\lambda}^2(1-\theta)^2}{2(2-\theta)}.$$

Substituting for $\widetilde{\lambda}$ from (S.6) and canceling $\kappa$ yields (S.2).

**Do Informed Laboratories Wish to be Uninformed?** If laboratory $\lambda \ge \widetilde{\lambda}$ deviates to being uninformed, then by posting a price $p$, the laboratory attracts all researchers who find matching with laboratory $\lambda$ at that price attractive. The laboratory must have beliefs over the researchers attracted by such a deviation. We assume pessimistic beliefs: the laboratory assumes that the lowest attribute researcher will match.

We begin by considering $\lambda = 1$, and suppose this laboratory chooses to be uninformed. If it were to charge $p = \phi - \frac{1}{2} + \theta$, the equilibrium price paid by researcher $\rho = 1$ to match with laboratory $\lambda = 1$, researcher $\rho = 1$ incentives are unchanged. But that match is no longer relevant (given our assumption on beliefs), since lower attribute researchers are willing to pay that price. The most profitable deviation is to charge a higher price in attempt to screen out lower attribute researchers.[6]

We now argue that if $\theta < \widetilde{\lambda}$, the most profitable deviation by laboratory $\lambda = 1$ is to charge such a high price that $\rho = \widetilde{\lambda}$ is indifferent, and that such a deviation is not profitable. Researcher $\rho \ge \widetilde{\lambda}$ has chosen attribute $\rho$ and has payoffs gross of costs of

$$\frac{\rho^2}{6} - \phi + \frac{\rho^2}{3} = \frac{\rho^2}{2} - \phi,$$

and is willing to match with the deviating laboratory $\lambda = 1$ at a price $p$ if $\theta\rho - p \ge \rho^2/2 - \phi$, i.e., if $\theta\rho - \rho^2/2 + \phi \ge p$. The laboratory's goal is to maximize the lowest $\rho$ satisfying this inequality through his choice of $p$. The quadratic on the left of the inequality is maximized at $\rho = \theta$ and is monotonically decreasing for larger $\rho$. This implies that if $\theta < \widetilde{\lambda}$, the

[6] At higher prices the highest attribute researcher prefers to match with laboratory $1-\varepsilon$, for $\varepsilon$ small. But since the laboratory believes he will match with the lowest attracted attribute, this is irrelevant.

optimal choice of $p$ makes researcher $\widetilde{\lambda}$ just indifferent ($p = \theta\widetilde{\lambda} - \widetilde{\lambda}^2/2 + \phi$); no researcher is willing to match at a larger $p$.

The laboratory does not find this deviation profitable if

$$\begin{aligned} &\frac{1}{2} + \phi - \kappa \geq (1-\theta)\widetilde{\lambda} + \theta\widetilde{\lambda} - \widetilde{\lambda}^2/2 + \phi \\ \iff &\frac{1}{2} - \kappa \geq \widetilde{\lambda} - \widetilde{\lambda}^2/2. \end{aligned}$$

Using (S.6) to eliminate $\widetilde{\lambda}$ in the inequality and rearranging, one obtains condition (S.3).

Lower index informed laboratories also have no incentive to become uninformed, though for some this deterrence involves a concern that the researcher will have an attribute less than $\theta^{1/2}\widetilde{\lambda}$, rather than $\widetilde{\lambda}$. Lower informed laboratories may find it optimal to become informed if they could guarantee no researcher with an attribute below $\widetilde{\lambda}$ would find the price attractive. However, this is impossible: By becoming uninformed, laboratory $\rho = \widetilde{\lambda}$ cannot deter lower attribute researchers without deterring all researchers. A (loose) upper bound on the payoff from deviating is obtained by assuming that at the price $p$ which makes the researcher with attribute $\widetilde{\lambda}$ just indifferent, the laboratory is guaranteed that the only additional researcher attribute attracted is $\widetilde{r} = \theta^{1/2}\widetilde{\lambda}$. It can be verified that even with such a payoff, the deviation is not profitable.