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Strategic Information Revelation

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We analyze the problem in which agents have non-public information and are to play an asymmetric information game. The agents may reveal some or all of their information to other agents prior to playing this game. Revelation is via exogenously specified certifiable statements. The equilibria resulting from various revelation strategies are used to determine equilibrium revelation of information. Sufficient conditions are provided for complete revelation of all private information. A number of examples are provided illustrating when revelation will or will not occur in commonly analyzed games.

1. INTRODUCTION

Many economic problems are characterized by the presence of both strategic behaviour on the part of individuals and by asymmetric information. Models encompassing both of these characteristics have been used successfully to examine questions in many fields, usually using the concept of Bayesian-Nash equilibrium as the solution concept. Typically the agents' information is given exogenously and is fixed throughout the analysis. Modelling the information structure as static is appropriate for some problems, but not for all, in particular, situations that are inherently dynamic. Most economic encounters allow for some interchange between the agents before the outcome is settled irrevocably. In such circumstances it is possible that at least one or more of the agents possessing non-public information may be able to reveal part or all of his information to one or all of the other agents in the economy during the course of their contact. To the extent that such a communication is believed by the other agents, the priors of the other agents will change and we will have what is in essence a new asymmetric information game (new in that the priors of the agents have changed). An agent with such a possibility of changing the priors of the other agents in this way would then presumably behave in a strategic manner with respect to his private information: reveal some or all of it if the result is an increase in his payoff.

We should be interested in the question of strategic information revelation since there are many models that "explain" particular phenomena via models that have asymmetrically informed agents. Many of the conclusions of these models depend on the asymmetry of information; if the asymmetry disappears so may the results. If agents behave strategically with respect to their private information, they may reveal all of their private information, eliminating the asymmetry.

We model the strategic revelation of private information by adding a first stage announcement game to a given game with asymmetric information. Agents' private information is modelled in the standard way of assigning a different type to each type of information that an agent might have. Agents are allowed to announce some or all of their private information by announcing a set of types, to be interpreted as "I am one of these types". We assume that certain exogenously specified announcements are certifiable, that is, can be proved by the agent making them.[1] For example, if the private information is whether a given agent owns a house, he may be able to certify that he owns one (by showing a deed, for instance) but may not be able to certify that he does not own a house. It is assumed that agents simultaneously make announcements and if an agent makes a certifiable announcement, other agents revise their beliefs according to Bayes rule. With these revised probability beliefs, the given asymmetric information game is played. We can associate to each set of beliefs, the equilibrium payoff (assuming uniqueness) of the original game with these beliefs. In this way the first stage information game can be considered as an asymmetric information game by itself and its Bayes–Nash equilibrium can be investigated.

In Section 3 we provide sufficient conditions on the information and game structure such that the unique sequential Bayes–Nash equilibrium of the information revelation game is complete revelation; that is, agents reveal all their private information. The conditions are quite strong, but there are some economic problems which are commonly modelled in a way which satisfies the conditions we show to be sufficient for revelation of private information. Also, while the conditions are not necessary for information revelation, for many problems failing to satisfy the conditions it can be shown that information revelation will not occur. We provide a number of examples that illustrate how information may not unravel in various cases that the conditions are not satisfied. This provides at least the beginning of an understanding of the circumstances under which information will not be revealed. The circumstances in which we may be sure that information will not be revealed is of importance in itself.

Our analysis applies to two specific areas of research. The first is the problem of information sharing among oligopolists. A number of recent papers (Clarke (1983), Gal-Or (1985, 1986), Novshek and Sonnenschein (1982), Palfrey (1985), Shapiro (1986) and Vives (1984), among them) have considered whether firms that have private information would have higher profits when the information is shared with other firms than when it is kept private. The problem is treated as a game with incomplete (asymmetric) information, using the Bayesian equilibrium solution concept. The equilibrium for the game with asymmetric information is compared to the equilibrium with symmetric full information. Since the welfare properties of the two equilibria differ, a very real public policy question emerges about whether to encourage or discourage information sharing among oligopolists.

The papers described above can be interpreted as investigating strategic information revelation; the firms calculate expected profits under both the sharing and the non-sharing regimes. Our approach differs from that taken in these papers in that the calculations in the previous literature were ex ante calculations of expected profits, the expectation being taken over all information that an agent might get. However, the ex ante calculation is usually at variance with the calculation made once the private information is known, at least for some realizations of the random variables. Ex ante an agent may decide that it

1. We do not treat the possibility of agents making non-certifiable announcements. More will be said on this subject below.

is better to share information than not but, upon getting his private information, finds that his expected profit given this particular private information (the expectation being over the others' private information) is lower than it would have been had his private information not been revealed. In this case the information would not be revealed if the agent could prevent it. To avoid this difficulty, most of the literature cited above assumes that the agents agree to give the private information directly to a central agency which distributes it to other agents. Alternatively, binding contracts among agents to share information might be feasible.

Consider the opposite case: the agents agree not to share information based on their (ex ante) expected profits in the sharing and the non-sharing regimes, yet an agent finds that for the particular information he received, his profits would have been higher had he shared. Again, some sort of commitment not to reveal the information would be necessary to enforce the agreement. A contract to prevent revelation might be very difficult to enforce. If the agents' private information were received by a central agency without the individual agent knowing it, there would be no problem. The possibility of such an agency may or may not be realistic.

We provide conditions that are sufficient to guarantee complete revelation of private information. While quite strong, these conditions are satisfied in many of the oligopoly models in which information sharing has been investigated. Thus, in the absence of some way to commit to not sharing information, the information will be revealed. Ex ante it may be in the agents' interest not to share, but the only sequential Bayes equilibrium involves complete sharing.

The second area of research involves a seller with private information about the quality of a good facing a potential buyer who is uninformed about the quality. The question of how much information the seller will reveal to the buyer in a sequential equilibrium has been studied by (among others) Farrell (1986), Farrell and Sobel (1983), Grossman (1981), Matthews and Postlewaite (1985), Milgrom (1981), and Milgrom and Roberts (1986). Ths focus of much of this literature has been to point out that even without laws mandating disclosure of information, sequential equilibria in the models studied reveal all relevant information to the consumer. We will discuss the relationship between the present paper and previous work in Section 6.

Perhaps more important than extending earlier work, our paper emphasizes how restrictive are the conditions which guarantee the revelation of information in equilibrium. Our work differs from most of the previous work in that we explicitly model the certifiable statements an agent may make. By making explicit which statements are certifiable, it becomes clear that it is quite restrictive to assume a sufficiently rich set of certifiable statements in many problems.

2. MODEL

Let $N = \{1, \ldots, n\}$ be the set of agents ($n \geqq 2$). The game is played in two consecutive stages numbered 1 and 2. In stage 1, agents engage in a game of information exchange. In stage 2, another game is played taking as given the private information that results from information exchange in stage 1. We shall analyze sequential equilibria of the two-stage game.

Each agent i has *private information* summarized by his *type* t_i, an element of T_i. For simplicity, T_i is assumed to be a finite set $\{t_i^1, \cdots, t_i^{l_i}\}$ whose elements are ordered as $t_i^1 < t_i^2 < \cdots < t_i^{l_i}$. For any variable (or any set or function), we denote its *profile* over all agents by the corresponding bold letter and its profile over all agents except that of agent i by the corresponding letter with subscript $-i$. Thus, t denotes the information profile $(t_i)_{i\in N} \in \times_i T_i = \mathbf{T}$ and t_{-i} denotes $(t_j)_{j\neq i} \in \times_{j\neq i} T_j = T_{-i}$.

At the beginning of stage 1, each agent i is informed of his true type $t_i \in T_i$. All other agents do not know agent i's true type. They know, however, the probability distribution of agent i's types, p_i over T_i for each i. The p_i's are assumed to be independent and to be common knowledge.

After the informational exchange that takes place in the first stage, agents have beliefs about other agents' types. Let Q_i be the set of all probabilities on T_i. A belief about agent i's type is a probability $q_i \in Q_i$. Belief q_i should be interpreted as other agents believing that the probability of agent i's true type being $t_i \in T_i$ is $q_i(t_i)$. Note that q_i, the beliefs about i, is common to all agents $j\neq i$. This means that agents $j\neq i$ interpret statements by i in the same way.

The second-stage game is defined as follows. For any agent $i\in N$, S_i is his *set of actions*. $\pi_i : \mathbf{S}\times \mathbf{T} \to R$ is his *payoff function* specifying his payoff $\pi_i(\mathbf{s}, \mathbf{t})$ when the action profile is $\mathbf{s}\in \mathbf{S}$ and the true information profile is $\mathbf{t}\in \mathbf{T}$. Agent i's *second-stage strategy* is a mapping $\sigma_i : T_i \to S_i$ that associates his choice of action $\sigma_i(t_i)$ to his own type $t_i \in T_i$. We denote by Σ_i the set of agent i's second stage strategies.

For any agent $i\in N$, his *expected payoff* of the second-stage game is defined by a mapping $\Pi_i : \mathbf{\Sigma}\times Q_{-i}\times T_i \to R$ such that, given a second-stage strategy profile $\boldsymbol{\sigma}$, beliefs about other agents' types q_{-i}, and his own type t_i, his expected payoff is defined as:

$$\Pi_i(\boldsymbol{\sigma}; q_{-i}, t_i) = E[\pi_i(\sigma_i(t_i), \sigma_{-i}(t_{-i}); t_i, t_{-i}) \mid q_{-i}]$$
$$= \textstyle\sum_{t'_{-i}\in T_{-i}} \times_{j\neq i} q_j(t'_j)\pi_i(\sigma_i(t_i), \sigma_{-i}(t'_{-i}), t_i, t'_{-i}),$$

where $E[\cdot \mid q_{-i}]$ is the expectation operator conditional upon belief q_{-i} and $\sigma_{-i}(t'_{-i}) = (\sigma_j(t'_j))_{j\neq i}$.

Given an information exchange in stage 1 that gives rise to a belief profile $\mathbf{q}$, a second-stage game is played. We call this game a "subgame" although this is a game that stems from an information set (in the extensive form representation of the entire game associated with each given information exchange in stage 1) rather than the one that stems from a node.

Given a belief profile $\mathbf{q}$, a *subgame equilibrium* is a second stage strategy profile $\boldsymbol{\sigma}^*$ such that, for all $i\in N$, $t_i \in T_i$ and $s_i \in S_i$,

$$\Pi_i(\boldsymbol{\sigma}^*; q_{-i}, t_i) \geqq \Pi_i(s_i, \sigma^*_{-i}; q_{-i}, t_i).$$

Given a belief profile $\mathbf{q}$, the subgame equilibrium is *unique* if for any two subgame equilibria $\boldsymbol{\sigma}^*$ and $\boldsymbol{\sigma}^{**}$, $\sigma^*_i(t_i) = \sigma^{**}_i(t_i)$ for any t_i for any i. We shall assume that for any possible belief profile, the subgame equilibrium is unique. This is admittedly a very restrictive and critical assumption. However, without this assumption problems arise from the selection of one of several equilibria in the second stage that could arise from a single vector of beliefs. Moreover, our proof relies on a property that is guaranteed under uniqueness but that may fail if we simply took a selection from a correspondence that was multiple-valued.

Two points should be made in defence of the assumption of uniqueness. The first is that a large class of games, including those referred to in the introduction, satisfies the assumption. Thus, an interesting set of problems falls within our analysis.

The second point is that in the case that there is not a single equilibrium, there will always be "coordination equilibria" involving communication. By coordination equilibria we mean that all agents play one equilibrium following one statement and a second equilibrium following a different statement. In this way there is always the possibility of communication altering the set of equilibria if there are multiple subgame equilibria.

Coordination equilibria may well involve no transmission of information, i.e., no belief may be altered by the statements made. Our purpose in this paper is to investigate how the possibility of communication may eliminate the asymmetry of information. The assumption of uniqueness allows us to focus on this question.

Having assumed uniqueness, the *subgame equilibrium expected payoff* to agent $i \in N$ of type $t_i \in T_i$ when belief profile is $\boldsymbol{q}$ is unambiguously defined. We shall write this payoff as $u_i(\boldsymbol{q}, t_i)$.

In stage 1, agents exchange information in the form of reporting a subset of T_i. Agents may choose not to participate in information exchange. We interpret this choice as agent i reporting the whole set T_i. When i's true type is $t_i \in T_i$, a report $x_i \subset T_i$ is said to be *truthful* if $t_i \in x_i$. We assume that a report is not believed unless its truthfulness is certified.[2] As we discussed in the previous section, some reports, though truthful, may not be certifiable. The report that an agent is of the type "he does not know anything" is such an example in many situations (see example 3 below). In general, which sets are certifiable depends upon the particular problem we are analyzing. In this paper, we shall assume that the set of certifiable reports is exogenously given and that certifiable reports can be made and certified without incurring any cost.

Formally, for any $i \in N$ let Δ_i be the collection of all non-empty subsets of T_i and $\Delta_i(t_i) = \{x_i \in \Delta_i \mid t_i \in x_i\}$, i.e. the set of truthful reports when his type is t_i. Let $\Delta_i^*(t_i) \subset \Delta_i(t_i)$ be the collection of truthful reports of i that are certifiable when his true type is t_i. We allow no revelation to be a possibility, by assuming $T_i \in \Delta_i^*(t_i)$. For $i \in N$ his feasible *reporting strategy* is a mapping $\rho_i : T_i : T_i \to \Delta_i$ satisfying $\rho_i(t_i) \in \Delta_i^*(t_i)$ for all $t_i \in T_i$. We denote by $x_i = \rho_i(t_i) \in \Delta_i$ agent i's report.[3]

After the first stage, the report profile is certified and becomes common knowledge. Thus, for example, if a singleton set is certifiable and such a set is reported and certified, then it becomes common knowledge that the agent who reported the singleton must be of the reported type.

Agents will make inferences about other agents' types from the report profile. An *inference function* about the type for an agent $i \in N$ is a mapping $b_i : \Delta_i \to Q_i$. Thus, given agent i's report $x_i \in \Delta_i$ and inference function b_i, other agents' beliefs about agent i's type are represented by a posterior probability distribution $b_i(t_i \mid x_i)$.

Our assumption that only certifiable reports are sent naturally gives rise to the following definition of consistent beliefs: An inference function profile $\boldsymbol{b}$ is said to be *consistent* with a reporting strategy profile $\boldsymbol{\rho}$ if for all $i \in N$, $\boldsymbol{t} \in \boldsymbol{T}$, and for all $\boldsymbol{x} \in \Delta$,

$$\text{(a)} \quad b_i(t_i \mid x_i) = 0 \quad \text{if } t_i \notin x_i,$$

and

$$\text{(b)} \quad b_i(t_i \mid x_i) = \frac{p_i(t_i)}{\sum_{t_i' \in \rho_i^{-1}(x_i)} p_i(t_i')} \quad \text{if } t_i \in \rho_i^{-1}(x_i).$$

(a) is simply a statement that agents' beliefs reflect the assumption that only a truthful report is certifiable, and hence the posterior beliefs should put full probability on the

2. The assumption that non-certifiable statements will not be believed separates this paper from the literature investigating how the possibility of such "cheap-talk" may shrink the set of equilibria (see, e.g. Farrell (1985), Grossman and Perry (1986), Okuno-Fujiwara and Postlewaite (1987), and Matthews, Okuno-Fujiwara and Postlewaite (1989). Our main results involve sufficient conditions for full revelation of all private information. Allowing non-certifiable statements will not alter these results; in this sense the assumption involves no loss in generality. We discuss these issues further in Section 6.

3. If agents were allowed to use random reporting strategies, the equilibrium described in the text would continue to be an equilibrium. We have not investigated whether additional equilibria might arise when random reporting strategies are used, but we expect not.

certified report. (b) is a statement that, for reports that are actually sent for some type, the beliefs are consistent with Bayesian updating.

An inference function profile $\boldsymbol{b}$ is called *sceptical* if for all $i \in N$, for all $t_i \in T_i$ and for all $x_i \in \Delta_i^*(t_i)$:

$$b_i(t_i \mid x_i) = \begin{cases} 1 & \text{if } t_i = \min\{t_i' \mid t_i' \in x_i\} \\ 0 & \text{otherwise.} \end{cases}$$

A *sequential equilibrium*[4] is a pair $(\boldsymbol{\rho}^*, \boldsymbol{b}^*)$

(a) $\boldsymbol{b}^*$ is consistent with $\boldsymbol{\rho}^*$, and
(b) for all $i \in N$, $t_i \in T$ and $x_i \in \Delta_i^*(t_i)$,

$$E[u_i(\boldsymbol{b}^*(\rho_i^*(t_i), \rho_{-i}^*(t_{-i}')), t_i) \mid p_{-i}] \geqq E[u_i(\boldsymbol{b}^*(x_i, \rho_{-i}^*(t_{-i}')), t_i) \mid p_{-i}].$$

A sequential equilibrium $(\boldsymbol{\rho}^*, \boldsymbol{b}^*)$ is said to *reveal private information completely* if for all $i \in N$ and for all $t_i \in T_i$, $b_i^*(t_i \mid \rho_i^*(t_i)) = 1$.

3. MAIN RESULTS

Before stating our main results, we need a few additional definitions. For any $i \in N$ and $t_i^k \in T_i$, the associated *degenerate belief on* t_i^k is $q_i^k \in Q_i$ satisfying:

$$q_i^k(t_i) = \begin{cases} 1 & \text{if } t_i = t_i^k \\ 0 & \text{otherwise.} \end{cases}$$

A belief about i, q_i, *stochastically dominates* an alternative belief, q_i', if for all $t_i \in T_i$,

$$\textstyle\sum_{t_i' \leqq t_i} q_i'(t_i') \geqq \sum_{t_i' \leqq t_i} q_i(t_i')$$

and strict inequality holds for at least one $t_i \in T_i$. It follows straight-forwardly that, given any non-degenerate belief, q_i, the degenerate belief associated with the highest (lowest, resp.) type in the support of q_i stochastically dominates (is stochastically dominated by, resp.) q_i.

The subgame equilibrium payoff is said to be *positive-monotone in beliefs* if the following property holds for any belief profile $\boldsymbol{q}$, any agent i and any alternative belief about i's type q_i':

If q_i' stochastically dominates q_i then for all $t_i \in T_i$,

$$u_i((q_i', q_{-i}); t_i) > u_i(\boldsymbol{q}; t_i).$$

The subgame equilibrium payoff is said to be *weakly positive-monotone in beliefs* if, for any i, t_i, $\boldsymbol{q}$ and q_i' where q_i is non-degenerate and q_i' is the degenerate belief associated with the highest (lowest, resp.) type in the support of q_i, $u_i(\boldsymbol{q}; t_i)$ is greater (smaller, resp.) than $u_i((q_i', q_{-i}); t_i)$.

Note that this definition of weak positive-monotonicity is equivalent to the following: for all i, t_i, $\boldsymbol{q}$, q_i', $u_i((q_i', q_{-i}); t_i) > u_i(\boldsymbol{q}; t_i)$ if q_i' stochastically dominates q_i and the intersection of the supports of q_i and q_i' is either empty or a singleton.

Assumption 1. For all $i \in N$ and for all $t_i \in T_i$, there exists $x \in \Delta_i^*(t_i)$ such that $t_i = \min\{t \mid t \in x\}$.

4. This concept was introduced by Kreps and Wilson (1982) although we formulate it slightly differently.

This is an assumption that the set of certifiable statements an agent can make is sufficiently rich. It states that each type of each agent can certify that he is in a subset for which his true type is minimum.

In the case where the private information that a player has is about some physical quantity, say his cost of producing a good, Assumption 1 may plausibly be satisfied. For some problems, a player may or may not have private information about such a physical characteristic, and further only he may know whether or not he has such private information. In such a case, one of the player's types may correspond to the case in which he does not have particular information. If a player cannot certify that he does not have particular information, Assumption 1 may fail to hold. Example 3 in the next section presents a case in which incomplete revelation results from such a failure of Assumption 1.

Theorem 1. *Under Assumption 1, if the subgame equilibrium payoff is weakly positive-monotone in beliefs, then the only sequential equilibrium is that of complete revelation with the sceptical inference function profile.*

Corollary. *Under Assumption 1, if the subgame equilibrium payoff is positive-monotone in beliefs, the same result follows.*

From this theorem (or corollary), we need only check whether the subgame equilibrium payoff function is (weakly) positive-monotone in beliefs in order to identify whether complete revelation occurs. However, determining whether the function has this property may not be straightforward. It would be more informative to identify a set of sufficient conditions on the underlying game structure that guarantees the property. The next theorem provides such a set of conditions.

Assumption 2 (*Dimensionality*).

(a) For all $i \in N$, S_i is the closed interval $[0, \bar{s}]$ of the real line.
(b) For all $i \in N$, $s_{-i} \in S_{-i}$, and $t \in T$, $\pi_i(0, s_{-i}, t) > \pi_i(\bar{s}, s_{-i}, t)$.

Most commonly analyzed problems satisfy this assumption, including single-good oligopoly models. Example 4 below considers a multi-good Cournot problem which violates this assumption in which incomplete revelation occurs.

Assumption 3 (*Interiority*). For any consistent belief profile $\boldsymbol{q}$, the subgame equilibrium $\boldsymbol{\sigma}^*$ is interior; that is, for any $i \in N$, $t_i \in T_i$, $\sigma_i^*(t_i) \in (0, \bar{s})$.

This assumption is straightforward. It requires the equilibrium in the subgame resulting from any consistent belief profile to be interior. While it is primarily a technical assumption, it is not completely innocuous. Example 4 of the next section presents a natural case in which the assumption fails and incomplete revelation results.

Assumption 4. For all $i \in N$, $s \in S$, and $t \in T$,

(a) π_i is concave and differentiable in s_i, and decreasing in s_{-i},
(b) (Strategic substitutes or *SS*)

$$\frac{\partial}{\partial s_i} \pi_i(s, t) \text{ is continuous and decreasing } in\ s_{-i},$$

(c) (Positive-monotonicity of best response functions under SS)

$$\frac{\partial}{\partial s_i}\,\pi_i(s, t) \text{ is increasing in } t_i \text{ and non-increasing in } t_{-i}.$$

Assumption 4 is, in a sense, the critical assumption. Part (a) is straightforward and enables us to say that an increase in s_j can be interpreted by i as j becoming more *aggressive*. Parts (b) and (c) are assumptions on the best response (or reaction) functions. Consider a particular agent; given his and others' types, a best response function (assuming all agents' types are known to him) is defined. Part (b) states that all these functions are downward sloping. In terms of Bulow *et al.* (1985), this is an assumption that second stage actions are *strategic substitutes.* Part (c) states that the best response functions for an agent's types are ordered in a particular way. As the agent's type increases, his best response function shifts out (making his behaviour more aggressive) while other agents' best response functions shift in or stay the same (making their behaviour weakly less aggressive.)

Positive monotonicity holds in many asymmetric information problems commonly studied, including many oligopolistic models. Even in these models, however, if the information structure becomes a little more complex, Assumption 4(c) may fail. Examples 6 and 7 in the next section illustrate how positive monotonicity may fail in quite reasonable problems, and as a result, give rise to incomplete revelation of private information in equilibrium. Example 8 demonstrates that positive monotonicity is not a necessary condition. In this example, complete revelation of information occurs despite the failure of positive monotonicity.

Theorem 2. *If $n = 2$, if Assumptions 2–4 hold, and if the subgame equilibrium is unique for any $q \in Q$ and $t \in T$, then the subgame equilibrium payoff is weakly positive-monotone in beliefs.*

Corollary. *If $n = 2$, if Assumptions 1–4 hold, and if the subgame equilibrium is unique for any beliefs and type profiles, then all sequential equilibria completely reveal private information. Moreover, the equilibrium inference function profile is sceptical.*

The following example illustrates Theorem 2 and its corollary.

Example 1. (Sequential equilibrium in which private information is revealed.)

This is one of the simplest examples in which the assumptions we have shown to be sufficient for revelation of the private information hold.

Consider a Cournot duopoly problem in which each firm (agent) has constant marginal cost. Firm 1's marginal cost is C and is known for certain. Firm 2's marginal cost can be one of two equally likely values—high, H, or low, L. Firm 2 knows its own marginal cost, while firm 1 knows only the possible values and the probabilities associated with the values. The two firms face a linear inverse demand function $p(x) = a - bx$ where x is the combined quantity that the firms produce. It is assumed that this structure is common knowledge. We will look for a Bayes–Nash equilibrium for the incomplete information game they will play. An equilibrium is a quantity that firm 1 produces, s_1, and a pair of quantities s_2^H and s_2^L for firm 2 depending upon its actual costs. These choices should be optimal for each of the firms given its information at the time that it must make its decision. Hence, s_1 is a best response to the quantity that firm 2 puts on the market which is taken to be random with value s_2^H or s_2^L, each with probability $\frac{1}{2}$. In Figure 1, the best response functions, or reaction curves, are shown. When firm 2's marginal cost is high its best response to firm 1's choice s_1 is shown as R_2^H. It is linear because we assume linear demand and constant marginal cost. Similarly, when firm 2's marginal cost is low its best response function is linear but is higher now due to its lower cost. It is shown as R_2^L. (Note that, in this example, low cost information corresponds to high type and high cost to low type).

In general, we would not be able to graph a best response function for firm 1 in the same way. The normal way of calculating a firm's optimal level of output given a fixed level of output of the other firm would not make sense here. Since firm 1 does not know firm 2's cost and firm 2's optimal output will depend upon its cost, firm 1 will essentially be facing a lottery: firm 2 will produce one output when its cost is low and a different output when its cost is high. Thus, in general, firm 1's best response function will be a function mapping pairs of outputs into its optimal response. But the fact that the demand and marginal cost functions are linear makes it possible for firm 1 to calculate its optimal output given only the expected output of the other firm. We have drawn firm 1's best response function R_1, where this should be interpreted in exactly this way: it represents firm 1's profit maximizing output given an expected output of firm 2. We have also drawn in the "average" of the two different best response functions for firm 2 given its possible costs; this is denoted R_2^A. The equilibrium for this example will then be an output of s_1 for firm 1 and outputs s_2^H and s_2^L for firm 2 for costs H and L, respectively. As can be seen from the diagram, s_2^H and s_2^L are each best responses to s_1 for firm 2 for each of its possible costs. For firm 1, s_1 is its best response to the average of these two values.

If firm 2 can certify its marginal cost to firm 1, it will have an incentive to do so when its costs are low. In this case the equilibrium will not be determined by the intersection of R_1 and R_2^A but rather by the intersection of R_1 and R_2^L since the latter will then be known to be the relevant best response function for firm 2. Hence the equilibrium is the pair (s_1, s_2^L). The movement from that part of the original equilibrium

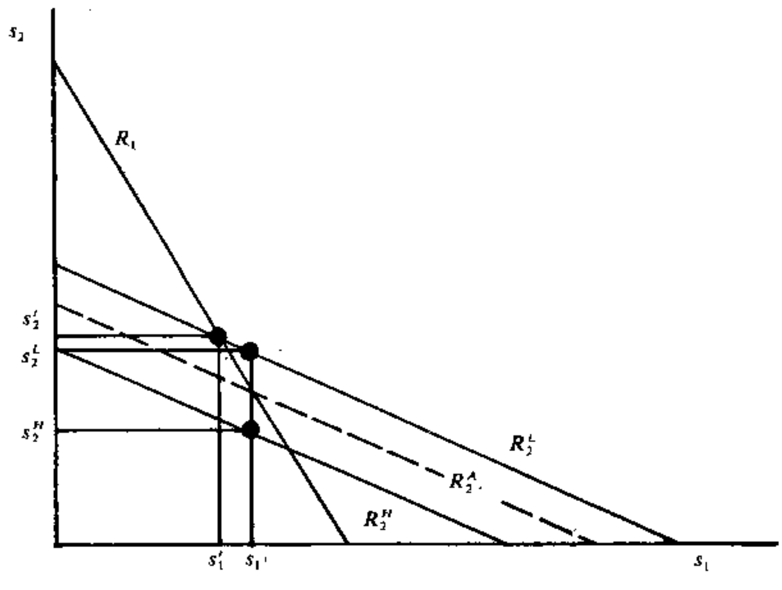

FIGURE 1

that was relevant when firm 2 had low cost to this new point involves firm 1 producing less and firm 2 producing more. The combined output will be lower than before since both points will be on R_2^L which has slope less than 1. Thus the price will be higher, which in conjunction with the increase in firm 2's output guarantees that firm 2's profit will be higher; this means that firm 2 has an incentive to reveal (and certify, of course) its marginal cost whenever it is low. The only reasonable inference that firm 1 can make when firm 2 does not reveal any information (in other words, announces that it is simply one of the two possible types) is to infer that its cost is high. With a first stage appended to the game in which firm 2 can announce and certify its cost to firm 1, the only sequential equilibrium will involve revealing all of firm 2's information to firm 1.

Let us turn our attention to Assumption 4 of the theorem. For expository purposes, this assumption is stated in a more restrictive form than necessary. As is easily seen from Example 1, the assumption guarantees the following property: if an agent makes other agents believe he is of higher type, (1) other agents' behaviour will become less aggressive, and (2) they will believe the agent has become more aggressive, inducing even less aggressive behaviour on their part as the subgame actions are strategic substitutes. These changes in other agents' behaviour will yield a higher profit to the given agent. The same property will hold even in the case of strategic complements if we alter part *c* of Assumption 4. More specifically, let:

Assumption 4′. For all $i \in N$, $s \in \boldsymbol{S}$, and $\boldsymbol{t} \in \boldsymbol{T}$,

(a) π_i is concave and differentiable in s_i, and decreasing in s_{-i},

(b) (Strategic complements or SC)

$$\partial/\partial s_i\, \pi_i(\boldsymbol{s}, \boldsymbol{t}) \text{ is continuous and increasing in } s_{-i},$$

(c) (Positive-monotonicity of best response function under SC)

$$\partial/\partial s_i\, \pi_i(\boldsymbol{s}, \boldsymbol{t}) \text{ is decreasing in } t_i \text{ and non-increasing in } t_{-i}.$$

We then have:

Theorem 3. *If $n = 2$, if Assumptions 2, 3, and 4′ hold, and if the subgame equilibrium is unique for any $\boldsymbol{q} \in \boldsymbol{Q}$ and $\boldsymbol{t} \in \boldsymbol{T}$, then the subgame equilibrium payoff is weakly positive-monotone in beliefs.*

The proof of Theorem 3 is essentially the same as Theorem 2 and will be omitted. Intuitively, the logic is obvious. By part (c) of Assumption 4′, if an agent's type increases and other agents are aware of this, they believe the agent's behaviour will become less aggressive and their own behaviour becomes less aggressive. Since the subgame actions are strategic complements, these changes make other agents' behaviour less aggressive and will provide a higher payoff to the given agent.

Example 2 below illustrates the case in which part (b) of Assumption 4 is changed to the case of strategic complements but part (c) is left intact. Alternatively, this example may be seen as satisfying Assumption 4′(a) and 4′(b) but not 4′(c). Positive monotonicity in beliefs fails in this example.

Example 2. (Incomplete revelation of private information due to strategic complementarity.) Consider a Bertrand duopoly in which two firms produce differentiated products with constant marginal cost. The demand function for each product is

$$q_i = 1 - bp_i + cP_j$$

where $0 < c < b$ and $i, j = 1, 2 (i \neq j)$. As usual, the second-stage strategy for each firm i, s_i, is considered as the negative of P_i. With this interpretation, Assumption 4(a) is satisfied but not 4(b). That is, the second-stage strategies are strategic complements instead of strategic substitutes.

Firm 1's marginal cost is c and is common knowledge. Firm 2's marginal cost can be one of two equally likely values—high, H, or low, L. This value is private information known only to firm 2. Following Assumption 3, we assume firm 2 can certify an upper bound on its marginal cost. That is, the sets of marginal costs it can certify are either $\{H, L\}$ or $\{L\}$, but not $\{H\}$. Intuitively, it may not be difficult for the firm to show that the marginal cost is L, that is, its private information is in $\{L\}$. It could do this by showing the production line runs very fast, for example. But running the line slowly doesn't guarantee that it cannot run faster, i.e. that its private information is in $\{H\}$.

Given this structure the best response function for each firm when the value of marginal cost is common knowledge is linear in prices. Figure 2 illustrates the problem diagramatically. Here, R_1 is firm 1's best response function while R_2^H (R_2^L, resp.) represents firm 2's best response function when its marginal cost is high (low, resp.) Following the previous example, R_2^A represents firm 2's average best response function.

When firm 2's marginal cost is not known by firm 1, the equilibrium is determined by R_1 and R_2^A. Namely, firm 1 will choose P_1^A while firm 2 will choose P_2^H or P_2^L depending on its private information. If 2's cost is low and if 2 certifies this information, the new equilibrium will be determined by the intersection of R_1 and R_2^L. Firm 2's price is now lower and 1's demand becomes smaller because of demand substitution. On the

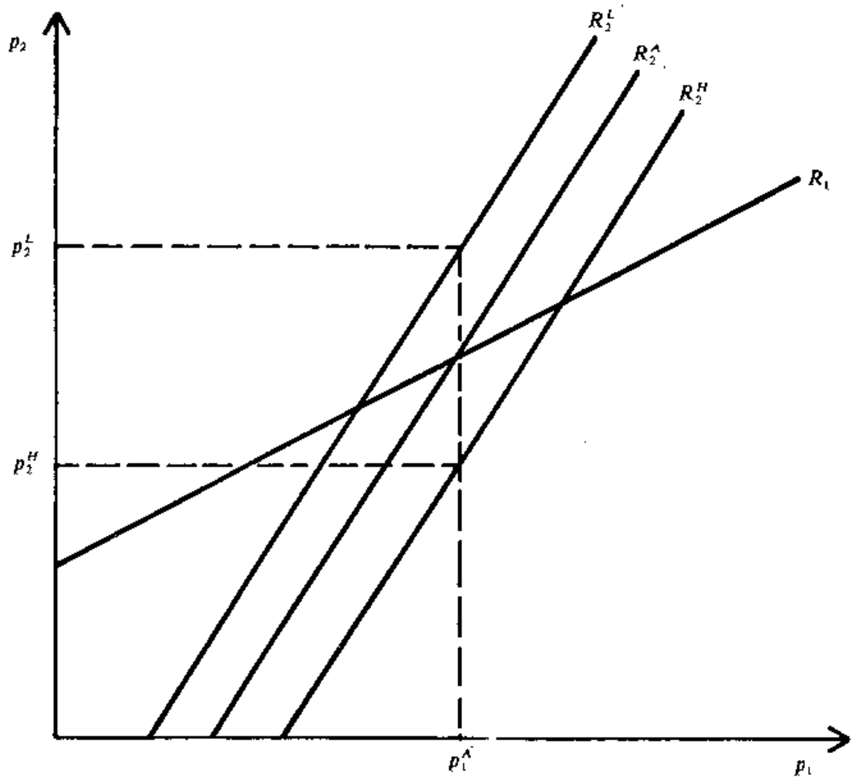

FIGURE 2

other hand if firm 2 can, somehow, persuade firm 1 to believe that its true marginal cost is high, the equilibrium will be determined by the intersection of R_1 and R_2^H. Firm 1's price will be higher than it is with no information revelation and 2's profit will increase.

Thus, when firm 2's marginal cost is high, firm 2 has an incentive to reveal this information. If firm 2 reveals that its marginal cost is high and successfully makes 1 believe it, 1's behaviour becomes less aggressive due to strategic complementarity. In Bertrand competition, less aggressive play is more profitable and firm 2 will obtain a higher profit by revealing its information.

However, by assumption, firm 2 cannot credibly certify that its own cost is high; even the low cost firm 2 will have the incentive to claim that its cost is high. Even though firm 2 may say its cost is high (and show to firm 1 that it can run the line quite slowly), it cannot persuade firm 1 of its truthfulness. A low cost firm 2 will have no incentive to reveal the information because doing so lowers its profit. The resulting equilibrium will involve no information revelation regardless of the level of firm 2's marginal cost.

In Example 2, because either Assumption 4(b) or 4′(c) is not satisfied, positive monotonicity in beliefs fails. That is, the lower type firm has the incentive to reveal private information, but under Assumption 1 such information is not certifiable. If, however, we assume all the singleton sets are certifiable, full revelation must result even in Example 2.

More precisely, the subgame equilibrium payoff is said to be *negative monotone in beliefs* if for all $\mathbf{q}\in \mathbf{Q}, i\in N, \mathbf{t}\in \mathbf{T}$, and $q_i'\in Q_i$, $u_i((q_i', q_{-i}), t_i) < u_i(q, t_i)$ whenever q_i' stochastically dominates q_i and the intersection of the supports of q_i and q_i' is either empty or a singleton. The equilibrium payoff is said to be *monotone in beliefs* if it is either positive-monotone or weakly negative-monotone in beliefs.

Assumption 1′. For all $i\in N$ and for all $t_i\in T_i$, $\{t_i\}\in \Delta_i^*(t_i)$.

Assumption 1′ is an assumption that each of the singleton sets is certifiable. Clearly this is stronger than Assumption 1.

Theorem 4. *Under Assumption 1′ if the subgame equilibrium payoff is weakly monotone in beliefs, then the only sequential equilibrium is that of complete revelation with the sceptical inferences.*

The proof is straightforward. If the equilibrium payoff is positive-monotone, the assertion holds trivially from the corollary to Theorem 1. If it is negative-monotone then reverse the order of types and apply the same corollary.

Under Assumption 1′, we can relax Assumption 4 to guarantee complete revelation.

Assumption 4(*d*). (Negative monotonicity of best response functions under *SS*) For all $i\in N, \mathbf{s}\in \mathbf{S}$, and $\mathbf{t}\in \mathbf{T}$

$$\partial/\partial s_i\, \pi_i(\mathbf{s},\mathbf{t}) \text{ is decreasing in } t_i \text{ and non-decreasing in } t_{-i}.$$

Theorem 5. *Suppose* $n=2$, *Assumptions* 2, 3, *and either* 4(a)–(c) or 4(a), 4(b), 4(d) *hold, and the subgame equilibrium is always unique. Then the subgame equilibrium payoff is weakly monotone in beliefs.*

The proof of this theorem is again straightforward and will be omitted. Furthermore, this theorem can be generalized to cover the case of strategic complements just as Theorem 3 is a generalization of Theorem 2.

We turn our attention to the case in which there are more than 2 agents. For $n > 2$, there is a good reason to believe a set of similar (possibly stronger) conditions would guarantee complete revelation. However, we are only able to provide a more stringent set of sufficient conditions; namely, we will replace Assumption 4 by:

Assumption 5 (*Linearity*). For all $i \in N$, $\pi_i(s,t) = c\{a_i(t) - \sum_{j \neq i} ds_j - s_i\}s_i$, where $a_i(t)$ is increasing in t_i and non-increasing in t_{-i}, c and d are parameters satisfying $c > 0$ and $2 > d > 0$.

Under Assumption 5, the best response function for i given others' choices, s_{-i}, and type profile $\boldsymbol{t}$ is linear. Moreover, this assumption is consistent with all the properties of Assumption 4. We note that Example 1 above, as well as several examples below satisfy this assumption.

Theorem 6. *For any finite n, if Assumptions* 2–5 *hold and the subgame equilibrium is unique for any* $\boldsymbol{q} \in \boldsymbol{Q}$ *and* $\boldsymbol{t} \in \boldsymbol{T}$, *then the subgame equilibrium payoff is weakly monotone in beliefs.*

Corollary. *For any finite n, if Assumptions* 1–5 *hold and the equilibrium is unique for any beliefs and type profiles, then all sequential equilibria completely reveal private information. Moreover, the equilibrium inference function profile is sceptical.*

4. EXAMPLES

In this section, we will provide further examples in which various subsets of the assumptions on the second-stage game structure hold. In these examples, sequential equilibria may or may not completely reveal private information. Through these examples, the role each assumption plays in the revelation of private information will become clear. The examples will also show how information will not be revealed in nonpathological situations when some of the assumptions do not hold.

Example 3. (Incomplete revelation of private information due to non-certifiability of some type.) This example will be the same as Example 1 except that we will allow the possibility that firm 2 might not know its own cost. Firm 2 then has 3 possible types—t_2^1, that it has high cost and knows it, t_2^2, that it does not know its cost (but knows that it is high or low), and t_2^3, that it has low cost and knows it. We will assume that each of the three types is equally likely. We will assume as before that when firm 2 knows its cost it can certify it to firm 1, say by conducting a laboratory experiment that reveals its technology and cost. (This might not be possible; it is an assumption.) We will assume, however, that if firm 2 is of type t_2^2 it cannot certify it. Again, this is an assumption although it does seem that it would be difficult to certify that one did not know something. The best response functions for firm 2 when it is of type t_1 or t_2 are exactly as in the previous case. When it is of type t_2, that is, when it does not know its own cost, firm 2 will maximize its expected profits. The assumptions of linear demand and constant marginal cost gives us a best response function in this case which is precisely the average of the two best response functions when it knows its costs. This will also be the average of the three best response functions over the three possible types that firm 2 can be. Firm 1's best response function is as it was in Example 1. The equilibrium for this example involves firm 1 producing s_1 and firm 2 producing s_2^1, s_2^2, or s_2^3 depending upon whether its type is t_2^1, t_2^2, or t_2^3 respectively.

As before if firm 2 is of type t_2^3 it will have an incentive to reveal (and certify) this to the other firm and raise its profits. Thus when firm 1 hears no announcement, it knows that it must not be facing t_2^3. Then when firm 2 is of type 2, it has an incentive to reveal itself, the problem has reduced itself to that in Example 1. The two types have different best response functions and the one with the higher best response function will increase its profits by revealing itself. But we have assumed that this type cannot certify its type; there is no way (by assumption) that it can prove that it is not a type 1, that its cost is high and it knows it. Thus there will be only partial revelation of private information: t_2^3 will announce its type and t_2^1 and t_2^2 will announce nothing. Type 2 could, of course, announce that he has type 2, but then so could type 1. Since it cannot be certified, it is equivalent to announcing nothing. The final equilibrium will then be of the form that type 3 of firm 2 reveals itself and a complete information Cournot game follows, while types 1 and 2 will not be separated and an incomplete information Cournot game follows in this case.

We should note that the failure of Assumption 1 causes the information to not be revealed here. Monotonicity, as well as our other assumptions of Theorem 1, hold for this example.

For the rest of the examples, we shall assume that all the singletons are certifiable.

Example 4. (No revelation of private information due to corner equilibrium in the second stage, i.e., a failure of Assumption 3.)

Let there be two firms playing a Cournot game in the second stage with inverse demand function $p(x) = 1 - x$. Firm 1 has costless production while firm 2 has constant marginal cost of either 3 or 0, each equally likely. Firm 1 knows firm 2's marginal cost, but firm 2 does not.[5] Firm 1's marginal cost is common knowledge.

If no information is revealed, it is as though firm 2 has a production cost of 3/2 per unit. Since the highest possible price per unit is 1, firm 2 chooses $s_2 = 0$ in equilibrium. Hence when no information is revealed, firm 1 chooses the monopoly quantity and makes monopoly profits. If firm 1 reveals its private information when firm 2's cost is low, the game will be a symmetric Cournot game in which firm 1's profit is lower than the monopoly profit level. Revealing the information when the cost is high gives rise to a situation similar to no revelation: firm 2 chooses $s_2 = 0$ and firm 1 makes monopoly profit. In either case firm 1's profit is no larger with revelation than without revelation. Thus no revelation is an equilibrium.

Example 5. (No revelation of private information due to a multi-dimensional strategy space at the second stage.) Suppose there are two firms, 1 and 2, and two goods, A and B. The demand functions for the two goods are uncertain. Firm 1 believes that each of the following cases is equally likely.

$$\{p_A = 1 - x_A, p_B = 0\}, \qquad \{p_A = 0, p_B = 1 - x_B\}, \qquad \{p_A = 1 - x_A, p_B = 1 - x_B\}.$$

5. It is not as far-fetched as it may seem that firm 1 knows firm 2's cost and firm 2 does not. One can imagine a firm in an industry that, through experience, has determined the costs of various technologies for producing a good. Firm 2 may be a potential entrant with access to a technology about which firm 1 knows more than firm 2.

Firm 2 knows which case is correct. We therefore associate firm 2's type with the demand configuration. Each firm has the same technology, characterized by the cost function $c(x_A, x_B) = \frac{2}{3}(x_A + x_B)$. The Bayes-Cournot equilibrium has firm 1 producing 0, since from its point of view, the maximum expected price for either good that could arise is 2/3 while its marginal cost is $\frac{2}{3}$. Thus firm 2 receives monopoly profits in the Bayes-Cournot equilibrium when its type remains unknown to firm 1. Whichever demand function arises (i.e. for whichever type firm 2 is), revealing this type leads to a symmetric Cournot equilibrium with the actual demand functions common knowledge. This yields lower profits for firm 2 than concealing its type; hence, not revealing its type is the unique equilibrium.

Note that this example does not satisfy monotonicity.

Example 6. (No revelation of private information due to a lack of monotonicity.) Suppose there are two firms, 1 and 2, and a single good. Assume that the demand for the good is represented by $p(x) = a - bx$. Firm 1's marginal cost is $mc_1(s_1) = 0$ while firm 2's is either $mc_2(s_2) = 1 - s_2$ or $mc_2(s_2) = 1 + s_2$. Assume that neither firm knows 2's cost function but that each gets one of two signals regarding the marginal cost function. The relation of the signals to firm 2's marginal cost is as in the table below.

		Firm 2 t^1	Firm 2 t^2
Firm 1	t^1	$1 - s_2$	$1 + s_2$
Firm 1	t^2	$1 + s_2$	$1 - s_2$

Each pair of signals is equally likely; thus, a single firm's signal gives no information about firm 2's marginal cost. With no further information, firm 2 simply maximizes expected profit and the Bayes-Cournot equilibrium is a deterministic outcome independent of signals. If firm 2 were to tell firm 1 its signal, nothing would change; only firm 2 can use additional information to alter its decision. If firm 2 learns firm 1's signal, however, it will know its own marginal cost and will make its output level dependent on this. Firm 2's choice is represented in Figure 3. The marginal revenue function associated with the residual demand function is shown along with firm 2's possible marginal cost functions, $mc_2(s_2) = 1 + s_2$ and $mc_2(s_2) = 1 - s_2$; also shown is $mc_2(s_2) = 1$, which is the expected marginal cost that firm 2 uses to choose an output level when its marginal cost is unknown. Its choice in this case will be s_2^U. If firm 2 learns its marginal cost its choice will be either s_2^H and s_2^L depending upon whether the "high" or "low" marginal cost has occurred. It can be shown that s_2^U is less than the expected value of s_2^H and s_2^L (at least for some parameters). Thus for any choice of output by firm 1, firm 2's expected optimal response when it learns its true marginal cost is higher than its optimal response to this output level without the information. We have illustrated the best response functions and the equilibria for each situation in Figure 4. Firm 1's reaction curve, independent of whether firm 2 knows its marginal cost, is R_1. Firm 2's reaction curve is R_2^U when it does not know its costs. R_2^H and R_2^L are firm 2's reaction curves when it learns its costs are "high" or "low" respectively; R_2^E is its expected quantity produced in response to firm 1's output. When firm 2 does not know its cost, s_1^U and s_2^U are the equilibrium quantities in the Bayes-Cournot equilibrium. If firm 2 knows its cost, firm 1's output drops to s_1 and firm 2's equilibrium output is s_2^H or s_2^L, depending upon its cost. With

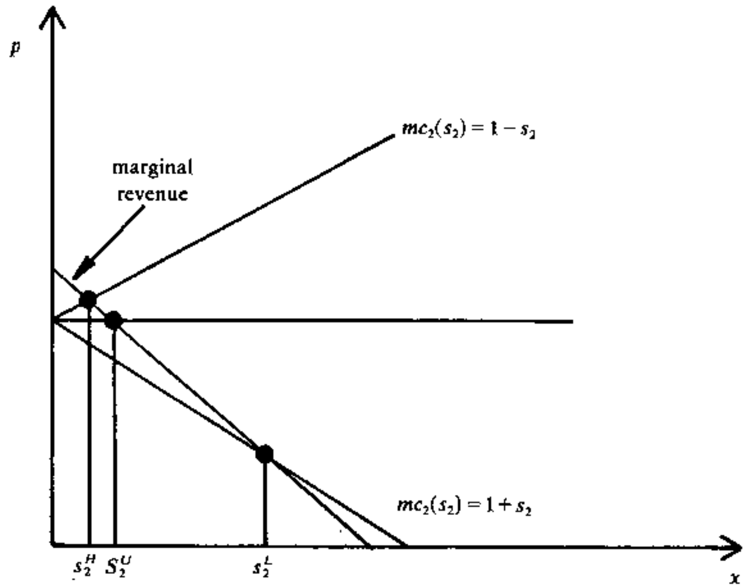

FIGURE 3

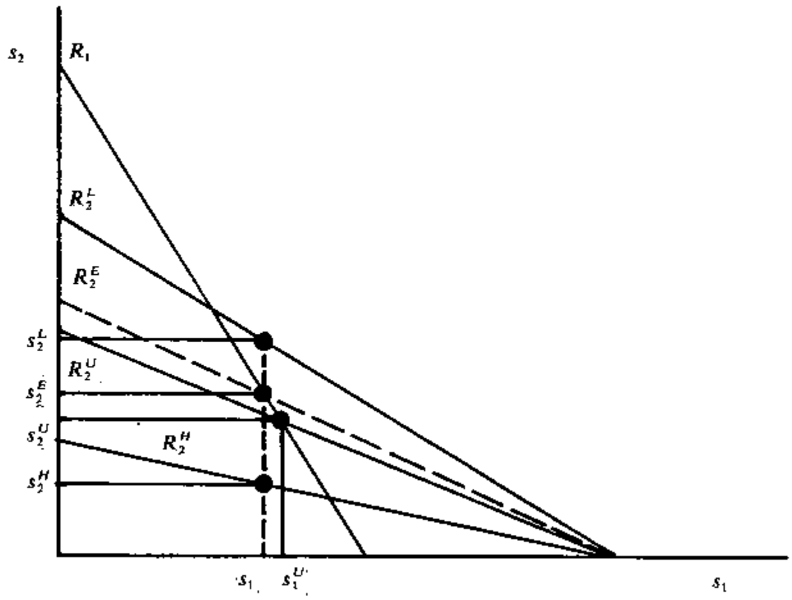

FIGURE 4

revelation of information, expected total quantity goes up, hence expected price goes down. Also firm 1's quantity goes down, and thus firm 1's expected profits go down with revelation. This is sufficient to certify that no information revelation is an equilibrium for the first stage.

Example 7. (Example in which information about a firm's cost is not revealed due to the impossibility of partially revealing the firm's private information; formally, monotonicity fails.) Again consider a Cournot duopoly problem with a linear demand function for a single good. Firm 1 can be of four types, t^1, t^2, t^3, or t^4; firm 2 has a single type. Both firms' marginal costs depend on firm 1's type as shown in the table below.

firm 1's type	t^1	t^2	t^3	t^4
firm 1's mc	H	H	L	L
firm 2's mc with technology 1	H	L	H	L
firm 2's mc with technology 2	L	H	L	H

The certifiable sets for firm 1 are the singleton sets and the entire set. The interpretation of the example is as follows. There are two technologies available for producing the good in question. It is common knowledge that exactly one of these two technologies is "good," that is, that it leads to a low marginal cost. The other technology will lead to a high marginal cost. Firm 1 has chosen and committed to one of the technologies, but firm 2 does not know which one. For example, when firm 1's type is t^1, technology 2 is the low-cost technology, but firm 1 has chosen technology 1. Firm 2's cost depends upon which technology it chooses (not knowing, of course, which is the better technology). Thus, the game involves firm 2 choosing a technology and an output level and firm 1 choosing only an output level.

If firm 1 reveals no information about its type, firm 2 will choose either technology and the two firms will play the ensuing incomplete information Cournot game. If firm 1 could reveal only its marginal cost, it is straightforward to show that when its cost is low, the profits in the equilibrium when it reveals this to firm 2 are higher than when it does not. Thus, the only sequential equilibrium would completely reveal firm 1's private information in this case. This is tantamount to firm 1 revealing the set containing types 3 and 4 when either of them is the true type. But if we assume that the certifiable sets are only the singleton sets, this is not possible. Firm 1 here is faced with the choice of revealing either nothing or its precise type. Revealing its type precisely when its marginal cost is low will lead to a choice by firm 2 of the technology which gives firm 2 a marginal cost exactly the same as firm 1's. This then gives rise to a symmetric, complete information Cournot game. The profits from the symmetric game may be lower than those in the game in which firm 2 chooses a technology and output level without knowing firm 1's type. Thus the only sequential equilibrium here involves no revelation of information.

This seems a very real problem in information sharing. The only way to certify the information that you would like to share involves revealing information that you would like not to share. Many firms would like to share with their competitors (and could certify) that their costs are low. Many times this can only be certified by demonstrating the technology and thereby losing whatever advantage over competitors it had to begin with.

Example 8. (Common value example in which monotonicity fails but private information is nonetheless revealed in equilibrium.) This example demonstrates that the monotonicity assumption is not a necessary condition for revelation of information.

Consider a Cournot duopoly problem with the two firms having costless production. They face a linear demand function $p(x) = a - bx$, where a is uncertain. Each of the two firms will get one of two signals which is correlated to this parameter. We will identify the firms' types with the signals they can receive. If both firms are of type 1, $a = 3$, if one is of type 1 and the other is of type 2, $a=4$, and if both firms are of type 2, $a=5$. The firms' types are independent and for each firm, its types are equally likely. Assumption 4 (and 4'), monotonicity, fails for this example, although all other assumptions are satisfied. For firm 1, $\partial\pi_1/\partial s_1(s, t)$ is increasing in t_1. Intuitively, when firm 2's best response function is higher, it would like to reveal this to firm 2. But to reveal this is to reveal that firm 1 received the favourable signal regarding demand, which would cause firm 2's best response function to shift out as well. Thus there are direct effects of revealing one's type (revealing that you are playing a best response function that is further out) and indirect effects (the shifting of your opponent's best response function because of the information about his payoff function contained in the revelation). Monotonicity essentially requires that these two effects go in the same direction. While the two effects go in opposite directions in this example, and hence our proposition does not apply, it turns out that the direct effects dominate and the sequential equilibrium reveals private information.

Example 9. (Application of our model to the quality revelation problem.) Our framework is sufficiently broad to encompass economic problems other than oligopoly models. One such problem is that of a monopolist selling a good of exogenous quality that is known to him but not to the consumer. We can identify the seller's types with his information, that is, with the quality of the good he is selling. The buyer's types are associated with different valuations for the good in question. More specifically, the seller is to choose a price and the buyer is to choose a quantity to purchase given that price. The buyer has a demand function that shifts out with higher quality. We think of the buyer as having started with a given demand function (actually there is a set of demand functions—one for each quality assessment he has). The buyer has already acquired some of the commodity, and the quantity is not known to the seller. We thus associate the quantity already acquired with the buyer's types. Larger quantities of already acquired goods shift the demand function the seller is facing inward. Under at least some plausible parameters, Assumption 4 will be satisfied (as well as the other assumptions) and the only sequential equilibrium of the game will involve complete revelation of each agent's private information.

In the case of a consumer with a known demand function, this problem is essentially that treated by Grossman (1981) and Milgrom (1981).

5. PROOFS

Proof of Theorem 1. Let $(\boldsymbol{p}^*, \boldsymbol{b}^*)$ be any sequential equilibrium. Suppose for some $i\in N$, $b_i^*(x_i)$ does not put full probability on the lowest element of x_i. Let $t_i^M(x_i)$ be the highest type in the support of $b_i^*(x_i)$. Then, agent i who finds his type to be $t_i^M(x_i)$ (or higher) will never report x_i, as he will be better off by reporting the set whose lowest element is his own type, for such a report will generate a belief that stochastically dominates the original belief. This implies that the beliefs associated with equilibrium reports must be degenerate and the only sequential equilibrium is that of complete revelation.

Suppose for some disequilibrium report x_i, $b_i^*(x_i)$ does not put full probability on the lowest type in x_i. Then the lowest type will find himself better off by sending this report than the equilibrium report he is supposed to send. For the belief associated with x_i stochastically dominates the belief associated with his equilibrium report that, by the previous paragraph, has full probability on his true type. Hence the equilibrium inference function profile must be sceptical.

On the other hand, if an inference function profile, $\boldsymbol{b}$, is sceptical and a reporting strategy profile, $\boldsymbol{p}$, completely reveals private information, the pair $(\boldsymbol{p}, \boldsymbol{b})$ obviously constitutes a sequential equilibrium. ‖

Proof of Corollary to Theorem 1. Straightforward. ‖

To prove theorem 2, we need the following definition and lemmas.

Definition. For any $i \in N$, agent i's *best response function* in subgames is a mapping $\phi_i : \Sigma_{-i} \times \boldsymbol{Q} \times T_i \to S_i$ such that:

$$\phi_i(\sigma_{-i}; \boldsymbol{q}, t_i) = \arg\max_{s_i \in S_i} E[\pi_i(s_i, \sigma_{-i}(t_{-i}); t_i, t_{-i}) \mid q_{-i}].$$

We shall denote the vector $(\phi_i(\sigma_{-i}; \boldsymbol{q}, t_i))_{t_i \in T_i}$ by $\phi_i(\sigma_{-i}; \boldsymbol{q})$. Note that actually ϕ_i does not depend upon q_i. We choose this definition for ease of notation.

Lemma 1. *Under Assumptions* 2 *and* 4 (*or* 4′), *for any* i, $\sigma_{-i} \in \Sigma_{-i}$ *and* $\boldsymbol{q} \in \boldsymbol{Q}$, *i's best response* $\phi_i(\sigma_{-i}; \boldsymbol{q}, t_i)$ *is increasing in* t_i *if its value lies in the interior of* $S_i = [0, \bar{s}]$.

Proof. By the definition of best response function, if $s = \phi_j(\sigma_{-j}; \boldsymbol{q}, t_j)$ then

$$E\left[\frac{\partial}{\partial s} \pi_j(s, \sigma_{-j}; t) \mid q_{-j}\right] = 0$$

always holds. The assertion is straightforward from Assumption 4. ‖

Lemma 2. *Under Assumptions* 2–4, *for any* i, $\boldsymbol{q} \in \boldsymbol{Q}$, $q_i' \in Q_i$ *and* $\boldsymbol{\sigma} \in \Sigma$, *if* $\boldsymbol{\sigma}$ *is the equilibrium of a subgame for the belief profile* $\boldsymbol{q}$ *and if* q_i' *stochastically dominates* q_i, *then for any* $j \neq i$ *and any* $t_j \in T_j$,

$$\phi_j(\sigma_{-j}; q_{-i}, q_i', t_j) < \phi_j(\sigma_{-j}; \boldsymbol{q}, t_j)$$

and the inequality is reversed if q_i *stochastically dominates* q_i'.

Proof. Note that $\partial \pi_j / \partial s$ is non-increasing in t_i and decreasing in s_i. Moreover, by the previous lemma, σ_i is increasing in t_i. Hence the assertion must follow. ‖

Lemma 3. *Suppose* $n = 2$ *and Assumptions* 2–4 *hold. For any* $\boldsymbol{q} = (q_1, q_2)$ *with* q_1 *being non-degenerate, define* t_1^m (t_1^M, *resp.*) *to be the lowest* (*highest, resp.*) *type in the support of* q_1. *Let* q_1^m (q_1^M, *resp.*) *be the associated degenerate belief. Then, if* (σ_1^*, σ_2^*), (σ_1^m, σ_2^m) *and* (σ_1^M, σ_2^M) *are the subgame equilibria associated with* (q_1, q_2), (q_1^m, q_2) *and* (q_1^M, q_2), *then*

$$\sigma_1^m(t_1^m) < \sigma_1^*(t_1^m) < \sigma_1^*(t^M) < \sigma_1^M(t^M).$$

Proof. The middle inequality follows from Lemma 1. We shall prove only the first inequality as the last inequality is proved in the same manner.

Note first that:

(a) since q_1^m is degenerate, for any σ_1, $\phi_2(\sigma_1; q_i^m, q_2)$ depends only upon $\sigma_1(t_1^m) = s_1^m$ and can be written as $\hat{\phi}_2(s_1^m; q_1^m, q_2)$.
(b) by the definition of subgame equilibrium and uniqueness, $\sigma_1^m(t_1^m)$ is the unique fixed point of the composite mapping

$$\psi(s_1^m; q_1^m, q_2, t_1^m) = \phi_1(\hat{\phi}_2(s_1^m; q_1^m, q_2); q_1^m, q_2, t_1^m).$$

Since q_1 stochastically dominates q_1^m, by the previous lemma,

$$\begin{aligned}\sigma_1^*(t_1^m) &= \phi_1(\sigma_2^*; q_1, q_2, t_1^m)\\ &< \phi_1(\phi_2(\sigma_1^*; q_1^m, q_2); q_1, q_2, t_1^m)\\ &= \phi_1(\hat{\phi}_2(\sigma_1^*(t_1^m); q_1^m, q_2); q_1^m, q_2, t_1^m)\\ &= \psi(\sigma_1^*(t_1^m); q_1^m, q_2, t_1^m).\end{aligned}$$

By the uniqueness of the fixed point, it follows that the fixed point of $\psi(\cdot; q_1^m, q_2, t_1^m)$ is larger than $\sigma_1^*(t_1^m)$. ‖

Proof of Theorem 2. Let (q_1, q_2) be given and assume q_1 is non-degenerate. Let $t_1^M(t_1^m$, resp.) be the highest (lowest, resp.) type in the support of q_1 and $q_1^M(q_1^m$, resp.) be the associated degenerate belief. By the previous lemma,

$$\sigma_1^m(t_1^m) < \sigma_1^*(t_1^m) < \sigma_1^*(t^M) < \sigma_1^M(t_1^M)$$

if $\boldsymbol{\sigma}^m$, $\boldsymbol{\sigma}^*$ and $\boldsymbol{\sigma}^M$ are the associated subgame equilibria. But then,

$$\sigma_2^m = \phi_2(\sigma_1^m; q_1^m, q_2) < \sigma_2^* < \sigma_2^M = \phi_2(\sigma_1^M; q_1^M, q_2)$$

by Lemma 2 and the fact that $\phi_2(\cdot; q_1^m, q_2)$ ($\phi_2(\cdot; q_1^M; q_2)$, resp.) is increasing in $s_1^m(s_1^M$, resp.). By Assumption 4(a), the assertion must hold. ‖

Proof of Theorem 3. Let a belief profile $\boldsymbol{q}$ and a second-stage strategy profile $\boldsymbol{\sigma}$ be given. Then for any $i \in N$, i's second-stage expected payoff when his true type is $t_i \in T_i$ is written as:

$$\begin{aligned}\pi_i(\sigma: \boldsymbol{q}, t_i) &= E[c\{a_i(t_i, t'_{-i}) - d\textstyle\sum_{j\neq i}\sigma_j(t_j) - \sigma_i(t_i)\}\sigma_i(t_i) \mid q_{-i}]\\ &= c\{\alpha_i(t_i, q_{-i}) - d\textstyle\sum_{j\neq i}\sum_{t_j\in T_j} q_j(t_j)\sigma_j(t_j) - \sigma_i(t_i)\}\sigma_i(t_i),\end{aligned} \tag{1}$$

where

$$\alpha_i(t_i, q_{-i}) = E[a_i(\boldsymbol{t}) \mid t_i, q_{-i}] = \textstyle\sum_{t_{-i}\in T_{-i}} x_{j\neq i} q_j(t_j) a_i(t_{-i}, t_i).$$

By interiority, the subgame equilibrium strategy σ^* given $\boldsymbol{q}$ must satisfy the following set of first-order conditions for all $i \in N$ and $t_i \in T_i$;

$$\alpha_i(t_i, q_{-i}) - d\textstyle\sum_{j\neq i}\sum_{t_j\in T_j} q_j(t_j)\sigma_j^*(t_j) - 2\sigma_i^*(t_i) = 0, \quad \text{or} \tag{2}$$

$$\alpha_i(t_i, q_{-i}) - d\textstyle\sum_{j\neq i}\sum_{t_j\in T_j} q_j(t_j)\sigma_j^*(t_j) - \sigma_i^*(t_i) = \sigma_i^*(t_i). \tag{3}$$

In view of (1) and (3), the subgame equilibrium payoff is written as:

$$u_i(\boldsymbol{q}, t_i) = \pi_i(\boldsymbol{\sigma}^*; \boldsymbol{q}, t_i) = c(\sigma_i^*(t_i))^2. \tag{4}$$

Thus, the subgame equilibrium payoff will increase if and only if his subgame equilibrium strategy $\sigma_i^*(t_i)$ increases as a result of change in beliefs.

Moreover, the equilibrium strategy configuration $\boldsymbol{\sigma}^*$ under $\boldsymbol{q}$ can be expressed as a solution to the matrix equation, $\boldsymbol{Q}\cdot\boldsymbol{\sigma}^* = \boldsymbol{\alpha}(\boldsymbol{q})$, where $\boldsymbol{Q}$, $\boldsymbol{\sigma}^*$ and $\boldsymbol{\alpha}(\boldsymbol{q})$ are defined as follows.

Let $\nu_i = T_i$, i.e. the cardinality of the set T_i, and let $\nu = \sum_{i\in N}\nu_i$. A $\nu\times\nu$ matrix $\boldsymbol{Q}$, which consists of n^2 submatrices $\boldsymbol{Q}_{ij}$ each of which is a $\nu_i \times \nu_j$ matrix, is defined as:

(i) $\boldsymbol{Q}_{ii} = 2\boldsymbol{I}_{\nu_i}$, and

(ii) $\boldsymbol{Q}_{ij} = d\boldsymbol{e}_{\nu_i}\boldsymbol{q}'_j$ if $i \neq j$,

where $\boldsymbol{I}_n$ is an n-dimensional identity matrix, $\boldsymbol{e}_n$ is an n-dimensional unit column vector, and $\boldsymbol{q}_i$ is a ν_i-dimensional column vector $\{q_i(t_i)\}_{t_i\in T_i}$. A $\nu\times 1$ vector $\boldsymbol{\alpha}(\boldsymbol{q})$, which consists of n subvectors $\boldsymbol{\alpha}_i(\boldsymbol{q})$ each of which is a $\nu_i\times 1$ vector, is defined as:

$$\boldsymbol{\alpha}_i(\boldsymbol{q}) = \{\alpha_i(t_i, q_{-i})\}_{t_i\in T_i} = \{E[a_i(\boldsymbol{t}) \mid t_i, q_{-i}]\}_{t_i\in T_i}.$$

Finally, with an abuse of notation, a $\nu_i \times 1$ vector, $\boldsymbol{\sigma}^*$, consisting of n subvectors $\boldsymbol{\sigma}_i^*$ each of which is a $\nu_i \times i$ vector is defined as:

$$\boldsymbol{\sigma}_i^* = \{\sigma_i^*(t_i)\}_{t_i \in T_i}.$$

Note that the inverse matrix of $\boldsymbol{Q}$ exists and we denote it as $\boldsymbol{R}$, for otherwise uniqueness is violated. Hence, the equilibrium $\boldsymbol{\sigma}^*$ is denoted as $\boldsymbol{\sigma}^* = \boldsymbol{R} \cdot \boldsymbol{\alpha}(\boldsymbol{q})$.

Moreover, in view of Theorems 8.3.3. and 9.3.3. of Graybill (1969), $\boldsymbol{R}$ can be written as follows:

(i) $\boldsymbol{R}$ is also partitioned into n^2 matrices of the same order as $\boldsymbol{Q}$,

(ii) $\boldsymbol{R}_{ii} = \frac{1}{2}\boldsymbol{I}_{\nu_i} + (\gamma - \delta)\boldsymbol{e}_{\nu_i}\boldsymbol{q}_i'$, and

(iii) $\boldsymbol{R}_{ij} = -\delta \boldsymbol{e}_{\nu_i}\boldsymbol{q}_j'$ if $i \neq j$,

where

$$\gamma = d/2(2-d) > 0 \quad \text{and} \quad \gamma > \delta = \frac{d(\frac{1}{2}+\gamma)^2}{1+dn(\frac{1}{2}+\gamma)} = \frac{\gamma}{4+(n-2)d} > 0.$$

It then follows that for any $t_i \in T_i$:

$$\sigma_i^*(t_i) = \tfrac{1}{2}\alpha_i(t_i, q_{-i}) + (\gamma - \delta)\boldsymbol{q}_i'\alpha_i(\boldsymbol{q}) - \textstyle\sum_{j \neq i} \delta \boldsymbol{q}_j' \alpha_j(\boldsymbol{q}).$$

Now suppose the belief profile changes from $\boldsymbol{q}$ to (q_i', q_{-i}) where q_i' stochastically dominates q_i. By Assumption 4′, $\alpha_i(t_i, q_{-i})$ is an increasing function of t_i and the second term of the right hand side of the equation increases. By the same assumption, $a_j(t)$ is non-decreasing in t_i for all $j \neq i$ and $\alpha_j(t_j, q_{-j})$ decreases or remains the same as a result of this belief change. Hence, $\sigma_i^*(t_i)$ increases whenever the proposed belief change occurs. In view of our earlier observation that the subgame equilibrium payoff increases if and only if the subgame equilibrium strategy increases, this proves the theorem. ‖

6. RELATION TO PREVIOUS WORK

Most of the previous work on information revelation assumed that agents were restricted to making truthful assertions; a person could withhold information (completely or partially), but he was restricted from making false assertions. The result of this assumption is that all assertions must be believed. This assumption, along with the assumption that *any* truthful assertion can be made drove the complete revelation results.

Clearly a person is able to make an assertion whether or not the assertion is truthful. Normally, the condition that an agent could not make false assertions should be an equilibrium conclusion rather than an assumption. If we assume that the statements are about physical phenomena, then in some problems private information could be ex ante certified as truthful. In this paper, we have assumed that only certifiably truthful assertions can be sent. We can interpret this restriction as allowing any assertion to be made, but only those that are certifiably truthful as changing beliefs. In this sense it is without loss of generality that we restrict agents to make only certifiably truthful assertions.

The departure from earlier work on information revelation is that we don't assume that all truthful statements can be made, but only certifiably truthful statements. We have argued that there may be some kind of private information a person may have that he could not certify, for example, a case in which person's type might correspond to his not knowing something. In our model, a person may have such private information and yet have no certifiable assertion that would convey this information. The possibility of such private information can upset the "unravelling" that results in complete revelation in other models, as shown in Example 3. In our view, a contribution of the present model is that we make explicit the assertions that a person can make that can be certified, and hence, must be believed.

A second way in which our work differs from most previous research on the information revelation problem is that we treat the case in which more than a single agent has private information. Of the literature mentioned above only Milgrom and Roberts (1986) treat the case of multiple agents with private information. For the case of multiple agents, our results giving conditions under which complete revelation of information occurs differ from theirs in several ways. First, we have more stringent conditions on our information structure; they make no assumptions that correspond to the monotonicity condition, for example. On the other hand, their results that guarantee complete revelation are restricted to the case that there is no outcome that is Pareto superior to the full information outcome. Our theorems guaranteeing complete revelation apply regardless of whether the full information outcome is "good" or not. As argued in the introduction, consideration of the communication possibilities is important because complete revelation may be the only equilibrium outcome even in the case that such revelation makes all agents strictly worse off than in the no communication case.

7. CONCLUSION

We should care about information sharing for two reasons. Some public policies, such as consumer information laws, are developed because normally operating competitive markets are perceived to provide too little information. On the other hand, some public policies are based on the belief that certain kinds of information sharing may be socially bad; the case of firms within an industry sharing certain information may fall into this category. Before policies are formed to correct problems of too little or too much information sharing in markets, we need models in which the agents in the market are presumed to make choices optimally about the amount of information they share with the other economic agents. As we have pointed out, whether information sharing is good or bad for the participants is a separate question from whether such information sharing can be part of a sequential equilibrium.

We have provided theorems giving sufficient conditions for information to be fully revealed in a sequential equilibrium. The conditions of the theorems, in particular monotonicity, are quite strong. The examples show that weakening these conditions may easily upset the conclusion that the information will be revealed in equilibrium. As in Example 2, incomplete information about whether some information is known or not known by other agents is typically not certifiable; this may well lead to less than full revelation of private information. Example 7 shows that if the information structure becomes complex, agents may prefer to reveal nothing to revealing all they know, if those are the alternatives.

The theorems and examples give a rough feeling for problems that may prevent full revelation of information in equilibrium. Some of the examples, such as Example 7, suggest that intermediaries may arise to collect and certify information and pass along to other agents an accurate but less revealing summary of an agent's private information. In this example, firm 1 would like to reveal its cost when it is low but will not do so if by doing so it reveals the optimal technology for the other firm. An intermediary (a manufacturers' association or a governmental agency?) might arise to solve this problem. The analysis of the role that such intermediaries might play in information sharing seems to us to be a very interesting problem.

As seen in some of the examples, sequential equilibria may lead to no information sharing even when it is ex ante efficient to do so. If information sharing is more easily accomplished within a firm than between firms, alternative firm structures may emerge as a result of the potential benefits of information sharing. Future research should investigate the relationship between the alternative institutional structures of firms and industries and the possibilities of information sharing.

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