Penn Institute for Economic Research
Department of Economics
University of Pennsylvania
3718 Locust Walk
Philadelphia, PA 19104-6297
pier@econ.upenn.edu
http://www.econ.upenn.edu/pier

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"On Price-Taking Behavior in Asymmetric Information Economies"

by

Richard McLean, James Peck and Andrew Postlewaite

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On Price-Taking Behavior in Asymmetric Information Economies [*,†]

Richard McLean
Rutgers University

James Peck
The Ohio State University

Andrew Postlewaite
University of Pennsylvania

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Abstract

It is understood that rational expectations equilibria may not be incentive compatible: agents with private information may be able to affect prices through the information conveyed by their market behavior. We present a simple general equilibrium model to illustrate the connection between the notion of informational size presented in McLean and Postlewaite (2002) and the incentive properties of market equilibria. Specifically, we show that fully revealing market equilibria are not incentive compatible for an economy with few privately informed producers because of the producers' informational size, but that replicating the economy decreases agents' informational size. For sufficiently large economies, there exists an incentive compatible fully revealing market equilibrium.

1. Introduction

In many markets of interest, agents are asymmetrically informed. Sellers of stock or automobiles often possess information that potential buyers do not have. In

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the presence of informational asymmetries, prices may reveal information to some agents. A particularly low price for shares in a company may signal to an uninformed agent that better informed agents are not buying the stock, or may be selling the stock. The notion of rational expectations equilibrium is one generally accepted extension of Walrasian equilibrium to economies with asymmetrically informed agents. As in the case of symmetric information, agents are assumed to maximize expected utility in a rational expectations equilibrium. In the rational expectations model, however, agents maximize expected utility not with respect to an exogenously given probability distribution. Instead, agents maximize expected utility with respect to an updated probability distribution that combines their initial information with the additional information conveyed by the prices.

Informational asymmetries can have serious consequences for the performance of an economy. While Walrasian equilibria are Pareto efficient under quite general conditions when agents are symmetrically informed, market outcomes in the simplest of economies can be inefficient when the agents are asymmetrically informed, as shown clearly in Akerlof's Lemon's paper (Akerlof (1970)). In at least one case, however, asymmetric information does not result in inefficiency. This is the case in which the rational expectations equilibrium is fully revealing, that is, when the price reflects all of the agents' information. If the price conveys all the information that agents have, then each agent's decision problem is equivalent to the problem that would be solved if all the information were publicly available. In this case, the welfare theorems assure that efficient outcomes are obtained.

Fully revealing rational expectations equilibria are ex-post efficient, but open the question of the reasonableness of the price-taking assumption. As in the case of Walrasian analysis of symmetric information economies, rational expectations equilibria assume that agents ignore the effect of their market behavior on prices. In economies with symmetrically informed agents, this assumption is sometimes justified by a heuristic argument that in large economies agents will not be able to affect the price. There is also a formal foundation for this argument that relies on the explicit modeling of agents' strategic possibilities in a general equilibrium setting and provides conditions under which the Nash equilibria of the strategic market game are approximately Walrasian.[1] Roughly, it can be shown that, in plausible strategic market games, agents will have little effect on the price of a good when they control a small portion of the good. This provides some justification that price-taking behavior is a plausible assumption for agents in large economies.

The situation with asymmetric information is more complicated since agents

[1] See, e.g., Mas Colell, Dubey and Shubik (1980) or Postlewaite and Schmeidler (1978, 1981).

can affect the price not only through the quantity of a good that they trade, but also through the information their trades reveal. This second channel through which agents can affect prices means that it is not enough that the quantity of a good that an agent controls is small relative to the aggregate quantity of that good. An agent with a small amount of a particular good may affect the price of that good because of the information he possesses. It is well understood that there may be a conflict between the information contained in rational expectations equilibrium prices and an agent's incentive to reveal, directly or indirectly, his information.[2]

This conflict should not in itself be surprising, since the incentive *not* to take prices as given exists even when agents are symmetrically informed. The most that one would hope for is that the effect of an agent's behavior on prices, via the information that his market behavior reveals, will be negligible in large economies. Palfrey and Srivastava (1986) considered a stochastic replication procedure for an economy in which the incentive compatibility problems associated with rational expectations equilibria asymptotically vanish. However, their stochastic replication procedure has the property that, with probability one, each agent's private information is duplicated as the number of agents increases. In a large economy, a single agent's information is redundant in the presence of the information of all other agents.

We are interested in situations in which a single agent's information is *not* redundant. The prototypical large economy that we envision is one in which preferences and technology depend on the state of the world, which is not directly known. Each agent has some information (a signal) regarding the relative likelihoods of states. When agents' signals are conditionally independent (given the true state), the signal of a single agent can still provide additional information about the true state, even in the presence of many agents. However, the incremental value of that signal vanishes as the number of agents becomes large. In this world, agents become "informationally small" as economies grows, but they never become "informationally irrelevant." There is a large literature analyzing competitive models that ignores the asymmetric information that must surely be present in any real-world problem. The usefulness of analyses that ignore such asymmetric information hinges on the belief that the incentive problems brought on by asymmetrically informed agents become negligible in large economies.

We present and analyze a simple general equilibrium example with asymmetrically informed agents similar to that described above. In the example, asym-

[2] See, e.g., Blume and Easley (1983).

metrically informed agents make production decisions based on their private information. Markets then open in which the produced goods are traded. When the number of producers is small, the fully revealing market equilibria are not incentive compatible; an agent's market behavior can reveal private information, and the revealed information can affect prices in ways detrimental to that agent. Consequently, when agents take into account the informational impact of their market behavior, the outcome may be different from the competitive outcome. However, when the economy is replicated in a natural way, agents become informationally small, where the notion of informational size is essentially that introduced in McLean and Postlewaite (2002). As a consequence of their asymptotically vanishing informational size, agents will have no incentive to manipulate prices in large economies.

We discuss within the example several of the issues that arise in modelling general equilibrium economies with asymmetric information, including the completeness of markets and multiple equilibria.

2. Example

The economy. There are two states of nature that are equally likely, θ_1 and θ_2 and two periods. There are two kinds of agents, producers and consumers, and three commodities: type 1 widgets, type 2 widgets, and money (denoted m).

Producers. There are $\overline{n}$ producers, each of whom can make exactly one widget using his own labor and chooses which type to produce in the first period. The producers' choice of widgets is simultaneous. Producers have identical state independent payoff functions defined simply as their final holdings of money. That is, they value neither widgets nor their own labor input.

Consumers. Each consumer is endowed with 20 units of money but no widgets. Consumers have the same utility function $u(\cdot)$ that depends on the state, the number x_1 of type 1 widgets consumed, the number x_2 of type 2 widgets consumed and the final holding of money as given in the table below:

$$\begin{aligned} u(x_1, x_2, m, \theta_1) &= m + 25x_1 \\ u(x_1, x_2, m, \theta_2) &= m + 10x_2 \end{aligned}$$

Note that, in state θ_i,only type i widgets yield positive utility.

Information. Prior to production, each producer receives a noisy signal of the state. The conditional distributions of the signal a producer receives in each state are given in the table below:

state	θ_1	θ_2
signal		
s_1	.8	.2
s_2	.2	.8

Producers' signals are conditionally independent.

Markets. There are no markets open in the first period. In the second period competitive markets open in which the widgets that have been produced can be exchanged for money. Since producers incur no opportunity cost in making widgets, each of the $\bar{n}$ producers makes a widget. We denote by n the number of widgets of type 1 produced in period 1 (hence, there are $\bar{n} - n$ type 2 widgets produced). If producers choose to make different types of widgets when they have observed s_1 than they make when they have observed s_2, the mix of widgets on the market in period 2 will convey information about the state θ. If consumers rationally take this information into account, the competitive price in period 2 will depend on the mix of widgets offered for exchange.

A strategy for a producer is a mapping $\sigma : \{s_1, s_2\} \to \{1, 2\}$ specifying which type of widget to produce as a function of the observed signal. We consider symmetric equilibria in which all producers employ the same (pure) strategy. If $\sigma(s_1) = \sigma(s_2)$, then no information will be conveyed by the mix of widgets on the market. However, if $\sigma(s_1) \neq \sigma(s_2)$, the number of widgets of type 1 will reveal the number of producers who received each of the two signals.

Consumers form expectations given the (common) strategy of the producers and the number of widgets of each type that are offered on the market in the second period. When $\sigma(s_1) \neq \sigma(s_2)$, Bayes rule uniquely determines the posterior distribution on Θ, but if $\sigma(s_1) = \sigma(s_2)$ there are producer choices lying off the equilibrium path where one or more producers produce the widget which was not the strategic choice for either of the signals. We denote by $\mu(\cdot|n) = (\mu(\theta_1|n), \mu(\theta_2|n))$, a consumer's beliefs when he observes n widgets of type 1 on the market. We assume that $\mu(\theta_1|n)$ is the Bayesian posterior probability on θ_1 when the number of widgets of type 1, n, is consistent with producers' (common) strategy σ, and unrestricted otherwise.[3] No restriction is placed on consumers' beliefs when n is not consistent with σ.

Let $J_{\bar{n}} = \{1, .., \bar{n}\}$. A price is a function $J_{\overline{n}} \to \mathbb{R}^2_+$ where $p(n) = (p^n_1, p^n_2)$ is the pair of prices for widgets 1 and 2 respectively when n widgets of type 1 are

[3]It will be consistent with σ if either (i) $\sigma(s_1) \neq \sigma(s_2)$; or (ii) $\sigma(s_1) = \sigma(s_2) = 1$ and $n = \bar{n}$; or (iii) $\sigma(s_1) = \sigma(s_2) = 2$ and $n = 0$.

produced in period 1. (The price of m is normalized to 1.) Given a price function $p(\cdot)$, consumers maximize expected utility (with respect to their beliefs $\mu(\cdot|n)$). A market equilibrium is a price $p(\cdot)$ and optimizing behavioral rules for producers and consumers for which markets clear.[4]

Definition: Given beliefs, $n \mapsto \mu(\cdot|n)$, a *market equilibrium (ME)* is a price function $p(\cdot)$ and a common producer strategy $\sigma(\cdot)$, for which

i. The symmetric strategy profile $(\sigma(\cdot), ..., \sigma(\cdot))$ is a Bayes equilibrium and

ii. For each n, consumer demand for widgets at price $p(n)$ is equal to number the of widgets produced.

When consumers' have large initial endowments of money, they will want to purchase a large number of widgets of a given type if the expected utility of that type of widget is greater than the price. Similarly, they will purchase none if the expected utility is below the price. Consequently, the only possible market clearing prices for sufficiently large m are those for which the market price of each widget is equal to the expected value of that widget. The expected value of widget 1 when there are n widget 1's on the market is (given producer strategy σ) $25\mu(\theta_1|n)$, which then must be the price of widget 1 when n widget 1's are offered on the market. Analogously, the expected value and the price of widget 2 when n widget 1's are offered is $10\mu(\theta_2|n)$.

We are interested in the existence (or nonexistence) of a *fully revealing market equilibrium* (FRME), that is a market equilibrium for which the equilibrium price reveals the private information that agents (producers) have. When $\sigma(s_1) \neq \sigma_2(s_2)$, the price reflects the number of widgets of type 1 on the market, which is the same as the number of producers who have observed signal s_1. Hence, a ME is fully revealing if the common producer strategy is separating. We will show that fully revealing ME cannot exist when the number of producers is too small, but they will exist if the number of producers is sufficiently large.

The case of a single producer

In the presence of a single producer, the problem reduces to the existence of a separating equilibrium in a simple sender-receiver game. In a separating

[4] We use the term *market equilibrium* rather than *rational expectations equilibrium* because producers' choices may not maximize expected profit at the given prices since their decisions must be taken prior to time at which the market opens. We discuss this further in the last section.

equilibrium, the producer chooses one type of widget when he observes signal s_1 and the other type when he observes signal s_2.

Suppose that $\sigma(s_1) = 1$ and $\sigma(s_2) = 2$. Note first that, given this strategy, the consumers' beliefs are $\mu(\theta_1|1) = .8$ if he sees widget 1 and $\mu(\theta_1|0) = .2$ if he sees widget 2. The equilibrium prices must therefore be $p_1^1 = 25\mu(\theta_1|1) = 20$, $p_2^1 = 10\mu(\theta_2|1) = 2$ and $p_1^0 = 25\mu(\theta_1|0) = 5$, $p_2^0 = 10\mu(\theta_2|0) = 8$. We claim that this proposed strategy is not an equilibrium. To see this, suppose the producer receives signal s_2. If the producer produces widget 2, his payoff will be $p_2^0 = 8$, while his payoff from deviating and producing widget 1 is $p_1^1 = 20$. Thus, there cannot be a separating equilibrium in which the producer chooses widget 2 when he receives signal s_2.

The same type of argument demonstrates that there cannot be a separating equilibrium in which the producer chooses widget 2 when he receives signal 1. Thus there cannot be a fully revealing equilibrium when there is a single producer. We note that welfare is maximized when the producer chooses widget 1 after receiving signal 1 and widget 2 after signal 2.

The case of two producers

Suppose there are two producers. We will show that as in the case of a single producer, there can be no separating equilibrium. Let $\sigma(\cdot)$ be the common strategy and suppose that $\sigma(s_1) \neq \sigma(s_2)$. In particular, suppose that $\sigma(s_1) = 1$ and $\sigma(s_2) = 2$. Finally, suppose that producer 1 receives signal s_2. If both producers are following this strategy the consumers' beliefs about state 1 following 0, 1 or 2 widget 1's being offered on the market are given in the table below.

Widget Production	Consumers' beliefs	Expected value of widget 1	Expected value of widget 2
2 widget $1'$s	$\mu(\theta_1|2) = .94$	23.5	.5
1 widget 1	$\mu(\theta_1|1) = .5$	12.5	5
0 widget $1'$s	$\mu(\theta_1|0) = .06$	1.5	9.4

As before, the market clearing price of widgets must be the expected value of the widget. If a producer chooses to produce a type 2 widget, then the most that he will get for this widget is 9.4. If he produces a type 1 widget, then his payoff is at least 12.5. Consequently it cannot be an equilibrium for producers to produce widget 1 following signal 1 and widget 2 following signal 2.

As in the case of a single producer, the calculations for the strategy which prescribes producing widget 1 following signal 2 and widget 2 following signal 1 are the same as in case 1. Consequently, this producer strategy will not be an equilibrium either.

In summary, when there are 2 producers there is no symmetric separating equilibrium. It is easy to see why: a consumer values widget 1 more highly in state θ_1 than he values widget 2 in θ_2. Producer 1 affects consumers' beliefs through his choice of widget. If consumers expect producers to choose widget 2 after seeing widget 2, a producer's payoff will be higher if he instead produces widget 1.

The case of many producers

When there are many producers, there will exist separating equilibria in which each producer chooses widget i after receiving signal s_i, $i = 1, 2$.[5] Suppose producers receive conditionally independent, noisy signals of the state that are accurate with probability .8 (that is, $P(s_i|\theta_i) = .8$). If $\overline{n}$ is large and if all producers follow the separating strategy proposed above, then, with high probability, approximately 80% of the widgets offered for sale will be of type 1 when the state is θ_1 (so that $n \approx .8\overline{n}$) and approximately 80% of the widgets offered for sale will be of type 2 when the state is θ_2 (so that $n \approx .2\overline{n}$). This observation is simply an application of the law of large numbers. If $\overline{n}$ is large and if approximately 80% of the widgets are of type 1 (i.e., if $n \approx .8\overline{n}$), a simple calculation verifies that the consumers' beliefs will ascribe probability close to 1 to state θ_1 (i.e., $(\mu(\theta_1|n), \mu(\theta_2|n)) \approx (1, 0)$). If $\overline{n}$ is large and if approximately 80% of the widgets are of type 2 (i.e., if $n \approx .2\overline{n}$), the same calculation verifies that the consumers' beliefs will ascribe probability close to 1 to state θ_2 (i.e., $(\mu(\theta_1|n), \mu(\theta_2|n)) \approx (0, 1)$). If the true but unobserved state is θ_1, then, with high probability, the price vector $(p_1^n, p_2^n) \approx (p_1^{.8\overline{n}}, p_2^{.8\overline{n}}) \approx (25, 0)$ and if the true but unobserved state is θ_2, then, with high probability, the price vector $(p_1^n, p_2^n) \approx (p_1^{.2\overline{n}}, p_2^{.2\overline{n}}) \approx (0, 10)$. When $\overline{n}$ is large, the production decision of a single producer has only a small effect on the ratio $\frac{n}{\overline{n}}$. If $\overline{n}$ is large, it follows that, with probability close to 1, any single producer who changes production from one type of widget to the other will have only a small effect on consumers' beliefs, and hence on the prices of the widgets.

Suppose that $\overline{n}$ is large and that producers are employimg the separating strategy. Consider a producer who receives signal s_1. He believes that the price of

[5] In addition, there will exist equilibria which are pooling, and hence, are not fully revealing. This is discussed in the last section.

a type 1 widget will be close to 25 with probability $P(\theta_1|s_1) = .8$ and close to 0 with probability $P(\theta_2|s_1) = .2$, yielding an expected price of 20 for type 1 widgets. On the other hand, he believes that the price of a type 2 widget will be close to 0 with probability $P(\theta_1|s_1) = .8$ and close to 10 with probability $P(\theta_2|s_1) = .2$, yielding an expected price of 2 for type 2 widgets. Consequently, a producer who observes signal s_1 will produce a type 1 widget.

A similar calculation will be made by a producer who receives signal s_2. He believes that the price of a type 1 widget will be close to 25 with probability $P(\theta_1|s_2) = .2$ and close to 0 with probability $P(\theta_2|s_1) = .8$, yielding an expected price of 5 for type 1 widgets. On the other hand, he believes that the price of a type 2 widget will be close to 0 with probability $P(\theta_1|s_2) = .2$ and close to 10 with probability $P(\theta_2|s_2) = .8$, yielding an expected price of 8 for type 2 widgets. Consequently, a producer who observes signal s_2 will produce a type 2 widget..

In summary, there will exist a fully revealing market equilibrium when the number of agents is sufficiently large. It will be an equilibrium for each producer to produce the widget that maximizes expected value given his own information alone if other producers are doing the same. Any deviation from this will have a vanishingly small effect on price as the number of producers becomes large, making such deviations unprofitable.

3. Modelling the Consumer

The concept, market equilibrium, models producers as strategic, specifying precisely what actions are available to them, but does not do the same for consumers. In this section, we model the second stage game as a Shapley-Shubik market game (Shubik (1973), Shapley (1974)), in which producers put goods and consumers put money on widget-1 and widget-2 trading posts, with the prices determined to clear the markets. We will consider limits, as the number of consumers approaches infinity, of symmetric perfect Bayesian equilibria in which all producers offer their widgets for sale. These equilibria will define beliefs, producer strategies, and prices that constitute a market equilibrium. We view this section as justifying the simpler model of section 2.

Wherever possible, we maintain the notation of section 2. The timing of the game is now as follows. At stage 1, producers observe their signal, either s_1 or s_2, and decide which widget to produce. We restrict attention to equilibria in which all producers adopt the same production strategy, σ, and supply their widget to the appropriate trading post. At stage 2, consumers observe the number of widgets of each type that were produced, where n denotes the number of type 1 widgets and $\overline{n} - n$ is the number of type 2 widgets. After observing n, consumers decide how much money to bid for type 1 widgets and type 2 widgets, at their respective trading posts.

Let h be the number of consumers, and let j index a particular consumer. For $j = 1, ..., h$, let $b_1^j(n)$ denote the amount of money that consumer j bids on the widget-1 trading post when the number of type-1 widgets produced is n, and let $b_2^j(n)$ denote the amount of money that consumer j bids on the widget-2 trading post when the number of type-1 widgets produced is n. A strategy for consumer j is a mapping, $\psi^j : J_{\overline{n}} \to \mathbb{R}_+^2$, such that $b_1^j(n) + b_2^j(n) \leq 20$ holds for all n.

The market clears according to the following allocation rule.

$$x_1^j(n) = \frac{n b_1^j(n)}{\sum_{j'=1}^h b_1^{j'}(n)}, \tag{3.1}$$

$$x_2^j(n) = \frac{(\overline{n} - n) b_2^j(n)}{\sum_{j'=1}^h b_2^{j'}(n)}, \quad \text{and} \tag{3.2}$$

$$m^j(n) = 20 - b_1^j(n) - b_2^j(n),$$

where $x_1^j(n)$ denotes consumer j's purchases of widget 1 when the number of type-1 widgets produced is n, $x_2^j(n)$ denotes consumer j's purchases of widget 2 when the number of type-1 widgets produced is n, and $m^j(n)$ denotes consumer j's money consumption when the number of type-1 widgets produced is n. The money received by a firm selling a particular widget is the price of that widget. These prices are given by

$$p_1^n = \frac{\sum_{j'=1}^h b_1^{j'}(n)}{n} \quad \text{and}$$

$$p_2^n = \frac{\sum_{j'=1}^h b_2^{j'}(n)}{(\overline{n} - n)}.$$

The allocation rule guarantees that all trade on a market takes place at the same price, which is the total amount of money bid divided by the total amount of widgets supplied. From (3.1) and (3.2), we see that the percentage of the widgets up for sale that consumer j purchases is equal to the percentage of the money that consumer j bids. If numerator and denominator are both zero in (3.1) or (3.2), then consumers do not receive any widgets. Therefore, we adopt the convention that $\frac{0}{0} = 0$ in (3.1) and (3.2). However, prices are a different story. If, say, there are no type-1 widgets produced and no money is bid for type-1 widgets, then the price of type-1 widgets is indeterminate. The resolution of this indeterminacy is irrelevant for the characterization of perfect Bayesian equilibrium, but could affect the comparison to market equilibrium. We will comment on this later.

We restrict attention to symmetric perfect Bayesian equilibria, in which all widgets produced are supplied to the market. To find an equilibrium, we find consumer j's best response to the common strategy played by all other consumers, $(b_1(n), b_2(n))$, after n type-1 widgets are produced. We then impose the condition that consumer j's best response is in fact $(b_1(n), b_2(n))$. Given beliefs, μ, the optimization problem for consumer j is to choose $(b_1^j(n), b_2^j(n))$ to solve

$$\begin{aligned} \max[20 - b_1^j(n) - b_2^j(n)] + \mu(\theta_1 \mid n) \frac{25 n b_1^j(n)}{b_1^j(n) + (h-1) b_1(n)} \\ + \mu(\theta_2 \mid n) \frac{10(\overline{n} - n) b_2^j(n)}{b_2^j(n) + (h-1) b_2(n)}. \end{aligned}$$

Computing the first order conditions, imposing $(b_1^j(n), b_2^j(n)) = (b_1(n), b_2(n))$, and simplifying, we have

$$\begin{aligned} b_1(n) &= \frac{\mu(\theta_1 \mid n) 25 n (h-1)}{h^2} \\ b_2(n) &= \frac{\mu(\theta_2 \mid n) 10(\overline{n} - n)(h-1)}{h^2}. \end{aligned}$$

Notice that the above equilibrium bids are uniquely determined, as long as we impose symmetry. Plugging the above bids into the formula for prices, we have

$$p_1^n = \left(\frac{h-1}{h}\right) 25 \mu(\theta_1 \mid n) \quad \text{and} \tag{3.3}$$

$$p_2^n = \left(\frac{h-1}{h}\right) 10 \mu(\theta_2 \mid n). \tag{3.4}$$

The prices in (3.3) and (3.4) are uniquely determined from the ratio of bids and offers, except for p_1^n when $n = 0$ holds, and p_2^n when $n = \overline{n}$ holds. In these cases, bids and offers are zero, but either n or $(\overline{n} - n)$ appear in both the numerator and denominator, and cancel each other. Thus, we will define the *prices associated with a symmetric PBE* by (3.3) and (3.4).[6]

[6] The prices given by (3.3) and (3.4), for the case of markets with zero supply and demand, would arise if we placed ε offers of widgets on each market, and let ε approach zero. See the discussion of virtual prices in, say, Dubey and Shubik (1978).

Proposition. *Consider a sequence* $(\sigma^h, \psi^h, \mu^h)_h$ *where* $(\sigma^h, \psi^h, \mu^h)$ *is a symmetric PBE for the game with* h *consumers and consider the associated sequence of prices,* $(p_1^{n,h}, p_2^{n,h})_h$*. If* $(\sigma^{h'}, \psi^{h'}, \mu^{h'})_{h'}$ *is a convergent subsequence, then* $(\lim_{h' \to \infty}(p_1^{n,h'}, p_2^{n,h'}), \lim_{h' \to \infty}$ *is a market equilibrium for beliefs* $\lim_{h' \to \infty} \mu^{h'}$.

Proof. From the definition of PBE, and because there are only four possible producer strategies, σ^h, there exists $\overline{h}, \overline{\sigma}$, and $\overline{\mu}$ such that $h' > \overline{h}$ implies:

(1) $\sigma^h = \overline{\sigma}$, and

(2) $\mu^h(\theta_1 \mid n) = \overline{\mu}(\theta_1 \mid n)$ and $\mu^h(\theta_2 \mid n) = \overline{\mu}(\theta_2 \mid n)$ for all n occurring with positive probability, given $\overline{\sigma}$. Thus, $\overline{\mu}$ is consistent with $\overline{\sigma}$, according to the criterion required for a PBE. This also implies that $\overline{\mu}$ is consistent with $\overline{\sigma}$, according to the criterion required for a market equilibrium.

>From (3.3) and (3.4), we see that, for $h' > \overline{h}$, the incentives for producers to deviate are exactly the same in the PBE as they are in a market equilibrium. Sequential rationality of $\overline{\sigma}$ in the PBE implies $\{\overline{\sigma}\}$ satisfies part (i) of the definition of a market equilibrium, given beliefs $\overline{\mu}$. The limiting price function, $p(n) = (25\overline{\mu}(\theta_1 \mid n), 10\overline{\mu}(\theta_2 \mid n))$, satisfies part (ii) of the definition of a market equilibrium. □

4. Discussion

Incomplete markets

In the example, market equilibrium cannot be fully revealing when the number of producers is small, but is fully revealing when the number of producers is sufficiently large. Because producers are informationally small in large economies, they cannot gain by attempting to manipulate prices. However, even for large economies, a fully revealing market equilibrium is not a rational expectations equilibrium. In a rational expectations equilibrium, producers can observe the prices of widget-1 and widget-2, infer the state of nature, and produce the widget corresponding to the correct state. In the example, the market equilibrium is fully revealing, but only after output has been produced. A producer will produce the wrong widget with probability .2, so a market equilibrium is not ex post efficient. Also, if we were to change the parameter for observing the correct signal from .8 to .6, then there is no fully revealing market equilibrium. Producers are informationally small, so there is no incentive to manipulate market prices, but producers receiving signal s_2 are better off gambling that their signal is wrong and producing widget 1.

Markets are incomplete in the example: there is no forward market in which producers can sell their planned output before producing it. As mentioned in the introduction, the structure of markets is crucial for the example. Suppose instead that the only market available operated in the first period, in which producers could offer widgets of either type for delivery in the second period. Whatever prices prevail in this forward market, all producers will wish to sell the same widget – the widget with the higher price. Thus, it is impossible that producers with different signals will behave differently. But when all producers behave identically regardless of their information, the price cannot reflect their information.[7] On the other hand, suppose that a securities market operated in the first period, on which producers could trade money, contingent on whether the number of widgets produced was greater than, or less than, $\frac{\overline{n}}{2}$. Now production could depend on the prices of securities, so that full revelation would lead to efficient production decisions. In future work, we will explore the conjecture that, in this more complete market structure, fully revealing market equilibria exist, and correspond to rational expectations equilibria.

One might imagine a non-tatonement process that reveals producers' information (for example, a bargaining process between buyers and sellers), with trade taking place only after revelation has taken place. Assuming such an unmodelled process is unsatisfactory, however. The point of the present exercise is to understand when agents' private information will be revealed when those agents are behaving strategically with respect to the revelation. Any interesting analysis addressing this issue must model the process by which agents' information is reflected in prices. In other words, it is necessary to specify exactly what actions agents can take and the mapping of their actions into prices and outcomes.[8]

Specifying that producers choose which widgets to produce, with prices and outcomes arising from competitive behavior subsequent to the choices, provides a precise and plausible mechanism by which informed agents' information is incorporated into prices. One can, of course, think of alternative mechanisms that link agents' actions and resulting outcomes, but the intuition in the example is quite general. Whatever the mechanism linking actions and prices, if strategic behavior is modelled by Bayes equilibria, the revelation principle applies. An agent's

[7] This is similar to the phenomenon in Grossman and Stiglitz (1980).

[8] See Dubey, Geanakoplos and Shubik (1987) for an early argument along these lines and the general treatment of the question in Jackson and Peck (1999).

incentive to misreport his information will be limited by the degree to which his report affects the expected price. Said differently, those agents whose information is likely to have a trivial effect on price have little to gain from misreporting that information. For many natural mechanisms, when the gains from altering behavior to affect the price are small, equilibrium actions will be close to actions that are optimal ignoring the effect on price.

Multiple equilibria

We have demonstrated the existence of a fully revealing incentive compatible ME when the number of producers was sufficiently large. This does not mean that all incentive compatible ME's are fully revealing. The nonrevealing ME in which sellers produce widget 1 regardless of their signal, at a price of $(12.5, 10)$ remains an incentive compatible ME. This will be a perfect Bayes equilibrium if consumers' beliefs following the disequilibrium choice of widget 2 by a producer were that that producer had seen signal s_1 with probability .5. Even if consumers beliefs were such that they believed that a producer who made this disequilibrium choice had seen signal s_2, this would have a negligible effect on the subsequent price when there are many producers. As a result, the return to a producer who chose to produce widget 2 would be lower than producing widget 1.

Our point is not that a large number of agents *necessarily* leads to information revelation but only that a large number (and the consequent informational smallness) makes the return to manipulation of prices through the information revealed vanish asymptotically.

Interim vs. ex post incentive compatibility

The revealing ME is incentive compatible because at the time the seller makes his decision about which widget to produce, a change will have a small effect on the price with high probability. This is because the law of large numbers implies that for "most" realizations of the sellers' signals, the posterior on Θ given the signals puts probability close to 1 on the true state, and any single deviation in the sellers' choice of widgets will have a small effect on the posterior. For some realizations, however, a single seller's change in the widget produced can have a nonnegligible effect on the posterior. Suppose that there are 1001 sellers, and the vector of signals s is such that 500 sellers receive s_1 and 501 sellers receive s_2. $P(\theta_1|s) = .4$ in this case. Consider, however, the vector of signals s' in which one s_2 is changed to an s_1; $P(\theta_1|s') = .6$. In other words, a single seller's change in the choice of the widget to produce causes a nontrivial change in the posterior distribution on Θ, given the change in inference resulting from the production change. The nontrivial change, of course, translates into a nontrivial change in the market price.

Regardless of the number of replicas, a single seller's actions will have a nontrivial effect on market prices for some realizations of the other agents' signals. However, when the number of sellers is large, the probability that the other sellers' signals are such that any given seller will have a nontrivial effect on the price is small. Since the potential gains from any change in price are bounded, the *expected* price change resulting from a change in production will be small when there are many sellers.

The presence of many other sellers makes a given seller informationally small. Given the other sellers' information, the given seller's signal provides little additional information, and the posterior distribution on Θ is not likely to be very sensitive to his information, and hence not likely to be sensitive to his market behavior.

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